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AERONAUTICS

SEVENTH ANNUAL REPORT

OF THE

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

1921

INCLUDING TECHNICAL REPORTS
NOS. 111 TO 132

WASHINGTON
GOVERNMENT PRINTING OFFICE
1923

LETTER OF SUBMITTAL.

To the Congress of the United States:

In compliance with the provisions of the act of March 3, 1915, establishing the National Advisory Committee for Aeronautics, I transmit herewith the seventh annual report of the committee for the fiscal year ended June 30, 1921.

I think there can be no doubt that the development of aviation will become of great importance for the purposes of commerce, as well as for national defense. While the material progress in aircraft has been remarkable, the use has not as yet been extensively developed in America. This has been due, in the main, to lack of wise and necessary legislation. Aviation is destined to make great strides, and I believe that America, its birthplace, can and should be foremost in its development.

I therefore urge upon the Congress the advisability of giving heed to the recommendations of the committee, the first and most important of which is that a bureau be established in the Department of Commerce for the regulation and development of air navigation.

WARREN G. HARDING.

THE WHITE HOUSE,
December 7, 1921.

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LETTER OF TRANSMITTAL.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
Washington, D. C., November 28, 1921.

THE PRESIDENT:

In compliance with the provisions of the act of Congress approved March 3, 1915 (naval appropriation act, Public No. 273, 63d Cong.), I have the honor to transmit herewith the Seventh Annual Report of the National Advisory Committee for Aeronautics, including a statement of its expenditures for the fiscal year ended June 30, 1921.

The progress in scientific research during the past year is described in the reports of the technical subcommittees and is exemplified by the technical reports (Nos. 111 to 132, inclusive) which have been published during the past year, summaries of which are contained in this report. The relative value of scientific research in the development of aeronautics is greater now than ever before, as the development of aviation in the absence of necessary legislation and without the stimulus of war is slow, and it would be more so were it not for the continuous prosecution of organized scientific research.

The problem of developing the uses of aircraft requires the Federal regulation of air navigation and the establishment and regulation of transcontinental airways, including the necessary airdromes and weather report stations.

Attention is especially invited to the Summary of General Recommendations of the committee, preceding the body of the report, and setting forth what, in the judgment of the committee, are the most important steps to be taken for the advancement of aeronautics. The committee lays special emphasis at this time on the advisability of the Federal Government giving aid in this great and promising field of development and urges that the most important single measure in the general policy outlined by the committee is the enactment of legislation establishing a bureau in the Department of Commerce for the regulation and development of air navigation.

Respectfully submitted.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
CHARLES D. WALCOTT, *Chairman.*

SUMMARY OF GENERAL RECOMMENDATIONS.

The more important general recommendations of the National Advisory Committee for Aeronautics are summarized as follows:

LEGISLATION FOR THE DEVELOPMENT OF AVIATION.

The most urgent need for the successful development of aviation at the present time, either for military or civil purposes, is the enactment of legislation providing for the Federal regulation of air navigation, and the establishment of airways and airdromes under Federal regulation. The Federal regulation should include the licensing of aviators, aircraft, and airdromes; the airways should consist of chains of landing fields providing supply and repair facilities and including the necessary meteorological stations, observations, and reports. If the Federal Government will establish and regulate transcontinental airways, as recommended, the committee is confident that air lines for the transportation of passengers or goods will be rapidly established by private enterprise in all parts of the country. The first national airways, however, should be carefully planned to serve military as well as civil needs. The committee reiterates its former recommendations as to the manner of accomplishing the desired results, and urgently recommends the establishment by law of a Bureau of Air Navigation in the Department of Commerce.

EXTENSION OF AEROLOGICAL SERVICE.

The committee emphasizes the importance of extending aerological service (under the Weather Bureau) along airways as established, and recommends that adequate provision of law be made for this service, which is so indispensable to the success and safety of air navigation.

POLICY TO SUSTAIN THE INDUSTRY.

Whatever may have been the faults or the shortcomings of the aircraft industry during or since the war, the fact remains that there must be an aircraft industry, and that it should be kept in such a condition as to be able to expand promptly and properly to meet increased demand in case of emergency. The Government, as the principal consumer, is directly concerned in the matter, and should formulate a policy which would be effective to sustain and stabilize the aeronautical industry and encourage the development of new and improved types of aircraft. In this respect the committee invites attention to the recommendation contained in its special report submitted to the President on April 9, 1921, published as House Document 17, and again recommends the adoption of a policy which, while safeguarding the interests of the Government, will tend to sustain and stabilize the industry.

IMPORTANCE OF MILITARY AVIATION.

Aviation is indispensable to the Army and to the Navy in warfare; and its relative importance will continue to increase. Other branches of the military services are comparatively well developed, whereas aviation is still in the early stages of its development. The demand for greatly reduced expenditures in the military and naval services should not apply to the air services. The committee recommends that liberal provision be made for the Army and Navy Air Services, not only that provision be made for the maintenance and training of personnel, but also that the funds be adequate to insure the fullest development of aviation for military and naval purposes.

SCIENTIFIC RESEARCH.

Substantial progress in aeronautical development, whether for military or commercial purposes, must be based upon the application to the problems of flight of scientific principles and the results of research. The exact prescribed function of the National Advisory Committee for Aeronautics is the prosecution and coordination of scientific research, and, while encouragement may be taken from the progress made, greater provision for the continuous prosecution of research on a larger scale is strongly recommended by the committee.

THE AIR MAIL SERVICE.

The Air Mail Service has demonstrated that airplanes can be utilized with certain advantages in carrying the mails. And it has done more than this, despite the handicap of using, military types of aircraft, poorly adapted to its work or to any civil or commercial purpose, in demonstrating that commercial aviation for the transportation of passengers or goods is feasible. There are several causes which are delaying the development of civil aviation, such as the lack of airways, landing fields, aerological service, and aircraft properly designed for commercial uses. The Air Mail Service stands out as a pioneer agency, overcoming these handicaps and blazing the way, so to speak, for the practical development of commercial aviation. As a permanent proposition, however, the Post Office Department, as its functions are now conceived, should no more operate directly a special air mail service than it should operate a special railroad mail service; but until such time as the necessary aids to commercial aviation have been established it will be next to impossible for any private corporation to operate under contract an air mail service in competition with the railroads. The National Advisory Committee for Aeronautics therefore recommends that provision be made for the continuation of the Air Mail Service under the Post Office Department.

HELIUM AND AIRSHIPS.

The United States has a virtual monopoly of the known sources of supply of helium, and these are limited. Experiments have been conducted by the Bureau of Mines with a view to the development of methods of production and storage, but as yet the problem of storage in large quantities has not been satisfactorily solved. Because the known supply is limited, because it is escaping into the atmosphere at an estimated rate sufficient to fill four large airships weekly, and because of the tremendously increased value and safety which the use of helium would give to airships, particularly in warfare, it is, in the opinion of the National Advisory Committee for Aeronautics, the very essence of wisdom and prudence to provide for the conservation of large reserves through the acquisition and sealing by the Government of the best helium-producing fields. Attention now being given to the development of types of airships to realize fully the advantages which the use of helium would afford should be continued. Such development would give America advantages, for purposes either of war or commerce, with which no other nation could successfully compete.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

2722 NAVY BUILDING, WASHINGTON, D. C.

Charles D. Walcott, Sc.D., *Chairman*,
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S. W. Stratton, Sc.D., *Secretary*,
Director, Bureau of Standards, Washington, D. C.
Joseph S. Ames, Ph.D., *Chairman Executive Committee*,
Director, Physical Laboratory, Johns Hopkins University, Baltimore, Md.
Thurman H. Bane, Major, United States Army,
Chief, Engineering Division, Air Service, Dayton, Ohio.
William F. Durand, Ph.D.,
Professor of Mechanical Engineering, Leland Stanford Junior University, California.
John F. Hayford, C. E.,
Director, College of Engineering, Northwestern University, Evanston, Ill.
Charles F. Marvin, M. E.,
Chief, United States Weather Bureau, Washington, D. C.
William A. Moffett, Rear Admiral, United States Navy,
Chief, Bureau of Aeronautics, Navy Department, Washington, D. C.
Mason M. Patrick, Major General, United States Army,
Chief of Air Service, War Department, Washington, D. C.
Michael I. Pupin, Ph. D.,
Professor of Electro-Mechanics, Columbia University, New York City.
D. W. Taylor, Rear Admiral, United States Navy,
Chief Constructor, Navy, Washington, D. C.
Orville Wright, B. S.,
Dayton, Ohio.

EXECUTIVE COMMITTEE.

Joseph S. Ames, *Chairman.*
S. W. Stratton, *Secretary.*

Thurman H. Bane,
John F. Hayford,
Charles F. Marvin,
William A. Moffett,
Mason M. Patrick,
D. W. Taylor,
Charles D. Walcott.

SEVENTH ANNUAL REPORT

OF THE

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Washington, D. C., November 28, 1921.

To the Congress:

In accordance with the provision of the act of Congress approved March 3, 1915, establishing the National Advisory Committee for Aeronautics, the committee submits herewith its seventh annual report. In this report the committee has described its activities during the past year, the technical progress in the study of scientific problems relating to aeronautics, the assistance rendered by the committee in the formulation of a policy regarding the Federal regulation of air navigation, the coordination of research work in general, the examination of aeronautical inventions, and the collection, analysis, and distribution of scientific and technical data. This report also contains a statement of expenditures, estimates for the fiscal year 1923, and recommendations as to the present requirements of aviation.

FUNCTIONS OF THE COMMITTEE.

The National Advisory Committee for Aeronautics was established by act of Congress, approved March 3, 1915. The organic act charges the committee with the supervision and direction of the scientific study of the problems of flight with a view to their practical solution, the determination of problems which should be experimentally attacked, their investigation and application to practical questions of aeronautics. The act also authorizes the committee to direct and conduct research and experimentation in aeronautics in such laboratory or laboratories in whole or in part as may be placed under its direction.

Supplementing the prescribed duties of the committee, its broad general functions may be stated as follows:

First. Under the law the committee holds itself at the service of any department or agency of the Government interested in aeronautics, for the furnishing of information or assistance in regard to scientific or technical matters relating to aeronautics, and in particular for the investigation and study of problems in this field with a view to their practical solution.

Second. The committee may also exercise its functions for any individual, firm, association, or corporation within the United States, provided that such individual, firm, association, or corporation defray the actual cost involved.

Third. The committee institutes research, investigation, and study of problems which, in the judgment of its members or of the members of its various subcommittees, are needful and timely for the advance of the science and art of aeronautics in its various branches.

Fourth. The committee keeps itself advised of the progress made in research and experimental work in aeronautics in all parts of the world, particularly in England, France, Italy, Germany, Austria, and Canada.

Fifth. The information thus gathered is brought to the attention of the various subcommittees for consideration in connection with the preparation of programs for research and experimental work in this country. This information is also made available promptly to the military and naval air services and other branches of the Government, and such as is not confidential is immediately released to university laboratories and aircraft manufacturers interested in the study of specific problems, and also to the public.

Sixth. The committee holds itself at the service of the President, the Congress, and the executive departments of the Government for the consideration of special problems which may be referred to it, such as organization of governmental activities in aeronautics, recommendations as to proper action under the Convention for the Regulation of International Air Navigation, questions of policy regarding the regulation and development of civil aviation, advanced education in aeronautical engineering, etc.

ORGANIZATION OF THE COMMITTEE.

The committee has 12 members, appointed by the President. The law provides that the personnel of the committee shall consist of two members from the War Department, from the office in charge of military aeronautics; two members from the Navy Department, from the office in charge of naval aeronautics; a representative each of the Smithsonian Institution, of the United States Weather Bureau, and of the United States Bureau of Standards; and not more than five additional persons acquainted with the needs of aeronautical science, either civil or military, or skilled in aeronautical engineering or its allied sciences. All members as such serve without compensation.

During the past year Rear Admiral William A. Moffett, Chief of the Bureau of Aeronautics in the Navy Department, was appointed by the President to membership on the committee to succeed Capt. T. T. Craven. The President has also appointed to membership on the committee Maj. Gen. Mason M. Patrick, who succeeded Maj. Gen. Charles T. Menoher as Chief of the Air Service of the Army.

The full committee meets twice a year, the annual meeting being held in October and the semiannual meeting in April. The present report includes the activities of the committee between the annual meeting held on October 7, 1920, and that held on October 6, 1921.

The present organization of the committee is as follows:

Charles D. Walcott, Sc. D., chairman.
S. W. Stratton, Sc. D., secretary.
Joseph S. Ames, Ph. D.
Maj. Thurman H. Bane, United States Army.
William F. Durand, Ph. D.
John F. Hayford, C. E.
Charles F. Marvin, M. E.
Rear Admiral William A. Moffett, Unites States Navy.
Maj. Gen. Mason M. Patrick, United States Army.
Michael I. Pupin, Ph. D.
Rear Admiral D. W. Taylor, United States Navy.
Orville Wright, B. S.

THE EXECUTIVE COMMITTEE.

For carrying out the work of the Advisory Committee the regulations provide for the election annually of an executive committee, to consist of seven members, and to include further any member of the Advisory Committee not otherwise a member of the executive committee but resident in or near Washington and giving his time wholly or chiefly to the special work of the committee. The present organization of the executive committee is as follows:

Joseph S. Ames, Ph. D., chairman.
S. W. Stratton, Sc. D., secretary.
Maj. Thurman H. Bane, United States Army.
John F. Hayford, C. E.
Charles F. Marvin, M. E.
Rear Admiral William A. Moffett, United States Navy.
Maj. Gen. Mason M. Patrick, United States Army.
Rear Admiral D. W. Taylor, United States Navy.
Charles D. Walcott, Sc. D.

The executive committee, in accordance with the general instructions of the Advisory Committee, exercises the functions prescribed by law for the whole committee, administers the affairs of the committee, and exercises general supervision over all its activities. The executive committee held regular monthly meetings throughout the year, and in addition held five special meetings, on the following dates: October 7, 1920; January 27, April 4, April 8, and June 30, 1921.

The executive committee has organized the necessary clerical and technical staffs for handling the work of the committee proper. General responsibility for the execution of the programs and policies approved by the executive committee is vested in the executive officer, Mr. George W. Lewis. In the subdivision of general duties he has immediate charge of the scientific and technical work of the committee, being directly responsible to the chairman of the executive committee, Dr. Joseph S. Ames. The assistant secretary, Mr. John F. Victory, has charge of administration and personnel matters, property, and disbursements, under the direct control of the secretary of the committee, Dr. S. W. Stratton.

SUBCOMMITTEES.

The executive committee has organized six standing subcommittees, divided into two classes, administrative and technical, as follows:

ADMINISTRATIVE.	TECHNICAL.
Personnel, buildings, and equipment.	Aerodynamics.
Publications and intelligence.	Power plants for aircraft.
Governmental relations.	Materials for aircraft.

The organization and work of the technical subcommittees are covered in the reports of those committees appearing in another part of this report. A statement of the organization and functions of the administrative subcommittees follows:

COMMITTEE ON PERSONNEL, BUILDINGS, AND EQUIPMENT.

FUNCTIONS.

1. To handle all matters relating to personnel, including the employment, promotion, discharge, and duties of all employees.
2. To consider questions referred to it and make recommendations regarding the initiation of projects concerning the erection or alteration of laboratories and the equipment of laboratories and offices.
3. To meet from time to time on the call of the chairman, and report its actions and recommendations to the executive committee.
4. To supervise such construction and equipment work as may be authorized by the executive committee.

ORGANIZATION.

Dr. Joseph S. Ames, chairman.
Dr. S. W. Stratton, vice chairman.
Prof. Charles F. Marvin.
J. F. Victory, secretary.

COMMITTEE ON PUBLICATIONS AND INTELLIGENCE.

FUNCTIONS.

1. The collection, classification, and diffusion of technical knowledge on the subject of aeronautics, including the results of research and experimental work done in all parts of the world.
2. The encouragement of the study of the subject of aeronautics in institutions of learning.
3. Supervision of the office of aeronautical intelligence.

4. Supervision of the committee's foreign office in Paris.

5. The collection and preparation for publication of the technical reports, technical notes, and annual report of the committee.

ORGANIZATION.

Dr. Joseph S. Ames, chairman.
Prof. Charles F. Marvin, vice chairman.
Miss M. M. Muller, secretary.

COMMITTEE ON GOVERNMENTAL RELATIONS.

FUNCTIONS.

1. Relations of the committee with executive departments and other branches of the Government.

2. Governmental relations with civil agencies.

ORGANIZATION.

Dr. Charles D. Walcott, chairman.
Dr. S. W. Stratton,
J. F. Victory, secretary.

AVIATION BOMBING TESTS.

The sinking of former German warships in the aviation bombing tests held off the capes of the Chesapeake in June and July, 1921, demonstrated that warships of all types are vulnerable to attack from the air, and that aircraft may be advantageously employed in the defense of our coasts and in engagements with enemy fleets on the high seas. Even in naval engagements on the high seas at such distances from shore that coast-defense aircraft could not participate, aircraft will be employed by the contending fleets. The Navy, therefore, should develop airplane carriers which must provide means for aircraft to take off and alight during an engagement. The Navy has developed a catapult launching device and is also experimenting with the deck-landing problem. It is essential, in the judgment of the committee, that Congress authorize the construction of specially designed airplane carriers.

FEDERAL REGULATION OF AIR NAVIGATION - AIR ROUTES TO COVER THE WHOLE UNITED STATES COOPERATION AMONG GOVERNMENT DEPARTMENTS CONCERNED WITH AVIATION.

The committee renews its previous recommendations for the establishment of a bureau of air navigation in the Department of Commerce, for the Federal regulation and licensing of air navigation, and to aid generally in the development of commercial aviation. The committee is not unmindful of the legal sentiment that a constitutional amendment should first be adopted before such legislation is enacted, on the ground that effective regulation of air navigation as proposed would otherwise be unconstitutional as violating the rights of private property and encroaching upon the rights of the States. To postpone such legislation until a constitutional amendment can be proposed and ratified would have the effect of greatly retarding the development of commercial aviation, with no assurance that sufficient popular interest would ever be aroused to accomplish such an amendment. The committee is of the opinion that the most effective course to be followed for the development of aviation would be to first enact the legislation deemed necessary for the Federal regulation of air navigation and the encouragement of the development of civil aviation, and let the question of the constitutionality of such legislation be tested in due course. In the meantime, there would be development in civil and commercial aviation, and if eventually the legislation which made possible such development should be definitely determined to be unconstitutional there would then, in all probability, be sufficient public interest in the subject and popular demand to adopt an amendment to the Constitution.

President Harding, in his first address to Congress on April 12, 1921, stated that the recommendation of the committee to provide for Federal regulation of air navigation ought to have legislative approval. There is virtual unanimity in aeronautical circles that Congress should establish a bureau of air navigation in the Department of Commerce for the Federal regulation of air navigation, for the establishment of airways, and in general for the development of civil and commercial aviation in all parts of the country.

On April 1, 1921, the President wrote the following letter to the chairman of the committee:

THE WHITE HOUSE,
Washington, April 1, 1921.

Dr. CHARLES D. WALCOTT,
Chairman National Advisory Committee for Aeronautics,
2722 Navy Building, Washington, D. C.

DEAR DR. WALCOTT: It is desired that you immediately organize a subcommittee of the National Advisory Committee for Aeronautics, with representatives from the War, Navy, Post Office, and Commerce Departments, and civil life; that the subcommittee take up vigorously and fully the question of Federal regulation of air navigation, air routes to cover the whole United States, and cooperation among the various departments of the Government concerned with aviation, reporting—

(*a*) What can and should be done without further legislative action.

(*b*) What legislative action and appropriations are necessary to carry into effect the recommendations of the subcommittee.

Very truly, yours,

WARREN G. HARDING.

A special meeting of the executive committee was promptly called, at which the subcommittee was organized in accordance with the President's request. This subcommittee held meetings in Washington on April 5, 6, 7, and 8, 1921. Its report was approved at another special meeting of the executive committee, held on April 8, and transmitted to the President on April 9. The President transmitted the report to the Congress with the following letter:

To the Senate and House of Representatives:

I transmit herewith for the consideration of the Congress a special report of the National Advisory Committee for Aeronautics, prepared at my request and dealing with Federal regulation of air navigation, air routes to cover the whole United States, and cooperation among the various departments of the Government concerned with aviation.

The attention of the Congress is invited to the statement of general considerations on a national aviation policy, and to the committee's recommendations for legislative action, which have my approval.

WARREN G. HARDING.

THE WHITE HOUSE, *April 19, 1921.*

The special report of the committee referred to is as follows:

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
Washington, D. C., April 9, 1921.

The PRESIDENT,
The White House.

DEAR MR. PRESIDENT: In accordance with your letter of April 1, 1921, addressed to Dr. Charles D. Walcott, chairman of the National Advisory Committee for Aeronautics, this committee organized a special subcommittee on Federal regulation of air navigation, as follows:

War Department: Maj. Gen. C. T. Menoher, United States Army; Maj. W. G. Kilner, United States Army.

Navy Department: Rear Admiral D. W. Taylor, United States Navy; Commander Kenneth Whiting, United States Navy.

Post Office Department: Mr. E. C. Zoll, Mr. C. I. Stanton.

Department of Commerce: Dr. S. W. Stratton, Mr. E. T. Chamberlain.

Representatives from civil life: Mr. Sidney Waldon, Mr. F. H. Russell, Mr. Glenn L. Martin.

Dr. Charles D. Walcott, chairman.

Mr. J. F. Victory, secretary.

This subcommittee has taken up, as you directed, the question of Federal regulation of air navigation, air routes to cover the whole United States, and cooperation among the various departments of the Government concerned with aviation, and, in addition, the two questions specified in your letter:

"(*a*) What can and should be done without further legislative action.

"(*b*) What legislative action and appropriations are necessary to carry into effect the recommendations of the subcommittee."

The report of this subcommittee is as follows:

The following general considerations on a national aviation policy are recommended:

1. Aviation is inseparable from the national defense. It is necessary to the success of both the Army and the Navy. Each should have complete control of the character and operations of its own air service.

2. Aeronautics is a comparatively new science capable of such tremendous and rapid development that it is of vital importance, in time of peace, to make the greatest possible progress in the science itself. Everything should be done to stimulate invention and to encourage the practical use of aircraft of all kinds and of all the equipment and appliances necessary or incidental thereto.

3. It is considered impracticable in time of peace to maintain a large armed air force, but it is considered imperative that we maintain a sufficient nucleus of available personnel, including organized reserves, and of adequate equipment of the most modern type as a foundation upon which to build at the outbreak of war.

4. It is essential that commercial aviation be fostered and encouraged in harmony with the military and naval aviation policies and programs. The development of aviation as a whole will be made with the minimum of expense to the Government through the adoption of a wise and constructive policy for the upbuilding of commercial aviation.

5. The air mail service is an important initial step in the development of civil and commercial aviation. It must be maintained and extended as rapidly as possible, not only to carry the mails but to become a potential war reserve.

6. It is a pressing duty of the Federal Government to regulate air navigation; otherwise independent and conflicting legislation by the various States will be enacted and hamper the development of aviation. For this purpose a bureau of aeronautics should be established in the Department of Commerce, by legislation similar to the Kahn bill as modified. (See draft of bill taken from Sixth Annual Report of the National Advisory Committee for Aeronautics, Appendix A.)

7. Approved policies with respect to the encouragement and development of commercial aviation should be carried out by the Department of Commerce.

8. The Army Air Service should be continued as a coordinate combatant branch of the Army. Its existing organization should be used in cooperation with the Navy, Post Office, and other governmental agencies in the prompt establishment of national continental airways and in cooperation with the States and municipalities in the establishment of local airdromes, landing fields, and other necessary facilities.

9. The Naval Air Service and the control of naval activities in aeronautics should be centralized in a bureau of aeronautics in the Navy Department.

10. The continuous prosecution of scientific research in aeronautics is now provided for by the National Advisory Committee for Aeronautics, established by law in 1915, and broad questions of policy regarding the coordination of the activities of all governmental agencies concerned with aeronautics should be referred to that committee for consideration and recommendation.

11. The National Advisory Committee for Aeronautics should have authority to recommend to the heads of the departments concerned on questions of policy regarding the development of aviation, and to recommend to departmental heads desirable undertakings or developments in the field of aviation. To provide for the more effective discharge of these functions, the chief of the air mail service of the Post Office Department and the chief of the proposed Bureau of Aeronautics in the Department of Commerce should be members of the committee.

12. Under this policy, there would be an Army Air Service under the Secretary of War; a Naval Air Service under the Secretary of the Navy, with its activities centralized in a Bureau of Aeronautics in the Navy Department; an air mail service under the Postmaster General; a bureau of aeronautics for the regulation of air navigation, under the Secretary of Commerce, and for carrying out such policies as may be adopted for the encouragement and upbuilding of civil and commercial aviation; a National Advisory Committee for Aeronautics for the continuous prosecution of scientific research in aeronautics, and, in an advisory capacity, the coordination of all aeronautical activities of the Government.

Referring specifically to the detailed questions under the three headings, namely, (1) "Federal regulation of air navigation," (2) "Air routes to cover the whole United States," (3) "Cooperation among the various departments of the Government concerned with aviation," the committee reports as follows:

1. Federal Regulation of Air Navigation.

(*a*) Federal regulation of air navigation can not be accomplished under existing laws. Smuggling and other illegal uses of aircraft can be prevented in a measure.

(*b*) It is recommended that a bureau of aeronautics be established in the Department of Commerce (substantially along the lines of the Kahn bill as modified) for the regulation of air navigation and for carrying out such policies as may be adopted for the encouraging and upbuilding of civil and commercial aviation, and that an estimate of $200,000 be submitted for the fiscal year 1922.

2. Air Routes to Cover the Whole United States.

(*a*) The Post Office Department is specifically authorized to establish an air route between New York and San Francisco. There is some question as to whether existing laws permit it to establish other routes.

The Army has no specific authority of law to establish air routes, but has charted seven important mail airways as follows:

1. One route from Augusta, Me., to Camp Lewis, Wash.
2. One from Washington, D. C., to San Francisco, Calif.

3. One from Savannah, Ga., to San Diego, Calif.
4. One from Augusta, Me., to Miami, Fla.
5. One from Camp Lewis, Wash., to San Diego, Calif.
6. One from Laredo, Tex., to Fargo, N. Dak.
7. One from Chicago, Ill., to Baton Rouge, La.

(Chart, Appendix B, shows these routes.)

(*b*) In order to enable the Army to carry forward its program of air routes to cover the whole United States, it is recommended that an appropriation of $2,000,000 be made available during a period of two years.

Attention is drawn to "Necessary aerological service and estimate of costs," Appendix C. It is recommended that such portions of the appropriations asked for as are necessary to give aerological service on the approximately 4,000 miles of air mail routes now in commission be made available, and that the funds to cover additional stations along the national continental air routes to cover the whole United States be made available as fast as the need is indicated by the Army and the Post Office Department.

It is recommended that legislation be enacted which will definitely authorize the Post Office Department to establish air mail routes between Chicago, Minneapolis, and St. Paul, and between Chicago and St. Louis, and such other air mail routes as may be determined by the Postmaster General as the need for them arises, taking full advantage, wherever practicable, of existing or contemplated airways.

3. Cooperation Among the Various Departments of the Government Concerned with Aviation.

(*a*) Cooperation among the air services of the Army, Navy, and Post Office with Coast and Geodetic Survey, Bureau of Fisheries, Coast Guard, Weather Bureau, Geological Survey, and forest patrol service is being carried on with excellent results, as shown in Appendix D.

It is recommended that the President direct the National Advisory Committee for Aeronautics to appoint a subcommittee composed of representatives of the War, Navy, Post Office, and Commerce Departments, and two civilians representing the aircraft industry, who shall survey the engineering and production facilities of the aircraft industry and shall recommend a policy calculated to sustain and develop the industry to meet the needs of the Government.

(*b*) Attention is drawn to the memorandum on forest fire patrol, Appendix E. It is recommended that the funds ($217,151) and personnel asked for be made available for the purpose specified.

In summing up this report, permit me to emphasize the immediate need of legislation to provide for—

First. A naval air service under the Secretary of the Navy, with its activities centralized in a bureau of aeronautics in the Navy Department.

Second. A bureau of aeronautics under the Secretary of Commerce for the regulation of air navigation and the encouragement and upbuilding of civil and commercial aviation.

Third. The development of a system of national continental air routes to cover the whole United States and to include the meteorological service essential thereto.

Fourth. The extension of the air mail service.

Fifth. Making the chief of the air mail service and the chief of the proposed bureau of aeronautics of the Department of Commerce members of the National Advisory Committee for Aeronautics.

Respectfully submitted.

National Advisory Committee for Aeronautics.
C. D. Walcott, *Chairman.*

Appendixes.

A. Draft of modified Kahn bill.
B. Continental airways and approximate costs.
C. Necessary aerological service and estimate of costs.
D. Cooperation among the various departments of the Government concerned with aviation.
E. Cooperation of War and Agricultural Departments and operating cost of forest fire patrol.

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Appendix A.

DRAFT OF MODIFIED KAHN BILL.

[Note.—The modified Kahn bill referred to is not here reproduced, having been previously printed in the committee's sixth annual report. The bill was introduced in the first session of the Sixty-seventh Congress by Representative Hicks of New York, and is now known as the Hicks bill (H. R. 271).]

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APPENDIX B.

CONTINENTAL AIRWAYS AND APPROXIMATE COSTS.

[By Air Service of the Army.]

Seven important main airways have been charted, and by utilizing Government airdromes already existing these airways make the air net complete for the United States.

One route from the State of Maine to the State of Washington covers the following stations:

Augusta, Me., municipal.
Plattsburg, N. Y., Government, not Air Service.
Binghamton, N. Y., municipal.
Buffalo, N. Y., private.
Cleveland, Ohio, private.
Detroit, Mich., Air Service.
Bryan, Ohio, municipal.
Chicago, Ill., Government, not Air Service.
Winona, Minn., municipal.
St. Paul, Minn., Government, not Air Service.
Fargo, N. Dak., municipal.
Portal, N. Dak., municipal.
Miles City, Mont., Government, not Air Service.
Missoula, Mont., municipal.
Spokane, Wash., private.
Camp Lewis, Wash., Government, not Air Service.

One transcontinental route from Washington, D. C., to San Francisco, Calif.:

Bolling Field, Anacostia, D. C., Air Service.
Moundsville, W. Va., municipal.
Dayton, Ohio, Air Service.
Speedway, Indianapolis, Ind., Air Service.
Rantoul, Ill., Air Service.
Des Moines, Iowa, municipal.
Omaha, Nebr., Government, not Air Service.
Hastings, Nebr., municipal.
St. Paul, Nebr., municipal.
Cheyenne, Wyo., Government, not Air Service.
Rawlins, Wyo., municipal.
Salt Lake City, Utah, municipal.
Elko, Nev., municipal.
Sacramento, Calif., Air Service.
San Francisco, Calif., Air Service.

One route from Savannah, Ga., to San Diego, Calif.:

Savannah, Ga., municipal.
Camp Benning, Ga., Air Service.
Montgomery, Ala., Air Service.
Baton Rouge, La., municipal.
Lake Charles, La., municipal.
Houston, Tex, Air Service.
San Antonio, Tex., Air Service.
Del Rio, Tex., Air Service.
Sanderson, Tex., municipal.
El Paso, Tex., Government, not Air Service.
Douglas, Ariz., Government, not Air Service.
Tucson, Ariz., municipal.
Phoenix, Ariz., municipal.
Yuma, Ariz., Government, not Air Service.
San Diego, Calif., Air Service.

One route north to south on the Atlantic coast:

Augusta, Me., already mentioned.
Boston, Mass., municipal.
Mitchel Field, Long Island, Air Service.
Bustleton, Pa., Government, not Air Service.
Aberdeen, Md., Air Service.
Baltimore, Md., municipal.
Washington, D. C., Air Service.
Langley Field, Va., Air Service.
Camp Bragg, N. C., Air Service.
Camp Jackson, S. C., Air Service.
Savannah, Ga., already mentioned.
Jacksonville, Fla., Government, not Air Service.
Daytona, Fla., municipal.
Carlstrom Field, Fla., Air Service.
Miami, Fla., Air Service.

One route along the Pacific coast:

Camp Lewis, Wash., already mentioned.
Portland, Oreg., municipal.
Red Bluffs, Calif., Government, not Air Service.
Sacramento, Calif., already mentioned.
Fresno, Calif., private.
Riverside, Calif., Air Service.
San Diego, Calif., already mentioned.

One middle route from north to south:

Laredo, Tex., Air Service.
San Antonio, Tex., already mentioned.
Dallas, Tex., Air Service.
Waco, Tex., municipal.
Wichita Falls, Tex., municipal.
Post Field, Okla., Air Service.
Little Rock, Ark., municipal.
Tulsa, Okla., private.
Leavenworth, Kans., Government, not Air Service.
Omaha, Nebr., Government, not Air Service.
Aberdeen, S. Dak., municipal.
Fargo, N. Dak., already mentioned.

One route from the Lakes to the Gulf:

Chicago, Ill., already mentioned.
Chanute Field, Rantoul, Ill., Air Service.
Belleville, Ill., Air Service.
Millington, Tenn., Air Service.
Jackson, Miss., municipal.
Baton Rouge, La., already mentioned.

In the establishment of these airways it is found that the various municipalities are usually glad to contribute the actual landing fields if the Federal Government will furnish the necessary equipment to operate such a field. The installation provided will depend largely upon the number of reserve personnel in the immediate vicinity that would rely upon this installation for their flying training.

The type installation is as follows:

1 wing cone with proper support and field marker T	$150
1 well with pump, 100-gallon tank	1,000
1 building, 25 by 100 feet, to contain the field-office lockers, meteorological office and emergency hospital and for use as a machine shop, supply storage, and workroom	25,000
1 oil tank, approximately 500 gallons, with pump	750
1 oil house, 10 feet square	850
1 gas tank with approximately 1,000 gallons capacity, with pump	1,000
1 hanger, steel, size 66 by 120 feet, with steel folding or sliding doors and cement floors	35,000
Total	63,750

It will be noted that in this type installation only one hanger is furnished. In many cases it will be necessary to have at least three hangars. In centers like Philadelphia and Boston this means the addition of $70,000 to the above amount.

It is estimated that to establish completely this proposed net would cost approximately $2,000,000. In the estimate for the fiscal year 1922 there was requested $1,000,000 as a start on this proposed scheme. It was desired to begin with the most important centers, such as Boston, Philadelphia, Baltimore, Chicago, St. Paul, Minn., Denver, Colo., Kansas City, and Seattle, Wash.

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Appendix C.

NECESSARY AEROLOGICAL SERVICE AND ESTIMATE OF COSTS.

A rational development of any program of navigation of the air must recognize meteorology and weather forecasting as a fundamental factor of great importance. The Weather Bureau was originally created by joint resolution of Congress approved February 9, 1870, placing the organization under the Chief Signal Officer of the Army, in the War Department, where it remained and developed for a period of about 20 years.

ORGANIC ACT.

The joint resolution was superseded by legislation entitled "An act to increase the efficiency and reduce the expenses of the Signal Corps of the Army, and to transfer the Weather Service to the Department of Agriculture." (Act. Oct. 1, 1920, ch. 1266, 266 Stat., 653.)

This organic act became effective July 1, 1891, and defines and outlines the functions and duties of the Weather Bureau very comprehensively and fully in the following language:

"Sec. 3. That the Chief of the Weather Bureau, under the direction of the Secretary of Agriculture, on and after July 1, 1891, shall have charge of the forecasting of weather, the issue of storm warnings, the display of weather and flood signals for the benefit of agriculture, commerce, and navigation, the gauging and reporting of rivers, the maintenance and operation of sea coast telegraph lines and the collection and transmission of marine intelligence for the benefit of commerce and navigation, the reporting of temperature and rainfall conditions for the cotton interests, the display of frost and cold-wave signals, the distribution of meteorological information in the interests of agriculture and commerce, and the taking of such meteorological observations as may be necessary to establish and record the climatic conditions of the United States, or as are essential for the proper execution of the foregoing duties."

With the practical development of navigation of the air a new demand was at once made upon the Weather Bureau for service in aid of aviation, and it is construed that the words of the organic act "for the benefit of agriculture, commerce, and navigation" obviously includes navigation of the air as well as of the oceans and inland waterways. Congress itself has in effect subscribed to this construction of the law, because in 1917 an appropriation was made for the conduct of this work under the following language:

"For the establishment and maintenance by the Weather Bureau of additional aerological stations, for observing, measuring, and investigating atmospheric phenomena in the aid of aeronautics, including salaries, travel, and other expense in the city of Washington and elsewhere."

This appropriation was first included in the act making appropriation for the support of the Army for the fiscal year ending June 30, 1918, and has subsequently been continued in the annual appropriation bills for the Weather Bureau of the Department of Agriculture.

AUTHORITY OF LAW NOW ADEQUATE.

It is believed the foregoing completely establishes that the Weather Bureau has full existing authority of law for the conduct of meteorological work of every character required for aeronautics, and under these laws the bureau has been conducting as large a program of work as is possible with the funds available.

METEOROLOGY AND WEATHER FORECASTING A UNIT.

In the organization of Government operations and activities with reference to meteorology and aviation, it is deemed to be a fundamental proposition that the whole problem of meteorological advices and warnings constitute a unit, for this service can only be administered when a comprehensive system of observations are available from a network of stations furnishing the fullest possible data concerning the conditions of the atmosphere, not only over the continents but over the oceans and at all altitudes accessible.

On this principle, prior to the war, the Weather Bureau received by international cooperation, reports from the entire Northern Hemisphere and published a daily weather map of the same. This service, suspended by the war, is now only partly restored. For 50 years the Weather Bureau has been an essential unit dealing with meteorology. Segregation of essential functions and activities, or the assignment of portions of its work to other organizations, can not be accomplished without loss of efficiency or great increase in cost and duplication and conflict of effort, because the fundamental system of complete observations and reports must be available for study and application by an agency attempting to furnish advices and forecasts.

POLICY AND PROGRAM.

Any program of meteorological work in aid of aeronautics is necessarily very intimately associated and interlocked with general meteorological work, forecasts, warnings, and advices of every character. For this service it is fundamentally essential to maintain a comprehensive network of stations over the entire continental area of the United States from which observations and reports are regularly and systematically received, including observations of the free air. Where practicable, by international cooperation and otherwise, such reports must be supplemented by reports from the other continents and the oceans.

In the specific interests of aviation this system of stations must include stations for the making of observations in the free air by the use of kites, balloons, and other agencies. At present only six completely equipped stations are in operation, supplemented by a small number of stations at which only pilot balloon observations are made. Reports furnish twice daily the direction and velocity of motion of the free air in different altitudes at from about 500 meters to about 3,000 or 4,000 meters above the surface of the earth. Results are telegraphed promptly to Washington twice a day and are used in furnishing daily forecasts and warnings, which are sent to the aviation branches of the Army, the Navy, the Post Office Department, aeronautical clubs, and all civilian interests who can utilize and desire the advices and information.

Present appropriations.—The amount of money expended for all purposes identified immediately with the exploration of the free air is at present $81,020, this being the total of the appropriation for the year 1920–21. For 1921–22 the same appropriation, $81,020, is available.

The above sum is supplemental to the general appropriations of the Weather Bureau for all purposes, and the general resources of the Weather Bureau are, of course, available to supplement the actual sum appropriated for aviation purposes.

RESULTS NOW BEING OBTAINED.

Prior to 1917 the Weather Bureau conducted a single observation station for the exploration of the upper air and published results of these observations more or less in detail in the bulletins of the Mount Weather Observatory. Subsequently it issued a five-year summary of these observations, giving details of upper air conditions. These results are very valuable and were greatly in demand by artillerists, aeronauts, and others during the period of the war and were quoted and referred to in connection with a great many problems dealing with free air conditions. Upon the establishment of five additional kite stations the observational data began to be published as promptly as possible in the supplements of the Monthly Weather Review, which also were in demand and used during the period of the war. From lack of funds the publication of the supplements had to be suspended, but the free air data are being analyzed and studied, and a three-year summary from the six aerological stations is in process of compilation. A considerable number of special studies of free air phenomena, as shown by observations, have been made and the results published in the Monthly Weather Review. Systematic forecasts of conditions for aeronauts were anonunced to begin December 1, 1918. These have been continued ever since and very considerably extended. The whole service of the Weather Bureau in aid of aviation is in a plastic form, subject to change and development as experience and needs indicate. Specific service was rendered in connection with the trans-Atlantic flights, both of the *NC-4* and the British airship, and on every occasion of any aeronautical event the Weather Bureau has been alert to make its advices and information directly available and beneficial in every way possible.

PROPOSALS FOR DEVELOPMENT.

Progress in the meteorological science and its applications requires an accumulation of observational data of atmospheric phenomena and conditions over a consideralbe period of time.

Additional stations needed.—Eight primary stations should be established at the earliest possible date, to bring the total up to 14. Including cost of equipment, salaries, telegraphing, and summarizing the results, these stations would

require an annual expenditure of about $22,500 each. The total cost for the 8 stations annually, would be $180,000. At least one additional station should be established each ensuing year until a total of 20 primary stations for the entire United States has been secured.

ACTUAL METEOROLOGICAL SERVICE ALONG AIR ROUTES.

Whenever an air route for regular traffic is established, the meteorological service necessary for that particular route should be set up even in advance of the time the air route is in actual practical operation. For each 1,000 miles of independent airway established and put in more or less complete operation the Weather Bureau should maintain a concentrated service of such route. This will require supplemental stations at intervals of about 250 miles, and the plan comprehends even additional reports from cooperative observers at intermediate points, giving visibility, local cloudiness, squalls, etc., so that at any time an aviator along the route would know the actual conditions at places toward which he may be flying. The cost of establishing and fully maintaining and operating these stations at intervals of 250 miles is estimated at $5,000 each, or a total of $25,000 for each independent 1,000 miles of air route. At the present time there are approximately 4,000 miles of air routes authorized and in more or less regular operation. Meteorological reports and information are now needed for these aeronautical operations. In addition, flying is practiced more or less regularly over additional courses also needing the benefits of meteorological advices and warnings.

SUMMARY.

The appropriations estimated to be necesasry to meet the present and prospective requirements of aeronautics within the next five years are as follows:

For 8 primary stations essential to fundamental studies of conditions in the free atmosphere	$180,000
Secondary stations for 4,000 miles of authorized air routes and for special and supplementary flying	220,000
Total	400,000

Annual extensions.

1 additional primary station per year for 6 years or until 6 are added	$22,500
Supplementary stations along established air routes for each 1,000 miles of continuous airway	20,000
Total	42,500

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APPENDIX D.

COOPERATION AMONG THE VARIOUS DEPARTMENTS OF THE GOVERNMENT CONCERNED WITH AVIATION.

WAR AND NAVY DEPARTMENTS.

The cooperation between the Air Services of the Army and Navy Departments is provided by a joint Army and Navy aeronautical board, whose duty is to prevent duplication and to secure coordination in the construction of aircraft experimental stations and all operating air stations used by the Army, Navy, or both. All questions relating to the development of new types of aircraft and weapons to be used by aircraft are referred to the aeronautical board to determine which department should properly be charged with its development. All matters of procurement and purchase of aircraft and the estimates for the appropriations for the Army and Navy aviation programs are presented to this board for review and recommendation before submission to Congress. Training facilities are interchanged and existing law provides for the interchange of material.

WAR, NAVY, AND POST OFFICE DEPARTMENTS.

The War, Navy, and Post Office Departments cooperate extensively in the following ways: Surplus aeronautical equipment in possession of the Army or Navy for which the Post Office Department has a definite use is transferred freely, the Post Office Department having the opportunity to obtain this material before it is offered for sale. Air mail stations, especially those on the New York-San Francisco airway, are used by the Army and Navy, and in turn the air mail service has the use of the Army fields, repair depots, and the facilities of the naval aircraft factory. Army, Navy, and air mail pilots in distress may obtain assistance and supplies from the air stations of each department without discrimination. There is complete interchange of technical and engineering data, operating reports, and general information. In addition, the air mail pilots and mechanics must have passed the Army or Navy aviation physical examination, and in case of questions as to the physical fitness of any air mail pilot or mechanic he is reexamined by the Military or Naval Air Service medical departments. Other governmental agencies, such as the Bureau of Standards, Bureau of Mines, and Weather Bureau, render the technical assistance for which they are especially fitted.

EXPERIMENTAL AND RESEARCH DEVELOPMENT.

Coordination of experimental and research work is provided for by the subcommittees of the National Advisory Committee for Aeronautics. The organization of the National Advisory Committee for Aeronautics provides for subcommittees of power plants for aircraft, materials for aircraft, and aerodynamics. Each department of the Government, as well as the different branches of the aircraft industry, are represented in the membership of the various subcommittees. The proposed and active research and experimental development of each governmental department is reported to the subcommittees, thus preventing unnecessary duplication. The subcommittees further provide means of exchange of information and ideas which permits the industry and the various departments to familiarize themselves with the research that is in progress.

NAVY AND DEPARTMENT OF COMMERCE.

(*a*) *Geodetic Survey.*—The Air Service of the Navy is now engaged in mapping the Mississippi Delta for the Coast and Geodetic Survey, and has completed the mapping of swamps in South Carolina for the same service.

(*b*) *The Bureau of Fisheries.*—The Navy has demonstrated to the Bureau of Fisheries the practicability of locating schools of fish and reporting their location to fishermen. This service has been extended to the point where it was shown that it would be practicable to maintain an airplane service for this purpose.

NAVY AND TREASURY DEPARTMENT.

(*a*) *Coast Guard.*—The Navy Air Service has cooperated with the Coast Guard in the training of pilots and the transfer of equipment to this organization.

NAVY AND DEPARTMENT OF AGRICULTURE.

(*a*) *Weather Bureau.*—The Weather Bureau has cooperated to the extent of its facilities in providing meteorological information to the Navy. A meteorological interdepartmental committee has been organized to coordinate the needs and services of all the governmental departments operating airplanes.

WAR DEPARTMENT AND DEPARTMENT OF AGRICULTURE.

(*a*) *Forest Service.*—An aerial survey is being made of the recent Olympic blow down of the State of Washington, where over 8,000,000,000 board feet of timber has been destroyed. Forest fire patrol will be continued and will require approximately 1,000,000 miles of flying per year.

(*b*) *Bureau of Farm Management.*—Photographs have been made of agricultural districts in order to obtain photographs by means of which farm management and development may be improved.

(*c*) *Bureau of Entomology.*—Various areas have been photographed for the Bureau of Entomology and experiments are being made for the purpose of locating rust spores in the upper air currents.

(*d*) *Weather Bureau.*—The Air Service has cooperated with the Weather Bureau in various capacities, especially in connection with obtaining meteorological information.

WAR DEPARTMENT AND TREASURY DEPARTMENT.

Aerial photographs of various areas are being made for the use of the United States Public Health Service.

WAR DEPARTMENT AND DEPARTMENT OF THE INTERIOR.

Geological Survey.—Geological surveys are being made of various areas such as Bibb County, Ga., Greater New York, New York Harbor, 2,708 square miles in North Carolina, some 4,000 square miles in the vicinity of Los Angeles, and many other areas totaling some 20,000 square miles. For the same department the Air Service is cooperating with the director of national parks and the chief of the Reclamation Service.

WAR DEPARTMENT AND DEPARTMENT OF COMMERCE.

Coast and Geodetic Survey.—Areas near the head of the Chesapeake Bay, Atlantic City, and Florida Reefs are being photographed from the air; also the coast of New Jersey, from Cape May to Sandy Hook. By means of aerial photographs a revision of the charts of the James River from Hampton Roads to Richmond and the coast lines of South Carolina, Georgia, and Florida are being accomplished. Aerial photographs of various forest areas, public buildings, rivers, and cities are being taken whenever facilities become available. Among the important examples of this type of work may be quoted: Forest areas of the Adirondack region and the State of Pennsylvania, public buildings and grounds of the District of Columbia, 1,200 miles of the Red River Basin between the States of Oklahoma and Texas, and the Tennessee River Basin. Photographs for the secretary of the Commission of Fine Arts are being made to aid him in planning the new botanic gardens.

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APPENDIX E.

COOPERATION OF WAR AND AGRICULTURAL DEPARTMENTS AND OPERATING COST OF FOREST FIRE PATROL.

[By Air Service of the Army.]

1. These figures do not include the overhead, such as cost of initial equipment or airplanes nor personal expenses and salaries of operating personnel, which are covered in the specific Army appropriations for that purpose.

(*a*) Year 1921, program covering California and Oregon alone.

Estimate of actual money expended.

(*a*) Maintenance of airplanes and their spare parts, 37 planes, at $200 per plane	$7,400.00
(*b*) Maintenance of engines and their spare parts, 50 engines, at $200 each	10,000.00
(*c*) Fuel and oil, based on 28 gallons per hour, at 35 cents per gallon, and 1½ gallons of oil, at $1 per gallon:	
Gasoline	35,660.80
Oil	5,994.00
Total	59,054.80

(*b*) Estimate for fiscal year 1922, extended forest fire patrol. This program will not be carried out due to lack of (*a*) appropriations, (*b*) personnel, (*c*) instructions from General Staff adopting first alternative (that is, a reduction) of the three alternatives set forth.

This would allow the forest fire patrol to include Washington, Idaho, Montana, and a small portion of Wyoming, a total of five squadrons, 160 officers and 660 enlisted men.

Estimated cost.

(*a*) Maintenance of airplanes and their spare parts	$54,026
(*b*) Maintenance of engines and their spare parts	47,253
(*c*) Cost of fuel and lubricants	115,872
Total	217,151

N. B.—This does not include the appropriation of $50,000 obtained by the Department of Agriculture for cooperation with the Air Service and forest fire protection and expended under the supervision of the Department of Agriculture and its officers.

AIRSHIP DEVELOPMENT.

Germany and England have produced the best demonstrations of the potential value of rigid airships, both for military and commercial purposes. Germany, in particular, has demonstrated the practicability of commercial airship passenger service. The French Government has laid out a progressive program of development. France has two rigid airships of the Zeppelin type, which were acquired from Germany through the Reparations Commission. The present French program calls for the early inauguration of a commercial airship line from Marseille or Toulon to Algiers. In Italy, the semirigid type of airship has been developed, both for military and commercial purposes.

America's first program of airship development contemplated the procuring by the Navy of a rigid airship from England—the ill-fated *R-38*, known in this country as the *ZR-2;* the procuring by the Army of the Italian semirigid airship *Roma;* and the construction by the Navy at Lakewood, N. J., of the first American rigid airship, to be known as the *ZR-1*.

The disaster to the *ZR-2* in Great Britain before delivery to this country threatened for a time the discontinuance of airship development in America. The National Advisory Committee for Aeronautics, in a special resolution addressed to the President and to the Secretaries of War and of the Navy in September, 1921, pointed out the tremendous advantage possessed by America by reason of its virtual monopoly of the known sources of supply of helium, and urged the continuance of our airship development program, stating that it would be contrary to the true American spirit to abandon a conservative program of development because of a single disaster. The committee renews its recommendations then made, that the development of airships be continued, and that sufficient funds be provided for experimental work for the, obtaining of definite information regarding the strength qualities of materials and girders used

in the construction of airships, and for the development and checking of the theories used in the general design of airships.

As the techinque of airship construction has not as yet been developed in this country, the committee believes it advisable, in the interests of rapid and economical progress, that the Goverment procure a rigid type of airship, either of German or British manufacture, embodying the latest developments, as far as possible, of both countries. Such an airship is an indispensable part of our program, and should be obtained to fill the vacancy caused by the loss of the *ZR-2*. The committee also renews its recommendation that the present program for the construction of the *ZR-1* at Lakewood, N. J., be prosecuted with renewed vigor.

HELIUM FOR AIRSHIPS.

Helium, next to hydrogen, is the lightest gas known. It has 92 per cent of the lifting power of hydrogen, and for military purposes possesses an inestimable advantage over hydrogen in that it is nonflammable. The natural-gas wells in the United States afford a practical monopoly of the known sources of supply. According to the latest estimates, helium is escaping into the atmosphere at the rate of one and a quarter million cubic feet a day, or at a rate sufficient to fill four large airships a week. At this rate, according to our present knowledge of helium-bearing gas, our great resources will have become dissipated within the next 20 years unless some appropriate measures are taken to preserve the sources of supply.

The refrigeration process is employed to obtain helium. In this process every constituent present in the natural gas is liquified except helium, which is expelled into suitable containers for storage. The application of this process to the extraction of helium has not been perfected, but the line of development is reasonably clear. The Army, the Navy, and the Bureau of Mines, acting in close cooperation (with the limited funds available), are carrying out certain developments which promise to solve the production problem.

The Bureau of Mines is also conducting experiments to determine whether underground-chamber storage is practicable and economical. If so, the problem may be resolved into one of conservation by the storage of helium in its natural state. Helium for current uses, however, will continue to be stored in high-pressure containers until used.

In connection with the consideration of an airship development policy, the committee presents as the crystalized opinion of the Government experts who have studied the helium problem that in helium the United States has exclusive possession of a valuable adjunct to national defense which will be wasted unless conservation is provided for without delay.

The National Advisory Committee for Aeronautics therefore recommends that the Government acquire and seal for future use the best helium-producing gas fields; that such experiments be continued as are involved in the development of an efficient and economical process for the extraction and repurification of helium; and that the Government continue experimental work in connection with the development of airships, and inaugurate without delay the use of airships inflated with helium. With large reserves of helium and the development of types of airships to fully realize the advantages to be derived from the use of helium, America would possess resources and knowledge with which no other nation could successfully compete.

THE ELECTROSTATIC PROBLEM FOR AIRSHIPS.

The question of the danger to airships from static charges of electricity and from lightning was referred to the committee, and a large amount of data was collected. In the end the procedure adopted by the Navy Department was investigated and approved.

THE AIR MAIL SERVICE.

In January, 1921, the committee submitted the following special report to the Congress through the President, setting forth its views as to the value to the Nation of the Air Mail Service of the Post Office Department:

To the Congress:

The executive committee of the National Advisory Committee for Aeronautics, at a meeting held on January 13, 1921, had under consideration the present status of the Air Mail Service of the Post Office Department and the probable effect on the development of aeronautics in America of the discontinuance of this activity, which has been threatened by the elimination on the floor of the House of the appropriation for its continuance.

It is the opinion of the National Advisory Committee for Aeronautics that the Air Mail Service is in effect an experimental laboratory for the development of the civil uses of aircraft, and, viewed from this angle alone, is worth what it costs over and above the value of the service it actually renders in the more rapid transportation of mail.

The Sixth Annual Report of the National Advisory Committee for Aeronautics contained a statement of a national aviation policy, in which the need of Federal encouragement for the development of commercial aeronautics war emphasized and the definite recommendation made that the Air Mail Service of the Post Office Department be furthes extended and developed.

The increasing relative importance of aircraft in warfare not only justifies but makes imperative the expenditure of public funds to aid the development of commercial aeronautics, on which military and naval aeronautics will be largely dependent. The Air Mail Service has given the best demonstration of the practicability of the use of aircraft for civil purposes, and, in the face of many obstacles, has accomplished remarkable results of real and permanent value to the Nation at relatively slight cost. It is the unanimous opinior of the National Advisory Committee for Aeronautics that the Air Mail Service of the Post Office Department should be continued.

The reasons for this special report are set forth in the following resolution, which was adopted without a dissenting vote on January 13, 1921:

"Whereas the Air Mail Service of the Post Office Department has given to the world the best demonstration of the practicability of the uses of aircraft for civil and commercial purposes, and has greatly promoted the commercial development of aircraft; and

"Whereas the development of the civil and commercial uses of aircraft is essential to the maintenance of an aircraft industry in such a condition as to permit the necessary expansion in time of war, unless the taxpayers of the country are willing to pay, through Army and Navy expenditures, practically the entire cost of maintaining the industry; and

"Whereas the civil development of aviation will aid military development and will in time bring about the establishment of air routes and landing fields more or less generally throughout the United States, which will be an important military asset in time of war; and

"Whereas the Air Mail Service has already contributed to the progress of civilization by improving the means of business communication and transportation; and

"Whereas the Army and Navy Air Services are by law limited to the development of types of aircraft for military and naval purposes which do not satisfactorily meet commercial needs; and

"Whereas the needs of other civil agencies of the Government, such as the Forest Service, the Coast Guard Service, the Coast and Geodetic Survey, require the development of nonmilitary types of aircraft; and

"Whereas the operation of the Air Mail Service is at the present time an essential part of the organized plan for the development of commercial aviation as contained in the national aviation policy formulated by this committee, and the cost of fostering such an organized plan is much less than the waste that would inevitably result from unprepared entry into war; and

"Whereas several European countries are supporting the development of commercial aviation, the discontinuance of the Air Mail Service will be a backward step in the development of aviation and will retard for a number of years the establishment of landing fields generally: Therefore, be it

"*Resolved*, That it is the opinion of the National Advisory Committee for Aeronautics that the Air Mail Service is in effect an experimental laboratory for the development of the civil uses of aircraft, and that, viewed from this angle alone, it is worth what it costs over and above the value of the service it actually renders in the more rapid transportation of mail;

"*Resolved further*, That the National Advisory Committee for Aeronautics submit a special report to the Congress through the President, stating that in its opinion the increasing relative importance of aircraft in warfare and the need for adopting and carrying through a sane, sound policy for the development of civil aviation, such as is outlined in the statement of a national aviation policy contained in the Sixth Annual Report of the National Advisory Committee for Aeronautics, not only justifies but makes imperative the expenditure of public funds to aid the development of commercial aeronautics; and that, as an existing agency which has, in the face of many obstacles, accomplished remarkable results of real and permanent value to the Nation, and at relatively slight cost, the Air Mail Service of the Post Office Department should be continued."

Respectfully submitted.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
JOSEPH S. AMES, *Chairman Executive Committee.*

The message of President Wilson, in transmitting the above report to the Congress under date of January 24, 1921, was as follows:

To the Senate and House of Representatives;

I transmit herewith for the consideration of the Congress a special report of the National Advisory Committee for Aeronautics, in which the committee sets forth its views as to the value to the Nation of the Air Mail Service of the Post Office Department, based on broad, general considerations of national interest and policy.

I concur in the opinions expressed by the National Advisory Committee for Aeronautics, and indorse its recommendation for the continuance of the Air Mail Service.

Subsequently, in April, 1921, the committee submitted to President Harding a special report dealing with Federal regulation of air navigation, air routes to cover the whole United States, and cooperation among the various departments of the Government concerned with aviation, which was published as House Document No. 17. In that report the committee recommended that the Post Office Department be authorized to extend the Air Mail Service by the establishment of air mail routes between Chicago, Minneapolis, and St. Paul, and between Chicago and St. Louis. This recommendation was based upon the advisability of affording greater encouragement for the development of civil aviation. Regardless of whether the Air Mail Service is to continue to be operated directly by the Government indefinitely, or as a private enterprise under contract with the Post Office Department, the service should be extended and developed. Eventually the service should be operated by private enterprise, that is, in so far as the air transportation is concerned; but before this will be economically practicable other necessary aids to commercial aviation, as recommended elsewhere in this report, should be afforded by the Federal Government.

THE AIR MAIL RADIO SERVICE.

The radio section of the Air Mail Service, organized to facilitate communications between fields and between fields and airplanes in flight, now has 16 land radio stations in operation. Communication between fields is almost instantaneous over a 16-hour period each day. Development has covered the practical application of airplane radio communication and radio direction finding. Airplane radio is being utilized as fast as appropriations permit.

In addition to air mail communications, this chain of radio stations provides an emergency communication system for the Post Office Department and other branches of the Government connecting all the important cities between Washington, New York, and San Francisco. It also forms the nucleus of a broadcasting system which now covers half the area of the United States.

CANADA'S CONTINUED COURTESY TO AMERICAN AIR PILOTS.

In June, 1920, the committee received from the State Department information to the effect that the Canadian Air Board had promulgated regulations permitting United States qualified aircraft and pilots to fly in Canada until November 1, 1920, on the same basis as if the United States had established air regulations as contemplated under the Convention for the Regulation of International Air Navigation. The Canadian Air Board has repeatedly extended this for successive periods of six months, and it remains a matter of courtesy pending the enactment of a separate treaty with Canada on the subject, because our Government has not ratified the Convention for the Regulation of International Air Navigation. In May, 1921, the Secretary of State requested the advice of the committee on the subject of further extensions of these courtesies by the Canadian Air Board. The committee, by resolution adopted at its meeting held on May 12, 1921, recommended that the State Department accept, on behalf of the Government of the United States, a further extension by six months of the period during which American pilots and aircraft will be permitted to fly in Canada under existing conditions; and that the State Department express to the Government of Canada the deep appreciation of the Government of the United States for the repeated courtesies of the Government of Canada in this matter, and at the same time express the hope that Federal legislation may be enacted at an early date which may lead to a more permanent and definite solution of the matter.

The committee at that time also reported to the Secretary of State that in its opinion the existing situation was highly undesirable, and served to emphasize the need for the early enactment of Federal legislation for the regulation of air navigation, and the committee recommended that the Secretary of State bring the matter to the attention of the President and the Congress with a view to securing the enactment of the necessary legislation.

INTERNATIONAL CONVENTION ON AIR NAVIGATION.

During the past year the committee has given consideration to a number of questions referred to it by the State Department, arising under the provisions of the International Convention of Air Navigation. The committee had, in September, 1919, and in May, 1920, recommended the ratification of this convention with certain reservations. In July, 1921, the committee, in response to an inquiry of the State Department, reaffirmed its opinion that the convention should be ratified in accordance with its previous representations and subject to the reservations recommended. If the International Convention on Air Navigation is not ratified by the United States, it is then the opinion of the National Advisory Committee for Aeronautics that a separate treaty regulating air navigation between the United States and Canada should be executed.

SAFETY CODE FOR AVIATION.

The extreme importance of establishing a safety code in aviation has been recognized by both the Army and the Navy and by manufacturers and others directly concerned with aviation. There is a general feeling, however, that great care should be exercised in the establishment of a safety code for aviation, to prevent, if possible, restrictions which will unnecessarily hamper the development of aviation.

As all national safety codes are established under the direction of the American Engineering Standards Committee, it was felt advisable in this case to have the safety code initiated by this body. The American Engineering Standards Committee, at its meeting on October 9, 1920, appointed a sectional committee for safety codes, with the Bureau of Standards and the Society of Automotive Engineers acting as joint sponsors for that committee. The following organizations were requested to designate representatives on the sectional committee for the safety code for aviation: War Department, Navy Department, Post Office Department, Coast Guard, National Advisory Committee for Aeronautics, National Safety Council, Underwriters' Laboratories, National Aircraft Underwriters' Association, United States Forest Service, Manufacturers Aircraft Association, Aero Club of America, American Society of Mechanical Engineers, and American Society of Safety Engineers.

The Bureau of Standards and the Society of Automotive Engineers, acting as sponsors, prepared a complete synopsis of a safety code for aviation, which was distributed to the various organizations to be represented on the sectional committee. A meeting for organization purposes was held in Washington on May 13, 1921, for the consideration of the scope and method of development of the proposed safety code for aviation. A meeting for organization was held in New York, September 7, 1921, at which a permanent organization was effected, the officers consisting of a chairman, Mr. H. M. Crane, Society of Automotive Engineers; vice chairman, Dr. Joseph S. Ames, National Advisory Committee for Aeronautics; and secretary, Mr. M. G. Lloyd, Bureau of Standards. Five subcommittees were appointed to deal with the subject matter of safety codes covering, respectively, the following subjects: Airplane structure, including design, construction, and test; power plants for aircraft, including design, construction, and test; equipment and maintenance of airplanes; lighter-than-air craft, including balloons, airships, and parachutes; airdromes and traffic rules, including landing fields, air ports, traffic rules, signals, and qualifications for pilots.

THE AERONAUTICAL BOARD OF THE ARMY AND NAVY.

The Aeronautical Board was appointed by the Secretary of War and the Secretary of the Navy, and is composed exclusively of Army and Navy officers. It has no official connection with the National Advisory Committee for Aeronautics, its functions being the consideration of military questions regarding the use of aeronautics in both services. The committee feels that there is a positive need for such a joint board; in fact, the present Aeronautical Board is a development of the Joint Army and Navy Technical Aircraft Board which was established during the war on the recommendation of this committee. There is no friction or duplication of functions whatsoever between the Aeronautical Board and this committee. On the contrary, a cordial contact has always existed in the work of the two organizations.

ASSISTANCE FROM ARMY AND NAVY.

The Army and the Navy Air Services have greatly contributed to the success of the research work in general conducted by the committee at the Langley Memorial Aeronatical Laboratory by placing at the service of the committee the necessary airplanes and accessories. The committee appreciates the cooperation given by the Army and Navy Air Services in the carrying out of research programs, and desires especially to acknowledge the many courtesies extended by the Army authorities at Langley Field, where the committee's laboratories are located.

WORK IN PROGRESS FOR THE ARMY AND THE NAVY.

As a rule, the technical subcommittees, including representatives of the Army and Navy Air Services, prepare programs of research work of general use or application, and these programs, when approved by the National Advisory Committee for Aeronautics, furnish the problems for solution by the Langley Memorial Aeronautical Laboratory. The cost of this work is borne by the committee out of its own appropriation. If, however, the Army Air Service or the Naval Air Service desires specific investigations to be undertaken by the committee, for which the committee has not the necessary funds, the regulations as approved by the President provide that the committee may undertake the work at the expense of either the Army or the Navy.

The work thus undertaken by the committee during the past year may be outlined as follows:

For the Engineering Division of the Air Service of the Army.—The committee's technical assistant in Europe purchased samples of foreign aeronautical instruments.

An investigation of the measurement of airplane oscillations with a kymograph was made at the Langley Memorial Aeronautical Laboratory.

A special type of accelerometer has been designed and constructed at the committee's laboratory.

A photographically recording air-speed meter, a special gyroscope for recording rate of turn, and a photographically recording multiple manometer for the measurement of the distribution of pressure on the different elements of an airplane have been designed and constructed at the committee's laboratory.

A study of fog landing devices has been undertaken with a view to reporting on the acoustical, electrical, and pressure methods.

For the Naval Air Service.—The design and development of an internal-combustion engine of the fuel-injection type has been undertaken at the committee's laboratory.

A research on the relative wing loading of biplane and triplane combinations, with variation in stagger, has been undertaken at the committee's laboratory.

An investigation of the aerodynamic properties of model aircraft to be tested in the new compressed-air wind tunnel has been inaugurated.

The committee's office of aeronautical intelligence has translated and prepared reports in English of aeronautical research and experimentation conducted in Germany during the war.

CONSIDERATION OF AERONAUTICAL INVENTIONS.

On the initiative of the inventions section of the Navy Department, an agreement has been reached between the department and the committee whereby all inventions of a general character relating to aeronautics which are received in the Navy Department are referred to the committee for consideration and proper action. The committee examines the invention, conducts the necessary further correspondence with the inventor, and if the invention is of prospective value makes a report to the Navy Department, a copy of which is sent to the Army Air Service. The agreement with the Navy Department was definitely approved by the Secretary of the Navy on April 12, 1921. In like manner, although without formal agreement, the committee considers inventions referred to it by the Army Air Service, and if any such inventions appear to be promising a copy of the report is sent to the Naval Air Service.

REPORTS FROM COMMERCIAL ATTACHÉS.

The Department of Commerce has, on request of the committee, instructed its commercial attachés and trade commissioners in England, France, Italy, Germany, and Holland to obtain reports and current information on developments in commercial aviation and forward same to this country for the use of the committee. Consular officers of the State Department, in all countries where any aeronautical activities exist, have been similarly instructed by the State Department. All the information that is received by the National Advisory Committee for Aeronautics is recorded in the committee's Office of Aeronautical Intelligence, and in this way becomes promptly available to the members of the committee and the governmental agencies concerned. If the information is not confidential, it is also made available to aircraft manufacturers.

THE UTILIZATION OF NONGOVERNMENTAL AGENCIES.

Whenever practicable, research authorizations when approved are distributed by the committee among governmental agencies. When, however, a certain investigation requires the use of facilities not possessed in any governmental establishment, or requires the services of a man not connected with the Government service, the committee contracts direct with the individual or institution concerned for a "special report" on the subject. In this way the committee has marshaled the services of educational institutions and eminent specialists in this and other countries.

GENERAL EDUCATION IN AERONAUTICAL ENGINEERING.

The committee is engaged in the preparation of problems in aeronautics and answers thereto, which will be distributed among professors in engineering courses at various universities with a view to having the engineering students generally acquire some knowledge of aeronautical engineering. The committee believes that the future development of aeronautics will be aided by even a slight familiarity with the problems involved on the part of engineering graduates generally.

OFFICIAL NOMENCLATURE FOR AERONAUTICS.

The committee's report No. 91, entitled "Nomenclature for Aeronautics," issued in August, 1920, was officially promulgated by the Secretary of the Navy and the Chief of Air Service of the Army for their respective services. This report supersedes all previous reports on this subject. It was prepared by the committee with a view to securing uniformity with reference to aeronautical terms in official documents of the Government and as far as possible in commercial publications. The report is prepared in classified and dictionary forms, and includes a list of aeronautical symbols.

BIBLIOGRAPHY OF AERONAUTICS.

The Bibliography of Aeronautics for the years 1910–1916 has been issued by the committee during the past year. The publication of this work by the committee was specifically authorized by law by act of Congress approved July 1, 1918. It covers the literature published from July 1, 1909, to December 31, 1916, and is a continuation of the work of the Smithsonian Institution issued as volume 55 of the Smithsonian Miscellaneous Collections, which covered the material published prior to June 30, 1909. Citations of the publications of all nations are included in the languages in which the publications originally appeared. The arrangement is dictionary form, with author and subject entry, and one alphabetical arrangement. Detail in the matter of subject reference has been omitted on account of cost of presentation, but an attempt has been made to give sufficient cross reference to make possible the finding of items in special lines of research. This volume comprises nearly 1,500 pages.

The committee has also prepared a Bibliography of Aeronautics for the years 1917, 1918, and 1919, which will be issued in one volume during the coming year. Beginning with the year 1920, the bibliography for each year will be prepared and issued annually by the committee.

INDEX OF AVIATION CHEMISTRY.

The committee has made arrangements with Dr. Edward C. Worden for the preparation for publication by the committee of an index of the voluminous report on aviation chemistry prepared for the Bureau of Aircraft Production by Dr. Worden and others immediately following the armistice. The report itself is too large to warrant publication. Ten typewritten copies are in existence, and it is proposed to make these available for general reference by placing them in libraries in different parts of the country. It is thought by the committee that the publication of an index of this report on aviation chemistry will suffice to make the whole available to those interested.

THE GENERAL REPORT ON AERONAUTIC INSTRUMENTS.

On recommendation of the subcommittee on aerodynamics, the National Advisory Committee for Aeronautics authorized the undertaking, at the Bureau of Standards, of the preparation of a general report on aeronautic instruments.

As authorized by the Committee on Aerodynamics, this report includes a complete account of the status of aeronautic instruments at the end of the war, and covers the subject in detail up to the beginning of the year 1920. Since the date of authorization nearly a year and a half has been required by the Bureau of Standards for the actual preparation of the manuscript, which has involved the cooperative effort of 17 different individuals, many of whom have since left the Government service.

The general report consists of eight separate technical reports, Nos. 125 to 132, including a report on the general characteristics of aeronautic instruments and problems and a bibliography of aeronautic instruments, and complete separate reports covering the following instruments, which are classified according to their use: Altitude instruments; aircraft speed instruments; direction instruments; power-plant instruments; oxygen instruments: and aerial navigation and navigating instruments.

The technical reports covering the above instruments present a systematic, illustrated description of American, British, French, Italian, Swiss, Dutch, Danish, Austrian, and German aircraft instruments, together with the methods of testing developed at the Bureau of Standards, and a brief statement of the results of the investigations. In the compilation of the material of the general report on aeronautic instruments separate papers have been written by experts on the respective types of instruments.

The general report is completed by a report by Dr. F. L. Hunt, Chief of the Aeronautic Instruments Section of the Bureau of Standards, on the recent developments and outstanding problems in aeronautic instruments. This report serves to bridge the gap between the information as to the various classifications of instruments, which was carried up to 1920, and the present, and contains information as to the most recent developments in aeronautic instrument design and construction.

QUARTERS FOR COMMITTEE.

The administrative offices of the National Advisory Committee for aeronautics are located in the Navy Building, Seventeenth and B Streets NW., Washington, D. C. The technical work of the committee, conducted by or under the supervision of the various subcommittees, has been carried on in various governmental laboratories and shops, including the Bureau of Standards and the committee's own field station at Langley Field, Va., known as the Langley Memorial Aeronautical Laboratory, and also in various laboratories connected with institutions of learning whose cooperation in the conduct of scientific research in aeronautics has been secured.

THE LANGLEY MEMORIAL AERONAUTICAL LABORATORY.

In previous annual reports the committee described the progress made in the development of its field station at Langley Field, Va., for the prosecution of scientific research in aeronautics. The station now comprises three principal units, namely, an aerodynamical laboratory or wind tunnel, an engine dynamometer laboratory, and a research laboratory building, the latter including administrative and drafting offices, machine and woodworking shops, and photographic and instrument laboratories. The research laboratory and the wind tunnel building are of permanent brick construction; the engine dynamometer laboratory is housed in a temporary four-section steel airplane hangar.

The committee has recently completed the construction of a factory type building of brick and steel, designed to house the new compressed-air wind tunnel. It is expected that the new wind tunnel will be in operation about July, 1922.

The Langley Memorial Aeronautical Laboratory occupies a plot of ground known as plot 16, Langley Field, Va., the plot having been set aside for the committee's use by the Chief Signal Officer of the Army in 1916, at the time the site was selected as a proposed joint experimental station and proving ground for the Army and Navy air services and the advisory committee. The use of that plot of ground was officially approved by the Acting Secretary of War on April 24, 1919. The four buildings at present constituting the Langley Memorial Aeronautical Laboratory have been erected by the committee pursuant to authority granted by Congress.

COMPRESSED AIR WIND TUNNEL.

On June 9, 1921, the executive committee of the National Advisory Committee for Aeronautics authorized the construction at the Langley Memorial Aeronautical Laboratory of a compressed air type of wind tunnel designed by Dr. Max Munk, technical assistant of the committee.

The utility of the present type of wind tunnel is limited by the fact that owing to a "scale effect" the results of tests on the small models, which are usually about 1/20 scale, are not immediately applicable to the full-size machine. Obviously it is very desirable to obtain, if possible, test results which are strictly proportional to those obtained in free flight. This condition may be realized by the use of a wind tunnel in which the air is compressed to about 20 atmospheres or more in order to compensate for the difference in the "scale" or Reynolds number for the model and for the full-size airplane.

The wind tunnel under construction has a diameter of 5 feet, the wind tunnel proper being placed within a steel cylinder 15 feet in diameter and 34 feet long. The steel cylinder has been tested for an internal pressure of 450 pounds per square inch and is designed for an average working pressure of 300 pounds per square inch.

The design of the cylinder further provides for a large door at one end and means for observing and operating the balance and setting wing angles from without the cylinder. The design of the balance has been carefully considered and due provision is made for the large forces to be measured.

The wind tunnel motor is 300 horsepower and the Reynolds number will be controlled by changing the air density rather than by changing the air speed. The air compressing units consist of two 300-horsepower compound compressors which compress the air to 115 pounds

per square inch. The air is compressed into a receiving chamber and is then compressed by a 175-horsepower duplex booster compressor to the desired pressure in the test chamber. With the compressor units selected it will require approximately one hour to fill the chamber with air at a pressure at 300 pounds per square inch and every provision is being made in the design to make it unnecessary to open the chamber until the model is completely tested.. Provision is also being made to maintain constant density so as to take care of temperature variations.

This tunnel when in operation will test models with a span of about 2 feet, but the results will be strictly comparable to similar data for a full scale machine, with a span of 30 feet, flying at 100 miles per hour. The construction of the models will therefore require special study and care.

OFFICE OF AERONAUTICAL INTELLIGENCE.

The Office of Aeronautical Intelligence was established in the early part of 1918 as an integral branch of the committee's activities. Its functions are the collection, classification, and diffusion of technical knowledge on the subject of aeronautics to the military and naval air services and civil agencies interested, including especially the results of research and experimental work conducted in all parts of the world. It is the officially designated Government depository for scientific and technical reports and data on aeronautics.

Promptly upon receipt, all reports are analyzed and classified, and brought to the special attention of the subcommittees having cognizance, and to the attention of other interested parties, through the medium of public and confidential bulletins. Reports are duplicated where practicable, and distributed upon request. Confidential bulletins and reports are not circulated outside of governmental channels.

To efficiently handle the work of securing and exchanging reports in foreign countries, the committee maintains a technical assistant in Europe, with headquarters in Paris. It is his duty to personally visit the Government and private laboratories, centers of aeronautical information, and private individuals in England, France, Italy, Germany, and Austria, and endeavor to secure for America not only printed matter which would in the ordinary course of events become available in this country, but more especially to secure advance information as to work in progress, and any technical data not prepared in printed form, and which would otherwise not reach this country.

The records of the office show that during the past year copies of technical reports were distributed as follows:

Committee and subcommittee members	2,039
Langley Memorial Aeronautical Laboratory	1,975
Paris office of committee	2,539
Army Air Service	2,296
Naval Air Service, including Marine Corps	4,159
Manufacturers	4,384
Educational institutions	4,308
Bureau of Standards	1,019
Miscellaneous	9,184
Total distribution	31,903

The above figures include the distribution of 13,080 technical reports and 7,108 technical notes of the National Advisory Committee for Aeronautics. Three thousand two hundred and twenty-eight written requests for reports were received during the year in addition to innumerable telephone and personal requests and 15,497 reports were forwarded upon request.

REPORT OF THE COMMITTEE ON AERODYNAMICS.

ORGANIZATION.

The committee on aerodynamics is at present composed of the following members:

Dr. John F. Hayford, Northwestern University, chairman.
Dr. Joseph S. Ames, Johns Hopkins University, vice chairman.
Maj. T. H. Bane, United States Army.
Dr. L. J. Briggs, Bureau of Standards.
Commander J. C. Hunsaker, United States Navy.
Dr. Franklin L. Hunt, Bureau of Standards.
Maj. H. S. Martin, engineering division, McCook Field.
Prof. Charles F. Marvin, Chief Weather Bureau.
C. I. Stanton, Air Mail Service.
Prof. Edward P. Warner, Massachusetts Institute of Technology, secretary.
Dr. A. F. Zahm, United States Navy.

FUNCTIONS.

The functions of the committee on aerodynamics are as follows:

1. To determine what problems in theoretical and experimental aerodynamics are the most important for investigation by governmental and private agencies.
2. To coordinate by counsel and suggestion the research work involved in the investigation of such problems.
3. To act as a medium for the interchange of information regarding aerodynamic investigations and developments in progress or proposed.
4. The committee may direct and conduct research in experimental aerodynamics in such laboratory or laboratories as may be placed either in whole or in part under its direction.
5. The committee shall meet from time to time on the call of the chairman and report its actions and recommendations to the executive committee.

The committee on aerodynamics by reason of the representation of the various organizations interested in aeronautics is in close contact with all aerodynamical work being carried out in the United States. In this way the current work of each organization is made known to all, thus preventing duplication of effort. Also all research work is stimulated by the prompt distribution of new ideas and new results which adds greatly to the efficient conduction of aerodynamic research. The committee keeps the research workers in this country supplied with information on all European progress in aerodynamics by means of a foreign representative who is in close touch with all aeronautical activities in Europe. This direct information is supplemented by the translation and circulation of copies of the more important foreign reports and articles.

The Aerodynamic Committee has direct control of the aerodynamical research conducted at Langley Field, the propeller research conducted at Leland Stanford University under the supervision of Dr. W. F. Durand, and some special investigations conducted at the Bureau of Standards and at a number of the universities.

WIND TUNNEL.

The committee's wind tunnel at Langley Field has recently had several changes made in it which have considerably improved the steadiness of flow. The most important of these is a new electrical system consisting of a synchronous motor-generator set which furnishes power direct to the wind tunnel motor. The speed of the wind tunnel motor is kept at a constant value within ± 0.2 of a per cent by means of automatic voltage regulators. The air flow has also been considerably improved by placing a series of vanes around the end of the exit cone so that the air escapes radially. A wire type of balance is now used in this tunnel for all speeds between 30 and 60 meters per second.

It has long been felt that the tests made in the wind tunnel with a model varying much from the usual type are unreliable because of the uncertainty of the scale correction. For this reason the committee is now constructing at Langley Field a compressed-air wind tunnel

with a throat diameter of 1.6 meters, a maximum speed of 25 meters per second, and a working pressure of 20 atmospheres. This wind tunnel will give a Reynolds number which is the same as for a full-sized airplane, and although the difficulties of supporting the model are great, the use of a comparatively low velocity and a high pressure have overcome the mechanical difficulties.

There are being constructed at the present time in the United States four other wind tunnels. At the Massachusetts Institute of Technology there are being erected a 1.25 meter and a 2.50 meter tunnel of the open-circuit type and with continuous throats. At McCook Field there is being constructed a high-speed 1.6 meter wind tunnel of the open-circuit type, which is designed for a velocity of 200 miles per hour. The Bureau of Standards at Washington is constructing a wind tunnel with a throat diameter of 3.25 meters. This wind tunnel is novel in that it is built in the open without any housing. The wind tunnel is well surrounded by trees and hills to prevent as far as possible the atmospheric conditions affecting the air flow.

The three-dimensional balance designed by Dr. A. F. Zahm for the Washington Navy Yard wind tunnel has proved very satisfactory. The weights of this balance are automatically actuated by electrically driven lead screws, and the time of making a test is much shorter than with other types of balances.

FREE FLIGHT.

The committee now has in use for aerodynamic research at Langley Field five airplanes; three *JN4H's*, one *VE-7*, and one Thomas-Morse *MB3*. The *JN4H* has been used by the committee extensively in experimental work, mainly because of its strength and the economy of operation. During the past year the flying time of the airplane has been 110 hours, representing 260 flights. Fifty-two per cent of the flying time has been used in actually making measurements in the air. No accidents of any kind have occurred with the committee's airplanes. One forced landing was made due to the sticking of the carburetor float during violent stunting, but the airplane was brought down without damage to itself or the instruments which it contained. Although complete airplanes have not as yet been constructed by the committee, a number of parts, such as wings, tail surfaces, etc., have been designed and constructed at Langley Field for use in free flight research.

INSTRUMENTS.

A number of new pieces of apparatus have been constructed for the wind tunnel, including a machine for forming plaster wings, a new micromanometer, a light balance for measuring the moments of control surfaces, and an instrument for measuring the rolling velocity of wings. It has become more and more evident as the discrepancies between free flight and model tests have been discovered, that it is necessary to produce in the wind tunnel a slip stream comparable with that on the full-sized airplanes. A very small flexible shaft has been developed which is able to drive the model airplane propeller up to speeds of 30,000 revolutions per minute, which corresponds to the normal speed of a full-sized propeller. The flexible shaft is so small that it disturbs the air flow inappreciably and in this respect is superior to an electric motor or a turbine.

It is realized that all free-flight data must be obtained by recording instruments, first, because events happen so rapidly that observations are difficult to make, and secondly, because the observer is under rather a nervous strain and can not take observations as accurately as he could in the laboratory. For this reason the committee has designed and constructed a considerable number of standardized recording instruments, electrically driven and synchronized, for taking records on interchangeable film drums. With these instruments the only duty of the observer is to change the drums at the end of the record, for the pilot can start and stop all of the instruments with a single switch. The following instruments have been constructed and used during the year:

(1) A new accelerometer more compact and accurate than the previous model.
(2) A recording air speed meter with a high natural frequency and small friction.
(3) A new model of a kymograph.

(4) A multiple manometer which will record on a moving film 30 simultaneous records of varying pressures. The natural frequency of this instrument is very high and the volume does not change appreciably with changes in pressure, which is a very important fact when recording pressures through long tubes.

(5) An instrument for recording angular velocities about a single axis.

(6) A control position recorder for three controls.

(7) A balance for measuring the forces on a trailing wing in flight.

The aeronautic instruments section of the Bureau of Standards has been engaged in an extensive program of research and development work on aircraft instruments in cooperation with the National Advisory Committee for Aeronautics, the Army, the Navy, and to a more limited extent with other Government agencies and private concerns. In addition to the experimental investigations and the development of new instruments a considerable amount of work has been carried out in connection with the routine testing of service instruments.

The investigation of the altitude effect on air speed indicators undertaken at the request of the National Advisory Committee for Aeronautics has been continued and extended. The experiments have been conducted in an improved wind tunnel with a 16-inch throat and mounted in one of the Bureau of Standards altitude chambers. With this apparatus valuable data have been obtained at speeds up to 100 miles per hour and under conditions of pressure and temperature corresponding to altitudes up to 30,000 feet.

Research concerning the action of diaphragms and Bourdon tubes undertaken at the request of the National Advisory Committee for Aeronautics has been continued with the purpose of determining the laws of deflection and of obtaining essential information of value in the design of instruments involving the use of diaphragms and Bourdon tubes.

A series of eight reports dealing with the various aeronautic instruments has been prepared for the National Advisory Committee for Aeronautics and will be found in the Seventh Annual Report.

At the request of the Army and the Navy, the development of the following instruments has been undertaken:

An improved aircraft sextant.
An improved compass.
An improved precision barometer.
A precision altimeter compensated for air temperature.
A precision barograph.
An improved rate of climb indicator.
An improved rate of climb recorder.
A combined statoscope and rate of climb indicator.
A synchronizing type ground speed indicator.
An astronomical position finder.
A horizontal angle indicator.
An improved centrifugal tachometer.
An air speed indicator for dirigibles.
A ballonet volume indicator for dirigibles.
Standard testing sets for field use.

Pursuant of the policy of following the latest developments in aeronautic instruments in foreign countries, a member of the aeronautic instruments section was detailed to investigate the recent developments in England, France, Italy, and Germany. This work was carried on in cooperation with the National Advisory Committee for Aeronautics representative in Europe and our military, naval, and commercial attachés, and much valuable information has been obtained.

A carbon pile tensiometer is being developed for the Navy which allows the accurate recording of tensions at a distance. An instrument has been devised by the Navy for the measurement of the ground speed of an airplane at frequent intervals of time on taking off or landing.

AEROFOIL TESTS.

During the past year the committee has conducted a large number of aerofoil tests in its 5-foot wind tunnel at Langley Field. The main object of these tests was to study the properties of thick aerofoils suitable for internal bracing. The tests were made at 35 meters per second, and in some cases as high as 60 meters per second as it was found that thick wings improve in efficiency with the speed more rapidly than thin wings. Some of the sections developed had at all angles a higher efficiency than the R. A. F. 15 section tested under the same conditions, and yet were more than three times as thick as that section in the center, while the maximum lift coefficients were approximately the same. A number of wings were tested which tapered in plan form, and it was found, contrary to expectations, that heavily tapered wings had the same center of pressure travel and practically the same efficiency as wings of uniform section.

The distribution of pressure was studied over 12 thick aerofoils of various types in order to determine the loading along the spars when the section varied along the span. A new method was devised for constructing pressure distribution models with comparatively little expense.

The effect of placing an aerofoil close to a flat surface representing the earth was thoroughly investigated both at the Massachusetts Institute of Technology and at the Washington Navy Yard. It was found that there was a remarkable increase in efficiency of the wing when close to a flat surface, which accounts for the fact that certain airplanes float for such long distances before landing.

Work has been continued in the McCook Field wind tunnel on various aerofoils at very high speeds, and a further study of vortex motion has been made.

Perhaps the most interesting work which has been carried out on aerofoils is that done by the committee in the testing of large aerofoils when suspended beneath a flying airplane. The aerofoil, constructed in the same way as an ordinary airplane wing of wood and fabric, is pulled up against the lower side of the fuselage in taking off, and when in the air is lowered down by means of a windlass to a distance of 20 or 30 feet or as far as is necessary to get out of the influence of the downwash. The magnitude of the resultant force is measured by a balance in the fuselage and the angle at which the wing trails back from the vertical measures the angle of the resultant. From these figures the lift and drag can be easily computed. At present only small wings of 6 feet span have been tested in this way, but it is evident from the great steadiness with which they trail beneath the airplane that accuracies probably as great as those obtained in the wind tunnel can be reached, although it is necessary to fly in smooth air for this kind of work. The results from the present apparatus although only of a preliminary character show such a good agreement with high speed wind tunnel tests of the same section that it is proposed to use a large bombing machine and trail wings of 30 feet span beneath it. Tests of this nature have not only the same Reynolds number, but also the same velocity, the same size, and the same amount of turbulence as the full-sized airplane, so that the results can be used by designers with perfect confidence.

A number of aerofoil sections have been tested, among which were several of the Göttingen series. The Washington Navy Yard tests check the Göttingen tests as closely as could be expected, the general types of the characteristic curves being very similar in every case. The Göttingen aerofoils tested were: Nos. *173*, *255*, *256*, and *322*.

Tests for scale effect have also been made on the *R. A. F. 15* and *R. A. F. 19* aerofoils.

STRUTS.

An interesting investigation has been made at the navy yard wind tunnel in Washington in the distribution of pressure over a strut. It is concluded that the total drag of the strut is the small difference between the upstream and downstream drags, so that a small error in measuring these will cause a huge error in the total.

STABILITY.

A very complete investigation has been made of the oscillations in flight of the *VE-7* and the *JN4H*, the latter airplane with a special tail plane to make it statically stable. The results on the whole are in poor agreement with the theory, due mainly, it is believed, to the fact that the oscillations are large, often over 60°, and that the slip stream has a considerable influence.

Considerable work has been done on static stability and it is becoming more and more evident that the aspect ratio of the tail plane has by far the greatest influence on the stability. It has also been found in actual flight that complete static stability may be obtained when the load is positive upon tail surfaces at all times. A study of the distribution of pressure over the tail surfaces of this surface in steady flight has given valuable information as to the functions of this surface in producing stability.

The lateral stability derivatives Y_v, L_v, and N_v have been determined in free flight for the *JN4H* and comparison has been made with the results from wind tunnel tests. On the whole the agreement is good, the discrepancies being mainly due to the influence of the slip stream and to the fact that in the model the control surfaces were assumed to be in a neutral position, whereas actually they were at a considerable angle.

A mechanical device has been constructed which will illustrate in every particular the dynamic and statical stability of an airplane. By the adjustment of weights the effect of changing the mass, the moment of inertia, the damping, etc., can be produced at will. As yet it has not been possible to obtain any quantitative value for stability with this instrument, but it is hoped that it may be used for quickly finding the stability properties of a new airplane from its known characteristics.

Tests have been made by the Navy on a series of balanced control surfaces with various types of balance. The characteristics of the type in which the axis is placed aft of the leading edge of the movable surface have been investigated at some length.

STRESSES IN FLIGHT.

The distribution of pressure was determined over the horizontal tail surfaces of a *JN4H* during all types of maneuver. In no case did the maximum loading on the tail exceed 6 pounds per square foot, and contrary to the usual expectations this load was in an upward direction. A theory has been devised which will give the loading on the tail surfaces in close agreement with the actual measurements.

The distribution of pressure over the rudder and fin have also been investigated on the same airplane and it was found that the heaviest loads occur in a roll where the loading may go as high as 10 pounds per square foot. It is interesting to notice from the standpoint of fuselage design that the maximum load on the horizontal tail surfaces, the maximum load on the rudder and fin, and the maximum load on the wings may all occur at the same time.

The recent development of very high speed airplanes has shown that very large unexpected loads may occur on the wing surface, several instances causing the stripping of the fabric or crippling of the trailing edge. A Thomas-Morse single seater which has a speed of over 160 miles an hour has been fitted up for measuring the distribution of pressure over the wing surfaces. It is hoped to determine the pressures both in steady flight and during violent maneuvering, and for this purpose the wings have been especially strengthened.

CONTROLLABILITY.

The measurement of and the design for controllability are very important problems and ones which have received but scant attention. In fact, the very definition of controllability is at the present time stated vaguely. The committee is now making an attempt to find some accurate quantitative means of measuring the controllability of various airplanes and to find the effect on controllability of various changes in control surfaces.

The desire for high speed has led many designers to eliminate the external bracing on the horizontal tail surfaces and for this reason a number of airplanes have been constructed with rather thick sections for the tail surfaces. Several airplanes of this type have been found by

pilots to be extremely sluggish in responding to the controls; that is, for a certain range about the neutral position the controls have no effect. This condition was investigated in the wind tunnel on a tail plane of this type, and it was found that the elevator must be moved several degrees on either side of its neutral position before the force on the tail is appreciably changed, due to the fact that the elevator seemed to be in the shadow of the thicker portion of the tail surfaces and could have no effect until it was turned out into the free air stream.

The angular velocity and angular accelerations have been measured on a *JN4H* during all types of maneuver, in order to provide designers with data which will be of use in construction of airplanes.

The subject of control, especially lateral control, at low flying speeds has received some attention. It is evident, however, that different airplanes, although varying only slightly in external characteristics, vary tremendously in the amount of lateral control which they have at the stalling speed, and an explanation of this would be of great value. The Navy has recently devised an entirely new type of lateral control which in wind tunnel tests shows great promise.

AIRSHIPS.

Several types of external-pressure pads developed by the Navy have been tested upon the wings of an airplane at Langley Field in order to assure that such openings when cemented to the outside of the wing will give the same pressure reading as a flush hole. One type of pad has proved to be very successful. The possibilities have been considered of measuring the pressure over the surface of an airship during accelerated flight, and as yet no satisfactory method has been devised for entirely eliminating the rather large errors due to the forces acting upon the air column in the long connecting tubes which are necessary in this experiment. The investigation, however, has not been abandoned, and it is believed that the difficulties will be overcome.

Extensive tests have been made on two models of the rigid airship *ZR-1*. These tests were made on the hulls, bare and with six types of control surfaces.

Tests have been conducted at the navy yard wind tunnel in Washington on the effect of fineness, ratio, and length of parallel middle body on airship forms.

PROPELLERS.

Experiments have been conducted in the wind tunnel to measure the drag of various propellers under various degrees of yaw and with different amounts of braking. The drags of propellers are rather small so that the possibility of the safe vertical descent of the helicopter without power does not look very probable if the usual type of propeller is used. Tests have also been conducted upon a helicopter propeller having blades which are automatically set at a constant angle of attack by means of individual tail planes.

An extensive investigation has been carried out at Leland Stanford University on the properties of propellers at angles of yaw. The results look very promising in connection with the horizontal travel of helicopters, as a considerable horizontal thrust may be obtained with no more power than is required in ordinary flight.

BOMBS.

The Bureau of Standards has been conducting a very extensive investigation of bombs and projectiles not only in their 150-mile wind tunnel but also in a 12-inch air stream from a high-power compressor where speed can be obtained above the velocity of sound. Some very interesting conclusions have been reached in connection with stream lining at very high speeds.

REPORT OF THE COMMITTEE ON POWER PLANTS FOR AIRCRAFT.

ORGANIZATION.

The committee on power plants for aircraft is at present composed of the following members:

Dr. S. W. Stratton, chairman.
Henry M. Crane, Society of Automotive Engineers.
Harvey N. Davis, Harvard University.
Dr. H. C. Dickinson, Society of Automotive Engineers, acting secretary.
Leigh M. Griffith, Langley Memorial Aeronautical Laboratory.
Capt. G. E. A. Hallett, United States Army.
Lieut. Commander S. M. Kraus, United States Navy.
George W. Lewis, National Advisory Committee for Aeronautics.
C. I. Stanton, Air Mail Service.
J. G. Vincent, Packard Motor Car Co.

FUNCTIONS.

The functions of the committee on power plants for aircraft are as follows:

1. To determine which problems in the field of aeronautic power plant research are the most important for investigation by governmental and private agencies.

2. To coordinate by counsel and suggestion the research work involved in the investigation of such problems.

3. To act as a medium for the interchange of information regarding aeronautic power plant research in progress or proposed.

4. To direct and conduct research on aeronautic power plant problems in such laboratories as may be placed either in whole or in part under its direction.

5. To meet from time to time on call of the chairman and report its actions and recommendations to the executive committee.

By reason of the representation of the Army, the Navy, the Post Office, and the industry upon this subcommittee, it is possible to maintain close contact with the research work being carried on in this country and to exert an influence toward the expenditure of energy on those problems whose solution appears to be of the greatest importance, as well as to avoid waste of effort due to unnecessary duplication of research. For purposes of detailed consideration, the activities of this committee can be advantageously grouped under the following heads:

Fuel-injection engine.
Supercharging compressors.
Cooling problems.
Radiation problems.
Carburetion problems.
Ignition problems.
Fuels and combustion.
Lubricants and lubrication.
Performance characteristics.
New engine types.
Extension of laboratory facilities.

FUEL INJECTION ENGINE.

The development of the aeronautic type of fuel-injection engine is one of the most important problems involved in the future evolution of large aircraft. As previously noted, the especial importance of such a development to the Bureau of Aeronautics of the Navy Department has resulted in their financial support of an extended research into the problems involved, the work being performed at the National Advisory Committee for Aeronautics research laboratory at Langley Field. Research on this general problem naturally divides into two main groups, largely because of the difference in type of apparatus required.

The study of the fundamental characteristics of fuel sprays, produced by different forms of nozzles and under various pressures and temperatures, is essentially a matter for the physical laboratory. Apparatus has been developed with which successive photographs of a single fuel injection spray are taken at a photographic frequency of approximately 1,200 per second. These are taken full size in groups of 15 upon a film, the film being mounted on a rapidly rotating drum. The illumination is secured by the mechanically timed discharge of 15 condensers,

the spark passing between magnesium points and the light being concentrated upon the spray by means of reflectors so placed that the illumination is by the dark-field method, thus insuring that only light which has been reflected or refracted by the fuel drops themselves can enter the camera lens.

The fuel injection spray is controlled by means of a specially designed cam-operated injection valve similar to the mechanism used upon stationary and marine fuel injection engines of the hydraulic injection type. The spray nozzle itself consists of a single opening in a nozzle plate, a set of such nozzle plates being available with holes varying in size from 0.004 to 0.024 inch. The most of the photographic study has been performed with nozzle openings to about 0.020 inch. The cam which operates the fuel injection valve is intermittently driven from a constant speed shaft, through one or more revolutions as desired, by means of a positive clutch of the general type used upon punch presses. This insures that any single injection is carried out at the same speed as in actual engine operation.

The fuel is supplied to the injection valve by a hand pump at any desired pressure up to 15,000 pounds per square inch. The injection takes place in a glass-walled chamber in which any desired air or nitrogen gas pressure up to 600 pounds per square inch may be obtained. The conditions therefore similate actual engine operation to a large extent, the element of high temperature being the only important factor neglected.

Studies have been made with water, light engine oil, and kerosene. Injection pressures up to 10,000 pounds and chamber gas pressures up to 150 pounds have been used. In addition to the series photographs on film, several hundred single photographs have been taken on glass plates, the series effect being secured by slightly altering the phase relation for each exposure. These latter are of good definition and depth, as the considerably increased amount of energy available for the single spark increased the intensity of illumination so greatly as to permit the camera lens to be stopped down to about one-sixteenth its former aperture.

In the case of the series records taken on film at the rate of 1,200 per second, the full development of the spray from the point at which the fuel first emerges from the nozzle occupies only three or four pictures, and the subsidence of the spray even less. Provision is therefore being made for considerably increasing the exposure frequency in order that the phenomena occurring during these two stages may be more thoroughly studied. Although the improved apparatus is not yet completed, it is quite evident from the work done that the frequency may be conveniently increased at least five times.

The work done to date has shown that no great difficulty will be experienced in producing photographs of such sharpness and depth that the characteristics of the spray are clearly shown and the spray development easily followed. It therefore only remains to secure similar photographs at higher speeds from sprays obtained with varying forms and sizes of nozzles, varying injection and spray chamber pressures, fuels or other liquids of various characteristics, and varying forms of cam opening and cam speed to enable the determination of the fundamental reasons for the differences in form, pulverization, penetration, and other characteristics. Much information has already been secured, although the investigation is still in its infancy.

The other group of the research activities on the fuel injection problems involves the application of the fundamental information obtained in the physical laboratory research to the actual operation of fuel injection engines in the dynamometer laboratory. In order to carry on this work to the best advantage, a very special single cylinder universal test engine has been designed and is being constructed. This engine is so arranged that the compression ratio can be changed throughout a wide range while the engine is running, as can also the opening and closing time and the lift of both inlet and exhaust valves. Each of these seven functions is controlled quite independent of the others, so that this engine is particularly well adapted to quickly determine the optimum adjustment for any condition of operation. The fuel injection devices to be applied are completely controllable in all their characteristics, and apparatus will be provided for measuring the important engine operation quantities, so that the results of tests on this equipment will provide complete data for the analysis and development of the high capacity and high speed fuel injection engine.

As the possibility of operating the present standard Liberty engine by some form of fuel injection system is attractive for some applications of that engine, work is also being carried on with a single cylinder Liberty which has been fitted with a special high compression piston and means for the application of various forms of fuel injection equipment. It is realized that the combustion chamber form is not well adapted to fuel injection, but it is believed that a specific power output and economy may be secured that is only slightly lower than with the standard Liberty.

SUPERCHARGING COMPRESSOR.

The Roots type of positive displacement blower as a compressor for supercharging aeronautic engines has been subjected to continual study throughout the year at the National Advisory Committee for Auronautics Research Laboratory at Langley Field. The original experimental machine has been in operation for a total time of approximately 75 hours, and tests have been made with it both as a separate unit and as mounted upon a Liberty engine. All of this work was done upon the dynamometer and very complete data was obtained of the characteristics of this device. The conditions were made to similate actual supercharging operation by connecting the suction of the compressor to a large depression tank in which the pressure was maintained at any desired value from atmospheric down to 6.5 pounds absolute.

In the case of the tests of the compressor alone, the discharge was direct to the atmosphere in the room and the compressor was directly driven by an electric dynamometer, so that very accurate power measurements were possible. Measurement of the air handled by the compressor was made by means of thin plate orifices conforming to the Durley standard, these orifices being placed at the entrance to a large chamber which in turn was connected to the depression tank through a control valve. The test performance of this compressor was very satisfactory. At a speed corresponding to normal engine speed, the overall efficiency compared to adiabatic compression was 73 per cent at a pressure ratio of 1.3 and 61 per cent at a pressure ratio of 2, these values being approximately 80 per cent of the maximum adiabatic efficiency which this type of compressor can possibly yield. The slip speed, or speed required to just maintain a given pressure ratio without discharge, was approximately 50 revolutions per minute at a ratio of 1.3 and 100 revolutions per minute for a ratio of 2, these low values resulting directly from the small clearances allowed.

Following the complete testing of the compressor alone, it was mounted upon a Liberty 12-cylinder aircraft engine, for which it had been designed, and tests made of the combined unit upon the dynamometer stand. When a receiver of 1.8 cubic feet capacity was placed next the supercharger in the air duct leading to the carburetors, the engine functioned well at all speeds, loads, and altitudes. As such a large receiver is extremely inconvenient to install in an airplane, it was deemed necessary to eliminate it from the final form of the apparatus. With a direct air duct connection, however, considerable difficulty was experienced due to resonance interference between the pressure impulses produced by the supercharger and the engine, this interference being so great at certain critical speeds as to prevent operation of the engine. After a number of experimental air duct arrangements had been tested, it was found that a small receiver immediately adjacent to the carburetor intakes had almost the same effect as a much larger receiver next the supercharger. Although the resonance interference has not been entirely eliminated, it has been confined to a small zone in an unimportant part of the speed range.

The following test figures are representative of the results which may be expected from service units of this type, when reasonably well maintained. At a normal engine speed and pressure ratio of 1.8 (approximately equivalent to an altitude of 16,000 feet), the supercharged unit gave within 11 per cent of its ground level power output, or within 12 per cent of the ground level power of a nonsupercharged engine. The test figures were corrected to the standard air temperature for that altitude and for the power increase due to the reduction of exhaust back pressure. This performance appears to be quite satisfactory, especially when compared to the 50 or 55 per cent power loss suffered by the nonsupercharged engine at that altitude.

The first experimental form of Root type supercharger will add something like 190 pounds to the total weight of a Liberty power plant, including the mounting and additional radiator, but it is believed that this can be reduced to under 150 pounds in subsequent models.

Following the completion of the dynamometer stand development, the supercharged engine is to be installed in a *DH-4* airplane and further tests conducted in free flight. A complete instrumental equipment will be fitted, in order that the flight test data shall be as comprehensive as possible.

Early in the year, some work was done with the positive gear driven centrifugal fan type of supercharging compressor, utilizing the apparatus built and partly developed by the American Expeditionary Forces in Paris during the last months of the war, but this work had to be laid aside on account of lack of personnel. The type shows considerable promise however, and it is the intention to again take it up at the earliest opportunity in order to definitely ascertain its value in comparison with the other types of superchargers which are more fully developed.

The engineering division of the Air Service has energentically continued its development of the General Electric exhaust turbine-driven centrifugal compressor and has built a number of units for experimental purposes. This type of supercharger is now a proved device, as is evidenced by its continuous use throughout the year with only minor changes. It has to its credit the world's record flight, reaching an indicated altitude of 40,800 feet. The engineering division is also developing the Sturtevant centrifugal blower type which is directly driven from the engine, but it has not as yet passed from the laboratory stage. They have also made a preliminary study of the gear driven centrifugal compressor of the heavy rotor type.

It is believed that the general problem of supercharging aeronautic engines to maintain power at altitude is perhaps next to the most important problem requiring immediate attention, and the committee's program calls for the energetic prosecution of research and development in this field.

COOLING PROBLEMS.

The problems incidental to the development of air cooled aviation engines of high performance are being studied in an experimental way at the McCook Field laboratory of the engineering division, Air Service. Much laboratory work of a more fundamental nature has also been carried on at the Bureau of Standards, the most of which is applicable to the radiation problem of the water cooled engine as well as to the direct air cooled type. It is hoped that it will be possible to concentrate increased research effort upon the problems of the air cooled engine, the importance of a satisfactory elimination of the radiator being well recognized by all.

New forms of radiator cores have been tested as they became available, and considerable progress has been made in the application of the laboratory research results to the logical design of radiators for specific installations. It is proposed to construct apparatus for the direct measurement of the air flowing through radiator cores in free flight when forming part of standard radiator installations on aircraft.

Several radiator cores of newer types, including a few specimens of the style finally adopted by the Air Service as a standard, were measured, and a report summarizing all the airplane radiator work done at the Bureau of Standards during the past four years has been prepared and is being published by the Bureau of Standards.

The Bureau of Standards is cooperating with the engineering division of the Army Air Service in a study of the fundamental relations between the heat removed from the hot metal of air-cooled cylinders and the power necessary to produce the air flow required for cooling. It is hoped that a number similar to the "figure of merit" used for airplane radiators can be determined for the ready intercomparison of air-cooled cylinder designs.

CARBURETION SYSTEMS.

Because of the excellent performance of the Maybach and B. M. W. engines under altitude conditions and the unique design of their carburetors, special investigations were made of the carburetors used on these engines. This work was done for the engineering division of the Army Air Service in the carburetor test plant of the Bureau of Standards. The results of this work have been collected in two reports, which it is hoped will be submitted to the committee for publication in the next annual report.

Two possible methods of inherent compensation of mixture ratio for altitude have been studied in the carburetor test plant of the Bureau of Standards. Both methods have given results which indicate that at full throttle a mixture ratio can be maintained independent of atmospheric pressure without aneroid control. Part throttle operation has as yet not been investigated.

During the course of the investigations in the altitude laboratory at the Bureau of Standards snow formation in the intake manifold was observed. This phenomena caused decidedly erratic and uncontrollable engine operation. As a similar phenomena could easily occur in actual flight, a short note on the subject was prepared and published by the committee as a technical note.

IGNITION PROBLEMS.

The routine testing of new forms of spark plugs and spark-plug insulators has been continued, both at the Bureau of Standards and at McCook Field. A gradual improvement of spark plugs has been noted, but much yet remains to be accomplished to insure the total elimination of spark,plug trouble. The experimental and mathematical investigation of the series gap has been continued, but final conclusions were not reached.

The program for the future provides for the continuance of routine tests of spark plugs and ignition systems generally, and also for the specific investigation of the effect of variation in ignition spark characteristics upon the engine performance.

Two reports on the theory of high tension magneto operation have been prepared by the Bureau of Standards and are being published by the committee. These reports are the result of experience gained in the magneto work previously carried out at the Bureau of Standards. One is an exact mathematical treatment and the other a more popular and nonmathematical treatment of the subject.

During the work on the high-compression engines in the altitude laboratory of the Bureau of Standards, it was noticed that the temperature of spark-plug electrodes was a major factor in determining the breakdown voltage of a given spark gap. A further investigation of this factor is being made in cooperation with the Army Air Service.

It has been observed at the Bureau of Standards that when an engine is operating on a very lean mixture on the verge of inflammability, the character of the ignition spark becomes important. Further investigation has shown that a very lean mixture ignited by a number or shower of sparks will miss fire less often than when ignited by a single spark. This subject is being further investigated in cooperation with the Army Air Service.

FUELS AND COMBUSTION.

The experimental investigation of compounded fuels has been continued by the Air Service at McCook Field and by the General Motors research laboratory at Dayton, Ohio. The value of such fuels for supercompressed engines or engines receiving their charge at a high temperature is thoroughly established. The investigation of compound fuels will be continued, as it is quite possible that their use may be obligatory in the high performance direct fuel injection engine, as well as in the high-compression carbureted engine.

The subject of the rate of flame propagation is under continuous experimental investigation, but has proved to be so complicated that it has not been possible as yet to draw definite conclusions from the work. This investigation is being carried on at the Bureau of Standards, and it is intended to devote an increased amount of attention to it during the coming year as an integral part of the general investigation of fuel characteristics.

Three independent methods for the laboratory measurement of reaction velocity in explosive gas mixtures have been developed at the Bureau of Standards. All the methods have been tried successfully with acetylene and illuminating gas, both of which have fairly well-known reaction velocities. These methods are, (1) the explosion of a mixture in a tube and either a photographic measurement of the rate of flame travel or the measurement of the energy liberated by means of a ballistic pendulum; (2) the measurement of reaction velocity by photography the contour of the inner core of a Bunsen flame; and (3) the photographic record of the combustion rate in a mixture inclosed in a spherical soap bubble and fixed at its center. A supply of carbon monoxide, which has a simple combustion reaction, has been obtained and it is planned to measure its rate of combustion by all three methods to establish their value, and then measure the combustion rates of hydrocarbon fuels.

The use of fuels of lower volatility has received considerable attention, more especially in connection with the development of the fuel injection engine. The greatly increased safety made possible by the substitution of such fuels for the present high volatility aviation gasoline, with the resulting indicated saving of lives and material, makes the subject one of the greatest possible importance. The program provides for the energetic prosecution of work upon this subject during the coming year.

LUBRICANTS AND LUBRICATION.

This subject has not received much attention. The program provided for the comprehensive study of the general subject of lubrication phenomena, with the object of determining the fundamental reasons for the unexplained differences in the lubricating value of oils which ordinary tests show to be quite similar. A lack of funds and personnel has, however, prevented carrying out this program, although it is hoped that the subject may be given proper attention during the coming year.

PERFORMANCE CHARACTERISTICS.

Although the work of determining the exact performance characteristics of existing engines does not definitely come under the classification of research, the Bureau of Standards has, by reason of its possession of ample altitude chamber equipment, carried out a number of performance tests of American and foreign engines for the double purpose of obtaining accurate information concerning the relative merits of the engines and also to determine calibration data which could be applied to the computation of the performance of aircraft in which these engines were subsequently fitted.

The laboratory work on a very extensive series of experiments in the altitude chamber of the Bureau of Standards on the effect of changes in compression ratio on power output and fuel consumption has been completed. The computation and analysis of the results is about 70 per cent completed, and a report giving the results of this work will be ready for publication in the next annual report. These experiments have included the effects of changes in mixture ratio, carburetor air temperature, and engine speed, as well as air pressure, and should be of great importance in the design of engines for aircraft service.

Two German engines, the 300-horsepower Maybach and the 185-horsepower B. M. W., have been tested in the altitude laboratory, in cooperation with the engineering division of the Army Air Service. The results of these tests have been submitted to the committee for publication, and furnish valuable data for the comparison of the performance of aircraft engines of successful foreign design with similar American designs.

Much publicity had been given to the high average economy attained in service by these two engines. An analysis of the action of these carburetors gave a very reasonable explanation for this economy. This explanation was therefore incorporated in a technical note submitted by the Bureau of Standards and published by the committee.

NEW ENGINE TYPES.

The possibilities of the double-piston two-cycle fuel injection automatic ignition engine have been kept prominently in mind during the past year, although it is realized that the development of such an engine must await the results of research into the problems of high-speed fuel injection and the application of such information to the development of a form of fuel-injection device that will answer the requirements of aviation engines. The research work on fuel injection has been described under the heading of "Fuel injection engines." It is proposed to begin active development of such an engine as soon as the fuel-injection research has produced results of sufficient value to justify that step. The possibility of altering the present four-cycle carburetted and electrically ignited engine so as to operate upon some form of fuel injection is being considered, and some work has been done in this direction, although it is realized that the conditions imposed by the present form of cylinder design will greatly increase the difficulties.

The air-cooled radial engine is being developed by McCook Field and the Navy Department, single-cylinder experimentation being relied upon to determine the most suitable form of cylinder and valve design. Some thought has been given to the advantages of operating large air-cooled engines by means of fuel injection, and this matter will receive attention during the coming year.

The engineering division laboratory at McCook Field has encouraged the development of a barrel type water-cooled engine, in which the cylinders lie parallel to the shaft, and a 350-horsepower unit of this type is now being constructed for experimental purposes. The possibility of reduced weight, head resistance, and vibration makes this type of considerable interest. Its development will be further encouraged during the coming year.

The engineering division has completed the development of its 18-cylinder 700-horsepower engine of standard broad-arrow type, and now has in hand the development of a 1,000-horsepower unit of this same form. An experimental model of the latter is now undergoing tests and has given results which indicate that little real difficulty is to be expected in its final perfection.

The Bureau of Aeronautics of the Navy Department has been giving considerable attention to high-power engines of the six cylinder in line type, a 400-horsepower unit being at the present time in process of development and a 1,000-horsepower unit having been designed and tested in the single-cylinder form.

EXTENSION OF LABORATORY FACILITIES.

The dynamometer laboratory equipment at Langley Field has been completely installed and made available for use, although it has been necessary to use one of the larger dynamometers as a temporary power plant for the generation of direct current for the operation of the laboratory. Contrary to expectations, no funds became available during the year for the construction of a permanent dynamometer building, so that it has been necessary to continue in the more or less temporary shelter provided by a standard Army hangar. Recently a 250-kilowatt motor generator set has been installed and connected to the central station of a neighboring town, so that it no longer is necessary to operate the temporary power plant, except for emergency requirements. The collection of engineering, physical, and electrical instruments and equipment has been increased during the year, although much remains to be accomplished in this direction.

The equipment at the Bureau of Standards has been materially refined as the result of operating experience, but no new major items have been added. The same comment applies to the engine laboratory facilities of the engineering division, Air Service, at McCook Field.

REPORT OF COMMITTEE ON MATERIALS FOR AIRCRAFT.

Following is a statement of the organization and functions of the committee on materials for aircraft:

ORGANIZATION.

Prof. Charles F. Marvin, chairman.
Dr. G. K. Burgess, Bureau of Standards, vice chairman.
Mr. Henry A. Gardner, Institute of Industrial Research.
Prof. George B. Haven, Massachusetts Institute of Technology.
Commander J. C. Hunsaker, United States Navy.
Mr. A. M. Hunt, American Magnesium Corporation.
Dr. Zay Jeffries, Aluminum Co. of America.
Mr. J. B. Johnson, Engineering Division, Air Service.
Prof. E. P. Warner, Massachusetts Institute of Technology.
Dr. Carlile P. Winslow, Forest Service.
Prof. H. L. Whittemore, Bureau of Standards, acting secretary.

FUNCTIONS.

1. To aid in determining the problems relating to materials for aircraft to be experimentally attacked by governmental and private agencies.

2. To endeavor to coordinate, by counsel and suggestion, the research and experimental work involved in the investigation of such problems.

3. To act as a medium for the interchange of information regarding investigations of materials for aircraft, in progress or proposed.

4. The committee may direct and conduct research and experiment on materials for aircraft in such laboratory or laboratories, either in whole or in part, as may be placed under its direction.

5. The committee shall meet from time to time on call of the chairman and report its actions and recommendations to the executive committee.

The committee on materials for aircraft, through its personnel acting as a medium for the interchange of information regarding investigations on materials for aircraft, is enabled to keep in close touch with research in this field of aircraft development.

Much of the research, especially in the development of light alloys, must necessarily be conducted by the industries interested in the particular development, and both the Aluminum Company of America and the American Magnesium Corporation are represented on the committee. In order to cover effectively the large and varied field of research on materials for aircraft three subcommittees were formed, as follows:

Subcommittee on metals (Dr. G. K. Burgess, chairman).
Subcommittee on woods and glues (Prof. H. L. Whittemore, chairman).
Subcommittee on coverings, dopes, and protective coatings (Mr. Henry A. Gardner, chairman).

Most of the research in connection with the development of materials for aircraft is financed directly by the Bureau of Construction and Repair of the Navy Department and the engineering division of the Army Air Service.

The Bureau of Construction and Repair not only conducts research at its aerodynamical laboratory at the Washington Navy Yard and at the naval aircraft factory in Philadelphia, but also apportions and finances research problems to the Bureau of Standards, the Langley Memorial Aeronautical Laboratory, the Institute of Industrial Research, and the Forest Products Laboratory.

SUBCOMMITTEE ON METALS.

Streamline wire.—This subcommittee has devoted much time to the preparation of a standard specification for the purchase of streamline stay wire similar to the Navy Specification No. 61. The first specification was issued in blue print form, Memo VII–1–18, to the members of the committee and others interested for criticism and suggestions.

This specification was revised and corrected in Memo VII-1-18A and after considering the criticisms of the largest manufacturers of these streamline wires in this country the committee is in a position to make a final report to the committee on materials for aircraft.

Due to the possibility of fatal accidents from the failure of a single stay wire on an airplane in flight, this subcommittee feels that only the best obtainable material should be used and that rigid inspection is necessary. For this reason provision is made in the specifications to insure a uniform high quality of material and a uniform treatment of each lot submitted for inspection so that the samples submitted to physical tests shall be representative of the lot from which they are taken. As an additional safeguard each wire is required to be submitted to a proof load within the elastic limit of the material.

The specifications contain a very carefully prepared series of sizes for the streamline sections having areas increasing in a geometrical series.

The allowable variations in dimensions were given careful consideration and the values recommended are believed to be those which will give wires which will be entirely satisfactory in use and not make the cost excessive.

The thread proposed for the terminals was the medium fit regular of the fine thread series recommended by the National Screw Thread Commission.

For protection of the wires from corrosion zinc plating was found to give the best results but spar varnish could be substituted.

This subcommittee has carefully considered the general subject of metals for aircraft and devoted all its energies to the investigation of those which appeared to be of the greatest importance. The possibilities in this field are very great and the facilities for fundamental research, particularly on the light alloys, should be greatly increased if this country is to undertake in quantity the development of either commercial or military all-metal aircraft.

Steel.—The fact that steel is the backbone of our material civilization with the vast amount of accumulated experience in regard to its manufacture and use, makes it highly desirable that its possibilities for use in aircraft be fully developed. It is low in cost and its resistanec to corrosion and fatigue, often the cause of failures, is fairly well known from experience supplemented by the results of laboratory experiments.

Steel strip has been produced which after heat treatment by the maker is formed and fabricated for aircraft parts. The heat treatment of thin steel sheets in this country is now impossible on a commercial scale and should be developed if this material is to be used for aircraft, automobile and other uses.

Prof. Moore has conducted at the University of Illinois for the National Research Council a series of fatigue tests on a large number of specimens of iron and steel. The specimens were tested in rotary beam machines of the Farmer type and in other type fatigue testing machines. Prof. Moore also used the thermometric method of determining the endurance limit. The preliminary reports indicate that there is a definite endurance limit which is proportional to the ultimate strength of the material.

An electrical method of heat testing long tubes of alloy steel has been perfected by Smead & Co. This material can be used in fuselage construction.

Light alloys.—Of the metals having a low specific gravity and comparatively high strength, the aluminum alloys similar to duralumin have been most generally used. At present this material can be obtained in the United States in many forms with the exception of tubes.

The alloys of magnesium are attracting attention due to their very light weight. Many of them have sufficient strength for many purposes but their resistance to corrosion must be more accurately known before they can be used with confidence.

These alloys have a wide field of usefulness particularly in aircraft, and the effect of variations in composition or in manufacture should be investigated and made available to all those interested. It is probable that alloys much more valuable than any known at present may be the result of properly directed research.

The methods of welding the light alloys as well as steel needs careful study and it is possible that other methods of joining parts may be largely replaced by welding.

Recent failures of metal aircraft structures show the need for laboratory investigations, not only on the materials, but also on the structural parts made from them. It is highly desirable to test full size girders, beams, and struts under the loading for which they are designed.

The effect of vibration on these structures may prove to be of great importance and should receive careful consideration.

The Bureau of Standards is continuing for the Bureau of Aeronautics, Navy Department, the tests on the corrosion of aluminum and its alloys in sea water and with various protective coatings, and unprotected specimens. Of the protective coatings the most promising seemed to be a light weight coating of varnish or oil. A coating of spar varnish remained for about ten months before scaling, and linseed oil gave good results for about a year. The test on basic, neutral and acid pigments indicate that there is no corrosion with any of the pigments after six months' exposure. The pigment coating has the disadvantage of being very high in weight as compared with the oil and varnish.

The Bureau of Construction and Repair, Navy Department, and the Air Service engineering division have both conducted extensive experiments as to the best material for the construction of gasoline tanks and fittings. The results of the tests of the effects of airplane fuels, dopes and doped fuels on the materials in the fuel system indicate that aluminum alloys are the least affected by the corrosive action of the fuels. Forged duralumin pipe fittings have been used and the aluminum alloy fuel tank proved superior to copper, tin, or terneplate as resisting the corrosive action of the fuel.

Considerable progress has been made in welding duralumin sheet in the construction of tanks and an aluminum solder has now been obtained that is satisfactory both for aluminum and aluminum alloys.

SUBCOMMITTEE ON WOODS AND GLUES.

Most of the research in the development of woods and glues for aircraft construction is carried on at the Forest Products Laboratory, Madison, Wis.

The problems studied include the properties of the woods of the United States and the conditions under which they are grown to secure the best results, the drying of the material, and methods of preventing decay, as well as the strength of structures made from wood, such as veneer, airplane wing ribs, etc. The development of a glue for joints in wood has been given a great deal of attention and the most favorable conditions of temperature, pressure, etc., have been determined. Waterproof glues are very desirable for aircraft construction and satisfactory glues of this type have been found.

Many problems for the Bureau of Construction and Repair of the Navy Department have been undertaken, such as the strength of screw fastenings and the making and testing for strength of structural parts of seaplanes and other aircraft.

The impact resistance of wood is an important property of this material in many cases, especially in airplanes. Many "crashes" are believed to be due to "brashness" of the wood in wing beams, and it seems desirable to give more importance to this subject. In addition to impact tests on small specimens, which have assumed considerable importance in England, it may be necessary to test full-size wings under conditions which approximate those found in service.

Very few tests of the fatigue resistance of woods have been made, but experience shows that they vary greatly in this respect. Certain woods, such as hickory for example, are found to be the best for the spokes of carriage wheels. There is little doubt that laboratory tests of fatigue would be of great assistance in selecting woods for aircraft construction which would secure the greatest safety with the least weight.

In the development of waterproof glues, the Forest Products Laboratory have obtained emulsions of animal glue and rubber in benzine. This glue may be more water resistant than animal glue. Tests have also been made with a casine glue which had been rendered insoluble by a treatment with formaldehyde. This glue was made under most favorable conditions and the results of tests show that it became soft and pliable after soaking in water for 11 days.

The indications are that this is the best we can expect of the water-resisting properties of casine glues.

A study has been made of the gluing pressure and temperatures to obtain the best results. Tests were made with three different glues and pressure of 25, 50, 150, 400, and 600 pounds per square inch. The results indicate that the quantity of glue spread and the exposure of the coated laminations appear to have a good deal to do with the amount of pressure necessary to obtain good joints. Medium and high pressures gave the best results.

From the preliminary tests of heating the wood before applying the glue results seem to indicate that warm wood and room give better results when low pressures are used.

The investigation by the Forest Products Laboratory for the Navy Department on the influence of internal stresses in laminated construction has been completed. The investigation covers three sources of internal stresses namely, the combination of plain-sawed and quarter-sawed material in the same construction, the gluing together of laminations of different moisture contents, and the gluing together of laminations of different densities. Nine species of wood were studied. The outstanding feature of the investigation is the decrease in magnitude of internal stresses with time and there is a strong indication that internal stresses die out under constant uniform atmospheric conditions.

Problems under way and not completed at the Forest Products Laboratory include a study of the influence of atmospheric and manufacturing conditions on airplane propellers, moisture resistant coatings, use of plywood in wing beams, study of torsion in box beams, and the design of beams for cantilever wings.

SUBCOMMITTEE ON COVERINGS, DOPES, AND PROTECTIVE COATINGS.

The Bureau of Aeronautics, Navy Department, has continued the investigation through the assistance of H. A. Gardner and the Bureau of Standards in developing a substitute for gold beater's skin for use with gas cells for rigid airships and possibly on the envelope of nonrigid airships. The films used include viscose, treated cellulose, gelatin compositions, casein, nitrocellulose, cellulose acetate, rubber, and tung oil. These have been tested singly and in combinations to determine permeability. The test results on certain of the films indicate that a gold beater's skin substitute is entirely possible.

Exposure tests of all specimens are to be made and provision is to be made by mechanical means to cause flexure of the film and duplicate service conditions.

The problem of obtaining a cheaper aircraft dope is being investigated by both the Army and Navy Air Services. The supplies of acetate dope, which was made during the war, are very limited and as the formula contains patented constituents the dope is very expensive.

TECHNICAL PUBLICATIONS OF THE COMMITTEE.

The committee on publications and intelligence has recommended the publication of 22 technical reports to be included in the seventh annual report. A summary of the technical reports published in the seventh annual report follows. The reports cover a wide range of subjects on which research has been conducted under the surveillance and cognizance of the various subcommittees, each report being approved by the subcommittee interested and recommended for publication to the executive committee. The technical reports presented represent fundamental research in aeronautics carried on at different aeronautical laboratories in this country, including the Langley Memorial Aeronautical Laboratory, the aeronautical laboratory at the Washington Navy Yard, the Bureau of Standards, and the Leland Stanford Junior University.

Considerable technical information is obtained by the committee that is of immediate interest to those interested in experimental and research problems in connection with aeronautics. To make this information immediately available, the National Advisory Committee for Aeronautics has authorized the committee on publications and intelligence to issue a series of "Technical Notes." In accordance with this authorization, the committee has issued 35 technical notes on subjects that were of immediate interest not only to research laboratories but also to airplane

manufacturers. A list of the technical notes issued during the year follows the general summary of the technical reports.

The first annual report of the National Advisory Committee for Aeronautics contained technical reports Nos. 1 to 7; the second annual report, Nos. 8 to 12; the third annual report, Nos. 13 to 23; the fourth annual report, Nos. 24 to 50; the fifth annual report, Nos. 51 to 82; the sixth annual report, Nos. 83 to 110, and since the preparation of the sixth annual report the committee has issued the following technical reports, Nos. 111 to 132.

Report No. 111, entitled "The Variation of Aerofoil Lift and Drag Coefficients with Changes in Size and Speed," by Walter S. Diehl, Bureau of Construction and Repair, United States Navy.—This report contains the results of an investigation into the effect of changes in size and speed upon aerofoil lift and drag coefficients. Certain empirical limitations to the interchangeability of v and l in the general equation of fluid resistance are pointed out and the existing methods of correcting for scale are criticized.

New methods of correcting for scale by means of simple formulae are derived and checked by comparison with test results. The drag coefficient at any given angle of attack within the range of steady flow is found to vary according to the expression

$$Dc_2 = Dc_1 - Dc_o \quad 1 - \left[\left(\frac{v_2\, l_2}{v_1\, l_1} \right) \right]$$

Where Dc_2 = Drag coefficient at $v_2\, l_2$

Dc_1 = Drag coefficient at $v_1\, l_1$

and Dc_o = Minimum drag coefficient at $v_1\, l_1$

Under similar conditions the lift coefficient is found to vary according to the expression:

$$Lc_2 = Lc_1 + .057 \log_{10} \frac{v_2\, l_2}{v_1\, l_1}$$

Where Lc_2 = Lift coefficient at $v_2\, l_2$

Lc_1 = Lift coefficient at $v_1\, l_1$

The applications and limitations of the above formulae are discussed and it is recommended that they be checked by special tests extending over a wide range of $v\, l$.

Report No. 112, entitled "Control in Circling Flight," by F. H. Norton and E. T. Allen, Langley Memorial Aeronautical Laboratory.—This investigation was undertaken by the National Advisory Committee for Aeronautics at the Langley Memorial Aeronautical Laboratory for the purpose of developing instruments that would record the forces and positions of all three controls, and to obtain data on the behavior of an airplane in turns. All the work was done on a standard rigged *JN4H* (airplane No. 2 of National Advisory Committee for Aeronautics, Report No. 70). It was found that the airplane was longitudinally unstable and nose heavy; that it was laterally unstable, probably due to too little dihedral; and that it was directionally unstable, due to insufficient fin area, this last being very serious, for in case of a loss of rudder control the airplane immediately whips into a spin from which there is no way of getting it out. On the other hand, it was found possible to fly quite satisfactorily with the rudder locked, and safely, though not so well, with the ailerons locked. The value of Y_v was obtained in free flight, and when the effect of the propeller was subtracted, the agreement with the model test was excellent, but with the propeller revolving at 1350 the value of Y_v was nearly doubled. The value of L_v and N_v were little affected by the slip stream, but their values do not agree with the model test.

Report No. 113, entitled "Tests on Air Propellers in Yaw," by W. F. Durand and E. P. Lesley, Leland Stanford University.—This report contains the results of tests to determine the thrust (pull) and torque characteristics of air propellers in movement relative to the air in a line oblique to the line of the shaft, and specifically when such angle of obliquity is large, as in the case of helicopter flight with the propeller serving for both sustentation and traction.

Report No. 114, entitled "Some New Aerodynamical Relations," by Max M. Munk, National Advisory Committee for Aeronautics.—This report contains three new relations extending the modern theory of aeronautics, intended to be applied in some later papers. They deal with phenomena in a frictionless fluid.

The first part contains a relation between the power absorbed by an aerofoil and the power absorbed by a propeller. In the second part the exactness of the ordinary formula for the induced drag of an aerofoil is examined and the error is determined.

In the third part the author shows that for the calculation of the air forces on bodies of considerable volume the imaginary sources and sinks equivalent to the flow around the body can be used in the same way as vortices are used for the calculation of lift and induced drag of wings.

Report No. 115, entitled "Bending Moments, Envelope and Cable Stresses in Nonrigid Airships," by C. P. Burgess, Bureau of Aeronautics, United States Navy.—No simple but comprehensive method of calculating the principal stresses in the envelope of a nonrigid airship has hitherto been described and published in the English language. The present report describes the theory of the calculations and the methods which are in use in the Bureau of Aeronautics, United States Navy. The principal stresses are due to the gas pressure and the unequal distribution of weight and buoyancy, and the concentrated loads from the car suspension cables.

The second part of the report deals with the variations of tensions in the car suspension cables of any type of airship, with special reference to the rigid type, due to the propeller thrust or the inclination of the airship longitudinally.

Report No. 116, entitled "Applications of Modern Hydrodynamics to Aeronautics," by L. Prandtl, Göttingen Laboratory.—The report gives rather briefly in Part I an introduction to hydrodynamics which is designed to give those who have not yet been actively concerned with this science such a grasp of the theoretical underlying principles that they can follow the subsequent developments. In Part II there follows a separate discussion of the different questions to be considered, in which the theory of aerofoils claims the greatest portion of the space. The last part is devoted to the application of the aerofoil theory to screw propellers.

Dr. Prandtl at the request of the National Advisory Committee for Aeronautics has used the same symbols in the formulæ as are used in Dr. Prandtl's German papers. These symbols are for the most part familiar to readers of the Technische Berichte. A table giving the most important quantities is at the end of the report. A short reference list of the literature on the subject and also a table of contents are added.

Report No. 117, entitled "The Drag of Zeppelin Airships," by Max M. Munk, National Advisory Committee for Aeronautics.—This report is a discussion of the results of tests with Zeppelin airships, in which the propellers were stopped as quickly as possible while the airship was in full flight. In this paper the author refers to the theory involved in these tests and calls attention to one scientifically interesting fact which can be derived from the tests and which has not yet been noted.

The most important general question concerning the tests is, of course: Does the negative acceleration of an airship with stopped propellers supply proper data for determining the drag of the airship when in uniform flight? This can not be absolutely answered in the affirmative, the two phenomena not being identical in principle. It is believed, however, that in this particular case the agreement is sufficient and that the data obtained from the test are the true quantities, or, at least, the approximate quantities wanted.

Report No. 118, entitled "The Pressure Distribution Over the Horizontal Tail Surfaces of an Airplane," by F. H. Norton, Langley Memorial Aeronautical Laboratory.—This work was undertaken by the National Advisory Committee for Aeronautics at the request of the Bureau of Construction and Repair of the United States Navy in order to determine as completely as possible the distribution of pressure over the horizontal tail surfaces of an airplane, and to analyze the relation of this pressure to the structural loads and the longitudinal stability. The investigation is divided into three parts, of which this is the first. The first part of the investigation is for the purpose of determining the pressure distribution over two horizontal tail surfaces in uniform free flight; the second part to conduct tests of similar tail planes in the wind tunnel; and the third part to determine the pressure distribution on the horizontal tail surfaces during accelerated flight on the full-sized airplane.

The general method used in this part of the investigation consists in determining the separate pressures at a large number of points on the tail surfaces of a *JN4H* airplane, by connecting small holes, opening on the tail surface, to the tubes of a multiple liquid manometer, which simultaneously measures the total number of pressures on one-half of the tail surface. The pressures are recorded by photographing the multiple manometer with an automatic camera which takes an exposure at each condition of air and engine speeds.

The tests in uniform free flight gave the following results:

1. Under no condition did the average tail load exceed 2.3 pounds per square foot.
2. The highest local load on the tail of the *JN4H* was 11 pounds per square foot.
3. The highest local load on the special tail was 25 pounds per square foot.
4. The torque exerted by the tail about the X-axis ranged from +1,200 inch-pounds (in the direction of the propeller rotation) to −1,600 inch-pounds.
5. The sealing of the crack between the elevator and tail plane has no appreciable effect on the distribution of pressure.
6. The inversion of the standard tail plane (flat surface up) gives a more uniform distribution of pressure as well as improving the stability.
7. The airplane was very stable with the special tail of high aspect ratio even with a center of gravity coefficient of 0.37.
8. The center of pressure travel on the wings, as determined by the integrated tail load, is farther forward than on the corresponding model.

Report No. 119, entitled "The Pressure Distribution Over the Horizontal Tail Surfaces of an Airplane—II," by F. H. Norton and D. L. Bacon, Langley Memorial Aeronautical Laboratory.—This investigation was undertaken by the aerodynamic staff of the National Advisory Committee for Aeronautics at Langley Field in order to determine whether the results obtained upon model tail surfaces can be used to accurately predict loads upon the full-sized tail; and also to find the distribution of load when large elevator angles are used, as the loads from such angles can not be obtained readily in free flight. The method consisted in using a metal horizontal tail surface inside of which small air passages, connecting with a series of holes in the surface, led the pressure off from the tail in rubber tubes. In this way the pressure at each of these holes was measured by a manometer at several angles of attack and several elevator settings. The results show that the model tests give a loading which is equivalent to the loading under similar conditions in the full-sized airplane and that the manner of distribution is quite similar in the two cases when there is no slip stream.

Report No. 120, entitled "Practical Stability and Controllability of Airplanes," by F. H. Norton, Langley Memorial Aeronautical Laboratory.—Thz effect of the characteristics of an airplane or balance, stability, and controllability, based on free flight tests, is discussed particularly in respect to the longitudinal motion. It is shown that the amount of longitudinal stability can be varied by changing the position of the center of gravity or by varying the aspect ratio of the tail plane, and that the stability for any particular air speed can be varied by changing the camber of the tail plane. It is found that complete longitudinal stability may be obtained even when the tail plane is at all times a lifting surface. Empirical values are given for the characteristics of a new airplane for producing any desired amount of stability and control, or to correct the faults of an airplane already constructed.

Report No. 121, entitled "The Minimum Induced Drag of Aerofoils," by Max M. Munk, National Advisory Committee for Aeronautics.—The "Minimum Induced Drag of Aerofoils" helps to explain the phenomenon of flight. It contains some theorems concerning the arrangement of airplane wings which are of considerable practical interest. In particular, it shows the theoretical reasons for the decrease of drag which accompanies all increase in the aspect ratio or lateral extension of a wing. The efficiency of a given arrangement of wings may be calculated from the formulæ derived in this paper.

Report No. 122, entitled "Preliminary Experiments to Determine Scale and Slip Stream Effects on a one-twenty-fourth Size Model of a *JN4H* Biplane," by D. L. Bacon, Langley Memorial Aeronautical Laboratory.—This work was undertaken at the Langley Field Aero-

dynamic Laboratory of the National Advisory Committee for Aeronautics to obtain results on a small model of a complete airplane which might be used for comparison with corresponding tests made in full flight. Somewhat similar tests have been previously made at various other laboratories; but as certain discrepancies exist between corresponding tests in different tunnels, it has been deemed advisable to obtain a direct comparison for this particular installation.

The present work covers tests on a one-twenty-fourth scale model at speeds varying from 6.7 m/sec. (15 m. p. h.) to 40.2 m/sec. (90 m. p. h.). A slip stream correction has been obtained by the use of a small belt-driven propeller mounted in front of the model, and force coefficients thus obtained are compared with the measurements of the same forces made in full flight on a geometrically similar airplane.

This report gives lift, drag, and longitudinal moment values obtained in tests of a particularly accurate model over a wide range of speeds. A measure of the slip stream corrections on lift and drag forces was obtained by the use of a power-driven model propeller.

Measurements were also made of forces and longitudinal moments for all angles from 0° to 360°.

Report No. 123, entitled "Simplified Theory of the Magneto," by F. B. Silsbee, Bureau of Standards.—This paper contains part of the results of ignition investigations being made for the National Advisory Committee for Aeronautics at the Bureau of Standards, and described a type of circuit which has been found useful for representing the action of the high-tension magneto. While this equivalent circuit is relatively simple, and consequently can be used as a basis for deriving definite mathematical formulas for induced voltages and similar quantities, it has been found experimentally to correspond quite closely in its performance with the highly complicated electrical circuits of an actual magneto. In the paper formulas are given for the voltage induced in the secondary under various conditions of operation, and a number of numerical examples are worked out showing the application of the equation to a variety of practical problems.

Report No. 124, entitled "Aerodynamic Characteristics of Aerofoils—II," by the National Advisory Committee for Aeronautics.—This collection of data on aerofoils has been made from the published reports of a number of the leading aerodynamic laboratories of this country and Europe. The information which was originally expressed according to the different customs of the several laboratories is here presented in a uniform series of charts and tables suitable for the use of designing engineers and for purposes of general reference.

It is a well-known fact that the results obtained in different laboratories, because of their individual methods of testing, are not strictly comparable even if proper scale corrections for size of model and speed of test are supplied. It is, therefore, unwise to compare too closely the coefficients of two wing sections tested in different laboratories. Tests of different wing sections from the same source, however, may be relied on to give true relative values.

The absolute system of coefficients has been used, since it is thought by the National Advisory Committee for Aeronautics that this system is the one most suited for international use, and yet is one for which a desired transformation can be easily made. For this purpose a set of transformation constants is included in this report.

Each aerofoil section is given a reference number, and the test data are presented in the form of curves from which the coefficients can be read with sufficient accuracy for design purposes. The dimensions of the profile of each section are given at various stations along the chord in per cent of the chord, using as datum the line shown on the curves. The shape of the section is also shown in reasonable accuracy to enable one to more clearly visualize the section under consideration, together with its characteristics.

The authority for the results here presented is given as the name of the laboratory at which the experiments were conducted, with the size of the model, wind velocity, and date of test.

Report No. 125, entitled "General Classification of Instruments and Problems, Including Bibliography," by Mayo D. Hersey, Bureau of Standards.—This report is Section I of a series of papers, comprising a general report on aeronautic instruments, published as Technical Reports

Nos. 125 to 132, inclusive, which contain the results of investigations relating to aeronautic instruments undertaken at the Bureau of Standards under research authorizations formulated and recommended by the subcommittee on aerodynamics and approved by the National Advisory Committee for Aeronautics.

As authorized by the committee on aerodynamics, these reports include a complete account of the status of aeronautic instruments at the end of the war and cover the subject in detail up to the beginning of the year 1920. Nearly a year and a half has been required for the actual preparation of the manuscripts, which represent the cooperative effort of seventeen individual authors, many of whom have left Government service. Report No. 132, by Dr. F. L. Hunt, now chief of the aeronautic instruments section of the Bureau of Standards, serves to bridge this gap by giving a brief statement of recent developments. The bibliography also has been kept complete up to the moment of going to press.

Technical Reports Nos. 126 to 131, inclusive, contain a systematic, illustrated description of American, British, French, Italian, Swiss, Dutch, Danish, Austrian, and German aircraft instruments, together with methods of testing developed by the Bureau of Standards, and brief statements of investigation results. In compiling the material for these reports, separate papers have been written by experts on the respective types of instruments, as, for example, altimeters, tachometers, or oxygen apparatus.

Subjects which are common to instruments in general are treated in this report. Throughout the series of reports emphasis has been placed on the description of successful types of instruments and the exposition of fundamental scientific principles, while the space devoted to investigations and developments of transitory interest has been reduced to a minimum. In this way it is expected that the reports will be of permanent value for reference.

This report is intended as a technical introduction to the series of reports on aeronautic instruments. It presents a discussion of those subjects which are common to all instruments. In the first place, a general classification is given, embracing all types of instruments used in aeronautics. The arrangement of information dealing with these various instruments throughout the reports is then briefly indicated as a guide to the reader. Finally a classification is given of the various problems confronted by the instrument expert and investigator. In this way the following groups of problems are brought up for consideration: First, problems of mechanical design; second, human factor; third, manufacturing problems; fourth, supply and selection of instruments; fifth, problems concerning the technique of testing; sixth, problems of installation; seventh, problems concerning the use of instruments; eighth, problems of maintenance; ninth, physical research problems. This enumeration of problems which are common to instruments in general serves to indicate the different points of view which should be kept freshly in mind in approaching the study of any particular instrument.

Report No. 126, entitled "Altitude Instruments," is made up of four parts. Part I on "Altimeters and Barographs," discusses briefly barometric altitude determinations with the view of explaining the methods of calibrations of altimeters and barographs used by various nations. A detailed description is given of all the principal types used and a discussion is given of results of investigations carried on at the Bureau of Standards during the past four years.

Part II, "Precision Altimeter Design." In this report the authors first developed a theory of aneroid design and then checked the results by experiments on an instrument constructed according to the theory. It was found that the most difficult error to eliminate or correct in an aneroid barometer is elastic lag or time effect.

Part III, "Statoscopes and Rate of Climb Indicators." The authors have made a detailed study of statoscopes, and for convenience, statoscopes are divided into two types, mechanical or diaphragm and bubble. The leak type rate-of-climb indicators receive major consideration in the discussion on calibration and tests.

Part IV, "Aerographs and Strut Thermometers." This paper contains a description of the principal types of thermometers and other aerographic instruments, including direct and distance reading strut thermometers. The paper concludes with a discussion of methods of testing and performance characteristics.

Report No. 127, entitled "Aircraft Speed Instruments," is presented in three parts. Part I, "Air Speed Indicators," Part II, "Testing of Air Speed Meters," and Part III, "Principles of Ground Speed Measurement."

Part I contains a discussion and description of the various types of air speed measuring instruments. The authors then give general specifications and performance requirements with the results of tests on air speed indicators at the Bureau of Standards.

Part II reports methods and laboratory apparatus used at the Bureau of Standards to make static tests. Methods are also given of combining wind tunnel tests with static tests. Consideration is also given to free flight tests.

Part III discusses the problem of finding suitable methods for the purpose of measuring the speed of aircraft relative to the ground. As yet no entirely satisfactory solution has been found.

Report No. 128, entitled "Direction Instruments." This report is divided into five parts, as follows:

Part I, "Inclinometers and Banking Instruments" points out the adequacy of a consideration of the steady state of gyroscopic motion as a basis for the discussion of displacements of a gyroscope mounted on an airplane, and develops a simple theory on this basis. Principal types of gyroscopic inclinometers are described and requirements stated.

Part II, "A New Type of Gyro Stabilizer." This paper describes a new type of stabilizing gyro mounted on top of a spindle by means of a universal joint, the spindle being kept in a vertical position by supporting it as a pendulum of which the bob is the driving motor. The paper also describes a new type of stabilized bomb sight in which only the cross wires of the sighting telescope are controlled by the gyro.

Part III, "The Testing and Use of Magnetic Compasses for Airplanes." Methods of tests and the difficulties in designing a satisfactory and reliable compass for aircraft use is considered in this paper.

Part IV, "Aircraft Compasses—Description and Classification." This paper contains a brief general treatment of the important features of construction of aircraft compasses and description of the principal types used. Mention is also made of several compasses now in the process of development.

Part V, "Turn Indicators." This report gives a brief history of the development of airplane turn indicators with detailed descriptions of all known types and makes. The results of laboratory and flight tests are given for the several available gyroscopic turn indicators.

Report No. 129, entitled "Power Plant Instruments." The report on power plant instruments is divided into five parts.

Part I, "Airplane Tachometers," gives a general discussion of the uses, principles, construction, and operation of airplane tachometers. Detailed description of all available instruments, both foreign and domestic, are given.

Part II, "Testing of Airplane Tachometers," a paper which describes methods of tests and the effect of various conditions encountered in airplane flight such as change of temperature, vibration, tilting, and reduced air pressure.

Part III, "Thermometers for Aircraft Engines." This paper describes the principal types of distance reading thermometers for aircraft engines, including an explanation of the physical principles involved in the functioning of the instruments and proper filling of the bulbs. Performance requirements and testing methods are given and a discussion of the source of error and results of tests.

Part IV, "Air Pressure and Oil Pressure Gauges." This paper gives methods of tests and calibration, also requirements of gauges of this type for the pressure measurement of the air pressure in gasoline tanks and the engine oil pressure on airplanes.

Part V, "Gasoline Depth Gauges and Flowmeters for Aircraft." The authors have described two types of gasoline gauges, the float type and the pressure type. Methods of testing and calibrating gasoline depth gauges are given. The Schroeder, R. A. E., and the Mark II Flowmeter are described.

Report No. 130, entitled "Oxygen Instruments." This report contains statements as to amount of oxygen required at different altitudes and the methods of storing oxygen. The two types of control apparatus—the compressed oxygen type and the liquid oxygen type—are described. Ten different instruments of the compressed type are described, as well as the foreign instruments of the liquid types. The performance and specifications and the results of laboratory tests on all representative types conclude this report.

Report No. 131, entitled "Aerial Navigation and Navigation Instruments." This paper outlines briefly the methods of aerial navigation which have been developed during the past few years, with a description of the different instruments used. Dead reckoning, the most universal method of aerial navigation, is first discussed. Then follows an outline of the principles of navigation by astronomical observation; a discussion of the practical use of natural horizons, such as sea, land, and cloud, in making extant observations; the use of artificial horizons, including the bubble, pendulum, and gyroscopic types. A description is given of the recent development of the radio direction finder and its application to navigation.

Report No. 132, entitled "Recent Developments and Outstanding Problems," by Franklin L. Hunt, Bureau of Standards. This report is Section VIII of a series of reports on aeronautic instruments (Technical Reports Nos. 125 to 132, inclusive) prepared by the Aeronautic Instruments Section of the Bureau of Standards under research authorizations formulated and recommended by the Subcommittee on Aerodynamics and approved by the National Advisory Committee for Aeronautics.

The preceding reports in this series have discussed in detail the various types of aeronautic instruments which have reached a state of practical development such that they have already found extensive use. It is the purpose of this paper to discuss briefly some of the more recent developments in the field of aeronautic instrument design and to suggest some of the outstanding problems awaiting solution. The different types of instruments will be considered as far as possible in the order in which they are discussed in the preceding papers.

LIST OF TECHNICAL NOTES ISSUED BY NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS DURING THE PAST YEAR.

No. 22. The Photographic Recording of Small Motions. By F. H. Norton, N. A. C. A.

23. Horizontal Buoyancy in Wind Tunnels. By A. F. Zahm, Bureau C. & R., Navy Department.

24. Development of the Inflow Theory of the Propeller. By A. Betz. Résumé translated from the German by Paris Office, N. A. C. A.

26. A Variable Speed Fan Dynamometer. By Karl D. Wood, Bureau of Standards.

27. Instrument for Measuring Engine Clearance Volumes. By S. W. Sparrow, Bureau of Standards.

28. Loads and Calculations of Army Airplanes. By Stelmachowski. Translated from Technische Berichte, Vol. III, No. 6, by Office of Naval Intelligence, U. S. Navy Department.

29. Progress made in the Construction of Giant Airplanes in Germany during the War. By A. Baumann. Résumé translated from the German by Paris Office, N. A. C. A.

30. Design of Recording Wind Tunnel Balances. By F. H. Norton, N. A. C. A.

31. Crippling Strength on Axially Loaded Rods. By Fr. Natalis. Translated from Technische Berichte, Vol. III, No. 6, by Prof. Pawlowski, University of Michigan.

32. Causes of Cracking of Ignition Cable. By F. B. Silsbee, Bureau of Standards.

33. The Effect of the Nature of Surfaces on Resistance, as Tested in Struts. By C. Wieselsberger. Translated from Zeitschrift fur Flugtechnik und Motorluftschiffahrt, Feb. 28, 1920, by Paris Office, N. A. C. A.

34. The 300 H. P. Aviation Engine. By Dr. A. Heller. Translated from Zeitschrift des Vereines Deutscher Ingenieure, by Paris Office, N. A. C. A.

35. The Optical Wing Aligning Device of the Langley Field Tunnel. By F. H. Norton, N. A. C. A.

No. 36. N. A. C. A. Langley Field Wind Tunnel Apparatus. The Tilting Manometer. By F. H. Norton, N. A. C. A.

37. The Determination of the Effective Resistance of a Spindle Supporting a Model Aerofoil. By W. E. Davidson and D. L. Bacon, N. A. C. A.

38. Measurements of Rudder Moments on an Airplane During Flight. By V. Heidelberg. Translated from Zeitschrift fur Flugtechnik und Motorluftschiffahrt, Vols. 21 and 22, by Paris Office, N. A. C. A.

39. High Thermal Efficiency in Airplane Service. By S. W. Sparrow, Bureau of Standards.

40. Effect of the Reversal of Air Flow upon the Discharge Coefficient of Durley Orifices. By Marsden Ware, N. A. C. A.

41. Influence of Span and Load per Square Meter on the Air Forces of the Supporting Surfaces. By A. Betz. Translated from Technische Berichte, Vol. 1, Sec. 4, by Lt. W. S. Diehl, Bureau C. & R., Navy Department.

42. The Determination of Downwash. By Lt. Walter S. Diehl, Bureau C. & R., Navy Department.

43. Note on the Resistance of Polished Cylinders (and cylindrical Wires) with Generatrices Perpendicular to the Airstream, by A. Toussaint. Translated from the French by Paris Office, N. A. C. A.

44. On the Resistance of Spheres and Ellipsoids in Wind Tunnels. By D. P. Riabouchinsky. Translated from Bulletin of the Aerodynamic Institute of Koutchino, by Paris Office, N. A. C. A.

45. Extract from a Report on the Resistance of Spheres of Small Diameter in an Airstream of High Velocity. By Capt. Toussaint and Lt. Hayer. Translated from the French by Paris Office, N. A. C. A.

46. Theory of the Ideal Windmill. By Wilhelm Hoff. Translated from the German by Paris Office, N. A. C. A.

47. Recent European Developments in Helicopters. Prepared by Paris Office, N. A. C. A.

48. Airplane Superchargers. By W. G. Noack. Translated from Zeitschrift des Vereines Deutscher Ingenieure, 1919, by Office of Naval Intelligence, Navy Department.

49. On the Resistance of the Air at High Speeds and on the Automatic Rotation of Projectiles. By D. Riabouchinsky. Translated from the French by Paris Office, N. A. C. A.

50. The Gordon Bennett Airplane Cup, 1920. By W. Margoulis, N. A. C. A.

51. Airplane Balance. By L. Huguet. Translated from "La Vie Technique and Industrielle," 1920, by Paris Office, N. A. C. A.

52. A New Method of Testing Models in Wind Tunnels. By W. Margoulis, N. A. C. A.

53. Similitude Tests on Wing Sections. By H. Kumbruch. Translated from the German by D. L. Bacon, N. A. C. A.

54. The Factors that Determine the Minimum Speed of an Airplane. By F. H. Norton, N. A. C. A.

55. Airplane Crashes; Engine Trouble. A Possible Explanation. By S. W. Sparrow, Bureau of Standards.

56. The Development of German Army Airplanes During the War. By Wilhelm Hoff. Translated from Zeitschrift des Vereines Deutscher Ingenieure, 1920. By office of Naval Intelligence, Navy Department.

RESEARCH PROGRAM AND ESTIMATES.

For the year 1923 the National Advisory Committee for Aeronautics has planned a program of research which the committee considers vital to progress in aviation in the United States and is still in keeping with the economieal policies now affecting the expenditure of governmental funds. The program has been recommended by the subcommittees on aerodynamics, power plants for aircraft, and materials for aircraft, and has been approved by the executive committee.

Aerodynamical research.—The use of the new compressed-air wind tunnel will make possible the determination of forces on an airplane and the determination of the interference effects of the structural members. The information obtained can be used directly by the designer, and the determination of the interference effects may have an important effect on the character of design.

The free-flight testing program provides for the determination of pressure distribution and stresses in airplanes at high speed and when violently maneuvered. For this particular research it may be necessary to construct a special airplane having double the usual factor of safety.

Statical and dynamical stability are to be investigated for various modifications of standard types of airplanes, with a view toward finding the relation between these characteristics and the variable factors in a given design. This investigation will attempt to find the relation between model tests and full-scale performance in such a manner that the designer will be enabled to interpret model-test results with confidence.

The full-scale investigation is concerned also with controllability. The effectiveness of control surfaces with systematic modification is to be given special study for the purpose of finding not only the effects of certain changes in design but also the amount of control required in any given case.

All of the full-scale work is planned with the view of making the results general and broad in their scope. This will be secured by repeating tests on different types of airplanes with automatic recording instruments.

The tests of aerofoils suspended from a balance placed in an airplane and tested in free flight shows promise. So far a model wing section with a 1-foot chord and a length of 6 feet has given such results that the work is being continued, and it is hoped to test a full-sized airplane wing suspended from a Martin bomber.

Preliminary tests have been made, and a satisfactory pressure pad has been developed for use in determining pressure distribution on an airship in flight. A special recording multiple manometer for these tests is now completed, and the results obtained will be of great value to the designer.

The research program in aerofoils provides for the continued study of designs and methods of construction that will make it possible to increase the lift characteristics of aerofoils. This problem is of great importance in providing for a lower landing speed and a quicker take-off. From a commercial and also a military point of view, the high lift wing will greatly increase the useful load carried or the greater flying range of the aircraft.

The committee asks for the sum of $141,134 to carry out research work in connection with aerodynamics.

Materials research.—The progress made during the past year in the development of all-metal aircraft has not been altogether satisfactory. The difficulty seems to be that as yet we have not developed suitable and economical methods of producing and fabricating aluminum alloys. The methods so far developed have been limited to the construction of the *ZR–I* and the Stout all-metal airplane. The Bureau of Standards is equipped and expects to conduct a series of investigations on the rolling and shaping of duralumin for aircraft construction. The heat treatment of the material will also be carefully studied in connection with the results of cold working.

The fatigue tests are to be continued to obtain reliable data for the designer, and vibration tests are to be made on various types of airplanes to determine some idea of the character of vibrations to be expected.

One of the most serious drawbacks of the present type of airplane is the cost of manufacture and the short life of the structure. Even in quantity production the cost is excessive, but if the materials used in the construction consisted entirely of metal, the parts would lend themselves better to quantity production, longer life would be assured, and storage conditions would be improved. Research on both steel and light alloys for aircraft will be carried on and an effort made to further interest manufacturers in the production of materials especially suitable for aircraft purposes.

Woods, glues, and protective coatings are not entirely satisfactory, and the cost of these materials for aircraft construction is excessively high. Research will be continued to produce a substitute for goldbeaters' skin, the present airplane dope, and new methods of wing rib and spar construction, especially in connection with the further development of plywood structures.

Aircraft power plants research.—The outstanding problem considered in connection with power plants for aircraft is the developing of a reliable engine that will use a low-grade fuel or a fuel with a much lower flash point than our present gasoline. With an engine of this type the fire hazard in aircraft would be greatly minimized and the reliability greatly increased by the elimination of the ignition system, the carburetor, and other accessories that are now responsible in a large part for engine failures. The progress made so far on this investigation is most encouraging and it is expected that within the year a successful experimental engine of this type will be produced. The engine will weigh more than the present type of aviation engine, but will lend itself to cheaper construction as the pressures and forces to be considered are about the same as now exist in aircraft engines.

The various types of superchargers will be investigated; five different types are to be tested. The use of the new altitude chamber at the Bureau of Standards will permit of the accurate determination of the characteristics of supercharged engines. Provision has been made so as to make it possible to test the exhaust turbine type supercharger in the altitude chamber and simulate conditions experienced at high altitude. The Moss type supercharger developed at McCook Field has been most successful, but it is hoped that a type considerably lighter and less complicated will result from the investigations during the coming year.

The radiator investigation will be continued especially with a view of determining from free flight and laboratory tests a type of radiator design giving the least air resistance and also providing for ease of installation and repair.

Research is planned for the further study of air-cooled cylinders, as it is realized that for certain types of aircraft the air-cooled engine in units up to 250 horsepower has a decided advantage.

The most severe limitation now placed on the further development of high-powered light-weight engine is due to preignition or erratic burning occurring when our present grades of gasoline are used. A study of the fuel problem will be carried on in connection with the study of the phenomena of combustion in an effort to control, if possible, the rate of combustion, which has been, so far, beyond control.

The estimate of the committee to cover the necessary power plant research for the fiscal year 1923 is $123,866.

Summary.—The committee's estimates for the prosecution of the programs of aerodynamical research, materials research, and aeronautic power plant research, as outlined above, total $265,000. To this should be added, under the committee on publications and intelligence, the work of the Office of Aeronautical Intelligence, in the collection, classification, and dissemination of scientific and technical reports and data on aeronautics, requiring the sum of $48,300, and for the general administration of the Washington office with its present personnel, the sum of $27,700, making the total estimates for the fiscal year 1923, $341,000. The appropriation for the fiscal year 1921 was $200,000, and for the present fiscal year the appropriation is $200,000. The continuous prosecution of a well-organized plan of scientific research is an essential factor in the development of the science of aeronautics, and the increased estimates of the committee for the fiscal year 1923 are made necessary by the increasing relative importance of scientific research in the general development of aeronautics.

FINANCIAL REPORT.

The appropriation for the National Advisory Committee for Aeronautics for the fiscal year 1921, as carried in the sundry civil appropriation act approved June 5, 1920, was $200,000 under which the committee reports expenditures and obligations during the year amounting to $199,959.21, itemized as follows:

Salaries (including engineering staff)	$66,881.22
Wages	33,955.04
Equipment	9,691.73
Supplies	27,783.99
Transportation and communication	912.58
Travel	6,081.65
Special investigations and reports	46,725.00
Construction of buildings	7,928.00
Total	199,959.21

CONCLUSION.

As has been stated before, the primary function of the committee is to pursue scientific investigations in aeronautics. This is necessarily the most important field in the wnole subject of aviation, if fundamental progress is to be made. When the results of physical and mathematical study are known, and not until then, better airplanes or other aircraft can be designed. Without these scientific results progress will be empirical and uncertain. No other activity in aviation at the present time in this country so much needs intelligent governmental support.

Respectfully submitted.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
JOSEPH S. AMES, *Chairman Executive Committee.*

REPORT No. 111

THE VARIATION OF AEROFOIL LIFT AND DRAG COEFFICIENTS WITH CHANGES IN SIZE AND SPEED

By WALTER S. DIEHL

Bureau of Construction and Repair, U. S. N.

REPORT No. 111.

THE VARIATION OF AEROFOIL LIFT AND DRAG COEFFICIENTS WITH CHANGES IN SIZE AND SPEED.

By Walter S. Diehl.

This report, prepared by Walter S. Diehl at the request of the National Advisory Committee for Aeronautics, contains the results of an investigation of existing scale correction data and the derivation of an original method for making these corrections rapidly and accurately. The following summary outlines briefly the subject as treated in the report.

SUMMARY.

1. General statement of the principle of dynamic similarity as applied to the problem of determining the variation of the lift and drag of an aerofoil with (variations in) the size and speed.

2. Interchangeability of v and l. Notes on limitations as found by experiment.

3. Review of existing scale correction data. Criticism and comments on the method employed by the N. P. L.

4. Determination of the variation of D_c with scale. It is shown that the minimum drag varies as $(vl)^{1.84}$ in a number of tests and must therefore be due almost entirely to a viscosity effect. Assuming that any increase in the drag coefficient, over the minimum, is due to inertia effects, the relation between the drag coefficients at any angle for the values of vl is

$$D_{c2}=D_{c1}-D_{c0}\ [1-\left(\frac{v_2l_2}{v_1l_1}\right)^{-0.16}]$$

where D_{c2} = drag coefficient at v_2l_2
D_{c1} = drag coefficient at v_1l_1
and D_{c0} = minimum drag coefficient at v_1l_1

The formula is checked by test results.

5. Variation of L_c with scale. It is found that at any given angle of attack the lift coefficients for the two values of vl bear the relation

$$L_{c2}=L_{c1}+\triangle L_c$$
$$=L_{c1}+.057\ \log_{10}\left(\frac{v_2l_2}{v_1l_1}\right)$$

where L_{c2} = lift coefficient at v_2l_2
and L_{c1} = lift coefficient at v_1l_1

The value of the constant 0.057 was determined from existing experimental data.

6. Applications and limitations. Method of applying formula. Discussion of limitations.

7. Conclusions.—It is recommended that the formulæ be checked by accurate tests made for this purpose and extending over a large range of vl.

INTRODUCTION.

A general expression for the resistance or reaction due to relative motion between a fluid and an immersed body may be written by application of the principle of dimensional homo-

geneity. If the motion be uniform and the fluid incompressible it is found that the reaction is given by

$$R = \rho v^2 l^2 f\left(\frac{vl}{\nu}\right) \tag{1}$$

where

ρ = density of the fluid,
v = relative velocity,
l = some linear dimension of the body,
μ = viscosity of the fluid,

and

$\nu = \frac{\mu}{\rho}$, the kinematic viscosity.

The derivation of the expression is due to Lord Rayleigh[1] and may be found in any treatise on aerodynamics. (See Bairstow, "Applied Aerodynamics," Ch. VIII.)

In most aeronautical engineering computations it is customary to neglect the variations of ν and to consider only variations of v and l. This is justified since model tests and flying are usually carried out under conditions which render ν substantially constant. The product of the velocity, v, in feet per second, and the chord, l, of an aerofoil in feet is then referred to as the "vl" or "scale" of the test or flight.

This investigation is concerned with the determination of the functions of vl which express the variation of lift and drag coefficients of an aerofoil, with particular reference to the application of model tests to full-size airplane performance.

Before discussing the previous work in this field it seems desirable to call attention to certain phenomena connected with the limitations of the interchangeability of v and l.

INTERCHANGEABILITY OF V AND L.

It should be noted that the condition of dynamic similarity, which may be expressed $v_1 l_1 = v_2 l_2$, presupposes geometrical similarity. This is equivalent to saying that geometrically similar aerofoils will give identical characteristic curves when tested at speeds inversely proportional to their chords.

This interchangeability of v and l and the dependence of aerofoil coefficients upon their product has been accepted for many years as being necessary from a physical standpoint. The validity of the assumption has sometimes been challenged but never disproved. On the other hand the results of various tests such as those made at N. P. L. on two geometrically similar aerofoils (Br. A. C. A., R. and M. No. 148) and at Göttingen on several series of geometrically similar aerofoils. (Kumbruch——Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 31, 1919) are to be taken as positive proof.

However it is well known to everyone who has had occasion to study the results of many aerofoil tests that there are certain limits within which it is necessary to keep both v and l, if the data are to be reliable. For instance if the velocity of the wind during a test be less than 30 f. p. s., or if the chord of the model be less than 3 inches, the flow is determined not only by the aerofoil section but also by the method of supporting the model and the quality of the air flow, or turbulence present in air stream. The upper limit to velocity depends chiefly upon compressibility and may arbitrarily be set at 200 f. p. s., at which speed the effect is of the order of 1 per cent.

It may therefore be said with some confidence that the laws connected with dynamic similarity apply to aerofoils subject to the limitations just mentioned.

N. P. L. SCALE CORRECTIONS.

By far the greatest amount of testing for scale effect has been done at the National Physical Laboratory. In a series of three reports of the British Advisory Committee for Aeronautics (R. and M. Nos. 72, 110, and 148) monoplane aerofoil characteristics for the R. A. F.–6 section

[1] Br. A. C. A., R. and M. No. 15.

and modifications are given for values of vl from 2.5 to 40. In another report (Br. A. C. A., R. and M. No. 196) biplane R. A. F.–6 characteristics are given for values of vl from 5 to 16.5. In addition to these systematic tests a large number of aerofoils have been tested at two or more speeds.

Although the data on scale effect thus accumulated are comparatively extensive, it appears that but little attention has been given to the actual determination of the laws involved. It has been customary since R. and M. No. 72 was published to plot scale tests with the aerofoil characteristics as ordinates and vl (or log vl) as abscissae, drawing a line for each angle of attack through the values of L_c, D_c, or L/D at the corresponding values of vl. This method will give satisfactory results only so long as the curves are used on similar aerofoils. On account of the great gap between the highest vl obtainable in the present wind tunnel tests and the vl of the average machine in flight, it leads to a conclusion which is in error. Figures 1, 2, and 3 are taken from a report by Mr. E. F. Relf, "An Empirical Method for the Prediction of Wing Characteristics from Model Tests, Compiled from Existing Experimental Data" (Br. A. C. A., R. and M. No. 450, June, 1918), and presumably represent the latest N. P. L. scale correction data. In the summary to this report the following conclusion is given: "With regard to the $\frac{vl}{v}$ correction it appears...that the model results can be directly applied without any great error, if the wing vl of the test is greater than 25 in ft.^{2}/sec." It had been stated in a previous report (Br. A. C. A., R. and M. No. 72, Sec. VI) that there was no scale effect above a vl of 40. Referring to figures 1, 2, and 3 it appears on first sight that there is ample basis for this conclusion. However, if the curves from figures 1 and 3 be replotted on a logarithmic scale as in figures 4 and 5, it immediately becomes evident that the effect of scale is operating according to the same law at $vl=40$ as at lower values of vl. It is in order to mention that this effect has been noted in a more recent report (Br. A. C. A., R. and M. No. 656, November, 1919), which states that "The drag coefficient of the model is still changing with speed at the highest speed of the experiments. The same is true of the lift coefficient to a very small extent."

The greatest difficulty experienced in applying the correction curves of figures 1, 2, and 3 occurs with high lift aerofoils, such as the RAF–19. In general it is found that the method is rather unsatisfactory on account of its limitations.

VARIATION OF D_c WITH SCALE.

An inspection of figure 5 will reveal two outstanding features. First, that the minimum drag coefficient, or rather the drag coefficient at an angle attack corresponding very nearly to the minimum drag, decreases as vl is increased in such a manner that all of the values lie on a straight line which has the slope -0.14. This indicates that the minimum drag varies as $(vl)^{1.86}$ instead of $(vl)^2$, and is consequently due almost entirely to skin friction, or more properly, perhaps, to a "viscosity effect." Second, that the higher values of D_c do not follow the same law, since the successive values of D_c, as vl is increased, lie on lines which are concave upward. Before drawing any conclusions from these observations it is desirable to examine a number of tests to see if the phenomena are universal.

Upon plotting to a logarithmic scale, as in figure 5, the drag coefficients from available tests, there is obtained in every case a group of lines very similar to those in figure 5. It is to be noted that the slope of the line representing minimum drag is slightly greater in the groups obtained from test data. This slope is quite uniform and of the average value -0.16. A specimen group is given in figure 6 to illustrate the general nature of all.

It can therefore be shown that the minimum drag of an aerofoil is almost entirely due to a viscosity effect, which is to say that

$$\text{minimum drag} \propto (vl)^{1.84}$$

$$\text{or minimum } D_c \propto (vl)^{-0.16} \qquad (2)$$

Now so long as the flow over the aerofoil is nonturbulent, the magnitude of the viscosity effect

can not change by any great amount. But it has been shown by Betz (Technische Berichte I–4, for translation see N. A. C. A., T. N. No. 41) that

$$D_{ci} = \frac{2L_c^2}{\pi}\left(\frac{S}{b^2}\right) \qquad (3)$$

where S = the area of the aerofoil,

b = the span,

and D_{ci} = the coefficient of the "induced drag." That is D_{ci} is a measure of the inertia reaction, in the direction of flight, experienced by aerofoil in imparting to the encountered air the downward deflection which produces the lift represented by L_c. Since D_{ci} is an inertia effect it must vary as $(vl)^2$. Consequently the total drag, which is assumed to vary as $(vl)^2$, has two components, the one varying as $(vl)^{1.84}$, the other as $(vl)^2$. The scale correction to the coefficient of total drag, D_c, must therefore be concerned only with that part of the drag which varies as $(vl)^{1.84}$, and since this part is due to an effect which renders it practically constant over the range of angles corresponding to steady flow it follows that the effect of a change in vl is to add to or subtract from each value of D_c a constant amount. This may be expressed in symbols as:

$$\text{at } v_1l_1,\ D_{c1} = D_{ci} + D_{cv} \qquad (4)$$

$$\text{at } v_2l_2,\ D_{c2} = D_{ci} + D_{cv} \pm \Delta D_{cv} \qquad (5)$$

or

$$D_{c2} - D_{c1} = \pm \Delta D_{cv} \qquad (6)$$

where D_{ci} = that part of D_c due to inertia effects and varying as $(vl)^2$.

D_{cv} = that part of D_c due to viscosity effects and varying as $(vl)^{1.84}$.

ΔD_{cv} = the correction to D_{cv} necessary to allow for the fact that D_{cv} varies as $(vl)^{1.84}$ instead of $(vl)^2$.

In order to obtain a definite check upon the above conclusions it is necessary to compare at each angle the drag coefficients obtained from tests on an aerofoil at two values of vl. There should be a difference between the two coefficients at each angle (within the limits previously stated), of ΔD_{cv}, which is given by

$$\Delta D_{cv} = D_{cv} - D_{cv}\left[\frac{(v_2l_2)^{1.84}}{(v_1l_1)^2}\right]$$

$$= D_{cv}\left[1 - \left(\frac{v_2l_2}{v_1l_1}\right)^{-0.16}\right] \qquad (7)$$

Since D_{cv} is substantially equal to the minimum value of D_c, which may be denoted by D_{co}, the above expression may be written

$$\Delta D_c = D_{co}\left[1 - \left(\frac{v_2l_2}{v_1l_1}\right)^{-0.16}\right] \qquad (8)$$

A number of tests have been compared on this basis in the manner illustrated by Table I, the results being tabulated in Table II. It is found that the values of $\triangle D_c$ are not only very nearly constant but that they check very closely with values given by equation 8. It is particularly to be noted that the aerofoil sections listed in Table II include every type from the double cambered RAF–20 to the deeply cambered RAF–19. The RAF–20 has a very low D_{co} and the RAF–19 a very high D_{co}, yet the calculated and observed values of $\triangle D_c$ agree very well in each case. This agreement is to be interpreted as a strong confirmation of the formula, which appears to be a very satisfactory approximation applying equally well to all aerofoils.

For convenience in making corrections and comparisons the expression $\left(\frac{v_2l_2}{v_1l_1}\right)^{-0.16}$ has been plotted in Fig. 8.

VARIATION OF L_c WITH SCALE.

There is very little to be learned from an inspection of figure 4 in regard to the variation of L_c with vl. Experimental data when plotted on logarithmic scales agree very well with

figure 4 but are disappointing on account of the low ranges of vl. A typical plot is given in figure 7 to illustrate the general appearance of test data.

If a careful study be made of the various tests it will be observed that the effect of increasing vl is to increase by a small amount each lift coefficient within the range of angles corresponding to nonturbulent flow. It will also be observed that the average increase in L_c is the same absolute quantity when vl is increased, for example, from 5 to 10 as from 10 to 20. That is to say, the average value of $\triangle L_c$ is proportional to the ratio $\left(\frac{v_2 l_2}{v_1 l_1}\right)$ and increases arithmetically as vl increases geometrically. $\triangle L_c$ should therefore be given by an expression of the form

$$\triangle L_c = K. \log \left(\frac{v_2 l_2}{v_1 l_1}\right) \tag{9}$$

where K is a constant to be determined from test data.

Table III contains data from a series of tests on two RAF–6a aerofoils with the corresponding values of $\triangle L_c$. The same method was employed on other tests to obtain the values of $\triangle L_c$ given in Table IV. The results are surprisingly consistent when consideration is given to the fact that $\triangle L_c$ is obtained as the differences between two nearly equal values of L_c, each subject under the best of conditions to an error of 2 per cent or more. The average of a number of readings should eliminate such errors, however.

Table IV contains all data used in the determination of K, which is found to be

$$K = .057$$

The value of the lift coefficient L_{c2} at a given angle of attack and $v_2 l_2$ is therefore given by

$$L_{c2} = L_{c1} + .057 \log_{10} \left(\frac{v_2 l_2}{v_1 l_1}\right) \tag{10}$$

where L_{c1} is the lift coefficient at the same angle and $v_1 l_1$.

It is to be noted that the value of K seems independent, not only of the type of aerofoil section but also of the arrangement, i. e., monoplane or biplane.

The method has been applied to tests on a complete model airplane (Br. A. C. A., R. and M. No. 656) with satisfactory results. It is unfortunate that free flight test data available for comparison are too erratic to be used, except at large angles where it checks very well with that calculated by 10.

APPLICATIONS AND LIMITATIONS.

In applying these corrections it is necessary to employ data obtained at a vl sufficiently high to eliminate all uncertainty in regard to steadiness of flow. There are so many factors which influence steadiness of flow that it is difficult to specify a lower limit to vl although in general it may be said that the results obtained from tests on a model of 3″ chord at 40 f. p. s. are reliable, but neither velocity nor chord should ever be less than these figures.

The application of the corrections should also be limited to the range of angles corresponding to steady flow roughtly from zero lift to maximum lift.

In regard to limitations, there have been tests on certain double cambered aerofoils in which the lift coefficient was found to decrease as vl was increased. Data are lacking to indicate the cause of this reversal, but since other double cambered aerofoils behave in the usual manner it is possible that the phenomena may be due to some special condition or type of flow.

In a few cases, the lift curves for tests at two or more values of vl on the same aerofoil coincide over a range of several degrees in the angle of attack. Special tests are required to indicate whether or not an individual correction of the same general form as equation 10 should be applied at each angle of attack in such cases.

CONCLUSIONS.

It has been shown that the value of the drag coefficient varies with vl according to the expression

$$D_{c2}=D_{c1}-D_{c0}\left[1-\left(\frac{v_2l_2}{v_1l_1}\right)^{-0.16}\right] \tag{11}$$

where D_{c1} is the drag coefficient at a given angle of attack and v_1l_1.

D_{c0} is the minimum drag coefficient at v_1l_1

and D_{c2} is the drag coefficient at the same angle of attack as D_{c1} but at v_2l_2.

It has also been shown that the lift coefficient at v_2l_2 is given by

$$L_{c2}=L_{c1}+.057\log_{10}\left(\frac{v_2l_2}{v_1l_1}\right) \tag{10}$$

where L_{c1} is the lift coefficient at v_1l_1 for the particular angle of attack under consideration.

The most obvious criticism of these formulæ is that they are based on low values of vl. The only tests at high values of vl, available for inclusion in this study were those made at Göttingen and reported by Kumbruch in the Zeitschrift für Flugtechnik und Motorluftschiffahrt, of May 31, 1919. Unfortunately the forces involved in the Göttingen tests were so large that the models deflected until the angles of attack were uncertain. The models were also of aspect ratio 2.5 and end plates were used to eliminate the tip losses. Although corrections were made for aspect ratio and the interference between the model and the walls of the tunnel it is felt that the effect of scale is not given by the final results.

It is recommended that the formulæ 10 and 11 be checked by tests extending over a large range of vl. Such tests should be made with more than usual care in measuring the angle of attack and wind velocity. The results so obtained should be checked with reliable free flight performance data.

TABLE I.—*Determination of D_c, monoplane RAF–6c.*

[Data from Br. A. C. A., R. and M. No. 110.]

α	D_c $v_1l_1=7.5$.	D_c $v_2l_2=12.5$.	$-\triangle D_c$.	
−6	0.0377	0.0370	0.0007	$\left(\frac{v_2l_2}{v_1l_1}\right)^{-0.16}=(1.67)^{-0.16}=0.923$.
−4	.0255	.0250	.0005	
−2	.0183	.0170	.0013	
0	.0152	.0138	.0014	
2	.0152	.0135	.0017	Minimum $D_{co}=.0152$.
4	.0190	.0173	.0017	
6	.0244	.0235	.0009	
8	.0331	.0313	.0018	$\triangle D=D_{co}[1-0.923]$.
10	.0418	.0398	.0020	
12	.0514	.0510	.0004	$.0152\times0.077$.
Average			.00124	$=.00117$.

TABLE II.—*Comparison of D_c, calculated and observed.*

(See Table I.)

v_1l_1	v_2l_2	$\frac{v_2l_2}{v_1l_1}$.	Min. D_c.	$\triangle D_c$ calculated.	Average $\triangle D_c$ observed.	Section.	Arrangement.	Reference.
7.5	12.5	1.67	0.0152	0.0012	0.0012	RAF–6c	Monoplane	R. and M. 110.
5	12.5	2.5	.0157	.0020	.0022	do	do	Do.
5	10	2.0	.0158	.0016	.0012	RAF–6	do	Do.
5	12.5	2.5	.0158	.0020	.0023	do	do	Do.
5	20	4.0	.0142	.0027	.0031	RAF–6A	do	R. and M. 148.
15	30	2.0	.0119	.0012	.0013	do	do	Do.
5	16.5	3.3	.0143	.0023	.0020	RAF–6	Biplane	R. and M. 196.
7	12.5	1.67	.0142	.0010	.0013	do	do	Do.
20	40	2.0	.0117	.0012	.0012	Propeller	Monoplane	R. and M. 362.
30	40	1.33	.0101	.0005	.0006	do	do	Do.
5	14.6	2.92	.0410	.0061	.0058	RAF–19	do	R. and M. 415.
5	10	2.0	.0410	.0041	.0047	do	do	Do.
5	10	2.0	.0083	.0008	.0007	RAF–20	do	Do.

TABLE III.—*Test data from R. and M. No. 148, showing method of obtaining* $\triangle L_c$.

α	$vl=5$ L_c	$vl=10$ L_c	$vl=15$ L_c	$vl=20$ L_c	$vl=30$ L_c	vl 30 − vl 5 $\triangle L_c$	vl 30 − vl 10 $\triangle L_c$	vl 30 − vl 15 $\triangle L_c$	vl 30 − vl 20 $\triangle L_c$
−4	−0.066	−0.064	−0.074	−0.066	−0.048	0.018	0.016	0.026	0.018
−2	−.061	+.010	+.022	+.037	+.060	.061	.050	.038	.023
0	+.068	.102	.127	.136	.145	.077	.043	.018	.009
2	.162	.207	.215	.214	.218	.056	.011	.003	.004
4	.269	.284	.283	.286	.293	.024	.009	.010	.007
6	.346	.355	.357	.358	.365	.019	.010	.008	.007
8	.413	.424	.431	.434	.441	.028	.017	.010	.007
10	.472	.489	.500	.505	.513	.041	.024	.013	.008
12	.537	.551	.563	.572	.585	.048	.034	.022	.013
14	.577	.584	.615	.616	.627	.050	.043	.012	.011
Average $\triangle L_c$						.0422	.0257	.0160	.0107

TABLE IV.—*Determination of K in the equation.*

$$\left[L_{c2}=L_{c1}+K.\log_{10}\left(\frac{v_2l_2}{v_1l_1}\right)\right]$$

No.	v_1l_1	v_2l_2	Average $\triangle L_c$	$\frac{(v_2l_2)}{(v_1l_1)}$	$\log_{10}.\left(\frac{v_2l_2}{v_1l_1}\right)$	K.	Section.	Reference.
1	2.5	5	0.0174	2	0.3010	0.0578	Monoplane RAF. 6	R. and M. No. 110.
	5	10	.0170	2	.3010	.0564		
	2.5	12.5	.0394	5	.6990	.0564		
2	5	30	.0422	6	.7782	.0542	Monoplane RAF. 6c	R. and M. No. 148.
	10	30	.0257	3	.4771	.0538		
	15	30	.0160	2	.3010	.0532		
	20	30	.0107	1.5	.1761	.0608		
3	5	10	.0173	2	.3010	.0572	Biplane RAF-6	R. and M. No. 196.
	5	16.5	.0329	3.3	.5185	.0635		
	7.5	12.5	.0138	1.67	.2219	.0622		
	7.5	16.5	.0217	2.2	.3424	.0632		
4	40	35	−.0034	.875	−.0581	.0585	Propeller aerofoil: $x/c=0.5$ $\frac{h}{c}=.075$	R. and M. No. 362.
	40	30	−.0063	.750	−.1239	.0510		
	40	25	−.0116	.625	−.2041	.0568		
	40	20	−.0176	.50	−.3010	.0585		
5	5	10	.0176	2	.3010	.0585	Monoplane RAF-19	R. and M. No. 415.
	5	14.6	.0238	2.92	.4654	.0511		
6	5	14.6	.0267	2.92	.4654	.0574	Monoplane RAF-20	R. and M. No. 415.
Average						.0570		

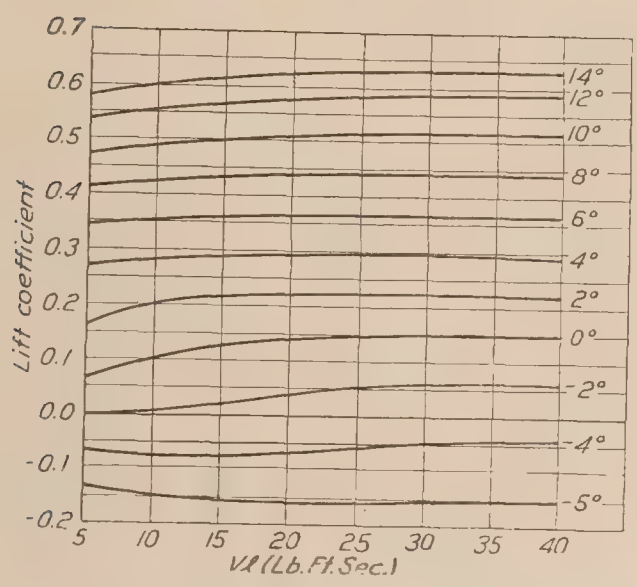

FIG. 1.—Correction to lift coefficient for scale effect. Reproduction of figure 19, B. A. C. A. R. & M. No. 450.

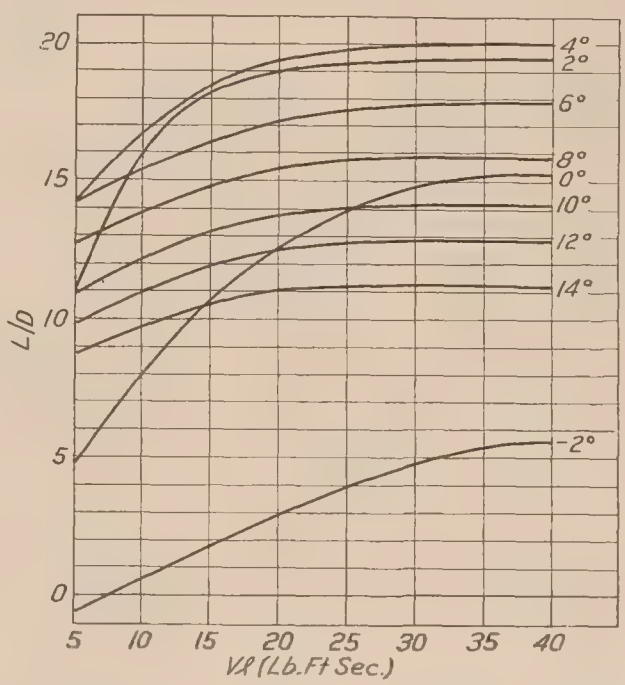

FIG. 2.—Corrections to L/D for scale effects. Reproduction of figure 20, B. A. C. A. R. & M. No. 450.

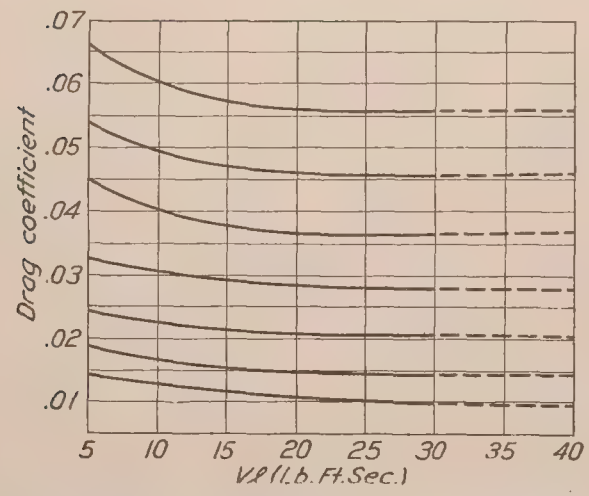

FIG. 3.—Correction to drag coefficient for scale effect Reproduction of figure 21, B. A. C. A. R. & M No. 450.

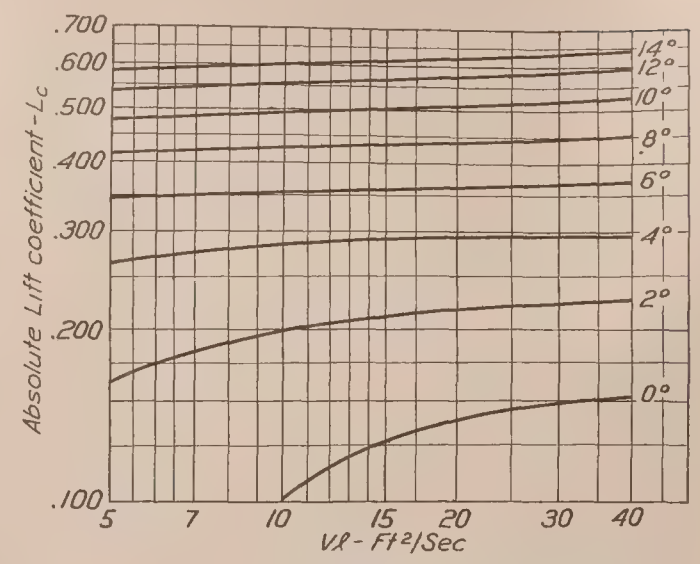

FIG. 4.—Variation of lift coefficient with Vl Re-p ot of figure 19, B. A. C. A. R. & M. No. 450. (See Fig. 1.)

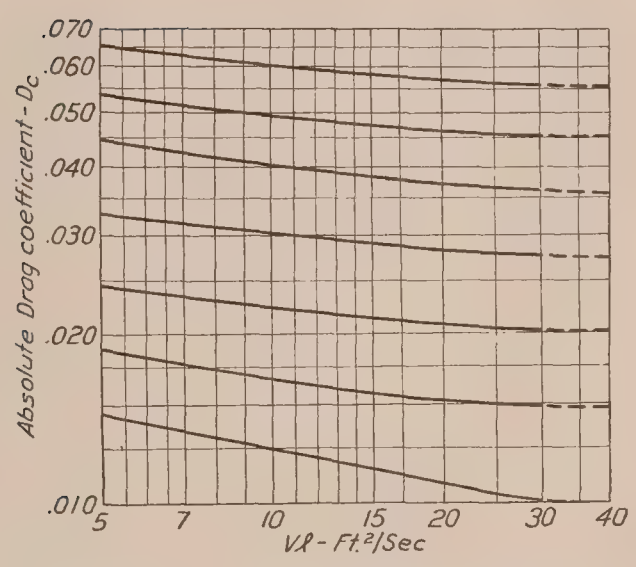

FIG. 5.—Variation of drag coefficient with Vl. Re-plot of figure 21, B. A. C. A. R. & M. No. 450. (See Fig. 3.)

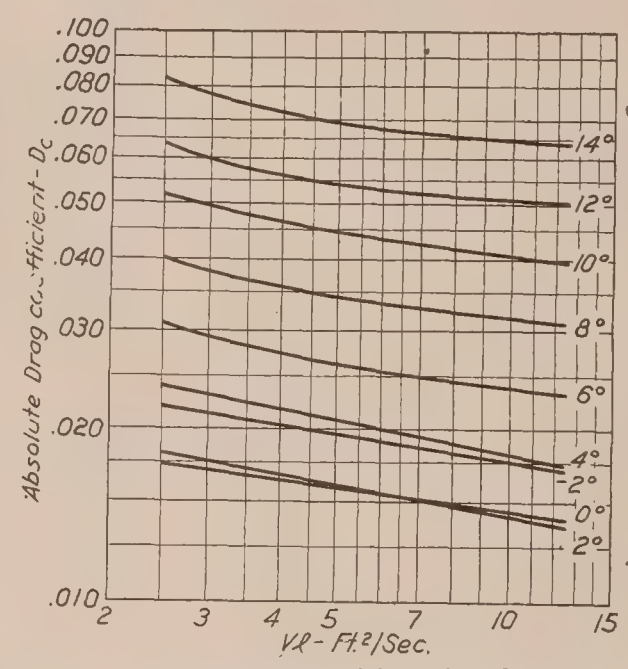

FIG. 6.—Variation of drag coefficient with Vl. Data from B. A. C. A. R. & M. No. 110. R. A. F. 6 Aerofoil.

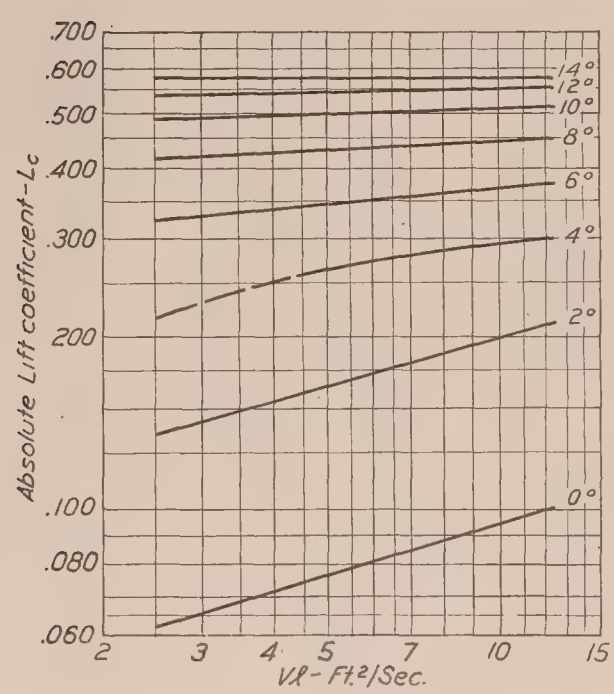

FIG. 7.—Variation of lift coefficient with Vl. Data from B. A. C. A. R. & M. No. 110. R. A. F. 6 Aerofoil.

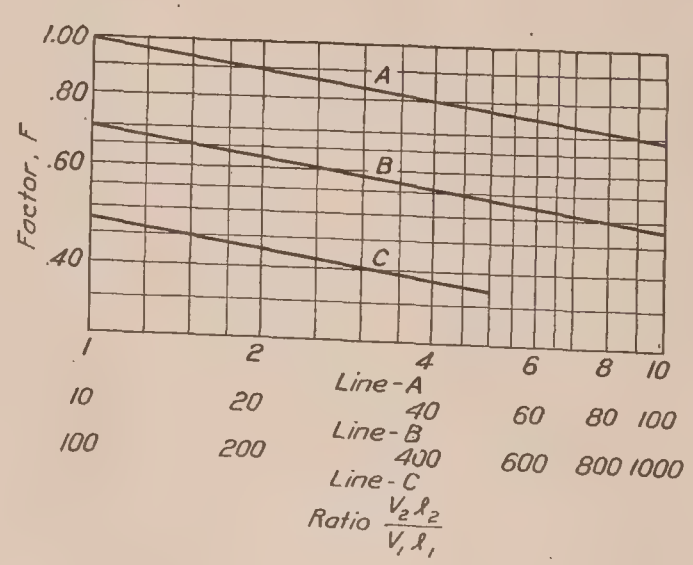

FIG. 8.—Evaluation of $F=\left(\frac{V_2 l_2}{V_1 l_1}\right)^{-0.16}$

REPORT No. 112

CONTROL IN CIRCLING FLIGHT

BY

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Langley Field, Va.

REPORT No. 112.

CONTROL IN CIRCLING FLIGHT.

By F. H. Norton and E. T. Allen.

SUMMARY.

This investigation was undertaken by the National Advisory Committee for Aeronautics at the Langley Memorial Aeronautical Laboratory for the purpose of developing instruments that would record the forces and positions of all three controls, and to obtain data on the behavior of an airplane in turns. All the work was done on a standard rigged JN4H (machine No. 2 of N. A. C. A., Report No. 70). It was found that the machine was longitudinally unstable and nose heavy; that it was laterally unstable, probably due to too little dihedral; and that it was directionally unstable, due to insufficient fin area, this last being very serious, for in case of a loss of rudder control the machine immediately whips into a spin from which there is no way of getting it out. On the other hand, it was found possible to fly quite satisfactorily with the rudder locked, and safely, though not so well, with the ailerons locked. The value of Y_v was obtained in free flight, and when the effect of the propeller was subtracted, the agreement with the model test was excellent, but with the propeller revolving at 1350 the value of Y_v was nearly doubled. The value of L_v and N_v were little affected by the slipstream, but their values do not agree with the model test.

GENERAL STABILITY AND CONTROLLABILITY.

It has been attempted to present this report from the standpoint of the engineer and pilot as well as from that of the physicist, for the more time that is spent on practical stability, the less important seems the theory of small oscillations as compared with the other phases of the problem. When one of the most used and well liked training planes is normally unbalanced and statically unstable with free controls in every particular, it would seem that the dynamical stability is at present of secondary importance. There are, of course, airplanes that are much more stable than the machine used in these tests, but the point that should be emphasized is that, generally speaking, a pilot does not know a stable from an unstable machine, and if the forces on the controls are small he is just as well satisfied with the unstable one as with the other. It seems to be the general impression among nonflyers that piloting an unstable plane is analogous to walking a tight rope, requiring constant vigilance and great dexterity, but the beginner learns to fly as quickly in an unstable machine as in a stable one. It is not intended to give the impression that stability is of no value, for this is certainly not the case; for a machine stable with free controls could be brought down safely if a control wire broke and it would be less tiring to the pilot in long flights.

The subject of stability and control is still in a very confused state for the reason that little has been done to bring together the work of mathematicians and physicists on one hand and engineers and pilots on the other. The former class is apt to theorize on conditions that do not actually exist in flight and the latter have no definite means of expressing control or stability quantitatively. However, the subject of stability and controllability may be logically divided into four parts.

The first is balance, that is, there should be no force on the controls when in the normal flying condition, for without the fulfilment of this requirement stability would have no significance. For example, a machine may be nose heavy and if left to itself dive under onto its back, yet if the pull on the stick were balanced by a spring the machine might be stable. This condition of course applies only to free controls.

The second division is what is commonly called static stability, and is present if forces are produced when the machine is displaced from its equilibrium position, which tend to return it to that position. This applies both with free and locked controls.

The third part is what is called dynamic stability, and can have significance only when the machine is in balance and is statically stable. A machine is said to be dynamically stable when any oscillation set up tends to damp out. This condition is easy to observe on almost any airplane by locking the elevators in the machine's stable region, which can be found by trial. If the throttle is closed for an instant until the nose starts to drop, and then opened to its former setting, the nose will soon rise, overshooting the equilibrium angle, and will continue to rise and fall regularly with a period of from 15 to 30 seconds. If stable, the amplitude of these oscillations will decrease; if unstable, they will increase. The same thing can be observed when making banked turns, but the stability characteristics are different under these conditions. Oscillations in roll and yaw are more difficult to observe, probably because a sufficient degree of static lateral stability has not been obtained. It may be said in general that dynamic stability is of the least importance compared with the other divisions of the subject, for if the machine is not statically stable it has no meaning, and if the machine is made statically stable it is practically always also dynamically stable.

The fourth part is lightness of control, especially in acrobatics; that is, a small force on, and a small movement of the controls should be required to produce large angular accelerations. Lightness of control is intimately connected with static stability, especially as regards longitudinal motions. There has been no satisfactory method of measuring this quality and its degree has been determined heretofore by the statements of pilots. By using three synchronized instruments recording respectively the control forces, the control positions, and the angular attitude of the machine, it should be possible to obtain a definite measure of controllability. This is a primary aim of the National Advisory Committee for Aeronautics in studying the subject.

APPARATUS AND METHODS.

The general method of recording the forces on the controls is shown in figure 1. An auxiliary rudder bar is connected to the regular bar by special springs (22). The rudder wires (24) are connected to this secondary bar, so that the relative movement between the two bars is a measure of the force applied. In the same way the relative motion between the handle and the top of the control column will measure the elevator and aileron forces. These forces can be read directly on the rudder scale (21) and the aileron and elevator scale (19), but in most cases the forces were transmitted to the recording instrument (23). In figure 2 is shown a detail view of the control head. The upper part with the handle is connected to the lower collar through the cantilever springs (18) and the relative motion is transmitted by the two bell cranks (16 and 17) mutually at right angles to the wires (20) running to the recording instrument.

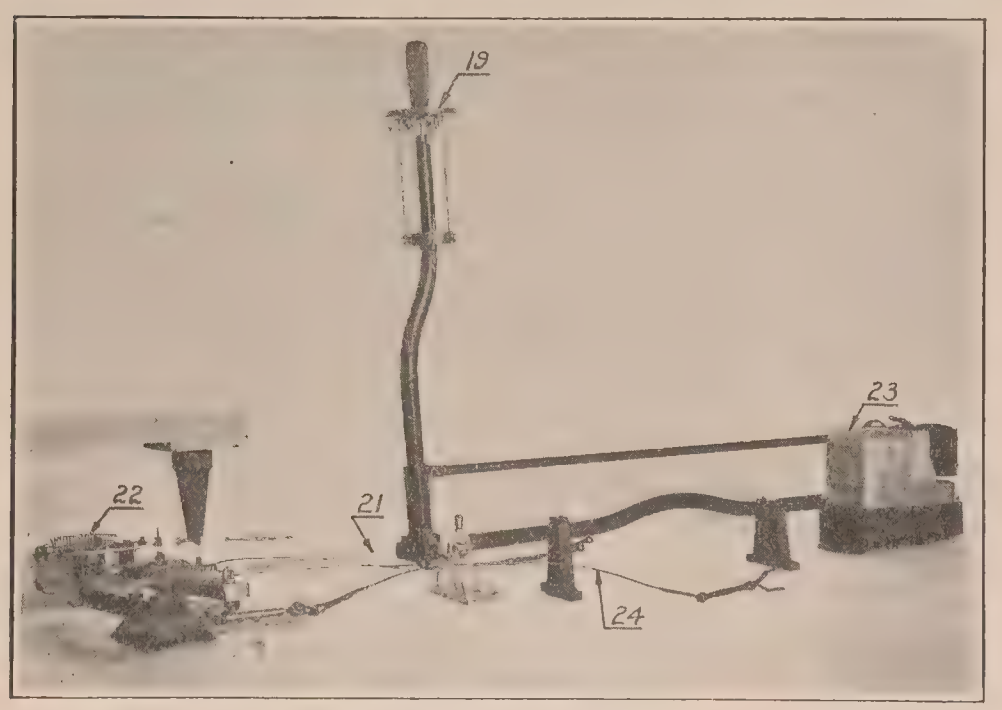


FIG. 1.—General assembly of apparatus for recording forces on the controls.

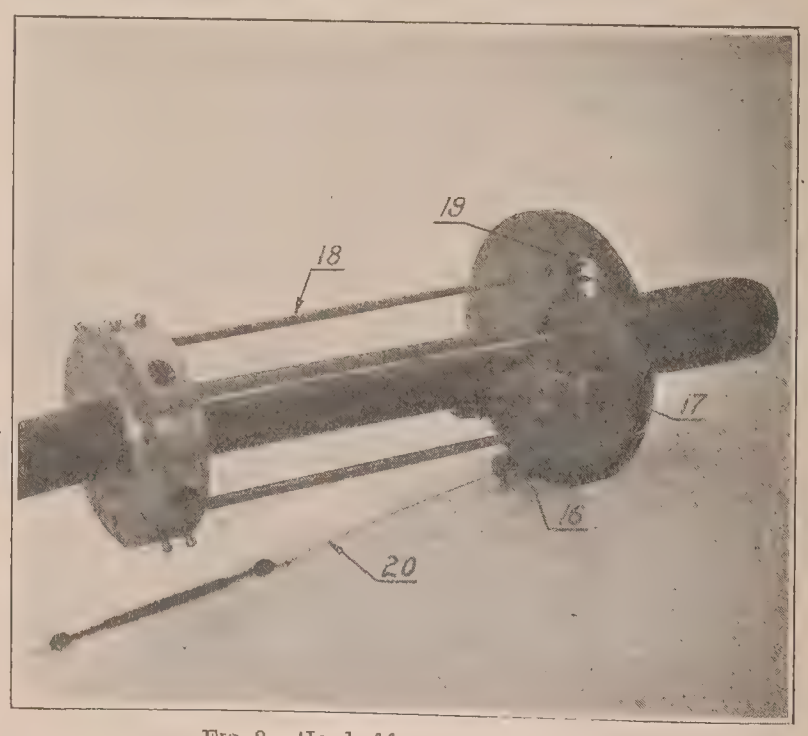


FIG. 2.—Head of force recording stick.

Figure 3 shows the interior of the instrument for recording the control forces; (6) is an electric motor to give a constant speed, but the governor in this case was removed, as the speed was of no importance in this work; (7) is a light holder, containing a special flash-light bulb and an adjustable prism to reflect the beam through the lens (9) onto the three mirrors (8) from which it is reflected to the photographic film. In order to distinguish between the three records a sectored disk (10) driven from the lower shaft by a belt (15) revolves slowly before the mirrors, making dotted lines on two of the records. The drum for holding the film is shown in figures 5 and 6 and is constructed like a plate holder, so that the shutter (25) is automatically opened by the pin (14) when the drum is placed on the instrument. At the same time

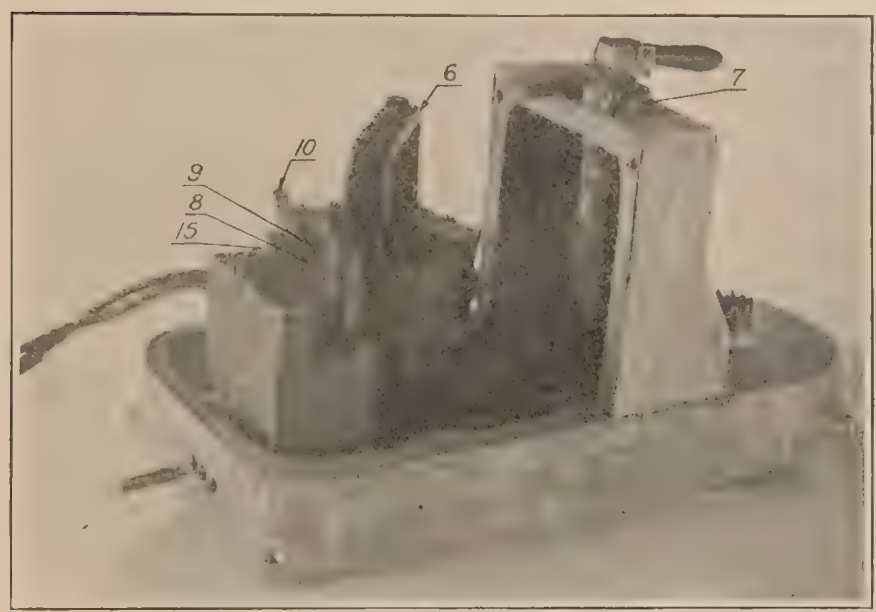


FIG. 3.—Instrument for recording forces.

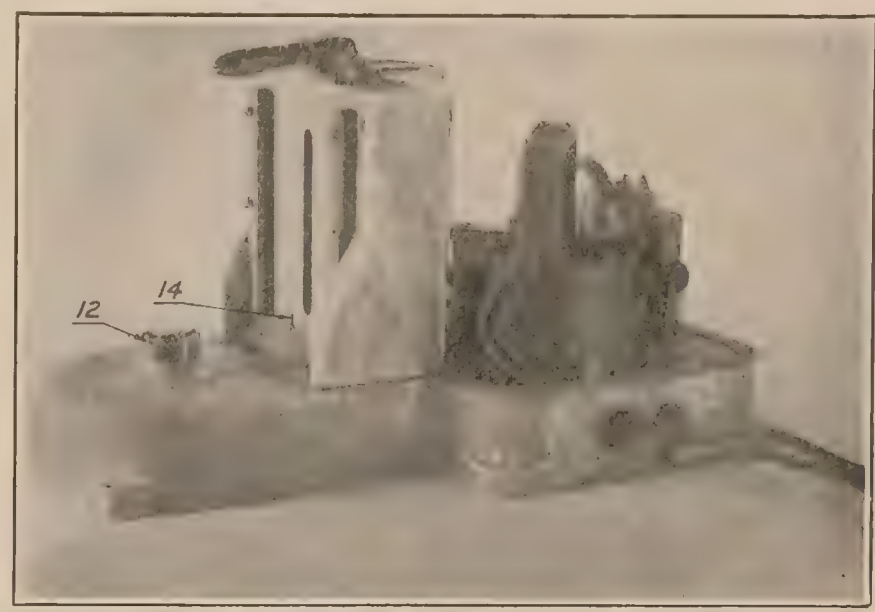


FIG. 4.—Another view of instrument for recording forces.

FIG. 5.—Film drum holder.

FIG. 6.—Film drum showing rollers for tightening film.

the clutch (12) is engaged to rotate the drum. The film is held on the inner drum (fig. 6) by the rollers (27) and the amount the drum has turned can be seen on the dial (28) through a red glass window. The underside of the instrument is shown in figure 7 with the bottom cover plate removed; (3) is the lower part of the motor frame, (2) is the drive shaft, (1) is the gear for rotating the film drum, and (4) are the springs to take up any backlash in transmission.

An identical instrument is being constructed to record the position of the controls, but as it could not be completed in time for these tests the scales shown in figure 8 were used and were read by the observer. The movement was transmitted from the control by throttle wire and a spring was placed in the system to take up the backlash. Each scale was carefully calibrated after it was in place.

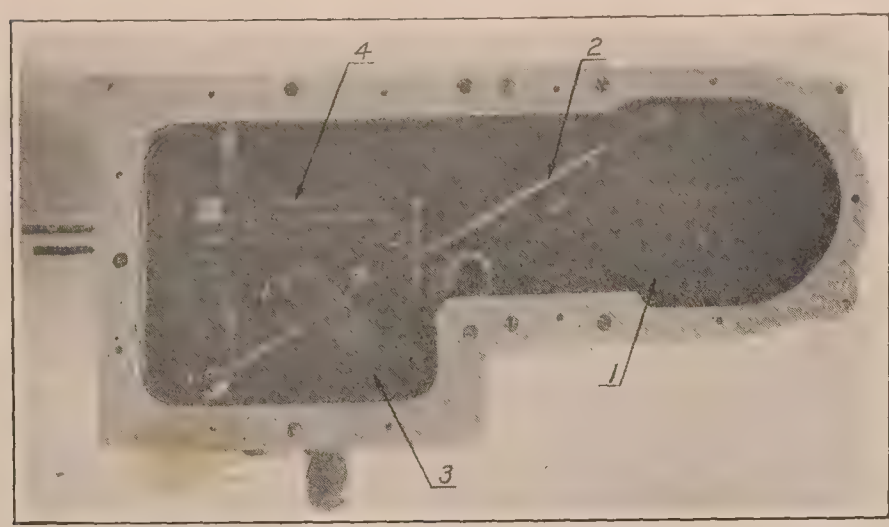


FIG. 7.—Underside of force recorder.

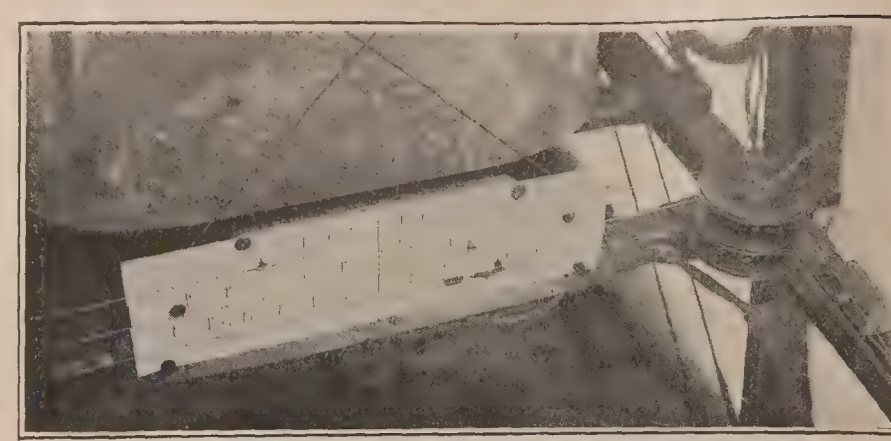
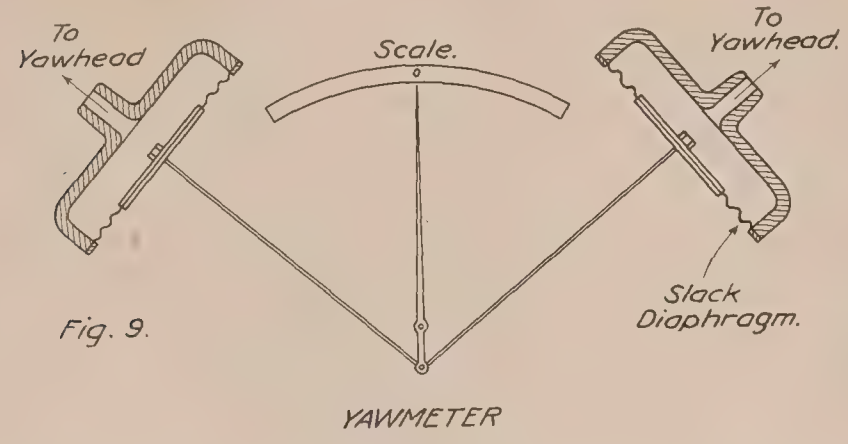
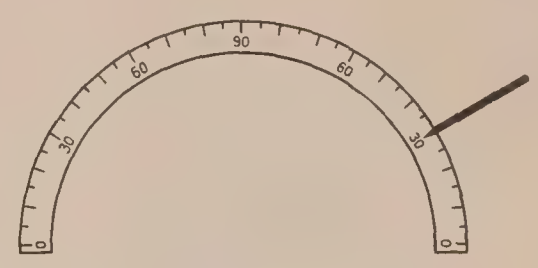


FIG. 8.—Scales for indicating the position of the controls.

The air speed was determined by the usual pitot-venturi instrument, and the error in bank by an especially sensitive bubble inclinometer. It was attempted to measure the yaw by the British type of yawmeter using slack diaphragms (fig. 9). This instrument, however, was found very unreliable, the scale being crowded together on one side of the zero and very open on the other and the readings varying with the speed of the airplane. The method finally adopted consists of a simple U tube half filled with alcohol and the free ends connected to the sides of a yaw head. In addition to this a graduated vane pivoted on a vertical axis was placed low down and ahead of the wings at the inner strut so that it could be read by the pilot. The method of procedure consisted in flying at the desired speed and yaw by means of the vane and then setting a movable pointer at the level of the liquid in the U tube and using this to fly by. Although this method seems a little indirect, it gave very satisfactory results.





Scale placed over cowling in front of pilot to determine angle of bank. Fig. 10.

The angle of bank was determined by a movable pointer on the cowling just astern of the motor, that could be set to any angle by a graduated circle (fig. 10).

The center of gravity of the machine was about 38 per cent back on the mean wing chord and was kept in one place by the addition of lead when a lighter observer went up.

A record of control forces obtained in turns is shown in figure 11, and it is evident that the records are quite distinguishable in this film. In figure 12 is shown a typical record of control forces in yaw, and in figure 13 a record of a short flight. In the latter record, as the air was quite bumpy, the illumination was not sufficient to give clear records. On this instrument only two dry cells were used on a 4-volt lamp, while to get the same illumination in the accelerometer four cells were necessary with this lamp. This is due to the shorter focal length of the lens and the larger mirrors in the force recorder.

The force recorded was calibrated by applying known forces on the stick and rudder bar by a pulley and weights, at the same time making a record on the film. The errors due to a movement of the controls were studied as well as the effect of one control acting on another, but in no case was the error greater than 1 pound, which was considered as close as any one set of readings could be duplicated. The angle scales were in some cases calibrated with various loads on the control surfaces, and in a few cases it was found necessary to make a small correction for deflection, so that the angle readings may be relied on to within one-half of a degree.

FIG. 11.—Control forces in a turn to the left and to the right.

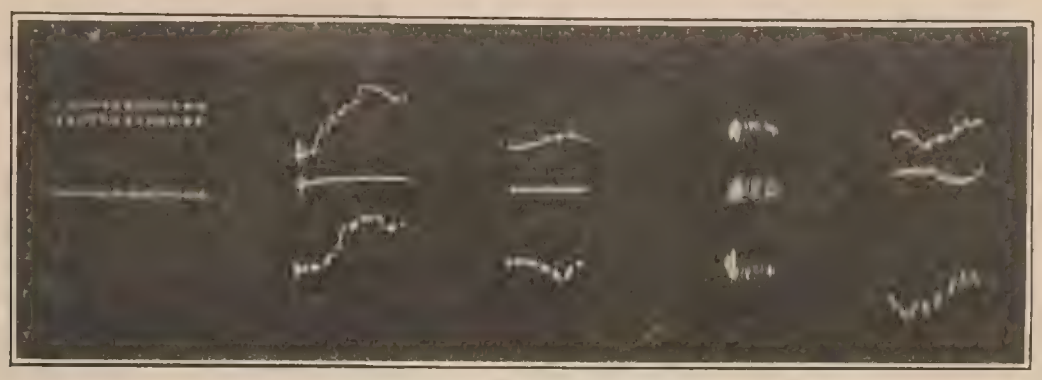


FIG. 12.—Control forces in yaw.

FIG. 13.—Control forces in bumpy air.

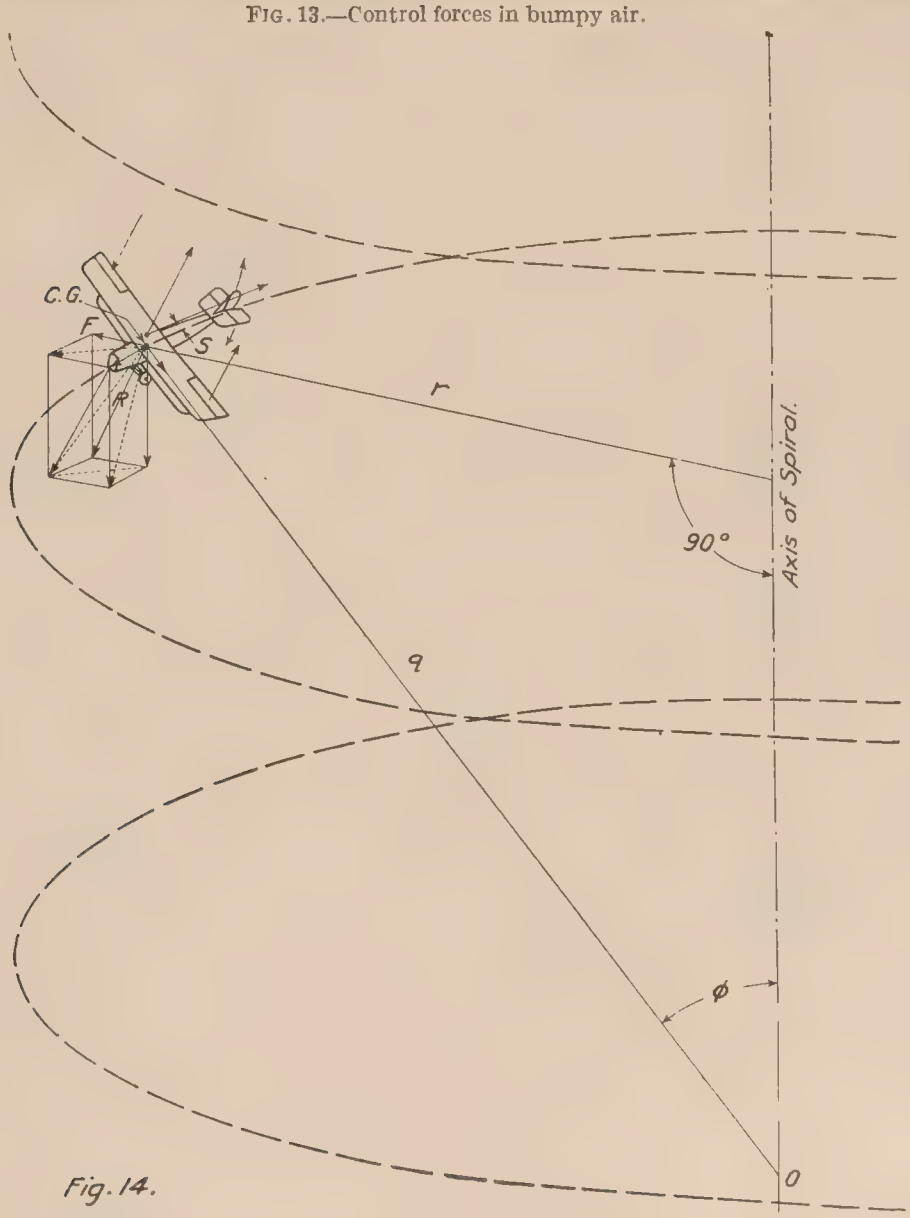


Fig. 14.

MOTION IN CIRCLING FLIGHT.

If m is the mass of the machine in slugs (weight in pounds divided by g or 32.2), ϕ the angle of bank from the horizontal, and F the centripetal force, we have:

$$F = mg \tan \phi = \frac{mv^2}{r}$$

or r, the radius of the circle $= \dfrac{v^2}{g \tan \phi}$

of V is the air speed in ft./sec.

R, the resultant force acting normal to the wing chord will be equal to $\dfrac{mg}{\cos \phi}$ as shown in figure 14, and the ratio of mg to R is given by $\dfrac{1}{\cos \phi}$, which is the acceleration experienced in a turn. In figure 15 is plotted this acceleration against angle of bank, as the acceleration should not vary with the air speed, so long as there is no skidding or side-slipping. In figure 16 are plotted the angle of attack of the airplane when flying at various speeds and banks taken from the full

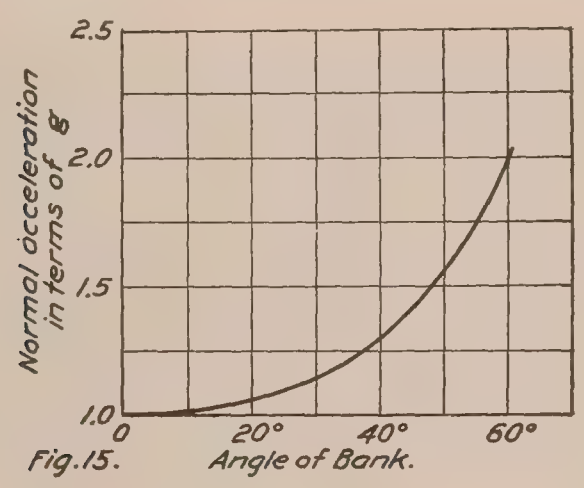


Fig. 15.

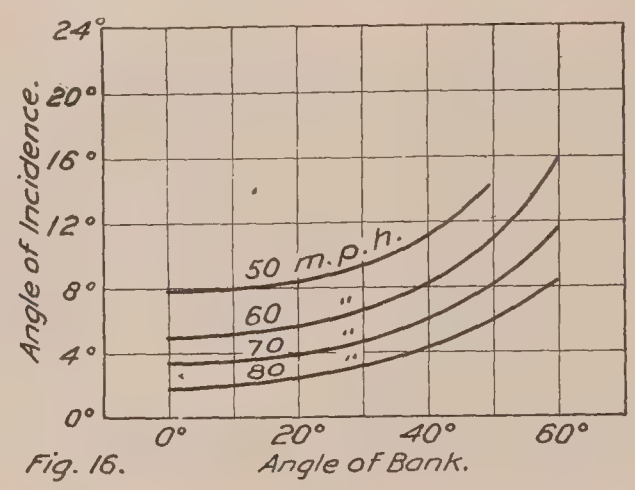


Fig. 16.

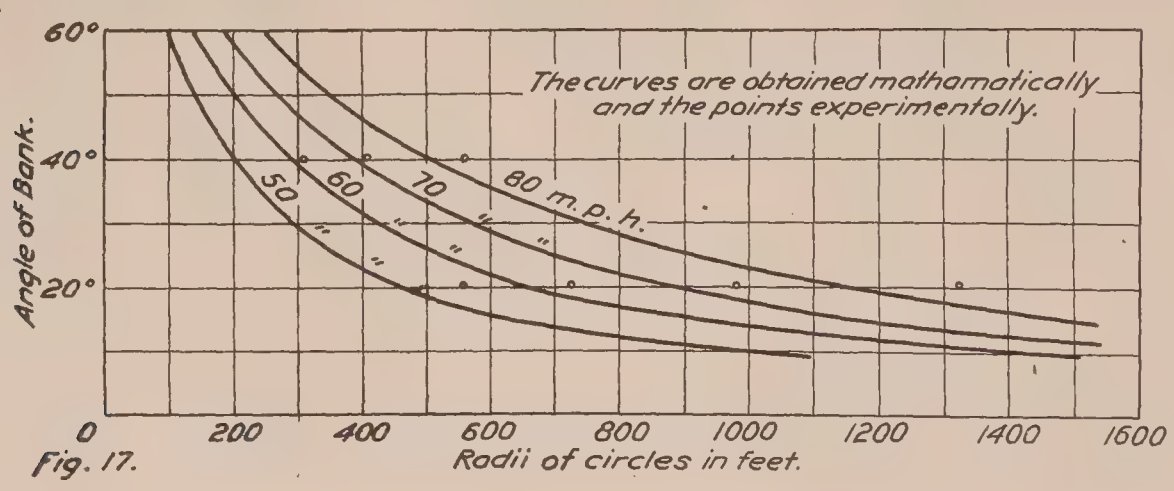


Fig. 17.

flight lift curves (the lift coefficient being computed from the known speed and load on the wings), and in figure 17 are plotted the theoretical radius of circles and the actual radii as obtained by taking the time for a complete circle. It is regretted that a camera obscura was not available to measure this distance accurately, but the results obtained check fairly well, and the discrepancy is probably due to the fact that the calibration of the air speed head is affected in banks. In all cases a true bank was held by means of the bubble inclinometer.

In a turn the outer wing has a higher velocity than the inner one, and its lift is therefore greater, producing a rolling moment which must be balanced by the moment of the ailerons. The outer wing will also have the higher resistance, and the resultant yawing moment is balanced by the rudder and in part, except at very small angles of attack, by the ailerons.

The distance S of the center of lift from the plane of symmetry (fig. 14) may be determined by an integration of the air forces.[1] As the velocity along the wing is proportional to its distance

[1] Kann, Technische Berichte Band III Heft 7.

x from point O in the axis of the turn, and the lift at any point is proportional to V^2, introducing $q=\frac{r}{\sin\varphi}$ (see fig. 14), we have

$$(q+s)\int_{q-a}^{q+s} x^2dx=\int_{q-a}^{q+a} x^3dx$$

where a is the half span,
which gives

$$S=\frac{2qa^2}{3q^2+a^2}$$

Hence, substituting for q its value,

$$S=\frac{2ra^2\sin\Phi}{3r^2+a^2\sin^2\Phi}$$

as a^2 is usually small compared with r^2 the question reduces to:

$$S=\frac{2}{3}\frac{a^2\sin\Phi}{r}=\frac{2}{3}\frac{a^2g\sin^2\Phi}{v\cos\Phi}\frac{2}{3}\frac{a^2v^2}{r\sqrt{v^4+g^2r^2}}$$

The rolling moment is RS, where R is the total normal force on the wings. For turns at constant altitude and correct bank,

$$R=W\sec.\Phi$$

$$L=RS=\frac{2}{3}\frac{a^2W\tan\Phi}{r}$$

Eliminating r and Φ in turn by introducing the speed of flight,

$$L=\frac{2}{3}\frac{a^2Wg\tan^2\Phi}{v^2}=\frac{2}{3}\frac{a^2Wv^2}{gr^2}$$

The moment Rs must equal the aileron moment, and as this increases as the square of the span, it is evident that machines having large spans must have relatively more powerful ailerons for the same angles of bank than small machines. If it is assumed that the force exerted by the ailerons is proportional to their angle from the mean position this should then be inversely proportional to the square of the radius of the turn if the speed remains constant. An examination of the experimental curves shows that this is true for turns of large enough radius for the a^2 term to be neglected.

The wing drag may be integrated in the same way as the lift, but as the greater part of the total drag is structural and aileron resistance, little would be gained, as the structural resistance will vary with different machines. It may be stated, however, that the decreased drag of the inner wing is almost balanced by the greater aileron resistance on that side, leaving very little for the rudder to balance, as evidenced by the fact that turns may be made with the rudder locked in neutral and with little or no side-slipping or skidding.

NOMENCLATURE.

Throughout this report the following terms and signs will be used. All directions are taken from the viewpoint of the pilot. It should be noticed that No. 5 is the reverse of that previously used, being changed for consistency.

1. In right, or positive roll, the left wing is raised.
2. In right, or positive yaw, the wind strikes the pilot's left cheek.
3. Aileron is positive when the trailing edge of the left aileron is down.
4. Rudder is positive when the trailing edge moves to the right.
5. Elevator is positive when the trailing edge is pulled up.
6. Aileron force is positive when force on the stick tends to give right aileron.
7. Rudder force is positive when the push is with right foot.
8. Elevator force is positive when the stick is pulled toward the pilot.

The forces on the controls are given in all the curves, and if it is desired to find the moments about the control surface hinges, the following conversion factors must be used:

1. Subtract $8\frac{1}{2}$ pounds (the static weight) from the stick force and multiply by 23.8 to obtain moments in inch pounds.
2. Multiply stick force by 56.0 to get aileron moments in inch pounds (sum of both ailerons).
3. Multiply rudder force by 10.5 to get moments in inch pounds.

CONTROL POSITIONS IN TRUE BANKS.

Before this work was commenced it was thought that there might be a considerable range of rudder and aileron position that would give equal flight properties to the airplane. While this was found to be true when flying without a lateral inclinometer in seemingly perfect banks, yet if the inclinometer was held exactly in the center the positions of the controls were uniquely determined. Due to the great difficulty in flying the machine at the correct bank and air speed

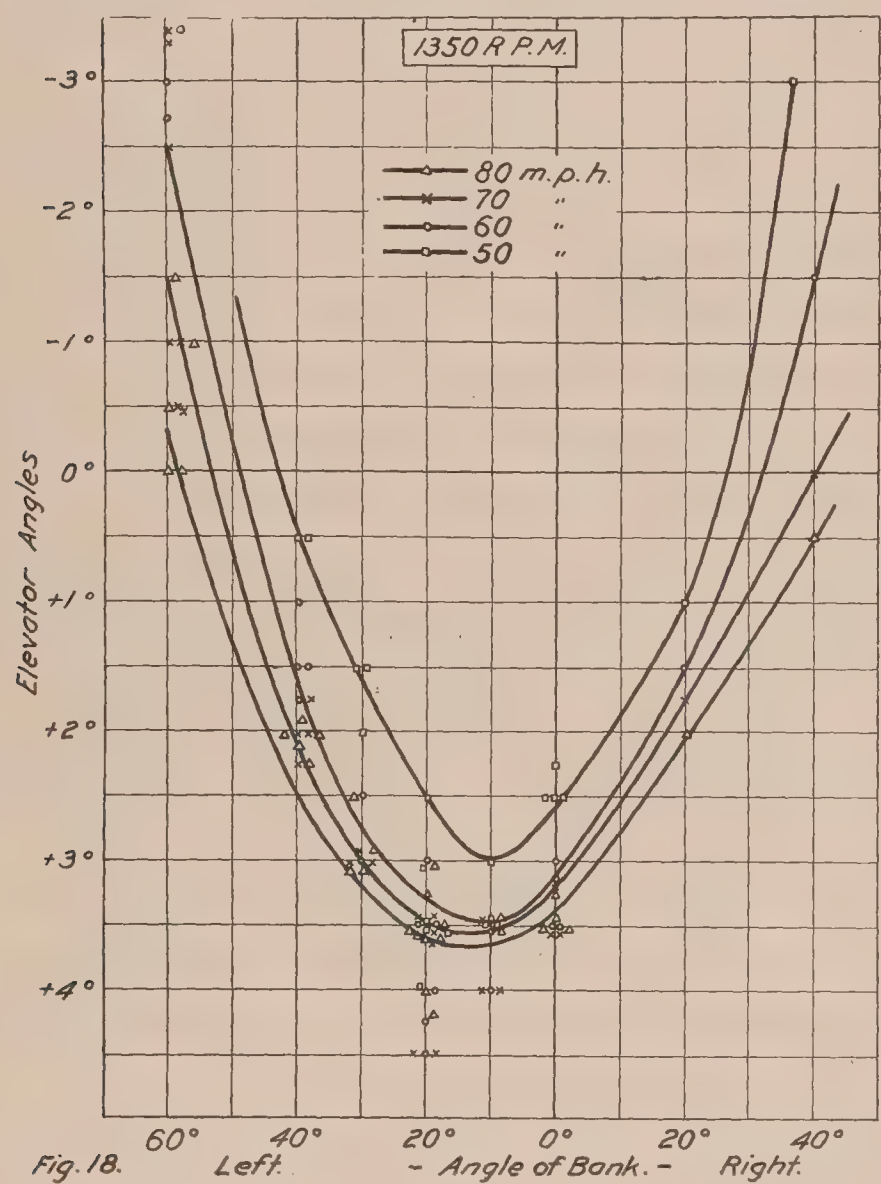


Fig. 18.

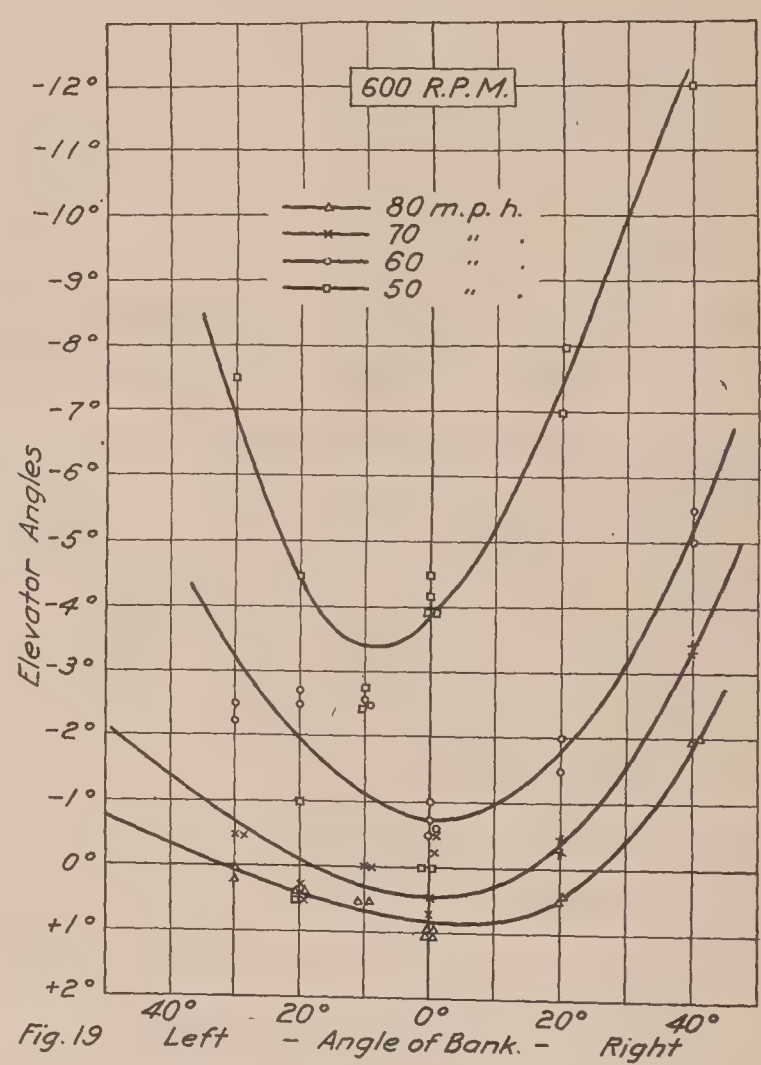


Fig. 19

at the same time that the readings were taken, it was found necessary to repeat each run several times. As the variation in control position is very slight, in many cases less than the error in reading, the points do not lie smoothly on a curve, but in all cases the curves obtained should be accurate to one degree. In order to investigate the effect of the slip stream, runs were made at 1,350 r. p. m. and 600 r. p. m., and the similarity in shape between the two sets is a check on the accuracy of the results.

The elevator is pulled back in any turn to keep the same flight speed, for the angle of attack is increased due to the greater apparent weight of the machine, and this must be balanced as in level flight by raising the elevator (figs. 18 and 19). The curves are nearly symmetrical about zero at 600 r. p. m., but at 1,350 r. p. m. more elevator is required in right banks than in left. The longitudinal, fixed control stability does not, however, seem to be otherwise affected, as the curves are very nearly parallel.

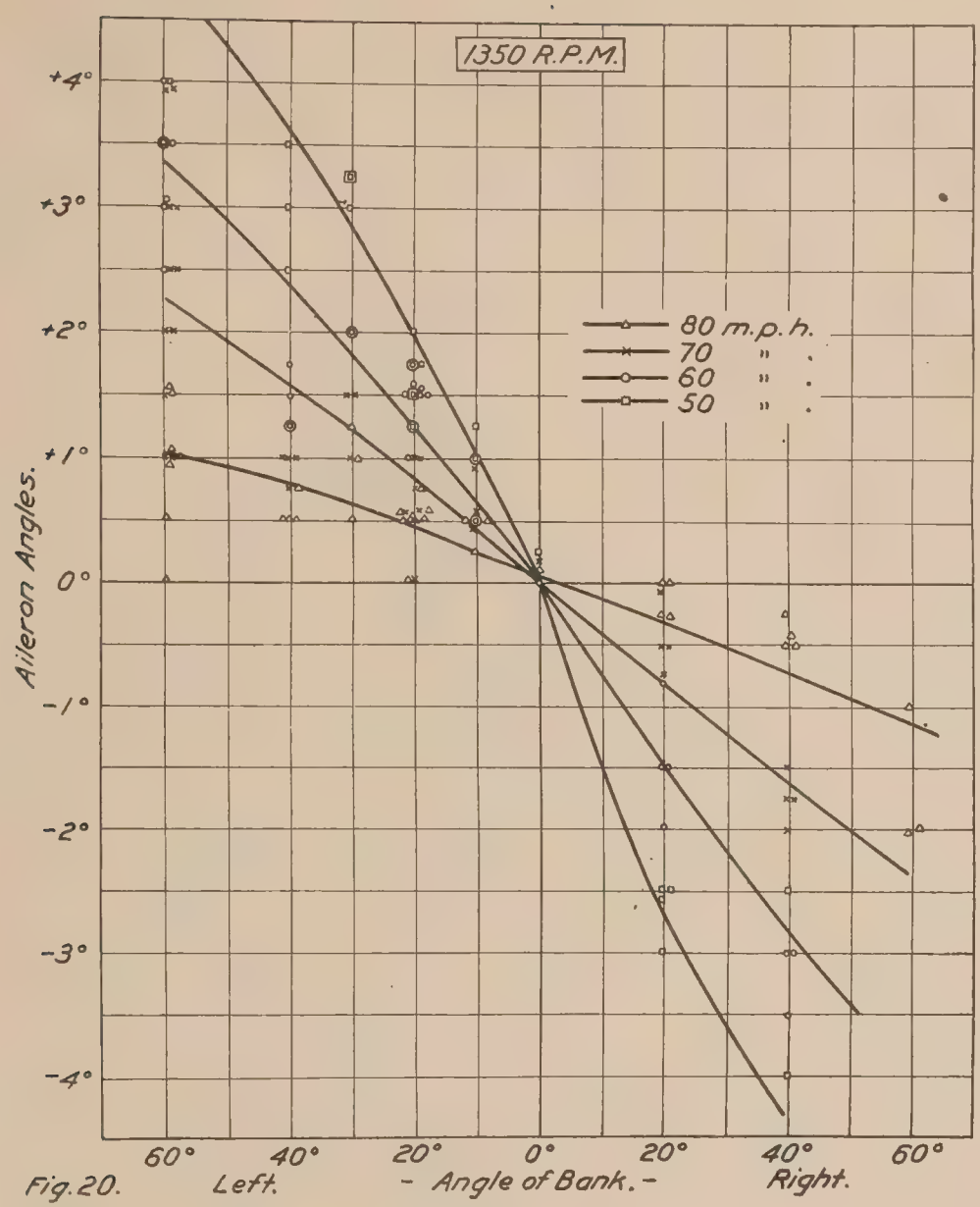


Fig. 20.

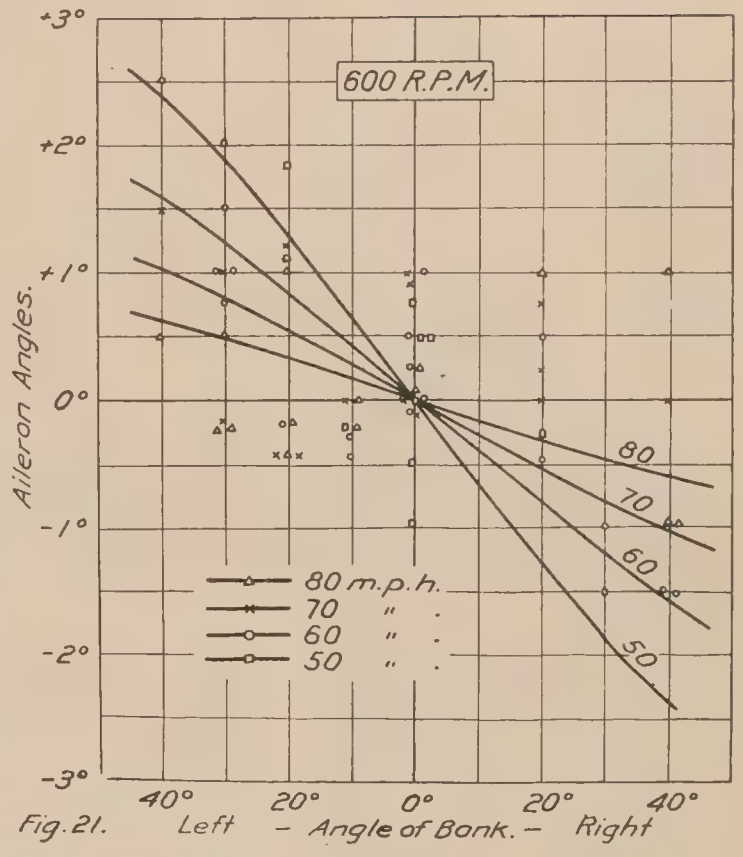


Fig. 21.

The aileron position curves (figs. 20 and 21) are nearly straight lines passing through zero angle at zero bank, and their slope increases inversely as the air speed; that is, the higher speeds require the smaller angles. The most important feature, however, is that opposite aileron is used to hold the bank from increasing; that is, the machine is laterally unstable with the fixed controls.

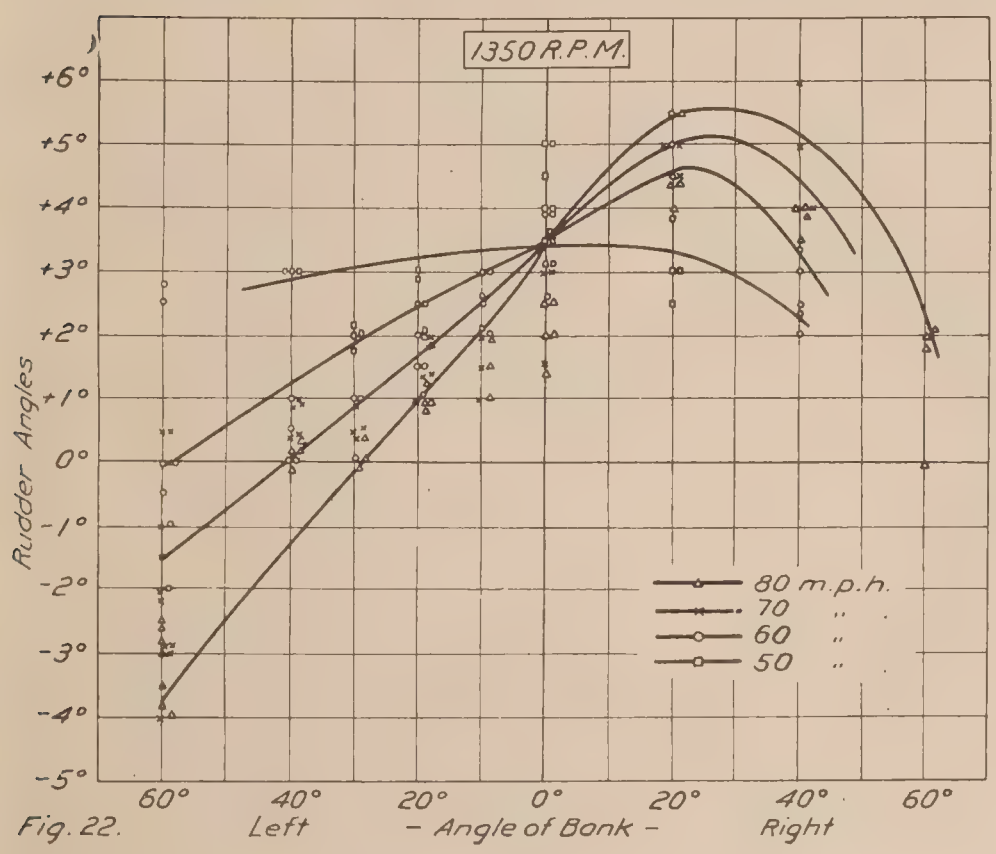


Fig. 22.

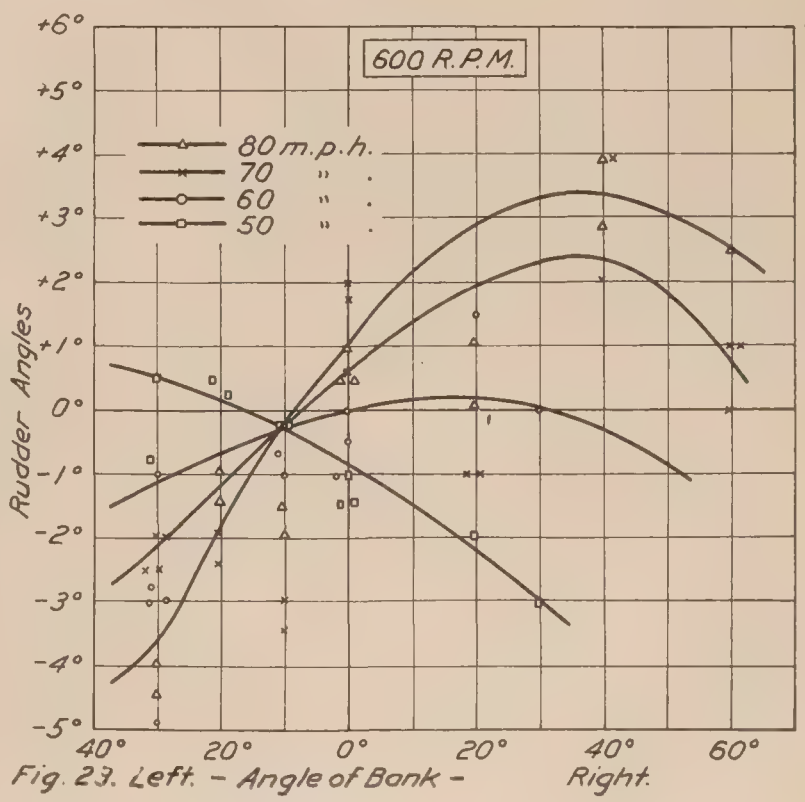


Fig. 23.

The rudder position curves (figs. 22 and 23) indicate directional stability with fixed rudder for small angles of bank. At high banks, especially right bank with the throttle open, the curves fall off rapidly as the rudder acts more and more as an elevator. The rudder is normally held $3\frac{1}{2}°$ right in order to balance the slip stream and propeller torque.

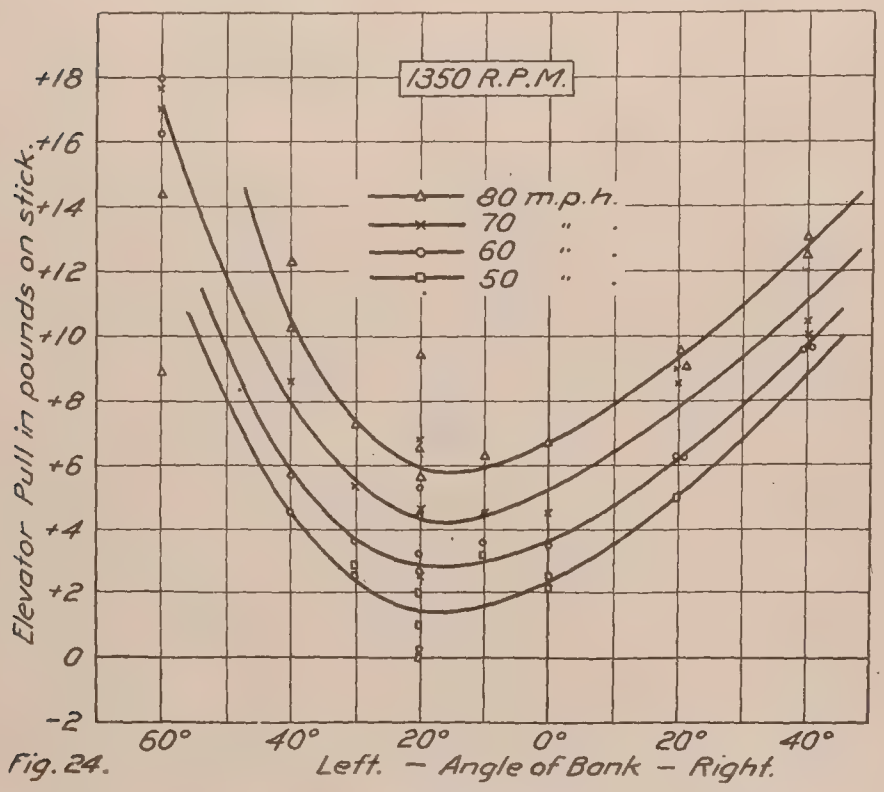


Fig. 24.

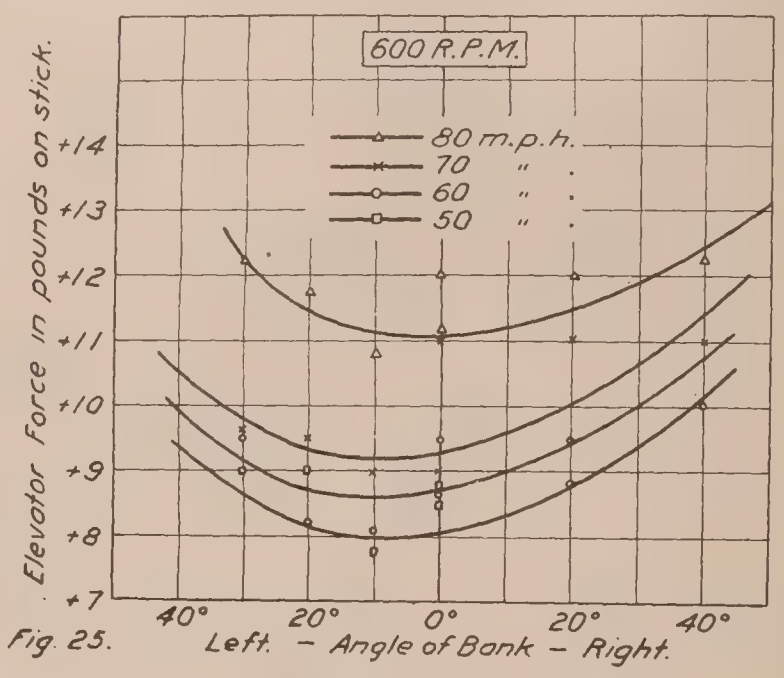


Fig. 25.

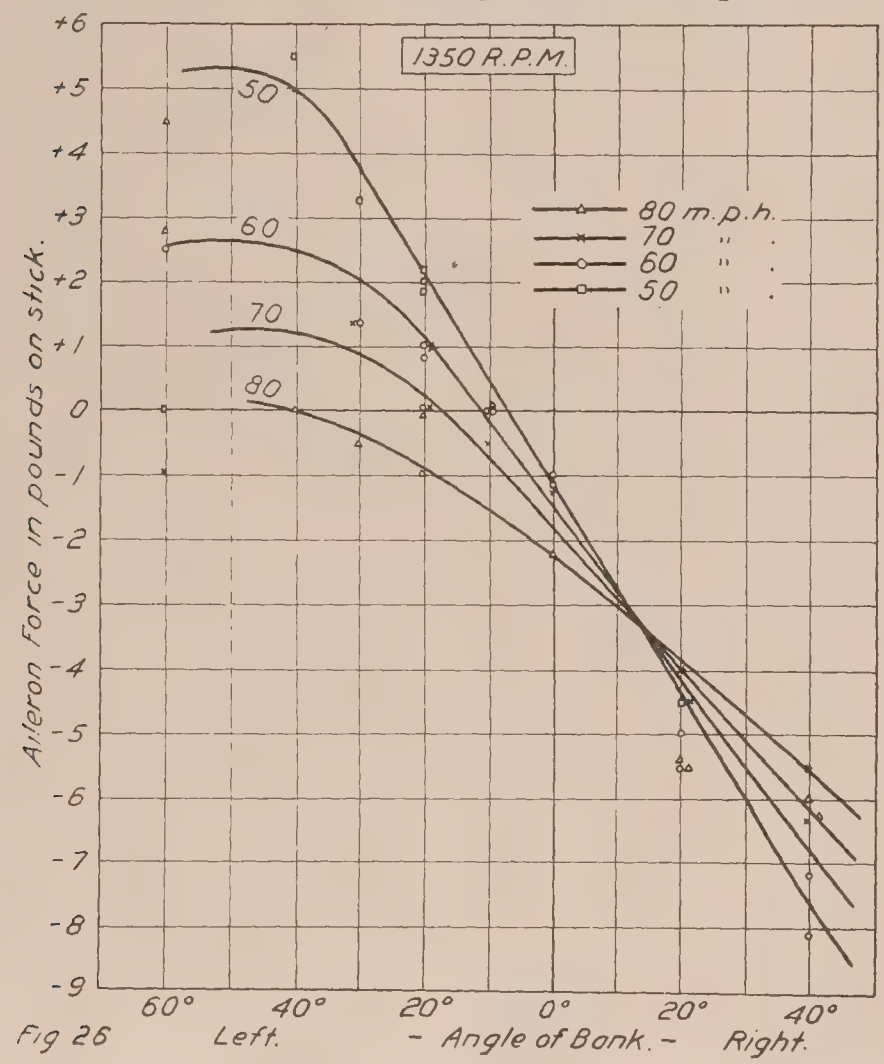


Fig 26

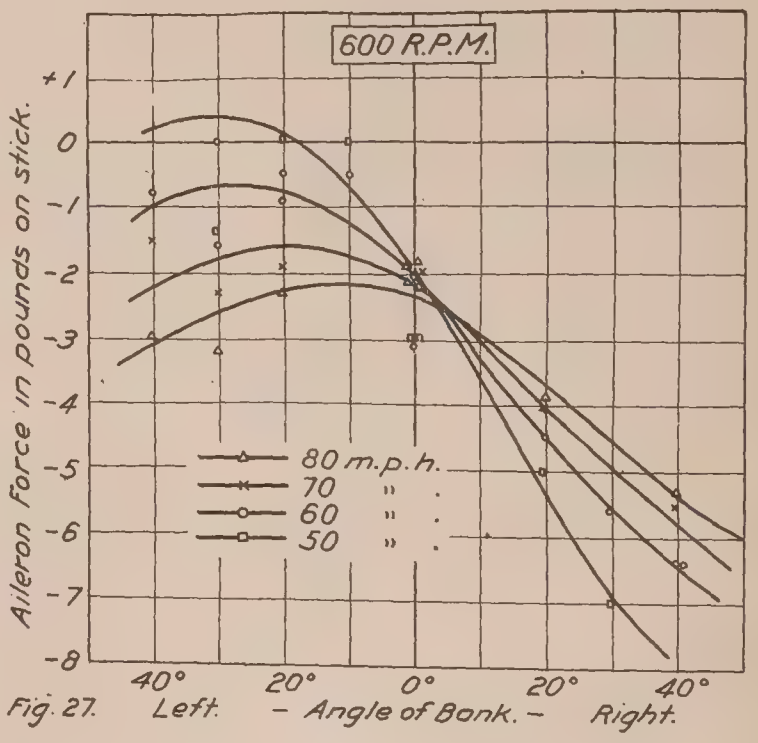


Fig. 27.

CONTROL FORCES IN TRUE BANKS.

The elevator force curves are nearly symmetrical and show increasing values with the angle of bank, as would be expected in view of the higher acceleration acting on the elevator (figs. 24 and 25). In the same way as with fixed controls the stability characteristics seem to be unaltered with free controls in any bank.

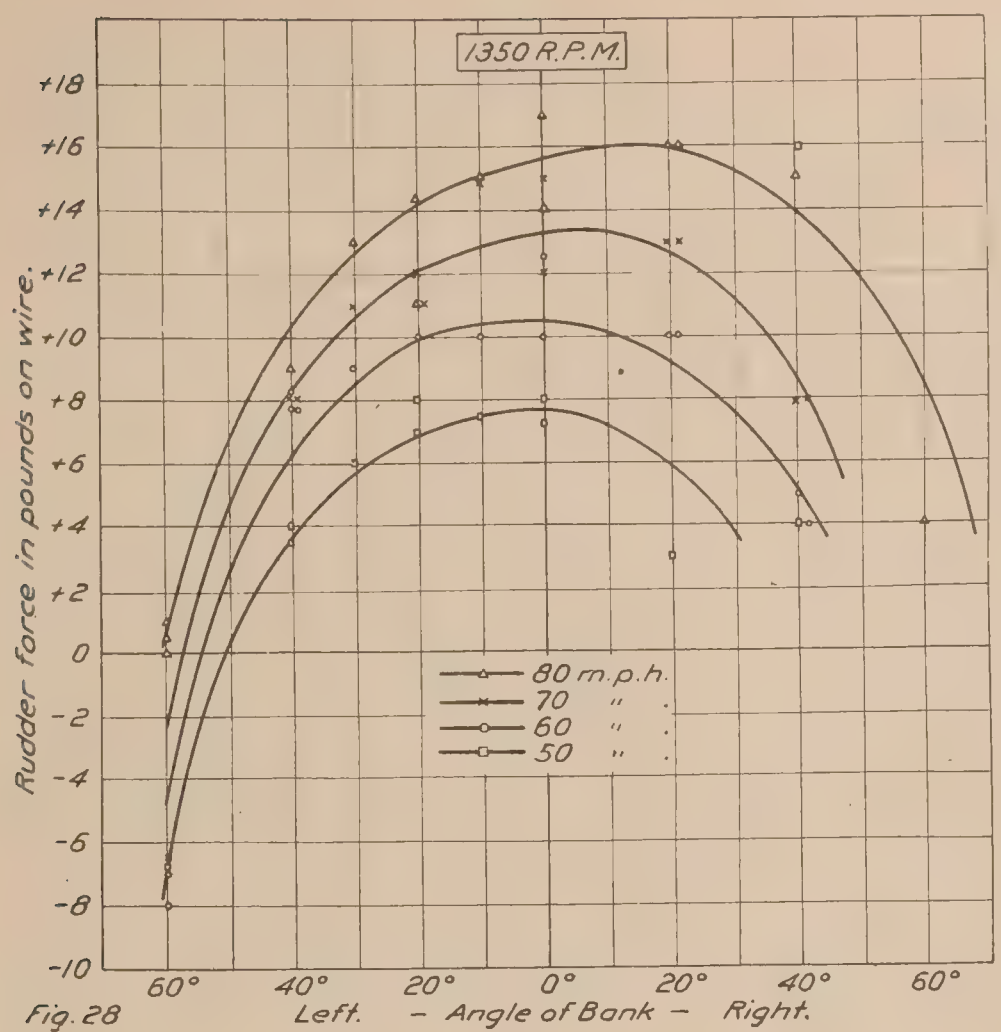


Fig. 28

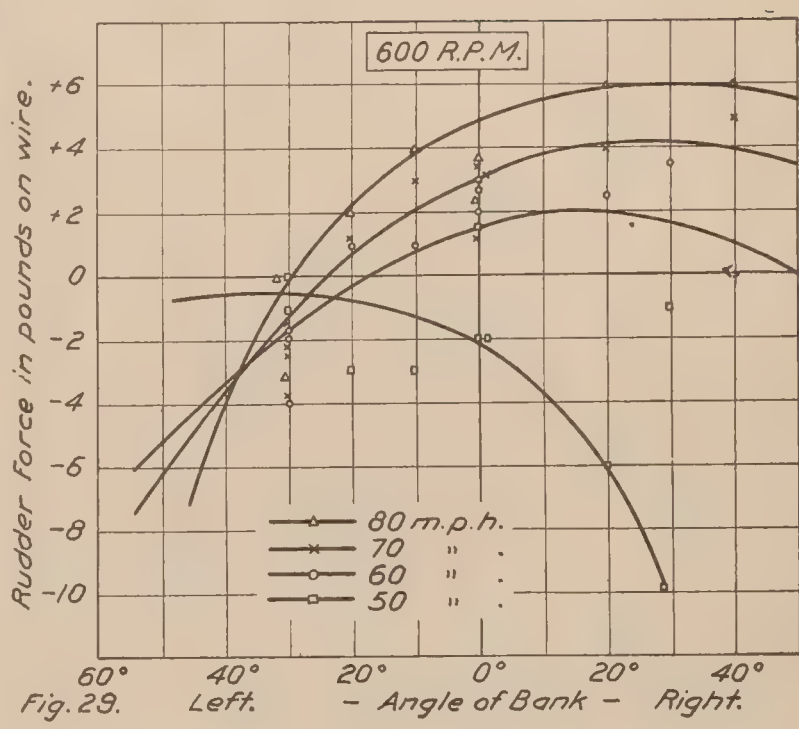


Fig. 29.

The aileron force curves (figs. 26 and 27) are similar to the corresponding position curves and indicate instability, but the slope of the curves increase as the air speed decreases.

The rudder force is normally of considerable magnitude in level flight at 1,350 r. p. m. (fig. 28) and increases with the air speed, but rather strangely falls off as the bank is increased either way. The force curves for 600 r. p. m. (fig. 29) are similar in shape, but cross at 30 left bank, and all the forces are of considerably less magnitude than with the motor on.

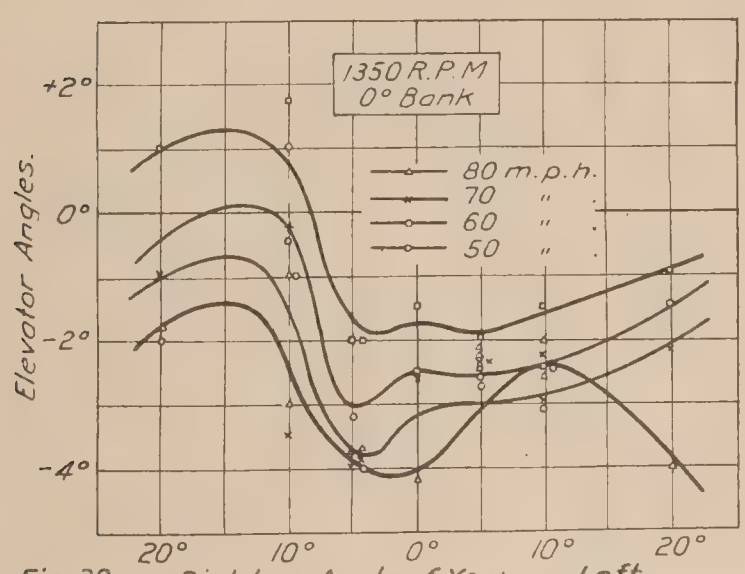


Fig. 30

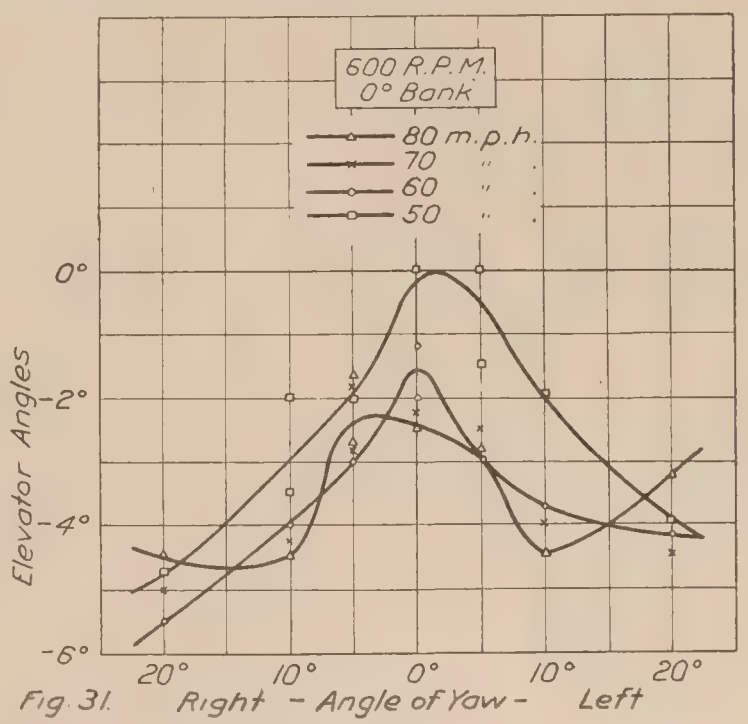


Fig. 31.

CONTROL POSITIONS IN YAW.

In figure 30 are plotted the elevator angles for various degrees of yaw at 1,350 r. p. m. The curves are nearly parallel, showing that the longitudinal stability with fixed controls is not affected by the angle of yaw. For angles of yaw up to 5° the stick is pushed forward slightly

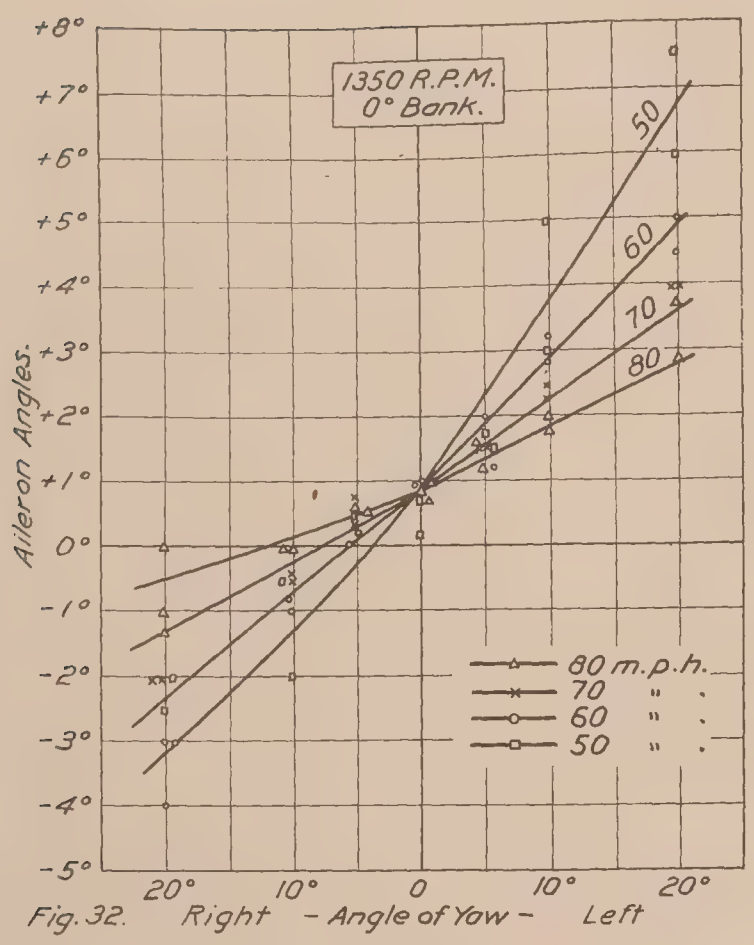


Fig. 32.

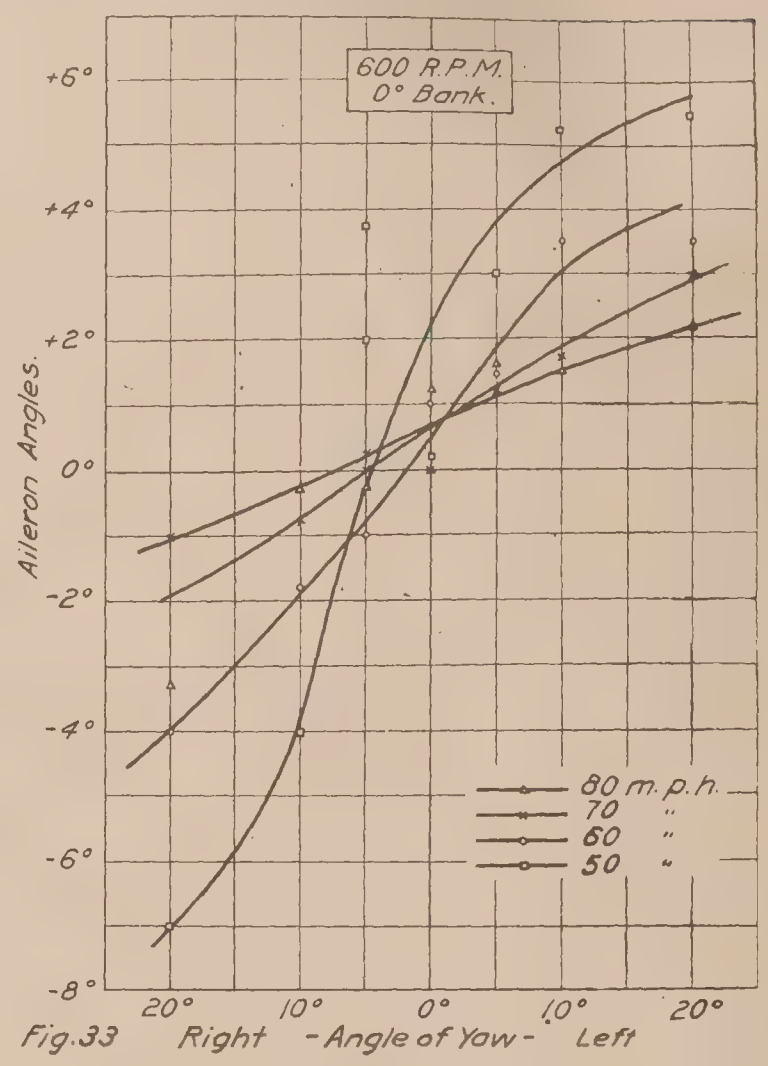


Fig. 33

to balance the machine, but at larger angles it is pulled back because the slipstream is deflected away from the tail. At 600 r. p. m., however (fig. 31), as there is no slipstream, the stick is pushed forward about proportionally to the angle of yaw, but at 80 m. h. p. the curves falls off at 20° yaw.

The curves of aileron angles against yaw are plotted in figures 32 and 33, and in general the aileron angle is proportional to the angle of yaw and inversely proportional to the air speed. At 600 r. p. m. there is a reversal of slope for the slow speeds at small angles of yaw.

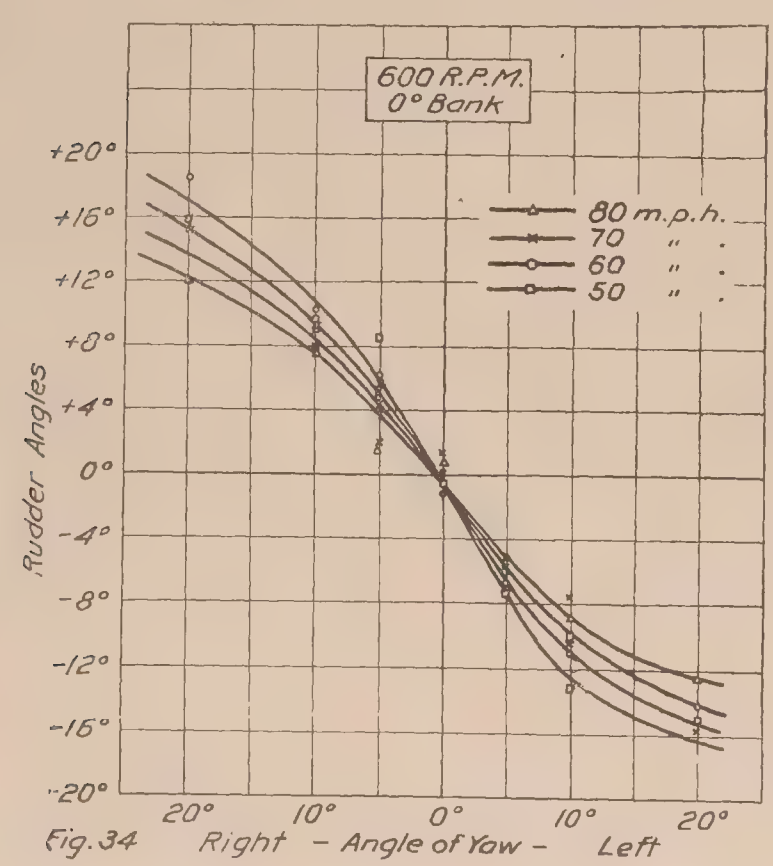


Fig. 34

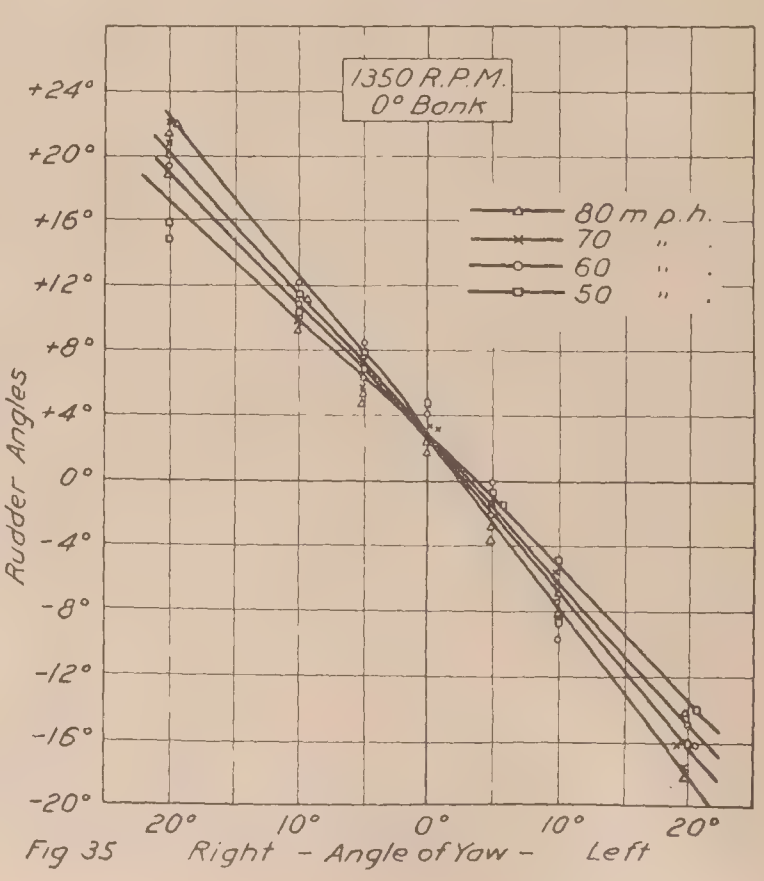


Fig 35

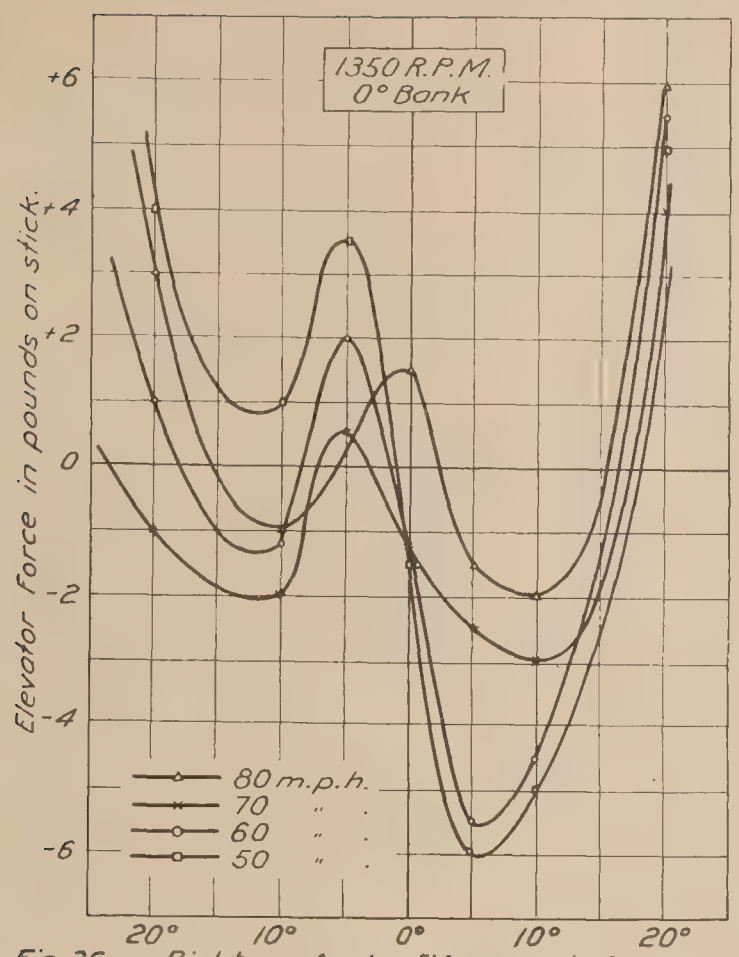


Fig. 36

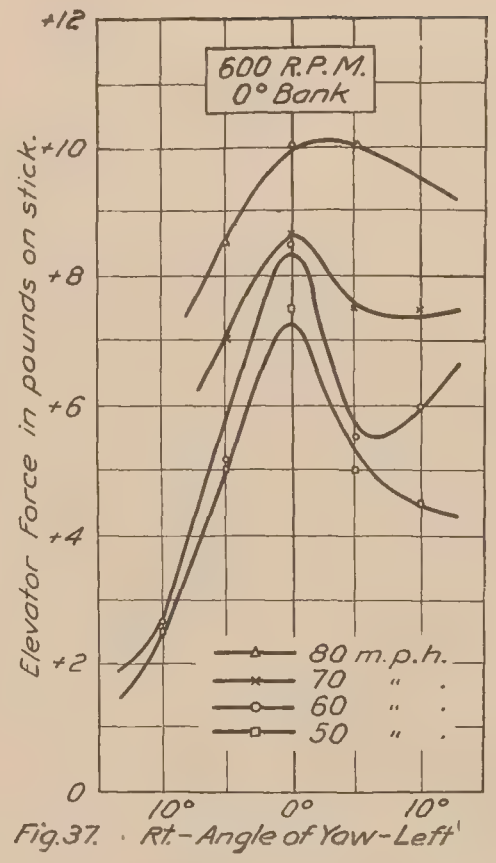


Fig. 37.

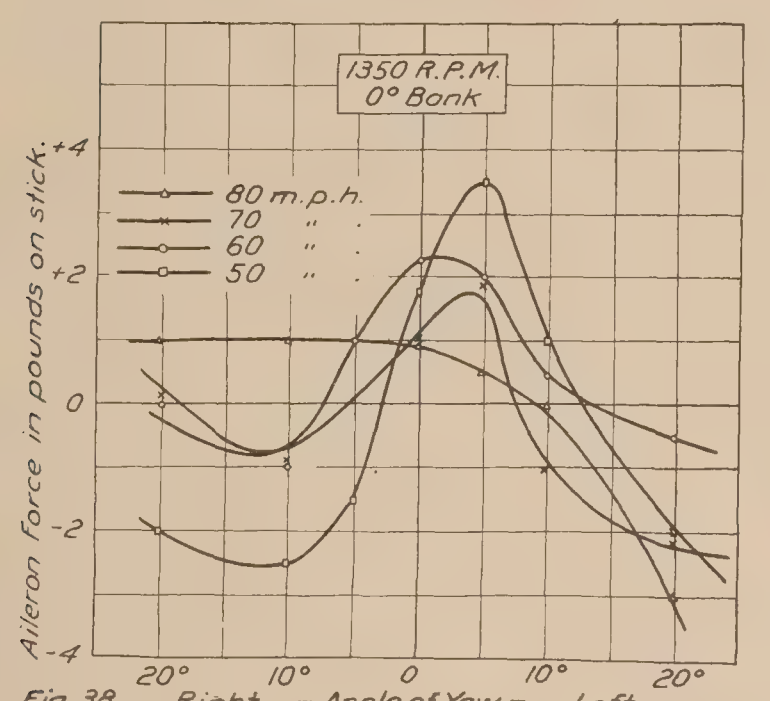


Fig. 38

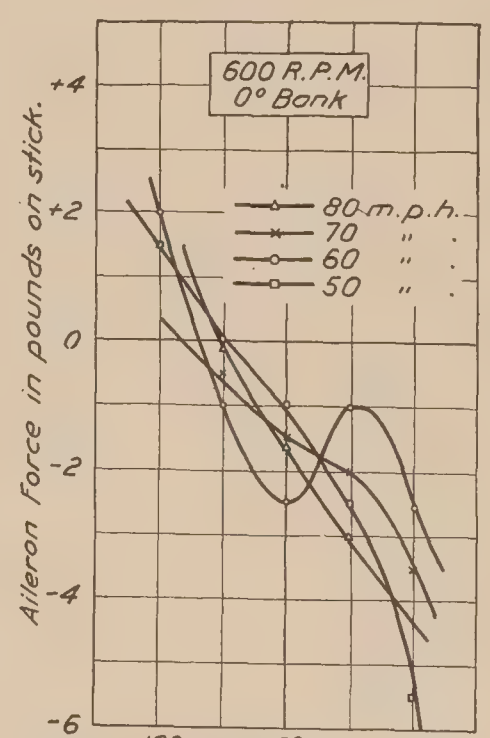


Fig. 39.

The curves of rudder position (figs. 34 and 35) show that the rudder must be kept over to hold the yaw, and the air speed has very little effect on this angle. On comparing the rudder with the ailerons it will be seen that the controls are crossed, as would be expected in a side slip.

CONTROL FORCES IN YAW.

The elevator forces (figs. 36 and 37) decrease with the angle of yaw up to about 10°, at which point they rapidly increase due to the tail coming out of the slipstream. In general the longitudinal stability with free controls is unaffected by yawing.

The aileron force curves are shown in figures 38 and 39 and indicate a positive slope at low speeds and small angles of yaw, especially with open throttle, but at large angles of yaw the slope is always negative, that is, right force is used to hold right yaw.

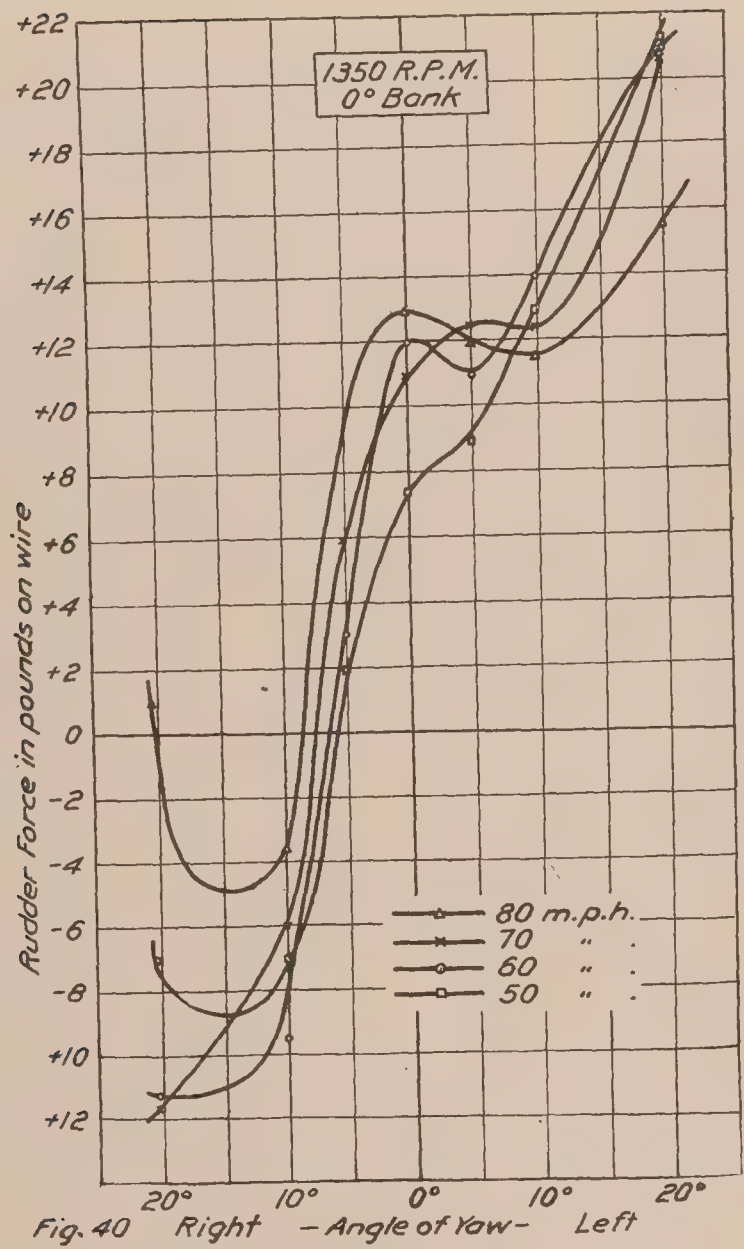


Fig. 40

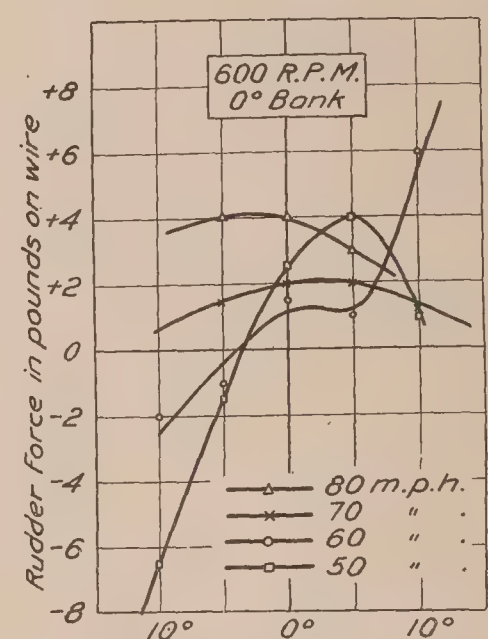


Fig. 41.

The rudder forces (figs. 40 and 41) are quite large at large angles of yaw, but what is most noticeable is the steep positive slope of the curves, indicating considerable directional instability a fact that will be discussed later. Most of the curves, however, have a small portion of negative slope at zero yaw, showing that at this angle there is a small stable region.

ACTUAL FLIGHT WITH FREE AND LOCKED CONTROLS.

Several flights were made to determine the behavior of the machine with locked and free controls. As shown in National Advisory Committee for Aeronautics Report No. 70, the region of longitudinal stability is confined to a small range from about 45 to 50 m. p. h. with locked elevator, so that it was necessary to conduct most of the work at this low speed, whereas the lateral stability would have been better at higher speeds.

With the elevators and ailerons locked the machine could be flown quite easily with the rudder, and gentle turns were successfully made. It was impossible, however, to fly for more than half a minute with all three controls locked, as a bump would send one wing tip down, a side slip ensuing that would not correct itself. The departure of this machine with locked controls from stability is, however, not very large.

It was found possible to fly not only safely but quite satisfactorily with the rudder locked. Turns could be made up to 80° banks smoothly, and with only a slight amount of side slip in coming out. The control seemed much better with the throttle open than when closed. With the ailerons locked the machine could be controlled and banks successfully made, but in coming out of the bank the side slip was excessive and a good deal of altitude must be lost before an even keel is reached. In this case the control is better with closed throttle.

In any flight condition, except rapid gliding flight, if the rudder was released the machine would begin to yaw and would soon whip into a very rapid spin from which there seems to be no

way of extricating it without the use of the rudder, so that any loss of rudder control would be an extremely serious matter. This dangerous feature could easily be remedied by the addition of a slightly larger fin. This emphasizes the fact that wind-tunnel tests for stability with free controls should be made with the movable surfaces removed or freely hinged. It was also noticed that the *C. G.* position had a marked effect on the directional stability with free controls, a small shift forward of its normal position making the machine almost completely directionally stable.

THE FREE FLIGHT DETERMINATION OF LATERAL DERIVATIVES.

For the computation of the stability characteristics of an airplane it is necessary to obtain experimental values of the resistance derivatives, which are 18 in number for a symmetrical airplane. The values of these derivatives are usually determined from the results of wind-tunnel tests. As the wind-tunnel tests do not, however, take into account the effect of the slipstream, and as the scale correction necessary to apply to the model results is unknown, it will be of great value to compute the resistance derivatives from data obtained in free flight. Unfortunately, it is quite difficult to determine all of the derivatives in this manner, and only a few can be obtained directly. It was one of the objects of this investigation to determine the value of Y_v, the lateral force, L_v the rolling moment, and N_v, the yawing moment, due to slide slipping, in free flight.

The method employed to determine Y_v consisted in finding the lateral acceleration experienced in various side slips and at different air speeds, and from this, the curve of lateral force against angle of yaw is plotted, and from the slope of this curve Y_v is computed. The lateral acceleration was determined from the readings of a bubble inclinometer, for in level flight the lateral acceleration is given by:

$$a = g \tan \alpha$$

where g is the acceleration of gravity and α is the angle read on the inclinometer.

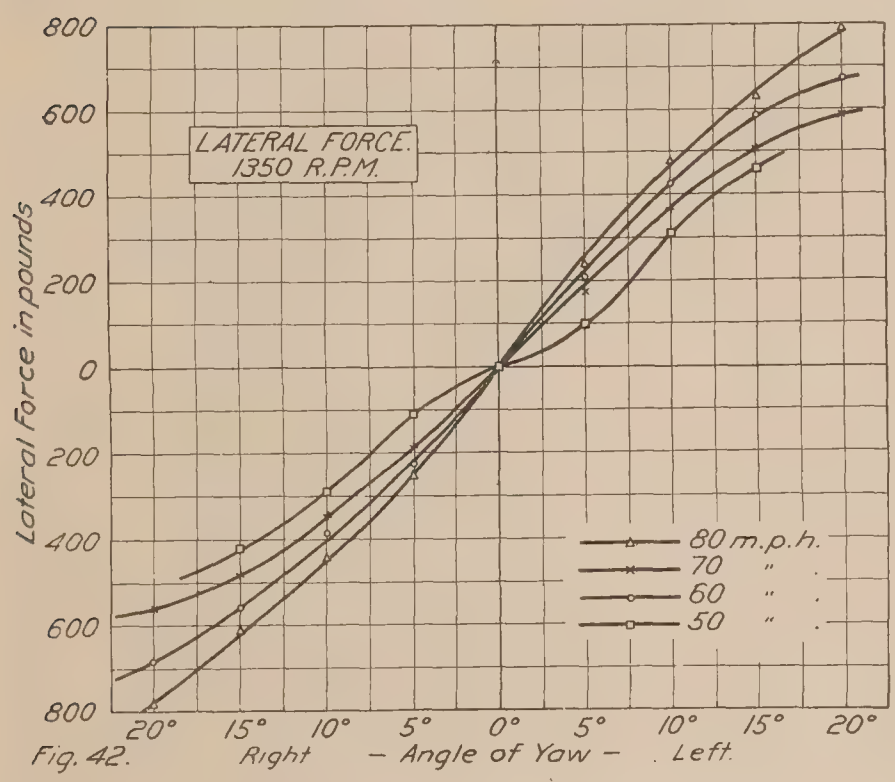


Fig. 42.

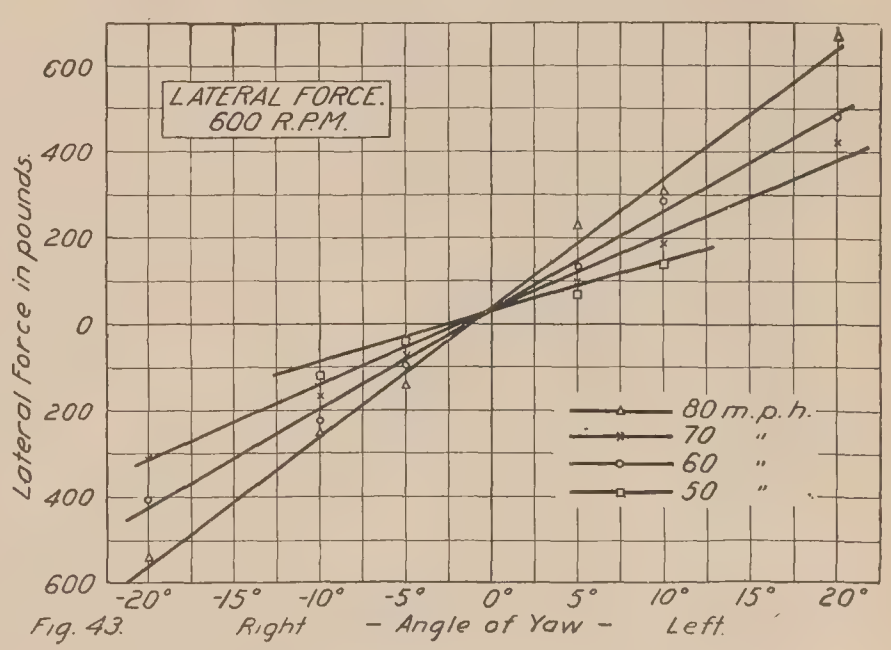


Fig. 43.

If the machine has an angle of bank ϕ, g will evidently be replaced by the resultant normal acceleration, R. Also, when the machine has an angle of pitch of θ, the term $\cos \theta$ must be introduced into the equation, so that it will then be:

$$a = g \frac{\cos \theta}{\cos \phi} \tan \alpha$$

As all the tests were made with the angle of bank at zero, and since the longitudinal angle in no case was enough to introduce any appreciable error, the simplified equation may be used.

There is one other correction, the deviation from the true reading when the air speed head is yawed. This correction was determined by a wind tunnel test of the instrument, and was found to be negligibly small.

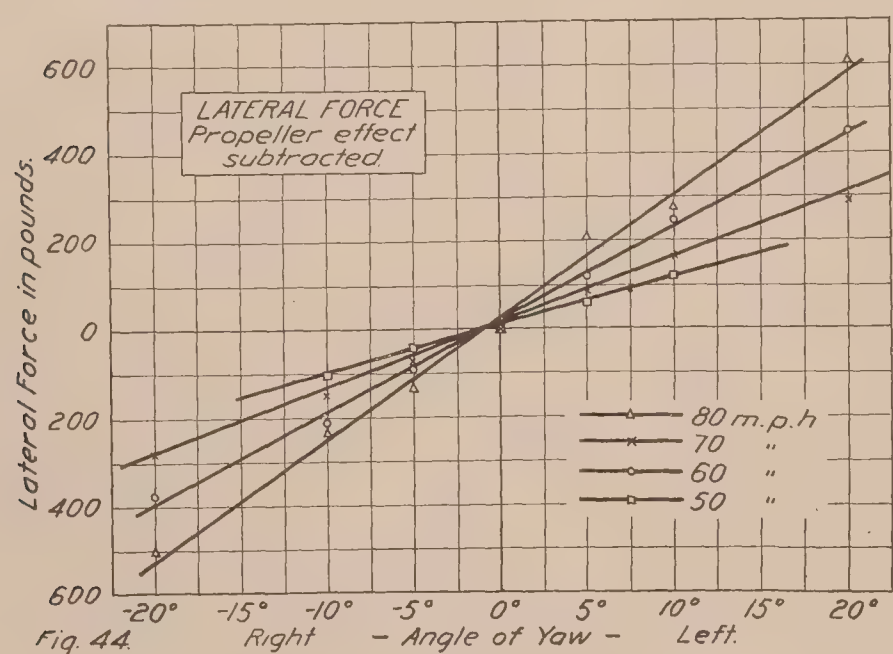


Fig. 44.

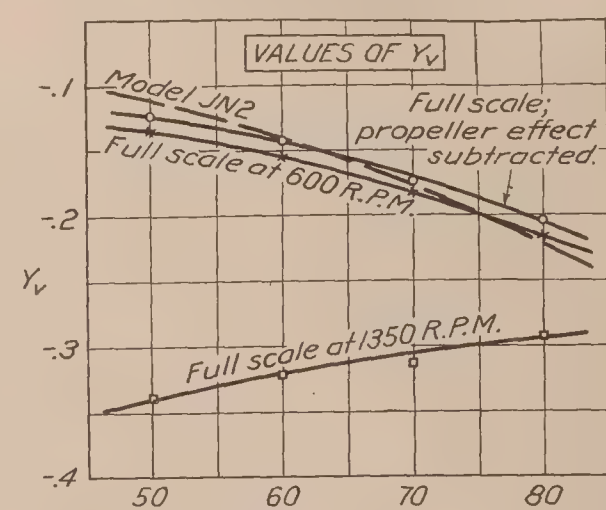


Fig. 45

In figure 42 are shown the curves obtained at 1,350 r. p. m. by plotting the lateral force in pounds, against angle of yaw, for different air speeds. In figure 43 is plotted the same thing at 600 r. p. m. It will be noticed that the lateral force is considerably smaller, being reduced by about 25 per cent. It might be thought that this difference was due to the fin effect of the propeller, but as will be shown later, this is not nearly enough to account for the difference. The discrepancy can only be accounted for, then, by the effect of the slipstream on the body, fin, and rudder.

It is desirable to determine Y_v without the effect of the propeller for comparison with model tests. This might be done by stopping the propeller in the air, but as it could not be stopped in the same position every time, it would lead to considerable errors. As the lateral force on a propeller can be computed very closely from tests run by the N. P. L., it was thought that the best results would be obtained by running the airscrew at approximately the speed of no thrust, and then subtracting its lateral force from the total of the whole machine. These corrected curves are shown in figure 44.

The derivative Y_v is equal to:

$$\frac{57.3}{Um} S$$

where U is the forward velocity in ft./sec.
m is the mass of the machine in slugs.
S is the slope of the lateral force curve in lbs./degree.

The values of Y_v as determined from the mean slope of the force curves are shown in figure 45, together with the curve obtained for the JN2 model,[2] showing a remarkably close agreement between the model and the full-sized machine with the propeller effect subtracted. The values of Y_v at 1,350 r. p. m. are, however, quite different, being nearly twice as large, and the slope of the curve is in the opposite direction. While these results give one greater confidence in the model tests for strictly comparable conditions, at the same time they emphasize the fact that all models should be tested with a revolving propeller to give actual slipstream effects, as the error introduced into stability calculations by neglecting the slipstream may be very large.

[2] Dynamical stability—Hunsaker.

The determination of L_v was accomplished by applying to the wing tips weights which were balanced by a certain movement of the ailerons. In this way it was possible to determine what angular setting of the ailerons corresponded to a given rolling moment and from these values the rolling moment produced in various degrees of yaw could be determined from the original curves shown in figures 32 and 33. As it would have been quite impossible to take off or land the machine with the large unbalanced weights on the wing tips it was necessary to apply equal weights on the wings and when in the air to release one of the weights so that the unbalanced force would remain. This was accomplished by placing on each wing tip a box with a hinged bottom containing sand so that the sand could be released from the cockpit, as shown in figure 46. Each one of the boxes was capable of holding 150 pounds and the machine was taken off with both boxes filled with a known weight of sand, and when ready to begin the test one box was emptied. When the test was completed the other box was emptied so that a landing could be made with no load in the boxes. Curves of rolling moment against aileron angle are shown in figure 47, and it will be seen that the lateral control on this machine is very powerful as an unbalanced load of 150 pounds on one wing tip can be easily supported, and this is all the more remarkable as this is a machine with ailerons only in the upper wing. The value of L_v is determined by the following formula:

$$L_v = \frac{57.3}{-Um} S$$

the symbols having the same meaning as before.

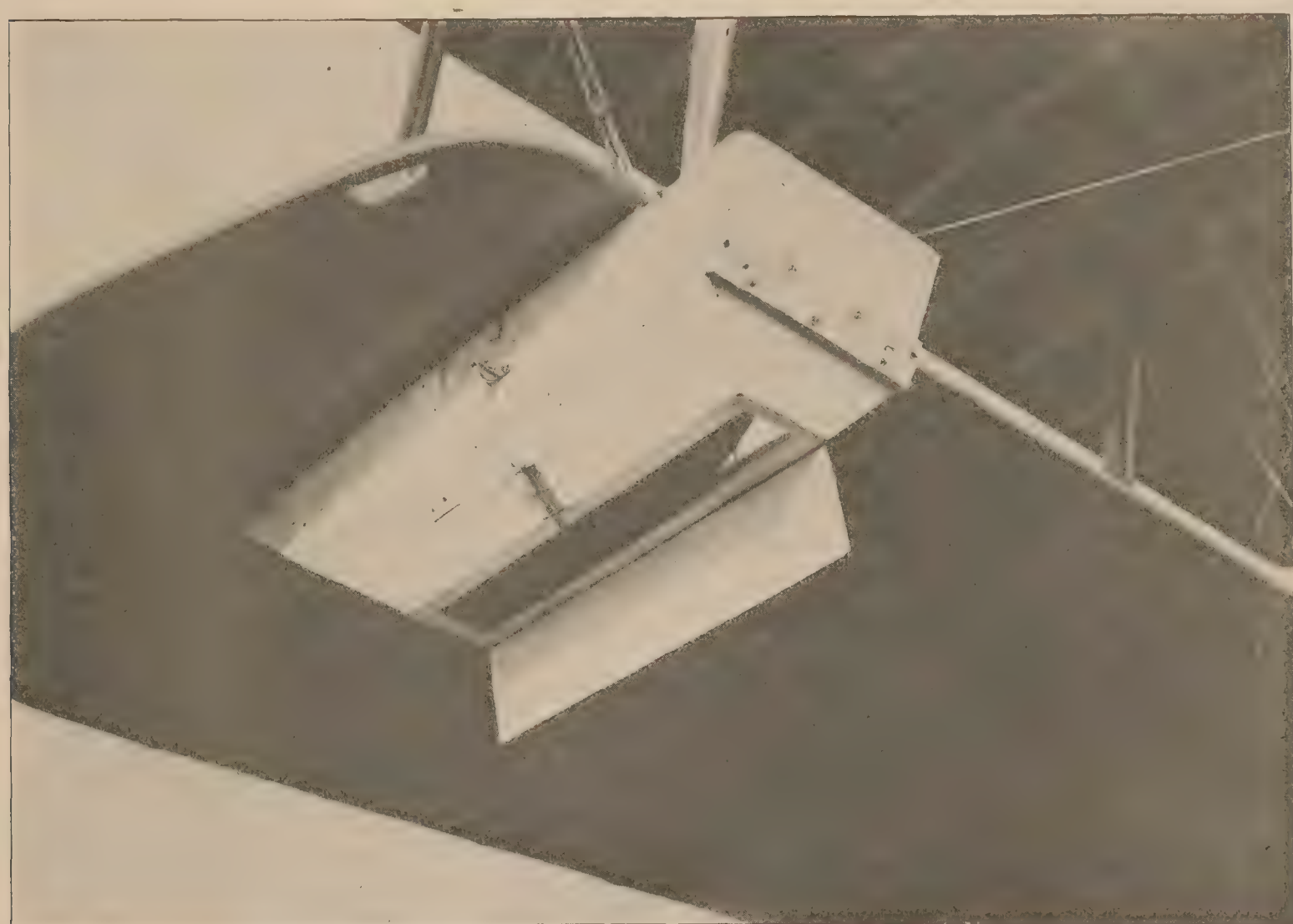


FIG. 46.—Method of applying a known rolling moment to the machine.

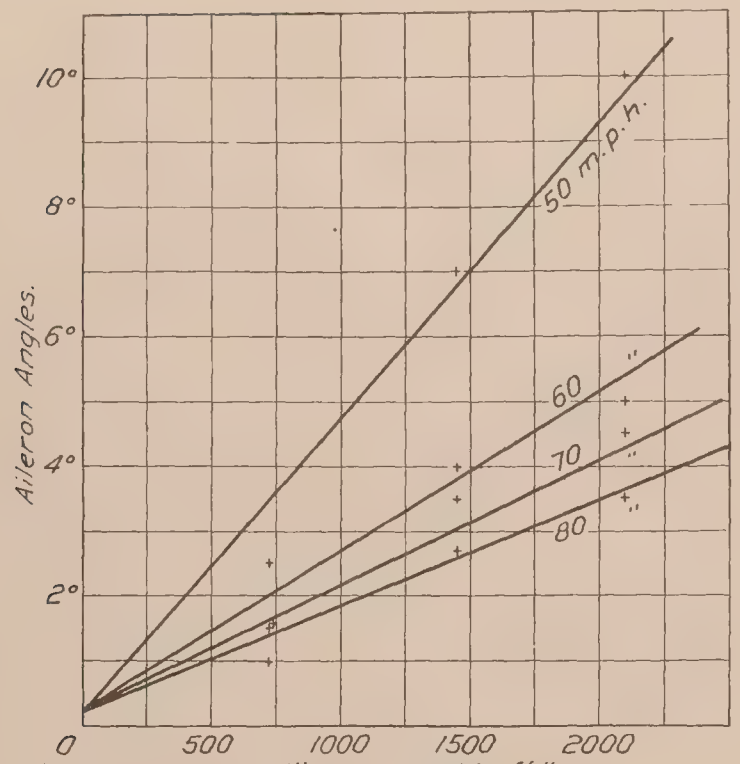


Fig 47

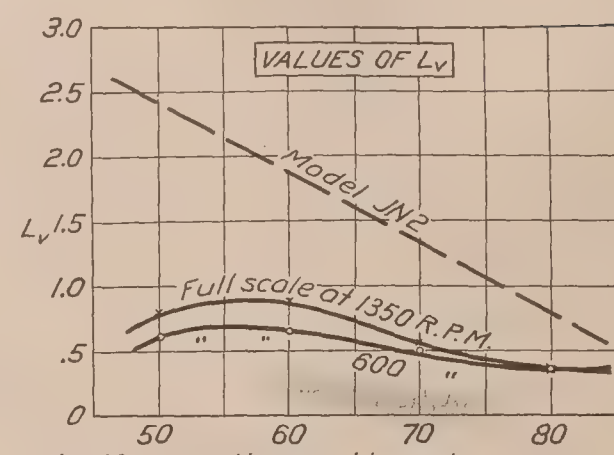


Fig. 48

In figure 48 there are plotted the curves for L_v with the motor on and the motor off and it will be seen that although the curves are quite consistent in shape, there is a slight difference between them, that is, the value with the motor on is slightly larger than the value with the motor off, but when comparing these values with the curve obtained from the model it will be seen that the full flight values are much lower than the latter, especially at the lower speeds. It is an interesting fact, however, that the curves from the full-sized test and from the model test seem to approach each other at the higher speeds and this can be explained by the fact that the ailerons at the high speeds on the full-sized machine are nearly at zero degrees, so that it approaches more and more the condition in which the model was tested, and it is believed that the discrepancy between the two tests is due to the fact that in the model ailerons were always kept at zero degrees, whereas, they should have been rotated to different angles for each angle of incidence of the machine.

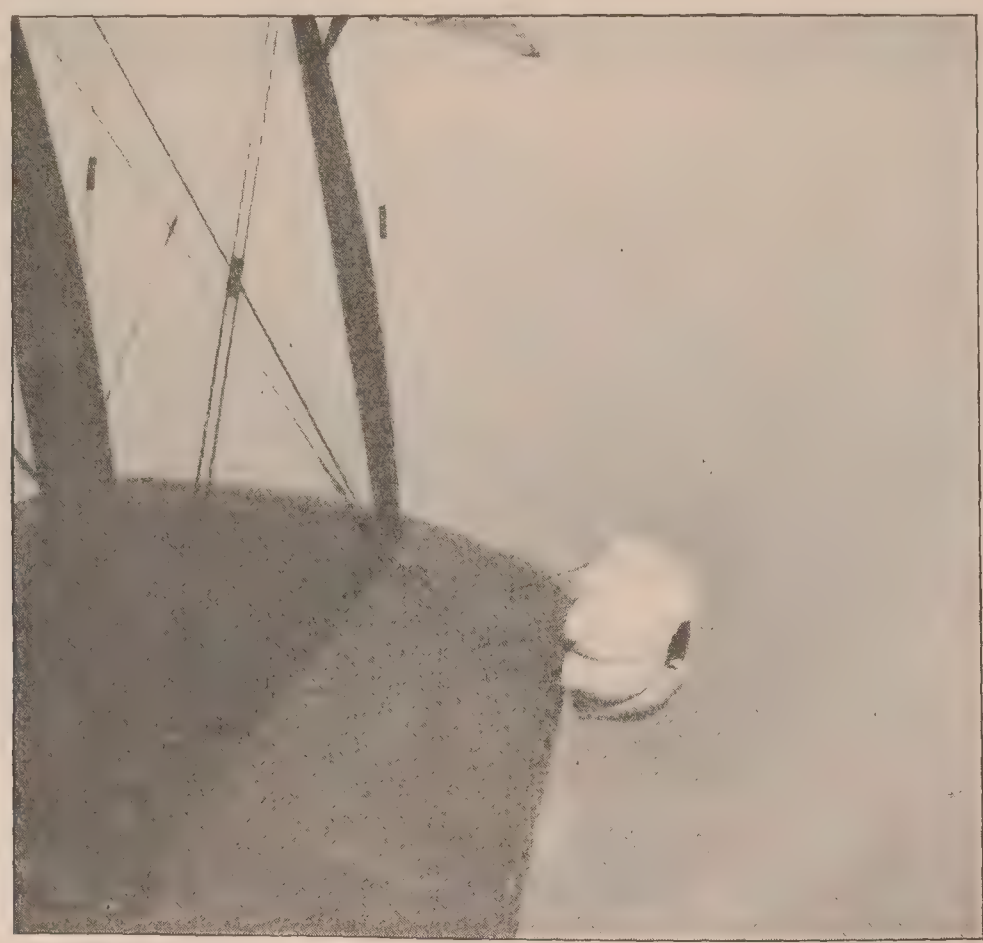


FIG. 49.—Method of applying a known drag to the wing tip by means of a small parachute.

The determination of N_v was accomplished by releasing small parachutes from one wing tip and measuring the pull of the parachute through a string which connected it to a spring scale in the cockpit, as shown in figure 49. In this way the moment for one degree change in rudder angle can be easily found and by using this constant the curve of rudder angle for various angles of yaw can easily be converted into yawing moment. The curves of yawing moment against rudder angle are shown in figures 50 and 51, each point representing the average of a large number of readings. The forces with the small parachute were so small that the accuracy

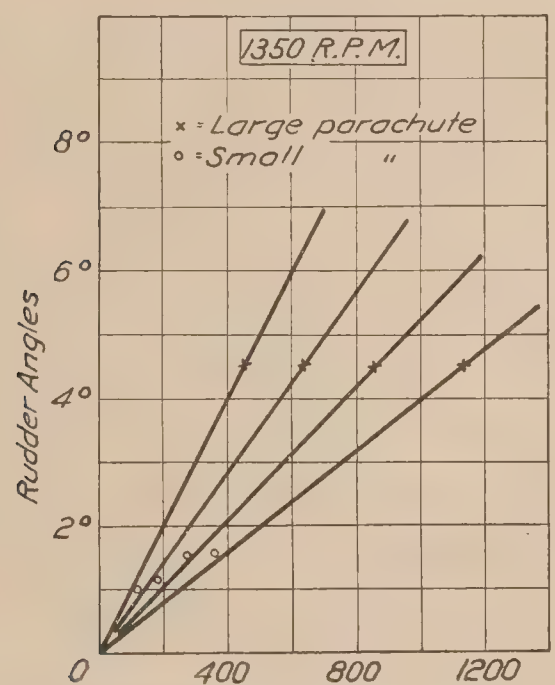


Fig. 50

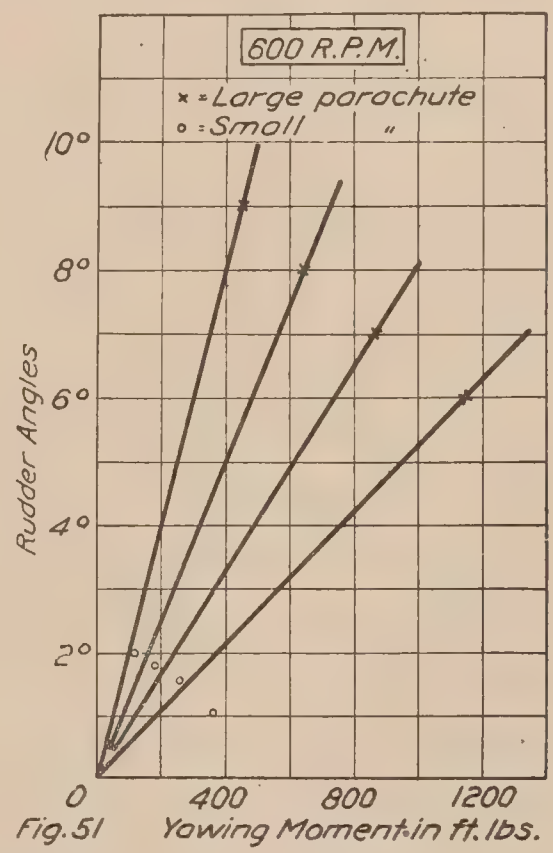


Fig. 51

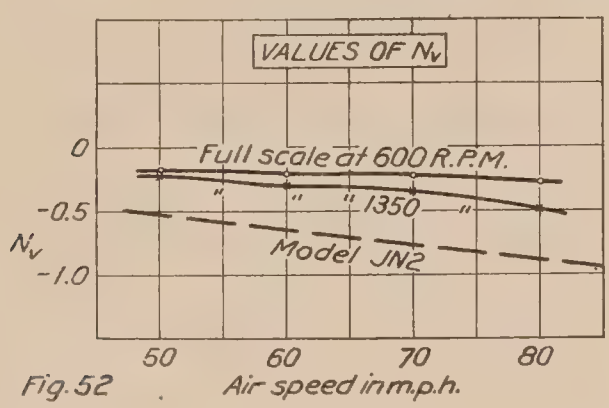


Fig. 52

of these readings is not great. N_v is found in the same way as L_v from the slope of the rudder position curves and the values obtained are plotted in figure 52. It is evident that the throttle setting makes very little difference with the value of N_v and that the values do not agree very closely with the model results, as the full scale results have a somewhat lower value than those of the model.

It should be realized when comparing these full flight stability derivatives with those from the model tests that the values obtained in full flight on account of the inherent errors in such testing can not be depended upon to better than 15 per cent and on the other hand the model test was made on a JN2 model, which is somewhat different from the JN4H, although the values of these stability derivatives should not be appreciably affected by this difference.

CONCLUSIONS.

The longitudinal stability is but little affected by either bank or side slip, although the forces and positions of the elevator may change to a considerable extent. In any bank the machine tends to increase its bank unless restrained both by force and by position of the ailerons. In a side slip the machine will continue to increase the angle of yaw unless restrained by the rudder. It is recommended that the lateral dihedral be increased, and that more fin surface be added, in which case the machine would probably be laterally stable at least through a small range about its symmetrical position. Also the leading edge of the fin should be moved to the left and a balanced portion added to the top of the rudder in order to neutralize the constant force on the rudder bar. The value of Y_v is greatly altered by the slipstream, so that the results from model tests can only be applied to gliding flight. The value of L_v is quite different at low speed from the model value, but agreement is approached at high speed. N_v is only slightly affected by the engine speed, and is not in close agreement with the model.

REPORT No. 113

TESTS ON AIR PROPELLERS IN YAW

By W. F. DURAND and E. P. LESLEY
Leland Stanford Junior University

REPORT No. 113.

TESTS ON AIR PROPELLERS IN YAW.

By W. F. Durand and E. P. Lesley.

This report was prepared by W. F. Durand and E. P. Lesley, for the National Advisory Committee for Aeronautics and contains the results of tests to determine the thrust (pull) and torque characteristics of air propellers in movement relative to the air in a line oblique to the line of the shaft, and specifically when such angle of obliquity is large, as in the case of helicopter flight with the propeller serving for both sustentation and traction.

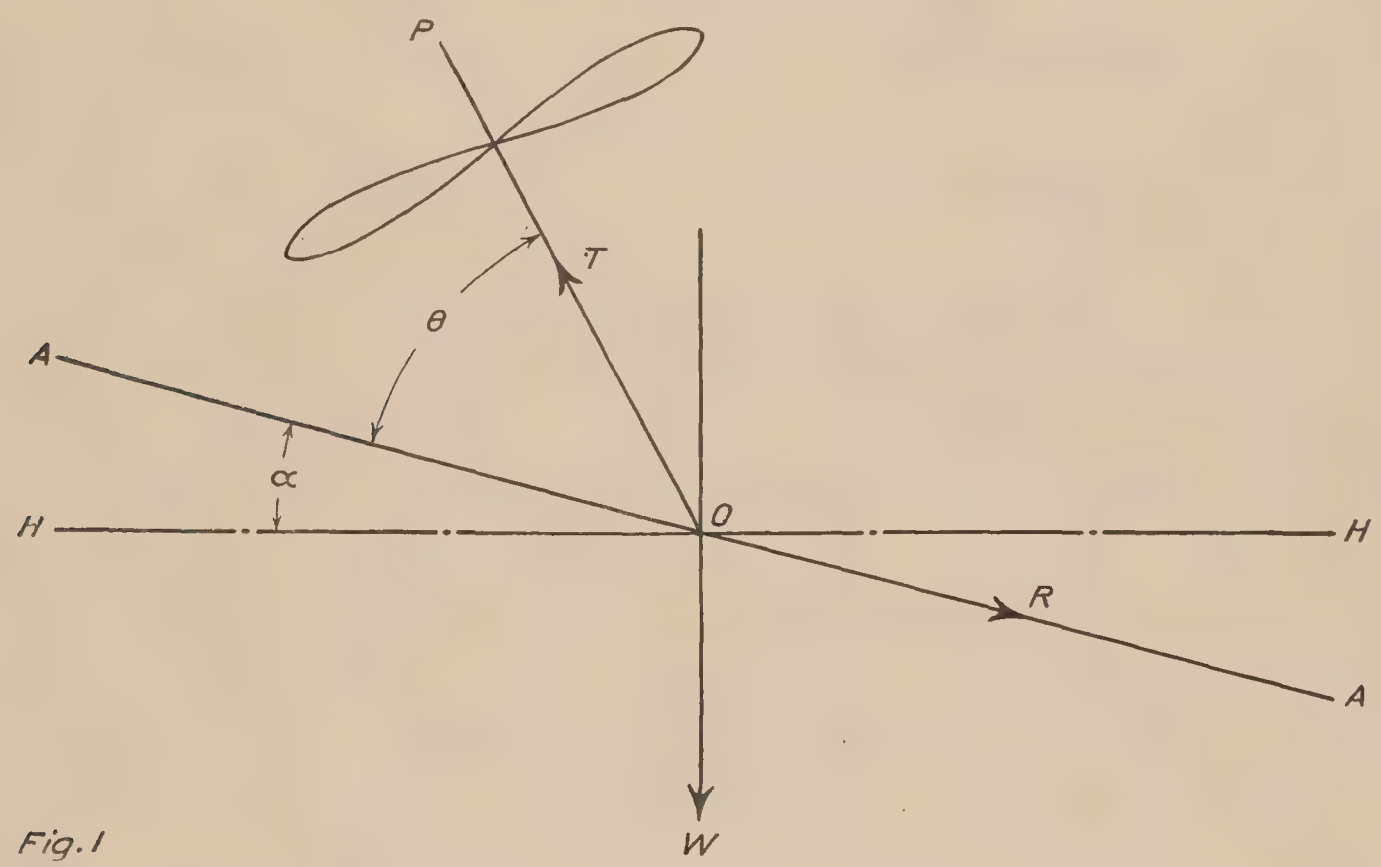


Fig. 1

Diagramatically let figure 1 represent such a propeller attached to an aircraft of some form with shaft OP inclined at angle θ to the line of motion AA, which is itself inclined at angle α to the horizontal HH.

Let T denote the traction on the shaft. Let R denote the traction resistance in the line of motion. Then assuming that T is the only force reaction of the propeller on the structure and assuming likewise conditions of right line nonaccelerated motion, we shall have

$$T \sin (\alpha + \theta) = W \tag{1}$$

$$T \cos \theta = R \tag{2}$$

The resistance R will be given by an equation of the form—

$$gR = k\Delta L^2 v^2 \tag{3}$$

Where k = coefficient depending on the geometrical form of the aircraft in question.
Δ = density of air.
L = Some standard or type lineal dimension of the aircraft.
v = velocity.

It will be noted that a coefficient k thus defined will be independent of the particular units employed so long as they form a homogeneous system (foot, pound, and second, or meter, kilogram, and second.)

We have then:

$$\frac{\sin(\alpha+\theta)}{\cos\theta}=\frac{W}{R}=\frac{gW}{k\Delta L^2v^2} \tag{4}$$

or—

$$gW=k\Delta L^2v^2\frac{\sin(\alpha+\theta)}{\cos\theta} \tag{5}$$

From these equations a number of elementary problems involving such motion may be solved, once we have the means of relating the force T to the general conditions of the problem.

The present investigation has then for its primary purpose the development of the relations between T and the operating conditions, including at the same time the similar relations for the torque Q.

With these relations established and suitable data known or assumed, the solution of problems arising in flight of this character and involving power, speed, etc., will be reduced to the same general program as in the usual case of airplane flight.

No further discussion of these phases of the problem will be given in the present report, the primary purpose of which is to furnish the data for the investigation of flight conditions of the character outlined.

LIMITATION IN SIGNIFICANCE OF RESULTS OBTAINED.

As noted in the preceding paragraph, the purpose of the investigation has been the determination of the thrust and torque characteristics of air propellers under conditions of movement sharply oblique to the line of the shaft.

These forces, however, do not represent the entire force reaction between the propeller and the aircraft. The latter will involve additional forces arising from two sources.

(1) A lateral or side thrust acting from the shaft to its bearings and due to the lack of symmetry of the air stream relative to the propeller shaft and the resulting lack of symmetry between the two opposite blades of the propeller in their relation to the air stream.

(2) A distortion of the lines of airflow relative to the aircraft, and due to the action of the propeller, thus modifying the free resistance of the aircraft moving with the attitude and speed assumed.

It is to be hoped that the character and significance of these secondary forces may be made the subject of a later study. It may, however, be noted in this connection that certain experimental difficulties are foreseen in such a study in connection with the elimination and determination of the lateral force (1) independent of (2) the effect due to distorted wind streams. These are, however, matters which are only incidental to the present report and are mentioned only to indicate the fact that the investigations reported on herein have been restricted to a study of the direct thrust and torque characteristics as noted.

MODELS SELECTED FOR TEST.

Twelve models, numbered 5, 6, 7, 8, 9, 10, 11, 12, 139, 144, 145, and 146 were selected. The principal characteristics of these models are shown in Table I. In the above table, blade shape No. 1 is shown in Report No. 14, N. A. C. A., figure 14, blade shape No. 2 in figure 15, while the various blade sections applicable to these forms are shown in figures 16, 18, 20, and 22, all of the same report. Models Nos. 5 to 12, inclusive, are among those tested in 1917, to determine performance coefficients under conditions of flight, and again in 1918, when standing thrust and power coefficients were derived.

TABLE I.—*Characteristics of model propellers.*

No.	Diameter (inches).	Pitch (inches).	Pitch ratio.	Mean blade width.	Shape of blade.	Blade section.
5	36	25.2	0.7	0.15*r*	1	1
6	36	25.2	.7	.20*r*	1	1
7	36	25.2	.7	.15*r*	2	1
8	36	25.2	.7	.20*r*	2	1
9	36	18.0	.5	.15*r*	1	1
10	36	18.0	.5	.20*r*	1	1
11	36	18.0	.5	.15*r*	2	1
12	36	18.0	.5	.20*r*	2	1
139	36	10.8	.3	.15*r*	2	1
144	36	10.8	.3	.20*r*	2	1
145	36	10.8	.3	.15*r*	1	1
146	36	10.8	.3	.20*r*	1	1

All of the above numbered propellers are, as shown in Table I, 3 feet in diameter, with uniform nominal pitch of driving face and are noncambered. There are thus represented among the models tested two forms, straight and curved; three pitch ratios, 0.7, 0.5 and 0.3; and two blade areas corresponding to mean blade width ratios of 0.15 and 0.20.

METHOD OF TEST.

The 0.3 pitch ratio models, Nos. 139, 144, 145, and 146, were first tested in the usual manner (described in Report No. 30, N. A. C. A.) to determine standing thrust and power coefficients. Tests to check those made in 1918 (Report No. 30) were also made on the other models. The dynamometer, after slight alterations, was then turned through an angle of 90° so that the propeller shaft was at a right angle to the wind stream through the experiment chamber. The following program was then conducted with each model. A wind velocity of about 30 feet per second was set up with the tunnel fan. The model was then rotated at various speeds from 10 to 50 revolutions per second and simultaneous observations of thrust, torque, or turning moment, revolutions, and wind velocity were made and recorded. Seven to ten observations, representing the same number of angular velocities, were made for each propeller. These tests were repeated with a wind velocity of about 45 feet per second and finally with one of about 60 feet per second. During a single series of observations the wind velocity was maintained approximately constant only. Because of the effect of the model itself in drawing air out of the stream and forcing it into the experiment chamber, there developed some increase in the experiment chamber pressure and a corresponding reduction in the velocity of the wind stream, especially at the higher angular velocities of the model.

Following the observations at an angle of 90°, the dynamometer was then set so that the propeller shaft and the axis of the tunnel formed an angle of 85° and the same program of tests, with the three wind velocities of about 30, 45, and 60 feet per second and seven to ten rotative speeds for each propeller, was carried out. This was repeated with angles of 80°, 70°, and 60°. The whole lot of experimental data thus represents, aside from the experiments to determine standing thrust and power coefficients, about 1,500 simultaneous observations of thrust, turning moment, angular velocity, and wind velocity, distributed about equally over five angles of approach, 90°, 85°, 80°, 70°, and 60°. For each turning moment or torque determination, an observation of torque zero was made.

REDUCTION OF DATA.

The data were reduced to the coefficients—

$$\frac{V}{ND},\quad C_t=\frac{g\times\text{thrust}}{\triangle N^2D^4},$$

$$\text{and } C_q=\frac{g\times\text{torque}}{\triangle N^2D^5}.$$

In $\frac{V}{ND}$, V is the wind velocity, N the rate of revolution, and D the diameter. In C_t and C_q, N and D have the same values as in $\frac{V}{ND}$ and Δ is the weight per unit volume of the me-

dium. It may be noted that these coefficients are independent of the units used provided the latter are homogeneous.

The coefficient C_t and ten times the coefficient C_q were plotted as ordinates, upon $\frac{V}{ND}$ as abscissæ, for each angle of approach. The result for propeller No. 8 at 80° is shown on Plate I. As may be noted, there is some dispersion of the plotted data from a single smooth curve, and the data for one wind speed indicate, in general, values of the coefficients somewhat different from those shown by the tests at the other wind speeds. Plate I is representative of the dispersion of the data from a fair curve in the average case. With some propellers it is somewhat greater and with others somewhat less. As an experiment the data on Plate I were changed in form and plotted as shown on Plate II. In this instance—

$$C'_t = \frac{g \times \text{thrust}}{\triangle V^2 D^2}$$

$$\text{and } C'_q = \frac{g \times \text{torque}}{\triangle V^2 D^3}.$$

As may be seen, these coefficients are of the form used previously in plotting data for tests of models under the condition of flight or of axial wind, and are derived from the coefficients shown on Plate I by dividing the values of C_t and C_q there indicated by the square of the corresponding $\frac{V}{ND}$. It is interesting to note that although mathematically the data as shown on Plates I and II are the same, the appearances are in the two cases very different.

The form as shown on Plate I was selected for presentation since coefficients without the factor V in the denominator are necessary in order to permit of graphic representation at $\frac{V}{ND}=0$.

In Plates III to XIV are shown the results as determined for the twelve propellers. Each curve as shown represents an average of points plotted. The actual marking of the spots is omitted in order to avoid confusion. The original observations, reduced to the coefficients C_t and C_q, are, however, given in Table II against the various values of $\frac{V}{ND}$ as resulting from the three different wind speeds employed. The curves of Plates III to XIV are in some instances slightly different from those at first drawn through the plotted data. These modifications were the result of drawing cross curves as shown on Plates XV to XXVI. For these latter plates the ordinates are as before, C_t and C_q, but the abscissæ are values of angle of approach.

DISCUSSION OF RESULTS.

Thrust.—As may be seen by reference to Plates III to XXVI, the effect of side wind at any angle of approach greater than a certain value for any one propeller is to increase the thrust developed at a fixed tip speed. For the propellers of 0.7 pitch ratio (propellers 5, 6, 7, and 8) the angle at which there is no change in thrust with change in wind speed is about 70°. For angles less than this the thrust varies inversely with wind velocity. For the 0.5 pitch ratio (propellers, 9, 10, 11, and 12) the angle of approach for approximately constant thrust with constant tip speed but variable wind velocity is between 72° and 74°. Not only is there greater variation in this angle for the 0.5 pitch ratio propellers than for those of 0.7 pitch ratio, but also the angle is, for a given propeller, usually less definite. For the 0.3 pitch ratio (propellers 139, 144, 145, and 146) the constant thrust angle lies between 73° and 78°, showing a wider variation than for the 0.5 pitch ratio as well as, in some cases, a more pronounced lack of definition.

For a constant value of $\frac{V}{ND}$ the thrust coefficient C_t varies directly with the angle of approach.

Torque.—The changes in torque with variation of side wind velocity are less marked than those in thrust. They appear to be in general somewhat less consistent. For the 90° angle

of approach it appears that the torque at first generally decreases with increase of wind velocity and later increases. An exception to this general rule is noted in the propeller No. 139 (Pl. XI) where the trend of the torque coefficient curve is slightly downward throughout its length. It may be noted, however, that for 85° and 80° the previously stated general rule appears to be followed. For the angle of 60° the torque coefficient curves on abscissæ of $\frac{V}{ND}$ are generally convex upward instead of downward as with 90°. For other angles the torque coefficient curves on abscissæ of $\frac{V}{ND}$ have a generally rising characteristic except at small values of $\frac{V}{ND}$ where in some cases they have very similar characteristics to the 90° curve.

An examination of Plates XIV to XXVI shows that with few exceptions there is an angle of approach for which the torque is practically constant for varying wind velocity. This angle is generally from 6° to 10° less than the angle for constant thrust. It may be also noted that while the torque coefficient curves of Plates III to XIV appear in some instances inconsistent, the cross curves on Plates XV to XXVI show in general similar characteristics. With few exceptions they are convex upward and show, with decreasing angle of approach, at first a rise and later a fall in the value of C_q.

PROPELLER No. 5.

60° yaw.			70° yaw.			80° yaw.			85° yaw.			90° yaw.		
$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.
0.620	0.1195	0.01146	1.76	0.1345	0.01434	1.15	0.1715	0.0130	1.58	0.2145	0.0150	0.000	0.151	0.01105
.413	.1290	.01092	1.27	.1445	.01366	.820	.1666	.01145	1.09	.1945	.01155	.527	.159	.0096
.352	.1394	.01122	1.06	.1467	.01311	.605	.1611	.01111	.954	.1834	.01155	.373	.151	.0105
.314	.1421	.01118	.850	.149	.01255	.484	.160	.01105	.745	.1734	.01133	.250	.150	.0105
.277	.1450	.01150	.770	.1522	.01245	.433	.1589	.01133	.650	.169	.01155	.181	.153	.0105
.250	.1471	.01172	.674	.1545	.01222	.424	.1589	.01133	.565	.1655	.01155	.159	.157	.01095
.240	.1490	.01172	.595	.1545	.01222	.390	.1577	.01145	.561	.1645	.01166	.145	.1735	.014
1.508	.0866	.01145	.572	.1323	.0120	.553	.1455	.01111	.543	.1556	.01072	1.05	.198	.0107
1.133	.1068	.01211	.411	.1378	.01145	.402	.1466	.01033	.414	.1534	.01061	.720	.173	.01065
.95	.1154	.01200	.325	.1434	.01122	.275	.1472	.0107	.282	.1511	.0105	.537	.1635	.01042
.776	.1271	.01200	.234	.1511	.01133	.212	.1495	.01088	.215	.1556	.01078	.447	.159	.01095
.665	.1311	.01345	.204	.1500	.01133	.191	.1545	.0111	.1915	.1495	.01031	.398	.158	.01093
.57	.1353	.01279	.178	.1489	.01145	.160	.1578	.01095	.186	.1561	.01078	.356	.157	.0112
.5525	.1384	.01245	.159	.1522	.01166	.138	.159	.01145	.158	.1578	.01122	.313	.157	.0112
1.027	.1038	.01180	1.278	.1328	.01366	1.95	.201	.01555	.173	.1528	.01116	1.91	.261	.01495
.7545	.1204	.01171	.882	.1406	.01250	1.38	.187	.01368	1.194	.1866	.01355	1.28	.209	.0125
.623	.1261	.01156	.690	.1455	.01212	1.08	.179	.01268	.827	.1745	.01139	.935	.189	.0113
.508	.1340	.01171	.597	.1466	.01172	.853	.171	.01168	.605	.1655	.01061	.772	.1745	.0112
.429	.1396	.01144	.471	.1490	.01122	.750	.168	.01142	.473	.1590	.01067	.672	.172	.01118
.3935	.1419	.01156	.430	.1566	.01145	.665	.167	.01178	.431	.1590	.01081	.613	.171	.01145
.371	.1455	.01195	.407	.1522	.01155	.572	.1645	.01178	.421	.1590	.0111	.531	.169	.01145
									.372	.1611	.01122			

PROPELLER No. 6.

0.572	0.1368	0.0129	1.277	0.1485	0.01346	0.548	0.1735	0.0121	1.694	0.2502	0.01659	0.605	0.1920	0.01155
.409	.1480	.01252	.894	.1570	.01324	.384	.1701	.01172	1.30	.222	.01445	.609	.1916	.0120
.365	.1500	.01239	.724	.1605	.01311	.278	.1695	.01184	.990	.211	.01341	.369	.1765	.01122
.311	.1532	.01232	.548	.1622	.01248	.218	.1673	.01200	.808	.200	.0130	.373	.1732	.01180
.2976	.1562	.01240	.477	.1636	.01227	.1875	.1682	.01222	.703	.191	.01266	.378	.1769	.01146
.270	.1585	.01248	.449	.1628	.01241	.1675	.170	.01245	.624	.187	.01279	.378	.1754	.01162
.262	.1596	.01262	.419	.1646	.01265	.1495	.1712	.0125	.540	.1845	.01273	.257	.1715	.01121
1.47	.0948	.01000	1.744	.1616	.01490	1.187	.1995	.01440	.537	.1777	.01181	.256	.1722	01153
1.117	.1142	.01185	1.26	.1600	.01370	.838	.1884	.01305	.421	.1752	.01141	.209	.1710	.01190
.955	.1234	.01222	1.028	.1616	.01350	.673	.1810	.01240	.292	.1678	.01160	.209	.1720	.01193
.77	.1335	.01245	.904	.1623	.01331	.544	.1780	.01230	.221	.1690	.01168	.178	.1731	.01215
.704	.1391	.01270	.789	.1635	.01322	.490	.1758	.01240	.188	.1697	.01204	.178	.1728	.01199
.611	.1441	.01276	.696	.1878	.01326	.445	.1755	.01242	.178	.169	.01212	.152	.1733	.01191
.566	.1471	.01281	.607	.1655	.01316	.402	.1752	.01256	.165	.1699	.01226	.156	.1731	.01212
1.037	.1117	.0124	.569	.1568	.01275	1.998	.248	.01704	1.240	.226	.01410	1.120	.2366	.01415
.755	.1332	.01242	.418	.1585	.01217	1.31	.209	.01476	.838	.202	.01345	1.140	.2405	.01492
.605	.1392	.01249	.331	.158	.01219	1.118	.204	.01423	.6195	.1842	.0120	.760	.2069	.0130
.4685	.1494	.01257	.258	.1621	.01225	.874	.194	.01320	.476	.179	.01180	.755	.2056	.01293
.417	.1518	.01257	.226	.1649	.01244	.781	.189	.01313	.4445	.1785	.01210	.539	.1879	.01245
.3925	.1528	.01248	.198	.1666	.01257	.677	.184	.01303	.417	.1784	.01239	.543	.1890	.01261
.387	.1542	.0127	.175	.1679	.01262	.589	.181	.01283	.388	.177	.01237	.461	.1828	.01229
												.453	.1828	.01229
												.361	.1792	.01206
												.348	.1735	.01187
												.293	.1770	.01203

Propeller No. 7.

60° yaw.			70° yaw.			80° yaw.			85° yaw.			90° yaw.		
$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.
0.580	0.1260	0.01149	0.552	0.137	0.0110	1.138	0.1760	0.01209	1.174	0.1879	0.01132	1.77	0.2455	0.01256
.399	.1354	.01092	.403	.1425	.0106	.798	.1640	.01080	.816	.1731	.01028	1.19	.1992	.01140
.3567	.1382	.01079	.3032	.145	.0105	.625	.1600	.01041	.593	.1620	.00972	.935	.1871	.01037
.294	.1394	.01055	.2362	.146	.0104	.484	.1548	.01015	.490	.1603	.00970	.757	.1743	.01020
.2733	.1418	.01061	.1999	.147	.0103	.435	.1548	.01021	.424	.1550	.00976	.666	.1702	.01034
.2595	.1428	.01012	.1790	.147	.01025	.415	.1540	.01035	.394	.1527	.00978	.612	.1673	.01013
.2285	.1404	.01072	.1528	.147	.0105	.374	.1533	.01033	.353	.1517	.00996	.544	.1615	.00999
1.49	.0921	.01151	1.710	.1455	.0143	1.872	.213	.01440	.532	.1567	.00984	.475	.1580	.01008
1.12	.1000	.01192	1.230	.1435	.0126	1.345	.185	.01240	.399	.1532	.00973	1.23	.2050	.01160
.951	.1178	.01167	.995	.1460	.0123	1.052	.173	.01142	.270	.1512	.00975	.868	.1802	.01074
.78	.1266	.01165	.845	.1480	.0118	.851	.167	.01085	.214	.1490	.00964	.636	.1732	.01007
.666	.1366	.01145	.722	.1480	.0115	.723	.163	.01060	.187	.1508	.00906	.547	.1624	.00992
.584	.1345	.01140	.625	.1480	.0112	.612	.159	.01065	.169	.1495	.00994	.433	.1558	.00992
.557	.1375	.01151	.553	.1480	.0112	.530	.160	.01060	.146	.1505	.00967	.399	.1544	.00996
1.024	.1038	.01202	1.279	.1377	.01285	.561	.152	.01065				.360	.1532	.00998
.742	.1233	.01160	.856	.1430	.01133	.373	.148	.00983				.287	.1522	.00999
.605	.1298	.01128	1.342	.1389	.0133	.271	.147	.00983				.562	.1190	.01007
.4405	.136	.0110	.908	.1420	.01202	.207	.146	.00898				.375	.1497	.01010
.398	.1383	.01111	.738	.1462	.0117	.177	.147	.00975				.276	.1493	.00982
.3675	.1379	.01091	.632	.1458	.01124	.155	.149	.0101				.214	.1503	.00962
.3585	.1399	.01100	.512	.1458	.0107	.139	.151	.0103				.179	.1506	.00980
			.431	.1440	.01047							.1585	.1502	.00990
			.399	.1460	.01041							.146	.1511	.00990
			.398	.1465	.01054							.000	.1434	.01009

Propeller No. 8.

1.619	0.0882	0.00988	1.784	0.162	0.01468	2.04	0.2450	0.0159	1.625	0.244	0.01535	0.	0.1598	0.0114
1.18	.1186	.01092	1.293	.157	.01406	1.452	.212	.0148	1.155	.221	.0137	.544	.1781	.01042
1.007	.1184	.01155	1.072	.1592	.0137	1.066	.1995	.0134	.858	.203	.01255	.373	.1738	.01088
.885	.1260	.01200	.907	.1595	.0134	.889	.1925	.0126	.712	.1915	.0119	.267	.1675	.01120
.718	.1340	.01189	.792	.163	.0132	.755	.1820	.0122	.625	.186	.01155	.221	.1671	.01144
.636	.1387	.01205	.707	.162	.0130	.645	.1810	.0121	.529	.1805	.0114	.134	.1675	.01130
.592	.1409	.01219	.609	.1623	.0127	.542	.1765	.01175	.491	.180	.0116	.176	.1681	.01132
1.068	.1173	.01198	1.305	.153	.0141	1.203	.200	.0143	1.260	.2241	.01389	.167	.1690	.01138
.765	.1294	.01182	.910	.159	.0131	.840	.187	.0129	.849	.1955	.01239	1.020	.2300	.01239
.624	.1372	.01183	.735	.157	.01245	.651	.178	.0116	.611	.1828	.01112	.719	.2020	.01153
.506	.1440	.01185	.628	.158	.0122	.558	.1695	.0114	.467	.1731	.01117	.528	.1821	.01078
.430	.1465	.01189	.500	.161	.01175	.455	.168	.0112	.433	.1753	.01121	.443	.1754	.01093
.393	.1487	.01182	.450	.162	.0117	.433	.169	.0113	.410	.1732	.01132	.380	.1703	.01093
.396	.1582	.01264	.422	.162	.01195	.392	.1685	.0114	.376	.1720	.01130	.327	.1720	.01098
.747	.1290	.01175	.554	.1615	.0128				.538	.1799	.0120	.297	.1711	.0111
.482	.1442	.01190	.419	.157	.01205				.404	.1733	.01109	1.707	.2725	.01417
.3915	.1475	.01181	.305	.157	.01170				.281	.1662	.01120	1.182	.2360	.01360
.3500	.1495	.01192	.238	.161	.01180				.219	.1654	.01140	.871	.2125	.01220
.3170	.1476	.01179	.2175	.161	.01190				.190	.1660	.01158	.701	.1978	.01179
.2781	.1550	.01190	.1878	.162	.01180				.168	.1647	.01152	.613	.1932	.01138
.254	.1562	.01220	.161	.164	.01210				.157	.1665	.01172	.545	.1864	.01113
												.512	.1853	.01117

Propeller No. 9.

1.393	0.0438	0.00659	1.553	0.0894	0.00875	1.720	0.1490	0.00955	1.355	0.167	0.0088	0	0.1180	0.00718
1.058	.0676	.00689	1.133	.1005	.00814	1.230	.1410	.00825	.978	.1535	.00734	.472	.1282	00670
.886	.0786	.00728	.880	.1056	.00805	.937	.1370	.00787	.743	.142	.00694	.337	.1225	.00686
.710	.0876	.00745	.730	.1161	.00783	.748	.134	.00725	.596	.1365	.00689	.257	.1219	.00683
.609	.0935	.00733	.622	.1132	.00778	.599	.131	.00713	.505	.1325	.00695	.2025	.1211	.00682
.531	.0984	.00738	.535	.1158	.00773	.519	.130	.00712	.447	.131	.00698	.183	.1202	.00677
.465	.1025	.00748	.514	.1170	.00788	.432	.129	.00740	.404	.1295	.00718	.158	.1217	.00694
.932	.0716	.0074	1.110	.0978	.00792	1.022	.132	.00805	1.116	.1591	.00776	.144	.1227	.00710
.675	.0875	.00707	.795	.106	.00940	.729	.129	.00738	.746	.1401	.00702	.131	.123	.00727
.523	.097	.00744	.603	.1115	.00777	.549	.1255	.00700	.551	.1309	.00694	.872	.1557	.00760
.4295	.103	.00722	.481	.113	.00713	.440	.123	.00693	.442	.1268	.00672	.541	.1367	.00719
.397	.1038	.00732	.389	.1165	.00724	.378	.123	.00695	.394	.1262	.00673	.413	.1290	.00685
.344	.1078	.00728	.359	.1180	.00726	.347	.125	.00708	.366	.1275	.00696	.330	.1273	.00671
.321	.1087	.00684	.332	.1175	.0075	.326	.125	.00723	.320	.1256	.00718	.281	.1273	.00699
.315	.1101	.00738	.516	.104	.00798	.471	.118	.00746	.491	.1270	.00802	.260	.1271	.00706
.511	.0901	.00739	.368	.109	.00738	.349	.117	.00702	.356	.1214	.00720	.238	.1272	.00725
.362	.0998	.00699	.306	.112	.00743	.255	.115	.00733	.255	.1191	.00709	.451	.1347	.00716
.315	.1003	.00719	.219	.114	.00733	.184	.118	.00710	.206	.1195	.00709	1.301	.1842	.00779
.288	.1070	.00727	.174	.1165	.00727	.157	.118	.00710	.159	.1206	.00716	.921	.1623	.00690
.249	.1090	.00731	.156	.119	.00738	.135	.121	.00724	.140	.1228	.00731	.701	.1480	.00640
.2185	.1100	.00736	.129	.119	.00754	.117	.122	.0075	.126	.1222	.00744	.614	.1440	.00692
.2105	.1152	.0077										.461	.1348	.00680
												.391	.1322	.00688

Propeller No. 10.

60° yaw.			70° yaw.			80° yaw.			85° yaw.			90° yaw.		
$\frac{V}{ND}$.	C_t	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.
1.346	0.0417	0.00564	1.510	0.098	0.00871	0.512	0.1345	0.00720	1.410	0.193	0.0101	1.503	0.2245	0.00961
1.03	.0652	.00680	1.135	.1075	.00861	.368	.1133	.00687	1.015	.1755	.00868	1.056	.1992	.00839
.815	.0815	.00713	.882	.1150	.00863	.250	.130	.00772	.7725	.1600	.00810	.865	.1820	.00795
.688	.0915	.00760	.710	.1180	.00841	.220	.130	.00780	.636	.1525	.00778	.703	.1671	.00771
.614	.0988	.00760	.611	.1225	.00844	.162	.1311	.00788	.565	.150	.00778	.621	.1618	.00756
.537	.1030	.00775	.535	.1245	.00858	.144	.1322	.00813	.4725	.1470	.00782	.528	.1570	.00772
.483	.1078	.00792	.475	.1260	.00850	.130	.1333	.00842	.422	.1445	.00780	.421	.1513	.00802
.925	.0733	.00708	1.115	.1045	.00873	.1020	.1510	.00847	1.04	.176	.00930	.481	.1552	.00786
.690	.0915	.00746	.805	.1145	.00820	.732	.1435	.00764	.728	.157	.00801	1.116	.2010	.00910
.531	.1015	.00783	.608	.1190	.00824	.556	.1385	.00768	.550	.1450	.00791	.770	.1732	.00836
.435	.1085	.00778	.486	.1230	.00818	.446	.1365	.00773	.443	.1425	.00785	.573	.1558	.00771
.378	.1125	.00779	.400	.1240	.00796	.388	.1355	.00779	.401	.1400	.00780	.441	.1555	.00763
.342	.1170	.00791	.378	.1255	.00822	.376	.1365	.00795	.343	.141	.00803	.376	.1490	.00749
.325	.1190	.00817	.356	.1270	.00871	.343	.1355	.00810	.340	.141	.00825	.334	.1432	.00764
.532	.1005	.00791	.528	.1185	.00819	1.783	.1765	.01065	.507	.1450	.0075	.273	.1412	.00803
.372	.1100	.00778	.583	.1210	.00814	1.275	.1585	.0094	.389	.1370	.00742	.507	.1516	.00773
.324	.1125	.00791	.293	.1220	.00805	.970	.1540	.00893	.277	.1320	.00780	.361	.1432	.00796
.290	.1155	.00782	.224	.1255	.00805	.757	.1490	.00825	.208	.1315	.00775	.258	.1350	.00776
.2455	.1195	.00780	.180	.1280	.00804	.605	.1450	.00797	.183	.1320	.00798	.201	.1342	.00778
.2185	.1230	.00801	.1596	.1295	.00821	.516	.1415	.00800	.1513	.1335	.00814	.169	.1343	.00782
.1970	.1230	.00817	.1368	.1300	.00842	.443	.1420	.00840	.134	.1335	.00828	.153	.1340	.00806
												.133	.1363	.00817
												0	.1298	.00817

Propeller No. 11.

1.379	0.0551	0.00697	1.442	0.1031	0.00869	1.56	0.159	0.00978	1.434	0.1698	0.00871	0	0.117	0.00685
1.024	.0735	.00749	1.129	.1072	.00824	1.104	.1434	.00792	1.071	.161	.00756	0	.1175	.00710
.807	.0849	.00768	.877	.1104	.00792	.852	.140	.00742	.809	.1500	.00709	.472	.1318	.00682
.672	.0932	.00762	.728	.1142	.00779	.694	.135	.00706	.676	.1422	.00669	.315	.1250	.00691
.597	.0972	.00757	.641	.1149	.00773	.577	.131	.00682	.573	.137	.00662	.217	.1214	.00662
.503	.1028	.00740	.549	.1175	.00762	.484	.1287	.00693	.481	.1339	.00672	.170	.1205	.00652
.460	.1047	.00737	.465	.1182	.00769	.426	.1258	.00696	.443	.1316	.00684	.1467	.1200	.00672
.908	.0791	.00834	1.125	.1019	.00872	1.07	.142	.00934	.952	.1565	.00778	.137	.1213	.00698
.677	.0919	.00793	.826	.110	.00814	.739	.132	.00762	.667	.1421	.0070	.1256	.1209	.00703
.516	.0990	.00755	.589	.1127	.00723	.556	.128	.00724	.534	.1342	.00678	.992	.1708	.00720
.394	.1052	.00746	.456	.1142	.00765	.444	.1268	.00702	.436	.1289	.00658	.742	.1540	.00658
.344	.1079	.00726	.384	.1140	.00741	.387	.1244	.00698	.392	.1268	.00678	.558	.1397	.00607
.323	.109	.00742	.381	.1150	.00735	.361	.1244	.00699	.349	.1142	.00676	.451	.1335	.00640
.308	.1092	.00736	.362	.1160	.00744	.322	.1244	.00712	.313	.1255	.00726	.382	.1295	.00654
.527	.0958	.00769	.332	.1166	.00735	.481	.123	.00727	.456	.1300	.0072	.350	.1284	.00648
.366	.1037	.0074	.526	.1108	.00794	.358	.1205	.00668	.364	.1263	.0068	.298	.1250	.00680
.317	.1051	.00720	.380	.1124	.00746	.245	.1182	.00666	.258	.1214	.00672	1.495	.200	.00687
.271	.1083	.00713	.299	.1139	.00736	.182	.1175	.00674	.202	.1198	.00692	1.103	.1779	.00730
.219	.1111	.00715	.219	.1149	.0069	.154	.1176	.00681	.1615	.1199	.00699	.836	.1615	.00680
.213	.1120	.00725	.182	.1161	.00702	.1356	.1189	.00693	.1386	.1192	.00698	.646	.1470	.00650
.190	.1123	.00738	.152	.1170	.00789	.121	.1198	.00722	.1208	.1198	.00718	.582	.1438	.00635
			.1286	.1172	.00682							.520	.1263	.00640
												.442	.1340	.00667

Propeller No. 12.

1.30	0.0478	0.00605	1.54	0.1045	0.00834	1.52	0.1732	0.00989	1.453	0.200	0.0101	1.66	0.2435	0.0102
1.01	.0698	.00678	1.145	.111	.0084	1.18	.160	.00922	1.012	.1752	.00936	1.165	.2095	.00914
.806	.0847	.00755	.894	.1155	.0088	.865	.1510	.00855	.82	.165	.00792	.852	.1830	.00811
.674	.0920	.00760	.739	.119	.00817	.705	.1432	.00804	.662	.1542	.00758	.700	.1705	.00732
.566	.1000	.00763	.627	.120	.00812	.595	.1405	.00811	.57	.1432	.00639	.614	.1622	.00703
.504	.1044	.00755	.553	.1211	.00812	.518	.1379	.00784	.511	.1415	.00749	.536	.1555	.00711
.455	.1069	.00784	.483	.1222	.0080	.448	.1332	.00777	.45	.139	.00765	.460	.1522	.00731
.884	.0799	.00761	1.035	1104	.00878	1.09	.160	.01028	1.02	.179	.00872	1.105	.2053	.00837
.668	.0944	.0078	.785	.1167	.00874	.76	.159	.00840	.713	.158	.00775	.757	.1745	.00792
.519	.1049	.00788	.600	.1211	.00845	.572	.138	.00820	.542	.1432	.00753	.585	.1545	.00722
.395	.1078	.00767	.479	.1220	.00811	.452	.1345	.00789	.421	.1366	.00735	.459	.1477	.00715
.352	.1105	.0077	.399	.1222	.00789	.396	.1342	.00768	.399	.1343	.00738	.403	.1445	.00715
.328	.1122	.00776	.370	.1211	.00783	.358	.1325	.00772	.357	.1332	.00748	.362	.1490	.00763
.318	.1128	.00786	.336	.1222	.00794	.319	.1320	.00786	.3325	.1321	.00764	.289	.1333	.00752
.52	.1030	.00889	.530	.1189	.00839	.465	.1389	.00833	.463	.1428	.00759	.493	.1510	.00750
.365	.1112	.0080	.397	.1222	.00811	.371	.1344	.00793	.367	.1372	.00765	.332	.1345	.00731
.318	.1122	.00789	.311	.1245	.00794	.254	.1255	.00761	.252	.1306	.00775	.260	.1322	.00755
.274	.1133	.00785	.221	.1220	.00781	.196	.1255	.00762	.190	.1283	.00760	.190	.1311	.00744
.251	.1144	.00761	.189	.1277	.00725	.1595	.1232	.00760	.1574	.1272	.00775	.1725	.1311	.00748
.218	.1168	.00783	.157	.1334	.00695	.1395	.1222	.00755	.1366	.1279	.00770	.149	.1311	.00765
.186	.1179	.00795	.140	.1278	.00663	.1251	.1244	.00772	.1277	.1259	.00785	.138	.1300	.00770
												0	.1255	.00784

Propeller No. 139.

60° yaw.			70° yaw.			80° yaw.			85° yaw.			90° yaw.		
$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.
1.163	0.00532	0.00403	1.64	0.0454	0.00454	1.55	0.0958	0.00504	1.16	0.115	0.00473	0	0.0738	0.00414
.890	.0263	.00374	.984	.0572	.00423	.956	.0915	.00422	.865	.107	.00391	0	.0772	.00407
.719	.0402	.00388	.787	.0630	.00427	.727	.0789	.00362	.684	.0985	.00346	0	.0748	.00401
.594	.0486	.00408	.661	.0665	.00412	.635	.0846	.00402	.564	.0932	.00355	0	.0738	.00363
.532	.0529	.00396	.548	.0699	.00412	.543	.0827	.00394	.513	.0909	.00373	0	.0750	.00378
.484	.0562	.00414	.492	.0705	.00412	.485	.0815	.00393	.474	.0878	.00377	0	.0754	.00387
.408	.0600	.00416	.455	.0719	.00413	.419	.0800	.00420	.421	.0848	.00375	0	.0753	.00386
.796	.0364	.00441	.936	.0568	.00454	.926	.0931	.00503	.465	.092	.00383	0	.0747	.00397
.602	.0494	.00412	.693	.0653	.00444	.669	.0818	.00435	.308	.0845	.0039	.423	.0936	.00346
.479	.0571	.00412	.538	.0687	.00421	.514	.0806	.00404	.268	.0821	.00362	.315	.0873	.00397
.386	.0622	.00406	.436	.0700	.00412	.408	.0802	.00388	.194	.0779	.0036	.236	.0821	.00353
.333	.0645	.00400	.372	.0738	.00364	.360	.0786	.00378	.157	.0767	.00364	.185	.0804	.00357
.301	.0662	.00416	.318	.0745	.00401	.319	.0768	.00376	.139	.0761	.00389	.155	.0796	.00372
.236	.0668	.00407	.292	.0748	.00404	.291	.0758	.00388	.121	.0758	.00406	.131	.0791	.00382
.458	.059	.00489	.469	.0702	.00447	.418	.083	.0041	.912	.1085	.00475	.112	.0792	.00393
.324	.0648	.00428	.318	.0700	.00396	.297	.0783	.00378	.656	.0988	.00373	.933	.1251	.00333
.259	.0734	.00410	.278	.072	.00375	.241	.0768	.00368	.480	.0905	.0037	.673	.1102	.00359
.242	.0688	.00406	.216	.0712	.00366	.201	.0755	.00383	.368	.08355	.00368	.522	.1003	.00336
.210	.0698	.00407	.1706	.0712	.00372	.1515	.0718	.00382	.325	.08105	.00373	.430	.0936	.00334
.191	.0700	.00408	.1482	.0713	.00372	.136	.0738	.00382	.305	.0795	.00388	.359	.0878	.00342
.172	.0692	.00404	.1262	.0711	.00392	.1182	.0725	.00389	.288	.07905	.00400	.317	.0860	.00353
												.286	.0829	.00369

Propeller No. 144.

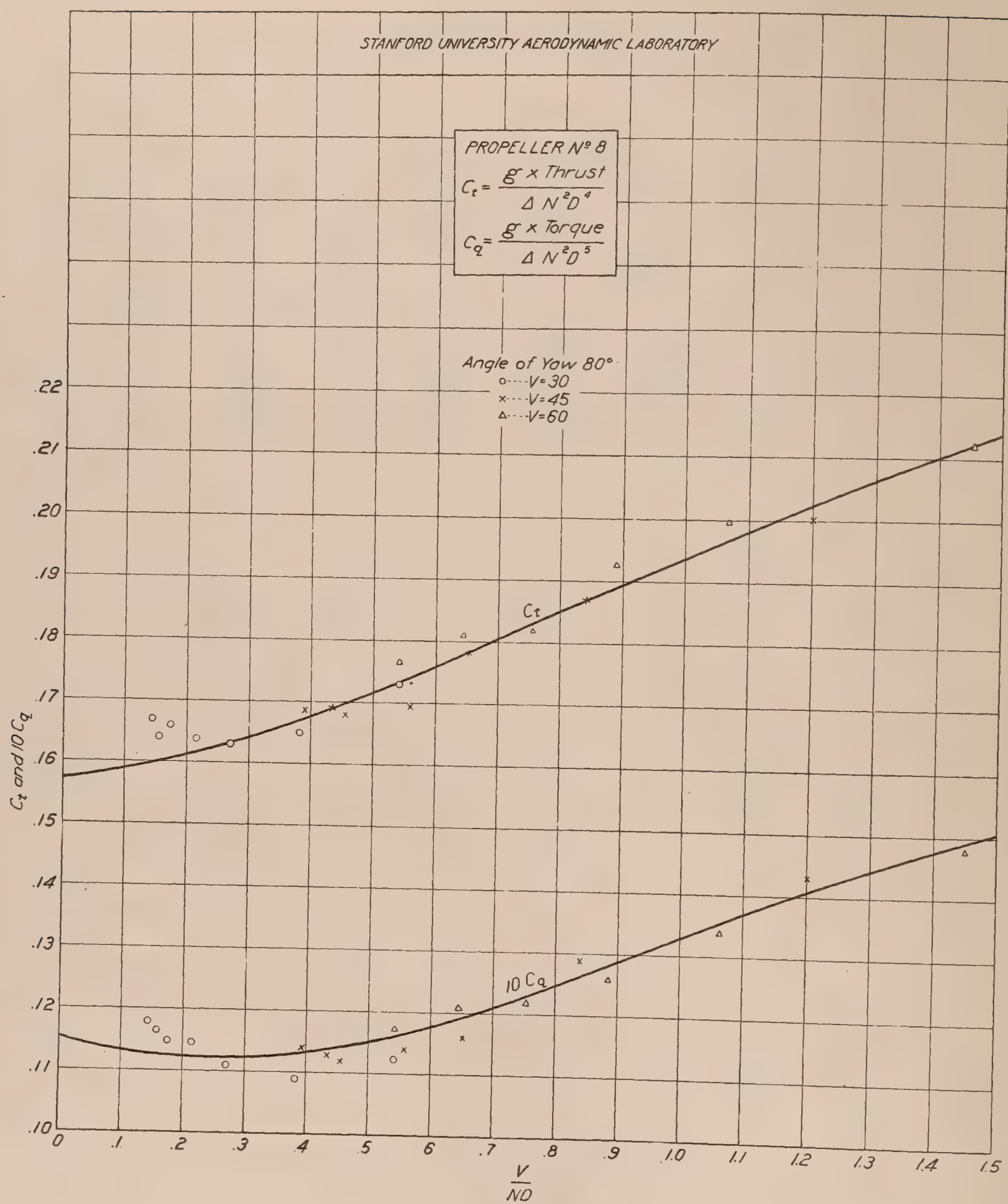
0.964	0.00293	0.0033	1.273	0.033	0.003615	1.261	0.0959	0.00468	0.517	0.0948	0.00402	0.913	0.1332	0.00379
.773	.02345	.003499	.975	.0468	.003670	.957	.0932	.00493	.332	.0841	.00382	.665	.1153	.00351
.623	.0385	.00354	.791	.0615	.003722	.766	.0909	.00416	.283	.0810	.00368	.514	.1027	.0033
.558	.0452	.003639	.691	.0613	.00377	.663	.0888	.00409	.227	.0800	.00334	.456	.0956	.00331
.526	.0482	.00366	.565	.0663	.00394	.584	.0880	.00404	.168	.0780	.00371	.340	.0913	.00352
.496	.0504	.003702	.515	.0688	.00390	.508	.0858	.00396	.144	.0780	.00402	.322	.0887	.00348
.467	.0531	.003815	.449	.0706	.00391	.429	.0848	.00410	.122	.0786	.00399	.292	.0878	.00361
.417	.0567	.00387	1.007	.0464	.00449	.931	.0926	.00503	1.209	.1201	.00465	1.258	.1413	.00384
.786	.0242	.00389	.732	.0594	.00387	.680	.0890	.00432	.892	.1114	.00399	.943	.1332	.00368
.602	.0411	.00357	.553	.0653	.00387	.521	.0856	.00420	.701	.1033	.00373	.747	.1197	.00360
.481	.0514	.00372	.467	.0683	.00383	.415	.0829	.00414	.572	.0978	.00361	.617	.1123	.00339
.386	.0577	.00383	.397	.0707	.00381	.350	.0811	.00408	.519	.0955	.00373	.524	.1038	.00334
.340	.0616	.00388	.326	.0722	.0038	.321	.0812	.00418	.471	.0936	.00388	.465	.1002	.00347
.299	.0643	.00388	.296	.0733	.0043	.299	.0812	.00395	.413	.0906	.00396	.411	.0962	.00366
.265	.0664	.00386	.471	.0673	.00431	.509	.0868	.00470	.843	.1110	.00488	.423	.0971	.00407
.466	.0524	.00446	.315	.0717	.00369	.318	.0807	.00392	.589	.0976	.00391	.309	.0870	.00387
.331	.0615	.00394	.281	.0714	.00371	.242	.0784	.0035	.448	.0913	.00368	.237	.0821	.0038
.266	.0656	.00385	.227	.0720	.00376	.179	.0769	.0036	.343	.0849	.00369	.181	.0804	.00377
.234	.0661	.00382	.171	.0732	.0038	.149	.0769	.0036	.310	.0835	.00370	.151	.0806	.00381
.214	.0681	.00381	.150	.0745	.00386	.132	.0779	.00390	.296	.0835	.00389	.132	.0799	.00399
.196	.0695	.00399	.123	.0748	.00387	.112	.0782	.0040	.282	.0834	.00395	.113	.0797	.00396
.167	.0710	.00384				.264	.0766	.00400						
						.204	.0775	.00383						
						.138	.0771	.00372						

Propeller No. 145.

1.02	0.0157	0.00449	1.334	0.0376	0.00447	1.278	0.0918	0.00515	1.193	0.1134	0.00518	1.331	0.1412	0.00428
.807	.0347	.0043	1.009	.0465	.00448	.963	.0912	.00474	.887	.1071	.00431	.950	.1266	.00365
.668	.0458	.00412	.813	.0630	.00459	.775	.0914	.00446	.700	.1014	.00390	.757	.1159	.00363
.564	.0543	.00437	.667	.0682	.00440	.671	.0911	.00436	.570	.0983	.00387	.627	.1081	.00349
.513	.0578	.0044	.577	.0720	.00439	.568	.0901	.00427	.508	.0965	.00396	.530	.1033	.00356
.483	.0597	.00434	.516	.0742	.00431	.491	.0889	.00419	.473	.0957	.00406	.458	.1005	.00371
.419	.0641	.00427	.442	.0758	.00428	.422	.0882	.00417	.422	.0935	.00423	.413	.0985	.00383
.800	.0343	.00469	.979	.0536	.00481	.942	.0897	.00492	.448	.0899	.00394	.901	.1213	.00341
.610	.0499	.00452	.718	.0645	.00446	.687	.0892	.00439	.309	.0844	.00386	.653	.1101	.00358
.481	.0585	.00441	.559	.0703	.00448	.524	.0886	.00422	.275	.0837	.00395	.518	.1019	.00361
.391	.0646	.00433	.468	.0729	.00431	.418	.0858	.00416	.209	.0829	.00397	.422	.0969	.00322
.341	.0676	.00433	.371	.0756	.00429	.361	.0849	.00417	.159	.0826	.00405	.344	.0930	.00365
.293	.0701	.00439	.330	.0768	.00423	.322	.0845	.00410	.141	.0829	.00418	.308	.0912	.00384
.267	.0725	.00446	.298	.0775	.00432	.304	.0852	.00419	.128	.0825	.00432	.290	.0924	.00400
.468	.0583	.00478	.479	.0692	.00467	.454	.0847	.00452	.805	.1047	.00455	.431	.0956	.00447
.338	.0667	.00437	.322	.0729	.00406	.306	.0811	.00395	.578	.0969	.00393	.310	.0881	.00414
.269	.0694	.00428	.283	.0746	.00408	.249	.0806	.00395	.445	.0920	.00388	.233	.0850	.00408
.243	.0716	.00429	.226	.0763	.00409	.201	.0807	.00402	.343	.0881	.00399	.175	.0843	.00402
.215	.0730	.00434	.165	.0777	.00405	.151	.0815	.00398	.313	.0874	.00391	.153	.0843	.00409
.185	.0750	.00427	.149	.0798	.00423	.137	.0817	.00416	.301	.0882	.00412	.137	.0849	.00418
.169	.0766	.00435	.125	.0807	.00444	.112	.0834	.00424	.278	.0876	.00411	.120	.0843	.00337

PROPELLER No. 146.

60° yaw.			70° yaw.			80° yaw.			85° yaw.			90° yaw.		
$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.	$\frac{V}{ND}$.	C_t.	C_q.
0.959	0.00655	0.00372	1.262	0.0339	0.00402	0.452	0.0884	0.00453	0.825	0.1126	0.00436	1.333	0.1540	0.00455
.776	.02555	.00395	.972	.0495	.00415	.307	.0844	.00426	.586	.1023	.00417	.958	.1395	.00385
.663	.0376	.00411	.774	.0590	.00424	.246	.0821	.00434	.450	.0964	.00407	.761	.1268	.00382
.582	.0458	.00425	.684	.0691	.00428	.176	.0832	.00433	.345	.0915	.00414	.615	.1125	.00360
.528	.0514	.00397	.553	.0713	.00443	.149	.0842	.00444	.318	.0914	.00432	.543	.1121	.00395
.495	.0545	.00400	.504	.0765	.00440	.128	.0855	.00447	.304	.0927	.00457	.552	.1152	.00406
.422	.0610	.00422	.442	.0764	.00440	.109	.0868	.00460	.282	.0924	.00465	.508	.1117	.00408
.766	.0281	.00368	.930	.0509	.00442	1.267	.0977	.00496	1.201	.1240	.00539	.520	.1129	.00404
.596	.0446	.00386	.700	.0532	.00418	.953	.0955	.00446	.891	.1162	.00469	.456	.1088	.00412
.473	.0551	.00396	.550	.0697	.00417	.774	.0956	.00446	.697	.1090	.00402	.395	.1040	.00449
.384	.0622	.00406	.436	.0728	.00420	.651	.0942	.00440	.569	.1035	.00413	.906	.1335	.00399
.327	.0671	.00423	.385	.0758	.00422	.561	.0944	.00447	.504	.1022	.00436	.650	.1182	.00369
.290	.0698	.00428	.334	.0772	.00423	.509	.0935	.00444	.460	.1016	.00444	.509	.1068	.00374
.267	.0724	.00440	.300	.0788	.00442	.432	.0931	.00459	.406	.0989	.00459	.405	.0994	.00386
.459	.0556	.00474	.292	.0723	.00387	.931	.0956	.00530	.449	.0958	.00416	.340	.0969	.00414
.330	.0646	.00431	.446	.0708	.00438	.676	.0922	.00464	.319	.0884	.00398	.329	.0974	.00430
.262	.0690	.00423	.318	.0743	.00401	.522	.0900	.00455	.271	.0872	.00419	.279	.0968	.00458
.237	.0707	.00416	.270	.0746	.00411	.413	.0884	.00439	.203	.0856	.00418	.433	.1084	.00410
.218	.0725	.00417	.209	.0778	.00414	.354	.0881	.00437	.153	.0861	.00438	.314	.0906	.00402
.185	.0756	.00433	.168	.0795	.00419	.323	.0889	.00443	.135	.0871	.00453	.242	.0850	.00418
.168	.0776	.00435	.148	.0810	.00426	.301	.0892	.00452	.121	.0873	.00473	.190	.0858	.00419
			.124	.0828	.00443							.158	.0864	.00126
												.136	.0873	.00450
												.122	.0888	.00464


STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 8
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
Angle of Yaw 80°
o----V=30
x----V=45
Δ----V=60
Ct
10 Cq
Ct and 10 Cq
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
0
.1
.2
.3
.4
.5
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.8
.9
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1.1
1.2
1.3
1.4
1.5
V/ND

PLATE 1.

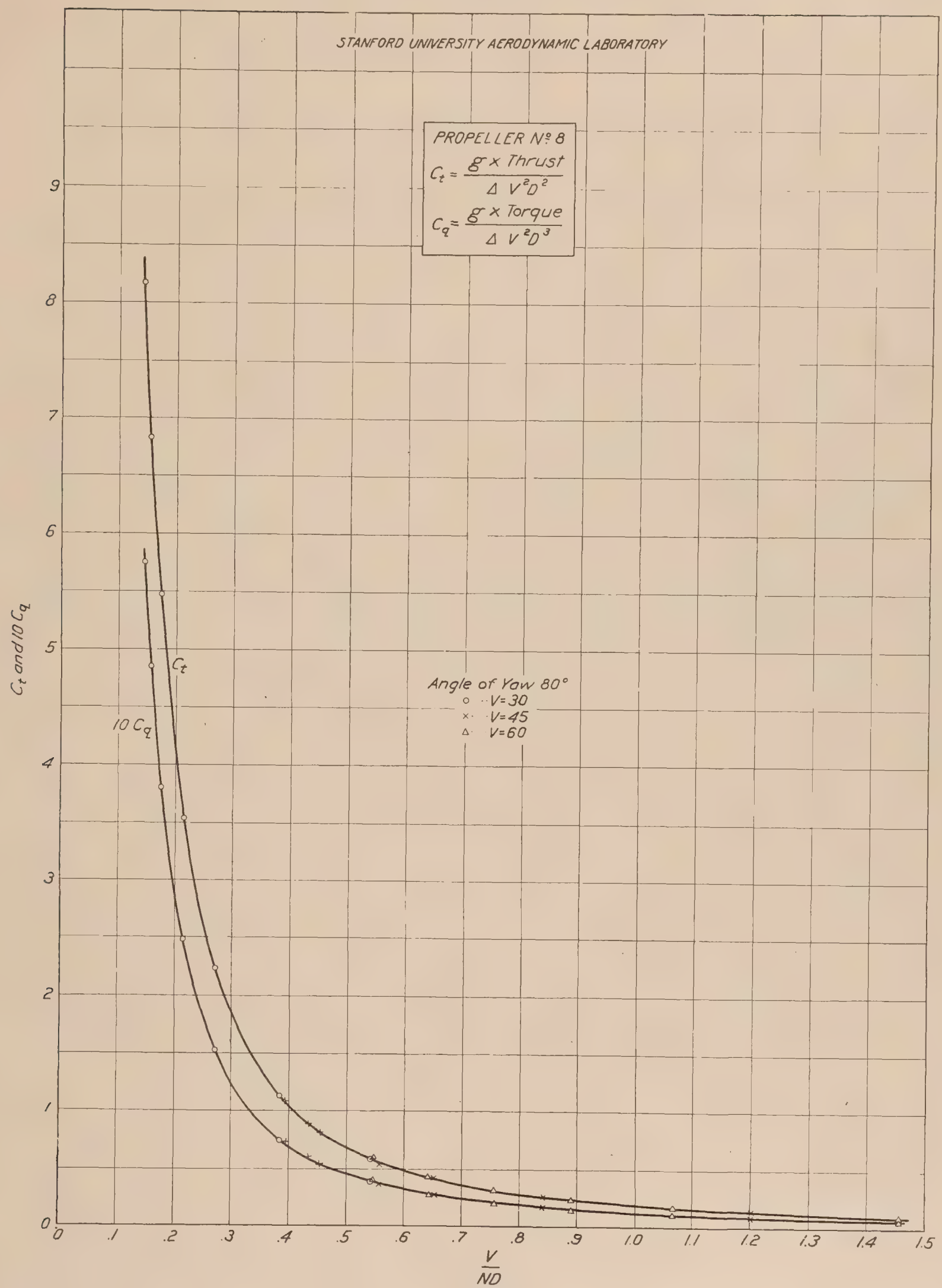

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER No 8
$C_t = \frac{g \times \text{Thrust}}{\Delta V^2 D^2}$
$C_q = \frac{g \times \text{Torque}}{\Delta V^2 D^3}$
Angle of Yaw 80°
○ V=30
× V=45
△ V=60
C_t
$10\,C_q$
C_t and $10\,C_q$
$\frac{V}{ND}$
0
1
2
3
4
5
6
7
8
9
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5

PLATE II.

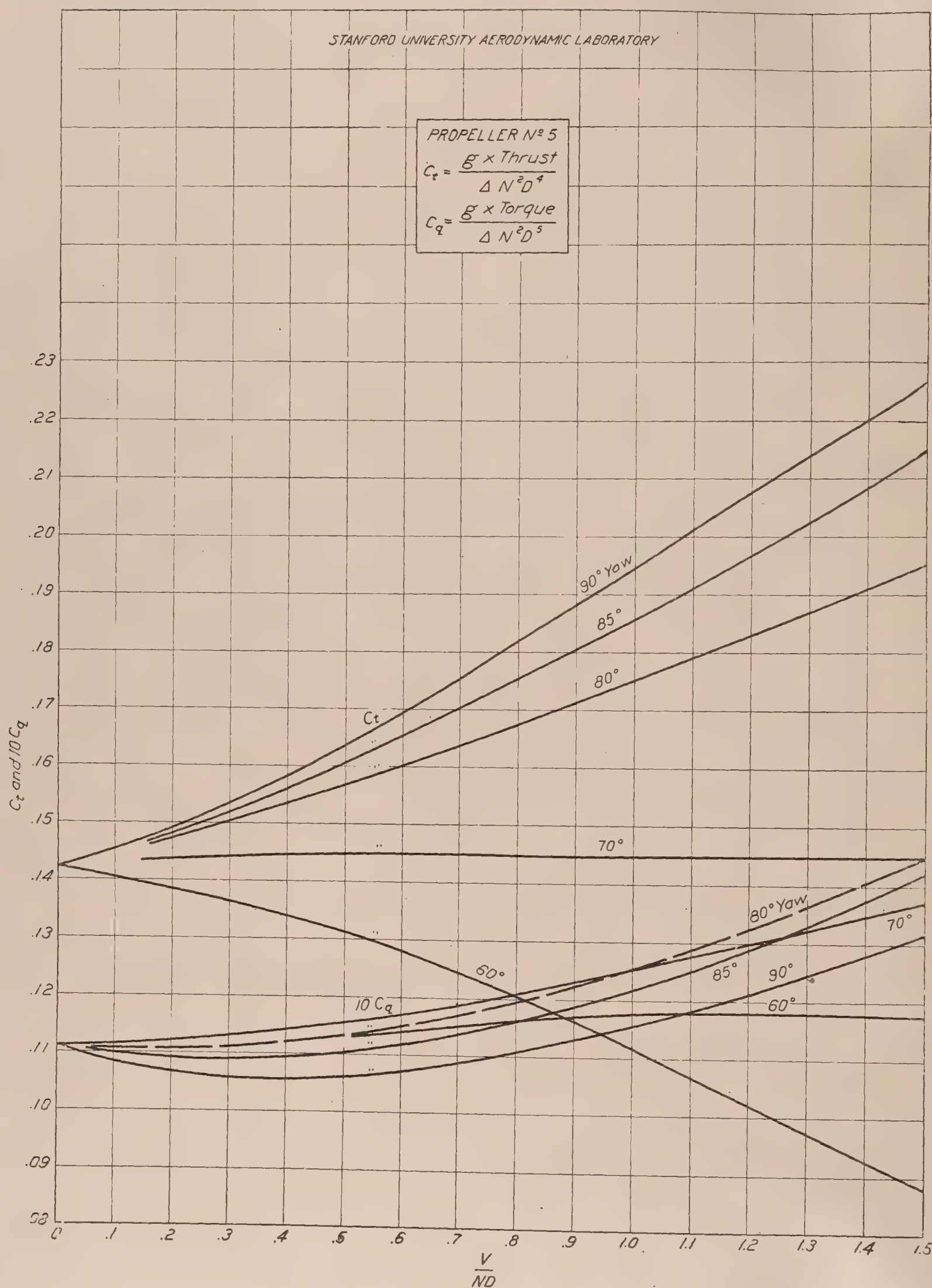

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 5
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
C_t and $10 C_q$
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{V}{ND}$
90° Yaw
85°
80°
Ct
70°
80° Yaw
70°
60°
85°
90°
10 Cq
60°

PLATE III.

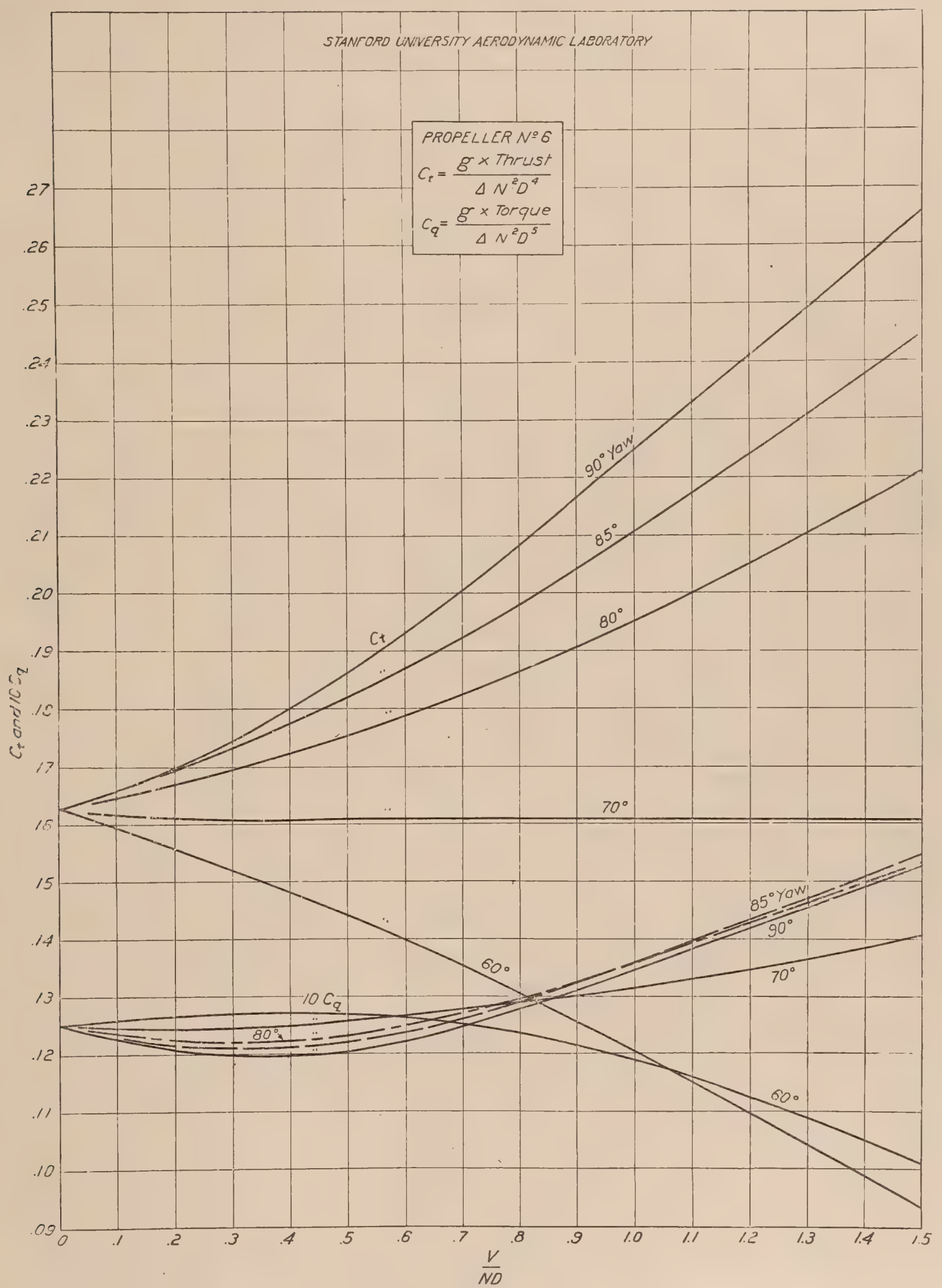

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 6
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
C_t and $10 C_q$
.27
.26
.25
.24
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{V}{ND}$
90° Yaw
85°
80°
70°
60°
C_t
85° Yaw
90°
70°
60°
$10 C_q$
80°

PLATE IV

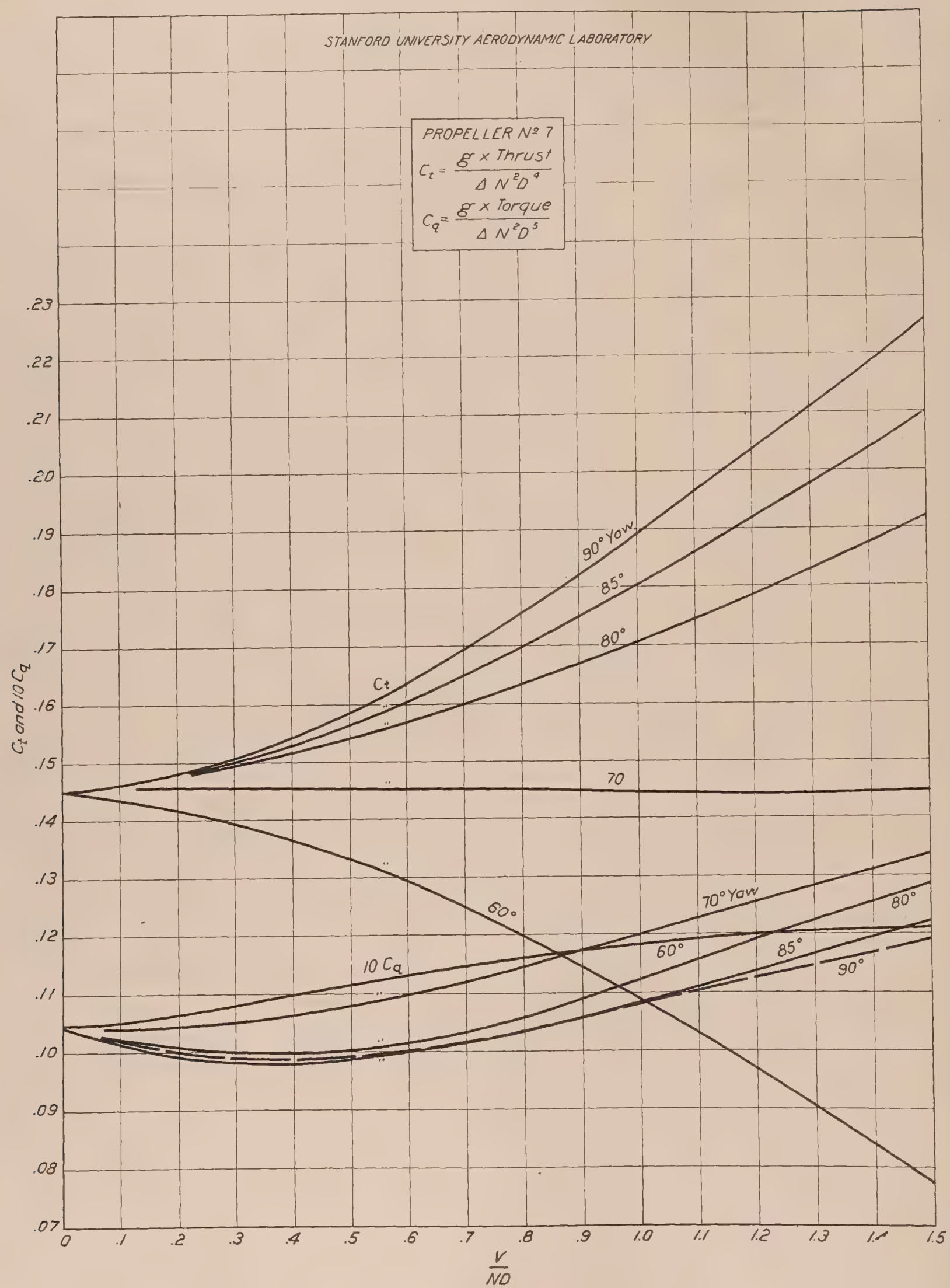

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 7
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
C_t and $10 C_q$
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{V}{ND}$
90° Yaw
85°
80°
C_t
70
60°
70° Yaw
80°
60°
85°
90°
10 C_q

PLATE V.

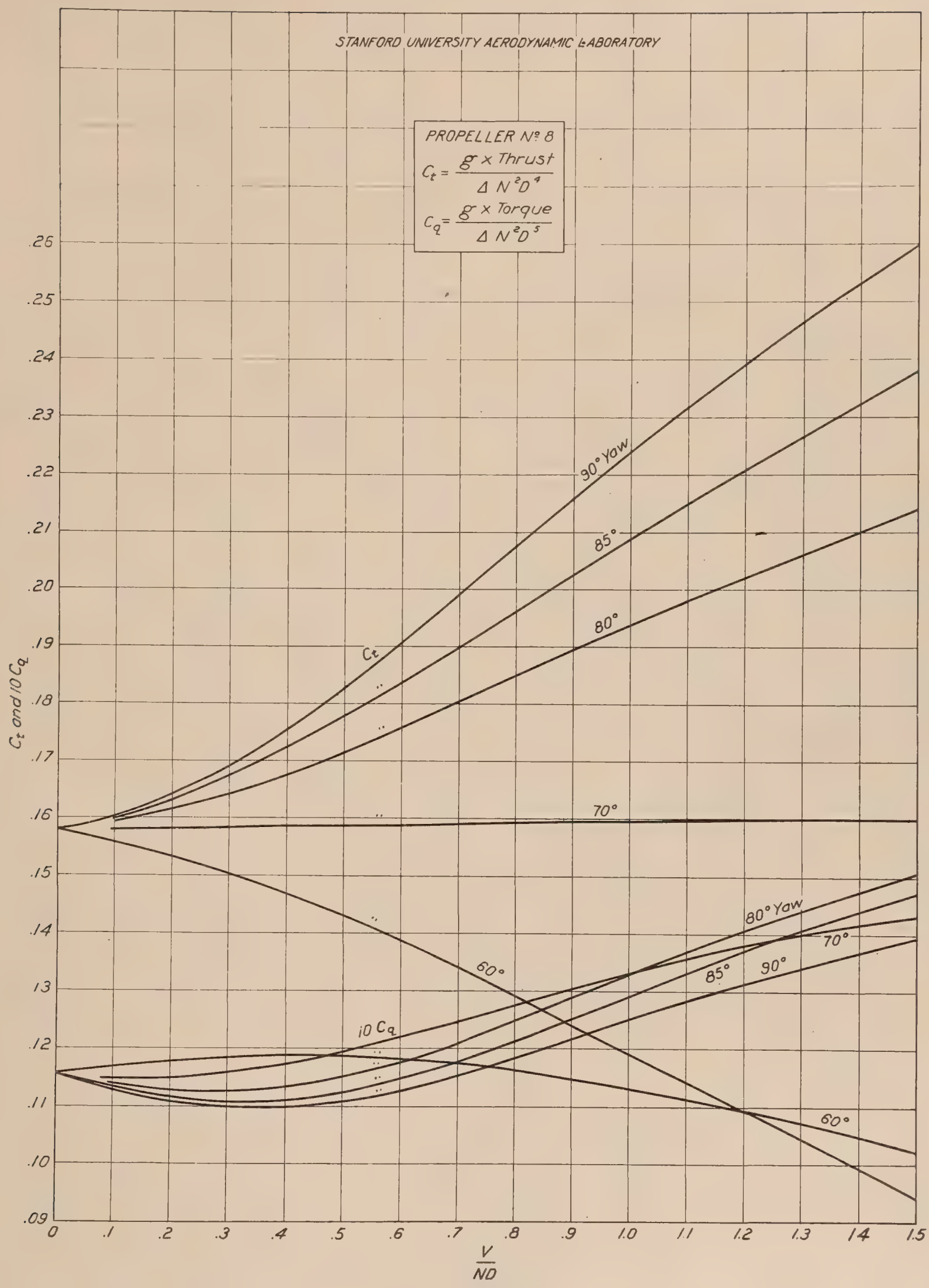

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 8
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
90° Yaw
85°
80°
70°
60°
C_t
80° Yaw
70°
85°
90°
60°
10 C_q
C_t and 10 C_q
.26
.25
.24
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{V}{ND}$

PLATE VI.

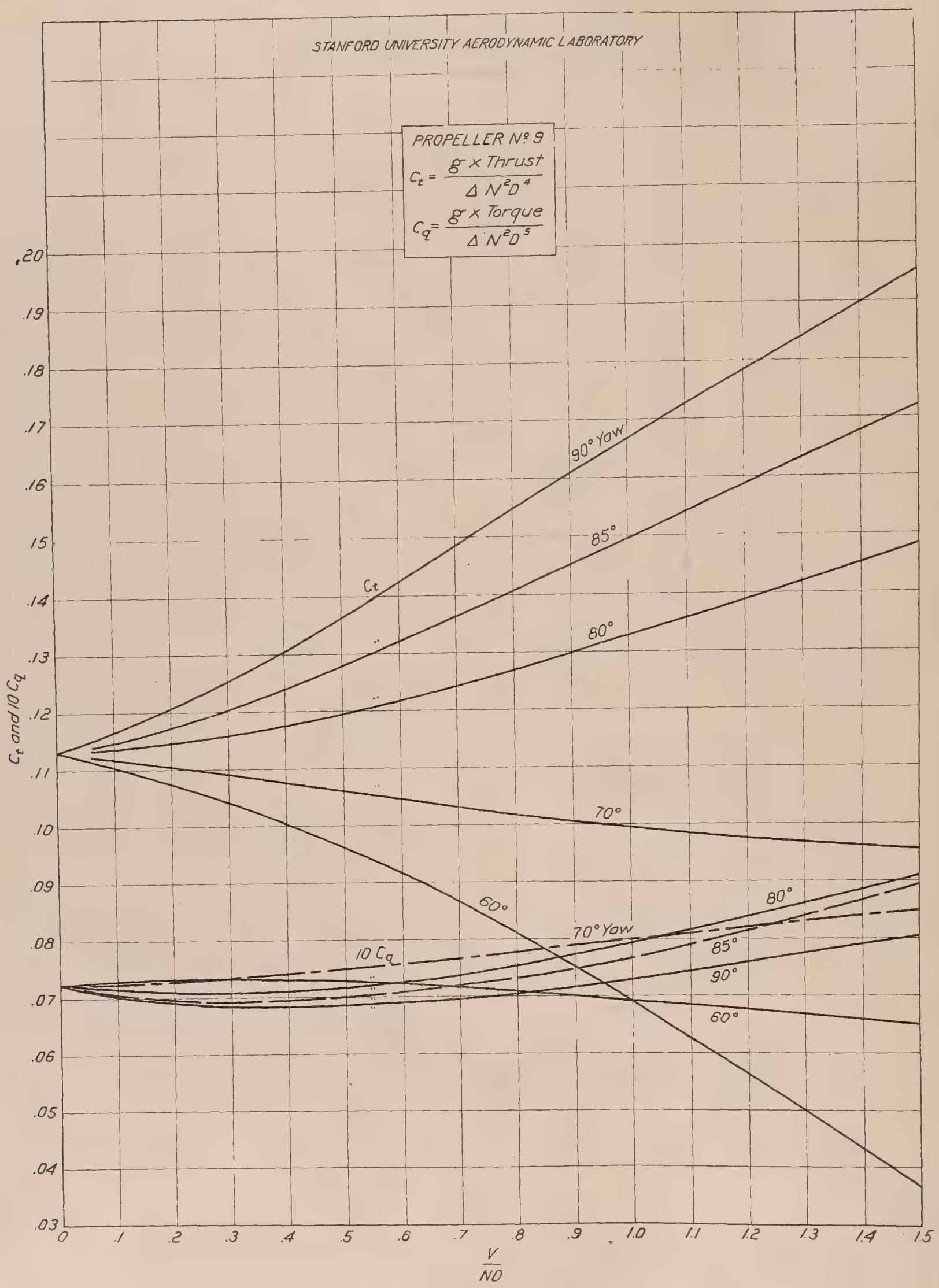

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER No. 9
$C_t = \frac{g \times \text{Thrust}}{\Delta N^2 D^4}$
$C_q = \frac{g \times \text{Torque}}{\Delta N^2 D^5}$
C_t and $10\,C_q$
$\frac{V}{ND}$
90° Yaw
85°
80°
70°
60°
80°
70° Yaw
85°
90°
60°
$10\,C_q$
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5

PLATE VII.

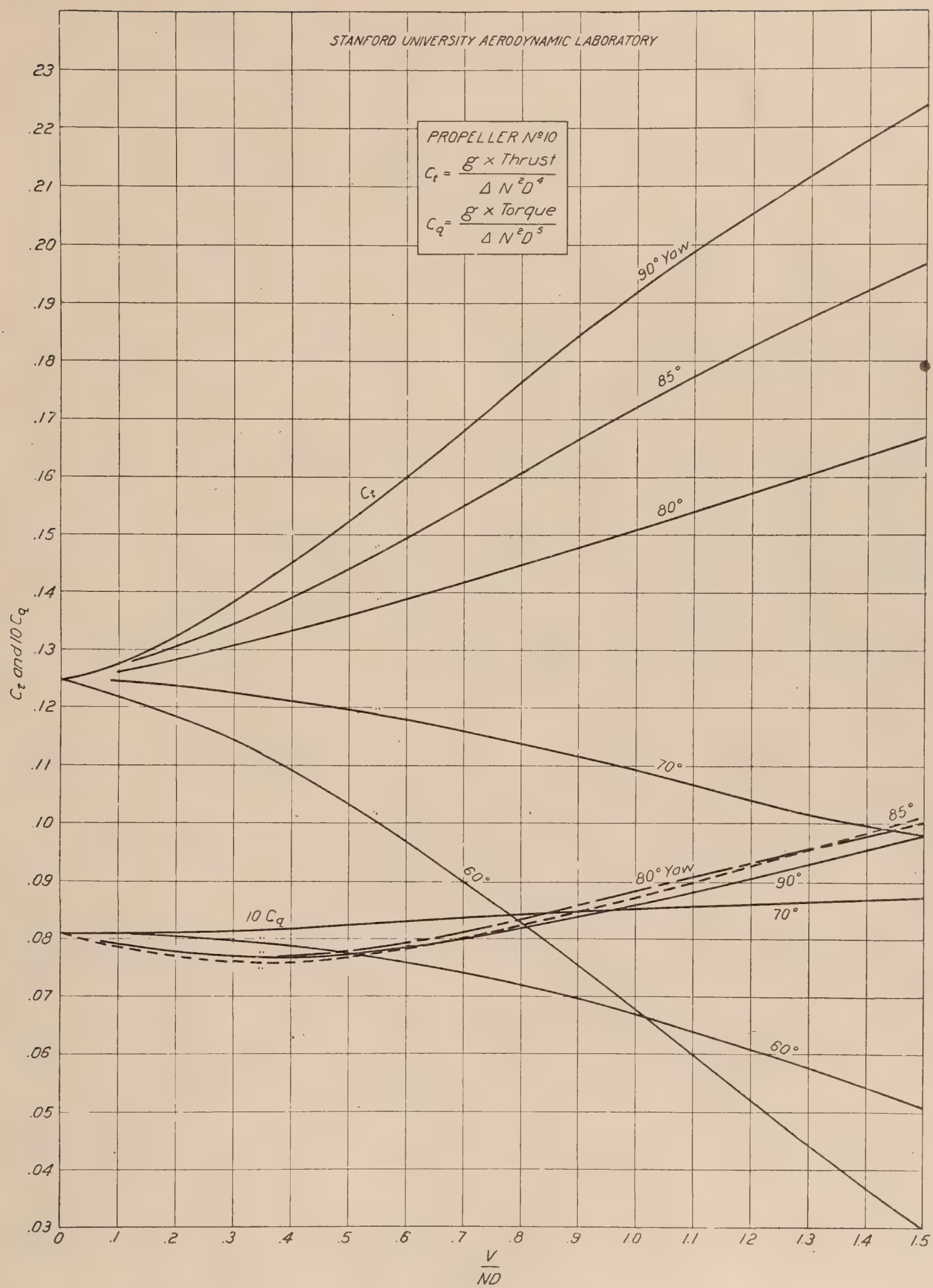

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº10
$C_t = \frac{g \times \text{Thrust}}{\Delta N^2 D^4}$
$C_q = \frac{g \times \text{Torque}}{\Delta N^2 D^5}$
90° Yaw
85°
80°
C_t
70°
85°
60°
80° Yaw
90°
70°
10 C_q
60°
C_t and 10 C_q
$\frac{V}{ND}$
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5

PLATE VIII.

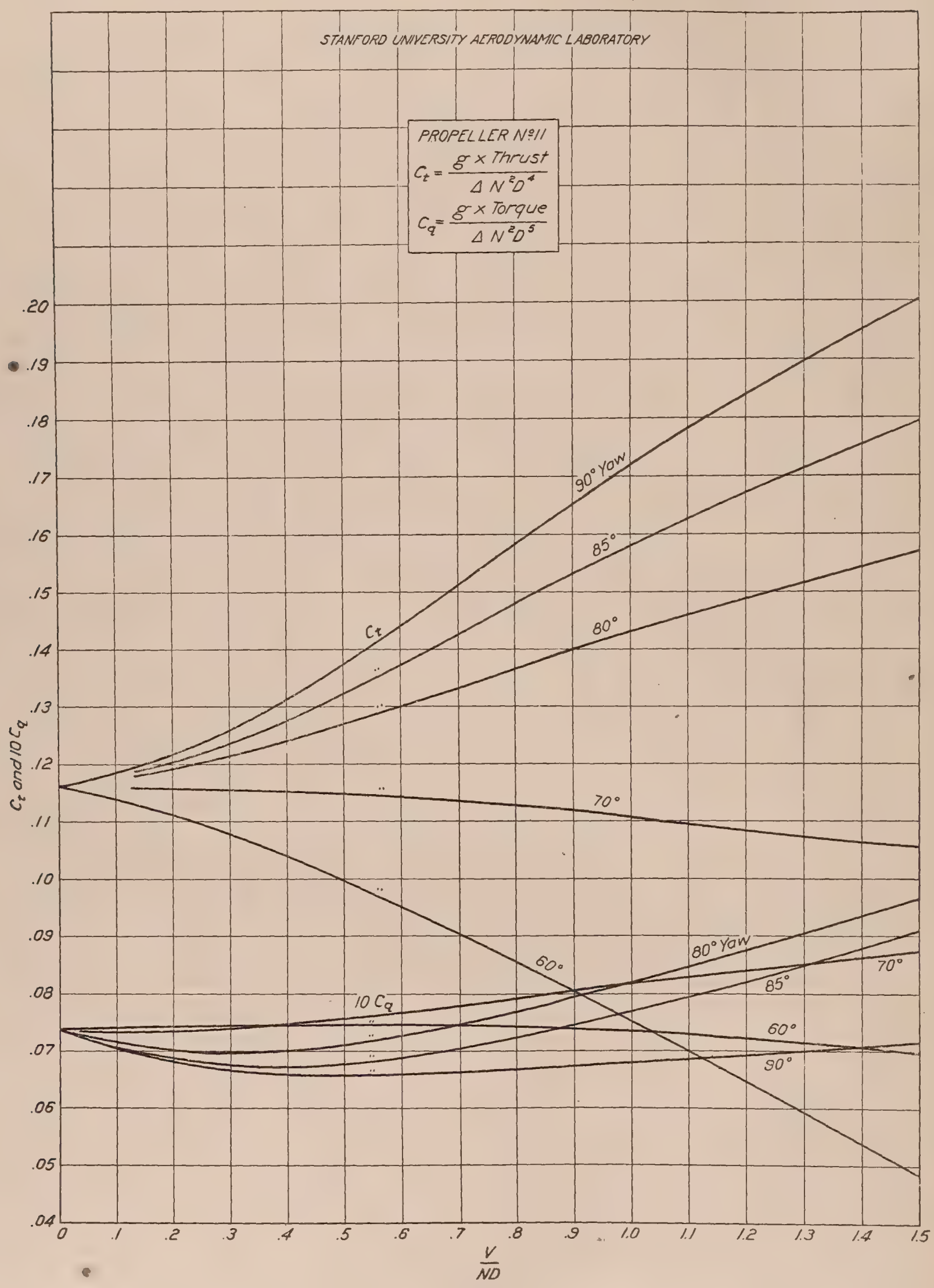

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº11
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
90° Yaw
85°
80°
70°
60°
80° Yaw
70°
85°
60°
90°
C_t
10 C_q
C_t and 10 C_q
$\frac{V}{ND}$
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5

PLATE IX.

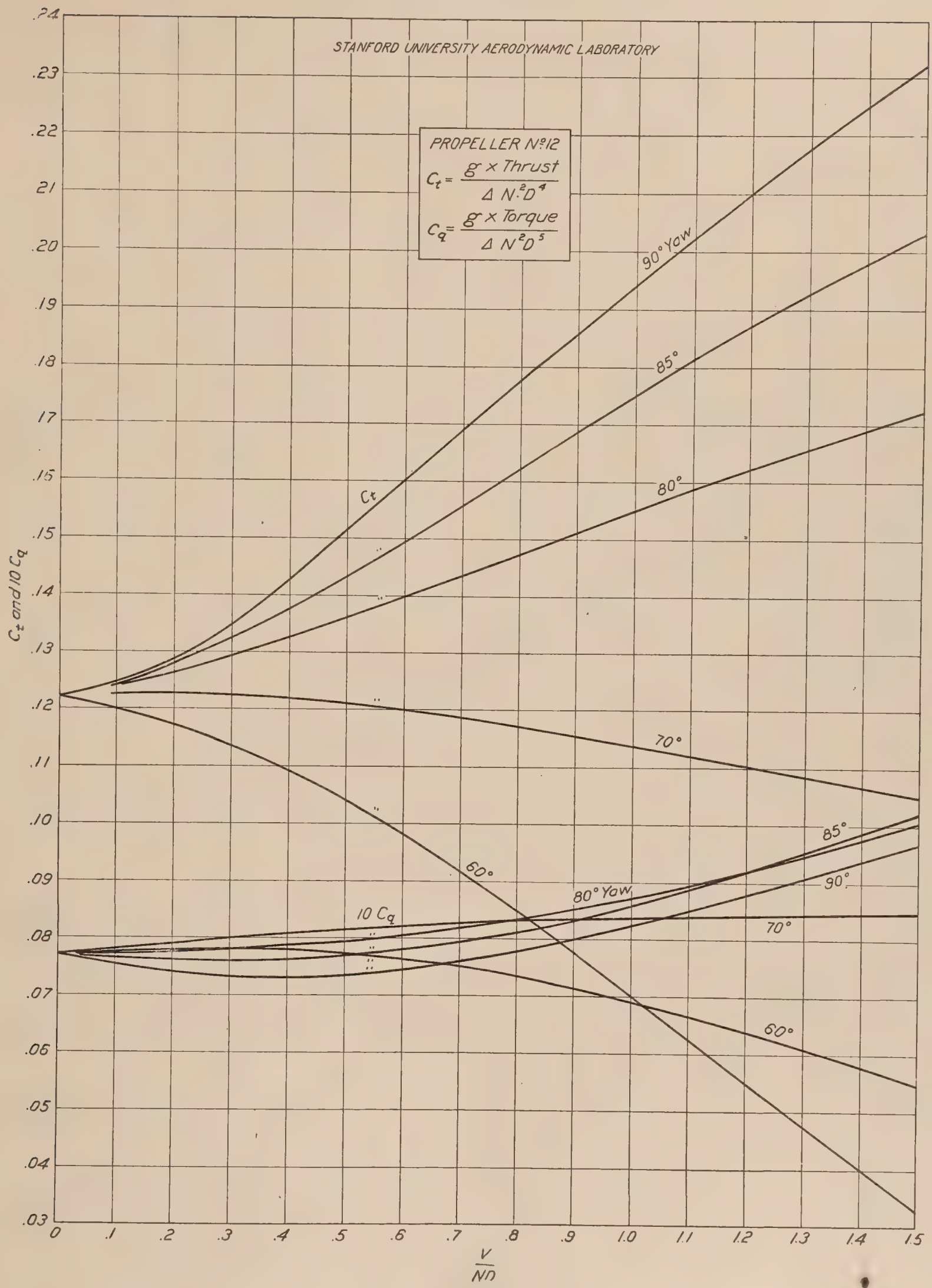

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº12
$C_t = \frac{g \times \text{Thrust}}{\Delta N^2 D^4}$
$C_q = \frac{g \times \text{Torque}}{\Delta N^2 D^5}$
90° Yaw
85°
80°
70°
60°
C_t
85°
90°
80° Yaw
70°
60°
10 C_q
C_t and 10 C_q
$\frac{V}{ND}$

PLATE X.

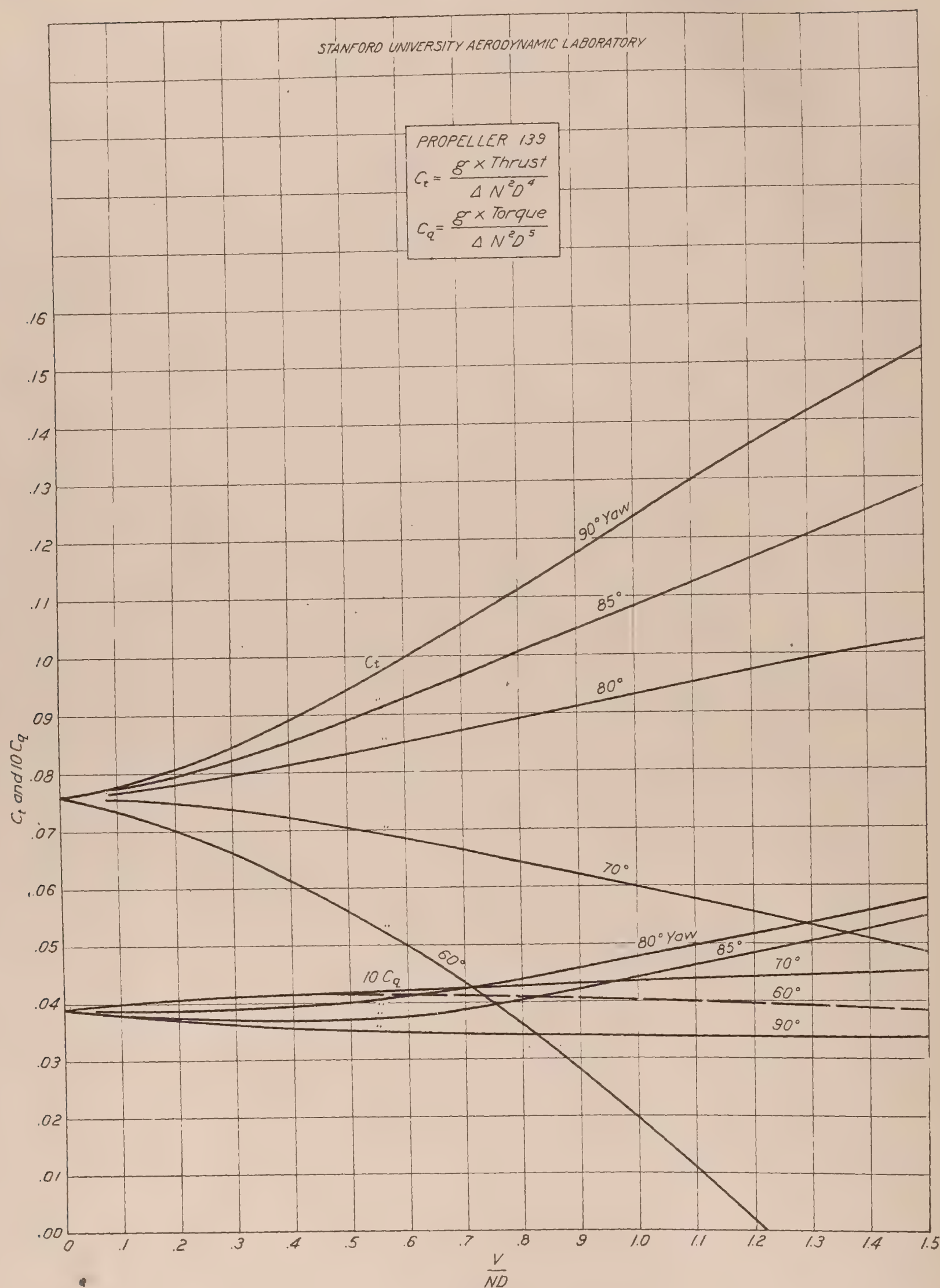

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER 139
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
C_t and $10 C_q$
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
.00
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{V}{ND}$
90° Yaw
85°
80°
70°
60°
C_t
80° Yaw
85°
70°
60°
90°
$10 C_q$

PLATE XI.

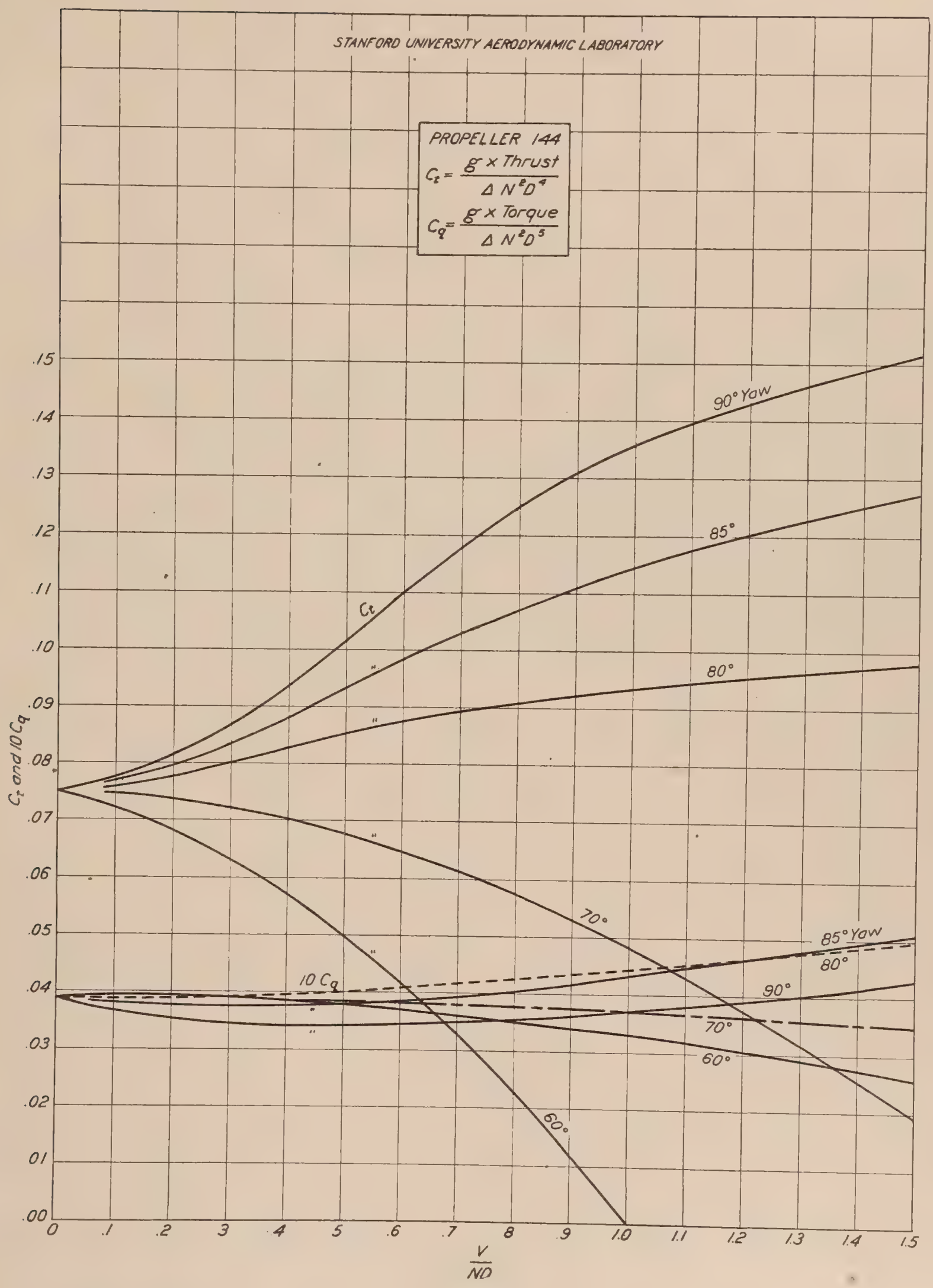

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER 144
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
C_t and $10 C_q$
$\frac{V}{ND}$
90° Yaw
85°
80°
70°
60°
85° Yaw
80°
90°
70°
60°
C_t
10 C_q
.00
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
.15
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5

PLATE XII.

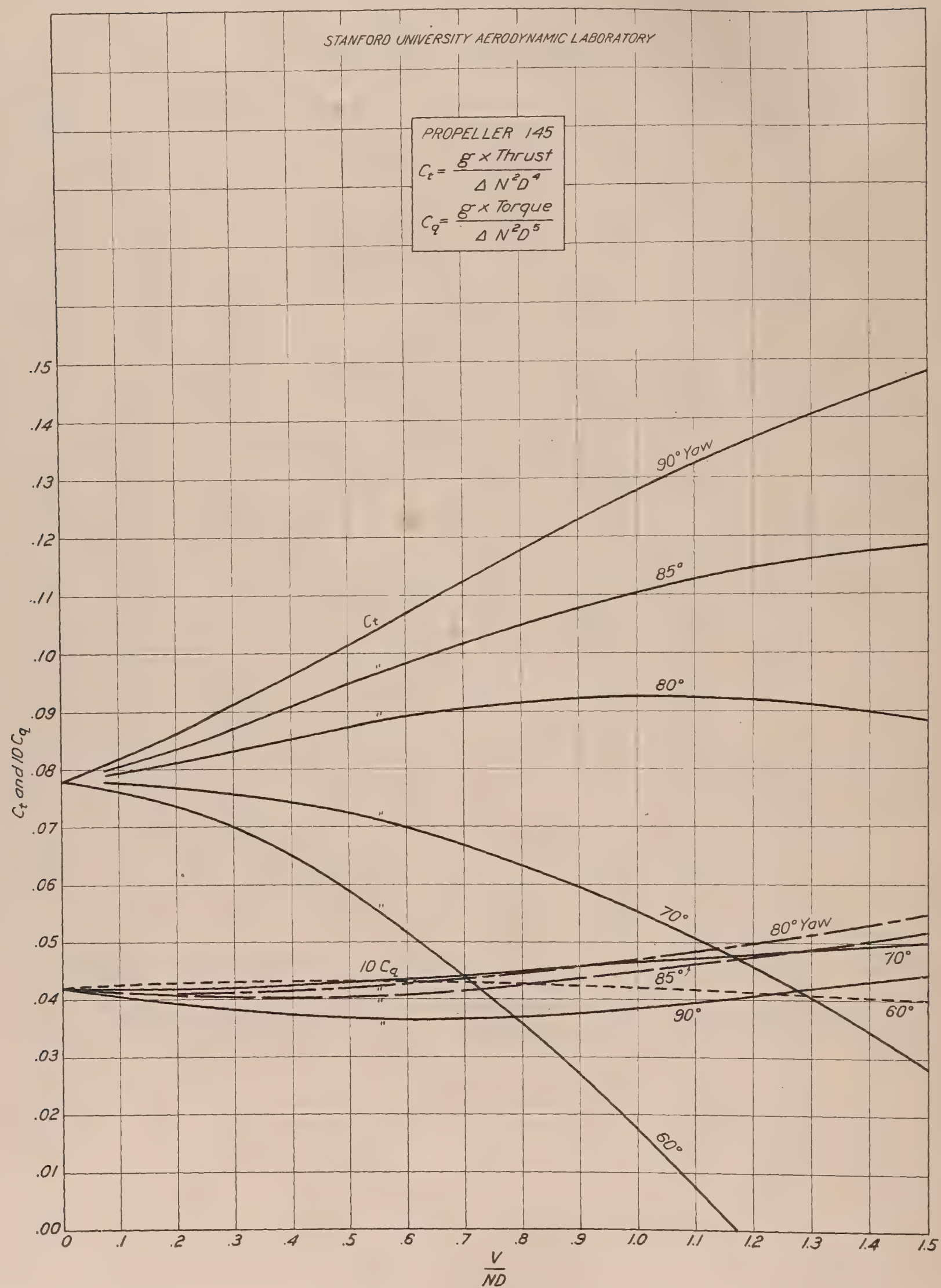

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER 145
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
90° Yaw
85°
80°
70°
60°
80° Yaw
70°
85°
90°
60°
C_t
$10\,C_q$
C_t and $10\,C_q$
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
.00
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{V}{ND}$

PLATE XIII.

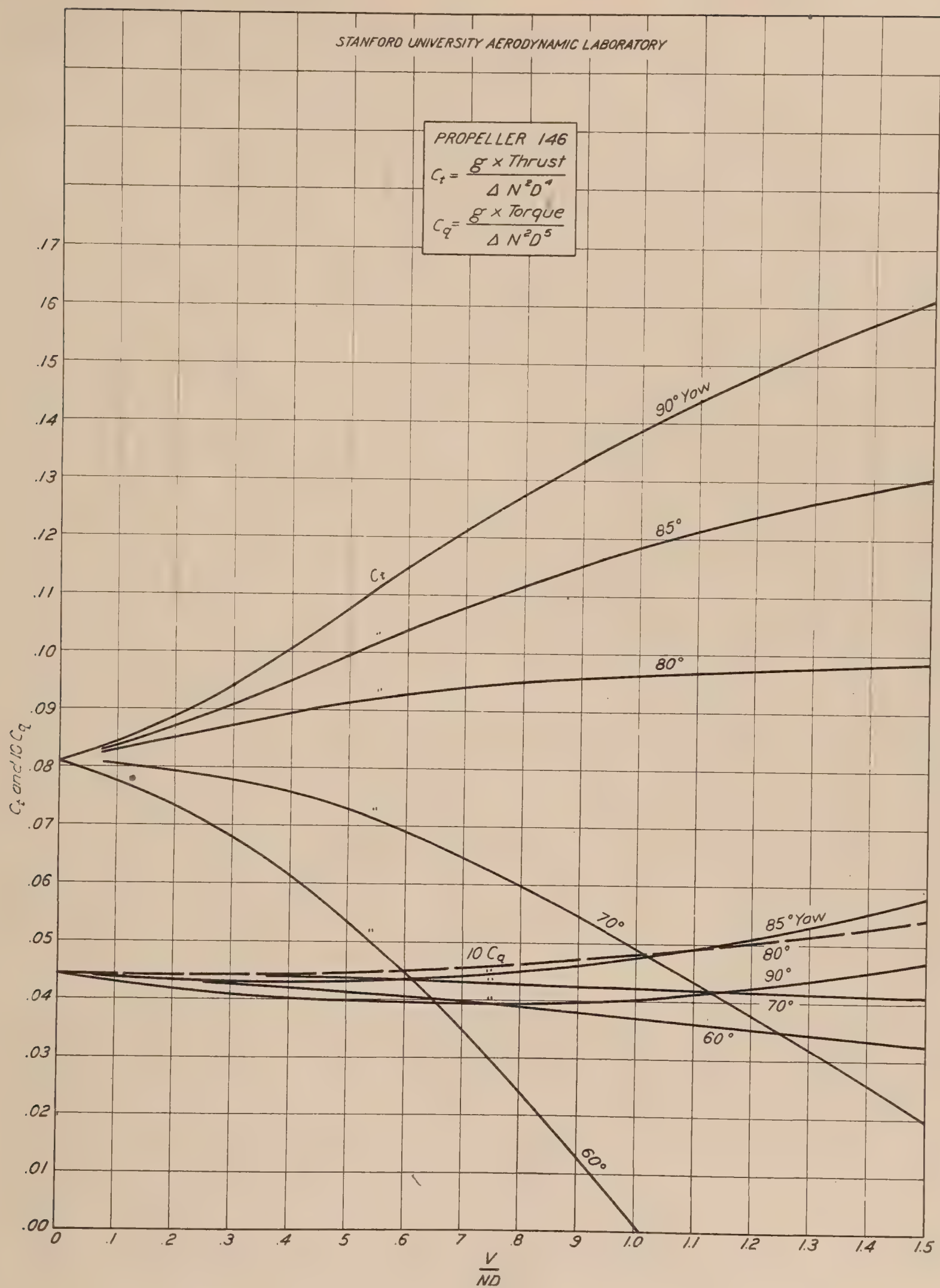

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER 146
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
90° Yaw
85°
80°
C_t
70°
85° Yaw
10 C_q
80°
90°
70°
60°
60°
C_t and 10 C_q
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
.00
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\frac{V}{ND}$

PLATE XIV.

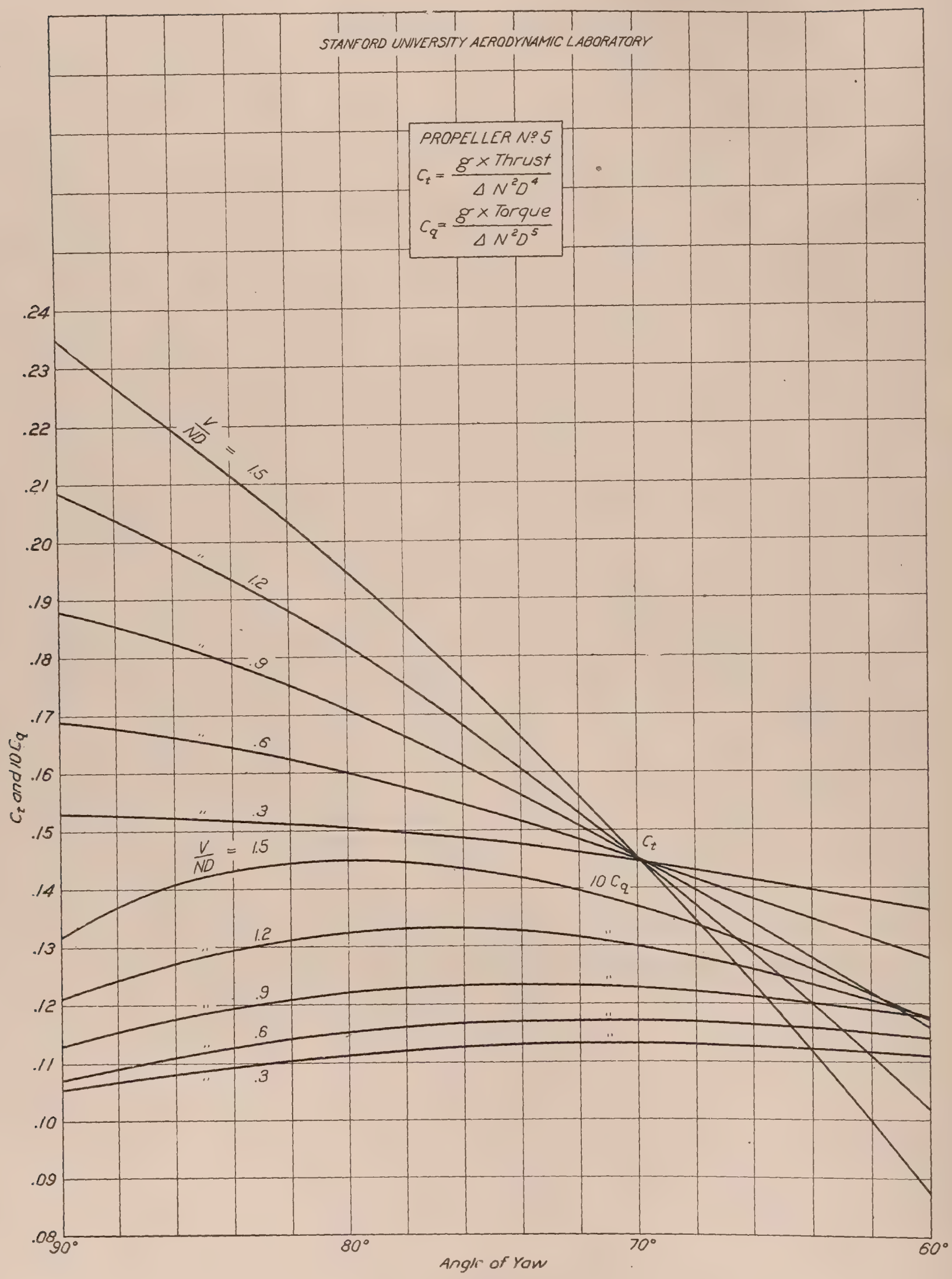

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 5
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
C_t
10 C_q
C_t and 10 C_q
.24
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
90°
80°
70°
60°
Angle of Yaw

PLATE XV.

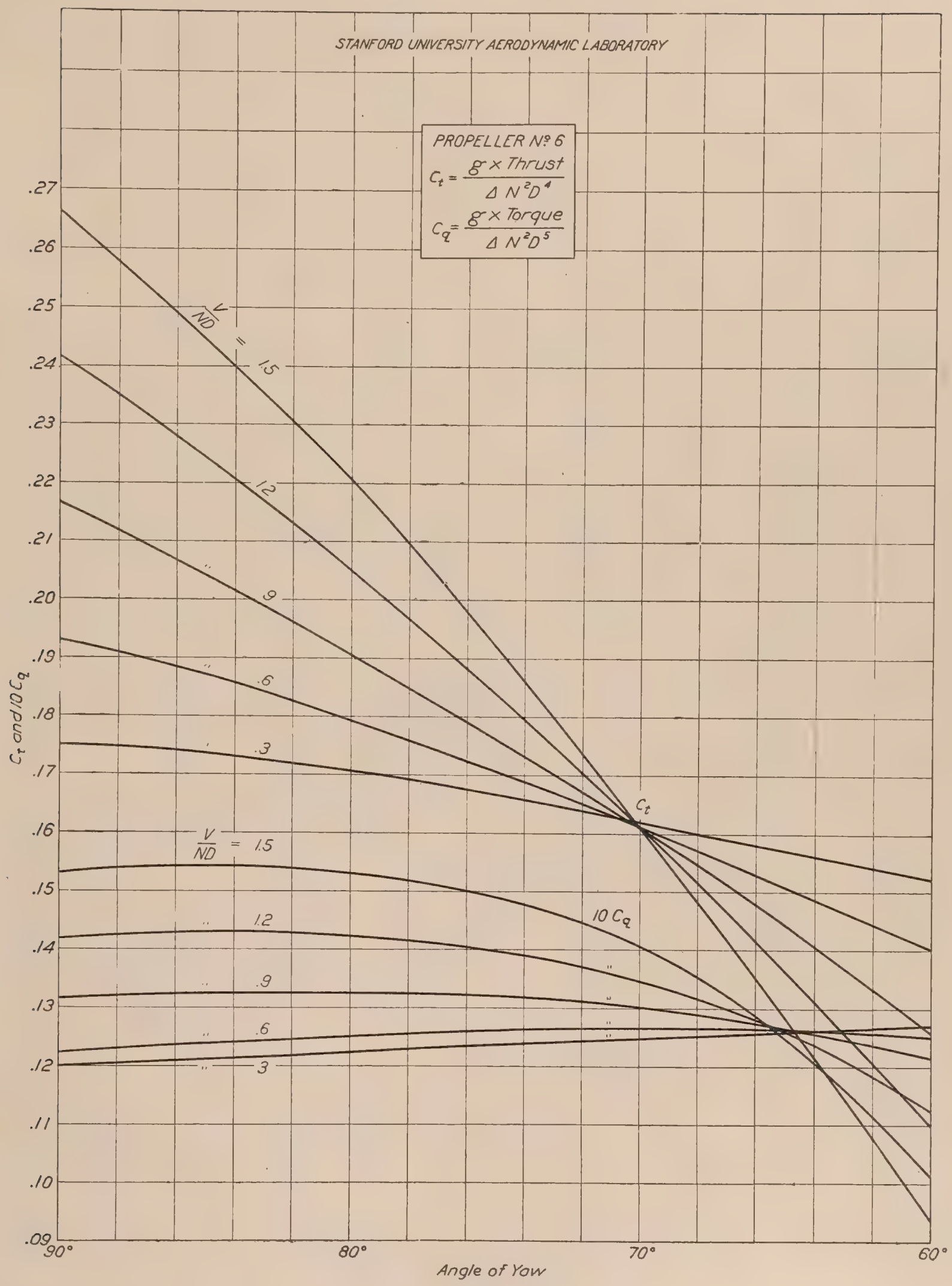

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 6
$C_t = \frac{g \times \text{Thrust}}{\Delta N^2 D^4}$
$C_q = \frac{g \times \text{Torque}}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
C_t
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
10 C_q
C_t and 10 C_q
.27
.26
.25
.24
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
90°
80°
70°
60°
Angle of Yaw

PLATE XVI.

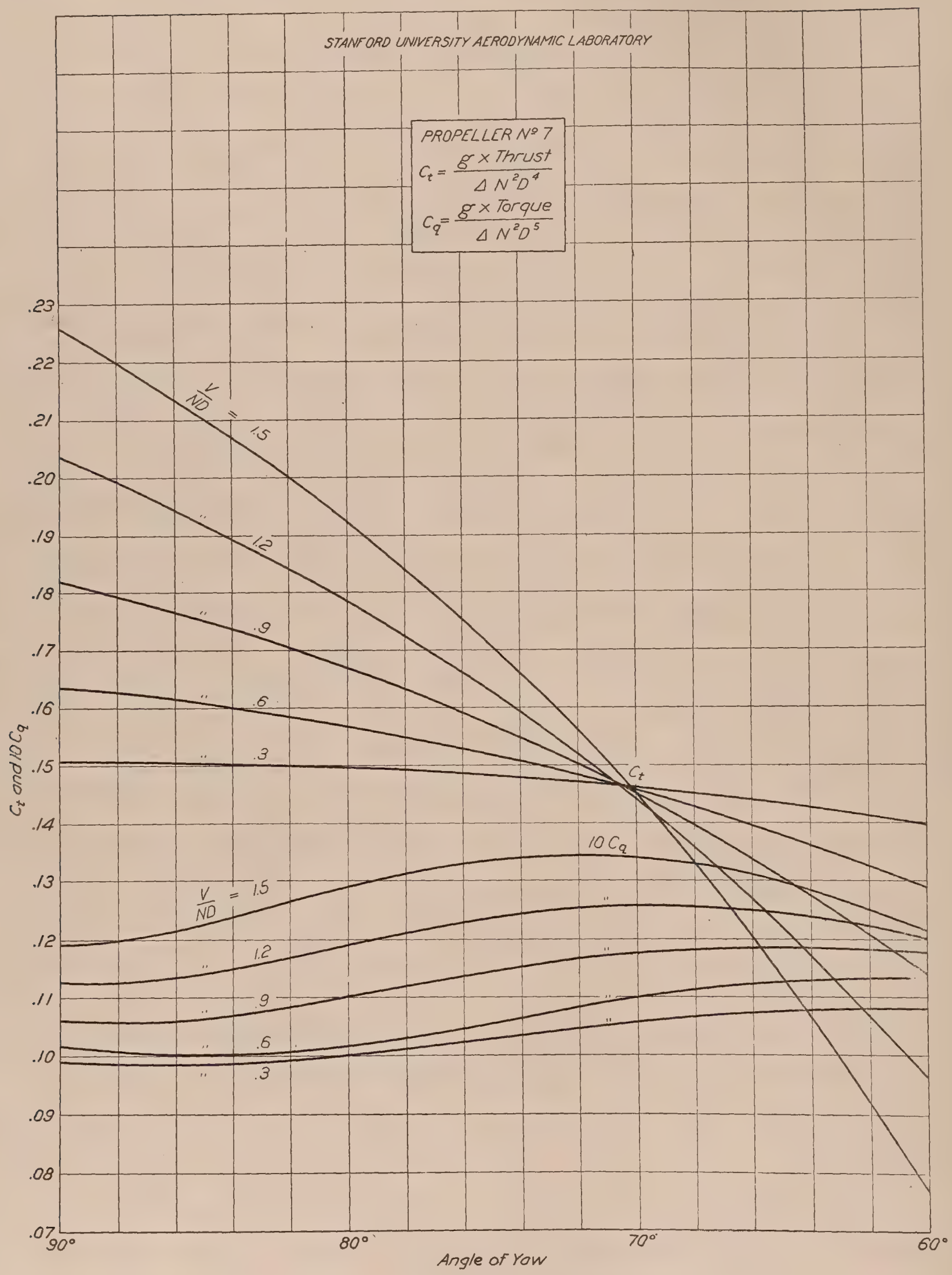

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 7
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
C_t
10 C_q
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
C_t and 10 C_q
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
90°
80°
70°
60°
Angle of Yaw

PLATE XVII.

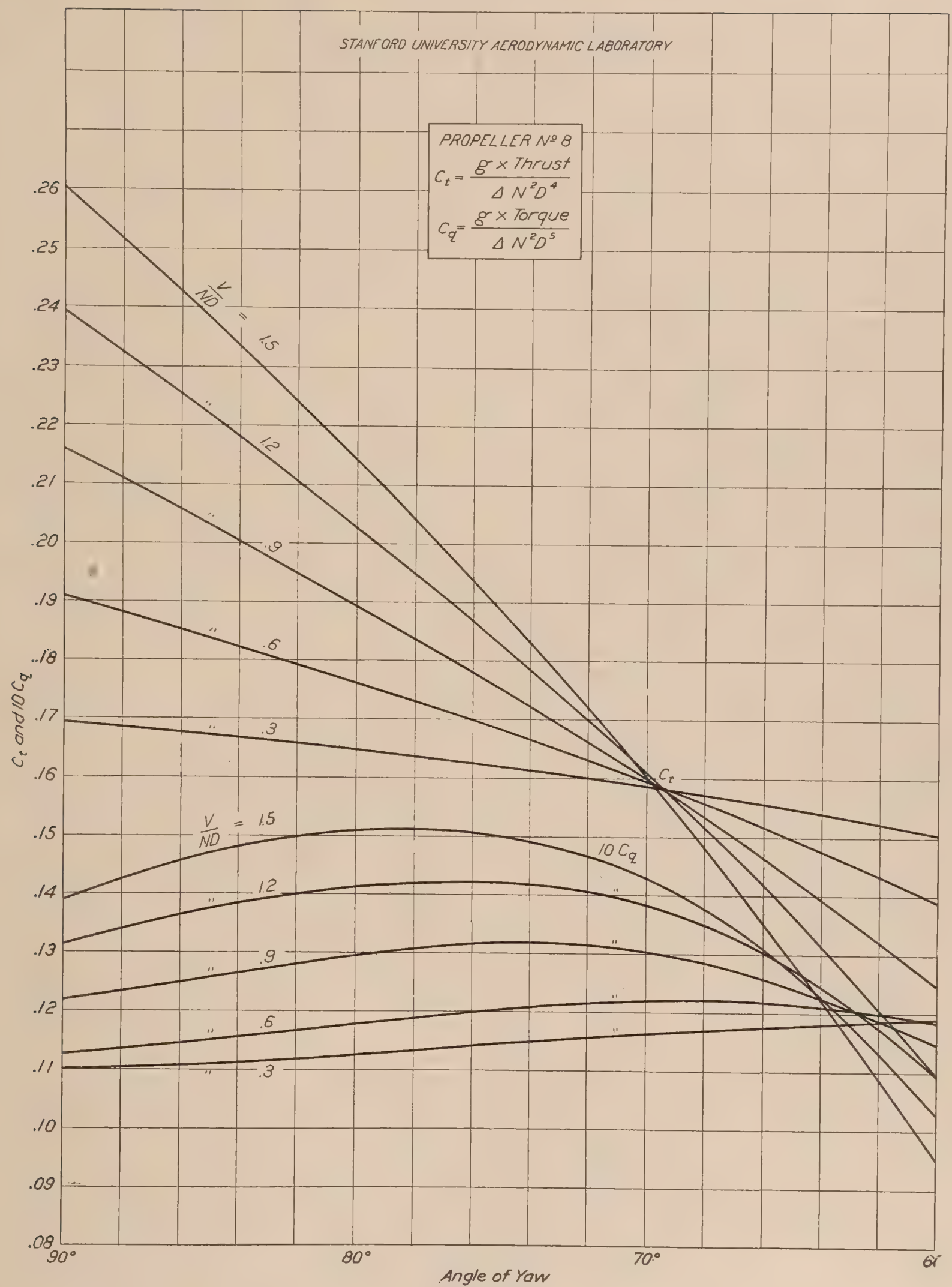

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº 8
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
C_t
$\frac{V}{ND}$ = 1.5
10 C_q
" 1.2
" .9
" .6
" .3
C_t and 10 C_q
.26
.25
.24
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
90°
80°
70°
60
Angle of Yaw

PLATE XVIII.

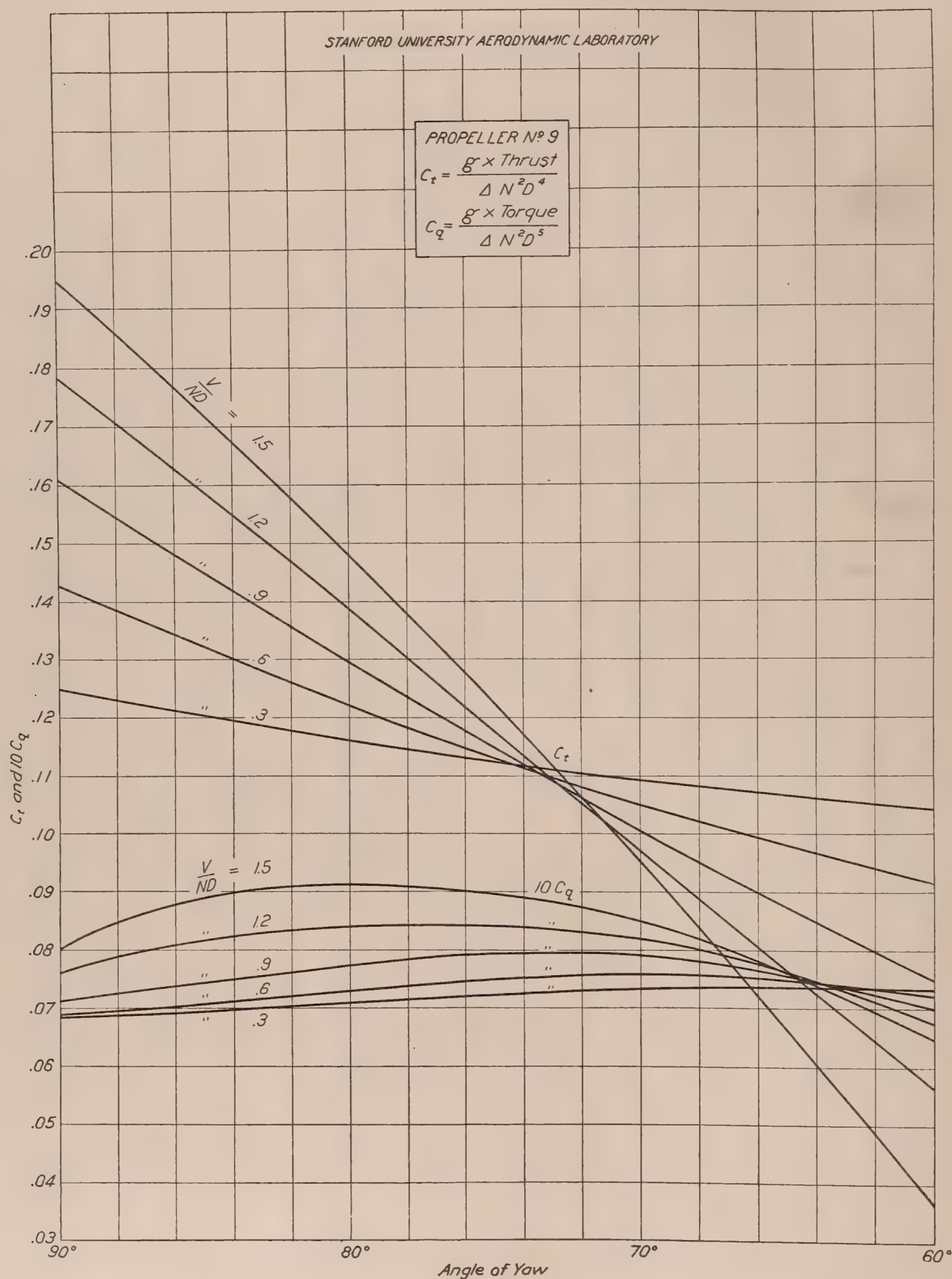

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER No. 9
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
C_t
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
10 C_q
C_t and 10 C_q
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
90°
80°
70°
60°
Angle of Yaw

PLATE XIX.

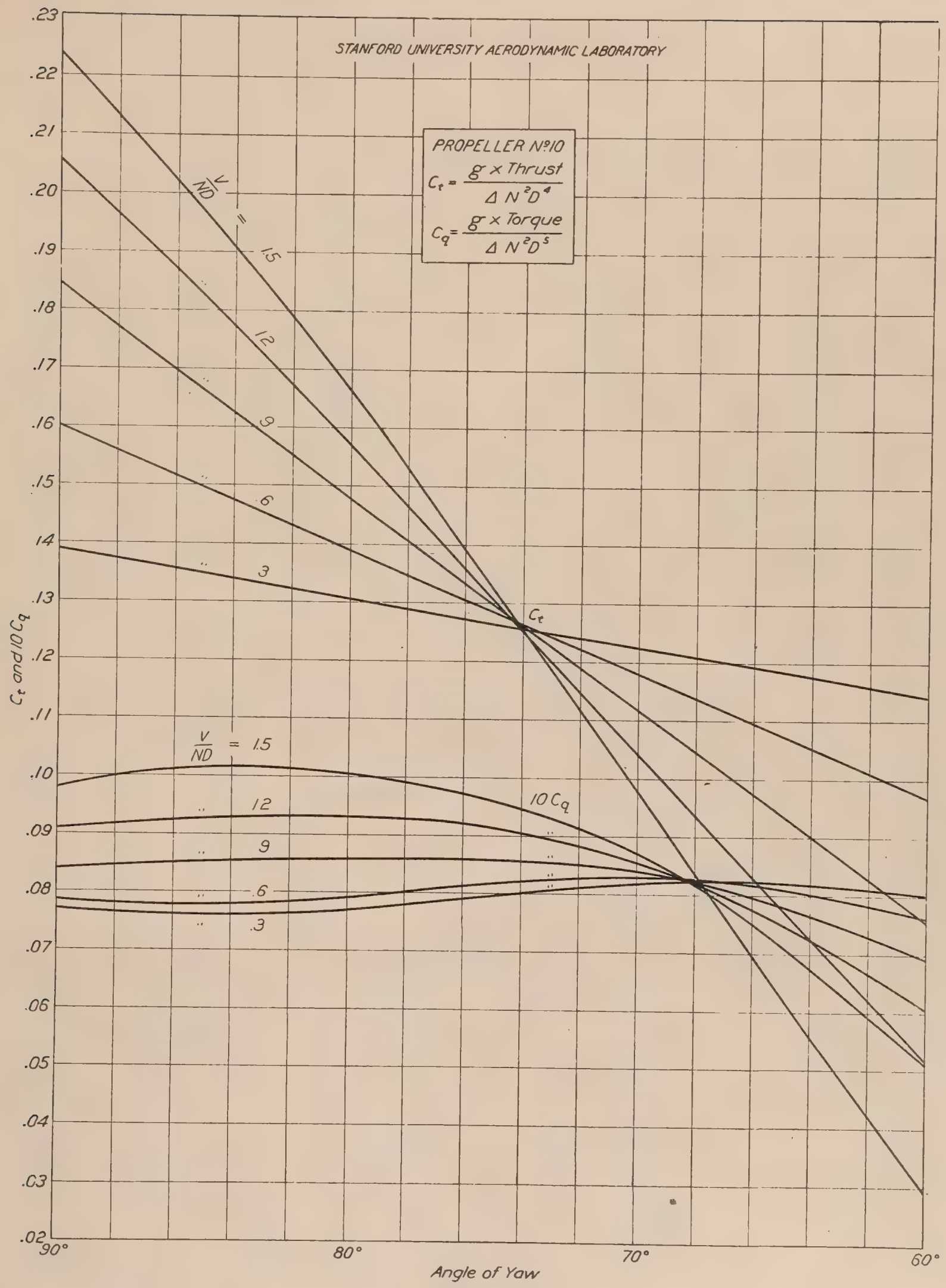
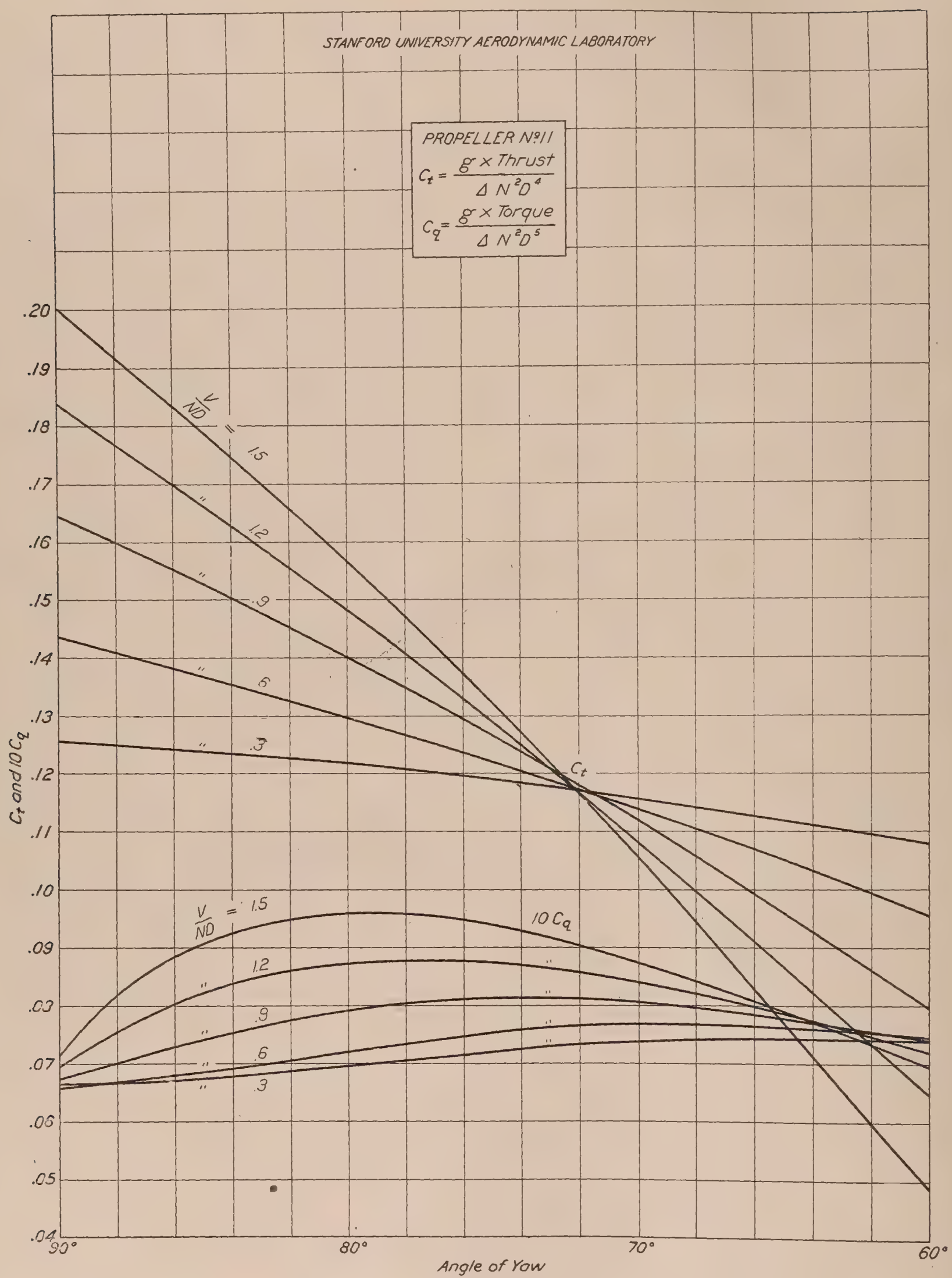

STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER No.10
$C_t = \frac{g \times \text{Thrust}}{\Delta N^2 D^4}$
$C_q = \frac{g \times \text{Torque}}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
12
9
6
3
C_t
$\frac{V}{ND}$ = 1.5
12
9
6
3
10 C_q
C_t and 10 C_q
.23
.22
.21
.20
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
90°
80°
70°
60°
Angle of Yaw

PLATE XX.


STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER No. 11
$C_t = \frac{g \times \text{Thrust}}{\Delta N^2 D^4}$
$C_q = \frac{g \times \text{Torque}}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
" .3
C_t
$\frac{V}{ND}$ = 1.5
" 1.2
" .9
" .6
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C_t and $10\,C_q$
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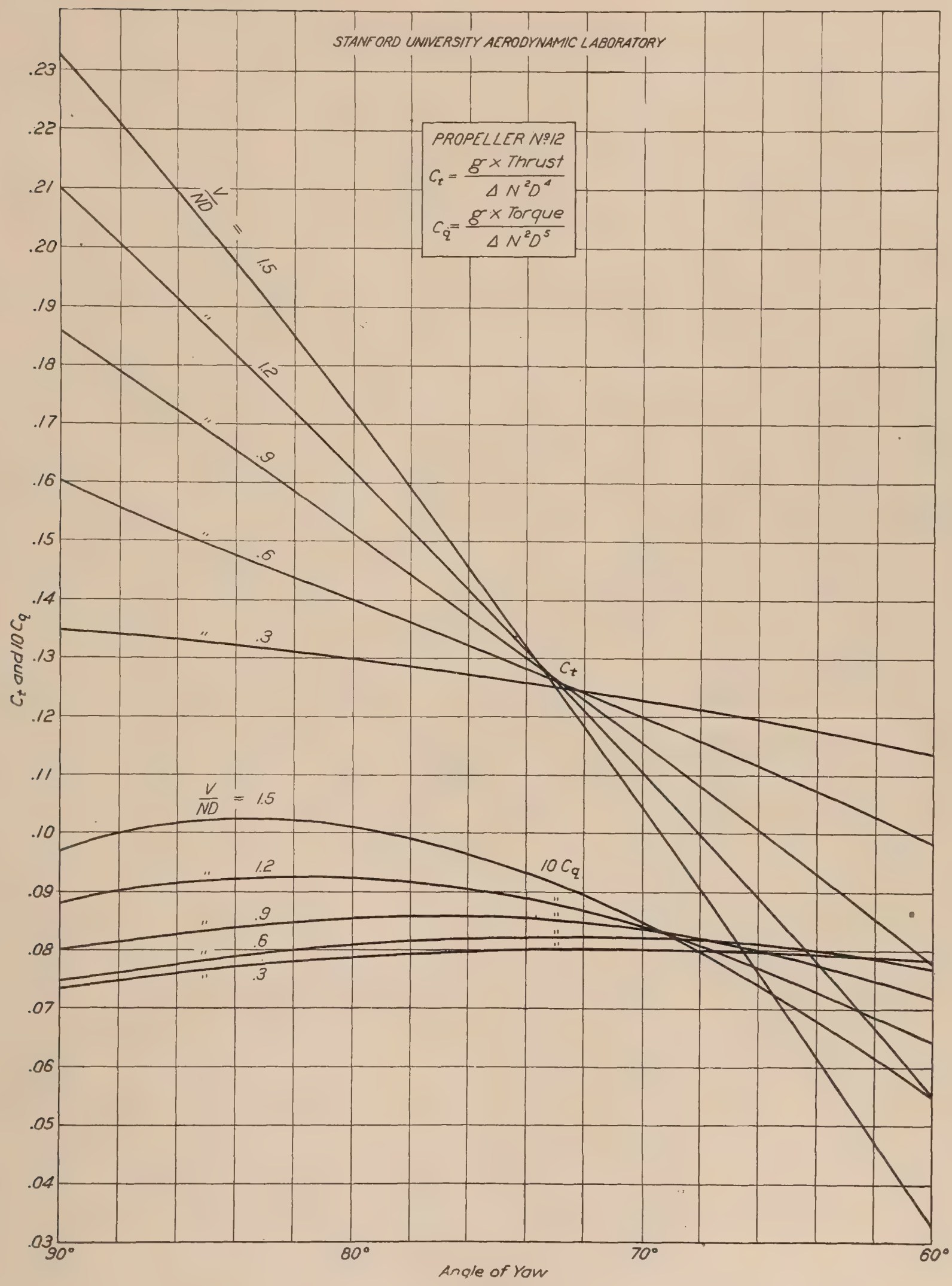
90°
80°
70°
60°
Angle of Yaw

PLATE XXI.


STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER Nº12
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$C_q = \frac{g \times \text{Torque}}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
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" .6
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C_t
$\frac{V}{ND}$ = 1.5
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C_t and 10 C_q
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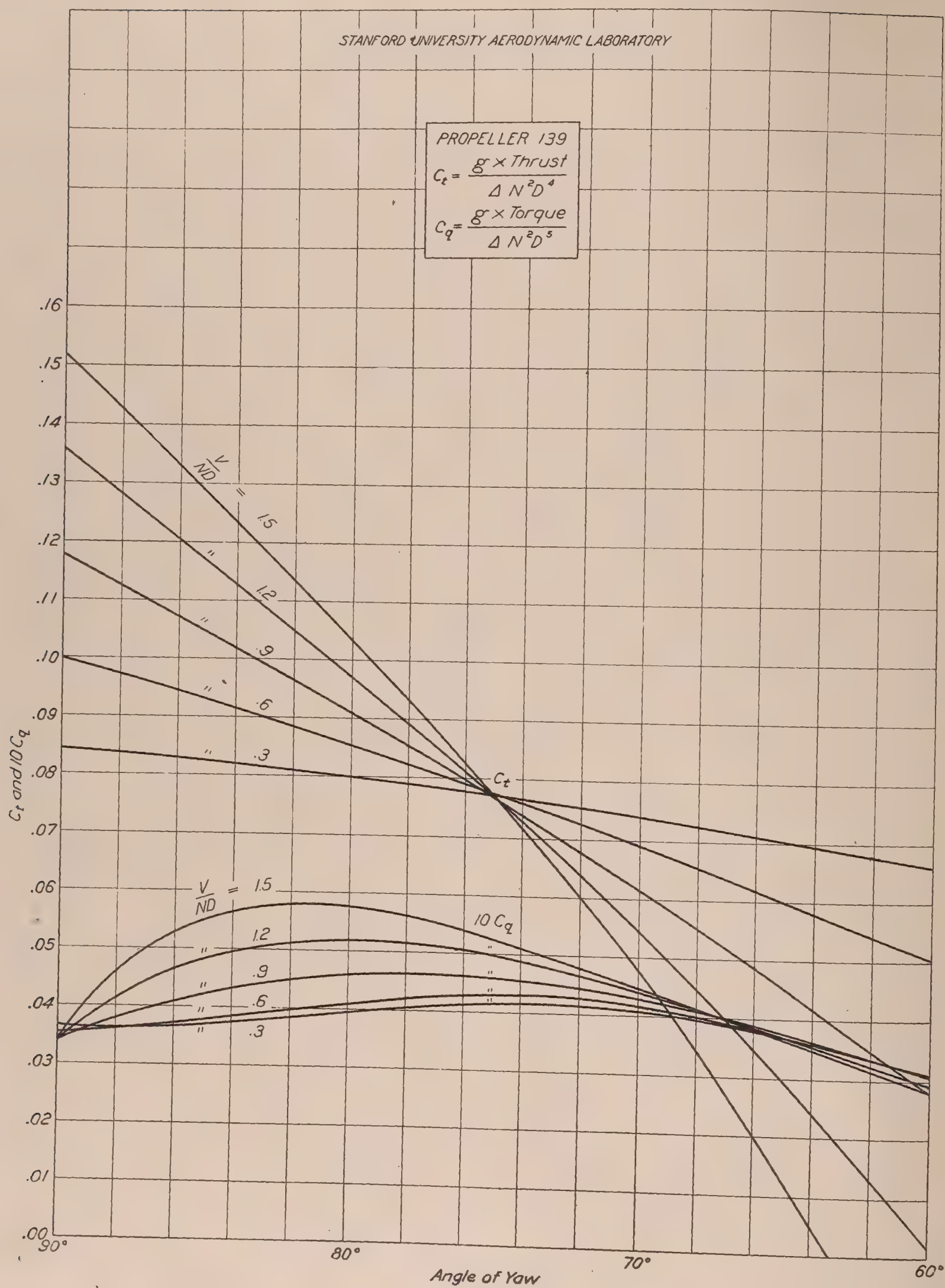
Angle of Yaw

PLATE XXII.


STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER 139
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
C_t and $10 C_q$
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$\frac{V}{ND}$ = 1.5
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C_t
$\frac{V}{ND}$ = 1.5
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" .6
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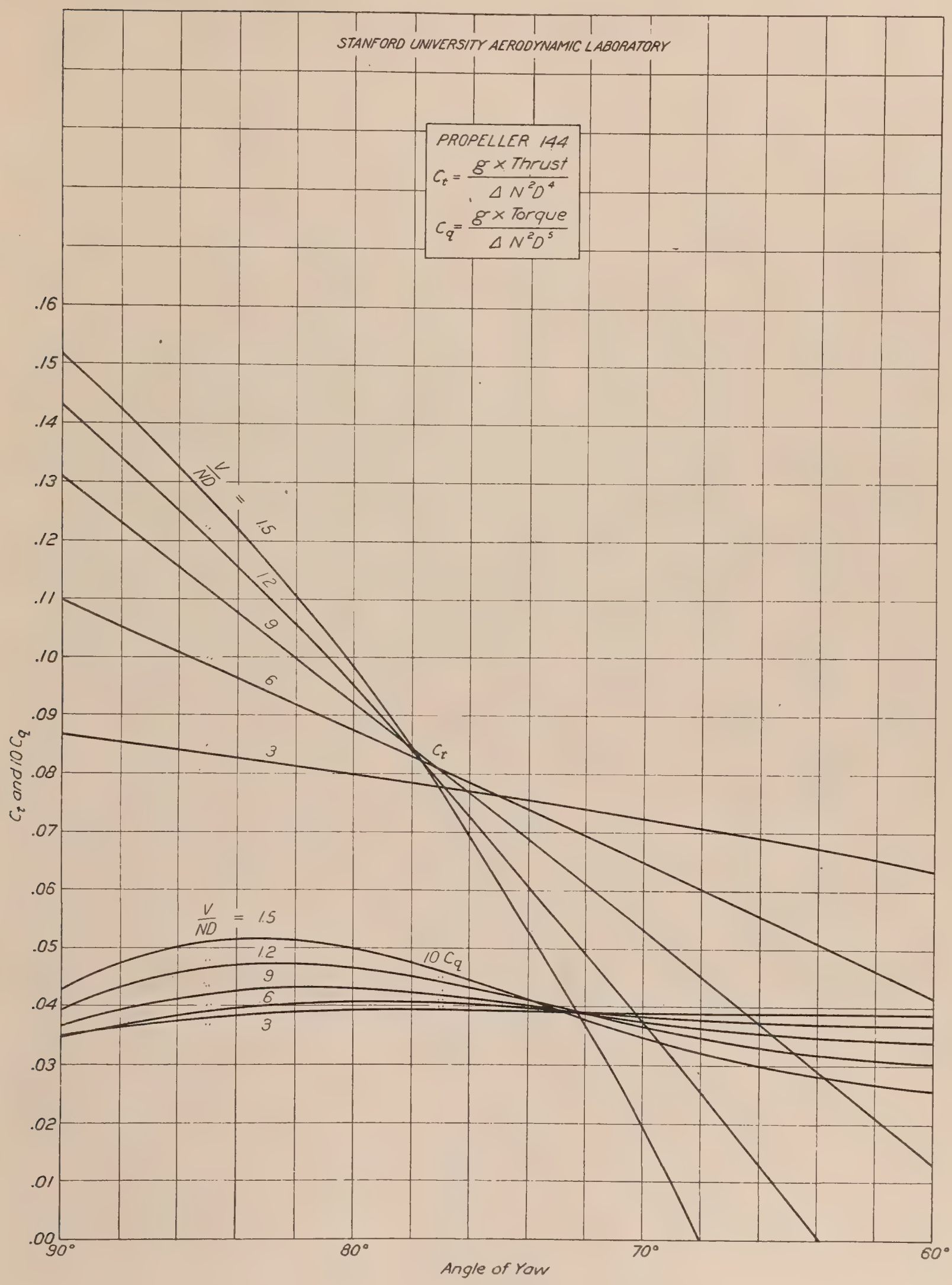
$10 C_q$
90°
80°
70°
60°
Angle of Yaw

PLATE XXIII.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PROPELLER 144
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
C_t and $10 C_q$
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$\frac{V}{ND}$ = 1.5
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C_t
$\frac{V}{ND}$ = 1.5
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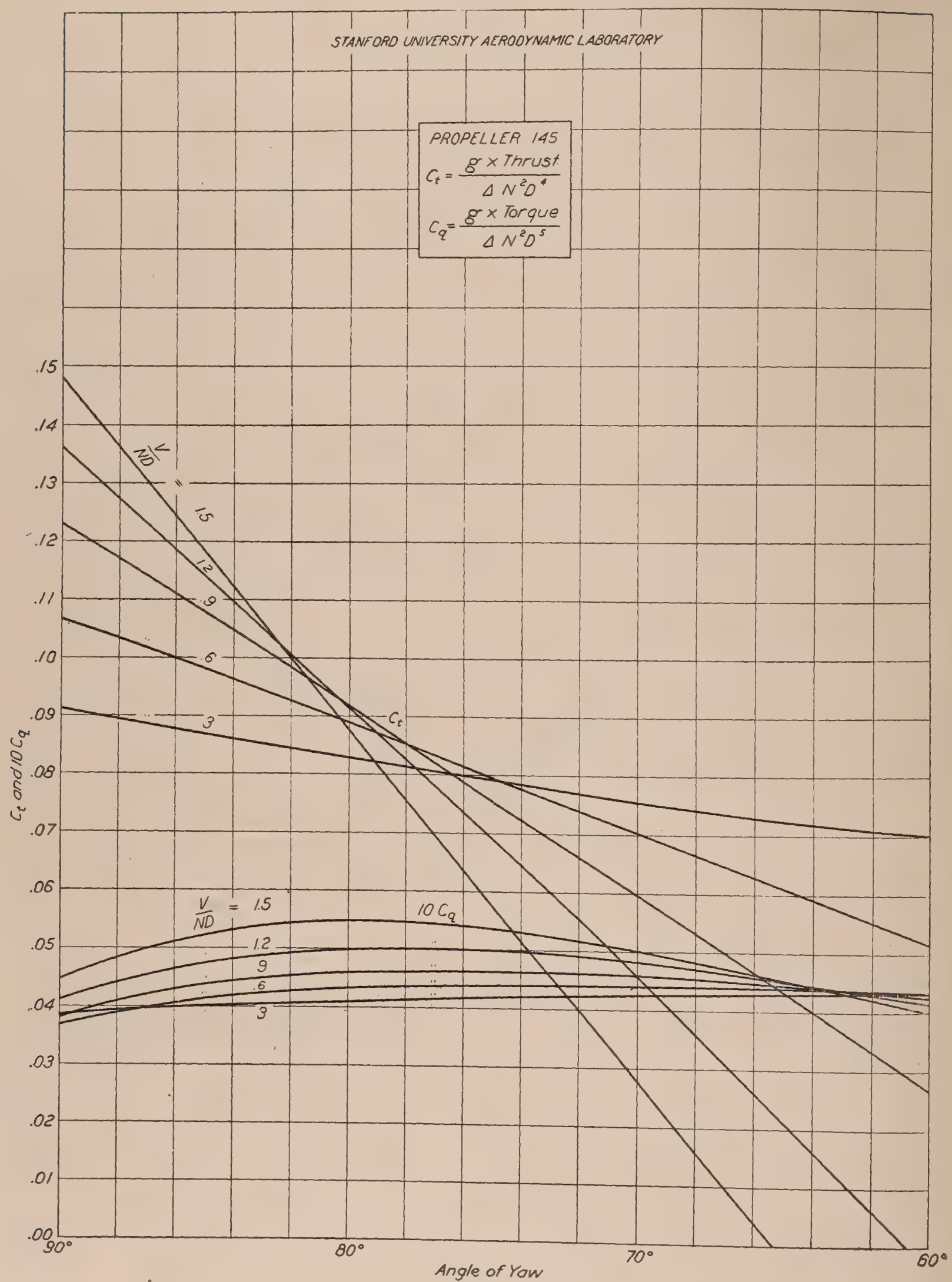
10 C_q
90°
80°
70°
60°
Angle of Yaw

PLATE XXIV.


STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER 145
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
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C_t
$10\,C_q$
C_t and $10\,C_q$
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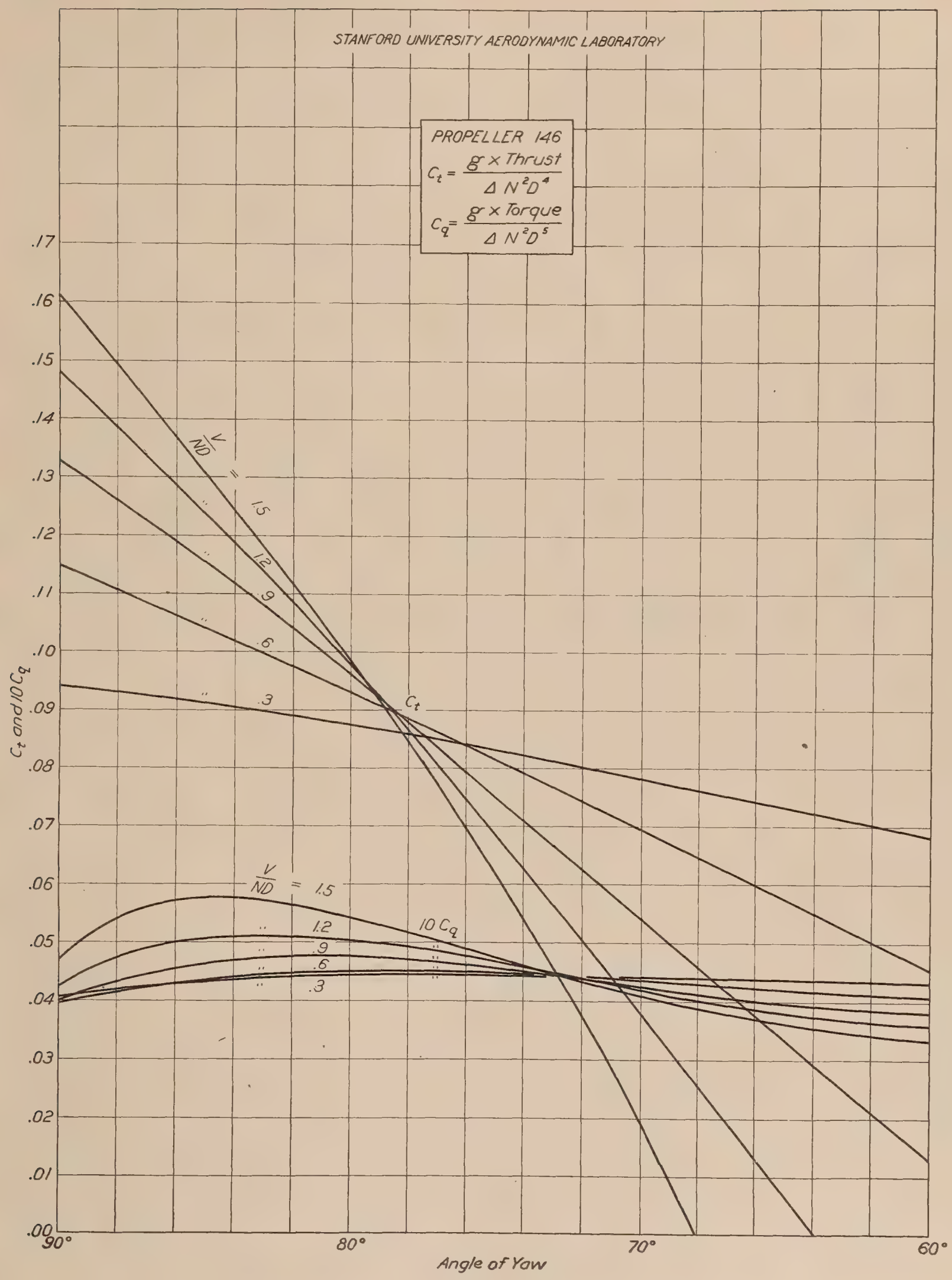
60°
Angle of Yaw

PLATE XXV.


STANFORD UNIVERSITY AERODYNAMIC LABORATORY
PROPELLER 146
$C_t = \frac{g \times Thrust}{\Delta N^2 D^4}$
$C_q = \frac{g \times Torque}{\Delta N^2 D^5}$
$\frac{V}{ND}$ = 1.5
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" .6
" .3
C_t
$\frac{V}{ND}$ = 1.5
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C_t and 10 C_q
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90°
80°
70°
60°
Angle of Yaw

PLATE XXVI.

REPORT No. 114

SOME NEW AERODYNAMICAL RELATIONS

BY

MAX M. MUNK

National Advisory Committee for Aeronautics

REPORT No. 114

SOME NEW AERODYNAMICAL RELATIONS

By Max M. Munk

RÉSUMÉ

This report was prepared for the National Advisory Committee for Aeronautics, and is a series of three notes, designed to extend the modern theory of aerodynamics and to develop it so that it may be applied to certain special problems in some later papers.

The motion of solid bodies in contact with each other is influenced by friction; but, nevertheless, it is often desirable to neglect this, and to make the necessary corrections later. Similarly, in treating the motion of a solid body through a fluid, it is desirable to begin with the case of a motion in which friction is neglected—i. e., with motion in a nonviscous fluid.

When two bodies are moving in a fluid, the disturbance in the fluid produced by one causes forces to act upon the other; and similarly, when there is but a single body in motion, any portion of it experiences a force due to the disturbances produced in the fluid by the other portions of the body. When the fluid is a nonviscous one, these forces may be calculated; and, from analogy with the phenomena of electrodynamics, they are called "induced" forces.

In the diagram let F be the *air* force acting upon the aerofoil; let v be the relative velocity of a particle of air close to the aerofoil; let V be the velocity of flight. Drawing the last two as vectors, it may be proved that F is perpendicular to the vector v and that the downwash w is in the line of F, so that $\bar{w}=\bar{V}-\bar{v}$, as shown. Further, I have proved that

$$w=kv$$

where $k=\frac{F}{\frac{1}{2}\pi b^2\rho v^2}$, b being the span of the aerofoil and ρ the density of the fluid.

Remembering that the dynamical pressure $q=\frac{1}{2}\rho v^2$, this may be written $k=\frac{F}{\pi b^2 q}$.

The lift is the component of F perpendicular to V; and the drag is the component parallel to V. That is:

$$L=F\frac{v}{V}$$

$$D=F\frac{w}{V}=\frac{w}{v}L=k\,L$$

As a first approximation in the calculation of D let us write $L=F$—i. e., identify v and V; hence $k=\frac{L}{\pi b^2 q}$; and therefore

$$D=\frac{L^2}{\pi b^2 q}$$

where $q=\frac{1}{2}\rho V^2$.

This formula for the induced drag is the one used by Prandtl and all recent writers on the subject of aeronautics.

In the first part of this paper use will be made of this relation and others derived, using the same approximation; in the second part the error introduced by making this approximation will be discussed; and in the third part a new theorem will be developed and its usefulness shown.

THE RELATIVE ABSORPTION OF POWER OF AIRPLANE WINGS AND PROPELLERS IN A NONVISCOUS FLUID

This note has particular reference to the comparison of the induced forces acting on a single pair of wings with those acting on a propeller having a large number of blades. In order to arrive at a proper basis for comparison it is convenient to consider power, instead of force. The wings require a certain amount of externally supplied power for a given performance. All of this power, which is to be considered as "induced," is transmitted to the air acted upon by the wings if the motion is steady. The power absorbed by the propeller, however, is partially recovered in the power utilized to propel the airplane. Only the difference between the power absorbed and the power utilized is transmitted to the surrounding air, and only this difference is to be considered as the induced power of the propeller.

Let L be the lift of the wings (or the thrust of the propeller); V the velocity of the airplane (or of the propeller), relative to the surrounding air; b the span of the wings (or the diameter of the propeller); and ρ the density of the air. As stated above, the induced drag corresponding to a velocity V is $D=\frac{L^2}{\pi b^2 q}$, hence the induced power absorbed by the wings is given by

$$P_{\text{ind}}=\frac{VL^2}{\pi b^2 q} \tag{1}$$

q being the dynamical pressure; that is, $q=1/2\rho V^2$. Similarly, the induced power of the propeller is given by

$$P_{\text{ind}}=\tfrac{1}{2}\,VL\left[\sqrt{1+\frac{L}{b^2 q\,\frac{\pi}{4}}}-1\right] \tag{2}$$

If the term $\frac{L}{b^2 q\,\frac{\pi}{4}}$ be less than unity, the expression within the brackets may be developed in a series. On this condition we have for the propeller—

$$P_{\text{ind}}=\frac{VL^2}{\pi b^2 q}-\frac{1}{16}\,\frac{VL^3}{\left[b^2 q\,\frac{\pi}{4}\right]^2}+-+ \tag{3}$$

Hence, if the thrust per unit area of the propeller disk be small in comparison with the dynamical pressure, the first term of the series gives a close approximation to the induced power. It therefore appears that, on this condition, the induced powers absorbed by wings and propeller agree.

It may therefore be stated that : If the thrust per unit area of the propeller disk be small n comparison with the dynamical pressure, then a wing and a propeller of equal span (or diameter), lift, and dynamical pressure require the same induced power.

This agreement is not accidental, but is due to the similar mechanical conditions of the two cases. The thrust of a propeller is produced by giving momentum to the air which passes through its disk. If the indraft substantially equal the velocity, then the mass of air passing through the disk per second is

$$V\,b^2\,\frac{\pi}{4}\,\rho$$

The lift of the wing is, however, produced by imparting momentum to the mass of air passing its plane in a direction perpendicular to the direction of flight, at that particular moment. But the share of each particle of air in the production of lift varies from point to point according to its position. With reference to the flow in the infinite plane perpendicular to the wing, it

behaves like a single line in a two-dimensional fluid accelerated perpendicular to its length. Let such a line have the length b, and let ρ be the density of the fluid. Then such an infinite two-dimensional fluid motion around the line has the same kinetic energy as the mass

$$b^2\rho\,\frac{\pi}{4}$$

moving with the same velocity as the line. That is to say, the whole infinite disk of air has the same mechanical effect as a circle with the diameter b cut out of the fluid, in the two-dimensional case, and moving like a rigid body with the line of length b.

Hence the equivalent mass of air flowing past the wing per second and producing the lift is

$$V\,b^2\frac{\pi}{4}\rho.$$

In the two cases the same mass of air produces the same lift (or thrust), and this is why the same induced power is required.

If the thrust be large, or if the velocity of the propeller be small, the theorem no longer holds true. Otherwise a stationary rotating propeller, as in a helicopter, would require an infinite amount of power. The original expression (instead of the development into series) for induced power must now be used, because the conditions which permitted the expansion in series are not maintained. If $V=O$, expression (2) becomes

$$P_{\text{ind}}=L\sqrt{\frac{L}{b^2\pi\frac{\rho}{2}}}$$

and it appears that the induced power of a propeller without forward velocity is equal to the induced power required by a single wing of span equal to the diameter of the propeller and moving with a velocity corresponding to a dynamical pressure of

$$q=\frac{L}{b^2\pi},\ \text{i. e.,}\ V^2=\frac{2L}{\pi b^2\rho}.$$

This expression is the fourth part of the thrust per unit area of the propeller disk. Therefore, a helicopter requires the same induced power as a wing of the same load and span, moving with a speed corresponding to a dynamical pressure equal to the fourth part of the thrust per unit area of disk. At the same time this required power equals the induced power absorbed by a wing with twice the load, the same span and a velocity corresponding to a dynamical pressure equal to the thrust per unit area of disk

MORE ACCURATE CALCULATION OF DRAG

Referring to the introduction, it is known that

$$L = F\frac{v}{V}$$

$$D = F\frac{w}{V} = kL, \text{ where } k = \frac{F}{\frac{1}{2}\pi b^2 \rho v^2}$$

in which F is the force acting upon a wing—

v is the velocity of a particle of air near the wing.
V is the velocity of flight.
w is the down-wash velocity.
b is the span of the wing.

In the deduction of these formulæ it has been assumed that the effective parts of the longitudinal vortices, which give rise to the lift, may be considered to be straight. This assumption certainly is allowable, and is approximately true.

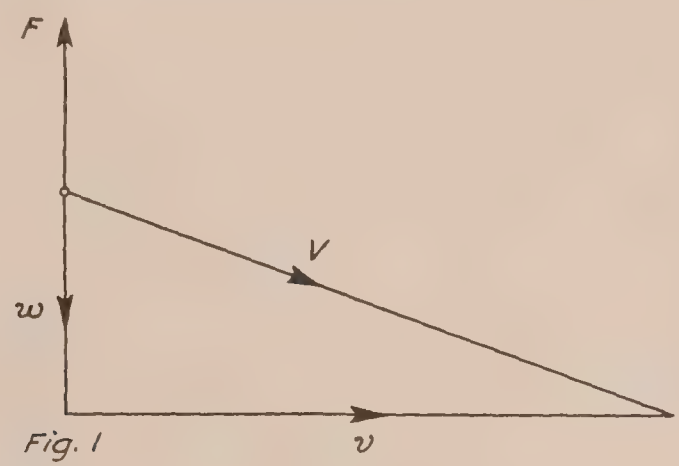

Fig. 1

Referring to the diagram,

$$V^2 = v^2 + w^2$$

and, since $w = kv$, $V^2 = v^2\,(1 + k^2)$. Therefore, since k is a small quantity, $V = v\,(1 + \frac{1}{2}k^2)$, or $v = V\,(1 - \frac{1}{2}k^2)$. Hence

$$L = F\,(1 - \tfrac{1}{2}k^2)$$
$$D = kF\,(1 - \tfrac{1}{2}k^2).$$

Let us now introduce a new coefficient, $k' = \frac{L}{\pi b^2 q^1}$ where $q^1 = \frac{1}{2}\rho V^2$—i. e., is the dynamical pressure corresponding to the velocity of flight. Hence, neglecting terms of high orders,

$$V = v + \tfrac{1}{2}k'^2 V$$
$$L = F - k'^2 L$$

and, therefore,

$$D = k'L\,(1 + 1.5k'^2).$$

In Part I we used

$$D = \frac{L^2}{\frac{1}{2}\pi b^2\,\rho V^2}; \text{ i. e., } D = k'\,L.$$

It is now seen that, using the second approximation, the induced drag is greater than this. Since k' is small, $1.5\,k'^2$ is small compared with unity; and therefore the error made in using the first approximation is not large. This is the reason why the simpler formula gives results agreeing well with experience.

Let us calculate the error for a practical case; e. g., span $b=35$ feet; lift $L=2{,}000$ pounds; velocity $V=100$ mi/hr. Then $k'=0.0203$; $k'^2=0.0004$, and the error $1.5\ k'^2=0.06$ of 1 per cent. Even if the density is decreased to one-half the normal value and the velocity to two-thirds of the previous value, k' therefore becoming 4.5 greater, the error is only 1.3 per cent.

As a consequence, the original simple formula,

$$D=\frac{L^2}{\pi b^2 q}, \text{ where } q=\frac{1}{2}\rho V^2$$

may be considered to be sufficiently exact for ordinary purposes. For extraordinary cases the more exact form may be necessary.

THE FORCES ACTING ON BODIES MOVING IN A PERFECT FLUID

The force acting on an aerofoil moving in a perfect fluid is calculated by taking into consideration the mutual effect of the velocity in the immediate neighborhood of the aerofoil and the intensity of the vortices imagined in the space occupied by the aerofoil. This leads to the result stated in the introduction. The resulting force on the aerofoil equals the product of the velocity referred to and the intensity of the vortex, and its direction is perpendicular to the velocity and the vortex, as shown.

It is not generally known, however, that these are not the only forces between a body and a perfect fluid moving around it; and that forces between them may exist even if the effect of the body must not necessarily be considered as due to hypothetical vortices occupying the space of the body. I shall deduce the following theorem: If the body may be replaced by sources and sinks, a source of intensity J will give rise to a force $-\bar{v}\ J\ \rho$, in which $\bar{v}$ is the velocity of the fluid and ρ its density, $\bar{v}$ indicating the vector character of the velocity.

This theorem is analogous to the one which states that if there is a hypothetical vortex of intensity J at a point in a fluid where the velocity is v, the force per unit length of the vortex is the vector product $\rho\ [\bar{v}\ \bar{J}]$.

In both theorems the fluid is considered to be incompressible, to be flowing irrotationally, and to be in a steady state. They can be proven for more general assumptions also.

In order to prove this theorem I shall first use general considerations and then shall develop the mathematical formulæ. Let us consider a liquid flowing in any manner inside a closed boundary, which may be thought of as a solid shell. There will be pressures acting on each element of the boundary, and these will be equal and opposite to the pressures on the outside if the same liquid is flowing around the immersed body, and has the same velocity at each point of the boundary as the fluid inside has. This flow inside may be due to vortices and to sources; but for these to give rise to a steady condition and also to exist as forming a distinct isolated system, certain conditions must be satisfied. In particular, for a vortex to remain stationary there must be certain applied forces, according to the well-known theorem of Kutta. If the vorticity at any point has the angular velocity w, and if at that point the velocity of flow is V, the force required to maintain a steady state is $\rho\ [\bar{V}\bar{w}]\ d\tau$, where ρ is the density and $d\tau$ is an element of volume. (This may be written $\rho\ [\bar{V}\bar{J}]\ dl$, where J is the strength of the vortex.) This force must be due to the action of the solid boundary; and, if the system is to be self-contained, this force must be equal and opposite to the force of the liquid acting on the boundary; i. e., must be equal to the force acting on the boundary due to the liquid flowing on the *outside*.

Again, as far as the flow inside the boundary is caused by sources, it is evident that, in order that the system may be self-contained, there must be both sources and sinks whose intensities are equal, and we must picture these as connected by thin tubes, so that the fluid disappearing at a sink may be brought to the source out of which it flows. Further, if these positive and negative sources are to be stationary, and thus give rise to a steady motion, certain forces must be applied. If the fluid entering any sink has the velocity V_1, and if, on emerging from the tube at the source, it has the velocity V_2, the strength of each being J (i. e., the volume output per unit of time), the momentum has been increased in a unit of time by an amount $\rho J(V_2 - V_1)$, consequently there must be a force applied; and, as before, this force must be due to the boundary, and must be equal to the force acting on the boundary due to the liquid flowing outside.

Thus, if in any problem the effect of an actual solid immersed in a stream of fluid is considered as due to an "equivalent" distribution of vortices or of sources in the space occupied by the solid, we have the value of the force acting on the solid.

The exact proof may be deduced mathematically. If V is the velocity at any point inside the boundary, the pressure is $-\frac{\rho}{2}V^2$; therefore the force against the boundary from within is $-\frac{\rho}{2}\int \bar{n}_1 V^2 dS$, where $\bar{n}_1$ is a unit vector normal to an element of surface dS, and drawn outwards. Therefore, if the velocity of the fluid outside is the same at each point as that inside, the force acting on the boundary from *without* is $+\frac{\rho}{2}\int \bar{n}_1 V^2 dS$. This may be transformed into an integral through the volume, viz:

$$\frac{\rho}{2}\int \bar{n}_1 V^2 dS = \frac{\rho}{2}\int \text{grad } V^2 d\tau = \rho\int \nabla \bar{V}\cdot \bar{V} d\tau \text{ using Gibbs's symbols.}^1$$

This holds true whether the velocity at the boundary has a component normal to it or not.

We must now introduce the condition that no fluid passes the surface, and therefore no momentum is transferred through it. If momentum were flowing through the boundary, the flow per unit of time would be $\rho\int \bar{V} V_n dS$. Thus must equal zero, therefore; and, as before, the integral may be transformed into a volume one, viz:

$$\rho\int \bar{V}\, V_n d\, S = \rho\int (\bar{V} \text{ div } \bar{V} + \bar{V}\nabla \bar{V})\, d\,\tau, \text{ using Gibbs's symbols.}^2$$

Since this equals zero, it may be subtracted from the previous integral giving the force, and hence

$$F = \frac{\rho}{2}\int \bar{n}_1\, V^2\, d\, S \begin{array}{l} = \rho\int(\nabla V.V - V\nabla V - \bar{V} \text{ div } \bar{V})\, d\tau \\ = \rho\int (\bar{V}\times \text{rot } \bar{V} - \bar{V} \text{ div } \bar{V})\, d\tau. \end{array}$$

This is a formal statement of the theorem.

The conclusions derived from this theorem agree with other results of hydromechanics and with the general laws of mechanics. A source and a vortex are the two elementary solutions of the partial differential equation determining the motion. Each of these solutions can be derived from the other, and it is possible to pass continuously from one to the other. For these reasons it would be impossible that for one of the solutions a theorem exists without there being a corresponding theorem for the other. The continuous transformation may begin with two distant two-dimensional vortices of opposite sign but of equal intensity (fig. 2). The distance may grow less and less till at last the two vortices form a double vortex, the intensity of each increasing at the same time in such a way that the product of the intensity and the distance is constant. The doublet vortex is identical with a doublet source, consisting of a rectilinear source and equal sink, whose axis is perpendicular to that of the vortex doublet. This source doublet may be continuously transformed into two separate sources, by increasing the distance of the source and sink and at the same time decreasing their intensity, so that the product of the intensity of each and the distance remains constant as before.

The velocity of the fluid in the neighborhood of these vortices or sources may be supposed to have a constant rate of change in the direction of the axis of the doublet vortices. The rate of change in the direction of the axis of the doublet sources is perpendicular to the first mentioned rate, if the velocity is irrotational and without divergence, and both have equal absolute magnitude. Then the force acting on the two vortices is constant during the whole transformation. It certainly would be absurd to expect this constant force suddenly to vanish either on proceeding from the doublet vortex to the doublet source, which is only another expression for the same phenomenon, or on transforming the doublet source to two separate sources. We are led therefore to attribute to each source the force given by the theorem.

---

[1] This is written in ordinary notation: $\rho\int\Big(V_x \text{ grad } V_x + V_y \text{ grad } V_y + V_z \text{ grad } V_z\Big)d\tau$.

[2] This is written in ordinary notation: $\rho\int \{\bar{V} \text{ div } \bar{V} + (i\bar{V} \text{ grad } \bar{V}_x + j V \text{ grad } \bar{V}_y + V \text{ grad } \bar{V}_z)\}\, d\tau$.

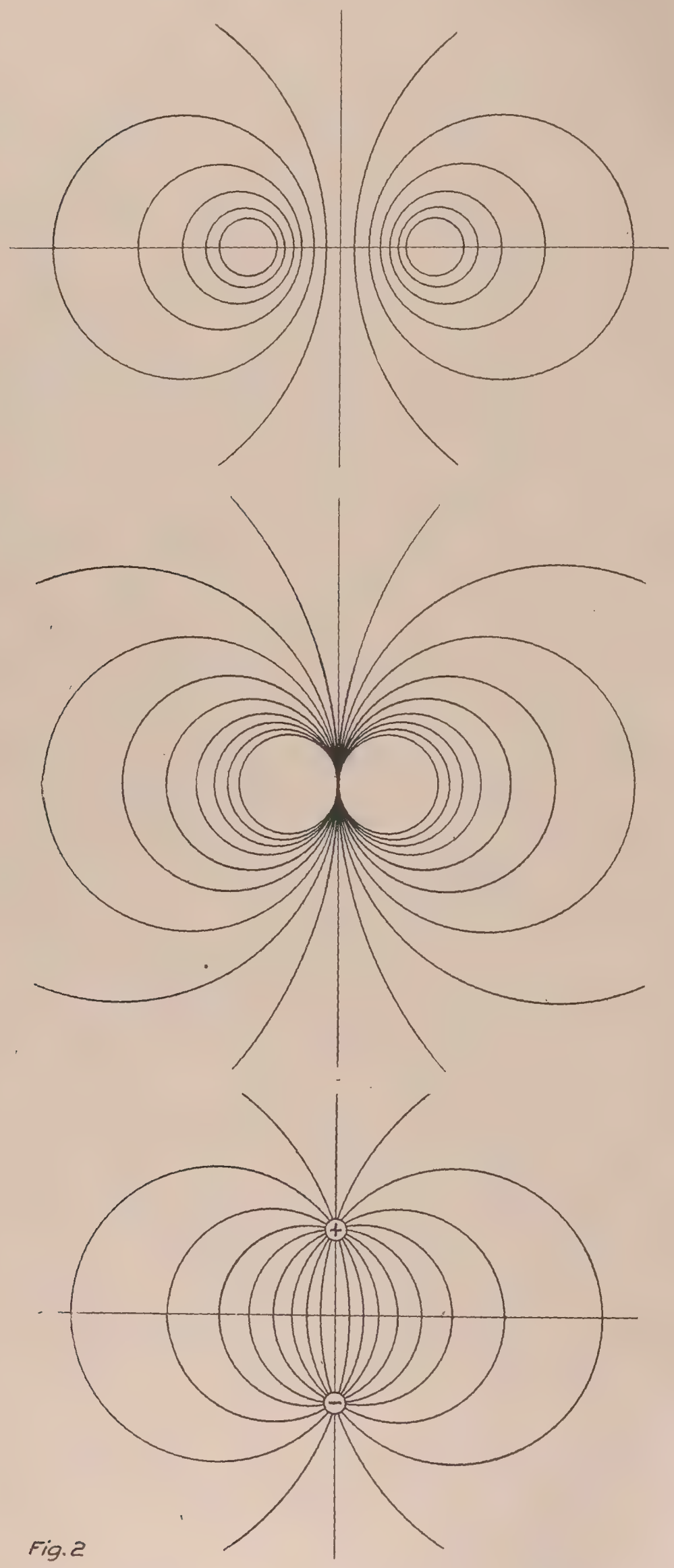

Fig. 2

The vortex and the source are the elements of the two possible kinds of discontinuities. If the fluid streams along a boundary with constant velocity on one side and with a different constant velocity on the other side (fig. 3) Euler's theorem states that the pressure on the two sides of the boundary is different. The difference is $\frac{1}{2}(V^2_2 - V^2_1)\rho$, the pressure being higher on the side of the smaller velocity. This boundary is to be represented by vortices perpendicular to the discontinuity of velocity and uniformly distributed over the boundary. They can be considered to be in fluid having the velocity $\frac{1}{2}(V_1 + V_2)$. Their intensity per unit of area is $(V_2 - V_1)$. The difference of pressure equals the product of the intensity per unit area, the velocity and the density, and is directed at right angles to the velocity and to the vortex. The force on the vortex is directed toward the space of the greater velocity.

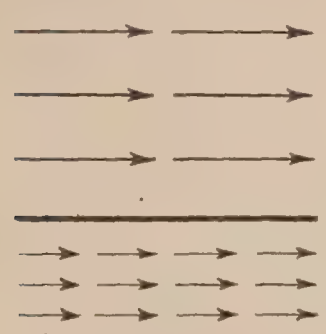

Fig. 3

The other kind of discontinuity occurs if the boundary is perpendicular to the two constant velocities (fig. 4). This is only possible if the boundary is occupied by sources, uniformly distributed, with the intensity $(V_2 - V_1)$ per unit of area. They again are in fluid having the velocity $\frac{1}{2}(V_2 + V_1)$ and the force acting on them is $\frac{1}{2}(V_2 + V_1)(V_2 - V_1)\rho$. This equals the pressure obtained by applying our theorem to the single sources which are distributed over the boundary.

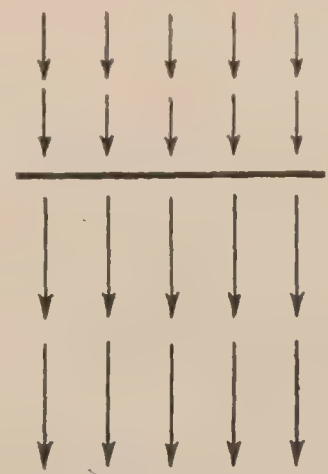

Fig. 4

CONCLUSIONS.

Consider two point sources with the intensities J_1 and J_2 at the distance R apart. The velocity in the neighborhood of the first source due to the second is

$$\frac{J_2\rho}{4\pi R^2}.$$

Hence, according to the theorem:

Two sources of the same sign produce attractive forces between them which are proportional to the product of their intensities, and inversely to the square of their distance apart. The magnitude of the force is

$$\frac{J_1 \cdot J_2 \cdot \rho}{4\pi R^2}.$$

Two sources of opposite signs produce repulsive forces according to the same law.

The consideration of the velocity in the neighborhood of a source due to an element of a vortex or in the neighborhood of this element due to a source show:

A source and an element of a vortex produce a force on each other proportional to the product of their intensities and inversely to the square of their distance. The force is perpendicular to the vortex and to the line of connection, and has the magnitude

$$\frac{J_1 \cdot J_2}{4\pi R^2} \cdot \sin\varphi$$

where φ is the angle between the direction of the vortex and the line joining it to the source.

These two facts are analogous to the theorem known before, which is derived from the relation between the force on a vortex, its velocity, and its intensity:

Two elements of vortices having the same direction produce repulsive forces on each other, which are proportional to the product of their intensities and inversely to the square of their distance apart. The magnitude of the force is

$$\frac{J_1 \cdot J_2}{4\pi R^2} \sin\varphi$$

where φ is the angle between the direction of the vortices and the line joining the elements. Elements having opposite directions produce forces of attraction according to the same law. [N. B.—These forces produced by or on vortices refer to elements of unit length.]

(For two dimensional problems the denominator would always be $2\pi R$.)

These statements are valid for steady motion. If there are no extraneous forces or only such as have a potential, the fluid motion involving vortices can not be steady if the vortex has a finite magnitude. The vortex moves with the fluid, hence it has no velocity relative to the fluid and therefore requires no extraneous force.[1] For sources a similar theorem is valid, for the same reason. It is difficult, however, to connect a physical meaning with it. The motion of the positive source would not even be stable.

A pair consisting of a source and a sink of equal intensities experiences a force proportional to the difference of the velocities at the two points. Passing to the limit, we see that the force on such a doublet is proportional to its intensity and to the rate of change of the velocity in the direction of its axis. These cases are of some practical importance, it being possible to represent moving bodies by such doublets.

If two such doublets are situated one behind the other, the two axes coinciding, the influence of the two nearest sources predominates. Two such doublets repel each other, with a force whose magnitude is

$$\frac{M_1 M_2}{4\pi R^4}$$

where M_1 and M_2 are the strengths of the doublets. The force varies inversely as the fourth power of the distance. It is to be expected, therefore, that it will not be of practical importance.

It may be mentioned that the forces between a single linear vortex and a doublet are inversely proportional to the square of the distance. The force and its direction also depend on the angle between the vortex and the axis of the doublet.

We will omit the consideration of peculiarities of higher order, as we do not think that we shall be forced to apply them.

APPLICATIONS

It has been noted by several writers[2] that the combination of two equal and opposite sources at a finite distance apart will, when placed in a fluid flowing parallel to the line joining the two sources, give rise to such a distribution of lines of flow that the total effect is the same as that produced by a body of the general shape of an airship.

Thus let the intensities of the sources be $+J$ and $-J$; let their distance apart be a; and let the velocity of flow parallel to the line joining the sources be v. Then the area of the greatest cross-section of the "equivalent airship" is, approximately,

$$S=\frac{J}{v+\frac{J^2}{\pi a^2}}$$

and therefore the greatest diameter is

$$d=\sqrt{S\frac{4}{\pi}}.$$

The length of the equivalent airship is approximately

$$l=a+2\sqrt{\frac{J}{4\pi v}}.$$

If $\frac{J}{v}$ is small compared with a^2, the airship is an elongated one; and it will be sufficient to assume that $S=\frac{J}{v}$, $l=a$; or, if l and S are given, that $J=vS$; $a=l$.

We will now apply this general theory to two problems.

1. The calculation of the force acting between two similar airships in motion in a perfect fluid, side by side, at a distance apart of their axes given by b.

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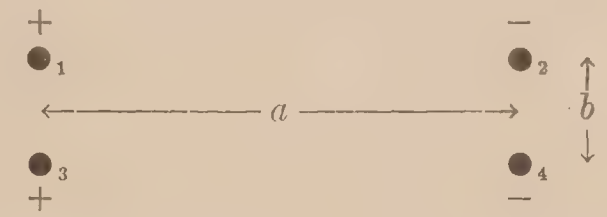
[1] Helmholtz's theorem.

[2] D. W. Taylor: Ship-shaped stream forms. Trans. British Inst. Naval Arch., Vol. 35, p. 385, 1894.

Solid stream forms and the depth of water necessary to avoid abnormal resistance of ship. Ib., Vol. 36, p. 234, 1895.

G. Fuhrmann: Theoretische und experimentelle Untersuchungen an Ballonmodellen. Jahrb. der Motorluftschiff-Studiengesellschaft. 1911–12.

Each airship may be represented by a pair of sources, as just noted.



Let us label them 1, 2, 3, 4; 1 and 3 are positive, 2 and 4 are negative. The attraction between 1 and 3 and between 2 and 4 is

$$f_{13}=f_{24}=\frac{J^2\rho}{4\pi b^2}=\frac{v^2S^2\rho}{4\pi b^2}.$$

The forces between 1 and 4 and between 2 and 3 are repulsions and their components perpendicular to the axes of the airships are

$$f_{14}=f_{23}=-\frac{v^2S^2\rho}{4\pi}\frac{b}{(b^2+l^2)^{3/2}}.$$

So the total attractive force on each airship is

$$F=2\frac{v^2S^2\rho}{4\pi}\left(\frac{1}{b^2}-\frac{b}{(b^2+l^2)^{3/2}}\right).$$

In a practical case let $l=600$ feet, $S=2{,}500$ square feet, $v=80$ mi/hr, $b=200$ feet. The calculated force comes out to be 800 pounds. Not only is this force very small, but in practice airships do not come so close together. So the result is to prove that the aerodynamical forces due to displacement are small.

2. The calculation of the longitudinal buoyancy of an airship model in a wind tunnel, due to the pressure gradient in the air stream. The force on the airship described as in the previous problem would be

$$F=vS\ (v_2-v_1)\ \rho=vS\rho\frac{dv}{dx}\cdot a=\frac{dp}{dx}\cdot\ \text{(volume)}.$$

This formula is the one used in practice and is sufficiently accurate for an elongated model. It can not, however, be applied to a short one. Consider, for instance, a sphere of radius r. It is "equivalent" to a doublet of the intensity $2\pi r^3\cdot v=\frac{3}{2}\,v$ (volume). The force acting on it $\frac{3}{2}\,\rho\,v\,\frac{dv}{dx}\cdot$ (volume) $=1.5\,\frac{dp}{dx}\cdot$ (volume). Hence the proper coefficient for a short airship lies between 1 and $1\frac{1}{2}$.

More complicated distributions of sources which are equivalent to different forms of airships may be found in the paper by Fuhrmann. This should be used in the calculation of the exact error, applying the theorem deduced in this paper.

REPORT No. 115

BENDING MOMENTS, ENVELOPE, AND CABLE STRESSES IN NON-RIGID AIRSHIPS

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Navy Department

REPORT No. 115.

BENDING MOMENTS, ENVELOPE, AND CABLE STRESSES IN NON-RIGID AIRSHIPS.

By C. P. Burgess.

SUMMARY.

No simple but comprehensive method of calculating the principal stresses in the envelope of a non-rigid airship has hitherto been described and published in the English language. The present report describes the theory of the calculations and the methods which are in use in the Bureau of Aeronautics, United States Navy. The principal stresses are due to the gas pressure and the unequal distribution of weight and buoyancy, and the concentrated loads from the car suspension cables.

The second part of the report deals with the variations of tensions in the car suspension cables of any type of airship, with special reference to the rigid type, due to the propeller thrust or the inclination of the airship longitudinally.

INTRODUCTION.

The Third Annual Report (1917) of the National Advisory Committee for Aeronautics contained a translation by Prof. Karl K. Darrow of the report of the investigations by two German engineers, Dr. Rudolf Haas and Alexander Dietzius, into "The Stretching of the Fabric and the Deformation of the Envelope in Non-Rigid Balloons." That report, prepared with characteristic German care and exhaustiveness, included a description of the theory and procedure for finding the stresses in the envelope when the bending moment is known, but failed to describe how the stresses in the car-suspension cables and hence the bending moment due to the weight of the car and its load may be determined when the system of suspension cables is at all complex. The writer was therefore asked by the National Advisory Committee for Aeronautics to prepare a more comprehensive report on the subject.

THEORY AND METHOD OF PROCEDURE.

The airship designer has an infinite choice of possible arrangements of the car-suspension system, and even when the layout of the cables is determined there is an infinite number of ways in which the load may be arbitrarily distributed among them, each distribution producing its own peculiar variation of the bending moment along the envelope, so that without a theory of the methods to be followed in the design of the suspension system but little can be done in the calculation of the stresses in the envelope. The present report is intended to furnish an account of a theory and method of procedure applicable to modern airships in which it is customary to suspend a short car close up to the envelope. When the bending moment is determined in accordance with this procedure, the stresses in the fabric of the envelope due to the bending may be determined to a good order of accuracy by application of the ordinary formula for the fiber stresses in a loaded beam, without recourse to the elaborate refinements and corrections described by Haas and Dietzius.

The hull or envelope of an airship may be regarded as a beam loaded by the forces of buoyancy acting upward or positively and the weights acting downward or negatively. The algebraic sums of these forces and of their moments about any point are zero when the airship

is in equilibrium. The buoyancy forces may be determined by the well-known methods of naval architecture from a curve of gross buoyancy. The theory and method of drawing such a curve are given in detail in the standard textbooks of naval architecture (e. g., Atwood's *Theoretical Naval Architecture*), but briefly it may be described as a curve drawn upon a base line representing the length of the ship and having ordinates which represent to a suitable scale the gross buoyancy per unit of length throughout the length of the ship. The total area inclosed by the curve gives to scale the gross buoyancy of the whole airship.

The determination of the downward acting forces is a much more complex matter than of the upward ones, because in addition to the weights on the envelope there are the vertical components of the forces in the car-suspension cables which, as already stated, may be distributed in an infinite number of ways.

The total bending moment in the envelope is not alone due to the vertical forces, and it will be shown that account must be taken of the bending moments resulting from the horizontal components of the suspension forces and also the so-called *gas pressure bending moment* which is due to the increase of gas pressure upward in the envelope, producing longitudinal forces which are greater above than below the neutral axis.

The following methods of assigning the tensions in the suspension cables and of computing the bending moments in the envelope have been devised in the Bureau of Construction and Repair. The reader will probably find that the various steps may be most readily understood by comparing with the text each successive curve or diagram shown in the accompanying figure of the load and bending moment diagram of a non-rigid airship.

The procedure is as follows:

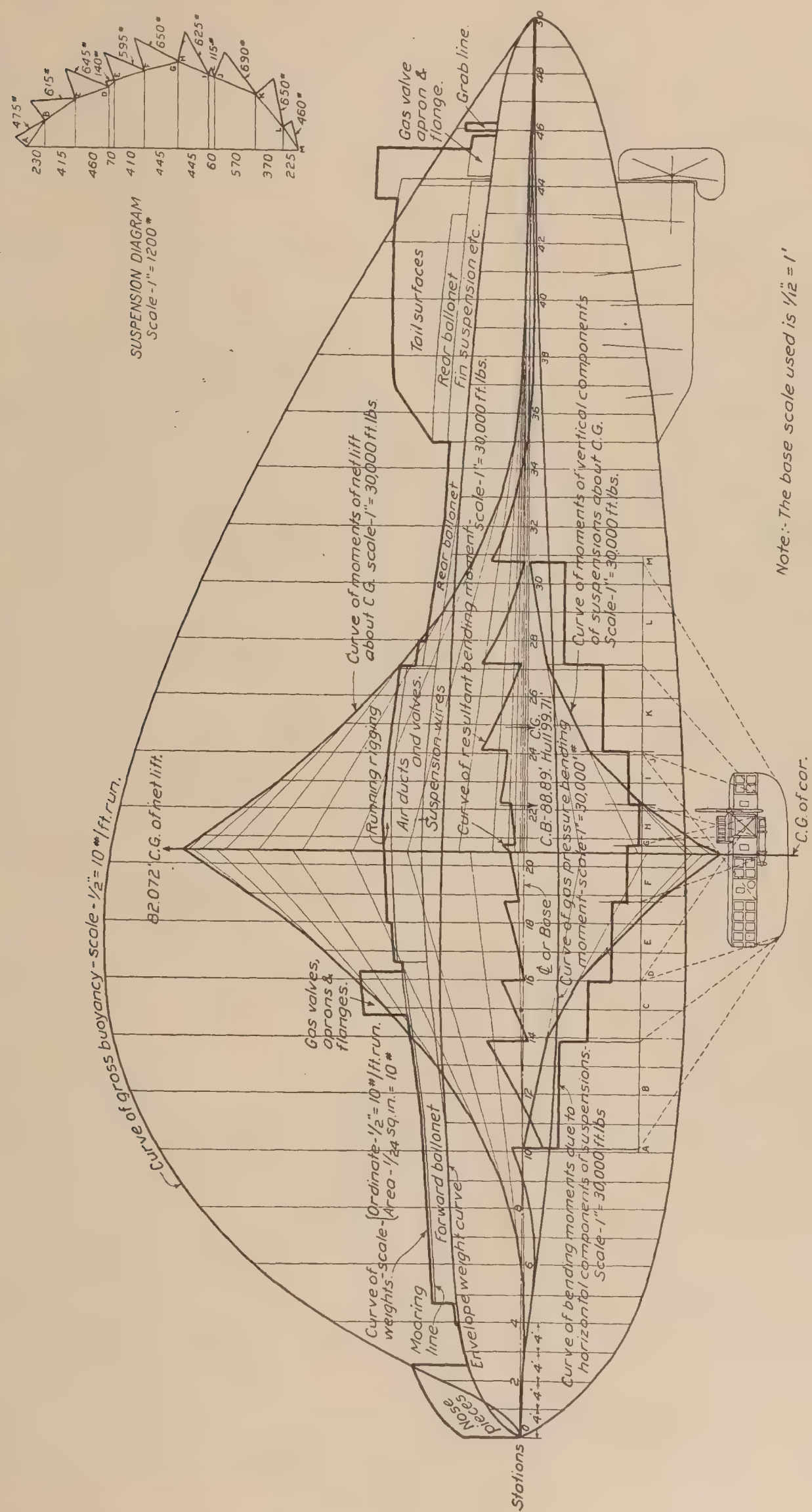
(*a*) A curve of gross buoyancy is drawn. For an envelope having circular cross sections the ordinate at any station is proportional to $k\pi R^2$, where R is the radius of the cross section and k is the lift of the gas per unit volume.

(*b*) A curve of envelope weights is laid off on the same base line and to the same scale as the curve of envelope weights. The area between the curves of gross buoyancy and envelope weights represents the net lift, i. e., the weight of the car and its load.

(*c*) The areas of the net lift curve between successive stations are found by a planimeter, and the longitudinal center of gravity of each of these areas estimated by eye; in most cases the mid-point of the station interval is assumed to be the center of gravity of the area. Moments of these areas are taken about some convenient station in order to calculate the center of gravity of the net lift. If the length of the envelope is divided up into 20 or more stations, this method of obtaining the moments and center of gravity of the area of the net lift curve is sufficiently accurate. The car should be placed so that its center of gravity is vertically under the center of gravity of net lift.

(*d*) The moments of each interval of the net lift curve are next taken about the center of gravity of net lift (referred to hereafter as the C. G.), and a moment curve of net lift drawn as if the envelope were a cantilever held by a single downward force at the C. G. This curve is plotted by laying off successively upon a vertical line through the C. G. the moments about that line due to each interval of the net lift curve, working inward from the bow and stern, and drawing straight lines successively from the points laid off to the previous line at a point vertically above the C. G. of the area for which the line being drawn represents the moment. The construction of this curve may be readily understood by inspection of the curve of moments of net lift about the C. G. in the accompanying load and bending moment diagram for a nonrigid airship.

(*e*) A diagram of the tensions in the car-suspension cables is constructed to fulfill the conditions that the sum of the horizontal components of the tensions is zero, and the sum of the vertical components equals the net lift, and the sum of the moments about the C. G. is zero. These conditions are obviously necessary if the airship is to be at rest in horizontal trim and static equilibrium. To simplify the problem of moments, the points of attachment of the suspensions to the envelope are all assumed to lie on one horizontal line. The necessary condition with regard to moments then becomes that the sum of the moments of the vertical com-



LOAD AND BENDING MOMENT DIAGRAM FOR A NON-RIGID AIRSHIP

FIG. 1.

ponents of the suspension about the C. G. equal zero. It can be seen at once from the suspension diagram whether or not the conditions in regard to the magnitude of the vertical and horizontal components of the suspension forces are fulfilled. The moments may also be obtained graphically, but it is more expeditious to use a combination of analytical and graphical methods, using the suspension force diagram to obtain the vertical component of the tension, and multiplying this component by the horizontal distance of the point of attachment of the suspension cable from the C. G. to obtain the moment. The leads of the suspension cables and the adjustment of the tensions among them are juggled by trial and error until the necessary conditions are fulfilled. The suspension diagram used in these calculations shows only the tensions in the vertical longitudinal plane. To find the actual tensions in the cables it is necessary to make a correction for the inclination of the cables to that plane.

Where there is a multiplicity of suspension cables the number of possible solutions is infinite. Besides the foregoing conditions which must be fulfilled there are others at which the designer should aim. The tensions in the car suspension should be chosen so as to produce as small a bending moment in the envelope as is practicable and consistent with other requirements, and although there is no direct means of knowing precisely what bending moments will result from any given design of suspension until the bending moment diagram is finished, a designer with a little experience in working out diagrams in this manner should have no difficulty in hitting upon a fairly good solution at the first attempt in any given case, and the second attempt should give entirely satisfactory results. Another condition of almost equal importance is that the weight of the car should be taken by wires as nearly vertical as possible, because inclined wires receive heavy loads when the airship inclines up or down to extreme angles, even if their tensions are quite small when the airship is in horizontal trim. These last two conditions may be found to be mutually inconsistent in some cases so that the designer is compelled to compromise. Another desirable condition, but usually of minor importance, is that the bending moments in the car should be small. With the short cars of modern airships this condition is easily realized.

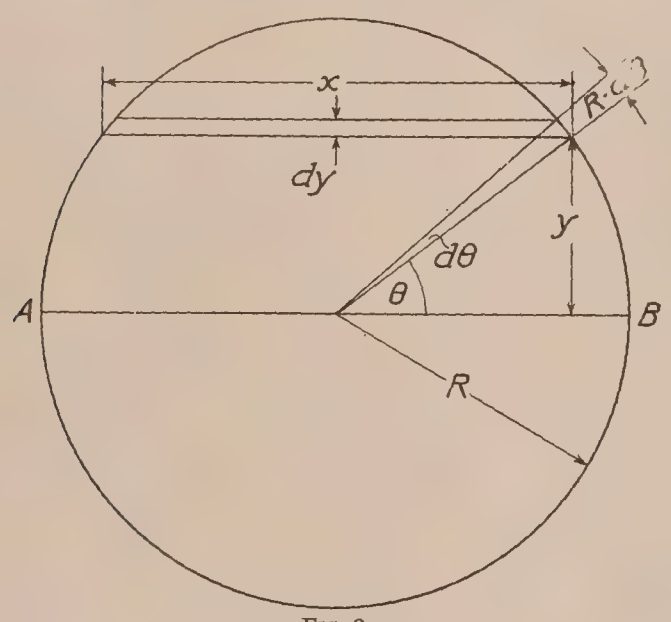


FIG. 2.

(*f*) A curve of the moments of the vertical components of the car suspension at the points of attachment to the envelope is constructed in a similar manner to the curve of moments of net lift. The line through the attachments of the suspension cables to the envelope is considered to be a cantilever supported by a single upward force at the C. G., and loaded with the vertical components of the suspensions.

(*g*) A curve is drawn of moments due to the horizontal components of the tensions in the suspension cables multiplied by the distance of the points of attachment below the longitudinal axis of the envelope. The ordinates of this curve are constant between the points of attachment of the suspension cables to the envelope, instead of continuously increasing toward the C. G. as in the curves previously drawn. This bending moment due to the horizontal components of the suspension forces has sometimes been neglected, and the tensions in the cables assigned with regard only to the vertical forces. In such cases the airship is subject to severe negative bending moment, causing the bow and stern to droop. The fact that the horizontal components of the suspension forces are the cause of an important bending moment makes it possible to support a short car close up to the envelope without requiring an excessive tension in the end suspension cables to keep the bow and stern of the airship from curving upward.

(*h*) A curve of gas pressure bending moments is constructed. This is a negative moment resulting from the increase of gas pressure upward. If, as shown in figure 2, an envelope having circular cross sections of radius R, x = width across the section at a distance y from the hori-

zontal diameter of the section, and k is the lift per unit volume of the gas, the increase in pressure upward produces differences of longitudinal forces on the section causing a moment about the horizontal diameter given by:

$$-M=\int_{-R}^{+R} Kxy^2dy$$

But

$$x=2\sqrt{R^2-y^2}$$

Therefore:

$$-M=\int_{-R}^{+R} 2ky^2\sqrt{R^2-y^2}dy$$

$$=\frac{\pi}{4}kR^4.$$

The curve of gas pressure bending moments is plotted in accordance with this formula.

(*i*) There are now four bending moment curves, a positive one due to the moment of the net lift about the C. G. and three negative ones. A final curve is drawn as the resultant of these four and its ordinates give the net bending moment in the envelope.

The accompanying figure shows the bending moments in a nonrigid airship worked out in accordance with this method. In spite of the fact that the car carries all fuel and is short and close to the envelope, the bending moment is nowhere large.

It was shown by Haas and Dietzius that when the bending moment and the moment of inertia of the cross-section of an airship envelop are known, the longitudinal stresses in the fabric due to bending may be computed by the ordinary beam-bending formula, $f=My/I$. The internal gas pressure produces tension in the envelope, and since fabric is incapable of sustaining compression it is essential that the compression due to bending shall not exceed the tension due to gas pressure.

In dealing with the stress and moment of inertia of an airship envelope it is customary to consider the cross section of the envelope as an infinitely thin ring, so that the stress is expressed as pounds/inch instead of as pounds/inch^2, and the dimensions of the moment of inertia are L^3 instead of L^4. In figure 2 the circle represents the cross section of the envelope of an airship, of radius R, and the moment of inertia about any diameter such as AB is given by:

$$I=4\int_0^{\frac{\pi}{2}} y^2Rd\theta$$

But

$$y=R\sin\theta$$

Therefore

$$I=4R^3\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta$$

$$=\pi R^3.$$

When the bending moment, M, at a circular-cross section is determined, the maximum unit stress in the fabric at that cross section due to the bending alone is when $y=R$, so that the maximum stress is given by:

$$f=\frac{My}{I}=\frac{M}{\pi R^2}.$$

The work of Haas and Dietzius is largely devoted to the methods of ascertaining the degree to which the cross section of the envelope departs from the designed circle. Owing to the stretch of the fabric the circumference increases about 3 per cent with modern three-ply fabrics, and the car suspension causes a distortion of the cross section increasing the height a further $2\frac{1}{2}$ per cent approximately.

The fabric is also reinforced by the overlap at the seams, amounting to about 5 per cent additional strength. The total effect is to make the moment of inertia of the actual cross section about 19 per cent greater than in the designed circular cross section. The increase in the

moment of inertia in any particular case may be calculated from the characteristics of the fabric and the stresses due to the gas pressure and the car suspension in accordance with the methods described in detail by Haas and Dietzius, but in ordinary practice the arbitrary correction of 19 per cent is sufficiently accurate.

The longitudinal stresses due to the internal gas pressure and the longitudinal components of the tensions in the car-suspension cables are superimposed on the stresses due to the bending moment.

The total gas pressure force upon any cross section is equal to the area of the section multiplied by the mean gas pressure, i. e., by the pressure at the mid-height of the envelope. The compressive force due to the suspension cables may be found from the suspension diagram. The resultant of these longitudinal forces does not lie along the neutral axis of the envelope, but the effect of this offset position has been taken care of in the methods already described for calculation of the bending moment.

The stress per unit width of fabric due to the longitudinal forces exclusive of bending is obviously equal to the forces divided by the circumference of the section with corrections made for the stretch and the effect of the seams. This stress is superimposed upon that due to the total bending moment.

In tapered portions of the envelope a correction must be applied to the calculation of the longitudinal stress. Let the longitudinal tangent to the envelope be inclined at an angle α to the longitudinal axis. Then the true longitudinal stress is obtained by dividing the apparent longitudinal stress by cos α. The apparent longitudinal stress is calculated by assuming the envelope to be cylindrical at the cross section considered.

Example: Find the total longitudinal stress per unit width of fabric at the top and bottom of an airship envelope at a cross section where the designed radius is 21 feet, the mean gas pressure is 2 inches of water, the total bending moment is $-50{,}000$ foot-pounds, the longitudinal component of the tensions in the suspension cables is 1,600 pounds, and $\alpha = 5°$.

The designed area of the cross section is $21^2 \times \pi = 1{,}390$ feet,^{2} and adding 9 per cent for the stretch of the fabric the actual area is 1,515 feet.^{2} The mean gas pressure is 2 inches of water $= 10.4$ pounds/ft.^{2} The total longitudinal force is $(1{,}515 \times 10.4) - 1{,}600 = 14{,}180$ pounds.

The circumference of the cross section is $2\,\pi \times 21 = 132$ feet, or 136 feet, allowing 3 per cent for the stretch. Neglecting the effect of α for the meantime, but allowing 5 per cent additional strength due to the seams, the unit stress in the fabric due to the total longitudinal force upon the cross section is

$$\frac{14{,}180}{1.05 \times 136} = 99.3 \text{ lbs./ft.}$$

The moment of inertia of the designed section is $\pi \times 21^3 = 29{,}200$ ft.,^{3} or $1.19 \times 29{,}200 = 34{,}800$ ft.,^{3} including the allowances for stretch, deformation, and seams. The distance of the top of the envelope from the neutral axis is $21 \times 1.03 \times 1.025 = 22.2$ ft., including the corrections for stretch and deformation. The unit stress due to the bending is given by:

$$f = \frac{My}{I} = \frac{50{,}000 \times 22.2}{34{,}800} = 31.9 \ \frac{\text{lbs.}}{\text{ft.}}$$

The bending moment is negative, producing tension in the top and compression in the bottom of the envelope. Adding together the stresses due to longitudinal forces and bending, and dividing by cos α in order to make the correction for the longitudinal taper, it is found that:

At the top of the cross section, the longitudinal tension is

$$\frac{99.3 + 31.9}{.9962} = 131.9 \ \frac{\text{lbs.}}{\text{ft.}} = 10.99 \text{ lbs./in.}$$

At the bottom of the cross section the longitudinal tension is

$$\frac{99.3 - 31.9}{.9962} = 67.6 \text{ lbs./ft.} = 5.63 \text{ lbs./in.}$$

TENSIONS IN CAR-SUSPENSION CABLES WHEN INCLINED LONGITUDINALLY.

It was shown in the preceding section that the airship designer is free within wide limits to assign arbitrarily the tensions in the car-suspension cables. Having chosen the tensions for any given condition, e. g., the airship at rest in horizontal trim with full load, the designer is not free to assign the change of tensions resulting from any change of load or trim or due to the application of another force to the car, such as propeller thrust.

The change in the cable tensions caused by any system of forces applied to the car may be calculated from the elastic changes in the lengths of the cable resulting from relative motion between the car and the hull, provided it is assumed that the points of attachment of the cables to the hull do not move relatively to each other, or move in a known manner. In rigid airships this assumption is justifiable. In non-rigids it is only approximately true owing to the flexibility of the envelope, and as the calculations are very laborious with the large number of suspension cables in such airships, it is generally better in practice to design the car suspension with an ample factor of safety in the normal condition, and with a good arrangement of martingales in accordance with approved practice. The martingales are suspension wires set at as flat an angle as practicable in order to take the longitudinal component of gravity when the airship is inclined longitudinally; and to be most effective there should be a pair leading forward from near the stern of the car, and another pair leading aft from near the bow.

The following method is suitable for an investigation into the changes in the tensions of the car-suspension cables of rigid airships, and may, if desired, be used as an approximation in non-rigid airships. Symmetry of the suspension in the horizontal direction usually makes it unnecessary to consider translation or rotation of the car with respect to the hull outside of the vertical plane, and in the following discussion such motions will not be considered, although the method may be extended to include them if required.

A small longitudinal movement of the car with respect to the hull produces in general a change not only in the longitudinal components of the suspension, but also in the perpendicular components and in the moments, so that the longitudinal motion will be accompanied by perpendicular motion and rotation. (Note: "Perpendicular" is used in this section instead of "vertical" to denote the direction in the vertical plane perpendicular to the longitudinal axis of the hull, in order to include cases when the airship is inclined longitudinally.) Similarly, rotation or perpendicular motion will in general be accompanied by the other two kinds of motion in the vertical plane. The problem is therefore to deduce three simultaneous equations which will give the net effect due to any change in the forces applied to the car.

Any change in the forces applied in the vertical plane may be resolved into three parts as follows:

Let A = Change in longitudinal force.
Let B = Change in perpendicular force.
Let M = Change in moment of the forces about a fixed point in the car.

The motion of the car with respect to the hull may be similarly resolved into three parts as follows:

Let x = Longitudinal motion.
Let y = Perpendicular motion.
Let ψ = Rotational motion.

The points of attachment of the suspension cables to the hull of the airship are assumed to be fixed relatively to each other.

The total change in the longitudinal components of the cable tensions due to a longitudinal movement of the car equal to some small unit of length (say 1 inch) must be calculated by adding together, with due regard to signs, the change in the longitudinal component of the tension in each cable.

From geometry and the laws of elasticity the change in the tension of a cable due to a longitudinal movement of 1 inch is given by:

$$T = \frac{Ea \cos\theta \cos\phi}{l}$$

Where T = Change of tension in pounds.
E = Modulus of elasticity of the cable in lbs./in.^{2}
a = Cross-sectional area of the cable in in.^{2}
l = Length of the cable in inches.
θ = Inclination of the cable to a plane through the longitudinal and transverse axes of the airship hull.
ϕ = Inclination of the cable to the verical plane.

By geometry the longitudinal component of T is $T \cos \theta \cos \phi$. The changes in the cable tensions and in the longitudinal and perpendicular components due to longitudinal and perpendicular movements of the car relatively to the hull are summarized in the following table:

Longitudinal component = Tension $\cos \theta \cos \phi$
Perpendicular component = Tension $\sin \theta \cos \phi$

For longitudinal movement x, tension $= \frac{Eax}{l} \cos \theta \cos \phi$

longitudinal component $= \frac{Eax}{l} \cos^2\theta \cos^2\phi$

perpendicular component $= \frac{Eax}{l} \sin \theta \cos \theta \cos^2\phi$

For perpendicular movement y, tension $= \frac{Eay}{l} \sin \theta \cos \phi$

longitudinal component $= \frac{Eay}{l} \sin \theta \cos \theta \cos^2\phi$

perpendicular component $= \frac{Eay}{l} \sin^2\theta \cos^2\phi$

The change of tension in a suspension cable due to a small rotation in the vertical plane about some fixed point in the car, such as the C. G., is given by:

$$T = \frac{Eap}{l} \sin \psi \cos \phi$$

Where p is the distance from the C. G. to the line of action of the cable, measured in the vertical plane. The longitudinal and perpendicular components of the tension due to rotation are the same as in the case of tension produced by longitudinal or perpendicular motion.

The change in moment due to a change in the cable tensions caused by any of the three types of motion is equal to $Tp \cos \phi$.

Let a_1 = Sum of changes in longitudinal components of cable tensions due to a longitudinal movement of 1 inch.
Let b_1 = Sum of changes in perpendicular components of cable tensions due to a longitudinal movement of 1 inch.
Let m_1 = Sum of changes in moments of cable tensions due to a longitudinal movement of 1 inch.
Let a_2 = Sum of changes in longitudinal components of cable tensions due to a perpendicular movement of 1 inch.
Let b_2 = Sum of changes in perpendicular components of cable tensions due to a perpendicular movement of 1 inch.
Let m_2 = Sum of changes in moments of cable tensions due to perpendicular movement of 1 inch.
Let a_3 = Sum of changes in longitudinal components of cable tensions due to a rotation of 1° about the C. G.
Let b_3 = Sum of changes in perpendicular components of cable tensions due to a rotation of 1° about the C. G.
Let m_3 = Sum of changes in moments of cable tensions due to a rotation of 1° about the C. G.

All of the above quantities must be calculated, and the longitudinal, vertical, and rotational movements of the car with respect to the hull may then be obtained by solution of the following

simultaneous equations of the first order which are derived directly from the definitions given above for the various forces, moments and motions:

$$\begin{aligned} a_1x + a_2y + a_3\psi &= -A \\ b_1x + b_2y + b_3\psi &= -B \\ m_1x + m_2y + m_3\psi &= -M \end{aligned}$$

After solving these equations for x, y and ψ, the total change of tension in each cable may be found by substituting the values of these variables in the preceding formulæ for cable tensions in terms of the variables and the known characteristics of each cable. To find the total tension in a cable, the change of tension must be added algebraically to the tension in the normal condition of the airship.

If the motion of the car relatively to the hull is sufficient to cause a cable previously taut to become slack, the effect of further motion must be calculated as if that cable were not present; i. e., the values of a_1, a_2, a_3, b_1, b_2, b_3, m_1, m_2, and m_3 must be modified by leaving out the cable which has become slack.

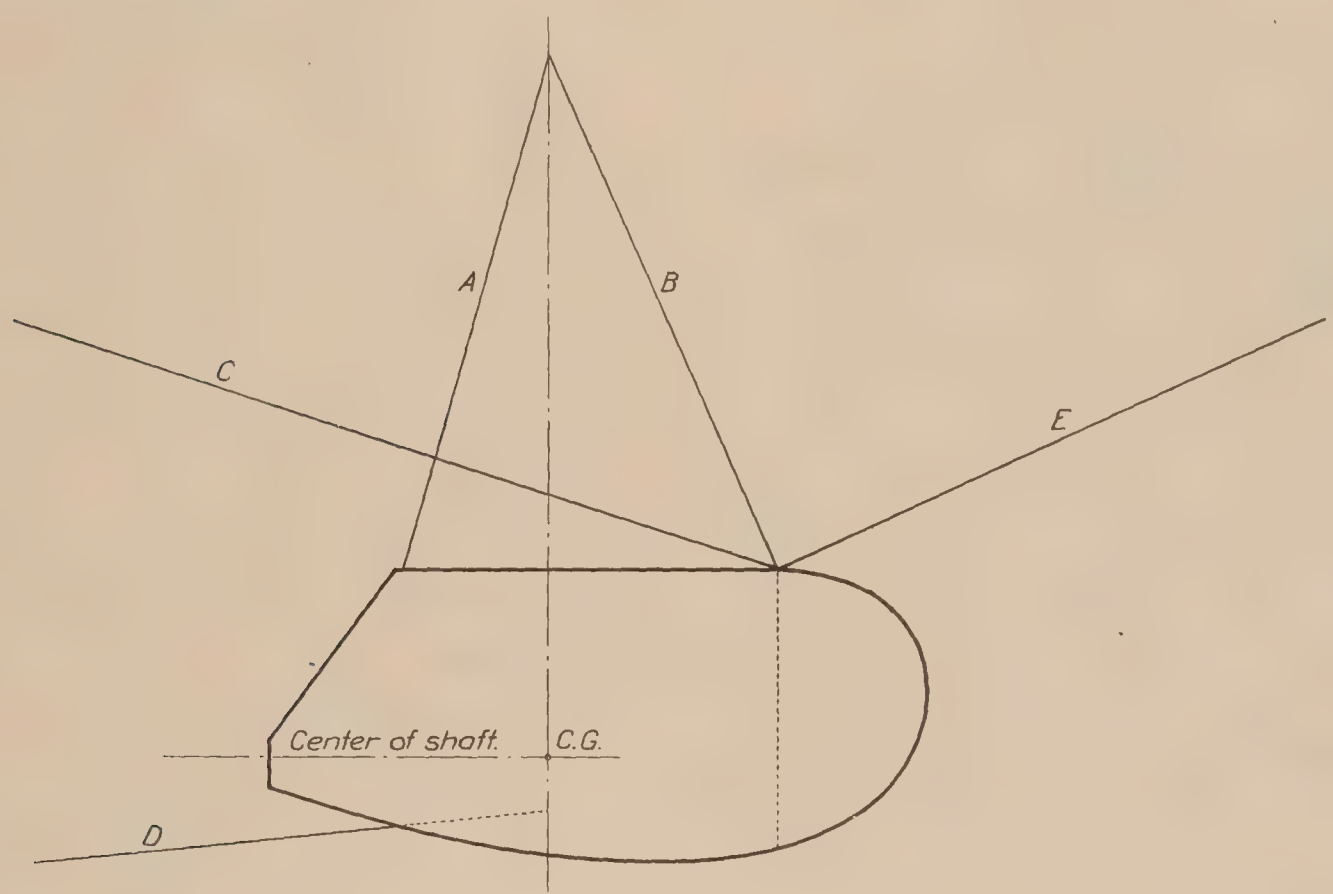


FIG. 3.

As a final check upon the accuracy of the work, force and moment diagrams should be constructed using the tensions found in the cables, and if the diagrams close it may be assumed that the work is correct.

Example: A side engine car of a rigid airship shown in the accompanying sketch (fig. 3) is suspended by five cables, designated A, B, C, D, and E. In the static condition with full load, the entire weight of the car, 2,800 pounds, is taken by the cables A and B, with the other three just barely taut. This arbitrary arrangement of the tensions in the cables is possible because A and B are secured to the hull of the airship at a point vertically over the C. G. of the car; but it should be noted that the division of the load between the cables is not determinate, and the designer is free to change the distribution of the load by assigning a tension to cables C and E, relieving B correspondingly.

Cables A, B, C, and E are all in the vertical plane. Cable D is inclined at an angle of 47° 45′ to that plane. In addition to the five cables there are three struts from the car meeting at a point on the main transverse frame of the airship abreast the C. G. of the car. These struts receive compression from the transverse component of any tension put upon the cable D, and

they preserve the alignment of the car in the vertical plane, but they do not hinder small movements in that plane. The product Ea is assumed to be constant for all cables.

If a forward thrust is applied to the car, cables C and D are put in tension, and the tensions in cables A and B are modified. To calculate the effects of such a thrust a table is prepared of the known quantities in the three simultaneous equations required.

The convention with regard to signs is that forward and upward movements and anticlockwise rotation are counted positive, and the converse negative; and changes in components of cable tensions tending to produce positive motions are also counted positive, and conversely.

Characteristics of suspension cables of side power car of a rigid airship.

	Cable.				Sum.
	A	B	C	D	
ϕ	0	0	0	47° 45′	
$\cos\phi$	1.0	1.0	1.0	.672	
θ	74° 15′	65° 30′	18° 15′	−5° 45′	
$\sin\theta$	.9625	.9100	.3132	− .1002	
$\cos\theta$	.2715	−.4147	−.9497	− .9950	
1, 100 x in	15.6	16.4	48.4	33.8	
p, 100 x in	− .550	.835	.720	− .158	
T due to forward motion 1″ $\frac{Ea}{100,000}$ lbs	− 17.4	25.3	19.6	19.8	
Longitudinal components $\frac{Ea}{100,000}$ lbs	− 4.72	−10.49	−18.60	− 13.22	− 47.03= a_1
Perpendicular components $\frac{Ea}{100,000}$ lbs	−16.74	23.00	6.14	− 1.33	11.07= b_1
Moment $\frac{Ea}{100,000}$ in. lbs	957	2110	1410	− 210	4267=m_1
T due to perpendicular rise of 1″ $\frac{Ea}{100,000}$ lbs	− 61.7	− 55.5	− 6.47	1.99	
Longitudinal components $\frac{Ea}{100,000}$ lbs	−16.74	23.00	6.14	− 1.33	11.07= a_2
Perpendicular components $\frac{Ea}{100,000}$ lbs	− 59.4	− 50.5	− 2.02	− 0.13	−112.05=b_2
Moment $\frac{Ea}{100,000}$ in. lbs	3390	− 4635	− 466	− 21	− 1732=m_3
T due to rotation of 1° $\frac{Ea}{100,000}$ lbs	61.7	− 89.1	− 26.0	5.5	
Longitudinal components $\frac{Ea}{100,000}$ lbs	16.72	36.95	24.70	− 3.68	74.69= a_3
Perpendicular components $\frac{Ea}{100,000}$ lbs	59.4	− 81.0	− 8.15	− 0.37	− 30.12= b_3
Moment $\frac{Ea}{100,000}$ in. lbs	− 3390	− 7445	− 1870	− 58	− 12763=m_3

Let it be assumed that the propeller exerts a forward thrust of 1,500 pounds, and since the line of action of this force passes horizontally through the C. G. of the car there is no change in the total moment about the C. G. or in the total perpendicular force. From the table, above, the coefficients of the variables are obtained, and the three simultaneous equations are written as follows:

$$-47.03x+11.07y+74.69\psi=-1,500\,\frac{100,000}{Ea}$$
$$11.07x-112.05y-30.12\psi=0$$
$$4,267x-1,732y-12,763\psi=0$$

Whence:

$$x=69\,\frac{100,000}{Ea}\text{ inches}$$

$$y=.63\,\frac{100,000}{Ea}\text{ inches}$$

$$\psi=23\,\frac{100,000}{Ea}\text{ degrees}$$

From the formulæ for the changes of tensions in the cables due to the three types of motions, the total change in the tension of each cable is given by:

$$T=Ea\,\frac{x}{l}\cos\theta\cos\phi+Ea\,\frac{y}{l}\sin\theta\cos\phi+Ea\,\frac{p}{l}\sin\psi\cos\phi=\frac{Ea}{l}\cos\phi\,(x\cos\theta+y\sin\theta+\psi p\sin 1°)$$

By substituting the values of the quantities in this equation for each cable, the changes of tension are found to be as follows:

In cable A, $T=180$ pounds.
In cable B, $T=-342$ pounds.
In cable C, $T=750$ pounds.
In cable D, $T=1{,}470$ pounds.

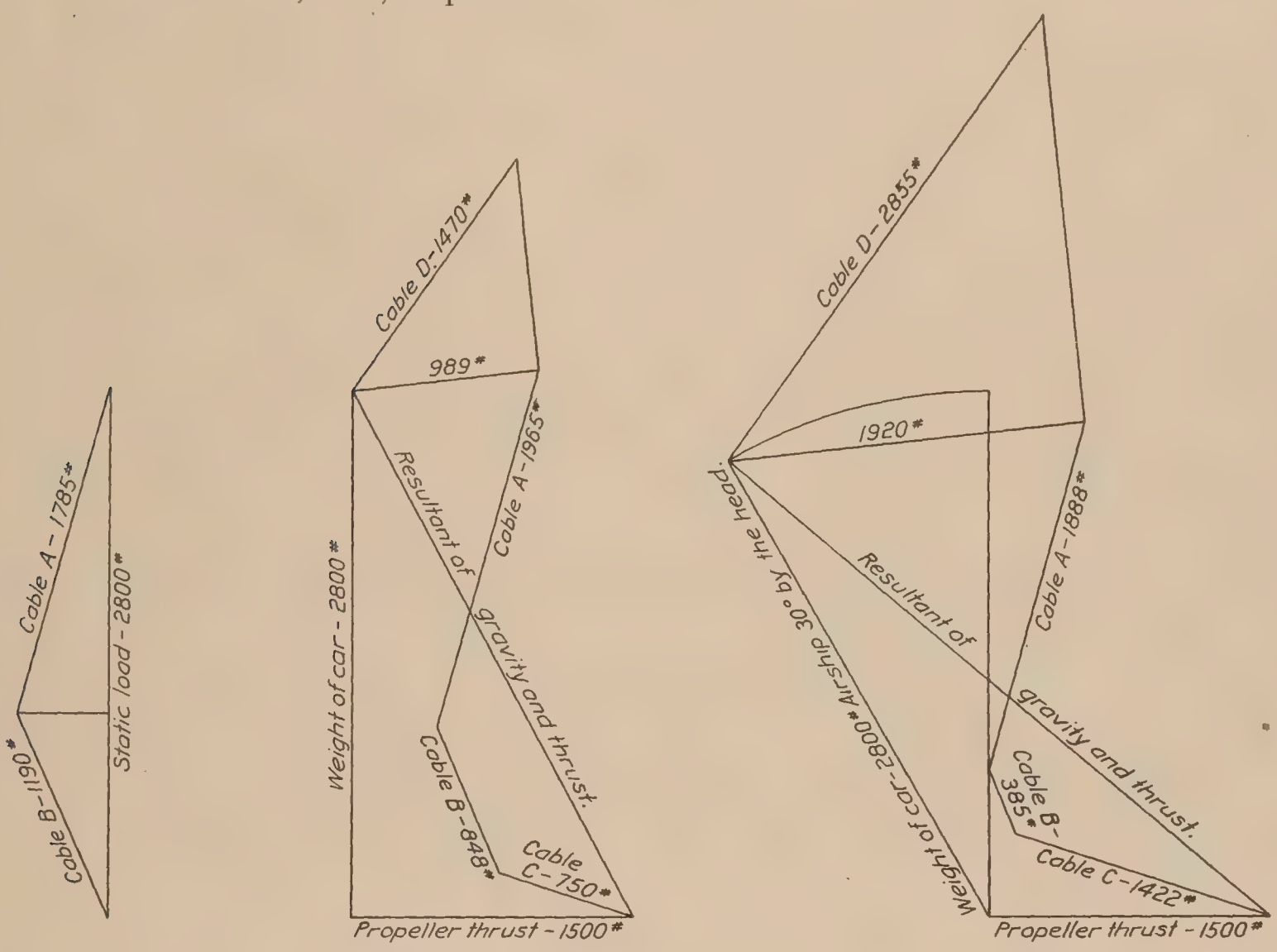


FIG. 4.

The tensions in cables A and B in the static condition are found from the force diagram in the accompanying figure to be 1,785 pounds and 1,190 pounds, respectively. The final tensions in the cables are:

In cable A, tension $=1{,}965$ pounds.
In cable B, tension $=848$ pounds.
In cable C, tension $=750$ pounds.
In cable D, tension $=1{,}470$ pounds.
In cable E, tension $=0$ pounds.

If the airship noses down to an angle of 30° by the head, gravity exerts a longitudina force of $2{,}800\times\sin 30°=1{,}400$ pounds, and the perpendicular force of gravity is reduced by $2{,}800\,(1-\cos 30°)=375$ pounds. If in this condition the propeller is exerting a forward thrust of 1,500 pounds, the three simultaneous equations to determine the movement of the car with respect to the hull are:

$$-47.03x+11.07y+74.69\psi=-2{,}900\ \frac{100{,}000}{Ea}$$

$$11.07x-112.05y-30.12\psi=-375\ \frac{100{,}000}{Ea}$$

$$4{,}267x-1{,}732y-12{,}763\psi=0$$

Whence:

$$x = 131.5 \frac{100,000}{Ea} \text{ inches}$$

$$y = 4.67 \frac{100,000}{Ea} \text{ inches}$$

$$\psi = 43.4 \frac{100,000}{Ea} \text{ degrees}$$

From these values of the variables the tensions in the cables may be calculated in the same manner as before, and the results are:

In cable A, tension = 1,888 pounds.
In cable B, tension = 385 pounds.
In cable C, tension = 1,422 pounds.
In cable D, tension = 2,855 pounds.
In cable E, tension = 0

Longitudinal forces to the rear due to reversing the propeller or inclination of the airship downward by the stern puts tension in the cable E, leaving cables C and D slack. The distribution of the load between cables A, B, and E is without redundancy of forces and may therefore be calculated by the conventional analytical or graphical methods for statically determinate problems.

CONCLUSION.

The theoretical calculations of the principal stresses in non-rigid airships due to the static forces now rest upon a basis nearly as well established as the parallel calculations for water-borne vessels. Much work remains to be done to determine the aerodynamic forces due to gusts of wind and the action of the rudders and elevators. Experimental apparatus is required to verify in actual flight the theoretical calculation of the stresses due to static forces, and more especially to determine the stresses due to dynamic forces which are very imperfectly susceptible of theoretical treatment.

Finally, there is need for a comprehensive literature to make a knowledge of the theory and practice of strength calculations for airships as accessible to the student as in the case of naval architecture.

REPORT No. 116

APPLICATIONS OF MODERN HYDRODYNAMICS TO AERONAUTICS

IN TWO PARTS

By L. PRANDTL
Göttingen University

Translated by the Staff of the National Advisory Committee for Aeronautics

REPORT No. 116.

APPLICATIONS OF MODERN HYDRODYNAMICS TO AERONAUTICS

By L. Prandtl.

PREFACE.

I have been requested by the United States National Advisory Committee for Aeronautic to prepare for the reports of the committee a detailed treatise on the present condition of those applications of hydrodynamics which lead to the calculation of the forces acting on airplane wings and airship bodies. I have acceded to the request of the National Advisory Committee all the more willingly because the theories in question have at this time reached a certain conclusion where it is worth while to show in a comprehensive manner the leading ideas and the results of these theories and to indicate what confirmation the theoretical results have received by tests.

The report will give in a rather brief Part I an introduction to hydrodynamics which is designed to give those who have not yet been actively concerned with this science such a grasp of the theoretical underlying principles that they can follow the subsequent developments. In Part II follow then separate discussions of the different questions to be considered, in which the theory of aerofoils claims the greatest portion of the space. The last part is devoted to the application of the aerofoil theory to screw propellers.

At the express wish of the National Advisory Committee for Aeronautics I have used the same symbols in my formulæ as in my papers written in German. These are already for the most part known by readers of the Technische Berichte. A table giving the most important quantities is at the end of the report. A short reference list of the literature on the subject and also a table of contents are added.

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APPLICATIONS OF MODERN HYDRODYNAMICS TO AERONAUTICS.

By L. Prandtl.

PART I.

FUNDAMENTAL CONCEPTS AND THE MOST IMPORTANT THEOREMS.

1. All actual fluids show internal friction (viscosity), yet the forces due to viscosity, with the dimensions and velocities ordinarily occurring in practice, are so very small in comparison with the forces due to inertia, for water as well as for air, that we seem justified, as a first approximation, in entirely neglecting viscosity. Since the consideration of viscosity in the mathematical treatment of the problem introduces difficulties which have so far been overcome only in a few specially simple cases, we are forced to neglect entirely internal friction unless we wish to do without the mathematical treatment.

We must now ask how far this is allowable for actual fluids, and how far not. A closer examination shows us that for the interior of the fluid we can immediately apply our knowledge of the motion of a nonviscous fluid, but that care must be taken in considering the layers of the fluid in the immediate neighborhood of solid bodies. Friction between fluid and solid body never comes into consideration in the fields of application to be treated here, because it is established by reliable experiments that fluids like water and air never slide on the surface of the body; what happens is, the final fluid layer immediately in contact with the body is attached to it (is at rest relative to it), and all the friction of fluids with solid bodies is therefore an internal friction of the fluid. Theory and experiment agree in indicating that the transition from the velocity of the body to that of the stream in such a case takes place in a thin layer of the fluid, which is so much the thinner, the less the viscosity. In this layer, which we call the boundary layer, the forces due to viscosity are of the same order of magnitude as the forces due to inertia, as may be seen without difficulty.[1] It is therefore important to prove that, however small the viscosity is, there are always in a boundary layer on the surface of the body forces due to viscosity (reckoned per unit volume) which are of the same order of magnitude as those due to inertia. Closer investigation concerning this shows that under certain conditions there may occur a reversal of flow in the boundary layer, and as a consequence a stopping of the fluid in the layer which is set in rotation by the viscous forces, so that, further on, the whole flow is changed owing to the formation of vortices. The analysis of the phenomena which lead to the formation of vortices shows that it takes place where the fluid experiences a retardation of flow along the body. The retardation in some cases must reach a certain finite amount so that a reverse flow arises. Such retardation of flow occurs regularly in the rear of blunt bodies; therefore vortices are formed there very soon after the flow begins, and consequently the results which are furnished by the theory of nonviscous flow can not be applied. On the other hand, in the rear of very tapering bodies the retardations are often so small that there is no noticeable formation of vortices. The principal successful results of hydrodynamics apply to this case. Since it is these tapering bodies which offer specially small resistance and which, therefore, have found special consideration in aeronautics under similar applications, the theory can be made useful exactly for those bodies which are of most technical interest.

[1] From this consideration one can calculate the approximate thickness of the boundary layer for each special case.

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For the considerations which follow we obtain from what has gone before the result that in the interior of the fluid its viscosity, if it is small, has no essential influence, but that for layers of the fluid in immediate contact with solid bodies exceptions to the laws of a nonviscous fluid must be allowable. We shall try to formulate these exceptions so as to be, as far as possible, in agreement with the facts of experiment.

2. A further remark must be made concerning the effect of the compressibility of the fluid upon the character of the flow in the case of the motion of solid bodies in the fluid. All actual fluids are compressible. In order to compress a volume of air by 1 per cent, a pressure of about one one-hundredth of an atmosphere is needed. In the case of water, to produce an equal change in volume, a pressure of 200 atmospheres is required; the difference therefore is very great. With water it is nearly always allowable to neglect the changes in volume arising from the pressure differences due to the motions, and therefore to treat it as absolutely incompressible. But also in the case of motions in air we can ignore the compressibility so long as the pressure differences caused by the motion are sufficiently small. Consideration of compressibility in the mathematical treatment of flow phenomena introduces such great difficulties that we will quietly neglect volume changes of several per cent, and in the calculations air will be looked upon as incompressible. A compression of 3 per cent, for instance, occurs in front of a body which is being moved with a velocity of about 80 m./sec. It is seen, then, that it appears allowable to neglect the compressibility in the ordinary applications to technical aeronautics. Only with the blades of the air screw do essentially greater velocities occur, and in this case the influence of the compressibility is to be expected and has already been observed. The motion of a body with great velocity has been investigated up to the present, only along general lines. It appears that if the velocity of motion exceeds that of sound for the fluid, the phenomena are changed entirely, but that up close to this velocity the flow is approximately of the same character as in an incompressible fluid.

3. We shall concern ourselves in what follows only with a nonviscous and incompressible fluid, about which we have learned that it will furnish an approximation sufficient for our applications, with the reservations made. Such a fluid is also called "the ideal fluid."

What are the properties of such an ideal fluid? I do not consider it here my task to develop and to prove all of them, since the theorems of classical hydrodynamics are contained in all textbooks on the subject and may be studied there. I propose to state in what follows, for the benefit of those readers who have not yet studied hydrodynamics, the most important principles and theorems which will be needed for further developments, in such a manner that these developments may be grasped. I ask these readers, therefore, simply to believe the theorems which I shall state until they have the time to study the subject in some textbook on hydrodynamics.

The principal method of description of problems in hydrodynamics consists in expressing in formulas as functions of space and time the velocity of flow, given by its three rectangular components, u, v, w, and in addition the fluid pressure p. The condition of flow is evidently completely known if u, v, w, and p are given as functions of x, y, z, and t, since then u, v, w, and p can be calculated for any arbitrarily selected point and for every instant of time. The direction of flow is defined by the ratios of u, v, and w; the magnitude of the velocity is $\sqrt{u^2+v^2+w^2}$. The "streamlines" will be obtained if lines are drawn which coincide with the direction of flow at all points where they touch, which can be accomplished mathematically by an integration. If the flow described by the formulas is to be that caused by a definite body, then at those points in space, which at any instant form the surface of the body, the components of the fluid velocity normal to this surface must coincide with the corresponding components of the velocity of the body. In this way the condition is expressed that neither does the fluid penetrate into the body nor is there any gap between it and the fluid. If the body is at rest in a stream, the normal components of the velocity at its surface must be zero; that is, the flow must be tangential to the surface, which in this case therefore is formed of stream lines.

4. In a stationary flow—that is, in a flow which does not change with the time, in which then every new fluid particle, when it replaces another particle in front of it, assumes its velocity, both in magnitude and in direction and also the same pressure—there is, for the fluid particles lying on the same stream line, a very remarkable relation between the magnitude of the velocity, designated here by V, and the pressure, the so-called Bernouilli equation—

$$p+\frac{\rho}{2}V^2=\text{const.} \tag{1}$$

(ρ is the density of the fluid, i. e., the mass of a unit volume). This relation is at once applicable to the case of a body moving uniformly and in a straight line in a fluid at rest, for we are always at liberty to use for our discussions any reference system having a uniform motion in a straight line. If we make the velocity of the reference system coincide with that of the body, then the body is at rest with reference to it, and the flow around it is stationary. If now V is the velocity of the body relative to the stationary air, the latter will have in the new reference system the velocity V upon the body (a man on an airplane in flight makes observations in terms of such a reference system, and feels the motion of flight as "wind").

The flow of incident air is divided at a blunt body, as shown in figure 1. At the point A the flow comes completely to rest, and then is again set in motion in opposite directions, tangential to the surface of the body. We learn from equation (1) that at such a point, which we shall call a "rest-point," the pressure must be greater by $\frac{\rho}{2}V^2$ than in the undisturbed fluid. We shall call the magnitude of this pressure, of which we shall make frequent use, the "dynamical pressure," and shall designate it by q. An open end of a tube facing the stream produces a rest point of a similar kind, and there arises in the interior of the tube, as very careful experiments have shown, the exact dynamical pressure, so that this principle can be used for the measurement of the velocity, and is in fact much used. The dynamical pressure is also well suited to express the laws of air resistance. It is known that this resistance is proportional to the square of the velocity and to the density of the medium; but $q=\frac{\rho}{2}V^2$; so the law of air resistance may also be expressed by the formula

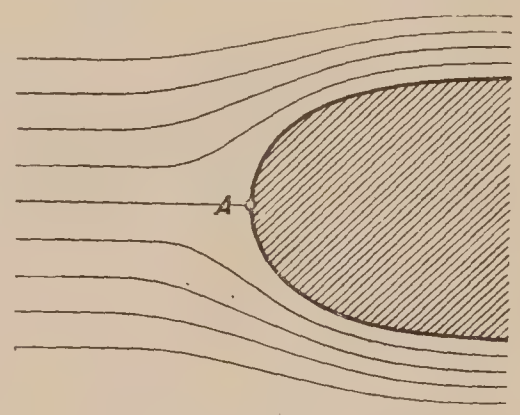

FIG. 1.—Flow around a blunt body.

$$W=c\,.\,F\,.\,q \tag{2}$$

where F is the area of the surface and c is a pure number. With this mode of expression it appears very clearly that the force called the "drag" is equal to surface times pressure difference (the formula has the same form as the one for the piston force in a steam engine). This mode of stating the relation has been introduced in Germany and Austria and has proved useful. The air-resistance coefficients then become twice as large as the "absolute" coefficients previously used.

Since V^2 can not become less than zero, an increase of pressure greater than q can not, by equation (1), occur. For diminution of pressure, however, no definite limit can be set. In the case of flow past convex surfaces marked increases of velocity of flow occur and in connection with them diminutions of pressure which frequently amount to $3q$ and more.

5. A series of typical properties of motion of nonviscous fluids may be deduced in a useful manner from the following theorem, which is due to Lord Kelvin. Before the theorem itself is stated, two concepts must be defined. 1. The circulation: Consider the line integral of the velocity $\int V\cos(V, ds).\,ds$, which is formed exactly like the line integral of a force, which is called "the work of the force." The amount of this line integral, taken over a path which returns on itself is called the circulation of the flow. 2. The fluid line: By this is meant a line which is always formed of the same fluid particles, which therefore shares in the motion of the

fluid. The theorem of Lord Kelvin is: In a nonviscous fluid the circulation along every fluid line remains unchanged as time goes on. But the following must be added:

(1) The case may arise that a fluid line is intersected by a solid body moving in the fluid. If this occurs, the theorem ceases to apply. As an example I mention the case in which one pushes a flat plate into a fluid at rest, and then by means of the plate exerts a pressure on the fluid. By this a circulation arises which will remain if afterwards the plate is quickly withdrawn in its own plane. See figure 2.

(2) In order that the theorem may apply, we must exclude mass forces of such a character that work is furnished by them along a path which returns on itself. Such forces do not ordinarily arise and need not be taken into account here, where we are concerned regularly only with gravity.

(3) The fluid must be homogeneous, i. e., of the same density at all points. We can easily see that in the case of nonuniform density circulation can arise of itself in the course of time if we think of the natural ascent of heated air in the midst of cold air. The circulation increases continuously along a line which passes upward in the warm air and returns downward in the cold air.

Frequently the case arises that the fluid at the beginning is at rest or in absolutely uniform motion, so that the circulation for every imaginable closed line in the fluid is zero. Our theorem then says that for every closed line that can arise from one of the originally closed lines the circulation remains zero, in which we must make exception, as mentioned above, of those lines which are cut by bodies. If the line integral along every closed line is zero, the line integral for an open curve from a definite point O to an arbitrary point P is independent of the selection of the line along which the integral is taken (if this were not so, and if the integrals along two lines from O to P were different, it is evident that the line integral along the closed curve OPO would not be zero, which contradicts our premise). The line integral along the line OP depends, therefore, since we will consider once for all the point O as a fixed one, only on the coordinates of the point P, or, expressed differently, it is a function of these coordinates. From analogy with corresponding considerations in the case of fields of force, this line integral is called the "velocity potential," and the particular kind of motion in which such a potential exists is called a "potential motion." As follows immediately from the meaning of line integrals, the component of the velocity in a definite direction is the derivative of the potential in this direction. If the line-element is perpendicular to the resultant velocity, the increase of the potential equals zero, i. e., the surfaces of constant potential are everywhere normal to the velocity of flow. The velocity itself is called the gradient of the potential. The velocity components u, v, w are connected with the potential Φ by the following equations:

FIG. 2.—Production of circulation by introduction and withdrawal of flat plate.

$$u=\frac{\partial\phi}{\partial x},\quad v=\frac{\partial\phi}{\partial y},\quad w=\frac{\partial\phi}{\partial z} \tag{3}$$

The fact that the flow takes place without any change in volume is expressed by stating that as much flows out of every element of volume as flows in. This leads to the equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{4}$$

In the case of potential flow we therefore have

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \tag{4a}$$

as the condition for flow without change in volume. All functions Φ (x, y, z, t), which satisfy this last equation, represent possible forms of flow. This representation of a flow is specially convenient for calculations, since by it the entire flow is given by means of the one function Φ. The most valuable property of the representations is, though, that the sum of two, or of as many as one desires, functions Φ, each of which satisfies equation (4a), also satisfies this equation, and therefore represents a possible type of flow ("superposition of flows").

6. Another concept can be derived from the circulation, which is convenient for many considerations, viz, that of rotation. The component of the rotation with reference to any axis is obtained if the circulation is taken around an elementary surface of unit area in a plane perpendicular to the axis. Expressed more exactly, such a rotation component is the ratio of the circulation around the edge of any such infinitesimal surface to the area of the surface. The total rotation is a vector and is obtained from the rotation components for three mutually perpendicular axes. In the case that the fluid rotates like a rigid body, the rotation thus defined comes out as twice the angular velocity of the rigid body. If we take a rectangular system of axes and consider the rotations with reference to the separate axes, we find that the rotation can also be expressed as the geometrical sum of the angular velocities with reference to the three axes.

The statement that in the case of a potential motion the circulation is zero for every closed fluid line can now be expressed by saying the rotation in it is always zero. The theorem that the circulation, if it is zero, remains zero under the conditions mentioned, can also now be expressed by saying that, if these conditions are satisfied in a fluid in which there is no rotation, rotation can never arise. An irrotational fluid motion, therefore, always remains irrotational. In this, however, the following exceptions are to be noted: If the fluid is divided owing to bodies being present in it, the theorem under consideration does not apply to the fluid layer in which the divided flow reunites, not only in the case of figure 2 but also in the case of stationary phenomena as in figure 3, since in this case a closed fluid line drawn in front of the body can not be transformed into a fluid line that intersects the region where the fluid streams come together. Figure 3 shows four successive shapes of such a fluid line. This region is, besides, filled with fluid particles which have come very close to the body. We are therefore led to the conclusion from the standpoint of a fluid with very small but not entirely vanishing viscosity that the appearance of vortices at the points of reunion of the flow in the rear of the body does not contradict the laws of hydrodynamics. The three components of the rotation ξ, η, ζ are expressed as follows by means of the velocity components u, v, w.

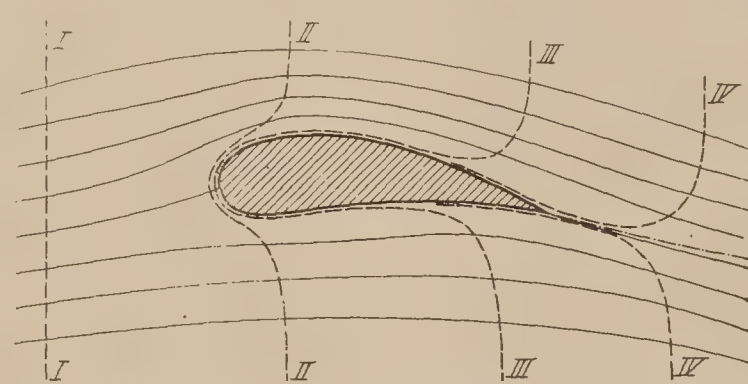

FIG. 3.—Successive positions of a fluid line in flow around a solid body.

$$\xi=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z},\ \eta=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x},\ \zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \tag{5}$$

If the velocity components are derived from a potential, as shown in equation (2), the rotation components, according to equation (5) vanish identically, since $\frac{\partial^2\Phi}{\partial z\partial y}=\frac{\partial^2\Phi}{\partial y\partial z}$

7. Very remarkable theorems hold for the rotation, which were discovered by v. Helmholtz and stated in his famous work on vortex motions. Concerning the geometrical properties of the rotation the following must be said:

At all points of the fluid where rotation exists the direction of the resultant rotation axes can be indicated, and lines can also be drawn whose directions coincide everywhere with these axes, just as the stream lines are drawn so as to coincide with the directions of the velocity. These lines will be called, following Helmholtz, "vortex lines." The vortex lines through the points of a small closed curve form a tube called a "vortex tube." It is an immediate consequence of the geometrical idea of rotation as deduced above that through the entire extent of a vortex tube its strength—i. e., the circulation around the boundary of the tube—is constant. It is seen, in fact, that on geometrical grounds the space distribution of rotation quite independently of the special properties of the velocity field from which it is deduced is of the same nature as the space distribution of the velocities in an incompressible fluid. Consequently a vortex tube, just like a stream line in an incompressible fluid, can not end anywhere in the interior of the fluid; and the strength of the vortex, exactly like the quantity of fluid passing per second through the tube of stream lines, has at one and the same instant the same value

throughout the vortex tube. If Lord Kelvin's theorem is now applied to the closed fluid line which forms the edge of a small element of the surface of a vortex tube, the circulation along it is zero, since the surface inclosed is parallel to the rotation axis at that point. Since the circulation can not change with the time, it follows that the element of surface at all later times will also be part of the surface of a vortex tube. If we picture the entire bounding surface of a vortex tube as made up of such elementary surfaces, it is evident that, since as the motion continues this relation remains unchanged, the particles of the fluid which at any one time have formed the boundary of a vortex tube will continue to form its boundary. From the consideration of the circulation along a closed line inclosing the vortex tube, we see that this circulation—i. e., the strength of our vortex tube—has the same value at all times. Thus we have obtained the theorems of Helmholtz, which now can be expressed as follows, calling the contents of a vortex tube a "vortex filament": "The particles of a fluid which at any instant belong to a vortex filament always remain in it; the strength of a vortex filament throughout its extent and for all time has the same value." From this follows, among other things, that if a portion of the filament is stretched, say, to double its length, and thereby its cross section made one-half as great, then the rotation is doubled, because the strength of the vortex, the product of the rotation and the cross section, must remain the same. We arrive, therefore, at the result that the vector expressing the rotation is changed in magnitude and direction exactly as the distance between two neighboring particles on the axis of the filament is changed.

8. From the way the strengths of vortices have been defined it follows for a space filled with any arbitrary vortex filaments, as a consequence of a known theorem of Stokes, that the circulation around any closed line is equal to the algebraic sum of the vortex strengths of all the filaments which cross a surface having the closed line as its boundary. If this closed line is in any way continuously changed so that filaments are thereby cut, then evidently the circulation is changed according to the extent of the strengths of the vortices which are cut. Conversely we may conclude from the circumstance that the circulation around a closed line (which naturally can not be a fluid line) is changed by a definite amount by a certain displacement, that by the displacement vortex strength of this amount will be cut, or expressed differently, that the surface passed over by the closed line in its displacement is traversed by vortex filaments whose strengths add up algebraically to the amount of the change in the circulation.

The theorems concerning vortex motion are specially important because in many cases it is easier to make a statement as to the shape of the vortex filaments than as to the shape of the stream lines, and because there is a mode of calculation by means of which the velocity at any point of the space may be determined from a knowledge of the distribution of the rotation. This formula, so important for us, must now be discussed. If Γ is the strength of a thin vortex filament and ds an element of its medial line, and if, further, r is the distance from the vortex element to a point P at which the velocity is to be calculated, finally if α is the angle between ds and r, then the amount of the velocity due to the vortex element is

$$dv=\frac{\Gamma\ ds \sin \alpha}{4\ \pi\ r^2}; \tag{6}$$

the direction of this contribution to the velocity is perpendicular to the plane of ds and r. The total velocity at the point P is obtained if the contributions of all the vortex elements present in the space are added. The law for this calculation agrees then exactly with that of Biot-Savart, by the help of which the magnetic field due to an electric current is calculated. Vortex filaments correspond in it to the electric currents, and the vector of the velocity to the vector of the magnetic field.

As an example we may take an infinitely long straight vortex filament. The contributions to the velocity at a point P are all in the same direction, and the total velocity can be determined by a simple integration of equation (6). Therefore this velocity is

$$v=\frac{\Gamma}{4\pi}\int_{-\infty}^{+\infty}\frac{ds\cdot\sin\alpha}{r^2}$$

As seen by figure 4, $s=h \operatorname{ctg} \alpha$, and by differentiation, $ds=-\frac{h}{\sin^2\alpha}d\alpha$. Further $r=\frac{h}{\sin \alpha}$; so that

$$v=\frac{\Gamma}{4\pi h}\int_0^\pi \sin \alpha d\alpha=-\frac{\Gamma}{4\pi h}[\cos \alpha]_0^\pi=\frac{\Gamma}{2\pi h} \tag{6a}$$

This result could be deduced in a simpler manner from the concept of circulation if we were to use the theorem, already proved, that the circulation for any closed line coincides with the vortex strength of the filaments which are inclosed by it. The circulation for every closed line which goes once around a single filament must therefore coincide with its strength. If the velocity at a point of a circle of radius h around our straight filament equals v then this circulation equals "path times velocity" $=2\pi h \cdot v$, whence immediately follows $v=\frac{\Gamma}{2\pi h}$. The more exact investigation of this velocity field shows that for every point outside the filament (and the formula applies only to such points) the rotation is zero, so that in fact we are treating the case of a velocity distribution in which only along the axis does rotation prevail, at all other points rotation is not present.

For a finite portion of a straight vortex filament the preceding calculation gives the value

$$v=\frac{\Gamma}{4\pi h}(\cos \alpha_1-\cos \alpha_2) \tag{6b}$$

This formula may be applied only for a series of portions of vortices which together give an infinite or a closed line. The velocity field of a single portion of a filament would require rotation also outside the filament, in the sense that from the end of the portion of the filament vortex lines spread out in all the space and then all return together at the beginning of the portion. In the case of a line that has no ends this external rotation is removed, since one end always coincides with the beginning of another portion of equal strength, and rotation is present only where it is predicated in the calculation.

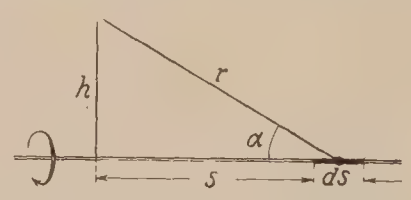

FIG. 4.—Velocity-field due to infinite rectilinear vortex.

9. If one wishes to represent the flow around solid bodies in a fluid, one can in many cases proceed by imagining the place of the solid bodies taken by the fluid, in the interior of which disturbances of flow (singularities) are introduced, by which the flow is so altered that the boundaries of the bodies become streamline surfaces. For such hypothetical constructions in the interior of the space actually occupied by the body, one can assume, for instance, any suitably selected vortices, which, however, since they are only imaginary, need not obey the laws of Helmholtz. As we shall see later, such imaginary vortices can be the seat of lifting forces. Sources and sinks also, i. e., points where fluid continuously appears, or disappears, offer a useful method for constructions of this kind. While vortex filaments can actually occur in the fluid, such sources and sinks may be assumed only in that part of the space which actually is occupied by the body, since they represent a phenomenon which can not be realized. A contradiction of the law of the conservation of matter is avoided, however, if there are assumed to be inside the body both sources and sinks, of equal strengths, so that the fluid produced by the sources is taken back again by the sinks.

The method of sources and sinks will be described in greater detail when certain practical problems are discussed; but at this point, to make the matter clearer, the distribution of velocities in the case of a source may be described. It is very simple, the flow takes place out from the source uniformly on all sides in the direction of the radii. Let us describe around the point source a concentric spherical surface, then, if the fluid output per second is Q, the velocity at the surface is

$$v=\frac{Q}{4\pi r^2}; \tag{7}$$

the velocity therefore decreases inversely proportional to the square of the distance. The flow is a potential one, the potential comes out (as line-integral along the radius)

$$\Phi = \text{const.} - \frac{Q}{4\pi r} \tag{7a}$$

If a uniform velocity toward the right of the whole fluid mass is superimposed on this velocity distribution—while the point source remains stationary—then a flow is obtained which, at a considerable distance from the source, is in straight lines from left to right. The fluid coming out of the source is therefore pressed toward the right (see fig. 5); it fills, at some distance from the source, a cylinder whose diameter may be determined easily. If V is the velocity of the uniform flow, the radius r of the cylinder is given by the condition $Q = \pi r^2 \cdot V$. All that is necessary now is to assume on the axis of the source further to the right a sink of the same strength as the source for the whole mass of fluid from the source to vanish in this, and the flow closes up behind the sink again exactly as it opened out in front of the source. In this way we obtain the flow around an elongated body with blunt ends.

10. The special case when in a fluid flow the phenomena in all planes which are parallel to a given plane coincide absolutely plays an important rôle both practically and theoretically. If the lines which connect the corresponding points of the different planes are perpendicular to the planes, and all the streamlines are plane curves which lie entirely in one of those planes, we speak of a uniplanar flow. The flow around a strut whose axis is perpendicular to the direction of the wind is an example of such a motion.

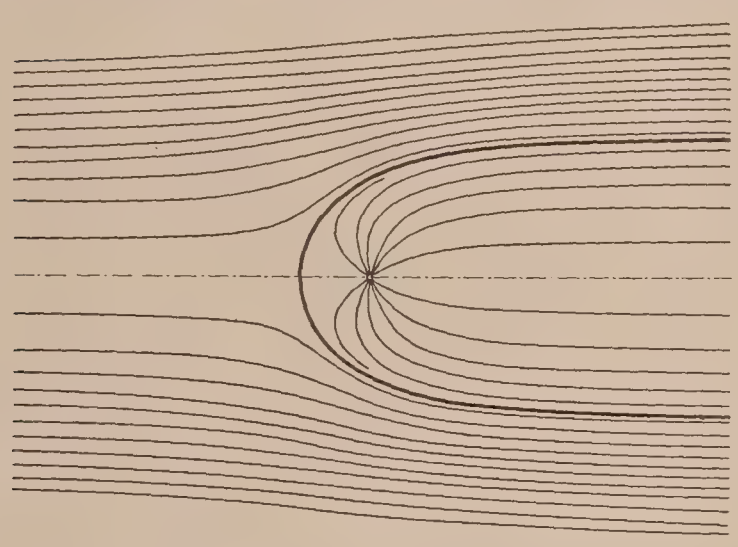

FIG. 5.—Superposition of uniform flow and that caused by a source.

The mathematical treatment of plane potential flow of the ideal fluid has been worked out specially completely more than any other problem in hydrodynamics. This is due to the fact that with the help of the complex quantities ($x+iy$, where $i=\sqrt{-1}$, is called the imaginary unit) there can be deduced from every analytic function a case of flow of this type which is incompressible and irrotational. Every real function, $\Phi(x, y)$ and $\Psi(x, y)$, which satisfies the relation

$$\Phi + i\Psi = f(x+iy), \tag{8}$$

where f is any analytic function, is the potential of such a flow. This can be seen from these considerations: Let $x+iy$ be put $=z$, where z is now a "complex number." Differentiate equation (8) first with reference to x and then with reference to y, thus giving

$$\frac{\partial\Phi}{\partial x} + i\frac{\partial\Psi}{\partial x} = \frac{df}{dz}\frac{\partial z}{\partial x} = \frac{df}{dz}$$

$$\frac{\partial\Phi}{\partial y} + i\frac{\partial\Psi}{\partial y} = \frac{df}{dz}\frac{\partial z}{\partial y} = i\frac{df}{dz} = i\frac{\partial\Phi}{\partial x} - \frac{\partial\Psi}{\partial y}$$

In these the real parts on the two sides of the equations must be equal and the imaginary parts also. If Φ is selected as the potential, the velocity components u and v are given by

$$u = \frac{\partial\Phi}{\partial x} = \frac{\partial\Psi}{\partial y}; \quad v = \frac{\partial\Phi}{\partial y} = -\frac{\partial\Psi}{\partial x} \tag{9}$$

If now we write the expressions $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ (continuity) and $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ (rotation) first in terms of Φ and then of Ψ, they become

$$\left.\begin{aligned}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}&=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=\underbrace{\frac{\partial^2\Psi}{\partial y\partial x}-\frac{\partial^2\Psi}{\partial x\partial y}}_{=o}\\ \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}&=\underbrace{\frac{\partial^2\Phi}{\partial y\partial x}-\frac{\partial^2\Phi}{\partial x\partial y}}_{=o}=-\frac{\partial^2\Psi}{\partial x^2}-\frac{\partial^2\Psi}{\partial y^2}\end{aligned}\right\}\tag{10}$$

It is seen therefore that not only is the motion irrotational (as is self-evident since there is a potential), but it is also continuous. The relation $\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=o$ besides corresponds exactly to our equation (4a). Since it is satisfied also by Ψ, this can also be used as potential.

The function Ψ, however, has, with reference to the flow deduced by using Φ as potential, a special individual meaning. From equation (8) we can easily deduce that the lines $\Psi=$ const. are parallel to the velocity; therefore, in other words, they are streamlines. In fact if we put

$$d\Psi=\frac{\partial\Psi}{\partial x}dx+\frac{\partial\Psi}{\partial y}dy=o,\text{ then }\frac{dy}{dx}=-\frac{\frac{\partial\Psi}{\partial x}}{\frac{\partial\Psi}{\partial y}}=\frac{v}{u}$$

which expresses the fact of parallelism. The lines $\Psi=$ const. are therefore perpendicular to the lines $\Phi=$ const. If we draw families of lines, $\Phi=$ const. and $\Psi=$ const. for values of Φ and Ψ which differ from each other by the same small amount, it follows from the easily derived equation $d\Phi+id\Psi=\frac{df}{dz}(dx+idy)$ that the two bundles form a square network; from which follows that the diagonal curves of the network again form an orthogonal and in fact a square network. This fact can be used practically in drawing such families of curves, because an error in the drawing can be recognized by the eye in the wrong shape of the network of diagonal curves and so can be improved. With a little practice fairly good accuracy may be obtained by simply using the eye. Naturally there are also mathematical methods for further improvement of such networks of curves. The function Ψ, which is called the "stream function," has another special meaning. If we consider two streamlines $\Psi=\Psi_1$ and $\Psi=\Psi_2$, the quantity of fluid which flows between the two streamlines in a unit of time in a region of uniplanar flow of thickness 1 equals $\Psi_2-\Psi_1$. In fact if we consider the flow through a plane perpendicular to the X-axis, this quantity is

$$Q=\int_{y_1}^{y_2}udy=\int_{y_1}^{y_2}\frac{\partial\Psi}{\partial y}dy=\int_{y_1}^{y_2}d\Psi=\Psi_2-\Psi_1.$$

The numerical value of the stream function coincides therefore with the quantity of fluid which flows between the point x, y and the streamline $\Psi=o$.

As an example let the function

$$\Phi+i\Psi=A(x+iy)^n$$

be discussed briefly. It is simplest in general to ask first about the streamline $\Psi=o$. As is well known, if a transformation is made from rectangular coordinates to polar ones r, φ, $(x+iy)^n=r^n(\cos n\varphi+i\sin n\varphi)$. The imaginary part of this expression is $ir^n\sin n\varphi$. This is to be put equal to $i\Psi$. $\Psi=o$ therefore gives $\sin n\varphi=o$, i. e., $n\varphi=o$, π, 2π, etc. The streamlines $\Psi=o$ are therefore straight lines through the origin of coordinates, which make an angle $\alpha=\frac{\pi}{n}$ with each other, the flow is therefore the potential flow between two plane walls making the angle α with each other. The other streamlines satisfy the equation $r^n\sin n\varphi=$ const. The velocities can be obtained by differentiation, e. g., with reference to x:

$$\frac{\partial\Phi}{\partial x}+i\frac{\partial\Psi}{\partial x}=u-iv=An(x+iy)^{n-1}=Anr^{n-1}\{\cos(n-1)\varphi+i\sin(n-1)\varphi\}$$

For $r=o$ this expression becomes zero or infinite, according as n is greater or less than 11, i. e., according as the angle α is less or greater than $\pi(=180°)$. Figures 6 and 7 give the streamlines for $\alpha=\frac{\pi}{4}=45°$ and $\frac{3}{2}\pi=270°$, corresponding to $n=4$ and $\frac{2}{3}$. In the case of figure 7 the velocity, as just explained, becomes infinite at the corner. It would be expected that in the case of the actual flow some effect due to friction would enter. In fact there are observed at such corners, at the beginning of the motion, great velocities, and immediately thereafter the formation of vortices, by which the motion is so changed that the velocity at the corner becomes finite.

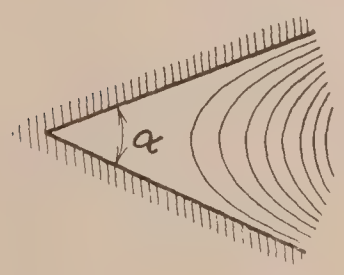

FIG. 6.—Uniplanar flow between plane walls making an angle $\alpha=45°$ with each other.

It must also be noted that with an equation

$$p+iq=\varphi(x+iy) \tag{11}$$

the x–y plane can be mapped upon the p–q plane, since to every pair of values x,y a pair of values p,q corresponds, to every point of the x–y plane corresponds a point of the p–q plane, and therefore also to every element of a line or to every curve in the former plane a linear element and a curve in the latter plane. The transformation keeps all angles unchanged, i. e., corresponding lines intersect in both figures at the same angle.

By inverting the function φ of equation (11) we can write

$$x+iy=\chi\ (p+iq)$$

and therefore deduce from equation (8) that

$$\Phi+i\Psi=f\ [\chi(p+iq)]=F\ (p+iq) \tag{12}$$

Φ and Ψ are connected therefore with p and q by an equation of the type of equation (8), and hence, in the p–q plane, are potential and stream functions of a flow, and further of that flow which arises from the transformation of the Φ, Ψ network in the x–y plane into the p–q plane.

This is a powerful method used to obtain by transformation from a known simple flow new types of flow for other given boundaries. Applications of this will be given in section 14

11. The discussion of the principles of the hydrodynamics of nonviscous fluids to be applied by us may be stopped here. I add but one consideration, which has reference to a very useful theorem for obtaining the forces in fluid motion, namely the so-called "momentum theorem for stationary motions."

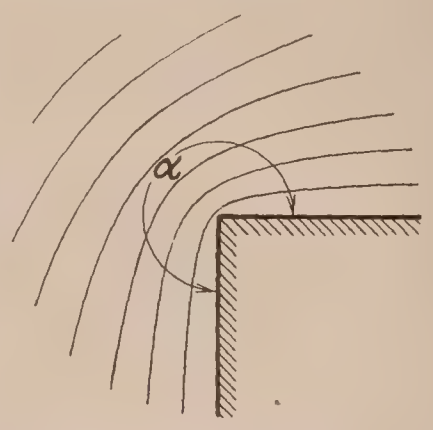

FIG. 7.—Uniplanar flow around plane walls making an angle 270° with each other.

We have to apply to fluid motion the theorem of general mechanics, which states that the rate of change with the time of the linear momentum is equal to the resultant of all the external forces. To do this, consider a definite portion of the fluid separated from the rest of the fluid by a closed surface. This surface may, in accordance with the spirit of the theorem, be considered as a "fluid surface," i. e., made up always of the same fluid particles. We must now state in a formula the change of the momentum of the fluid within the surface. If, as we shall assume, the flow is stationary, then after a time dt every fluid particle in the interior will be replaced by another, which has the same velocity as had the former. On the boundary, however, owing to its displacement, mass will pass out at the side where the fluid is approaching, and a corresponding mass will enter on the side away from which the flow takes place. If dS is the area of an element of surface, and v_n the component of the velocity in the direction of the outward drawn normal at this element, then at this point $dm=\rho dS\ .\ v_n\ dt$. If we wish to derive the component of the "impulse"—defined as the time rate of the change of momentum—for any direction s, the contribution to it of the element of surface is

$$dJ_s=v_s\frac{dm}{dt}=\rho dS\ .\ v_n v_s \tag{13}$$

With this formula we have made the transition from the fluid surface to a corresponding solid "control surface."

The external forces are compounded of the fluid pressures on the control surface and the forces which are exercised on the fluid by any solid bodies which may be inside of the control surface. If we call the latter P, we obtain the equation

$$\Sigma P_s = \iint p \,.\, \cos(n, s) \,.\, dS + \rho \iint v_n v_s dS \tag{14}$$

for the s component of the momentum theorem. The surface integrals are to be taken over the entire closed control surface. The impulse integral can be limited to the exit side, if for every velocity v_s on that side the velocity v_s' is known with which the same particle arrives at the approach side. Then in equation (13) dJ is to be replaced by

$$dJ - dJ' = (v_s - v_s') \frac{dm}{dt} = \rho dS v_n (v_s - v_s') \tag{13a}$$

The applications given in Part II will furnish illustrations of the theorem.

REPORT No. 116.

APPLICATIONS OF MODERN HYDRODYNAMICS TO AERONAUTICS.

By L. Prandtl.

PART II.

APPLICATIONS.

A. DISTRIBUTION OF PRESSURE ON AIRSHIP BODIES.

12. The first application of hydrodynamical theory to be tested by experiment in the Göttingen Laboratory referred to the distribution of pressure over the surface of models of airships. We can construct mathematically the flow for any number of varieties of sectional forms of bodies of revolution of this kind if we place along an axis parallel with the direction of the air current any suitable distribution of sources and sinks, taking care that the total strength of the sources and sinks are the same. According to the intensity of the uniform motion which is superimposed upon the flow from the sources, we obtain from the same system of sources and sinks bodies of different thicknesses. In order to obtain the smoothest possible shapes, the sources and sinks are generally distributed continuously along the axis, although single-point sources are allowable.

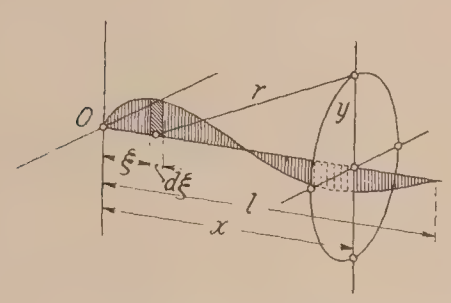

Fig. 8.—Explanation of quantities used in calculation of streamline shapes.
Abscissae: Position of source.
Ordinates: Intensity of source per unit length, i. e. $f(\xi)$.

In the case of continuously distributed sources and sinks the method of procedure is briefly this: The abscissas of the single sources are denoted by ξ, the intensity of the source per unit of length by $f(\xi)$, in which positive values of $f(\xi)$ denote sources, negative values sinks. The condition that makes the stream from the sources self-contained is expressed by the equation

$$\int_0^l f(\xi)d\xi = o.$$

By simply adding the potentials due to the single elementary sources $f(\xi)\ d\xi$, i. e., in this case by integrating them, the total flow due to the sources will be given by the potential defined by the following formula

$$\Phi_1(x,y) = -\frac{1}{4\pi}\int_0^l \frac{f(\xi)d\xi}{r} \tag{15}$$

in which $r=\sqrt{(x-\xi)^2+y^2}$, and y is the perpendicular distance from the axis of the point for which the potential is calculated, x is the abscissa along the axis measured from the same origin as ξ. (See fig. 8.) There must be added to this potential that due to the uniform flow with the velocity V, viz, $\Phi_2 = Vx$. The total potential is then $\Phi = \Phi_1 + \Phi_2$; and therefore the velocity parallel to the axis is $u = \frac{\partial\Phi}{\partial x} = V + \frac{\partial\Phi_1}{\partial x}$ and the sidewise (radial) velocity is $v = \frac{\partial\Phi}{\partial y} = \frac{\partial\Phi_1}{\partial y}$

In order to calculate the streamlines one could perform an integration of the direction given by u and v. These lines are obtained more conveniently, in this case also, by means of the stream function. (See sec. 10.) In the case of flow symmetrical with reference to the axis, such as is here discussed, one can take as stream function the quantity of fluid flowing inside the circle drawn through the point x, y in a plane perpendicular to the axis and having its center

on the axis. The amount of fluid delivered by the sources which lie up the stream is purposely deducted from this. It is not difficult to see that all points of the $X-Y$ plane, through whose parallel circles the same amount of fluid flows per second—after deduction of the sources—must lie on one and the same streamline, for evidently there is no flow, either in or out, through the surface formed by the streamlines drawn through the points of any one parallel circle (since the flow is along the surface); therefore the quantity of fluid flowing within this surface is constant, so far as it is not increased by the sources. From the meaning of the stream function, to determine which the velocity must be integrated over a surface, it follows that the stream function of a flow due to two or more causes is at every point the sum of the stream functions of the several partial flows. For a continuous distribution of sources therefore the stream function ψ is obtained by an integration exactly as was the potential. According to our premise the surface of the body is designated simply by the value $\psi=o$. The formulas are obtained as follows:

The flow from a simple source through a circle passing through a point lying to the right of the source is, writing $r=\sqrt{x^2+y^2}$,

$$\Psi'=\int_o^y u.2\pi y dy=\int_o^y \frac{Qx}{4\pi r^3}\,2\pi y dy=\frac{Qx}{2}\int_o^y \frac{ydy}{r^3}=\frac{Q}{2}\left(1-\frac{x}{r}\right)$$

From this, in accordance with what has been said, the quantity Q must be subtracted, so that

$$\Psi=\Psi'-Q=-\frac{Q}{2}\left(1+\frac{x}{r}\right) \tag{16}$$

For points lying to the left of the source we obtain from the integral

$$\Psi''=-\frac{Q}{2}\left(1+\frac{x}{r}\right)$$

which coincides with formula (16); this holds, then, everywhere.

For the assumed continuous distribution of sources we obtain

$$\Psi_1=-\frac{1}{2}\int_o^l f(\xi)\left(1+\frac{x-\xi}{r}\right)d\xi \tag{17}$$

in which $r=\sqrt{(x-\xi)^2+y^2}$. To this stream function of the sources must now be added that due to the parallel flow

$$\Psi_2=V\pi y^2 \tag{18}$$

Putting the total stream function $\Psi_1+\Psi_2=\Psi$ equal to zero, gives the equation of the surface of the body around which the flow takes place. Putting $\Psi_1+\Psi_2=C$ gives any other streamline. It is evident that, with the same distribution of sources, a whole group of body surfaces can be obtained, depending upon the choice of the ratio of the intensity of the sources to the strength of the parallel flow.

The determination is best made practically by graphical methods, for instance, by laying off the curves $x=$const. in a system of coordinates consisting of y and $-\Psi$, which can be obtained at once from a calculation by tables for the stream function Ψ_1. If we intersect these curves by parabolas corresponding to the equation $-\Psi=V\pi y^2-C$, we obtain at once a contour (for $C=o$), or some external or internal streamline (for $C>o$ or $C<o$). The parabola may be drawn upon transparent paper, and then by displacing the parabola along the Ψ axis we can at once obtain from figure 9 the values of y corresponding to any x.

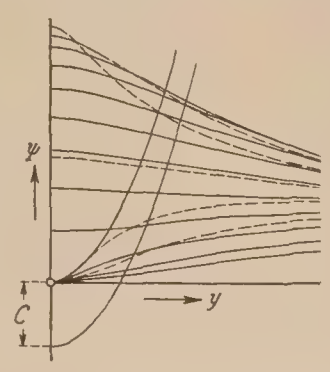

FIG. 9.—The curves are for different values of $x=$ const.

In this manner a former colleague of mine, who unfortunately fell immediately at the beginning of the war, Dr. G. Fuhrmann, calculated the shapes of bodies corresponding to a series of source distributions, and on the one hand he determined the distribution of pressure over the surface of these bodies by means of the Bernouilli equation (see sec. 4)

$$p=p_0+\frac{\rho}{2}\{V^2-(u^2+v^2)\}^1 \tag{19}$$

[1] The velocities u and v may be obtained from the potential, but also from the stream function ψ; for $u=\frac{1}{2\pi y}\frac{\partial\psi}{\partial y}$ and $v=-\frac{1}{2\pi y}\frac{\partial\psi}{\partial x}$.

and on the other he constructed models according to these drawings and measured the pressure distribution over them when placed in a wind tunnel. The agreement was altogether surprisingly good, and this success gave us the stimulus to seek further relations between theoretical hydronamics and practical aeronautics. The work of Fuhrmann was published in Jahr-b. der Motorluftschiff-Studien Gesellsch., Volume V, 1911–12 (Springer, Berlin), and contains a large number of illustrations. Four of the models investigated are shown here. The upper halves of figures 10 to 13 show the streamlines for a reference system at rest with reference to the undisturbed air, the lower halves the streamlines for a reference system attached to the body. The distribution of the source intensities is indicated on the axis. The pressure distributions are shown in figures 14 to 17. The calculated pressure distributions are indicated by the lines which are drawn full, the individual observed pressures by tiny circles.[2]

It is seen that the agreement is very complete; at the rear end, however, there appears a characteristic deviation in all cases, since the theoretical pressure distribution reaches the full dynamical pressure at the point where the flow reunites again, while actually this rise in pressure, owing to the influence of the layer of air retarded by friction, remains close to the surface.

As is well known there is no resistance for the theoretical flow in a nonviscous fluid. The actual drag consists of two parts, one resulting from all the normal forces (pressures) acting on the surface of the body, the other from all the tangential forces (friction). The pressure resistance, which in this case can be obtained by integration of the pressure distribution over the surface of the body, arises in the main from the deviation mentioned at the rear end, and is, as is known, very small. Fuhrmann's calculations gave for these resistances a coefficient, with reference to the volume of the body, as shown in the following table:[3]

Model	I	II	III	IV
k_1 =	0.0170	0.0123	0.0131	0.0145

This coefficient is obtained from the following formula:

$$\text{Drag } w_1 = k_1 \; U^{2/3} \; q$$

where U designates the volume and q the dynamical pressure.

The total resistance (drag) was obtained for the four models by means of the balance; the difference between the two quantities then furnishes the frictional resistance. The total drag coefficients were:[3]

Model	I	II	III	IV
k =	0.0340	0.0220	0.0246	0.0248

With greater values of VL than were then available for us, the resistance coefficients become nearly 30 per cent smaller. For purposes of comparison with other cases it may be mentioned that the "maximum section" was about 2/5 of $U^{2/3}$. The surface was about seven times $U^{2/3}$; from which can be deduced that the total resistance of the good models was not greater than the friction of a plane surface having the same area. The theoretical theorem that in the ideal fluid the resistance is zero receives in this a brilliant confirmation by experiment.

B. THEORY OF LIFT.

13. The phenomena which give rise to the lift of an aerofoil may be studied in the simplest manner in the case of uniplanar motion. (See sec. 10.) Such a uniplanar flow would be expected obviously in the case that the wing was unlimited at the sides, therefore was "infinitely

[2] In the wind tunnel there was a small pressure drop in the direction of its length. In order to eliminate the effect of this, the pressures toward the fore had to be diminished somewhat and those aft somewhat increased.

[3] After deduction of the horizontal buoyancy.

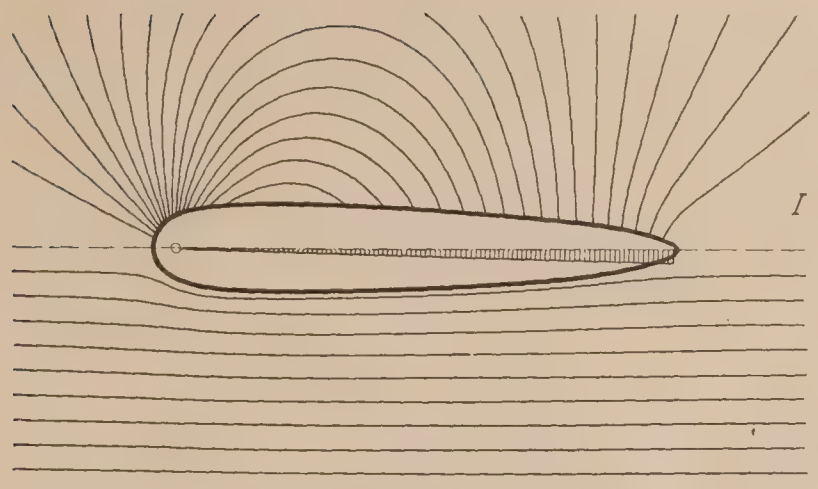

FIG. 10.

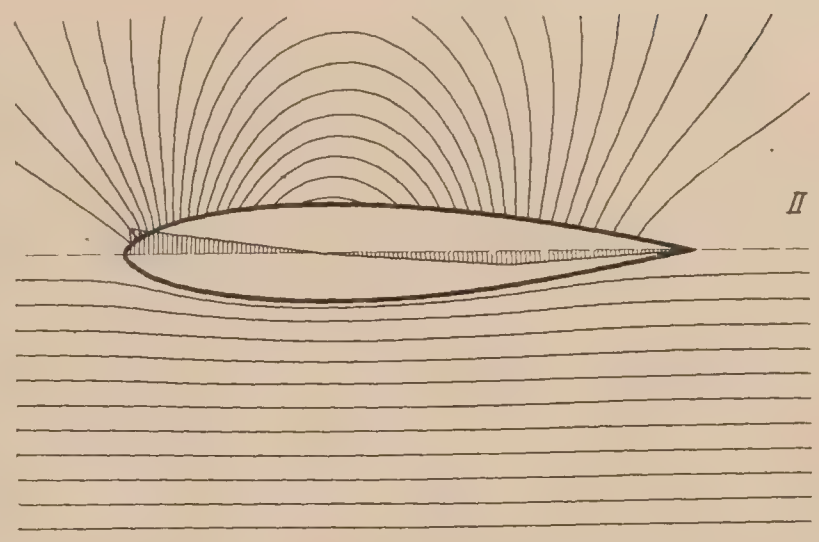

FIG. 11.

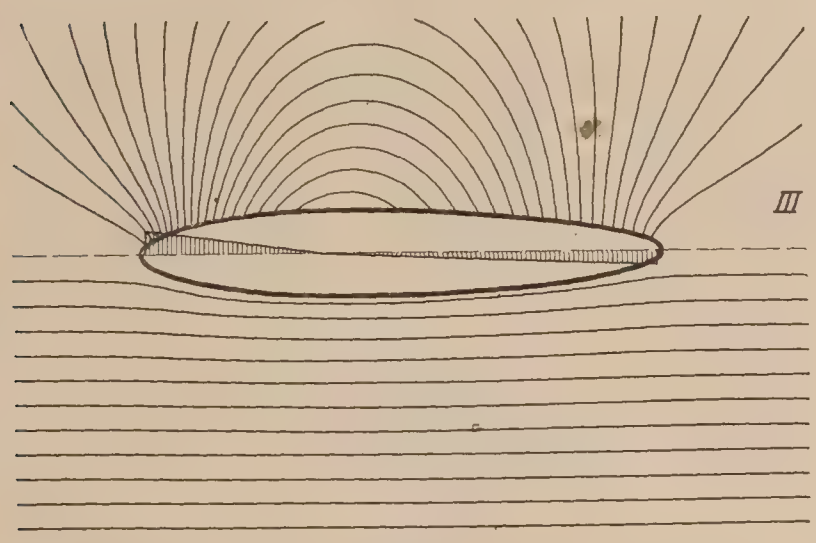

FIG. 12.

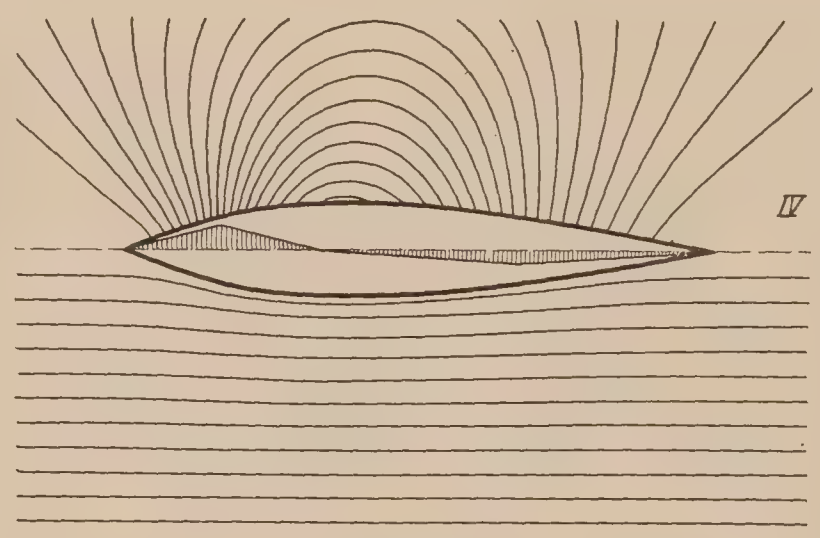

FIG. 13.

Four airship models as derived by Fuhrmann by combination of sources and uniform flow. Distribution of sources indicated on axis. Upper half: Streamlines relative to undisturbed air. Lower half: Streamlines relative to airship.

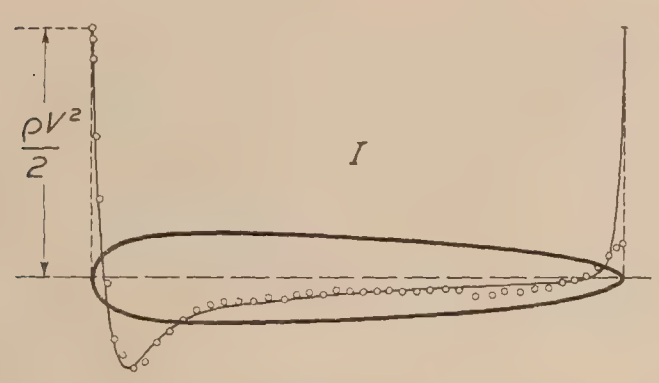

FIG. 14.

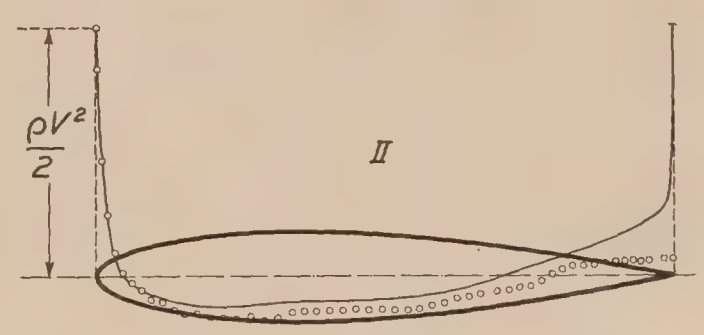

FIG. 15.

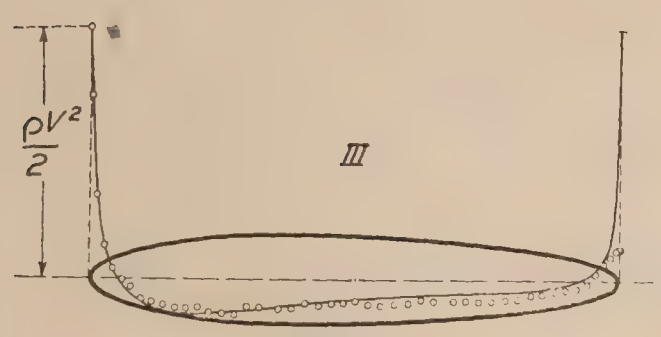

FIG. 16.

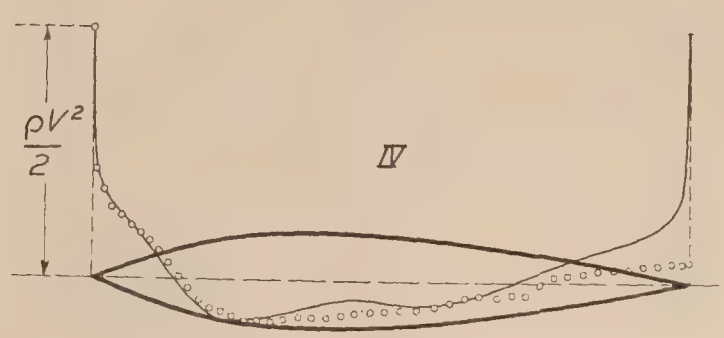

FIG. 17.

Pressure distribution over airships of figures 10 to 13. Full lines represent calculated values; small circles, points as found by observation in a wind-tunnel.

long," and throughout exhibited the same profile and the same angle of attack. In this case all the sections will be alike in all respects and each one can be considered as a plane of symmetry. The infinitely long wing plays an important part therefore in the considerations of the theoretical student. It is not possible to realize it in free air, and marked deviations from the infinitely long wing are shown even with very long wings, e. g., those having an aspect ratio of 1:10. In laboratories, however, the infinitely long wing, or uniplanar flow, may be secured with good approximation, if a wing having a constant profile is placed between plane walls in a wind tunnel, the walls running the full height of the air stream. In this case the wing must extend close to the walls; there must be no gap through which a sensible amount of air can flow. We will now discuss such experiments, and first we shall state the fundamental theory of uniplanar flow.

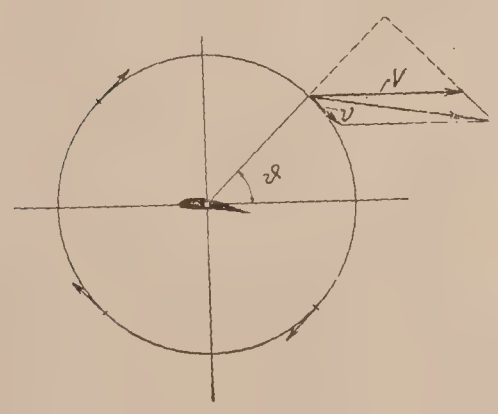

FIG. 18.—Deduction of the Kutta formula, uniplanar flow around infinite wing.

Since, as explained in section 4, in a previously undisturbed fluid flow, the sum of the static and dynamic pressures is constant: $p+\frac{\rho}{2}V^2=\text{const.}$, in order to produce lift, for which the pressure below the surface must be increased and that above diminished, such arrangements must be made as will diminish the velocity below the wing and increase it above. The other method of producing such pressure differences, namely, by causing a vortex region above the surface placed like a kite oblique to the wind, by which a suction is produced, does not come under discussion in practical aeronautics owing to the great resistance it sets up. Lanchester has already called attention to the fact that this lifting current around the wing arises if there is superimposed upon a simple potential flow a circulating flow which on the pressure side runs against the main current and on the suction side with it. Kutta (1902) and Joukowski (1906) proved, independently of each other, the theorem that the lift for the length l of the wing is

$$A=\rho\Gamma\, Vl \tag{20}$$

in which Γ is the circulation of the superimposed flow. It may be concluded from this formula that in a steady fluid flow lift is not possible unless there is motion giving rise to a circulation. In uniplanar flow in an ideal fluid this lift does not entail any drag.

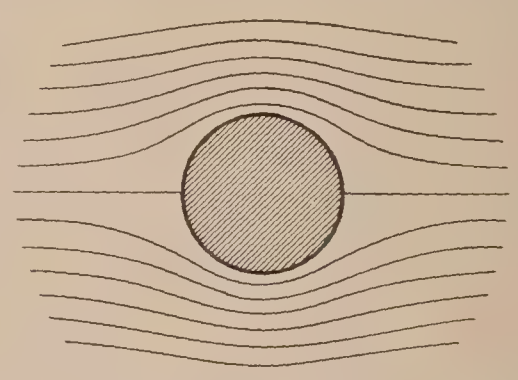
FIG. 19.—Uniplanar uniform flow around circular cylinder.

The proof of the Kutta-Joukowski formula is generally deduced by applying the momentum theorem to a circular cylinder of large radius whose axis is the medial line of the wing. The circulatory motion, which could be obtained numerically close to the wing only by elaborate mathematical processes, is reduced at a great distance from the wing to a motion which agrees exactly with the flow around a rectilinear vortex filament (see sec. 8), in which, therefore, the single particles describe concentric circles. The velocity around a circle of radius R is, then, $v=\frac{\Gamma}{2\pi R}$. For an element of surface $l.\ Rd\theta$ (see fig. 18) the normal component of the velocity is $V\cos\theta$, the mass flowing through per second $dm=\rho lRV\cos\theta d\theta$. If we wish to apply the momentum theorem for the vertical components, i. e., those perpendicular to the direction of V, then this component of the velocity through the element of surface must be taken. This, obviously, is $v\cos\theta$, taken positive if directed downward; the total impulse, then, is

$$J=\int v\cos\theta dm=\rho lRVv\int_0^{2\pi}\cos^2\theta d\theta.$$

The integral equals π, and therefore introducing the value of v

$$J=\frac{1}{2}\rho V\Gamma l.$$

Since the resulting impulse is directed downward (the upward velocity in front of the wing is changed into a downward one behind the wing), this means that the reaction of the fluid against the wing is a lift of the wing upward. The amount of the impulse furnishes, as is seen by referring to formula (20), only half the lift. The other half comes from the pressure differences on the control surfaces. Since, for a sufficiently large R, v can always be considered small compared with V, neglecting $\frac{\rho}{2} v^2$, the pressure p is given, according to the ernouilli equation, by

$$p = p_0 + \frac{\rho}{2} V^2 - \frac{\rho}{2}\left\{(V + v \sin \theta^2 + v^2 \cos^2 \theta\right\} \cong p_0 - \rho V \sin \theta.$$

A component of this, obtained by multiplying by $\sin \theta$, acts vertically on the surface element $lRd\theta$. The resulting force D is, then,

$$D = \rho l R V v \int_0^{2\pi} \sin^2 \theta d\theta.$$

This integral also equals π, so that here also

$$D = \frac{1}{2} \rho V \Gamma l$$

its direction is vertically up. The total lift, then, is

$$A = J + D = \rho V \Gamma .$$

14. For the more accurate analysis of the flow around wings the complex functions (see sec. 10) have been applied with great success, following the procedure of Kutta. Very different

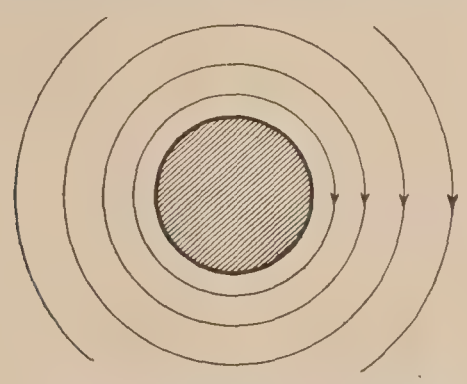

FIG. 20.—Uniplanar flow around circular cylinder considered as a columnar vortex of strength Γ.

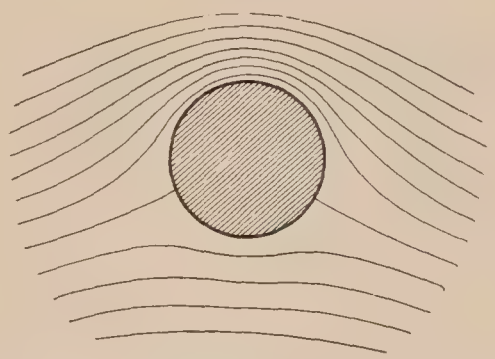

FIG. 21.—Superposition of two preceding flows.

methods have been used. Here we shall calculate only one specially simple case, in which the flow will be deduced first around a circular cylinder and then calculated for a wing profile by a transformation of the circular cylinder and its flow, using complex functions.

The flow around a circular cylinder has long been known. If the coordinates in the plane of the circle are p and q, and if we write $p + iq = t$, the potential and stream functions for the ordinary symmetrical flow around the circular cylinder are given by the very simple formula

$$\Phi_1 + i\Psi_1 = V\left(t + \frac{a^2}{t}\right)^{5} \tag{21}$$

It is easily seen by passing to polar coordinates that, for $r = a$, $\psi_1 = o$, and that therefore the circle of radius a is a streamline. Further, for the p axis, $\psi_1 = o$, i. e., this is also a streamline. The whole flow is that shown in figure 19. To this flow must be added the circulation flow expressed by the formula

$$\Phi_1 + i\Psi_2 = \frac{i\Gamma}{2\pi} \log t^{6} \tag{22}$$

which, as shown in figure 20, is simply a flow in concentric circles with the velocity $\frac{\Gamma}{2\pi r}$. The combination of the two flows, i. e., the flow for the sum of the expressions in equations (21) and (22), is shown in figure 21. It is seen that the rest point is moved down an amount de. By a suitable choice of the circulation this can be brought to any desired point.

[5] $t + \frac{a^2}{t} = \left(r + \frac{a^2}{r}\right) \cos \vartheta + i \left(r - \frac{a^2}{r}\right) \sin \vartheta$

[6] $i \log t = -\vartheta + i \log r.$

We must now discuss the transformation of this flow to a wing profile. For this purpose manifold means are possible. The simplest is furnished by a transformation according to the equation

$$z=x+iy=t+\frac{b^2}{t}.$$

By this the circle of diameter $AB=2b$ in the t plane (as we shall for brevity's sake call the p, q plane) is transformed into a straight line A' B' of the length $4b$ along the X axis, and concen-

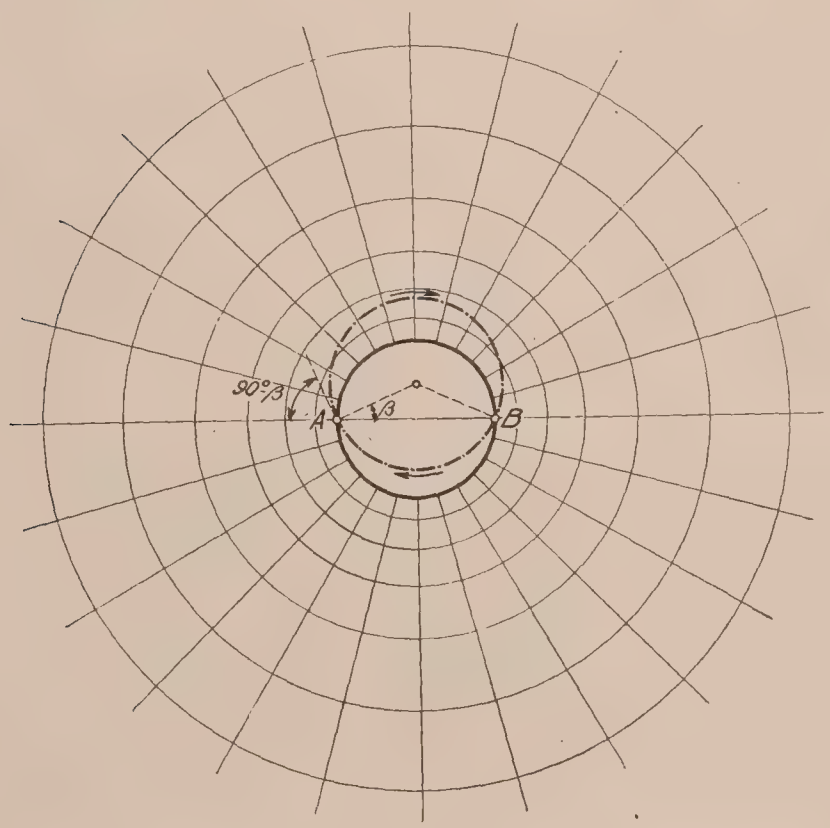

FIG. 22.

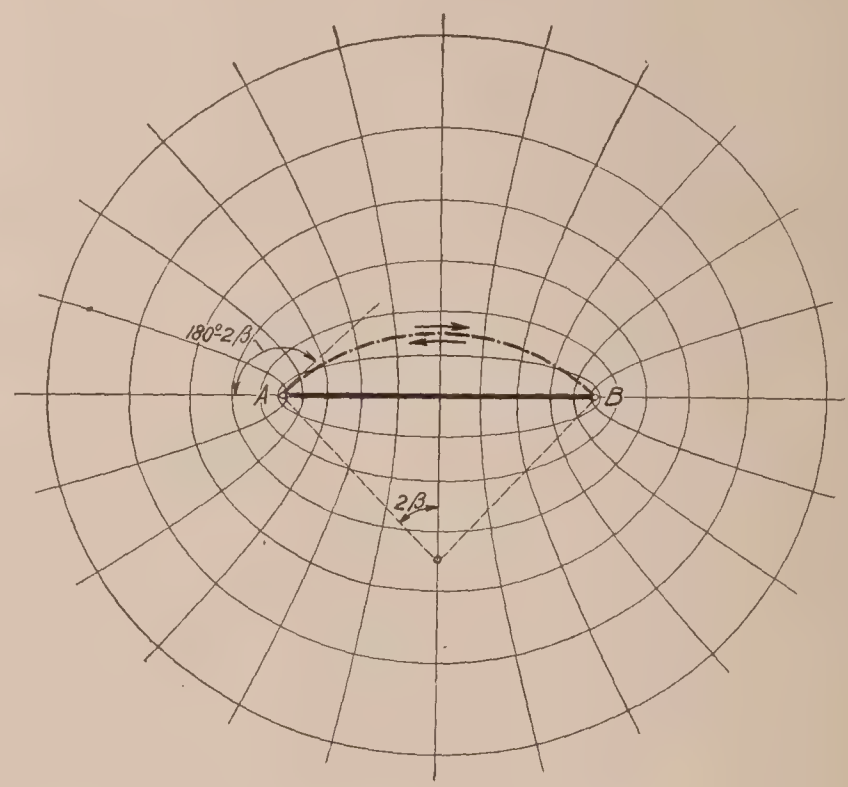

FIG. 23.

Conformal transformation of z plane into t plane by $z=t+\frac{b^2}{t}$.

tric circles around the former become ellipses, the radii become hyperbolas. All the ellipses and hyperbolas have their foci at the ends of the straight line, this forming a confocal system. Figures 22 and 23 illustrate the transformation. It may be mentioned, in addition, that the interior of the circle in figure 22 corresponds to a continuation of the meshwork in figure 23 through the slit A' B', whose form agrees with the meshwork as drawn. Any circle through the points AB is thereby transformed into an arc of a circle passed over twice, having an angle subtended at the center equal to 4β.

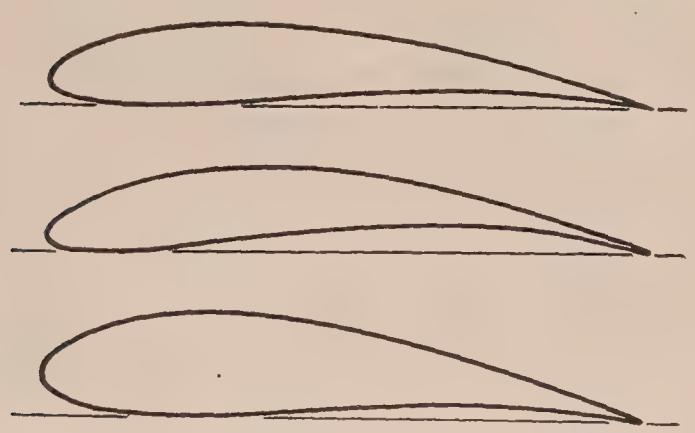

FIG. 24.—Illustrations of Joukowski sections.

Many different results may now be obtained by means of this mapping, according to the position which the circle, around which the flow takes place according to equations (21) and (22), bears to the diameter AB of the circle of figure 22. If the diameter AB is made to coincide with any oblique diameter of the circular section of the cylinder, we obtain a flow around an oblique plate whose angle of attack coincides with the inclination of the line AB. If the diameter AB is selected somewhat smaller, so that both points lie inside the circle symmetrically on the diameter, the flow around ellipses is obtained. If, however, the diameter AB coincides with a chord of the circle around which the original flow was, which, for example, may lie below the center, the flow around a curved plate forming an arc of a circle is obtained. By selection of various points in the interior of the original circle forms of diverse shapes are obtained. The recognition of the fact that among these forms very beautiful winglike profiles may be found we

owe to Joukowski. These are obtained if the point B is selected on the boundary of the original circle and the point A inside, and somewhat below the diameter through the point B. Figure 24 gives illustrations of such Joukowski profiles.

In order that the flow may be like the actual one, in the cases mentioned the circulation must always be so chosen that the rear rest point coincides with the point B, or, respectively, with the point on the original circle which lies nearest this point. In this case there will be, after mapping on the z plane, a smooth flow away from the trailing edge, as is observed in practice. It is therefore seen that the circulation must be taken greater according as the angle of attack is greater, which agrees with the observation that the lift increases with increasing angle of attack.

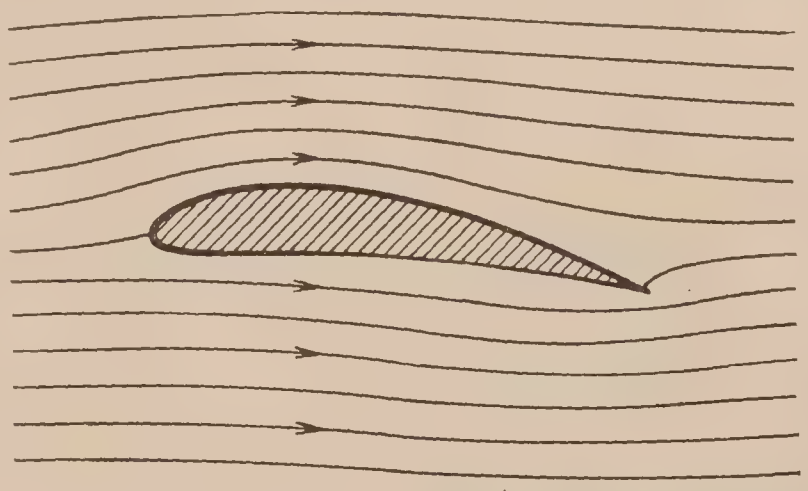

FIG. 25.—Transformation of simple potential flow, figure 19.

The transformation of the flows shown in figures 19 to 21 into wing profiles gives illustrations of streamlines as shown in figures 25 to 27—figure 25, simple potential flow; figure 26, circulation flow; figure 27, the actual flow around a wing obtained by superposition of the two previous flows.

We are, accordingly, by the help of such constructions, in the position of being able to calculate the velocity at every point in the neighborhood of the wing profile, and with it the pressure. In particular, the distribution of pressure over the wing itself may be calculated.

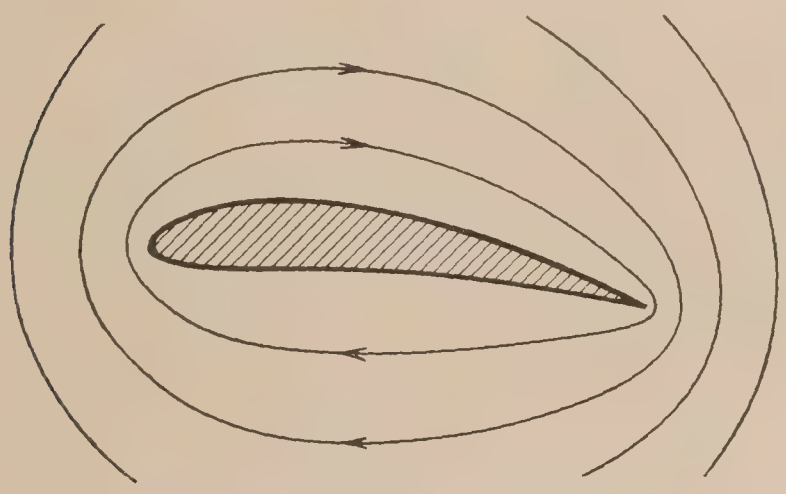

FIG. 26.—Transformation of circulatory flow, figure 20.

My assistant, Dr. A. Betz, in the year 1914 worked out the pressure distribution for a Joukowski wing profile, for a series of angles of attack, and then in a wind tunnel measured the pressure distribution on a hollow model of such a wing made of sheet metal, side walls of the height of the tunnel being introduced so as to secure uniplanar flow The results of the measurements agreed in a very satisfactory manner with the calculations, only—as could be well explained as due to friction—the actual circulation was always slightly less than that calculated for the same angle of attack. If the pressure distributions would be compared, not for the same angles of attack, but for the same amount of circulation, the agreement would be noticeably better. The pressure distributions are shown in figures 28 to 30, in which again the full curves correspond to the measurements and the dashes to the calculated pressures. Lift and drag for the wing were also obtained by the wind-tunnel balance. In order to do this, the middle part of the wing was isolated from the side parts, which were fastened to the walls of the tunnel by carefully designed labyrinths, so that within a small range it could move without friction. The result of the experiment is shown in figure 31. The theoretical drag is zero, that obtained by measurement is very small for that region where the wing is "good," but sensibly larger for too large and too small angles of attack. The lift is correspondingly in agreement

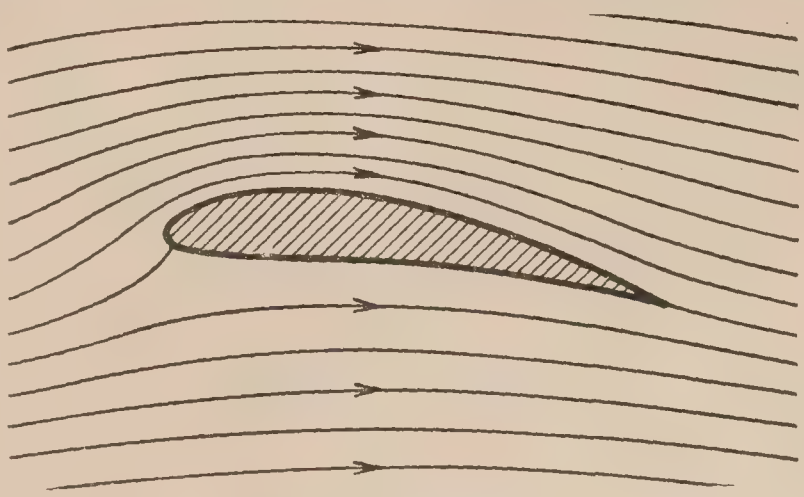

FIG. 27.—Transformation of superposition of the two flows, figure 21.

with the theoretical value in the good region, only everywhere somewhat less. The deviations of drag as well as of lift are to be explained by the influence of the viscosity of the fluid. The agreement on the whole is as good as can be expected from a theory which neglects completely the viscosity.

For the connection between the angle of incidence α and the circulation which results from the condition discussed above calculations give the following result for the lift:

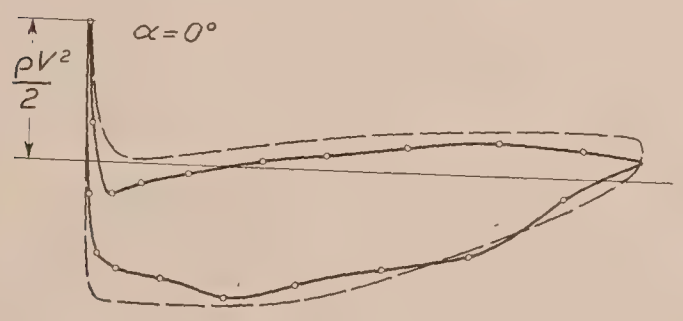

FIG. 28.

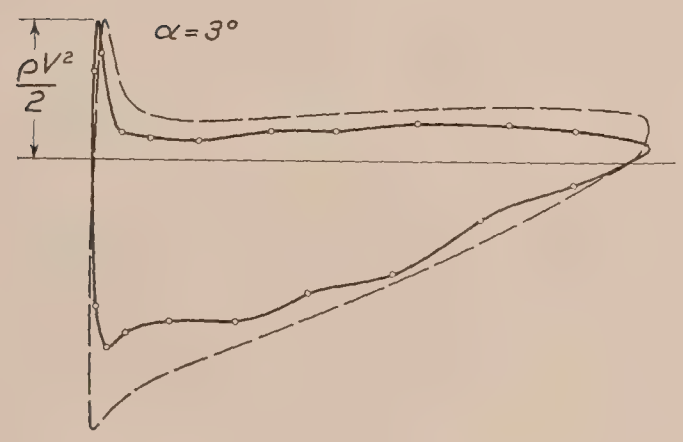

FIG. 29.

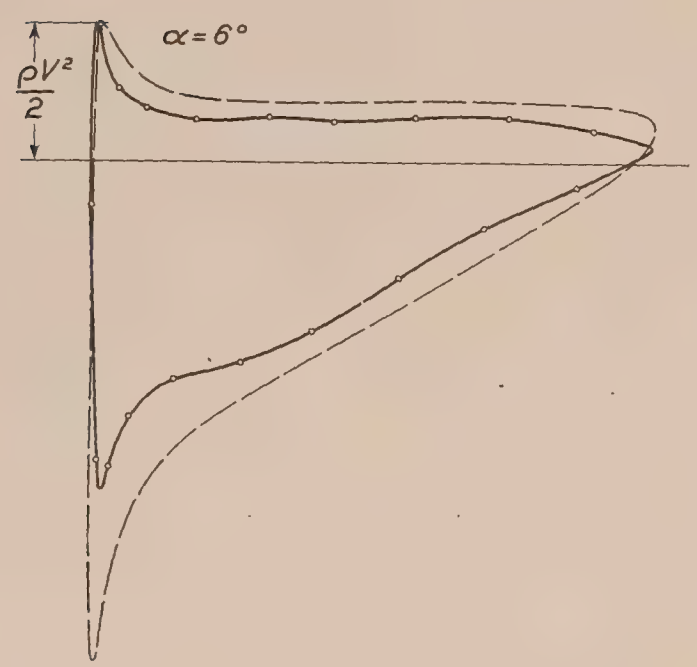

FIG. 30.

Pressure distribution over a Joukowski wing, different angles of attack. Full lines give results of wind-tunnel tests; dashed lines, calculated values.

(1) The Kutta theory gives for the thin plane plate the formula

$$A = bt.\pi\rho V^2 \sin\alpha \tag{23}$$

The lift coefficient C_a is defined by the equation

$$C_a = \frac{A}{Fq} \text{ where } q = \frac{1}{2}\rho V^2$$

and therefore

$$C_a = 2\pi \sin\alpha \tag{24}$$

(2) For the circularly curved plates having an angle of arc 4β subtended at the center (see figure 23) we have, according to Kutta, if α is the angle of attack of the chord,

$$C_a = 2\pi \frac{\sin(\alpha+\beta)}{\cos\beta} \tag{25}$$

which, for small curvatures, becomes $2\pi \sin(\alpha+\beta)$; this can be expressed by saying that the lift of the circularly curved plate is the same as that of a plane which touches the former at a point three-fourths of the distance around the arc from its leading edge.

For the Joukowski profiles and for others the formulas are less simple. v. Mises showed in 1917 that the increase of C_a with the angle of attack, i. e., $\frac{dC_a}{d\alpha}$, is greater for all other profiles than for the flat plate, and is the greater the thicker the profile. But the differences are not marked for the profiles occurring in practice.

The movement of the center of pressure has also been investigated theoretically. With the plane plate, in the region of small angles, it always lies at one-fourth of the width of the plate; with circularly curved thin plates its position for small angles is given by the following law:

$$x_0 = -\frac{t}{4}\frac{\tan\alpha}{\tan\alpha+\tan\beta} \tag{26}$$

in which t is the chord of the plate, and x_0 is the distance measured from the center of the plate. The fact that the movement of the center of pressure in the case of "good" angles of attack of the profiles agrees with theory is proved by the agreement of the actual pressure distribution with that calculated. In the case of thin plates a less satisfactory agreement as respects pressure distribution is to be expected because with them in practice there is a formation of vortices at the sharp leading edges, while theory must assume a smooth flow at this edge.

15. That a circulatory motion is essential for the production of lift of an aerofoil is definitely established. The question then is how to reconcile this fact with the proposition that

the circulation around a fluid line in a nonviscous fluid remains constant. If, before the motion begins, we draw a closed line around the wing, then, so long as everything is at rest, the circulation certainly is zero. Even when the motion begins, it can not change for this line. The explanation of why, in spite of this, the wing gains circulation is this: At the first moment of the motion there is still no circulation present, the motion takes place approximately according to figure 25, there is a flow at high velocity around the trailing edge. (See sec. 10.) This motion can not, however, continue; there is instantly formed at the trailing edge a vortex of increasing intensity, which, in accordance with the Helmholtz theorem that the vortex is always made up of the same fluid particles, remains with the fluid as it passes on. (See fig. 32.) The circulation around the wing and vortex, taken together, remains equal to zero; there remains then around the wing a circulation equal and opposite to that of the vortex which has gone off with the current. Therefore vortices will be given off until the circulation around the wing is of such a strength as to make the fluid flow off smoothly from the trailing edge. If by some alteration of the angle of attack the condition for smooth flow is disturbed, vortices are again given off until the circulation reaches its new value. These phenomena are completed in a comparatively short distance, so the full lift is developed very quickly.

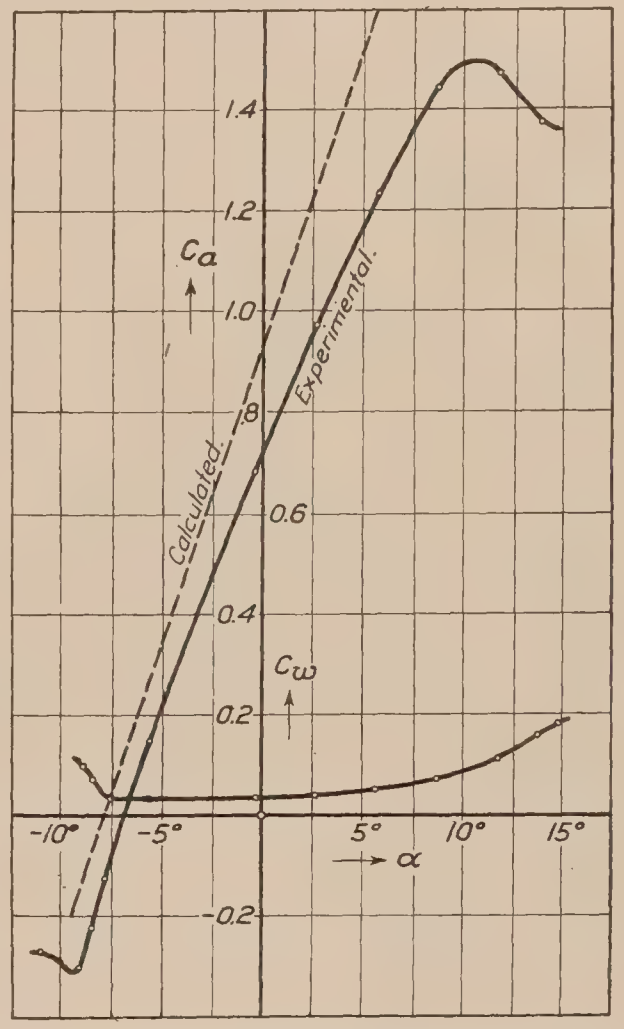

FIG. 31.—Values of lift and drag coefficients of a Joukowski wing as obtained in wind-tunnel tests and by theory.

In the pictures of flow around a wing, e. g., figure 27, one sees that the air in front of the wing flows upward against the reaction of the lift. The consideration of momentum has shown that half of the impulse is due to the oncoming ascending current. This fact needs some further explanation. The best answer is that given by Lanchester,[7] who shows that for the production of lift the air mass at any time below the wing must be given an acceleration downward. The question he asks is: What kind of a motion arises if for a short time the air below the wing is accelerated downward, then the wing is moved forward a bit without pressure, then the air is again accelerated, and so on? The space distribution of the accelerations is known for the case of a plane plate, infinitely extended at the sides, accelerated from rest; the pattern of the acceleration direction is given in figure 33. It is seen that above and below the plate the acceleration is downward, in front of and behind the plate it is upward opposite to the acceleration of the plate, since the air is escaping from the plate. Lanchester asks now about the velocities which arise from the original uniform velocity relative to the plate owing to the fact that the plate, while it gives rise to the accelerations as shown in figure 33, gradually comes nearer the air particle considered, passes by it, and finally again moves forward away from it. The picture of the velocities and streamlines which Lanchester obtained in this way and reproduced in his book was, independently of him, calculated exactly by Kutta. It is reproduced in figure 34. It is seen that as the result of the upward accelerations of the flow away from the wing

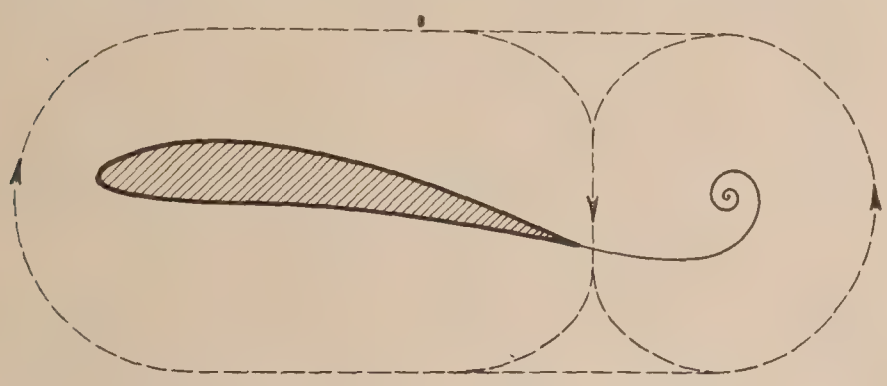

FIG. 32.—Production of circulation around a wing due to vortices leaving trailing edge.

[7] Aerodynamics I, § 110–118.

there is an upward velocity in front of the plate, a uniform downward acceleration at the plate itself due to which the upward velocity is changed into a downward one, and finally behind the plate a gradual decrease of the downward velocity on account of the acceleration upward.

C. THE FINITE WING.

16. It has been known for a long time that the aspect ratio of an aerofoil had a great effect on its properties. One could therefore have expected that, on account of the vanishing of pressure at the side edges, the intensity of the lift must decrease toward the edge, so that its average value for the same angle of attack must be smaller for small values of the aspect ratio than for large ones. But the observed influence of aspect ratios is sensibly greater than could be explained in this way. We must therefore investigate whether an explanation of this phenomenon can be found, if we apply to the finite aerofoil in some proper manner the results which are known to hold for uniplanar flow.

FIG. 33.—Acceleration diagram around an infinitely long flat plate accelerated at right angle to its surface.

It is easily seen that vortices in the free fluid must here be taken into account. For it is certain that circulation is present around the middle of the wing, because no lift is possible without circulation. If a closed line drawn around the middle of the wing, around which, therefore, there is circulation, is displaced sideways over the end of a wing, it will certainly no longer show circulation here when it is beyond the wing. From the theorem that the circulation along a closed line only changes if it cuts vortex filaments, and that the amount of the change of the circulation equals the sum of the strengths of the vortex filaments cut (see sec. 8), we must conclude that from each half of a wing vortex filaments whose strengths add up to Γ must proceed, which are concentrated mainly near the ends of the wing. According to the Helmholtz theorem we know further that every vortex produced in the fluid continues to move with the same fluid particles. We may look upon the velocities produced by the wing as small compared with the flight velocity V, so that as an approximation we may assume that the vortices move away from the wing backwards with the rectilinear velocity V. (If it is wished, we can also improve the considerations based upon such an assumption if the motion of the vortices of themselves relative to the air is taken into account. This will, however, be seen to be unnecessary for practical applications of the theory.)

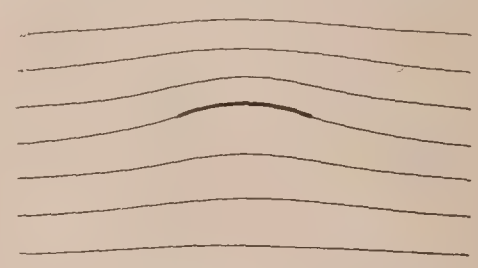

FIG. 34.—Streamlines around an infinitely long curved plate.

In order now to obtain the simplest possible scheme, we shall assume that the lift is uniformly distributed over the wing; then the total circulation will arise only at the ends, and continue rearwards as free vortices. The velocity field of an infinitely long wing, as we saw, was the same at great distances as that of a rectilinear vortex filament instead of the wing. We shall assume that the corresponding statement holds for the finite wing. We thus obtain, for the velocity field around a finite wing, a picture which is somewhat crude, it is true, if we take for it the velocity distribution due to a vortex filament of corresponding shape.

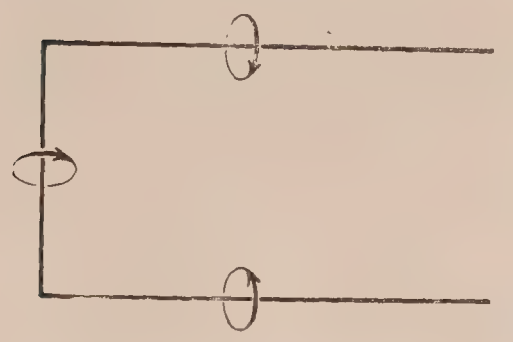

FIG. 35.—A finite wing, considered as due to vortices replacing the wing.

It may be mentioned here that, on account of there being the same laws for the velocity field of a vortex filament and the magnetic field of an electric current (see sec. 8), the velocity near a finite wing can also be investigated numerically by calculating the direction and intensity of the magnetic field produced near an electrical conductor shaped as shown in figure 35 due to an electrical current flowing in it.

The principles for the calculation of this velocity field have been stated in section 8; the total velocity is made up out of three partial velocities which are caused by the three rectilinear vortex portions. As is seen without difficulty, for the region between the vortices the flow is downward, outside it is upward.

17. This approximation theorem is specially convenient if the conditions at great distances from the wing are treated. With its help we can explain how the weight of an airplane is transferred to the ground. In order to make the flow satisfy the condition that at the ground components of velocity normal to it are impossible, we apply a concept taken from other branches of physics and superimpose the condition of an image of the airplane, taking the earth as the mirror. On account of symmetry, then, all velocity components normal to the earth's surface will vanish. If we use as our system of coordinates one attached to the airplane, we have then the case of stationary motion. If we take the X axis in the direction of the span of the wing, the Y axis horizontal in the direction of flight and the Z axis vertically down, and if u, v, w are the components of the additional velocity due to the vortices, then calling p_0 the undisturbed pressure and p' the pressure difference from p_0, and neglecting the weight of the air, Bernouilli's equation gives us

$$p_0+p'+\frac{\rho}{2}\,[u^2+(v-V)^2+w^2]=p_0+\frac{\rho}{2}V^2$$

If this is expanded and if u^2, v^2, and w^2 are neglected as being small of a higher order, there remains the simple equation

$$p'=\rho\ Vv \tag{27}$$

For the determination of the pressure distribution on the ground we must now calculate the value of v. Let us assume the vortices run off the wing in an exactly horizontal direction (actually, their path inclines downward slightly), in which case they do not contribute to v. There remains then only the "transverse vortex" of the length l (effective span) and the circulation Γ. We will assume that the span of the wing is small in comparison with the distance h of the airplane from the ground. In that case we can treat the transverse vortex as if it were a single vortex element. We obtain, then—see figure 36—at a point A, with the coordinates x and y, a velocity perpendicular to the plane ABF, of the amount

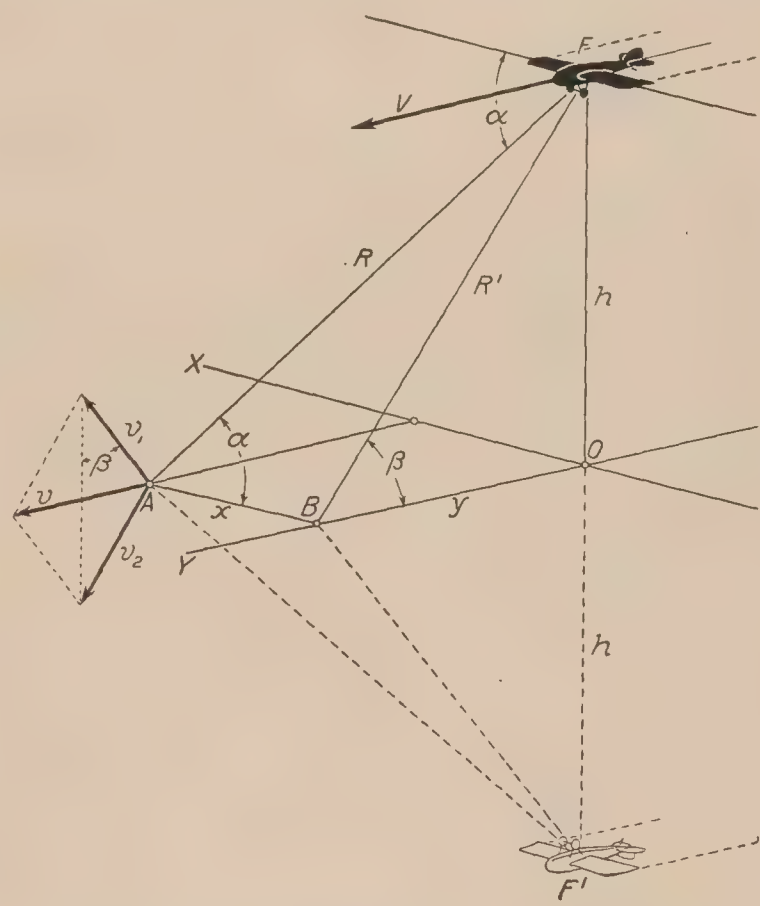

FIG. 36.—Application of method of images to airplane flying near the ground.

$$v_1=\Gamma\ l\ \frac{\sin\alpha}{4\ \pi\ R^2}$$

The image of the airplane furnishes an equal amount perpendicular to the plane ABF'. If β is the angle between the plane ABF and the XY plane, then the actual velocity at the ground, as far as it is due to the transverse vortex, will be the resultant of v_1 and v_2. It is therefore $v=2v_1\sin\beta$, or, if we write $\sin\alpha=\frac{R'}{R}$, $\sin\beta=\frac{h}{R'}$ (see fig. 36)

$$v=\frac{\Gamma\ l\ h}{2\ \pi\ R^3} \tag{28}$$

If we take into account the fact that, according to the Kutta-Joukowski formula (20), $\rho\ \Gamma\ V\ l=A$, equations (27) and (28) lead to the relation

$$p'=\frac{A\ h}{2\ \pi\ R^3} \tag{29}$$

If this is integrated over the whole infinite ground surface, it is seen that the resultant force due to the pressures on the ground has exactly the amount A. It is thus proved that the

pressure distribution due to the circulation motion transfers to the ground exactly the weigh of the airplane. The distribution of the pressure, which according to formula (29) is axially symmetrical with reference to the foot of the vertical line drawn from the airplane, is shown in figure 37. The pressure maximum is $p_1 = \frac{A}{2\pi h^2}$. Its amount, even for low heights of flight, is very small, since the surface over which the pressure is distributed is very large.

18. Applications of an entirely different kind may be made of the velocity field which belongs to the vortex of figure 35. For instance, an estimate may be made as to the magnitude of the downward velocity component at any point of the tail surfaces, and in this manner the influence of the wings upon the tail surfaces may be calculated. If in accordance with the Kutta-Joukowski formula the lift is written $A = \rho \Gamma V l$, in which, taking account of the fact that a

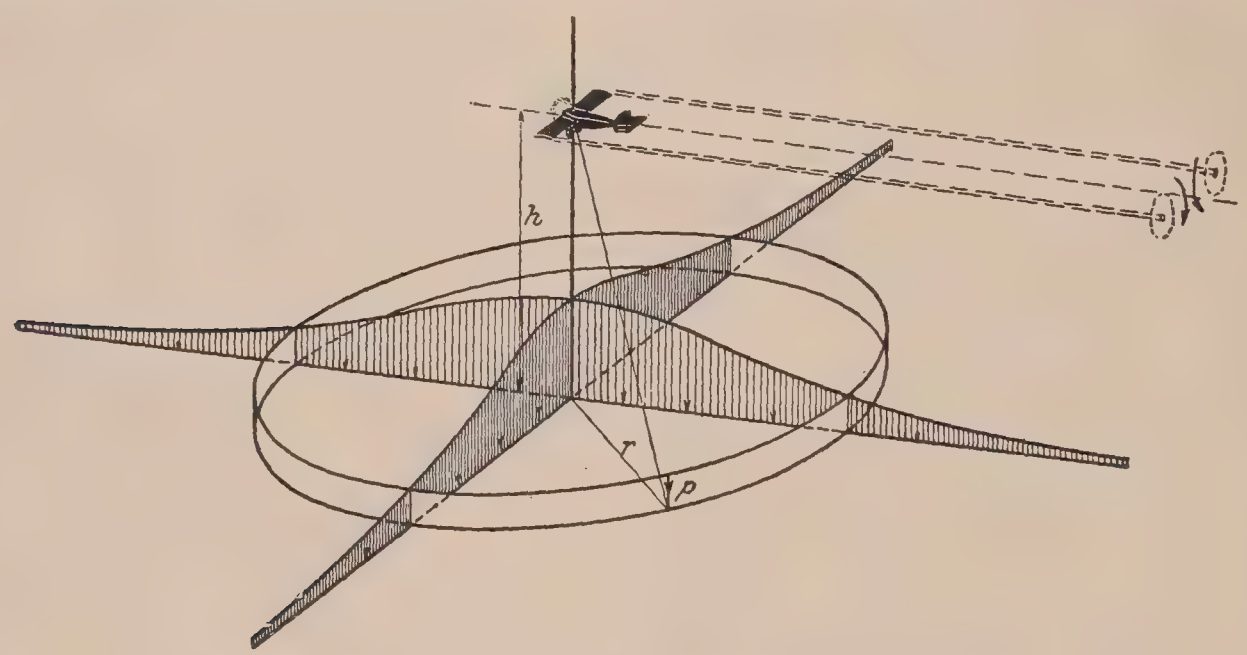

FIG. 37.—Distribution of pressure on ground caused by airplane flying near it.

portion of the vortices flow off within the ends of the wing, l, can be taken somewhat less than the actual span b, then at a distance d, behind the wing, the velocity component downward is

$$w = 2\frac{\Gamma}{4\pi}\left\{\frac{1}{\frac{l}{2}}\left(1+\frac{d}{a}\right)+\frac{1}{d}\frac{l/2}{a}\right\} = \frac{\Gamma}{\pi l}\left(1+\frac{a}{d}\right) \tag{30}$$

in which $a = \sqrt{\left(\frac{l}{2}\right)^2 + d^2}$.

If the flight velocity is V, this gives for the inclination of the downward sloping air-current $\tan \varphi = \frac{w}{V}$. We proved this relation in the year 1911 and found an approximate agreement with observation.

The principle made use of above has been applied with profit to the calculation of the influence of one wing of a biplane upon the other wing and has given a method for the calculation of the properties of a biplane from the properties of a single wing as found by experiments. The fundamental idea, which is always applied in such calculations, is that, owing to the vortex system of one wing, the velocity field near the wing is disturbed, and it is assumed that a wing experiences the same lift as in an undisturbed air stream if it cuts the streamlines of the flow disturbed by the other wing in the same manner as a monoplane wing cuts the straight streamlines of the undisturbed flow. As is easily seen, the wing profile must in general be slightly turned and its curvature slightly altered, as is shown in figures 38 and 39. By the rotation of the wing the direction of the resultant air force acting on it is turned through an equal angle. If the magnitude of the velocity as well as its direction is also changed, this must be expressed by a corresponding change in the resultant air force.

As an illustration we will treat briefly the case of a biplane without stagger. The most important component of the disturbance velocity w is again the vertical one; in the plane of the mean lift lines of the biplane it is affected only by the pair of vortices running off the wings, since the transverse vortex of one wing causes only an increase (or decrease) of the velocity of flow at the other wing. We are concerned here only with the calculation of that downward disturbance velocity due to the vortices from the wing not under investigation, since the other vortex system is present with the monoplane and its influence has already been taken into account in the experiments on a monoplane.

The total velocity due to a portion of a vortex proceeding to infinity in one direction, in the plane perpendicular to the vortex at its end, is, as may be deduced easily from the formula in section 8, exactly half of the corresponding velocity in the neighborhood of a rectilinear vortex filament extending to infinity in both directions. This can also be easily seen from the fact that two vortex filaments, each extending to infinity in only one direction—but oppositely in the two cases—form, if combined, a single filament extending to infinity in both directions. The total velocity caused at the point P by the vortex A, see figure 40, is $\frac{\Gamma_1}{4\pi r}$, where $r=\sqrt{x^2+h^2}$; its vertical component is

$$w_A=\frac{\Gamma_1}{4\pi r}\cdot\frac{x}{r}.$$

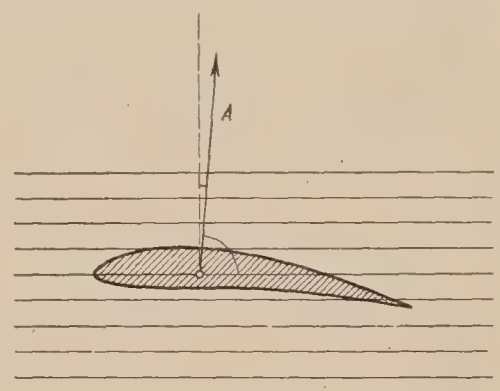

FIG. 38.

The vertical component due to the vortex B is

$$w_B=\frac{\Gamma_1}{4\pi r'}\cdot\frac{l_1-x}{r'}$$

where $r'=\sqrt{(l_2-x)^2+h^2}$.

Therefore the vertical component due to both vortices is

$$w=\frac{\Gamma}{4\pi}\left(\frac{x}{r^2}+\frac{l_1-x}{r'^2}\right) \qquad (31)$$

If we assume that the lift is uniformly distributed over the effective span l_2, which again we shall take as somewhat less than the actual span, then, since every element of the wing must be turned through the angle φ according to the formula $\tan\varphi=\frac{w}{V}$, the direction of the air force must be turned also, which means a negligible change in the lift, but an increase in the drag of this wing which must be taken into account.

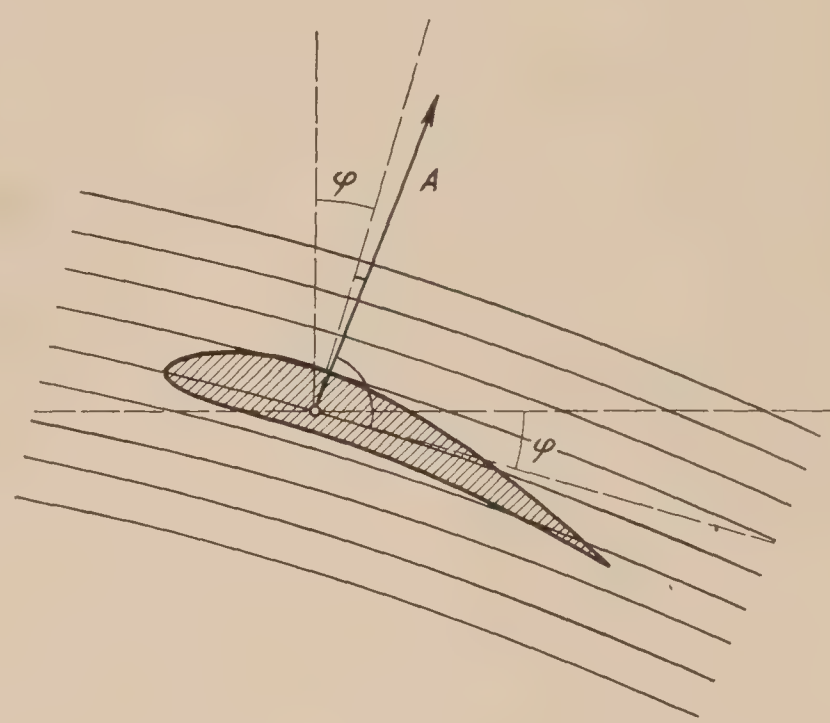

FIG. 39.

Influence of one wing of a biplane upon the other; rotation of wing profile, alteration of its curvature.

It is essential then in this calculation that we pass from a condition for a monoplane to one in which the wing when part of a biplane has the same lift as when considered as a monoplane. The angle of attack for which this condition will arise can be estimated afterwards from the average of the angles φ.

19. The contribution of vortex A to the increase of the drag of the upper wing in figure 40 is evidently

$$W'=\int_0^{l_2}\frac{w}{V}\frac{A_2}{l_2}dx=\frac{A_2}{l_2}\frac{\Gamma}{4\pi V}\int_0^{l_2}\frac{xdx}{r^2}=\frac{A_2}{l_2}\frac{\Gamma}{4\pi V}\log\frac{r_2}{r_1}$$

The contribution of vortex B is, by symmetry, the same. In accordance with equation (20), we can put $\Gamma=\frac{A_1}{l_1\rho V}$ and thus obtain for the increase of the drag of the upper wing

$$W_{12}=\frac{A_1A_2}{l_1\,l_2}\,\frac{\log r_2/r_1}{2\pi\rho V^2} \tag{32}$$

By the symbol W_{12} is meant that it is the drag produced by wing 1 upon wing 2. One can convince himself easily that the drag W_{21} which wing 2 produces upon wing 1 has the same magnitude. Therefore the total increase of drag due to the fact that two monoplanes which produce the lifts A_1 and A_2 are combined to form a biplane, the two lifts remaining unchanged (the angles of incidence of course being changed), is

$$W_{12}+W_{21}=2\,W_{12}=\frac{A_1A_2}{2\pi l_1\cdot\,l_2q}\log\frac{r_2}{r_1} \tag{33}$$

in which, as always $q=\frac{1}{2}\rho V^2$.)[8]

Upon the change in the magnitude of the velocity, which in accordance with the approximation used depends only upon the disturbance velocity v in the direction of flight, only the transverse vortex of the other wing has an influence. For any point this influence, according to our formula, is given by

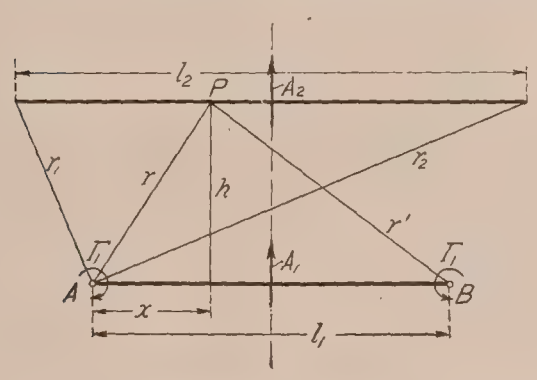

FIG. 40.—Velocity at a point P due to the tip vortices.

$$v=\frac{\Gamma}{4\pi h}\left(\frac{x}{r}+\frac{l-x}{r'}\right) \tag{34}$$

in which r and r^1 have the same meaning as before. The upper wing experiences due to the lower an increase in velocity, the lower one experiences due to the upper a decrease in velocity, to which correspond, respectively, an increase or a decrease in lift as shown by the usual formulæ. If we wish to keep the lifts unchanged, as required in the treatment given above, it is necessary to change the angles of attack correspondingly.

The effective change in the curvature[9] of the wing profile will, for simplicity's sake, be discussed here only for the medial plane of the biplane, i. e., for $x=\frac{l_1}{2}$. It is obtained in the simplest manner by differentiating the angle of inclination of the air current disturbed by the other wing, which is, remembering that $\tan\varphi=\frac{w}{V}$,

$$\frac{1}{R}=\frac{d}{dy}\left(\tan\beta\right)=\frac{1}{V}\frac{dw}{dy} \tag{35}$$

Outside of the vertical plane, owing to the disturbing wing, three vortices contribute to the magnitude of w. A side vortex contributes, at a point at the height h and the distance y in front of the transverse plane, a velocity v' perpendicular to r'' of the amount $\frac{\Gamma_1}{4\pi r}\left(1-\frac{y}{r''}\right)$, and therefore its share of w is

$$w_1=\frac{\Gamma_1x}{4\pi r^2}\left(1-\frac{y}{r''}\right)=\frac{\Gamma_1l_1}{8\pi r^2}\left(1-\frac{y}{r''}\right)$$

The transverse vortex contributes

$$w_2=-\frac{\Gamma_1}{4\pi r}\cdot\frac{l_1}{r''}\frac{y}{r'''},$$

in which the meaning of r'' and r''' may be seen from figure 41. The total w is, accordingly,

$$w=2w_1+w_2=\frac{\Gamma l_1}{4\pi}\left(\frac{1}{r^2}-\frac{y}{r^2r''}-\frac{y}{r''r'''^2}\right)$$

[8] The mutual action of two wings placed side by side can also be calculated from the considerations stated above, and results in a decrease of the drag. This decrease is of a similar kind to that which arises in the theory of a monoplane by an increase in the aspect ratio.

[9] By change in curvature of the wing is meant that if the flow were to be kept straight and the curvature changed, the forces on the wing would be changed exactly as they are on the actual wing owing to the change in the flow.—Tr.

The differential of this with reference to y, for the value of $y=o$, is, since then $r''=r$ and $r'''=h$

$$\left(\frac{dw}{dy_0}\right)=-\frac{\Gamma l_1}{4\pi r}\left(\frac{1}{r^2}+\frac{1}{h^2}\right).$$

the curvature sought is, then, according to equation (35),

$$\frac{1}{R}=\frac{\Gamma}{4\pi V}\frac{l_1}{r}\left(\frac{1}{r^2}+\frac{1}{h^2}\right) \tag{36}$$

Calculations of the preceding nature were made in 1912 by my assistant, A. Betz, so as to compare experiments with monoplanes and biplanes and to study the influence of different angles of attack and different degrees of stagger of the two wings of a biplane upon each other. The influence upon the drag was not known to us at that time, and the calculation was carried out so as to obtain the changes in the lift due to w, to v and to the curvature of the streamlines. In this connection the change of the lift of a monoplane when flying near the earth's surface was also deduced, by calculating the influence of the "mirrored wing" exactly as was that of the other wing of a biplane. All that was necessary was to change some algebraic signs, because the mirrored wing had negative lift. The theory of these calculations was given by Betz in the Z. F. M., 1914, page 253.

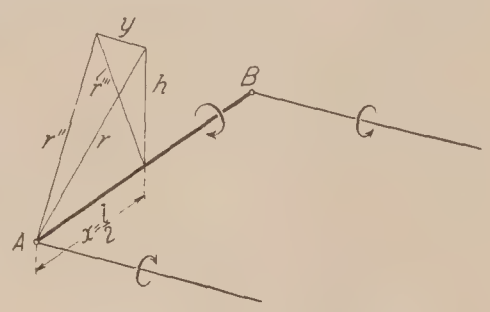

FIG. 41.—Curvature of wing-profile at its middle point due to velocities caused by transverse and tip vortices.

The results of the theory of Betz, from a more modern standpoint, such as adopted here, were given in the Technische Berichte, volume 1, page 103 et seq. There one can find the discussion requisite for the treatment of the most general case of a biplane having different spans of the two wings and with any stagger. In the case of great stagger it appears, for example, that the forward wing is in an ascending air current caused by the rear wing; the latter is in an intensified descending current due to the forward wing and the vortices flowing off from it. Corresponding to this, if the angle of attack is unchanged, the lift of the forward wing is increased, and that of the rear one weakened; at the same time the ratio $\frac{\text{Drag}}{\text{Lift}}$ experiences a decrease for the forward wing and a marked increase for the rear one.

For a wing in the neighborhood of the ground, owing to the influence of v there is a decrease of lift, and conversely there is an increase of lift due to the influence of w, provided the angle of attack is kept constant, but as the result an evident decrease in the ratio $\frac{\text{Drag}}{\text{Lift}}$. Owing to this last it is seen why in the early days of aeronautics many machines could fly only near the ground and could not rise far from it. Their low-powered engines were strong enough to overcome the diminished drag near the ground but not that in free air.

D. THEORY OF THE MONOPLANE.

20. If we extend the principles, which up to this point have been applied to the influence of one wing upon another, to the effect upon a single wing of its own vortices, it can be said in advance that one would expect to find in that case effects similar to those shown in the influence of one wing of a biplane upon the other, i. e., the existence of lift presupposes a descending flow in the neighborhood of the wing, owing to which the angle of attack is made greater and the drag is increased, both the more so the closer to the middle the vortices flowing off at the ends are, i. e., the smaller the aspect ratio is. One might propose to apply the theory previously given for biplanes by making in the formulas of this theory the gap equal to zero. Apart from the fact that the formulas developed do not hold for the immediate neighborhood of the vortex-producing wing, but must be replaced by more accurate ones, this certainly is not the proper path to follow, for, in the earlier treatment, we have taken the undisturbed monoplane as the object with which other cases are to be compared and have asked what drag, what change in angle of attack, etc., are caused by adding a second wing to this monoplane. To proceed

according to the same method, we must seek for the theory of monoplanes another suitable object of comparison. As such, the infinitely long wing will serve. Where the discussion previously was about change of angle of attack, increase of drag, etc., we intend now to refer these to the infinitely long wing as a starting point. Since in the theoretical nonviscous flow the infinitely long wing experiences no drag, the total drag of such a wing in such a fluid must be due to vortices amenable to our calculations, as the following treatment will show. In a viscous fluid drag will arise for both wings, infinitely long or not, which for those angles of attack for which the profile is said to be "good" is, according to the results of experiment, of the order of magnitude of the frictional resistance of a plane surface.

The carrying out of this problem is accompanied with greater difficulties than the calculation for a biplane as given. In order to obtain the necessary assistance for the solution of the problem, we shall first be obliged to improve the accuracy of our picture of the vortex system.

The density of the lift (lift per unit length) is not constant over the whole span, but in general falls off gradually from a maximum at the middle nearly to zero at the ends. In accordance with what has been proved, there corresponds to this a circulation decreasing from within outward. Therefore, according to the theorem that by the displacement of the closed curve the circulation Γ can change only if a corresponding quantity of vortex filaments are cut, we must assume that vortex filaments proceed off from the trailing edge wherever Γ changes. For a portion of this edge of length dx the vortex strength is therefore to be written $\frac{d\Gamma}{dx}\,dx$, and hence per unit length of the edge is $\frac{d\Gamma}{dx}$. These vortex filaments flowing off, closely side by side, form, taken as a whole, a surface-like figure, which we shall call a "vortex ribbon."

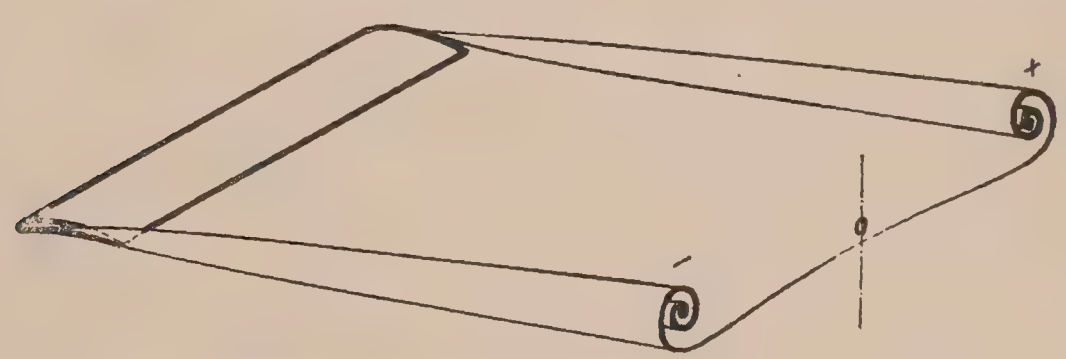

FIG. 42.—Change in shape of vortex ribbons at great distances behind the wing.

For an understanding of this vortex ribbon we can also approach the subject from an entirely different side. Let us consider the flow in the immediate neighborhood of the surface of the wing. Since the excess in pressure below the wing and the depression above it must vanish as one goes beyond the side edges of the wing in any manner, there must be a fall in pressure near these edges, which is directed outward on the lower side of the wing and inward on the upper. The oncoming flow, under the action of this pressure drop, while it passes along the wing, will receive on the lower side an additional component outward, on the upper side, one inward, which does not vanish later. If we assume that at the trailing edge the flow is completely closed again, as is the case in nonviscous flow, we will therefore have a difference in direction between the upper and lower flow; the upper one has a relative velocity inward with reference to the lower one, and this is perpendicular to the mean velocity, since on account of the Bernouilli equation in the absence of a pressure difference between the two layers the numerical values of their velocities must be the same. This relative velocity of the two flows is exactly the result of the surface distribution of vortices mentioned above (as the vortex theory proves, a surface distribution of vortices always means a discontinuity of velocity between the regions lying on the two sides of the surface). The relative velocity is the greater, the greater the sidewise pressure drop, i. e., the greater the sidewise change in lift. The picture thus obtained agrees in all respects with the former one.

21. The strengths of our vortex ribbon remain unchanged during the whole flight, yet the separate parts of the ribbon influence each other, and there takes place, somewhat as is shown in figure 42, a gradual rolling up of the ribbon, as a closer examination proves. An exact theoretical investigation of this phenomenon is not possible at this time; it can only be said that the two halves of the vortex ribbon become concentrated more and more, and that finally at great distances from the wing there remain a pair of vortices with rather weak cores.

For the practical problem, which chiefly concerns us, namely, to study the reaction of the vortices upon the wing, it is not necessary to know these changes going on at a great distance, for the parts of the vortex system nearest the wing will exercise the greatest influence. We shall therefore not consider the gradual transformation of the vortex ribbon, and, in order to make the matter quite simple, we shall make the calculation as if all the vortex filaments were running off behind in straight lines opposite to the direction of flight. It will be seen that, with this assumption, the calculations may be carried out and that they furnish a theory of the monoplane which is very useful and capable of giving assistance in various ways.

If we wish to establish the method referred to with greater mathematical rigor, we can proceed as follows: Since the complete problem is to be developed taking into account all circumstances, we shall limit ourselves to the case of a very small lift and shall systematically carry through all calculations in such a manner that only the lowest power of the lift is retained, all higher powers being neglected. The motion of the vortex ribbon itself is proportional to the total circulation, therefore also proportional to the lift; it is therefore small if the lift is small. If the velocities caused by the vortex ribbon are calculated, first for the ribbon in its actual form, then for the ribbon simplified in the manner mentioned, the difference for the two distributions will be small compared with the values of the velocity, therefore small of the second order, i. e., small as the square of the circulation. We shall therefore neglect the difference. Considerations of this kind are capable of deciding in every case what actions should be taken into account and what ones may be neglected. By our simplifications we have therefore made the problem linear, as a mathematician says, and by this fact we have made its solution possible. It must be considered a specially fortunate circumstance that, even with the greatest values of the lift that actually occur with the usual aspect ratios, the independent motion of the vortex ribbon is still fairly small, so that, in the sense of this theory, all lifts which are met in practice may still be regarded as small. For surfaces having large chords, as, for instance, a square, this no longer holds. In this case there are, in addition, other reasons which prove that our theory is no longer sufficiently accurate. This will be shown in the next paragraph.

It has already been mentioned that the infinitely long wing will serve as an object of comparison for the theory of the monoplane. We shall formulate this now more exactly by saying: Every separate section of the wing of length dx shall bear the same relation to the modified flow due to the vortex system as does a corresponding element of an infinitely long wing to the rectilinear flow. The additional velocities caused by the vortex system vary from place to place and also vary in the direction of the chord of the wing, so that again we have to do with an influence of curvature. This influence is in practice not very great and will for the sake of simplicity be neglected. This is specially allowable with wings whose chords are small in comparison with their spans, i. e., with those of large aspect ratio. If one wishes to express with mathematical exactness this simplifying assumption, it can be said that the theory of an actual wing of finite chord is not developed, but rather that of a "lifting line." It is clear that a wing of aspect ratio 1:6 may be approximated by a lifting line, specially if one considers that actually the lift is concentrated for the greatest part in a region nearer the leading edge. It is easily seen, however, that a surface in the form of a square can be approximated only poorly by a lifting line.

If we assume a straight lifting line, which lies in a plane perpendicular to the direction of flight, the flow due to the vortices, which according to the Biot-Sarvart law, is caused by its own elements, will not produce any velocities at the lifting line itself except the circulation flow around it, which would also be present for an infinitely long lifting line having the same circulation as at the point observed. All disturbance velocities at a point of the lifting line, which are to be looked upon as deviations from the infinitely long lifting line, are due therefore to the vortices which run off and hence can be calculated easily by an integration.

A qualitative consideration of the distribution to be expected for the disturbance velocities along our lifting line shows at once that—just as was the case for a biplane—the chief thing is the production of a descending current of air by the vortices. If we wish to retain the lift of the same intensity as with the infinite wing, the angle of attack must be increased, since the descend-

ing air stream added to the wind due to flight causes a velocity obliquely downward. In addition, the air force, as before, must be turned through the same angle, so that a drag results. The rotation will be the greater, the greater the lift and the closer to the middle of the wing the main production of vortices is. The drag must therefore increase both with increasing lift and with decreasing span.

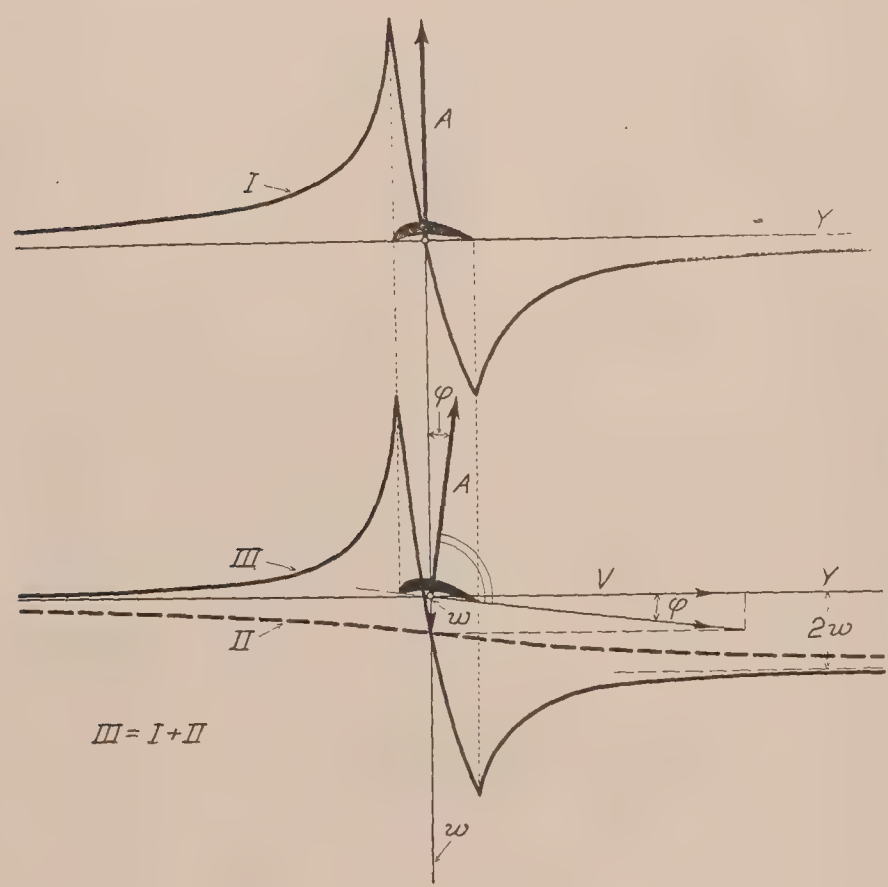

FIG. 43.—Wing having finite, but small, chord. Distribution of vertical velocity component along a line parallel to direction of flight.
I. Infinitely long wing.
II. The downward velocity produced by vortices flowing off.
III. Finite wing, sum of I and II.

A picture of what occurs with a wing of finite but small chord is given in figure 43. There the change is shown of the vertical velocity component along a straight line parallel to the direction of flight through the middle of the wing; in the upper part of the diagram, for the infinitely long wing, in the lower part, for the finite wing. We see from Curve I the rising flow in front of the wing, its transformation into a descending one at the wing itself and the gradual damping of the descending component due to the upward pressure drop behind the wing. (See sec. 15.) The corresponding curve for the finite wing is Curve III. It is derived from I by adding to the latter the descending velocity II. We recognize the rotation of the profile as well as that of the lifting force, which was originally perpendicular, through the angle φ where $\tan \varphi = \frac{w}{V}$ and w is the velocity downward at the location of the center of pressure (i. e., at the lifting line). If we follow the method of Lanchester, as described in section 15, the downward velocity w can also be looked upon as a diminution of the ascending flow at the leading edge of the wind due to the absence of the sidewise prolongation of the wing, i. e., to the deviation from an infinitely long wing which was the basis of the treatment in section 15. Discussions very similar to this are given in Lanchester, Volume I, section 117.

It may be seen from the figure that at great distances behind the wing the descending velocity is $2w$, which agrees with the relation already mentioned that the velocities due to a straight vortex filament extending to infinity in both directions are twice those due to a filament extending to infinity in one direction only, for points in the plane perpendicular to this vortex passing through its end point.

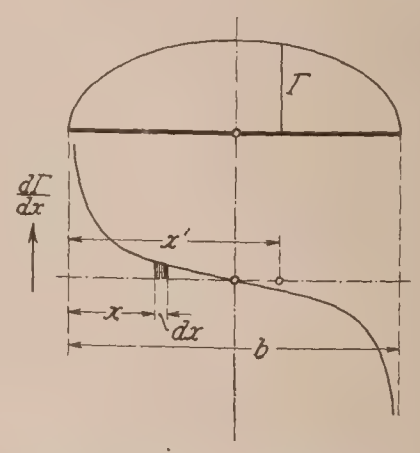

FIG. 44.—Velocity at a point x' due to vortex leaving wing at point x.

22. The mathematical processes involved in carrying out the theory outlined above become the most simplified if one considers as known the law, according to which the lift is distributed over the wing. We shall call this the "first problem." The calculation is made as follows: The distribution of lift is the circulation expressed as a function of the abscissa x. The strength of the vortex filament leaving an infinitely small section dx is then $\frac{d\Gamma}{dx} \cdot dx$. This produces at a point x', according to what has been already explained, a vertical velocity downward or upward of the amount

$$dw = \frac{1}{4\pi} \cdot \frac{d\Gamma}{dx} \cdot \frac{dx}{x' - x}$$

In this $x'-x$ takes the place of r in section 8. If the circulation falls to zero at the ends of the wing, as is actually the case, then all the vortices leaving the wing are of this kind. The whole added velocity at the position x', assuming that the function $\Gamma(x)$ is everywhere continuous, is

$$w=\frac{1}{4\pi}\int_0^b \frac{d\Gamma}{dx}\cdot\frac{dx}{x'-x} \tag{37}$$

We must take the so-called "chief value" of the integral, which is indeterminate at the point $x=x'$, i. e., the limiting value

$$\lim_{\epsilon\to 0}\left(\int_0^{x'-\epsilon}+\int_{x'+\epsilon}^b\right)$$

must be formed, as a closer examination shows. We can do this by calculating, instead of the value of the velocity at the lifting line, which is determined by the preponderating influence of the nearest elements, the value of w for a point a little above or below the lifting line. It is seen that this last is not indeterminate and that by passing to a zero distance from the lifting line it reaches the above limit. Concerning this excursus, important in itself, the preceding brief remarks may be sufficient.

After the calculation of the integral of (37), the downward velocity is known as a function of the abscissa x' (which we later shall call x). We then also know the inclination of the resultant air flow, $\tan\varphi=\frac{w}{V}$; the lift $dA=\rho\Gamma V\,dx'$, acting on the section dx', therefore contributes to the value of the drag

$$dW=\tan\varphi\cdot dA=\rho\Gamma(x')\cdot w(x')\,dx'$$

since it is inclined backward by the small angle φ. The total drag is therefore

$$W=\rho\int_0^b \Gamma(x')\cdot w(x')\cdot dx'=\frac{\rho}{4\pi}\int_0^b\int_0^b\frac{\Gamma(x')\frac{d\Gamma}{dx}dx\,dx'}{x'-x} \tag{38}$$

For a long time it was difficult to find suitable functions to express the distribution of lift, from which a plausible distribution of w would be obtained by equation (37). After various attempts it was found that a distribution of lift over the span according to a half ellipse gave the desired solution. According to this, if the origin of coordinates is taken at the center of the wing,

$$\Gamma=\Gamma_0\sqrt{1-\left(\frac{x}{b/2}\right)^2}\text{, hence }\frac{d\Gamma}{dx}=\frac{-\Gamma_0 x}{\frac{b}{2}\sqrt{\left(\frac{b}{2}\right)^2-x^2}}$$

The "chief value" of the integral

$$\int^{1+}\frac{t\,dt}{(t'-t)\sqrt{1-t^2}}\text{ is equal to }\pi$$

and therefore the integral of equation (37) is equal to $\frac{\Gamma_0\pi}{b/2}$, and thus is independent of x' and constant over the whole span. Hence

$$w=\frac{\Gamma_0}{2b}$$

The value of Γ_0 is obtained from

$$A=\rho V\int_{-b/2}^{+b/2}\Gamma dx=\rho V\Gamma_0\int_{-b/2}^{+b/2}\sqrt{1-\frac{x}{b/2}}\cdot dx=\rho V\,\Gamma_0\frac{\pi}{4}\,b,$$

giving

$$\Gamma_0=\frac{4A}{\pi\rho Vb}.$$

Hence

$$w=\frac{2A}{\pi\rho Vb^2} \tag{39}$$

Since w is constant there is no need of calculating the drag by an integral, for it is simply

$$W=\frac{w}{V}A=\frac{2A^2}{\pi\rho V^2b^2}=\frac{A^2}{\pi qb^2} \tag{40}$$

The calculation can also be performed for distributions of circulation given by the following general formula:

$$\Gamma = \sqrt{1-\xi^2}\,(\Gamma_0 + \Gamma_2 \xi^2 + \Gamma_4 \xi^4 + \ldots) \tag{41}$$

in which $\xi = \frac{x}{b/2}$.

According to the calculations of A. Betz

$$w = \frac{1}{2b} \Sigma n \left\{ \Gamma_{2n} \sum_{m=0}^{m=n} \xi^{2m} \left[(2n+1)\, p_{n-m} - 2n p_{n-m-1} \right] \right\} \tag{42}$$

and

$$W = \frac{\pi \rho}{4} \Sigma_i\, \Sigma_k \left\{ \Gamma_{2i}\, \Gamma_{2k} \sum_{m=0}^{m=k} q_{i+m} \left[(2k+1)\, p_{k-m} - 2k p_{k-m-1} \right] \right\} \tag{43}$$

in which the numbers p and q have the meaning

$$p_n = \frac{1.3 \ldots (2n-1)}{2.4 \ldots 2n}; \quad q_n = \frac{p_n}{2_n + 2}, \quad p_0 = 1, \; p_{-1} = 0$$

The elliptical distribution of lift, apart from its simplicity, has obtained a special meaning from the fact that the drag as calculated from equation (40) proved to be the smallest drag that is imaginable for a monoplane having given values of the total lift, the span and the velocity. The proof of this will be given later.

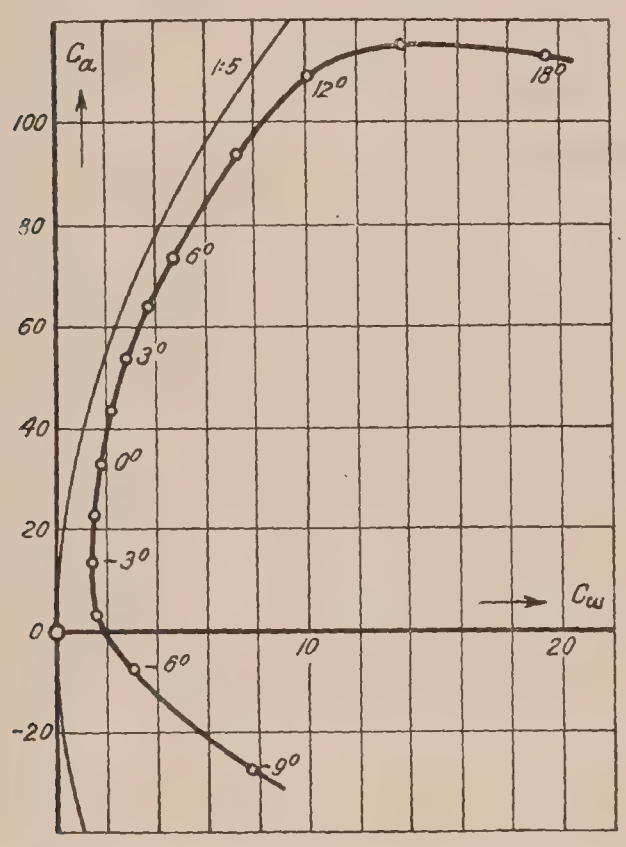

FIG. 45.—Polar diagram showing theoretical drag and observed drag.

It was desirable to compare this theoretical minimum drag with the drags actually obtained. As far back as 1913 this was done, but, on account of the poor quality of the profiles then investigated, all that was done was to establish that the actual drag was greater than the theoretical. Later (1915) it was shown, upon the investigation of good profiles, that the theoretical drag corresponds very closely to the relation giving the change of the observed drag as a function of the lift. If we plot in the usual manner the theoretical drag, as given in formula (40) as a function of the corresponding lift, we obtain a parabola, which runs parallel with the measured "polar curve" through the entire region for which the profile is good. (See fig. 45.)

This process was repeated for wings of different aspect ratios, and it was proved that for one and the same profile the difference between the measured and the theoretical drags for one and the same value of the lift coefficient had almost identically the same value in all cases.

This part of the drag depends, however, upon the shape of the profile, and we have therefore called it "profile drag." The part of the drag obtained from theory is called "edge drag," since it depends upon the phenomena at the edges of the wings. More justifiably the expression "induced drag" is used, since in fact the phenomena with the wings are to a high degree analogous to the induction phenomena observed with electric conductors.

Owing to this fact that the profile drag is independent of the aspect ratio, it became possible from a knowledge of the actual drag for one aspect ratio to calculate it for another. To do this, we pass from the formula (40) for the drag to the dimensionless lift and drag coefficients, by letting $\frac{A}{Fq} = c_a$ and $\frac{W}{Fq} = c_w$: we obtain then for the coefficient of the induced drag the relation

$$c_{wi} = \frac{c_a^2 F}{\pi b^2} \tag{44}$$

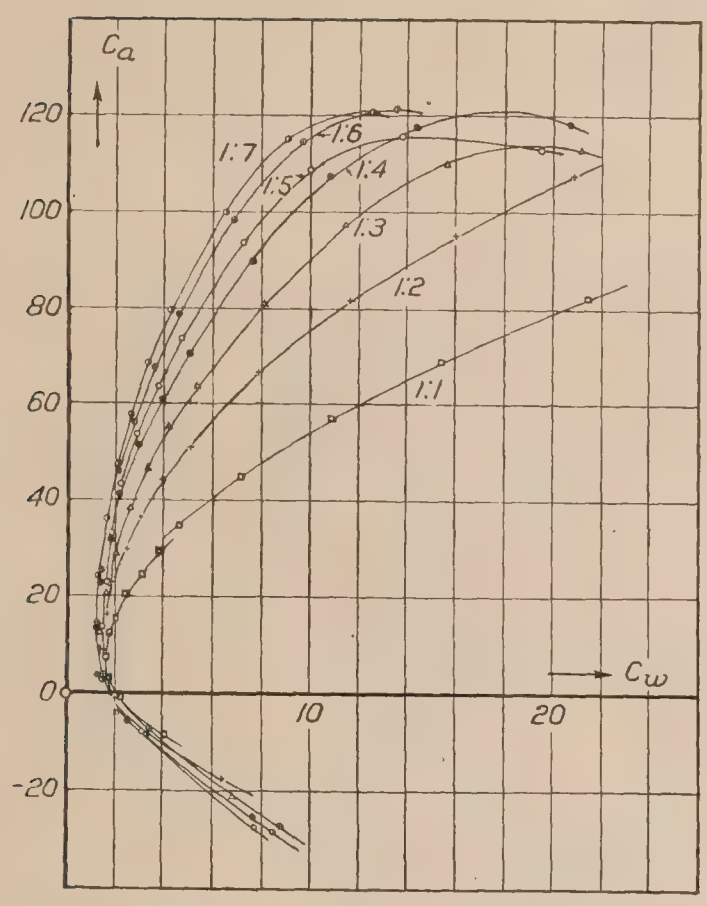

FIG. 46.—Polar diagrams for seven wings, aspect ratios 1:7, 1:6, etc., 1:1.

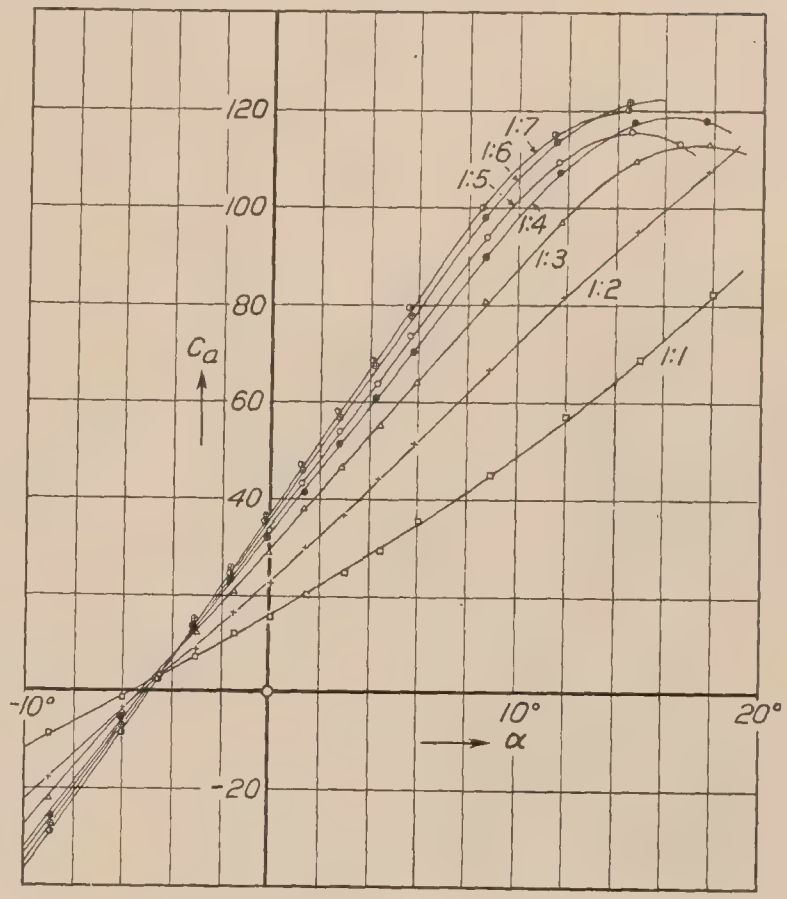

FIG. 47.—Lift coefficients plotted as function of angle of attack for aspect ratios 1:7 to 1:1.

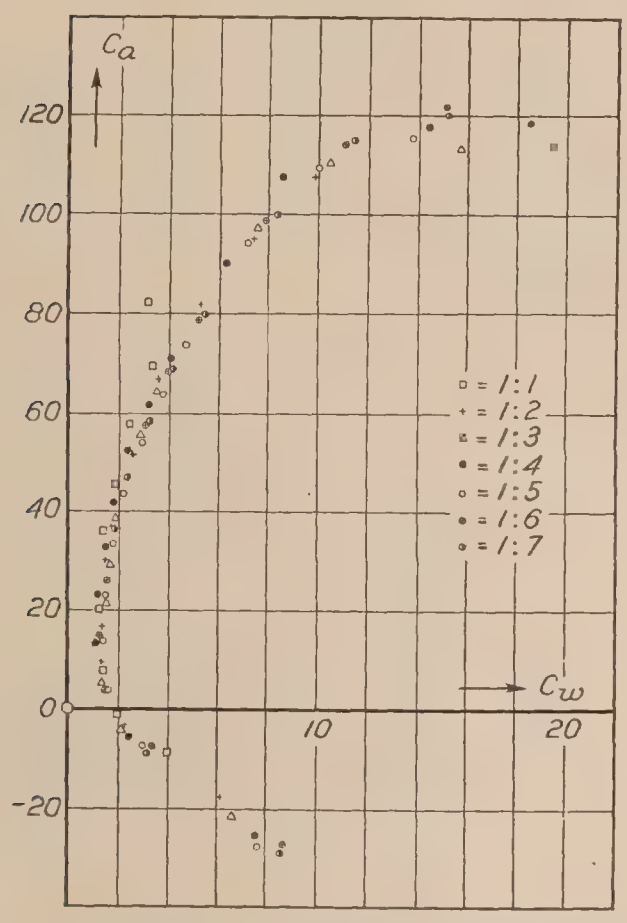

FIG. 48.—Polar diagrams reduced from observations on aspect ratio 1:5.

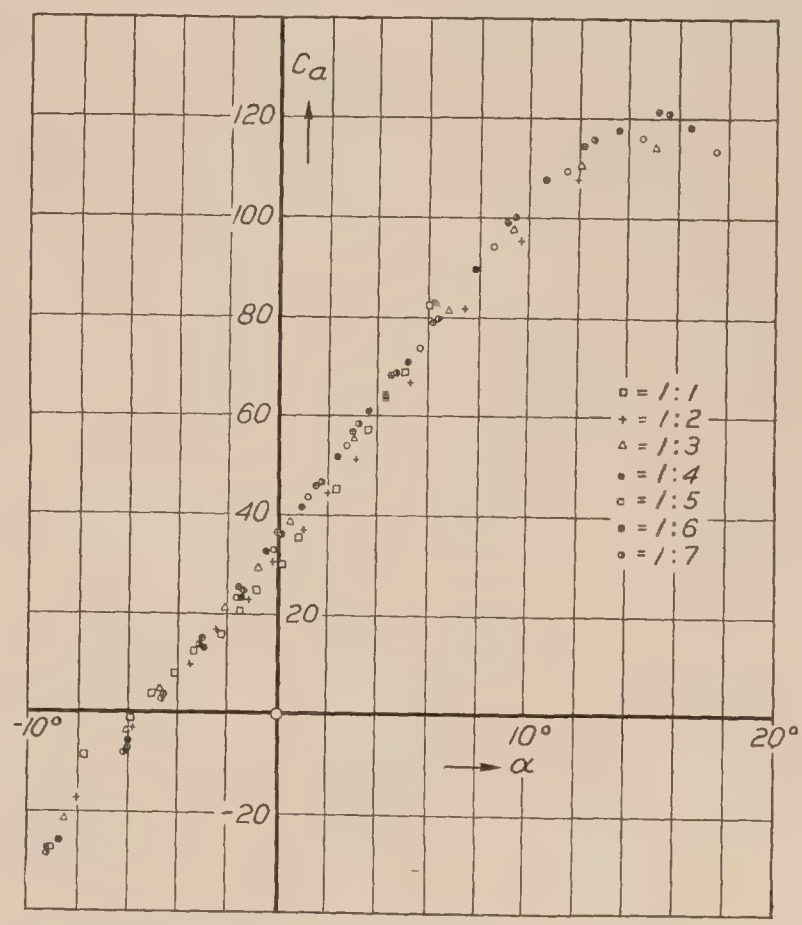

FIG. 49.—Lift coefficients as function of angle of attack, reduced for aspect ratio 1:5.

The profile drag may then be written $c_{wo}=c_w-c_{wi}$. If this drag coefficient depends only upon the lift coefficient, then it would be evident, since it would be the difference between the measured and the theoretical drag coefficients, that, for the polar curves of two different wings having $c_{a1}=c_{a2}=c_a$,

$$c_{w1}-\frac{c_a^2F_1}{\pi b_1^2}=c_{w2}-\frac{c_a^2F_2}{\pi b_2^2},$$

and therefore

$$c_{w2}=c_{w1}+\frac{c_a^2}{\pi}\left(\frac{F_2}{b_2^2}-\frac{F_1}{b_1^2}\right) \tag{45}$$

In a similar manner a calculation for the angle of attack may be made if we presuppose an elliptical distribution of lift. According to our assumption there is a close connection between the lift of the separate elements of the wing and the "effective" angle of attack, which is the same as the angle of attack of an infinitely long wing. This effective angle of attack, according to our earlier considerations, is the angle of attack of the chord with reference to the resultant air current. It is therefore $\alpha'=\alpha-\phi$. If we substitute $\tan\phi=\frac{w}{V}$ for ϕ, and introduce in equation (39) again the lift coefficient, instead of using the lift, we obtain for the comparison of two wings, expressing the fact that the effective angle of attack is to be the same for two equal lift coefficients, the relation

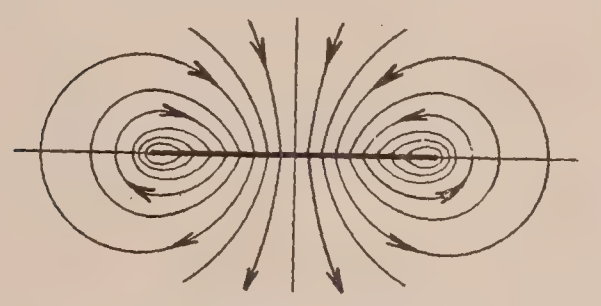

FIG. 49a.—Uniplanar flow in case of elliptic distribution of lift on a lifting line.

$$\alpha_1-\frac{c_a}{\pi}\frac{F_1}{b_1^2}=\alpha_2-\frac{c_a}{\pi}\frac{F_2}{b_2^2},$$

which leads to the transformation formula

$$\alpha_2=\alpha_1+\frac{c_a}{\pi}\left(\frac{F_2}{b_2^2}-\frac{F_1}{b_1^2}\right) \tag{46}$$

These formulas have been found to hold for distributions of lift which do not deviate too much from elliptical ones, although strictly speaking they apply only to the latter. The fact that the type of distribution does not have a marked effect is based upon the consideration that both in the calculation of drag and in that of the mean effective angles of attack we are concerned with average results. For the calculation of the drag one can also introduce the thought that no quantity varies much in the neighborhood of its minimum. Closer investigation of the square cornered wing has shown that, if the aspect ratio is not too small, the lift distribution does not deviate greatly from the elliptic type, and that the theoretical drag for usual aspect ratios at the most is 5 per cent greater than for the elliptic distribution. As an example of these formulas we shall take four figures from the book published by the Göttingen Institute (Ergebnisse der Aerodynamischen Versuchsanstalt, 1, Lieferung, 1921). The first and second figures show the polar curves, and the connection between lift coefficient and angle of incidence for seven wings of aspect ratio[10] 1:7 to 1:1. The last two figures give the results of calculating these experimental quantities from the results for the wing having the aspect ratio 1:5. It is seen that, apart from the aspect ratios 1:1 and 1:2 practically no deviations are present. The fact that the square can not be correctly deduced from the aspect ratio 1:5 need not excite surprise, since the theory was developed from the concept of the lifting line, and a square or a wing of aspect ratio 1:2 can scarcely be properly approximated by a lifting line. On the other hand it is a matter of surprise that an aspect ratio of 1:3 can be sufficiently approximated by the imaginary construction of a lifting line. The deviations in the case of the square are moreover in the direction one would expect from a lift distribution expanded over the chord. A quantitative theory is not available in any case at the present time.

23. If the lift distribution is not given, but, for example, the downward velocity, then the method of treatment followed hitherto may be used, by developing the downward velocity in a power series and determining the constants of the series given above for the lift from the constants

[10] The American practice is to define aspect ratio as the ratio of span to chord, which would involve taking the reciprocals of the ratios given in the text. Tr.

of this power series, by the solution of linear equations. By this the lift distribution and everything else are known.

Another method for the solution of this "second problem" will be obtained by the following consideration: The velocities at a distance behind the wing, on account of the connection mentioned so often between a vortex filament extending to infinity in one direction only and one extending to infinity in both directions, are twice as great as those in the cross section of the lifting line, if we do not take into account the change in shape of the vortex ribbon. We therefore have here, neglecting this change in shape, an illustration of a two-dimensional fluid flow (uniplanar flow), for which the vertical velocity components at the point where the wing is reached are specified. For the simple case that the vertical velocity w is constant, as was found to be true for the elliptical lift distribution, the shape of the flow that arises has been known for a long time. It is given in figure 49a. It is the same as that already considered, in another connection, in section 15. The picture of the streamlines show clearly the velocity discontinuity between the upper and lower sides of the vortex ribbon, indicated by the nick in the streamlines, and also the vortical motion around the two extreme points of the vortex ribbon, corresponding to the ends of the wing.

Any problems of this kind can therefore be solved by means of the methods provided by the potential theory for the corresponding problem of two-dimensional fluid flow. We can not go into these matters more closely at this time; by a later opportunity some special relations will be discussed, however.

A "third problem" consists in determining the lift distribution for a definite wing having a given shape and given angle of attack. This problem, as may be imagined, was the first we proposed; its solution has taken the longest, since it leads to an integral which is awkward to handle. Dr. Betz in 1919 succeeded after very great efforts in solving it for the case of a square-cornered wing having everywhere the same profile and the same angle of attack. The way the solution was obtained may be indicated briefly here. We start, as before, from the relation

$$\alpha = \alpha' + \phi \cong \alpha' + \frac{w}{V}$$

By equation (37) w is expressed in terms of the circulation. The effective angle of attack α' can be expressed in terms of Γ, since, according to the assumptions made before the lift, distribution, which is proportional to Γ, depends directly upon α'. The relation between α' and Γ can be assumed to be given sufficiently exactly for our purposes by a linear expression

$$\Gamma = Vt\,(c_1\,\alpha' + c_2) \tag{47}$$

in which t is the length of the chord (measured in the direction of flight). By the introduction of the factor Vt, c_1 and c_2 are made pure numbers. The numerical value of c_1, which is the more important, can be expressed, if c_{a_∞} is the lift coefficient for the infinitely long wing at the angle of attack α', by the relation

$$c_1 = \frac{1}{2}\,\frac{dc_{a_\infty}}{d\alpha'}$$

In fact

$$c_{a_\infty} = \frac{A}{Fq} = \frac{\rho\,\Gamma\,Vl}{lt.\,\frac{1}{2}\,\rho\,V^2} = \frac{2\Gamma}{Vt} = 2\,(c_1\alpha' + c_2)$$

For a flat-plate theory proves that $c_1 = \pi$, for curved wings it has a slightly greater value. If, according to what has gone before, we express α' by Γ and w by $\frac{d\Gamma}{dx}$ and write

$$\frac{d\Gamma}{dx} = f(x) \text{ and therefore } \Gamma = \int_0^x f(x)\,dx$$

we obtain after a simple calculation the integral equation

$$\int_0^x f(x)\,dx + \frac{c_1 t}{4\pi}\int_0^b \frac{f(x')\,dx'}{x - x'} = Vt\,(c_1\alpha + c_2) = \text{const.} \tag{48}$$

A solution of this equation can be obtained by expanding Γ as in equation (41) and developing then all the expressions in power series of $\xi=\frac{x}{b/2}$. For every power of ξ there is then a linear equation between the quantities Γ_0, Γ_2, etc. There is a system of equations, then, with an indefinite number of equations for an infinite number of unknowns, the solution of which in this form is not yet possible. The aspect ratio of the wing appears in these equations as a parameter, and it is clear that the solution for a small aspect ratio, i. e., $\frac{b}{t}$ is easier than for a large one. Dr. Betz proved that a development in powers can be made for the unknowns in terms of a parameter containing the aspect ratio. The calculations which are contained in the dissertation[11] of Dr. Betz (1919) are very complicated and can not be reproduced here; but certain results will be mentioned. The Betz parameter L has the meaning

$$L=\frac{2b}{c_1\,t}=\frac{4b}{t}\,\frac{d\alpha'}{dc_{a\infty}}$$

In the application to surfaces which are investigated in wind tunnels the value $\frac{dc_a}{d\alpha}$ is known, not $\frac{dc_{a\infty}}{d\alpha'}$. For this case theory gives a relation which can be expressed approximately

$$L\cong 3.85\,\frac{b}{t}\,\frac{d\alpha}{dc_a}-1.3.$$

We can thus obtain the value of $\frac{d\alpha'}{dc_{a\infty}}$ from the connection mentioned.

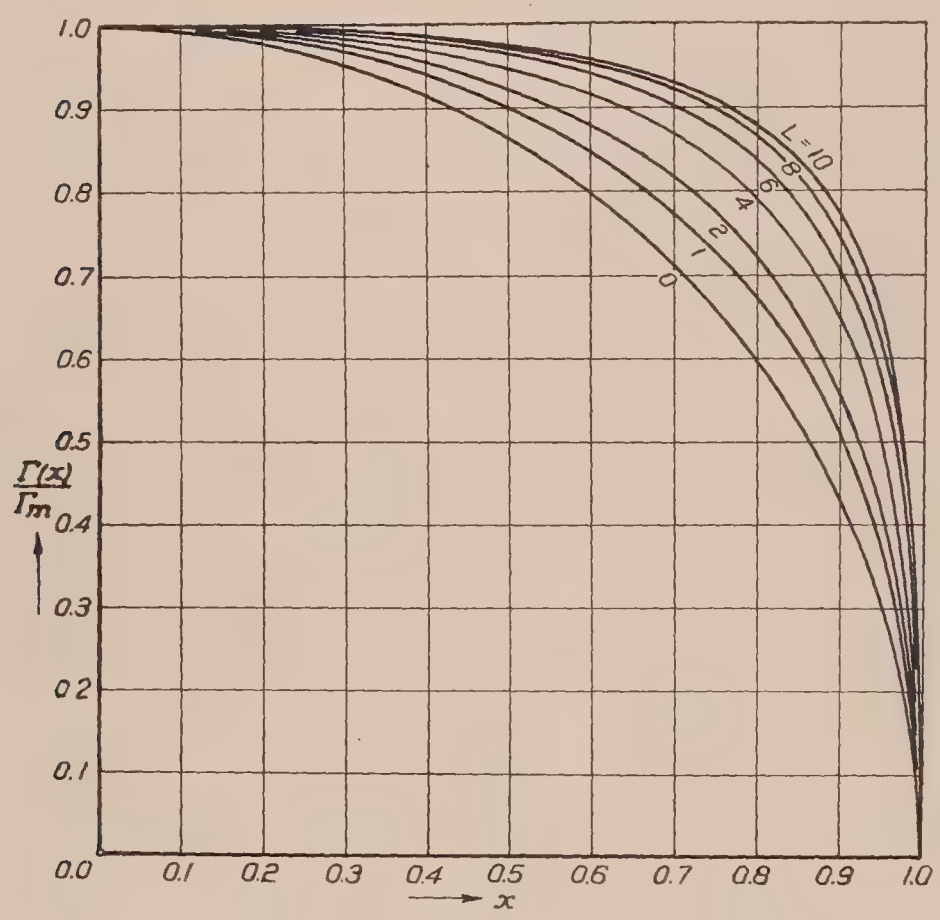

FIG. 50.—Change in distribution of lift, as a function of L, the parameter of Betz.

The distribution of lift density ever the span is elliptical for very small aspect ratios and for greater ratios becomes more and more uniform; for very elongated wings it approaches gradually a rectangular distribution. Figure 50 shows this change in the distribution depending upon L.

The drag of the wing with the rectangular distribution is greater naturally than with the elliptic distribution, since this gives the minimum of drag, yet the differences are not very great; for instance, for $L=4$, $\left(\frac{b}{t}\cong 6\right)$ it is about 5 per cent greater than that of the elliptical distribution. An approximation formula, according to the values obtained by Betz, is

$$W\cong\frac{A^2}{\pi q b^2}\,(0.99+0.015\,L).$$

This is applicable for values of L between 1 and 10.

The distribution of lift, downward velocity, and drag upon a very elongated wing is shown qualitatively in figure 51. It is seen that the downward velocity and the drag gradually accumulate at the ends of the wing. This gives also the correct transition to an infinitely long wing, with which for interior positions the lift is constant, and the downward velocity and the drag are equal to zero, while, as we know, near the ends these last quantities always assume finite values.

[11] Printed in extracts in Heft 2 of the "Berichte u. Abhl. der Wiss. Ges. f. Luftf." Munich, 1920 (R. Oldenbourg).

E. IMPROVED THEORY OF AIRPLANES HAVING MORE THAN ONE WING.

24. The knowledge obtained in the theory of a monoplane can be applied also to multiplanes and furnishes here a series of remarkable theorems. We shall limit ourselves to the theory of the first order, as designated in the theory of monoplanes, therefore we shall neglect the influences of v. Further, we shall not take into account the effect of curvature—i. e., we shall consider the separate wings replaced by "lifting lines." For the sake of simplicity we shall limit ourselves to multiplanes with wings which are straight and parallel to each other. The generalization of the theorems for nonparallel wings, corresponding to the deduction given in "Wing theory II," will then be stated without proof.

Let us first solve the introductory problem of calculating the vertical velocity w produced by a lifting line at a point A which lies off the lifting line. At the beginning let us assume that this point lies in the same "transverse plane" (plane perpendicular to the direction of flight). According to our assumption as to the location of A, the action of the transverse vortex is zero. With reference to the longitudinal vortices it is to be remembered that the velocity $\frac{1}{4\pi}\frac{d\Gamma}{dx}\cdot\frac{dx}{a}$ produced by a longitudinal vortex of strength $\frac{d\Gamma}{dx}dx$ is perpendicular to the line a (see fig. 52), and therefore must be multiplied by $\sin\beta$ to obtain the vertical component. We arrive at the downward velocity, therefore, by integrating over the lifting line, viz:

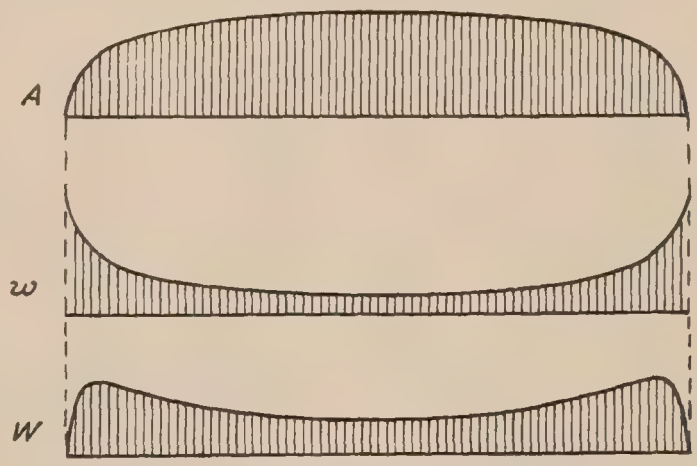

FIG. 51.—Distribution of lift, down-wash and drag for a long wing.

$$w=-\frac{1}{4\pi}\int_0^b\frac{d\Gamma}{dx}\left(\frac{\sin\beta}{a}\right)dx \tag{49}$$

This relation can be brought into another form by a partial integration. Since at both wing ends $\Gamma=o$, we have

$$w=\frac{1}{4\pi}\int_0^b\Gamma\frac{d}{dx}\left(\frac{\sin\beta}{a}\right)dx$$

But

$$\frac{d}{dx}\left(\frac{\sin\beta}{a}\right)=\frac{d}{dx}\left(\frac{}{a^2}\right)=\frac{a^2-2x^2}{a^4}=\frac{1-2\sin^2\beta}{a^2}=\frac{\cos 2\beta}{a^2}$$

so that we have

$$w=\frac{1}{4\pi}\int_0^b\Gamma\frac{\cos 2\beta}{a^2}dx \tag{50}$$

With the aid of this relation we can write down immediately the value of the drag which arises owing to a second wing being under the influence of the disturbance caused by the first wing lying in the same transverse plane. Let us call w_{12} the disturbance velocity at a point A on the second wing. According to the results of section 22, the drag then is

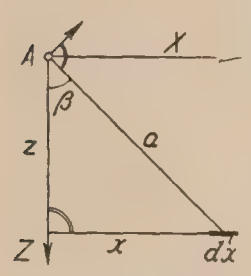

FIG. 52.—Velocity at a point A off the lifting line, but in the transverse plane, due to the vortex system.

$$W_{12}=\rho\int_0^{b^2}\Gamma_2 w_{12}dx$$

or, if the value of w_{12} as given by equation (50) is substituted,

$$W_{12}=\frac{\rho}{4\pi}\int_0^{b_1}\int_0^{b_2}\Gamma_1\Gamma_2\frac{\cos 2\beta}{a^2}dx_1dx_2 \tag{51}$$

The double integral, as one sees, is perfectly symmetrical in the quantities associated with both wings 1 and 2. We conclude from this that the drag which wing 1 experiences owing to the presence of wing 2 is of the same amount as the drag calculated here, that, therefore,

$$W_{12}=W_{21}.$$

In the more general case of two curved lifting lines lying in a transverse plane, a formula is obtained which differs from equation (51) only in having cos $(\beta_1+\beta_2)$ in place of cos 2β in which β_1 and β_2 are the angles which the line a makes with the normals on the two lifting elements connected by a, and in having $ds_1 ds_2$ in place of $dx_1 dx_2$. The relation $W_{12}=W_{21}$ therefore holds in this case also. This mutual relation, which was discovered in a different manner by my assistant, Dr. Munk, is of importance in various applications. Since it plainly is not necessary for the lifting elements taken as a whole to belong to a single surface, the theorem may be stated:

If, out of a lifting system all of whose elements lie in a transverse plane any two groups are selected, the portion of the drag experienced by group 1 due to the velocity field of group 2 is exactly of the same amount as that experienced by group 2 due to the velocity field of group 1.

We can interpret the partial integration performed above by saying that the velocity w appears by it as built up out of the contributions by merely infinitesimal wings having the length dx and the circulation Γ, while previously we have always built it up out of the actions of the separate vortices $\frac{d\Gamma}{dx}dx$. The integrand of equation (50) in fact agrees with the velocity which is caused by two vortex lines of equal but opposite strengths Γ lying at a distance dx apart. The double integral in equation (51) can, from this point of view, be looked upon as the sum of the actions of the vortex strips of all the elements dx_1 on all the lifting elements dx_2.

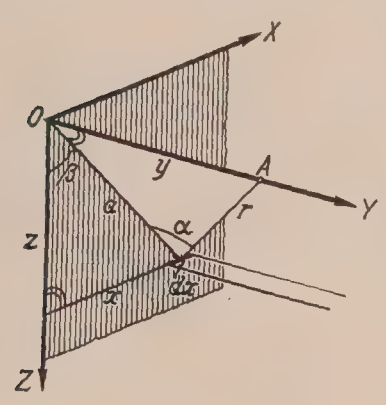

FIG. 53.—Velocity at a point A not in the transverse plane, due to the vortex system.

The objection might be raised that equations (50) and (51) cease to be applicable if the value $a=o$ appears, since this gives an expression of the form $\infty-\infty$. They are not, therefore, suited for the calculation of the velocity w at the wing itself. In this case we must return to equation (49), and take the "chief value" of the integral; or, the value of w_{12} and of W_{12} for a lifting line that lies very close must be calculated, and then we can obtain our final result by passing to the limit for coinciding lifting lines. As is seen from this, the relations $W_{12}=W_{21}$ hold also for lifting lines coinciding in space, which, besides, may have any arbitrary lift distribution.

The mutual drag need not, as has already been mentioned, always be positive. For instance, it is negative for two wings placed side by side, since then each wing is in an ascending current caused by the other, and the total drag is therefore less than the sum of the mutual drags which each of the wings would have at a greater distance apart. The behavior of certain birds which in a common flight space themselves in a regular phalanx can be explained by reference to this.

25. In order to be able to treat the case of staggered wing systems, the next problem is to calculate the velocity field due to a lifting element of the length dx together with its pair of vortices at a point A which may now lie off the transverse plane, and at a distance y from it. (See fig. 53.) The origin of coordinates will be taken at the projection of the point A upon the transverse plane, and the X axis parallel to the direction of the element. Using the abbreviations

$$a^2=x^2+z^2,\ r^2=a^2+y^2,$$

the velocity produced at the point A by one of the two vortices, by formula (6b), is given by $\frac{\Gamma}{4\pi a}\left(1+\frac{y}{r}\right)$; the component in the direction of the Z axis, to which we here again limit ourselves, is, then, putting $\sin\beta=\frac{x}{a}$.

$$w_1=\frac{\Gamma x}{4\pi a^2}\left(1+\frac{y}{r}\right).$$

The pair of vortices produces then a velocity which may be written as the difference of the effects of two vortices which are close together:

$$dw_1=\frac{\partial w_1}{\partial x}dx=\frac{\Gamma dx}{4\pi}\left[\left\{\frac{a^2-2x^2}{a^4}\left(1+\frac{y}{r}\right)-\frac{x^2y}{a^2r^3}\right\}\right]$$

To this must be added the contribution of the transverse vortex

$$dw_2 = \frac{\Gamma dx \,.\, y}{4\pi r^3}$$

The sum of these two velocities, if the two angles defined in figure 53 are introduced, amounts to

$$dw = \frac{\Gamma dx}{4\pi}\left\{\frac{\cos 2\beta}{a^2}\,(1+\sin\alpha) + \frac{\sin\alpha \,.\, \cos^2\beta}{r^2}\right\} \tag{52}$$

With the help of this formula we can now calculate at once the drag experienced by a lifting element situated at the point A and parallel to the former, whose length is dx_2 and circulation Γ_2. If the first element is given the index 1, this drag is

$$d^2 W_{12} = \frac{\rho\Gamma_1\Gamma_2 dx_1 dx_2}{4\pi}\left[\left\{\frac{\cos 2\beta}{a^2}\,(1+\sin\alpha) + \frac{\sin\alpha\cos^2\beta}{r^2}\right\}\right] \tag{53}$$

As is easily seen, the drag produced on the lifting element 1 by the lifting element 2 is obtained if in place of α and β the values $\alpha+\pi$ and $\beta+\pi$ are introduced. Therefore it is

$$d^2 W_{21} = \frac{\rho\,\Gamma_2\,\Gamma_1\,dx_2\,dx_1}{4\pi}\left[\left\{\frac{\cos 2\beta}{a^2}\,(1\text{-}\sin\alpha) - \frac{\sin\alpha\cos^2\beta}{r^2}\right\}\right] \tag{53a}$$

It is seen from this that the two parts of the drag are equal only if $\alpha = o$, that is if the two elements lie in the same transverse plane. Yet in the general case the sum $d^2 W_{12} + d^2 W_{21}$ is independent of α, therefore independent of the amount of stagger. The sum of the two mutual drags leads thus to the same formula as that already derived for nonstaggered wings. If we again pass to the general case of nonparallel lifting lines, in which again ds_1 and ds_2 are to be written in place of dx_1 and dx_2, we obtain as may be proved by performing the calculation, the relation

$$W_{12} + W_{21} = \frac{\rho}{2\pi}\int\int\frac{\Gamma_1\,\Gamma_2\,ds_1\,ds_2\cos(\beta_1+\beta_2)}{a^2} \tag{54}$$

As is evident, this sum remains unchanged if the two lifting groups are displaced in the direction of flight. Since the total drag of a lifting system is composed of such mutual drags as calculated above and of the proper drags of the separate wings, which likewise are not changed by a displacement of the wing in the direction of flight, the following theorem may be stated:

The total drag of any lifting system remains unchanged if the lifting elements are displaced in the direction of flight without changing their lift forces.

This "stagger theorem" was likewise proved by Munk. For a proper understanding of this theorem it must be mentioned expressly that, in the displacement of the separate lifting elements, their angles of attack must so be changed that the effective angles of attack and therefore the lifting forces remain unaltered.

This theorem, which at first sight is surprising, may also be proved from considerations of energy. Let us remember that, by the overcoming of the drag, work is done, and that in a nonviscous fluid, such as we everywhere assume, this work can not vanish. Its equivalent is, in fact, the kinetic energy that remains behind in the vortex motions in the rear of the lifting system. This energy depends only upon the character of these vortices, not upon the way in which they are produced. If we neglect, as we have throughout, any change in shape of the vortex system, then, in fact, the staggering of the separate parts of the lifting system can not have any influence upon the total drag.

26. For the practical calculation of the total drag of a multiplane, we have then the following: The total drag consists of the sum of all the separate drags and of as many mutual drags as there are combinations of the wings in twos. If the nature of the lift distribution over all the separate wings is specified, then the proper drags are proportional to the square of the separate lifts; the mutual drags, to the product of the lifts of the two wings in question. If the coefficients of this mixed quadratic expression are all known, then one can solve without difficulty the problem: For a specified total lift, to determine the distribution of lift over the separate wings which will make the total drag a minimum.

In order to know these coefficients to a certain degree I calculated them for the case of two straight lifting lines whose middle points lie in the same plane of symmetry, with the assumption that the lift over each separate wing is distributed according to a half ellipse. The results are given in my paper "The induced drag of multiplanes" in Volume III, part 7 of the Technische Berichte. For this purpose the velocity w for the entire neighborhood of a wing in the transverse plane was first calculated by formula (49), and then the integrals for the mutual drags were obtained by planimetry. To show the analogy with equation (40) this may now be expressed by the formula

$$W_{12}=\sigma\frac{A_1\,A_2}{\pi\,q\,b_1\,b_2} \tag{55}$$

by means of which the numerical factor σ can be expressed as a function of the two variables $\frac{h}{\frac{1}{2}(b_1+b_2)}$ and $\frac{b_2}{b_1}$. Calculation gave the following table:

TABLE I.—*Values of* σ

$\frac{2h}{b_1+b_2}$	0	0.05	0.1	0.15	0.2	0.3	0.4	0.5
$\frac{b_2}{b_1}$ = 1.0	0.780	0.655	0.561	0.485	0.370	0.290	0.230
.8	.690	.600	.523	.459	.355	.282	.225
.6	.540	.485	.437	.394	.315	.255	.210

The curve of the function σ is given in figure 54. For the most important case, viz, for two wings of equal span, I have developed an approximation formula which is

$$\sigma=\frac{1-0.66\frac{h}{b}}{1.055+3.7\frac{h}{b}} \tag{56}$$

It may be used from $\frac{h}{b}=0.05$ to 0.5.

The total induced drag of a biplane is then, if b_1 is the greater span and if the ratio $\frac{b_2}{b_1}$ is designated by μ

$$W=W_{11}+2\,W_{12}+W_{22}=\frac{1}{\pi q b^2}(A_1{}^2+2\sigma\mu A_1A_2+\mu^2A_2{}^2) \tag{57}$$

Simple calculation shows that for a given A_1+A_2 this drag is a minimum for

$$A_2:A_1=(\mu-\sigma):\left(\frac{1}{\mu}-\sigma\right) \tag{58}$$

The value of the minimum is fouud to be

$$W_{\min}=\frac{(A_1+A_2)^2}{\pi q b_1{}^2}\cdot\frac{1-\sigma^2}{1-2\sigma\mu+\mu^2} \tag{59}$$

The first factor of this formula is the drag of a monoplane having the span b_1 and the lift A_1+A_2. Since $\sigma<\mu$, the second factor of the formula is always less than 1, i. e., the induced drag of a biplane is less than that of a monoplane which in the same span carries the same load. For a "tandem," i. e., an arrangement of two wings one behind the other, the stagger theorem shows an equivalence with two coinciding wings, i. e., a monoplane. Among the different biplanes having prescribed span b_1 and prescribed gap h, that one is the most favorable in which the second wing also has the span b_1. The most favorable ratio of the two lifts is then 1 : 1 and the second factor of equation (59) becomes equal to $\frac{1}{2}(1+\sigma)$.

These statements must not, however, be misunderstood; they refer only to the comparison of such wing systems as have the same value for the greatest span. Naturally, for every biplane a monoplane may be found with somewhat greater span than that of the biplane, which at the same total lift has the same induced drag as the biplane.

This last remark leads us to apply also to the biplane the deduction formulas obtained for monoplanes. All that is necessary is to replace the biplane of span b_1 by a monoplane of a somewhat greater span kb_1, which with reference to the drag—and, on the whole, with reference to the angle of attack—is equivalent to the biplane. If again we pass from the lift and drag to their coefficients c_a and c_w, the formula connecting the drags of any two lifting systems 1 and 2 is

$$c_{w_1} - c_{w_2} = \frac{c_a^2}{\pi}\left(\frac{F_1}{(k_1 b_1)^2} - \frac{F_2}{(k_2 b_2)^2}\right) \tag{60}$$

in which, as is easily seen, the factor k, for a biplane having the most favorable distribution of lift, is the reciprocal of the square root of the second factor in formula (59).

The tests of this formula with biplanes have shown that, when by giving a special shape to the wing the lift distribution was made elliptical, there was good agreement with the calculations from monoplane experiments; with biplanes having the usual square-cornered wings, on the other hand, there was a discrepancy, which is to be attributed to the fact that the lift distribution on these biplanes deviates too far from an elliptic one. We can, however, retain the same transformation formula if the factor k is determined empirically for every wing system; it is found to be somewhat smaller than according to the theory given above. The experiments on this point are not yet completed, so more accurate values can not as yet be given. The earlier Göttingen experiments were worked up by Dr. Munk, to whom this last idea is due, in the paper "Contribution to the aerodynamics of the lifting parts of airplanes" in the Technische Berichte, Volume II, page 187.

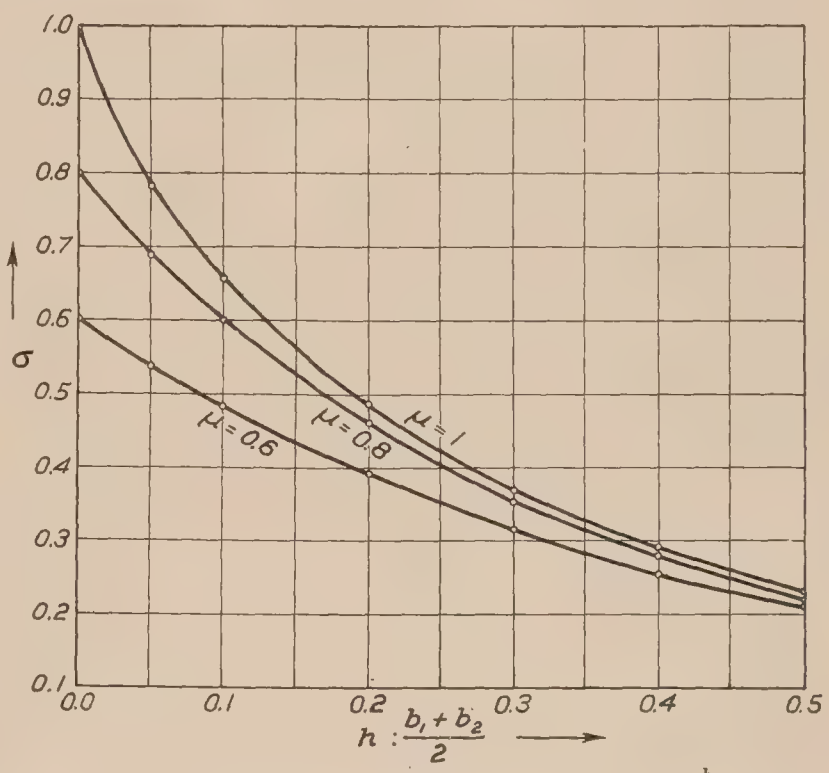

FIG. 54.—Curve of the function σ for biplanes whose span ratio, $\mu = \frac{b_2}{b_1}$, is varied.

27. In the previous section I have treated the problem of finding the minimum of the induced drag of a multiplane, under very definite assumptions concerning the distribution of lift over each separate wing. The strict minimum problem is however different, viz:

To determine for a given front view of a lifting system that distribution of lift over all the lifting elements which will make the induced drag a minimum for a specified total lift.

In this statement of the problem, the expression "a given front view"—i. e., more exactly stated, a given projection of the lifting system upon a plane perpendicular to the direction of flight—is used to mean that the wing chord is of secondary consideration, and does not need to be determined until later when the selection of suitable angles of attack is made.

The general solution of this problem was also given by Dr. Munk. It will be deduced here in a simpler manner than that given in Munk's dissertation, where the solution was obtained by the calculus of variations. By means of the stagger theorem mentioned in section 25 the wing system will be referred back to the corresponding nonstaggered system. For this, as we showed, the relation $W_{12} = W_{21}$ holds. We shall now introduce—with the simplifying assumption that all the lifting elements are parallel to each other—a variation of the lift distribution by adding at any one place a lift δA and at the same time taking away an equal amount at some other place, so that on the whole the lift, which is prescribed, remains unchanged. We must now consider the change in the induced drag caused by this variation. If there is superimposed upon the lift distribution an additional air force δA distributed over a short portion dx, there arises therefrom, in addition to the drag proper of the added lift—which, however, if sufficiently small is of the second order—a mutual drag, because on the one hand the added lift finds itself

in a flow having the downward velocity w, which is due to the lifting system, and on the other the lifting system is in the velocity field of the added lift. The first of these two drags, as is easily seen, is $\delta A\frac{w}{V}$; the other part, according to our theorem, has the same value; so the total drag is twice this. What condition, now, must be satisfied by the sum of the mutual drags caused by our twofold change of the lift distribution in order to obtain the absolute minimum of the induced drag? The answer is, evidently, that we will have the minimum only if by no change of this kind can the drag be further diminished. The sum of the induced drags, therefore, can in no case be negative; also it may not be positive, because in that case by a reversal of the signs of the changes which we selected we could make the sum negative. Only the value zero is therefore allowable. Hence, if w is the vertical velocity at point 1 and w_2 that at point 2, we have the relation

$$\delta A_1\frac{w_1}{V}+\delta A_2\frac{w_2}{V}=o$$

and, therefore, since $\delta A_1=-\delta A_2$,

$$w_1=w_2.$$

Since this holds for all the lifting elements, we have obtained the answer. The lift distribution which in the given wing system, for a specified total lift, causes a minimum of drag is that which leads to the same downward velocity at all the lifting elements. With monoplanes the elliptical lift distribution leads to a constant downward velocity w. We recognize from this that the elliptical distribution in fact is that distribution of lift which causes the least drag for a monoplane.

The theorem can, besides, be extended easily to the case of nonparallel lifting elements lying in a transverse plane. If w_n is the velocity in the transverse plane perpendicular to the lifting element and ϵ is the angle between the direction of w_n and that of the given total lift, then, as may be shown without difficulty, $w_n=w_o \cos\epsilon$ for all the elements. (If $\epsilon=o$, and hence $\cos\epsilon=1$, the statement made above again appears.)

28. A way to solve the problem of finding the lift distribution for a prescribed distribution of the vertical velocity has been indicated already in section 23. The velocity field left behind in the air by the lifting surface is, approximately, according to the remark made before, a uniplanar flow around the vortex system produced by the lifting system in its motion, and this vortex system may be regarded, as a first approximation, as a solid body in the fluid. In the minimum case this figure, according to the results of section 27, moves like a *rigid* body, not alone in the case of parallel lifting elements, but also in the general case, for the general minimum condition, $w_n=w_o \cos\epsilon$, expresses directly that the normal velocity of the fluid at an element of the rigid figure moving in the direction of the lift coincides with the normal component of the velocity w_o of the rigid figure itself. The problem is thus reduced to a perfectly definite one treated in the hydrodynamics of uniplanar fluid motion.

This uniplanar flow can be brought into relation, in a specially clear manner, with the pressure distribution existing on the wing system. The wing system, during its motion along its path, imparts to one portion of the air after the other the velocities which we have learned to know as the result of the vortices flowing off from the wings. This transmission of velocity is the result of the spreading out of the pressure field of the wing system over the air particles one after another. In order to simplify the phenomenon for ourselves we can now imagine that these velocities are produced at the same moment by a sort of impulse phenomenon over the whole path of the lifting system. To produce this impulse it is necessary to have a solid figure of the shape of the geometrical region passed over by the lifting system (i. e., of the shape of the vortex surfaces which it leaves behind). If we are concerned with a system of least drag, this figure moves as a rigid body; otherwise it would also experience a change of shape due to the impulse. The final velocity w^* of the figure coincides with the motion of the vortex surfaces at a great distance from the lifting system, and is therefore to be put equal to $2w$. For a monoplane having elliptical distribution our figure is therefore an infinitely long flat plate of the breadth b.

By the production of the velocity during the impulselike acceleration an increase in the pressure p_1 arises below the plate (in the case of multiplanes, under each of the plates corresponding to the separate wings) and at the same time a decrease in pressure p_2 above the plate (or the plates). We can now compare in a very simple manner the total action of the pressure differences at each point of a plate during the time of the impulse with the total action of the pressure differences of the wing in its forward movement at the point of the medium in question. If the resulting motion is the same in both cases, then the pressure differences integrated through the proper times must have the same values. If in the impulse phenomenon lasting a time τ a portion of the fluid of length l is considered, and if therefore the action of the lift $\frac{dA}{dx}dx$ in the time $t=\frac{l}{V}$ required to pass over the length l is to be compared, the following relation must hold for the conditions on a strip of width dx:

$$ldx\int_0^\tau (p_1-p_2)\,dt=\frac{dA}{dx}dx\cdot\frac{l}{V} \tag{61}$$

A formula connected with our previous relations can be obtained by a transformation of the left-hand term. According to a known extension of the Bernouilli equation for accelerated motion we have

$$\rho\frac{\partial\Phi}{\partial t}+\frac{\rho v^2}{2}+p=f(t).$$

For our impulse phenomenon the arbitrary time function $f(t)$ is a constant, since at the points of the fluid lying far away from the impinging plate the pressure does not change. If the impulse is sufficiently quick, then during the short time of impulse τ the acceleration and the pressure differences will be very large, and therefore the term $\frac{\rho v^2}{2}$ may be neglected in comparison with the other two, since it itself does not exceed moderate values. We obtain therefore the simplified relation

$$\rho\frac{\partial\Phi}{\partial t}+p=\text{const.}=p_0$$

which, if at the beginning everything is at rest, $(\Phi_0=o)$ may be integrated to

$$\rho\Phi=\int_0^\tau (p_0-p)\,dt \tag{62}$$

we can therefore write, in equation (61), the expression $\rho(\Phi_2-\Phi_1)$ in place of $\int_0^\tau (p_1-p_2)\,dt$. The potential differences $\Phi_2-\Phi_1$ which here appears is, according to the connection between potential and circulation (see sec. 5), nothing but the circulation Γ for a closed curve which passed around one edge of the vortex ribbon and intersects our vortex ribbon at x, the point considered. This circulation is again nothing but the circulation around the wing at the point x. If the factor ldx is omitted from both sides of equation (61), it takes, as a result of this transformation, the form

$$\frac{dA}{dx}=\rho(\Phi_2-\Phi_1)\,V=\rho V\Gamma \tag{63}$$

We have thus proved in an entirely independent way, as we see, the Kutta-Joukowski theorem for a wing element, which previously we took over, without proof, from the infinitely long wing.

The relations deduced in the previous paragraphs permit, in the case of a constant w, the formation of general theorems for w and W in place of (39) and (40). By integration of (63) the total lift is at once obtained

$$A=\rho V\Sigma\int(\Phi_2-\Phi_1)\,dx \tag{64}$$

The values of Φ in this formula are proportional to the velocity w^* of the vortex ribbon, that is, are dependent upon A. Quantities which are independent of A are derived if the potentials Φ are divided by w^*. In this way we obtain the potentials for a velocity of the vortex

ribbon equal to 1. The potential, being the line integral of the velocity, has the dimension velocity times length; the potential ϕ for $w^*=1$ has therefore the dimension of a length, and hence

$$\Sigma\int(\phi_2-\phi_1)dx$$

is a surface, which will in what follows be called F', which depends only upon the geometrical properties of the projection of the wing system upon a plane perpendicular to the direction of flight, therefore upon the front view of the wing system; and which evidently for geometrically similar front views is proportional to the square of the span. By introducing F' into equation (64) we have, since $\Phi=\phi w^*$,

$$A=\rho Vw^*F' \tag{65}$$

From this we may immediately deduce w^*, and thereby also the downward velocity at the point of the wing system

$$w=\frac{1}{2}w^*=\frac{A}{2\rho VF'} \tag{66}$$

If this value is introduced into the relation $W=\frac{w}{V}A$, we have

$$W=\frac{A^2}{2\rho V^2F'}=\frac{A^2}{4qF'} \tag{67}$$

The evaluation in the manner of the flow of figure 49 gives a potential Φ, if the span of the wing is set equal to b, which has the value $\sqrt{(b/2)^2-x^2}$ at the plate. The geometrical expression of this value gives a circle having the span b as diameter, therefore $F'=\frac{\pi b^2}{4}$. Using this value formula (67) passes over in fact into formula (40).

It may also be noted that a uniform velocity can be superimposed upon the uniplanar flow here discussed, whose discontinuity in potential at the rigid figure representing the vortex ribbon causes the surface F', without thereby changing the relation for F', for the potential discontinuity between the lower and upper sides, with which we are here concerned, is not changed by the superimposed uniform motion. We may now choose the velocity of the uniform motion exactly opposite and equal to the velocity w^* of the rigid figure, and thereby secure the condition that in the new flow the rigid figure is at rest and is surrounded by a flow which at infinity has the velocity w. The forces which the rigid figure experiences by the production of this motion, and which are connected intimately with the so-called "apparent mass," are what we have here set in parallel with the wing-lift.

The surface F' supplies in addition a very simple mechanical connection between the velocity w on the one hand and the lift and drag on the other. According to equation (65)

$$A=\rho F'Vw^*$$

$$WV=Aw=\rho F'V\frac{w^{*2}}{2}$$

where in the second equation use has again been made of the relation $w^*=2w$. Now $\rho F'V$ is the mass of air flowing per second through the section F'. If in order to simplify the whole problem it is once assumed that all the air particles within the section F' experience the full deviation w^* but that all outside are entirely undeviated, then exactly the correct lift and the correct work due to drag are obtained by application of the impulse theorem and the energy theorem. For the lift is now equal to the mass of the fluid deviated per second times the vertical velocity imparted to it, therefore equal to the impulse imparted to the medium. Also, in the same manner, the work done per second by the drag WV is the product of the mass of the fluid passing per second times the half square of the deflection velocity, and therefore equal to the kinetic energy left behind in the medium. This relation is indeed best suited to establish the phenomena of the theory of airplanes in a course for students who are only slightly skilled in mathematics. The fact that for a monoplane the circle having a diameter equal to the span comes out as the surface F' is one that will appear most plausible to the laity.

29. Of the theory pictured in the preceding section, according to which the determination of the induced drag in the case of the most favorable lift distribution is reduced to a problem of the potential theory, manifold applications have already been made. Especially, Dr. Grammel and K. Pohlhausen have treated, at my instigation, the case of the biplane made up of two straight monoplanes of the same span, and also, on the other hand, that of a monoplane having a longitudinal slot. The calculations in both cases are solved by means of elliptic integrals. I have given the formulas in my Wing Theory II. It may be sufficient here to state the practical final result, which is referred to the magnitude of the surfaces F'. These surfaces are best expressed for biplanes in terms of the corresponding surfaces of the monoplane having the same span. In fact, the ratio $F' : \frac{\pi}{4} b^2$, as is easily seen, equals the square of the factor k, introduced in section 26, by which the span must be increased in order to have a monoplane of the same induced drag. The values of k^2 for the biplane are obtained from the following table. The gap of the biplane, i. e., the distance apart of the two wings, is designated by h.

TABLE 2.

Values of $k^2 = F' : \frac{\pi}{4} b^2$

$h/b =$ ...	0.	0.05	0.10	0.15	0.2	0.3	0.4	.....	0.5	∞
$k^2 =$	1.000	1.156	1.212	1.289	1.352	1.461	1.550	.....	1.626	2.000

The values given in the table may be expressed by the approximation formula

$$k^2 \cong \frac{1.027 + 3.84 h/b}{1 + 1.63 h/b} \tag{68}$$

In the case of the monoplane having a slot a suitable comparison wing is obtained by shoving the two halves of the monoplane together until the slot is closed. If b is the original span and d is the width of the slot, this monoplane has evidently the span $d - d$. We shall therefore form the ratio $F' : \frac{\pi}{4} (b - d)^2$ and again designate it by k^2. Calculations gave the following values:

TABLE 3.

Values of $k^2 = F' : \frac{\pi}{4} (b-d)^2$.

$\frac{d}{b}$	0.000	0.001	0.010	0.0316	0.100	0.250	0.500	1.000
k^2.....	1.000	0.762	0.676	0.6200	0.568	0.528	0.506	0.500

It is seen that even very narrow slots produce an important increase in the induced drag. For a very wide opening k^2 falls to one-half, as may be deduced easily from the fact that now, instead of one monoplane, we have two monoplanes of half the span. The values given in the table may be expressed by the approximation formula

$$k^2 = 1 - \frac{1}{2\sqrt{1 + 0.35 (\log_{10} b/d)^2}} \tag{69}$$

Figures 55–57 show, at the left, the uniplanar w^* – flow and, at the right, the surfaces F', for a monoplane, a biplane, and a monoplane with a slot.

F. AEROFOILS IN A TUBE OR IN A FREE JET.

30. To draw conclusions from the experimental results obtained in a tube bounded by solid walls or in a free jet from a nozzle, it is very useful to know the influence of the neighboring walls and of the boundaries of the jet upon the phenomena at the aerofoil. We wish indeed to know the behavior of the aerofoil in an air space infinitely extended in all directions; and the problem therefore arises to introduce a method for passing by calculation from the case which prevails in the experiments to that of the unlimited air space. For this purpose we shall next state clearly the boundary conditions which exist at solid walls parallel to the direction of the

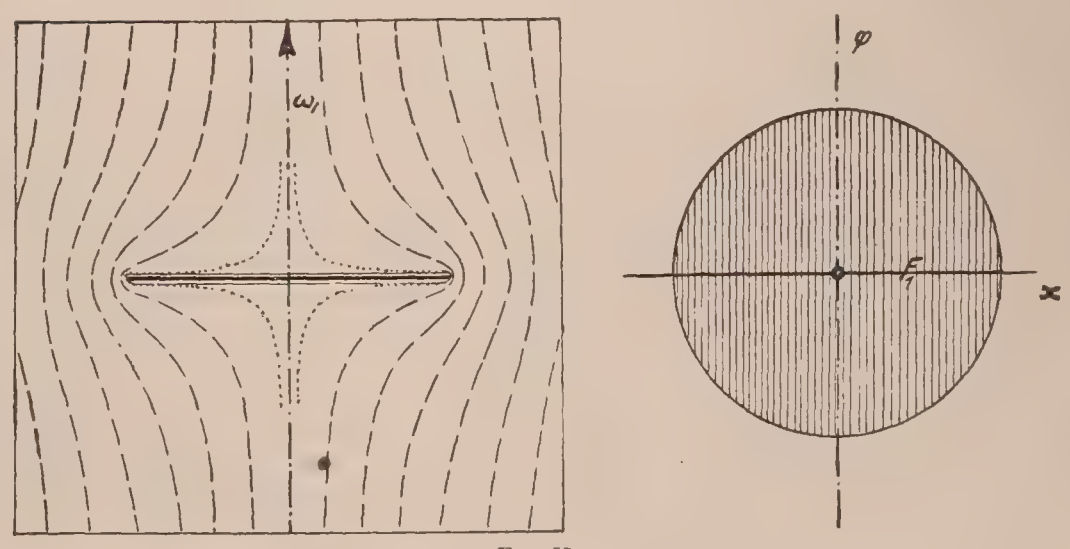

FIG. 55.

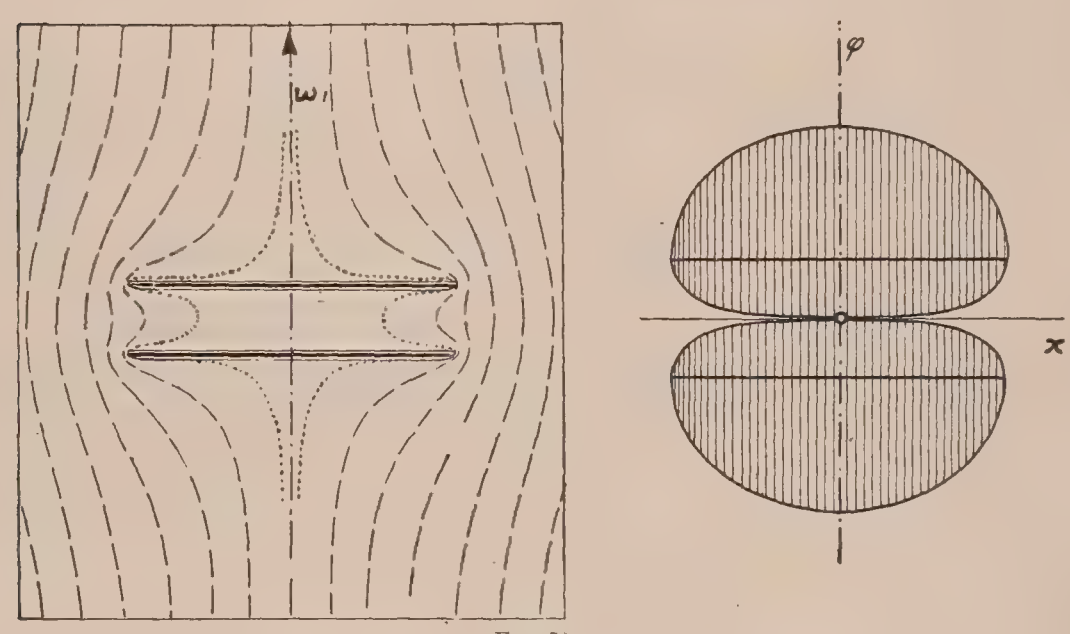

FIG. 56.

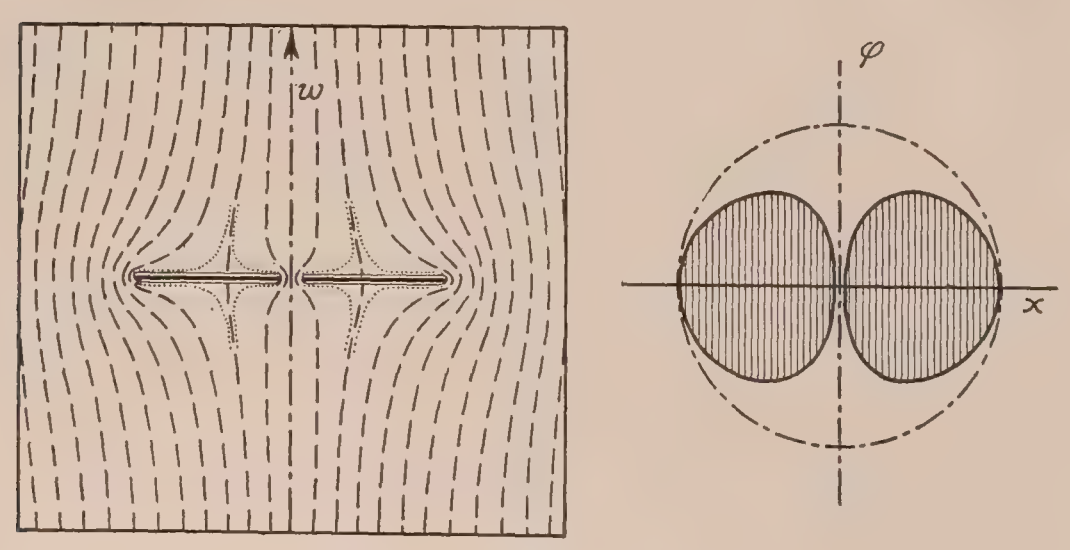

FIG. 57.

Left: Flow behind a monoplane, a biplane, and a monoplane with a slot, with reference to an observer moving downwards with the velocity W^* of the vortex system.
Right: Corresponding surfaces F_1.

wind and at the free boundary of a jet. At solid walls, the velocity components normal to the wall, w_n, must equal zero; on a free jet boundary, on the other hand, the pressure is to be put equal to that of the surrounding quiescent air layer, and therefore is constant. We can transform these last relations as follows, keeping within our theory of the first order. According to the Bernouilli theorem, if V is the undisturbed wind velocity, and u, v, and w are the additional velocities

$$p+\frac{\rho}{2}\left[u^2+(V+v)^2+w^2\right]=p_0+\frac{\rho}{2}V^2$$

or, since $p=p_0$,

$$u^2+v^2+w^2+2Vv=o.$$

If we neglect the squares of the disturbance velocities as being small of the second order, we have as the approximate initial condition for the free jet $v=o$. We proceed a step farther upon the path indicated to us by the approximation theory of the first order if we prescribe the value $v=o$, not for the actual jet boundary, but for that cylinder which is given by the surface of the undeviated jet. By doing this the boundary condition for the free jet becomes very similar, in a formal way, to that for the solid walls.

The two problems can now be solved in the following manner: We consider first the velocity field for the unlimited air space, according to the exposition previously given. This field offers, both in the case of the tube and in that of the free jet, contradictions with our boundary conditions at the walls or the jet boundary. We must superimpose a velocity field which in the interior of the region considered is free of singularities and which on the boundaries has velocities opposite to those velocity components, the vanishing of which is prescribed by the boundary condition. It is easily seen that by the superposition of this second velocity field on the original one the boundary conditions are satisfied exactly. The influence of this second field upon the aerofoil is now exactly that influence which we are seeking, and which we can calculate from the results of the theory of aerofoils as soon as this second field is known.

The additional velocity field corresponds to a pure potential motion; we have, therefore, the problem of determining its potential Φ. In the case of solid walls we are thus led to the problem of finding the potential for a given region (the interior of the tube) when the normal component w_n of the flow is given at the boundary of the region. This is the so-called "second boundary value problem" of the potential theory. The corresponding problem for a jet, as we shall see at once, leads to the "first boundary value problem," in which at the boundary the values of the potential itself are prescribed. According to what has been said above our region is a cylinder whose generating lines are parallel to the velocity V, hence, parallel to the Y axis, and for each point on the boundary the relation $\frac{\partial\Phi}{\partial y}=-\bar{v}$ (in which the dashes indicate boundary values) is prescribed. Integrating this relation for each generating line gives

$$\bar{\Phi}\,(y)=-\int_{-\infty}^{y}\bar{v}dy$$

If we go sufficiently far upstream every influence of the aerofoil vanishes; therefore for $y=-\infty$, $\Phi=o$; and hence $y=-\infty$ is taken as the lower limit of the integral. By this, then, we obtain the boundary values of the potential $\bar{\Phi}\,(y)$.

The complete calculation of the added potential Φ for the entire interior of the tube or jet is fairly difficult. If we concern ourselves, however, only with our main problem, to determine the corrections which must be applied to our experimental results, then we can again assume that the velocity components perpendicular to the axis of the tube in the plane of our aerofoil are half as large as at a great distance behind it. This consideration, which proceeded from the comparison of a vortex filament proceeding to infinity in one direction only with one proceeding to infinity in both directions, holds here exactly as well as in the cases discussed previously. We can therefore pass here as before from the space problem to a uniplanar one if we calculate the phenomena far behind the aerofoil. Our boundary conditions for the uniplanar problem are, for the tube, $\bar{w}_n=o$, for the free jet, $\Phi=\text{const}$. The last condition may be interpreted specially

conveniently if use is made of the method of treatment of section 28. Since in this, for the end of the impulse phenomenon, $\Phi=\int_0^\tau (p_0-p)\,dt$, $\overline{\Phi}=\text{const.}$ means simply that $\overline{p}=\text{const.}$, which, indeed, was the original boundary condition for the free jet.

31. The conditions stated in the preceding section can be secured most easily for a jet, or tube, of a circular cross section. In this case the added motion is obtained very simply by assuming for every vortex flowing off an equally strong one outside the circle, at the point outside corresponding to the one inside according to the reciprocal radii. If the direction of rotation of the external vortex is taken the same as that of the interior one, then at the points of the circle the boundary condition for a free jet is obtained; and, if opposite directions of rotations are taken, then the boundary condition for a tube is satisfied. This may be expressed by saying that there is combined with the aerofoil another obtained by reflexion according to reciprocal radii, whose circulation at corresponding points is the same in absolute value as that of the actual aerofoil, and for the jet it has the same sign, but for the tube the opposite sign.

The exact calculation has been made for a straight monoplane in the middle of the jet, assuming the lift to be distributed according to a half ellipse. If b is the span of the monoplane and D the diameter of the jet, then the disturbance velocity w' caused by the jet boundary at the distance x from the middle of the jet is

$$w'=\frac{A}{\pi D^2\rho V}\left(1+\frac{3}{4}\xi^2+\frac{5}{8}\xi^4+\frac{35}{128}\xi^6+\text{etc.}\right) \tag{70}$$

in which $\xi=2xb/D^2$.

The added drag calculated from this velocity according to equation (38) is found to be

$$W'=\frac{A^2}{\pi D^2\rho V^2}\left[1+\frac{3}{16}\left(\frac{b}{D}\right)^4+\frac{5}{64}\left(\frac{b}{D}\right)^8+\cdots\right] \tag{71}$$

A similar calculation for a uniform lift distribution gave for the first term in the formula for the drag the same value as in equation (71). It appears that the other terms of the series have but little importance with the usual ratios, so that we can limit ourselves to the first term. An approximation treatment shows, further, that any small wing system, in the middle of the circular jet gives rise to the same expression. We can therefore write for the total induced drag of the wing system in a jet of cross section F_0, if the surface F' is again introduced from section 28,

$$W=\frac{A^2}{4q}\left(\frac{1}{F'}+\frac{1}{2F_0}\right) \tag{72}$$

For a tube of circular cross section the same disturbance effect is found, but with the opposite sign; and therefore we have the approximation formula for the drag

$$W=\frac{A^2}{4q}\left(\frac{1}{F'}-\frac{1}{2F_0}\right) \tag{72a}$$

The correction, owing to the consideration of the finite cross section of the jet, is for the ratios ordinarily used not small. For $\frac{b}{D}=\frac{1}{2}$, it is already one-eighth of the induced drag. Formula (71) gives 0.1262 instead of 0.125; the corresponding formula for uniform distribution gives 0.127. It is seen, therefore, that the differences are not great, and that the approximation formula (72) is satisfactory for most cases.

For a tube of rectangular cross section the calculations would have to be made in such a manner that the aerofoil was mirrored at all the walls an infinite number of times, like a checkerboard. Further development of the calculation leads to elliptic functions. It has not yet been carried through. One can assume, however, that for a tube having a square cross section the influence of the walls will be of about the same magnitude as for the circle having an equal area.

G. APPLICATION OF THE THEORY OF AEROFOILS TO THE SCREW PROPELLER.

32. The fundamental ideas of the aerofoil theory can be applied step by step to the screw propeller. For the elements of the blades the Kutta-Joukowski formula holds, viz, that the air force is perpendicular to the velocity c of the element with reference to the air and that, per unit length of the blade, it has the value $\rho\Gamma c$. Corresponding to what has gone before, vortices will arise at the blade, having a vortex strength per unit length equal to $\frac{d\Gamma}{dx}$. If we wish again to construct a theory of the first order, that is, if we agree to consider as small the air forces and the velocities produced by them, then again the proper motion of the vortices will be small and therefore in a first approximation may again be neglected. The vortices then have the shape of screw lines and form vortex ribbons which—if for the sake of simplicity we assume straight radial blades—have the shape of ordinary screw surfaces.

The calculation of the velocity field of a screw vortex is markedly more complicated than that of a rectilinear vortex and leads to functions which thus far have not been studied in detail. In spite of this it is possible, as Dr. Betz has shown, to prove a series of general theorems very similar to those of Munk for multiplanes. Since the velocity c is not the same at the separate blade elements, we must speak of the "work lost" where Munk speaks of drag. The work applied for the motion of the propeller is composed of two parts—useful work + work lost. The latter in our ideal case, where friction is excluded, is transformed completely into kinetic energy of the air. The kinetic energy stands again in close connection with the vortex system produced by the propeller. Betz proved, among others, the following theorems:

(1) If two elements of a propeller blade lie upon the same radius at distances x and ξ from the axis, then the work lost at the point ξ due to the disturbance velocity caused by the air force at the point x is equal to the work lost at the point x owing to the disturbance velocity caused by the air force at the point ξ.

(2) This theorem must be somewhat modified for two elements which do not lie on the same radius. It reads: The work lost at the point ξ due to the disturbance velocity caused by the air force at the point x is of the same amount as the work which would be lost at the point x if the screw vortex proceeding from the element at ξ were to pass out forward in the prolongation of the actual vortex instead of going backward.[12]

(3) This last theorem leads at once to the following relation for the sum of the two amounts of work lost: The total work lost due to the mutual action of the air forces by two blade elements at points x and ξ is the same as the work which would be lost at one point alone if the screw vortex proceeding from the other point were to extend to infinity both forward and backward.

It is easily seen that this theorem is perfectly analogous to the stagger theorem of section 25, for if the vortex of the inducing element extends in both directions, then the position of the element itself on its own vortex strip is immaterial as far as the velocity field produced is concerned.[13] It is therefore true of screws that nothing is changed in the total energy-loss if blade elements are displaced in any way, without change of their air forces, along the relative streamlines passing through them (i. e., in this case, screw lines). This naturally is connected again with the fact that the total amount of the energy loss depends only upon the final distribution of the vortex systems, not upon the relative position of the places where the separate vortices arise.

Theorem No. 3 will be of use to us also in what follows. It can be made clearer by the following consideration. The field of the vortex ribbon of a lifting element dies away very quickly forward of the element, but in the rear it extends over the entire length of the path traversed. If the sum is formed of the two mutual losses in work of two elements at the points x and ξ, we can proceed, owing to the stagger theorem, to displace one of the two elements along its screw line so far backward that its velocity field is no longer appreciable at the position of the other

[12] In this the sense of rotation of the transverse vortex is to be reversed.

[13] The transverse vortex in this case cancels out completely in the determination of the velocity field, since it appears twice with opposite senses of circulation.

undisplaced element. At the same time, however, the influence of the latter element upon the first is increased since its vortex ribbon, viewed from the new position of the first element, extends as far forward as backward. The sum of the two mutual losses in work is reduced in this manner to the loss which the velocity field of the front element produces upon the displaced one.[14]

(4) The most important of Betz's theorems, from a practical standpoint, furnishes the complete analogy to Munk's theorem concerning the wing system having the least drag, and, corresponding perfectly to the statements in sections 27 and 28, may be expressed thus: The flow behind a propeller having the least loss in energy is as if the screw surfaces passed over by the propeller blades were solidified into a solid figure and this were displaced backward in the nonviscous fluid with a given small velocity. The potential difference between the front and rear sides of a screw surface at one and the same point furnishes, then, again the circulation Γ of the corresponding point of the propeller blade.

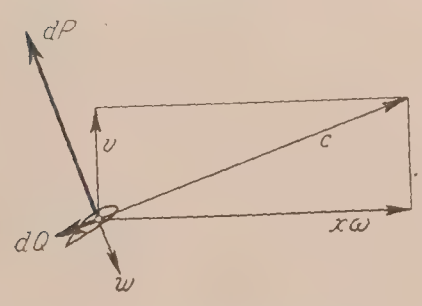

FIG. 58.—Velocity field near a blade of a screw-propeller.

A short proof of theorem 4 will be given. For this purpose the principal equations for the action of a screw must first be deduced. The screw is imagined to be displaced with the velocity v relative to the air, and to rotate at the same time with the angular velocity ω. A blade element at the distance x from the axis has then, with reference to the air which in the theory of the first order may be assumed to be at rest, the velocity c, with the components v and $x\omega$. (See fig. 58.)

If no vortex were produced, then, with the assumption of a nonviscous fluid, an air force dP would arise, which, according to the Kutta-Joukowski theorems, would be perpendicular to the velocity c and would have the value, for a blade element of length dx,

$$dP = \rho \Gamma c dx \tag{73}$$

The force dP is decomposed into two components, of which the one in the direction of v interests us specially, since it is applied to the screw. This component is

$$dS = dP \ . \ \cos \epsilon = \rho \Gamma x \omega dx \tag{74}$$

The total thrust, if there are n blades, is then

$$S = \rho \omega \sum_{1}^{n} \int_{0}^{r} \Gamma \ x \ dx \tag{75}$$

The other component

$$dT = dP \sin \epsilon = \rho \Gamma v \ dx \tag{76}$$

furnishes a contribution as a torque to the rotation moment. It is seen at once that $dS \ . \ v = dT \ . \ x \ \omega$, i. e., the useful thrustwork is equal to the work done by the torque hitherto used in our calculations. This depends immediately upon our assumption that the force dP is perpendicular to the velocity c. But the screw blades actually produce a vortex system and we must ask as to the reaction of the vortex system upon the phenomena of a screw. We shall assume, exactly as in the aerofoil theory, that we turn the blade profile in such a manner that the lifting forces desired by us actually come into play. Since we are interested here merely in the loss in work caused by the vortex system, we have to do only with the drag components caused by the vortex system. This depends, exactly as before, upon the velocity component perpendicular to the velocity of motion of the element, which in this case equals c. We shall again designate it by w. The added velocity component w furnishes a drag in the direction of motion equal to

$$dQ = dP \frac{w}{c} = \rho \Gamma w \ dx \tag{77}$$

The loss of work per second is therefore

$$dQ \ . \ c = \rho \Gamma dx \ . \ w \ . \ c \ (= dP \ . \ w)$$

[14] This process of thought can be applied, naturally, in the same way to aerofoils and furnishes a convenient deduction for the sum of the drags $W_{12} + W_{21}$.

If now, according to figure 58, we put $c = v/\sin E$, our problem is to make a minimum the total loss of work

$$L = \rho v \sum_{1}^{n} \int_{0}^{r} \frac{\Gamma w dx}{\sin \epsilon} \tag{78}$$

The variation of this quantity must therefore be put equal to zero. We shall proceed, for this purpose, similarly to the way Munk's theorem was deduced in section 27. We shall change by small amounts the circulation at two places, which may reach from x_1 to $x_1 + dx_1$ and from x_2 to $x_2 + dx_2$, in such a manner that the total thrust remains unchanged. According to equation (75) we must make

$$\delta\Gamma_1 \,.\, x_1 dx_1 + \delta\Gamma_2 \,.\, x_2 dx_2 = o \tag{79}$$

Exactly as before the condition for the minimum is obtained if the loss of energy due to our added circulation remains unchanged. In order to calculate the loss, let us make use of theorem No. 3 and assume that the added wing forces are brought into action far behind the propeller so that the loss is merely the product of the added air force by the velocity w^* which arises from the vortices of the propeller, and therefore for the first element is equal to $\rho v \delta\Gamma_1 \frac{w_1^*}{\sin \epsilon} dx_1$. Omitting the constant factor, we obtain as the minimum condition—

$$\delta\Gamma_1 \frac{w_1^*}{\sin \epsilon_1} dx_1 + \delta\Gamma_2 \frac{w_2^*}{\sin \epsilon_2} dx_2 = o,$$

from which is derived, making use of equation (79)

$$\frac{w_1^*}{x_1 \sin \epsilon_1} = \frac{w_2^*}{x_2 \sin \epsilon_2} = \text{const.} \tag{80}$$

We must compare this condition with that obtained for the velocity components normal to a rigid screw surface, when this surface is moved backward with the velocity w'. We then have (see fig. 59):

$$w_n = w' \cos \epsilon.$$

But on a screw surface the pitch h is connected with the angle of pitch ϵ and the radius x by the relation $h = 2\pi x \tan \epsilon$.

Multiplying this last equation with that for w_n, and solving, we get

$$w_n = \frac{2\pi w' x \sin \epsilon}{h}.$$

and therefore

$$\frac{w_n}{x \sin \epsilon} = \text{const.} \tag{81}$$

On comparing (81) with (80) it is seen that by a suitable choice of w' the value of w_n can always be made to agree with that of w^*, which proves Betz' theorem.

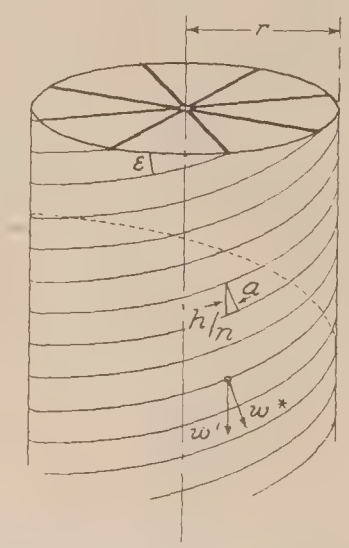

FIG. 59.—Idealized vortex system of a screw-propeller.

33. In order to learn more accurately the nature of the distribution of circulation which we are seeking, we shall proceed as if the velocity field at great distances from the screw is produced by having the velocity w' in the direction of the axis imparted impulsively to the rigid figure composed of the screw surfaces. In a purely qualitative way one can see that with any system of screw surfaces having a small pitch the air in the interior of the system is actually accelerated backward, with, of course, the appearance of tangential velocity components whose intensity is a function of the angle ϵ and is greatest for $\epsilon = 45°$. At the axis itself there is neither an axial nor a tangential acceleration. Less simple are the conditions near the outer boundary of the screw surface where a flow around the edges of the surfaces occurs.

In order to obtain a quantitative statement, we shall for simplicity's sake next think of a screw having a large number of blades. Our rigid figure consists, then, of a very large number

of screw surfaces, lying close together, and therefore the air is led with difficulty into the interior. When the impulse occurs it escapes in the direction of the normals to the screw surface. The radial velocity components w_r will be appreciable only in the neighborhood of the outer boundary of the screw surfaces; further in, we may put it equal to zero approximately. For the tangential components w_t and the axial components w_a, the relations hold, as is easily seen,

$$w_t = w^* \sin \epsilon = w' \cos \epsilon \sin \epsilon$$

$$w_a = w^* \cos \epsilon = w' \cos^2 \epsilon.$$

The angle may be expressed by writing

$$\tan \epsilon = \frac{h}{2\pi x} = \frac{v}{x\omega} = \frac{r'}{x} \tag{82}$$

In this, for brevity's sake, $\frac{h}{2\pi} = \frac{v}{\omega}$ is put equal to r' (r' is that radius for which the pitch of the screw, $\tan \epsilon$, $=1$). Then

$$\sin \epsilon = \frac{r'}{\sqrt{r'^2 + x^2}} \text{ and } \cos \epsilon = \frac{x}{\sqrt{r'^2 + x^2}} \tag{83}$$

and hence

$$w_t = w' \cdot \frac{r'x}{r'^2 + x^2} \text{ and } w_a = w' \frac{x^2}{r'^2 + x^2} \tag{84}$$

We must now determine the circulation around the separate blades as a function of the radius x. For a screw with n blades the total circulation of the vortices inside the circle of radius x coincides with the line integral for the closed circle of radius x; this circulation must evidently equal $n\Gamma$, where Γ is the circulation of one of the screw blades at the point x. From this we have

$$\Gamma = \frac{2\pi x \cdot w_t}{n} = \frac{2\pi r' w'}{n} \cdot \frac{x^2}{r'^2 + x^2} \left(= \frac{h w_a}{n} \right) \tag{85}$$

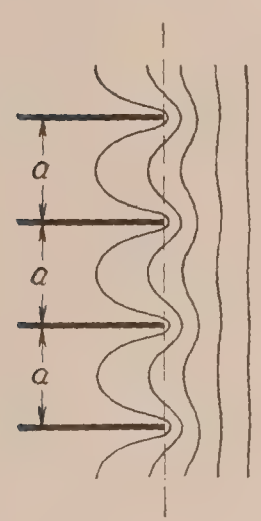

FIG. 60.—Conformal transformation $z = \frac{a}{\pi} \log \frac{1}{2} \left(t + \frac{1}{t} \right)$.

The curve for Γ according to equation (85) is shown in curve I of figure 62.

At the ends of the blades we would expect to have a decrease of the circulation of the same character as found for aerofoils. An approximate treatment can be devised in the following way: We imagine an infinite series of aerofoils which have a distance apart a and are not staggered and which extend infinitely far toward the left. We inquire what is the most favorable distribution of lift near the ends of these aerofoils. The distance a is then to be made equal to the perpendicular distance apart of the edges of two consecutive blades of the screw, i. e., according to figure 59,

$$a = \frac{h}{n} \cos \epsilon_1 = \frac{2\pi r'}{n} \cdot \frac{r^2}{r'^2 + r^2} \tag{86}$$

The problem now, according to the procedure of section 28, may be solved by seeking the potential flow around the edges of the corresponding family of planes and by determining the discontinuity of potential at the planes. This problem may be solved without difficulty by means of conformal representations. (See sec. 10.) It can be shown that the plane with the straight cuts as shown in figure 60, which we shall call the z plane, may be transformed into the unit circle (t plane) by the formula

$$z = \frac{a}{\pi} \log \frac{1}{2} \left(t + \frac{1}{t} \right) \tag{87}$$

The flow of figure 58 is transformed thereby into circulation flow around the unit circle; in fact

$$\Phi + i\Psi = iC \log t$$

After a short calculation, by elimination of t, we have

$$z = \frac{a}{\pi} \log \cos \frac{\Phi + i\Psi}{C} \tag{88}$$

For the surface around which the flow takes place, and which is given by the streamline $\psi = o$, we have, therefore,

$$x = \frac{a}{\pi} \log \cos \frac{\Phi}{C}$$

or

$$\Phi = \pm C \cos^{-1} e^{\frac{x\pi}{a}} \qquad (89)$$

which gives the real values for negative values of x. The potentials thus obtained or the velocity equal to 1 of the free flow $\left(\text{to obtain which } C \text{ must be put equal to } \frac{a}{\pi}\right)$, which, according to what has gone before, give us the surface F', form a picture such as is shown in figure 61. By means of this one can form a definite judgment as to how the circulation, and with it the thrust also, decreases at the blade tips. We can replace the shaded portions of Figure 61 by a straight line, having an equal area below it which, in accordance with the integration performed, must lie behind the blade tips at the distance

$$a' = a \log \frac{2}{\pi} = 0.2207a \qquad (90)$$

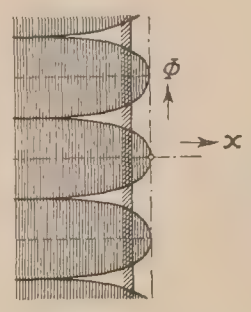

FIG. 61.—Potential obtained by the flow of figure 60.

We conclude from this that, with screws also, the decrease of circulation at the blade tips has about the same effect as if the screw had a radius diminished by $0.2207a$, and then the air would be considered uniform in every circle of radius x (as would be the case for a screw having an infinite number of blades).

The properties found for the inner portion of the screw and for its edge may be combined into a single formula, which can be applied as an approximation formula also for screws having a small number of blades. This formula is obtained by multiplying the value in equation (85) by the expression $\frac{2}{\pi} \cos^{-1} e^{\frac{-\pi(r-x)}{a}}$, which for large values of $\frac{r-x}{a}$ takes the value 1. (For $-x$ of formula (89) $r-x$ is here substituted, as is obvious.) Thus we obtain the formula

$$\Gamma = \frac{4w'r'}{n} \cdot \frac{x^2}{r'^2 + x^2} \cdot \cos^{-1} e^{\frac{-\pi(r-x)}{a}} \qquad (91)$$

The curve of Γ according to equation (91) is given in figure 62, for a 4-blade screw and for $r' : r = 1 : 5$, which correspond to average conditions in practice.

The whole deduction holds, as has already been remarked, for screws which are not heavily loaded. For screws with heavy loads an improvement can be introduced by calculating the pitch of the screw surfaces formed by the vortices, corresponding to the state of flow prevailing in the circular plane of the screw. Instead of writing $\tan \xi = \frac{v}{x\omega}$, we must write, more exactly, $\tan \xi = \frac{V + \frac{w_a}{2}}{x\omega - \frac{w_t}{2}}$, in which V is the velocity of flight, since in the screw disk plane half of the final disturbance velocities is already present. A useful approximation is obtained if, retaining our formulae, v is put equal to $V + \frac{w'}{2}$, and therefore r' is put equal to $\frac{V + \frac{w'}{2}}{\omega}$.

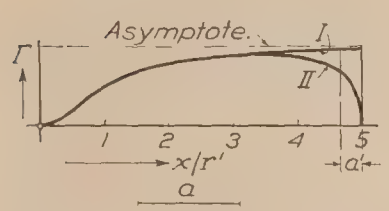

FIG. 62.—Distribution of circulation. I. Infinite number of blades. II. Four blades a is the distance of two vortices. (See Fig. 59.)

After the circulation is known, the distribution of thrust and torque may be calculated easily by means of equations (74) and (76), and thus, following the method used in the aerofoil theory, the requisite widths of the blades and angles of attack may be determined in order that for a given working condition (i. e., r' and w' given), in which the screw is to have the most favorable performance, all the information may be deduced from the theory. By taking into

account the more exact velocity relations in the propeller-disk plane this information may be improved.

The aerofoil theory has numerous further applications. An investigation of curved flights specially of the moment—important in discussions of stability—around the longitudinal axis in the case of a wing moved in a circle, is at present being made, also the calculation of the moment of a warped wing. A series of not unimportant single questions must wait for a further improvement of the theory, e. g., various conclusions specially concerning properties of profiles, influence of curvature, etc., can be reached, if we pass from the lifting line to the case of a load distributed also along the chord; for the treatment of a wing set oblique to the direction of flight the assumption of a load distributed along the chord is necessary since in this case the conditions contradict the "lifting line." Investigations of this kind, which can be accomplished only by very comprehensive numerical calculations, were begun during the war but since then, owing to a lack of fellow workers, have had to remain unfinished. A similar statement also applies to the calculations of a flapping wing already begun, in which one is likewise forced to assume the lift distributed along the chord, since otherwise the result is indefinite. Therefore much remains to be done.

MOST IMPORTANT SYMBOLS.

ρ =density.
V =velocity of the airplane.
u, v, w=velocity components in the X, Y, Z directions. (In the case of an airplane X is in the direction of the span of the wings, Y is in the direction of flight, Z is vertical.)
$q=\frac{\rho V^2}{2}$=dynamical pressure.
b =span of a wing ("Breite").
t =chord of a wing ("Tiefe").
h =gap of a biplane ("Höhe").
F =area of surface (=b . t) ("Fläche").
A =lift ("Auftrieb").
W =drag ("Widerstand").
$c_a=\frac{A}{Fq}$=lift coefficient (=2 K_y "absolute").
$c_w=\frac{W}{Fq}$=drag coefficient (=2 K_x "absolute").
α =angle of attack.
Γ =circulation.
Φ =velocity potential.
ψ =stream function.

LIST OF THE MOST IMPORTANT LITERATURE.

Abbreviations: Z. F. M.=Zeitschrift für Flugtechnik und Motorluftschiffahrt.
TB =Technische Berichte der Flugzeugmeisterei.

A. GÖTTINGEN PUBLICATIONS.

L. Prandtl: Tragflügeltheorie I. und II. Mitteilung. Nachr. von der Kgl. Gesellschaft der Wissenschaften. Math.-phys. Klasse 1918, S. 451, u. 1919, S. 107. (Aerofoil Theory, I and II Communications. Nachr. Kgl. Gesellschaft der Wissenschaft. Math.-phys. Classe, Göttingen, 1918, p. 451, and 1919, p. 107.)

—— Tragflächen-Auftrieb und -Widerstand in der Theorie. Jahrb. der Wissenschaftlichen Gesellschaft f. Luftfahrt, V. 1920, S. 37. (Aerofoil Lift and Drag in Theory. Jahrb. d. Wissens. Gesells. f. Luftfahrt, V. 1920, p. 37.)

—— Der induzierte Widerstand von Mehrdeckern. TB Bd. III, S. 309. (The Induced Drag of Multiplanes. TB. Vol. III, p. 309.)

G. Fuhrmann: Theoretische und experimentelle Untersuchungen an Ballonmodellen. Jahrbuch 1911/12 der Motorluftschiff-Studiengesellschaft, S. 65. (Theoretical and Experimental Investigations of Models of Airship Bodies. Jahrb. 1911/12 d. Motor Luftschiff-Studiengesells, p. 65.)

A. Betz: Die gegenseitige Beeinflussung zweier Tragflächen. Z. F. M., 1914, S. 253. (The Mutual Influence of Two Aerofoils. Z. F. M., 1914, p. 253.)

—— Untersuchung einer Joukowski'schen Tragfläche. Z. F. M., 1915, S. 173. (Investigation of a Joukowski Aerofoil. Z. F. M., 1915, p. 173.)

—— Einfluss der Spannweite und Flächenbelastung auf die Luftkräfte von Tragflächen. TB, Bd. I, S. 98. (Influence of Span and Wing-Loading upon the Air forces of Aerofoils. TB, Vol. I, p. 98.)

—— Beiträge zur Tragflügeltheorie mit besonderer Berücksichtigung des einfachen rechteckigen Flügels. Dissertation, Göttingen, 1919. Auszug in Beiheft II der Z. F. M., 1920, S. 1. (Contributions to the Theory of Aerofoils with Special Reference to the Simple Rectangular Wing. Dissertation, Göttingen, 1919. Extract in Beiheft. II Z. F. M., 1920, p. 1.)

—— Schraubenpropeller mit geringstem Energieverlust, mit einem Zusatz von L. Prandtl, Nachr. v. d. Kgl. Gesellschaftder Wissenschaften, Math.-phys. Klasse 1919, S. 193. (The Screw Propeller having the Least Loss of Energy, with an Appendix by L. Prandtl. Nachr. Kgl. Gesellschaft der Wissenschaft, Math.-phys. Class, Göttingen, 1919, p. 193.)

—— Eine Erweiterung der Schraubenstrahltheorie. Z. F. M., 1920, S. 105. (An Extension of the Theory of Screw Jets. Z. F. M., 1920, p. 105.)

M. Munk: Beitrag zur Aerodynamik der Flugzeugtragorgane. TB, Bd. II, S. 187. (Contribution to the Aerodynamics of Lifting Airplane Members. TB., Vol. II, p. 187)

—— Isoperimetrische Aufgaben aus der Theorie des Fluges. Dissertation, Göttingen, 1919. (Isoperimetric problems from the Theory of Flight. Dissertation, Göttingen, 1919.)

C. Wieselsberger: Beitrag zur Erklärung des Winkelfluges einiger Zugvögel. Z. F. M., 1914, S. 225. (Contribution to the Explanation of the Formation Flight of Migratory Birds. Z. F. M., 1914, p. 225.)

—— Experimentelle Prüfung der Umrechnungsformeln. In "Ergebnisse der Aerodynamischen Versuchsanstalt 1. Lieferung," München, 1921, S. 50. (Experimental Examination of the Formulae for Relations between Aerofoils. In "Ergebnisse der Aerodynamischen Versuchsanstalt," Part I, Munich, 1921, p. 50.)

B. WORKS ON THE TWO DIMENSIONAL PROBLEM.

W. M. Kutta: "Auftriebskräfte in strömenden Flüssigkeiten." Illustr. aeronaut. Mitteilungen, 1902, S. 133. Ausführlichere Abbandlungen in den Sitzungsber. der Bayr. Akad. d. Wiss., Math.-Phys. Klasse 1910, 2. Abh. und 1911, S. 65. (Forces of Lift in Flowing Fluids. Illustr. aeronaut. Mitteilungen, 1902, p. 133. More detailed papers in the reports of the proceedings of the Bavarian Acad. of Sciences, Math.-Phys. Class 1910, 2d report, and 1911, p. 65.)

N. Joukowski: Ueber die Konturen der Tragflächen der Drachenflieger. Z. F. M., 1910, S. 281. "Aerodynamique" aus dem Russischen übersetzt von Drzewiecki, Paris, 1916. (On the Contours of Airplane Wings. Z. F. M., 1910, p. 281. "Aerodynamics" translated from the Russian by Drzewiecki, Paris, 1916.

R. Grammel: "Die hydrodynamischen Grundlagen des Fluges." Braunschweig, 1917. (The Hydrodynamic Principles of Flight. Brunswick, 1917.)

R. v. Mises: Zur Theorie des Tragflächenauftriebs. Z. F. M., 1917, S. 157, 1920, S. 68 und 87. (The Theory of Lift of Wings. Z. F. M., 1917, p. 157, 1920, pp. 68 and 87.)

After this memoir was written, two papers, by R. Fuchs and E. Trefftz, on the theory of aerofoils appeared, both of which discuss the theory of a monoplane and that of airplanes of least drag. These papers are published in the "Zeitschrift für Angewandte Mathematik und Mechanik," 1921, Heft 2 u. 3, Berlin.

REPORT No. 117

THE DRAG OF ZEPPELIN AIRSHIPS

By MAX M. MUNK

National Advisory Committee for Aeronautics

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INTRODUCTION.

This report was prepared for the National Advisory Committee for Aeronautics, and is a discussion of the results of tests with Zeppelin airships, in which the propellers were stopped as quickly as possible while the airship was in full flight. Some details of these tests are described in a paper by V. Soden and Dornier.[1] They were continued after that publication and cover a series of interesting types. In this paper I intend to refer to the theory involved in these tests and to one scientifically interesting fact which can be derived from them and which has not yet been noted.

The chief general question concerning these tests is, of course: Does the negative acceleration of an airship with stopped propellers supply proper data for determining the drag of the airship when in uniform flight? This can not absolutely be answered in the affirmative, the two phenomena not being identical in principle. We believe, however, that in this particular case the agreement is sufficient and that the data obtained from the test are the true or, at least, the approximate quantities wanted. We have several strong reasons for our opinion and will proceed to discuss them.

MOTION IN A NONVISCOUS FLUID.

Consider in the first place what motion of the airship is to be expected. It is generally believed—and the following tests confirm the belief to a certain degree—that the drag of an airship can be represented by an expression of the form.

$$D = A \cdot V^2 \rho/2 \tag{1}$$

where A is a constant which has the dimension of an area and may therefore be called the area of drag; V is the velocity of flight; and $\rho/2$ is half the density of the surrounding air. The mass of the floating ship is equal to the mass of the displaced fluid and is therefore

$$M = v \cdot \rho \tag{2}$$

where v is the displacement of the ship. Hence, according to the general law of mechanics, the motion after the propellers stop is determined by the equation

$$-d/dt(V \cdot v\rho) = A V^2 \cdot \rho/2 \tag{3}$$

in which, however, the influence of the retardation on the drag itself is not yet taken into consideration. By integrating (3) two times we obtain successively

$$V = (2v/A)/(t + c_1) \tag{4}$$

$$\frac{X + c_2}{(2v/A)} = \log \frac{t + c_1}{c_3} \tag{5}$$

where c_1, c_2, and c_3 are three constants of integration determinable by the initial conditions.

[1] V. Soden and Dornier, Mitteilungen des Luftschiffbau Zeppelin in Friedrichshafen, Die Bestimmung des Schiffswiderstandes durch den Fahrtversuch, Zeitschrift für Flugt. und Motorl., 1911.

We chose the situation of $x=0$ and the origin of time $t=0$ so that c_1 and c_2 are zero and c_3 is the unity of time; and we have then

(6*) $$V=\frac{(2v/A)}{t}$$

(7*) $$\frac{X}{(2v/A)}=\log t$$

$(2v/A)$ has the dimension of a length, characteristic of the motion of the ship. In this paper we will call it the "characteristic length" of the ship and denote it s.

(8) $$s=(2v/A) \qquad \text{(Definition)}.$$

Then we obtain

(6) $$V=s/t$$

(7) $$X=s \log t.$$

At any moment the ship moves with a velocity such as would be necessary to cover the constant length s in the time elapsed from a constant origin of time.

We proceed now to take into account the difference between the drag in uniform flight and that in retarded flight; and shall consider, in the first place, the conditions of flight in a nonviscous fluid.

In such a fluid a uniformly moving body would in general have no drag at all. When a solid is moving in a fluid, the latter possesses kinetic energy proportional to the square of the velocity of the solid. Consequently, when the solid is retarded, the fluid itself must lose kinetic energy; and this means that the force opposing the motion of the fluid must have an equal reaction on the solid. This reaction is in such a direction as to oppose the change, i. e., it is a negative drag, tending to accelerate the solid. This energy is given the fluid by the force necessary to put the body into motion and is given back if the body is being retarded. The effect of this kinetic energy of the fluid is the same as if the body had a constant increment of mass, additional to its own mass. The force in question is perfectly taken into account if the displacement v in (8) is increased by a corresponding increment of volume.

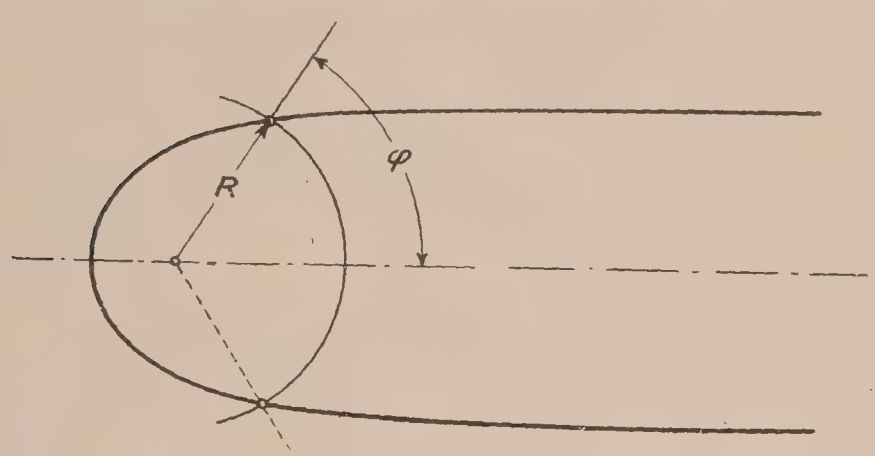


Fig. 1.

There is no difficulty in calculating this increment as exactly as desired. We will, however, confine ourselves to the simplest proper assumption, believing this to be quite sufficient for the present purpose, and for the conclusions we are about to make. We will limit ourselves to the case of a very long airship, so that the influence of each end on the other is small and may be neglected. Moreover, we shall assume the ends to be so shaped as to be capable of representation by the combination of the flow from a point source and the constant velocity V[2]. The intensity of the point source must be

(8) $$I=r^2\pi V$$

where r is the radius of the greatest section of the ship. Let the point source be situated at the origin of a system of polar coordinates R, φ. The fluid passing in unit of time through a spherical segment, $R=$constant, within the cone $\varphi=$constant is composed of two parts, one due to the constant velocity V, and the other to the point source. The first part is $(R \sin \varphi)^2\pi . V$ and the second part is $\frac{1}{2}(1-\cos \varphi) . r^2\pi . V$. If the edge of the spherical segment coincides

[2] Compare Report No. 114.

with the surface of the airship in question, the entire fluid passing, that is to say, the sum of these two expressions is equal to the intensity of the source $r^2 \pi V$., whence we obtain the equation of the airship body.

(9) $$r^2\pi V = (R \sin \varphi)^2 \pi V + \frac{1}{2}(1-\cos \varphi) r^2 \pi V$$

or, transformed,

(10) $$R = r \cdot \frac{\cos \varphi/2}{\sin \varphi}$$

As was to be expected, the shape is independent of the velocity.

Now we proceed to calculate the kinetic energy of the fluid outside the airship body, assuming the airship moving in air at rest. The motion of the air is entirely represented by the point source. At the distance R the velocity is

$$\frac{r^2\pi V}{4\pi R^2}$$

and the potential is

$$\Phi = -\frac{r^2\pi V}{4\pi R}.$$

Through a spherical zone between the two cones $\varphi = \varphi_1$ and $\varphi = \varphi_1 + d\varphi$, with the area $2R^2\pi \sin \varphi_1 d\varphi_1$ the fluid passing in unit of time is $d\psi = \frac{1}{2} r^2 \pi V \sin \varphi_1 d\varphi_1$.

The space integral of the kinetic energy can easily be transformed into a surface integral.[3] Twice the kinetic energy can be represented by the integral

(11) $$2T = \rho \int\int \Phi d\psi$$

which is to be performed over the surface of the body. Substituting in (11) the expressions Φ and ψ before mentioned, and replacing R by the right side of (10), it appears that

(12) $$2T = \frac{1}{8} r^3 \pi V^2 \cdot \rho \int_0^\pi \frac{\sin^2 \varphi}{\cos \varphi/2} d\varphi$$

The integrant in (12) can be transformed into $4 \frac{\sin^2 \varphi/2 \,.\, \cos^2 \varphi/2}{\cos \varphi/2} d\varphi$ the integral of which is $\frac{4}{3} \sin^3 \varphi/2$. Hence

(13) $$2T = \frac{1}{6} r^3 \pi V^2 \rho$$

Each end of the airship gives rise to the same kinetic energy, so that $2T$ is the total energy. This equals one-fourth of the energy which a sphere of the fluid would have if moving with velocity V, whose radius is the radius of the largest cross section of the airship. This gives us the apparent increment of mass of the airship, and is equivalent to about $2\frac{1}{2}\%$ of the entire volume.

The error due to our two assumptions with respect to the shape of the ship and to its length is not great. The share of a particle of the fluid in contributing to the energy decreases as the third power of R, and is small to the same degree that R is great when compared with $\frac{1}{2}r$, the distance of the point source from the head of the ship. Nor do we believe that the influence of the shape of the ship is great. In any case, the increment of the mass is so small when compared with the entire mass of the ship that it little matters whether the error in the increment is a little smaller or greater. It is only the order of magnitude of the increment that we intended to calculate.

[3] See Lamb. Hydrodynamics, section 61.

The exact calculation of the increment for several forms, however, would be useful too. It could be based with advantage on a valuable paper of Fuhrmann[4] on a similar theme. The integration could be performed graphically.

MOTION IN A VISCOUS FLUID.

The preceding calculation shows the increment of mass to be about $2\frac{1}{2}$ per cent of the mass of the ship. Accordingly, the force on the ship due to the retardation of the surrounding flow is only $2\frac{1}{2}$ per cent of the drag due to the viscosity of the air. Hence also the change in the distribution of pressure is small. If the fluid motion near the surface is stable, we can not expect it to be affected by so small a change in the distribution of pressure. The tests on airships show that within certain limits the drag is proportional to the square of velocity. This points to stability of flow for these particular ships. In this case we do not even expect a small change of the flow and of the drag.

The quantity deduced above is, so to speak, the influence of the motion in frictionless air on the friction. In reality the matter is more complicated. The ship leaves behind it a stream of air, following with a velocity less than the velocity of flight; this may be called the "wake."

It is possible to obtain a certain notion of the magnitude of the effects. Generally the velocity of the air in the wake can not surpass the velocity of flight. But even if it were as great, the air would remain in the neighborhood of the ship, there would be no space for the new wake to be formed, steady motion could not occur, and the phenomenon would not agree with the facts. Let V be the velocity of flight and V' the average velocity of the following stream of air. This air occupies a cylinder with the radius r' behind the ship. Then the volume of this cylinder filled with air in the unit of time is $r'^2\pi(V-V')$, and its momentum is

$$M=r'^2\pi(V-V')V'\rho \tag{14}$$

The radius r' has a minimum if V' is $\frac{1}{2}V$. Let us assume this for the present so that $M=\frac{1}{4}V^2\rho\pi r'^2$. If the coefficient of drag with respect to the section πr^2 is about .08, as it appears to be in the following tests, we would obtain for the momentum given the wake $.08\,V^2\frac{\rho}{2}\pi r^2$. Hence,

$$.08\,V^2\frac{\rho}{2}\cdot r^2\pi=\tfrac{1}{4}V^2\rho r'^2\pi$$

and therefore,

$$\frac{r'}{r}=.40.$$

Tests with airship models show that the distribution of pressure agrees with the theoretical value about up to $r'=\frac{1}{2}r$ at the rear end. For this reason it is probable that the mean velocity of the air in the wake is indeed not very different from half the velocity of flight. It would be interesting and important to determine it more exactly by model tests. As far as we know such tests have not yet been published. For the present purpose the exactness of our assumption is sufficient.

If the ship is retarded, the air in the wake, owing to its momentum, meets air with a less velocity, and pushes it aside. At last the air overtakes the ship. We will calculate what velocity the ship has at this moment of meeting. The following air has traveled the distance

$$\tfrac{1}{2}V_1(t_2-t_1)$$

V_1 being the velocity of the ship at the time t_1, and t_2 being the instant of the meeting. According to equation (7) the ship has traveled in the same time

$$s\log\frac{t_2}{t_1}$$

[4] Georg Fuhrmann: Theoretische und experimentelle Untersuchungen an Ballonmodellen. Jahrbuch der Motorluftschiff Studienges. 1911/12.

That is,

$$\frac{1}{2} V_1 (t_2 - t_1) = s \log \frac{t_2}{t_1}.$$

Hence from equation (7)

$$\frac{1}{2} V_1 \left(\frac{s}{V_2} - \frac{s}{V_1} \right) = s \log \frac{V_1}{V_2}$$

or

$$\log \frac{V_1}{V_2} = \frac{1}{2} \left(\frac{V_1}{V_2} - 1 \right)$$

The solution of this equation is $\frac{V_1}{V_2} = 3.5$. If the velocity has decreased to the 3.5th part of its value the air running after the ship overtakes it.

This air in the wake has the same direction as the ship, and therefore its momentum would decrease the drag of the ship and its retardation. The test, however, would show such an effect only if the pitot tube attached to the airship opens into air at rest, as in normal flight. In this case, when the air of the wake overtakes and surrounds the ship, the indicated velocity would cease to decrease before the velocity zero is reached.

If, on the other hand, the pitot tube is within the moving air, the test would show a more or less sudden increase of retardation, the following air having perhaps a greater velocity than the ship itself. This retardation, however, ceases soon, or at least decreases considerably.

We do not believe that the following air meets the ship at all. The distance the ship covers between the beginning of the retardation and the meeting with the following stream of air would be according to equation (7)

$$x_2 - x_1 = s \log \frac{t_2}{t_1} = s \log \frac{V_1}{V_2} = s \cdot \log 3.5.$$

The tests show s to be about 11,000 feet, and $\log 3.5 = 1.25$; so that the distance would be about 13,700 feet = 2.5 miles. We can hardly imagine that in the greater number of tests the course of the ship was so exactly straight and the wind so uniform that after about 2 miles the ship has not left her path by half its diameter, i. e., by 40 feet. The stream of following air seems more likely to be dissolved during so long a distance. But even if it should meet the ship with undiminished velocity, the pitot tube would not be within it at each test. The cylinder of the radius $\frac{1}{2} r$ when distributed around a cylinder of the radius r occupies a tube with a thickness of wall $r\ (\sqrt{1.5} - 1) = .22\ r$, or about 8 feet. The distance between the tube and the ship was in practice about 10 feet.

THE RESULTS OF THE TESTS.

We are now prepared to consider the results of the tests and to examine them with respect to the possibilities mentioned. Each of the curves represents a single test. $1/V$ is plotted against the time. According to equation (6) the tangent of the angle between the direction of the curve and the vertical axis of coordinates is the characteristic length of the ship. If the drag is proportional to the square of the velocity this length s is constant and the curve is a straight line. If the plotted points are lying on a uniformly curved line, the coefficient of drag changes continuously; if the curve has a sudden break, the coefficient changes discontinuously and suddenly from one value to another. On the right hand of each diagram a scale for the velocity in mi/h. is added.

L. Z. 10 is one of the oldest ships. Its velocity was only 19 *m/sec.* (*42.5 mi/h*).[5] All points lie as exactly on a straight line as can be expected. The same can be said of the test with *L 43*. *L 59* is the only ship the test with which gave a slightly curved line. The cofficient of drag was not constant during the test as with the two first-mentioned ships, but slowly increased as the velocity decreased.

[5] Jaray, in the paper "Studien zur Entwicklung der Luftfahrzeuge," in Zeitschr. f. Fl. u. Motorl. gives 21 m/s; we agree with V. Soden and Dornier in the above-mentioned papers. In the mentioned paper some other details of the ships can be found.

The remaining ships, six different ones, the tests of which were made at different times and independent of each other, show a very remarkable result. The points lie on two different straight lines, which cut each other at a definite angle. Look, for instance, at the curve of *L 44*. The first and the last points are somewhat irregular; and we think that at the first point the engines were not yet completely stopped and at the last point the dynamical pressure was too small to be obtained correctly. The other points coincide with the two straight lines mentioned; the first line is given by 7 points and the second by 4 points. We think that the genuineness of the broken line obtained with six ships out of nine can not be doubted. It is not improbable even that *LZ 10* also would have given a broken line if the test could have been begun at a higher velocity. We do not know whether *L 43* and *L 59* also have discontinuities outside the range of the test.

The curves show that the velocity decreases steadily at the end of each test, and that the retardation, but for one sudden discontinuity, is very regular and in accordance with equation (6). Neither does the velocity become constant at a definite value, nor can we find an increase of the retardation followed by a decrease. We are unable to find any indication of the following stream of air meeting the ship. We have but one explanation for the result of the test. The retarding ship suddenly changes its coefficient of drag, it being increased. Nor is this in disagreement with other experiments of aerodynamics. On the contrary, there is scarcely any body, if there is at all, the motion of the fluid around which or within which does not suddenly and discontinuously change under some particular conditions. We state that the airships investigated are no exceptions to this rule, but that such a discontinuity happens and the coefficient of drag suddenly changes if the ships are retarding and their velocity is 70 to 80 mi/h.

The reason is the same as in other cases, a sudden change of the motion of air, which has become unstable. It is not quite certain whether this change would also happen if the ships would be accelerated or if they were in uniform flight. We believe this change would occur either at the same velocity or at a velocity in the neighborhood.

Our opinion is supported by the particular values of the characteristic lengths obtained. These values are such that only the first ones, those obtained at the higher velocity, agree with the magnitude of the absorbed power of the ship.

According to the definition (8) $A=2v/s$, where v, the displacement of the ship is properly to be increased by $\pi/3\ r^3$, according to equation (13). This improvement is not very considerable. This area of drag A still contains the area of drag of the propellers, which we estimate to be 6.5 square feet per engine. After subtracting a corresponding value, we obtain an improved area of drag of the ship which may also be denoted by A. The required power then is

$$P=A\cdot q\cdot V\cdot \eta \tag{15}$$

where q is the dynamical pressure and η the efficiency of the propellers. The efficiency can be calculated by using this equation and its value is put into the table for the two characteristic lengths obtained in the columns headed η_1 and η_2.

The density of air was assumed to be that of sea level under mean conditions. It was not so, of course, the ship actually flying at some height. The publication of Soden and Dornier hints at a height of about 1,450 feet. But the power of the engines decreases as much as the density, and the result of the calculation remains unaltered. Only the last ship, *L 70*, is in exception, the engines of which are supercompressed. For this ship we assumed a height of flight of about 2,600 feet and put into a table a correspondingly increased value of the horsepower.

The values obtained for the efficiency, however, are rather uncertain. In (15) V occurs in the third power, the dynamical pressure q containing the square of V. Small differences of the velocity therefore considerably change the result, and the velocity of the ships is by no means very exactly known.

We think, however, that the values obtained from s_1, i. e., from the characteristic length at higher velocity, give better values. With the ships *L36*, *L44*, *L57*, and *L59* there are greater differences of the two lengths. In all these cases the efficiency obtained from the smaller characteristic length is too high.

The chief part of the theoretical loss of a propeller is [6]

$$1-\eta=\frac{\sqrt{1+c_p}-1}{\sqrt{1+c_p}+1};\quad c_p=\frac{T}{D^2\frac{\pi}{4}\cdot q}=\frac{N\cdot\eta}{D^2\frac{\pi}{4}\cdot q\cdot v}$$

where T denotes the thrust, N the required power, D the diameter, q the dynamical pressure, and η the efficiency. For one propeller of *L57*, for instance, we would obtain

$$c_p=\frac{240\text{ hp}\cdot 550\cdot .69}{75\text{ sq. ft.}\cdot 64.5^2\text{ (mi/h)}^2\cdot\frac{1}{390}\cdot 64.5\text{ mi/hr. }1.47}=1.77;$$

$$1-\eta=\frac{\sqrt{2.77}-1}{\sqrt{2.77}+1}=25\%$$

The loss as calculated from s_1 is $1-0.69=31\%$, which is in excess of the theoretical loss. The remaining loss of 14 per cent is well explained by the limited number of blades, by the rotation of the propeller stream, by the drag of the blades, and by the friction of the gear. The efficiency due to the second length s_2 is so high that it does not even account for the theoretical loss. The other ships give a similar result.

It is also possible to estimate the effective area of wing of the cars, struts, ropes, and, last but not least, radiators. This drag was considerably higher with the older ships than with the newer ones. In spite of it we assumed a drag of 6.5 square feet per engine, and subtracted this amount from the area of drag. The remainder still contains a part of these resistances with the older ships. The percentage, however, is not high. The area of the drag of the airship body, so obtained, is divided by the two-thirds power of the deplacement. An absolute coefficient for the drag of the airship body results. It is put into the last column of the table.

The coefficients obtained look reasonable. The first ship, *L10*, has a very old-fashioned shape and accordingly a high drag. The next three ships, *L33*, *L36*, and *L43*, are similar and also belong together with respect to their coefficients. Their coefficients differ indeed, but their mean, .028, forms a distinct difference from the other ships. The next two ships, *L44* and *L46*, are similar to *L70*. They all have the same coefficient, .020. *L57* and *L59* have a more slender form, they are longer, and accordingly their coefficient is somewhat higher. This fact indicates also that the skin friction is an essential part of the drag. The value of s obtained for *L33* seems less reasonable than the values obtained for the other ships.

TABLE OF TESTS.

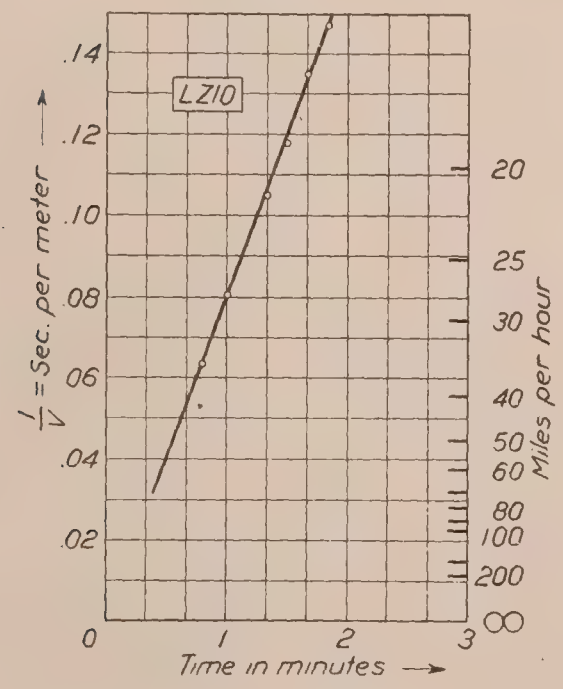
	No. of airship.	Name.	Displacement.	Diameter.	Increment.	Length.	s_1	s_2	Discontinuity at—	Number of engines.	Horsepower.	Maximum velocity.	A	η_1	η_2	D_c
			Cu. ft.	*Feet.*	*Cu. ft.*	*Feet.*	*Feet.*	*Feet.*	*mi/h.*			*mi/h.*	*Sq. ft.*			
1	10	L Z 10	706,000	45.9	12,600	460	2,460	2,460	.....	3	450	42.5	565	67	.....	0.069
2	76	L 33	2,140,000	78.3	62,800	645	9,800	9,340	21	6	1,440	63.0	409	49	53	.025
3	82	L 36	2,140,000	78.3	62,800	645	7,900	4,240	22	6	1,440	63.0	519	62	120	.029
4	92	L 43	2,140,000	78.3	62,800	645	7,900	7,900	.....	5	1,200	60.5	525	56	.....	.030
5	93	L 44	2,140,000	78.3	62,800	645	10,900	7,800	26	5	1,200	64.0	371	56	80	.020
6	94	L 46	2,140,000	78.3	62,800	645	11,100	8,560	25	5	1,200	65.0	364	58	77	.020
7	102	L 57	2,640,000	78.3	62,800	745	11,300	9,720	29	5	1,200	64.5	445	69	81	.022
8	104	L 59	2,640,000	78.3	62,800	745	11,800	7,580	Steady.	5	1,200	64.4	425	66	105	.021
9	112	L 70	2,400,000	78.3	62,800	694	10,800	9,160	33	7	2,000	77.3	405	65	78	.020

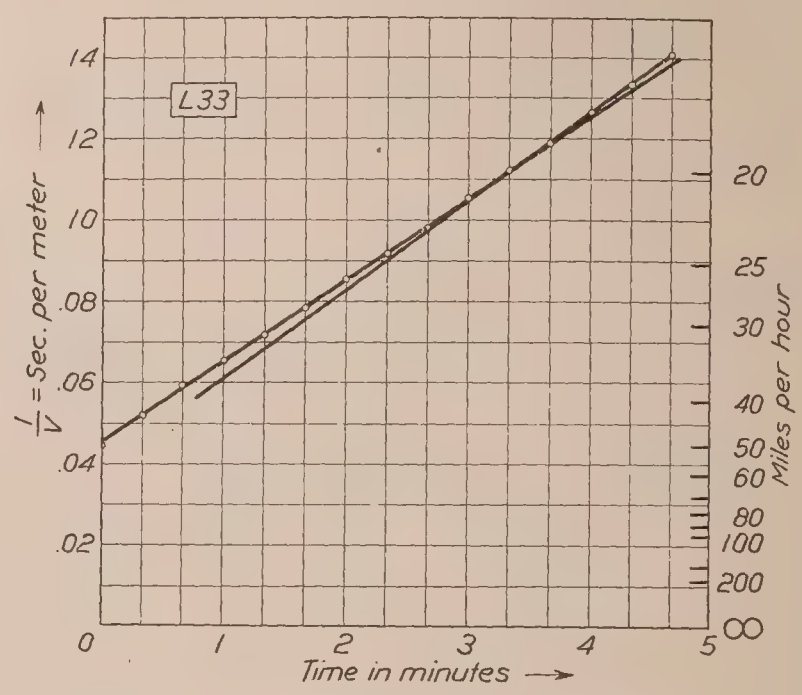
CONCLUSION.

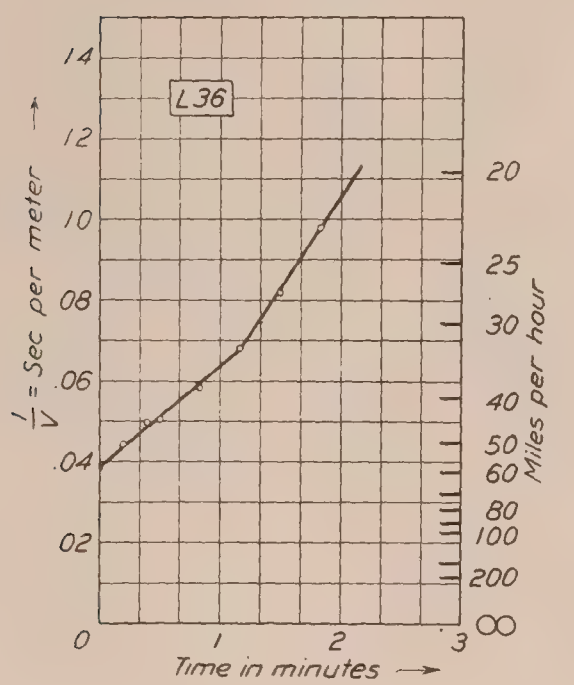
These are special results, and the tests show also one general result. If the drag coefficient changes considerably during the tests, it is almost sure that it would change on enlarging a model to its hundredfold linear dimension or to the millionfold volume. Under these circumstances it is quite useless to make model tests for the determination of the drag of particular airships, if in some way the effect of the change of scale can not be eliminated. It

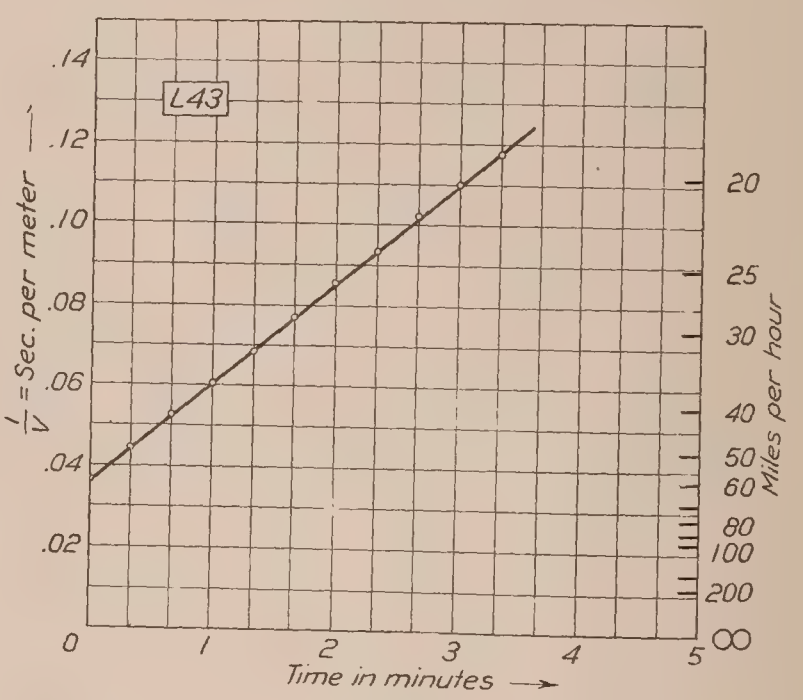
[6] See Report No. 114.

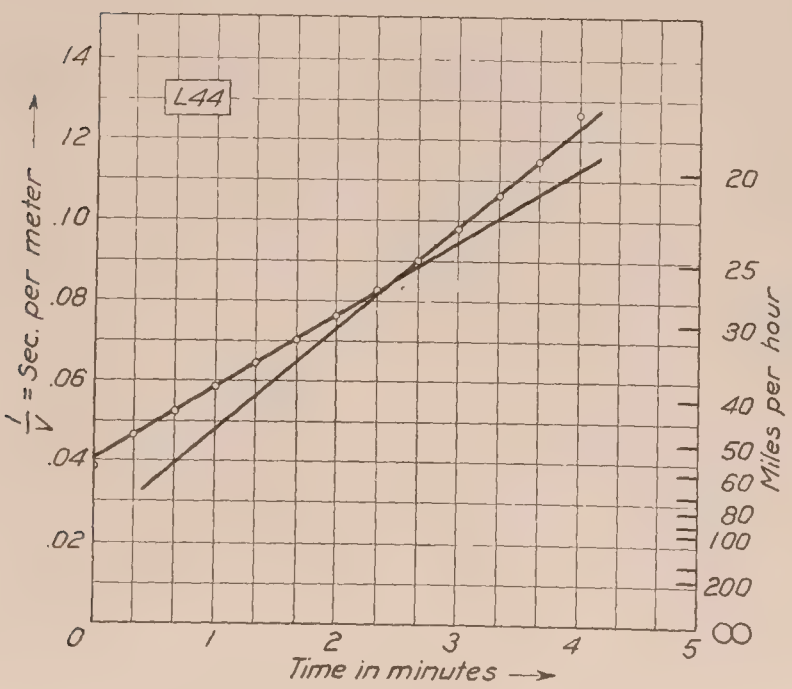
may be that in particular cases the coefficient obtained by the model test happens to agree with the full-sized coefficient; but that proves nothing. Different motions of the air may produce the same coefficient of drag. In consequence of the scale effect, it never is certain that one airship form is better than another, if the corresponding model gives a smaller drag at a Reynolds number 100 times as small.

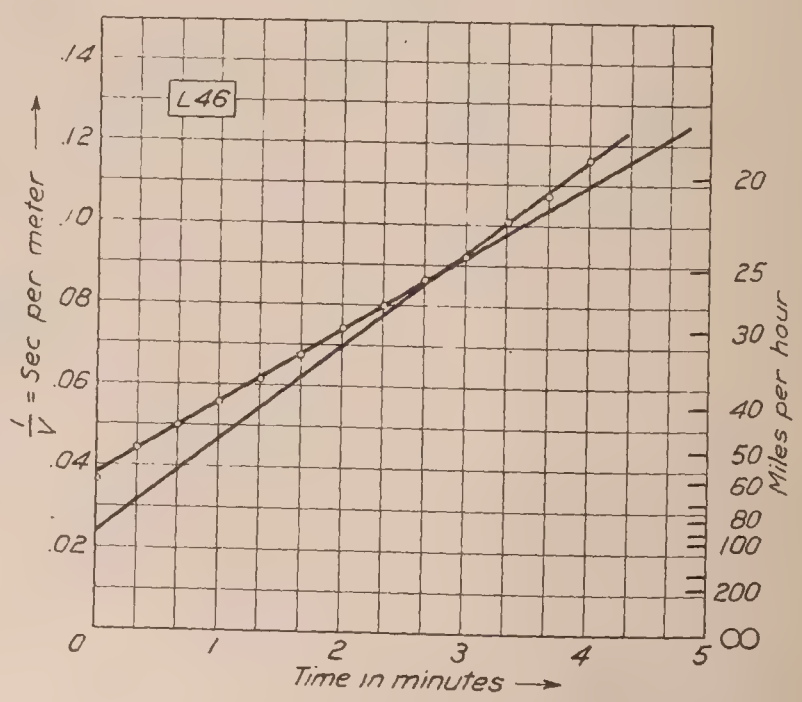


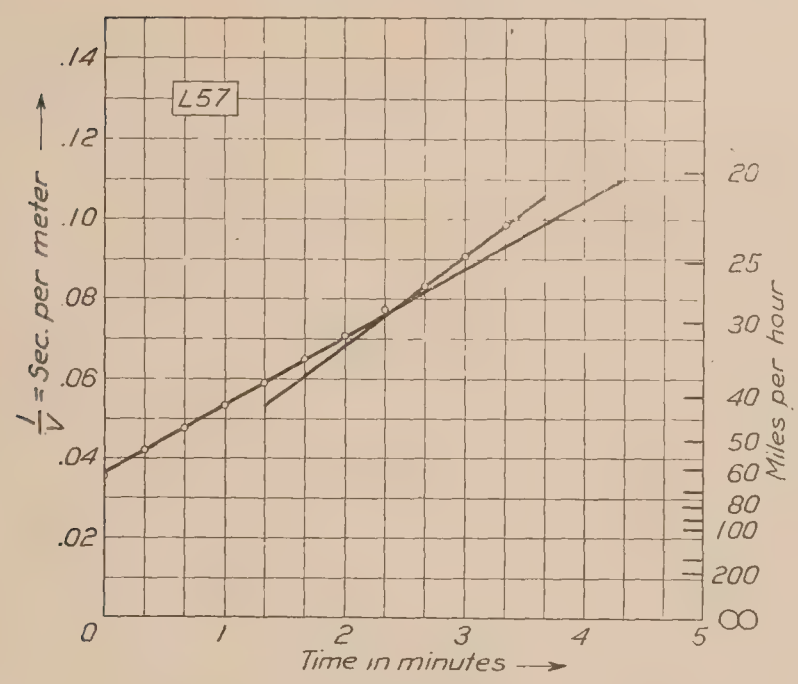











L57
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Time in minutes
1/V = Sec. per meter
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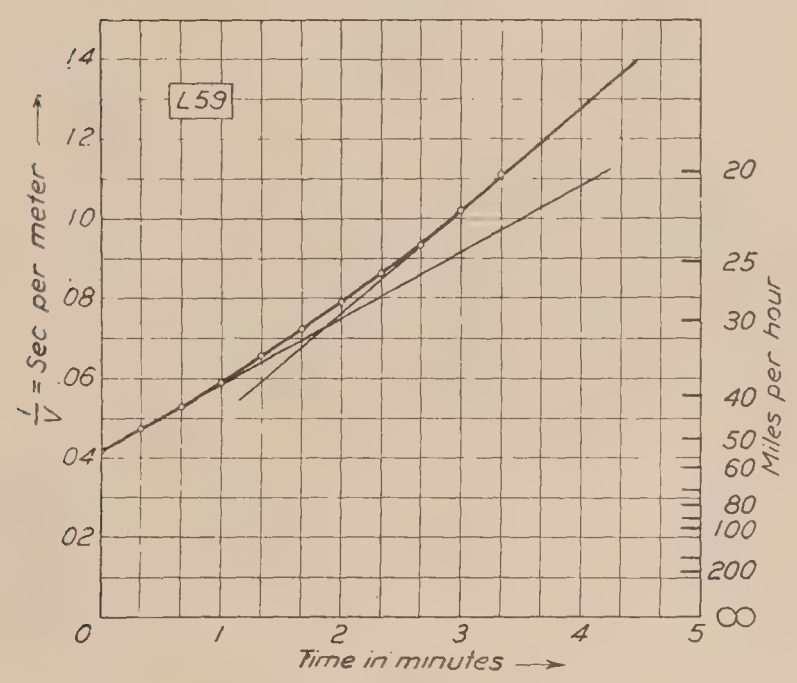
Miles per hour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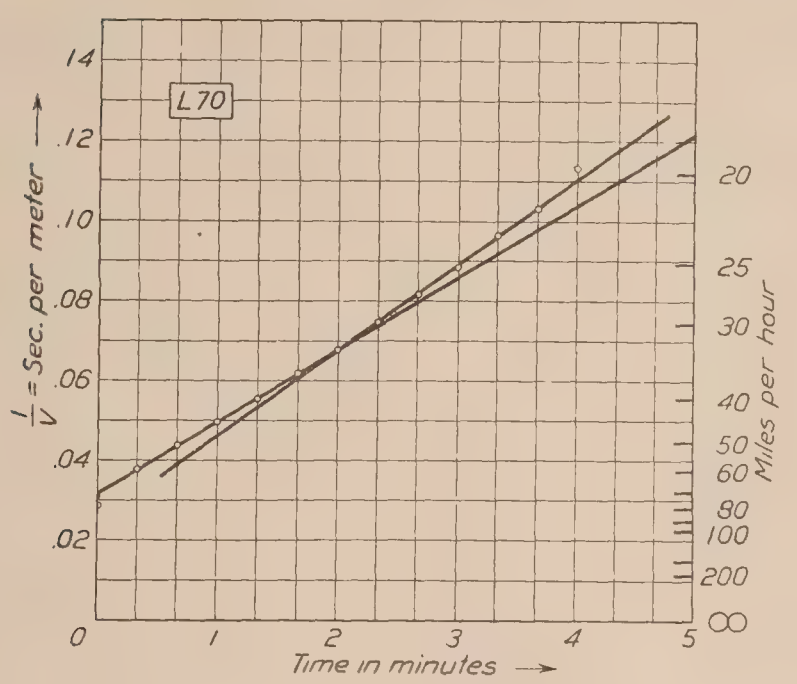
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Time in minutes
1/V = Sec. per meter
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Miles per hour

REPORT No. 118

THE PRESSURE DISTRIBUTION OVER THE HORIZONTAL TAIL SURFACES OF AN AIRPLANE

By F. H. NORTON
Langley Memorial Aeronautical Laboratory

REPORT No. 118.

THE PRESSURE DISTRIBUTION OVER THE HORIZONTAL TAIL SURFACES OF AN AIRPLANE.

By F. H. Norton.

UNIFORM FREE FLIGHT.

SUMMARY.

This work was undertaken by the National Advisory Committee for Aeronautics at the request of the Bureau of Construction and Repair of the United States Navy in order to determine as completely as possible the distribution of pressure over the horizontal tail surfaces of an airplane, and to analyze the relation of this pressure to the structural loads and the longitudinal stability. The investigation is divided into three parts, of which this is the first. This part is for the purpose of determining the pressure distribution over two horizontal tail surfaces in uniform free flight; the second part to conduct tests of similar tail planes in the wind tunnel; and the third part to determine the pressure distribution on the horizontal tail surfaces during accelerated flight on the full-sized airplane.

The general method used in this part of the investigation consists in determining the separate pressures at a large number of points on the tail surfaces of a JN4H airplane, by connecting small holes, opening on the tail surface, to the tubes of a multiple liquid manometer, which simultaneously measures the total number of pressures on one-half of the tail surface. The pressures are recorded by photographing the multiple manometer with an automatic camera which takes an exposure at each condition of air and engine speeds.

These tests in uniform free flight gave the following results:

1. Under no condition did the average tail load exceed 2.3 pounds per square foot.
2. The highest local load on the tail of the JN4H was 11 pounds per square foot.
3. The highest local load on the special tail was 25 pounds per square foot.
4. The torque exerted by the tail about the X-axis ranged from +1,200 inch-pounds (in the direction of the propeller rotation) to −1,600 inch-pounds.
5. The sealing of the crack between the elevator and tail plane has no appreciable effect on the distribution of pressure.
6. The inversion of the standard tail plane (flat surface up) gives a more uniform distribution of pressure as well as improving the stability.
7. The airplane was very stable with the special tail of high aspect ratio even with a center of gravity coefficient of 0.37.
8. The center of pressure travel on the wings, as determined by the integrated tail load, is farther forward than on the corresponding model.

INTRODUCTION.

PREVIOUS WORK.

There has been comparatively little previous work done on the pressure distribution on the horizontal tail surfaces, either in free flight or on models. Perhaps the most notable work is that done by the British on the tail plane of a DH4, but unfortunately the pilot and observer were killed before the work was more than half completed, so that only the left half of the tail plane was explored. Some work has also been done by the Germans on this subject, but they have limited themselves to a few points, so that their results are not at all complete.

SCOPE OF PRESENT INVESTIGATION.

This part of the investigation includes the complete study of the pressure distribution over two horizontal tail surfaces, one a standard JN4H tail plane, and the other of special construction, which was thicker and with a plan form of higher aspect ratio. The pressures

were recorded at flight speeds of 45 miles per hour to 100 miles per hour, and at engine speeds of 600, 900, 1,200, and 1,400 revolutions per minute, in order to get all of the conditions that would occur in flight. At the same time various modifications were made in the airplane, such as moving the position of the center of gravity horizontally, and varying the tail plane in angle, section, and plan form. The number of holes on the tail were so numerous (over 200) that the total pressure on the tail under any conditions could be obtained very accurately by integration of the pressure distribution curve. The results are plotted in a large variety of ways in order to bring out as clearly as possible the exact distribution of pressure.

REFERENCES.

Below are given the references to the most important investigations on this subject:

Distribution of the Pressure over the Tail Plane of a DH4 Airplane. R. & M., No. 552, British Advisory Committee for Aeronautics.

Stabilizer and Elevator Pressure Distribution. Report of the Experimental Department, Airplane Engineering Division, U. S. A., December, 1918. Work conducted at M. I. T.

Flosendruckmessungen, Technische Berichte, October 15, 1915.

ACKNOWLEDGMENTS.

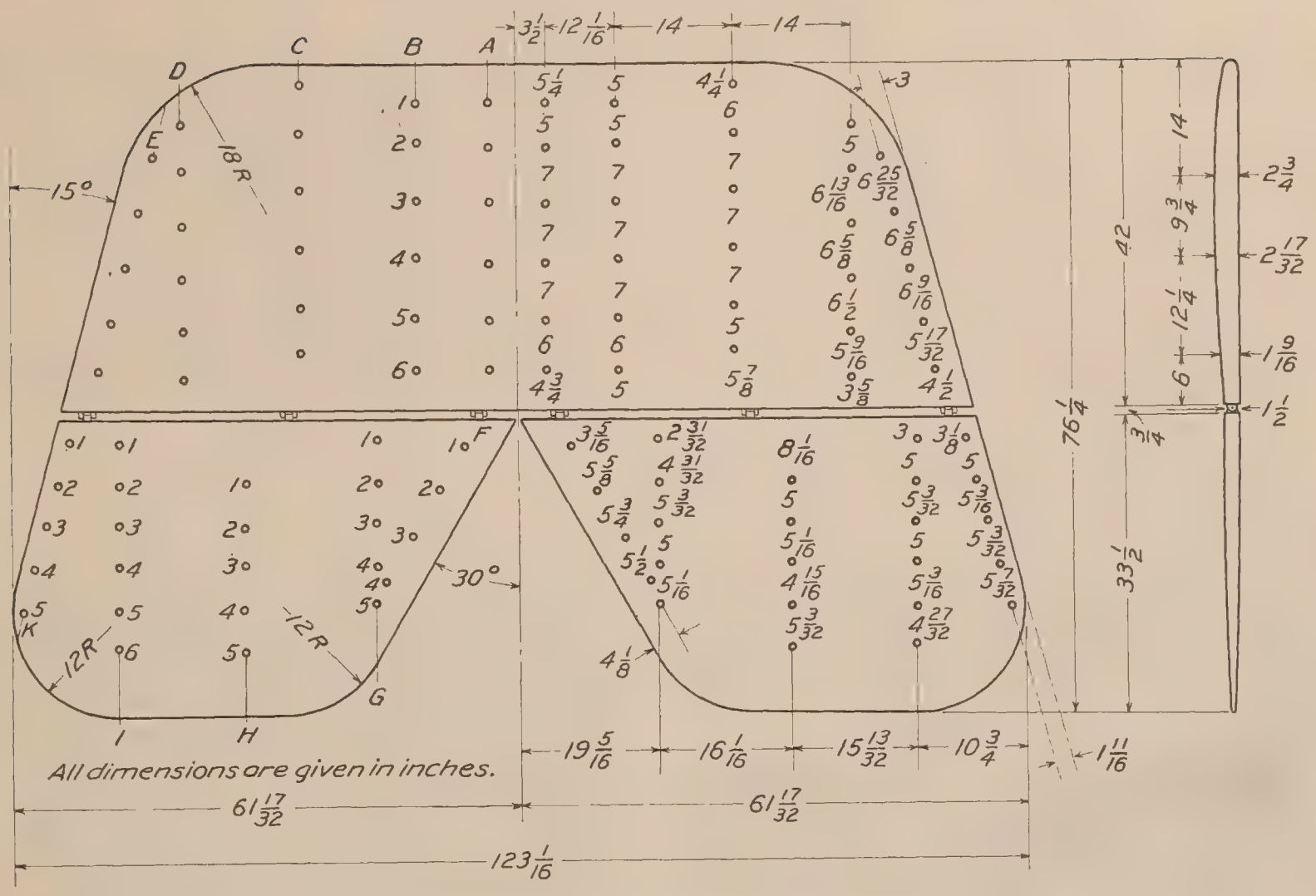
The piloting for this research was done by Mr. R. G. Miller and by Mr. E. T. Allen, and the computations and plotting were done by Mr. Gardiner, Mr. Hoot, Mr. Thaden, and Mr. Bennett, all of the Committee's staff.

APPARATUS AND METHODS.

TAIL SURFACES.

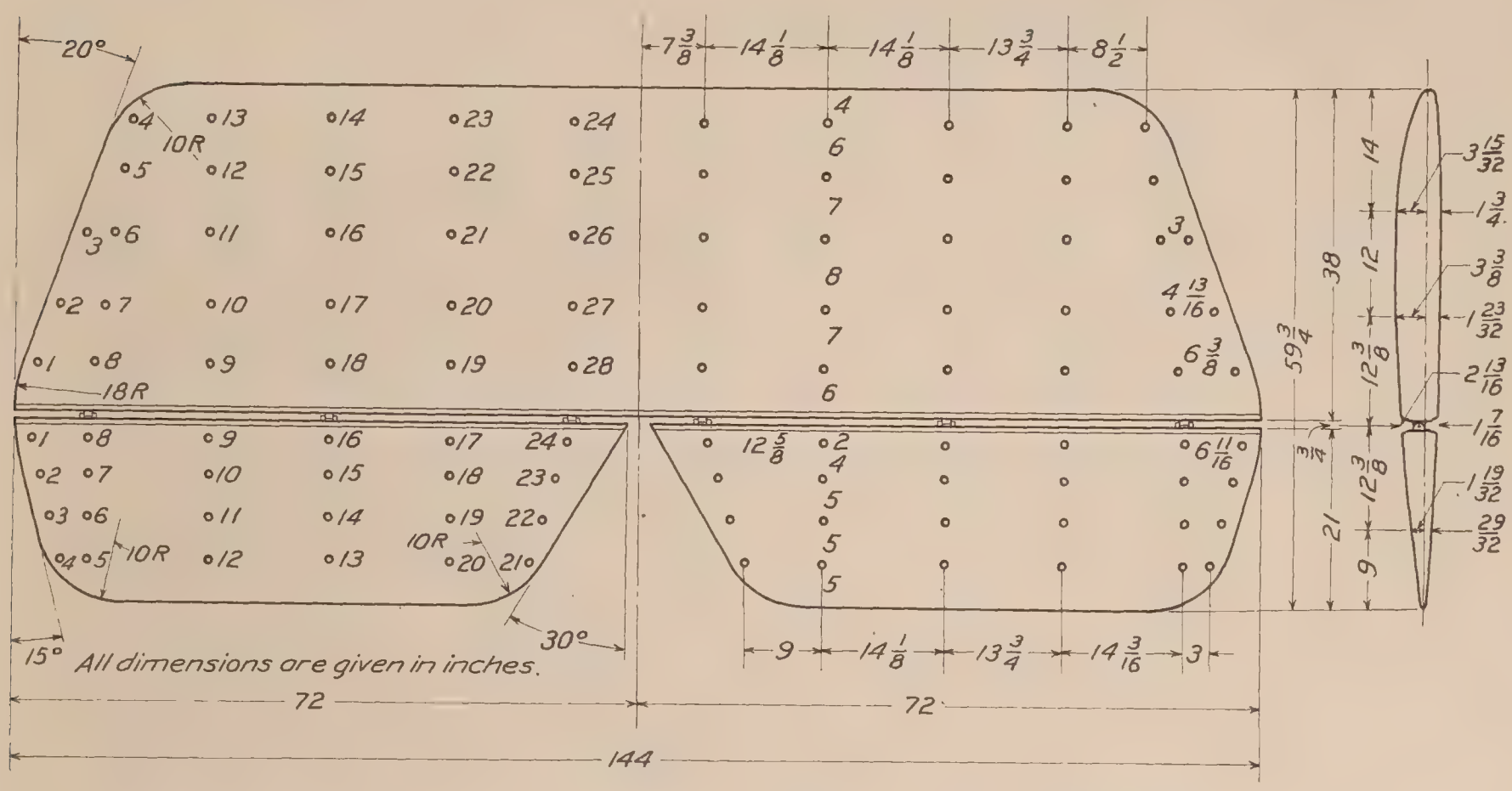
The location of the holes in the standard JN tail surface is shown in figure 1 and it is clearly seen that the holes are spaced closely enough to give a very accurate representation of the actual pressure distribution. As this tail was a standard one, it was unfortunately impossible to place the holes in regular rows along the span, necessitating a considerable amount of extra computation. In another test it would be considered advisable to rebuild the ribs, if necessary, in order to obtain an even spacing. The total number of holes on the right-hand side of the tail of both upper and lower surface were connected to the multiple manometer so that simultaneous readings were taken on all of these holes. As soon as the tests were completed on the right-hand half of the tail plane the tubes were removed and placed in the left-hand half and the tests were repeated. Figure 2 shows the layout of the special tail plane which was constructed at the Naval Aircraft Factory. Pressure tubes were attached to both right and left sides of this tail plane, but only one side could be connected to the manometer at a time, as with the other tail. As this tail plane with its tubes was 50 pounds heavier than the standard tail surface, it was necessary to place a considerable amount of lead in the nose of the airplane in order that it might balance in flight, and although it was designed with sufficient strength without external bracing, the usual tail struts and wires were attached as an extra precaution.

In figure 3 is shown the method of connecting the rubber tubes to the pressure holes. A thin brass strip was run along the cap of each rib and the pressure tubes were soldered into this strip of brass; after running through the cap strip of the rib they were connected onto the corresponding rubber tubes. The tubes running to the elevator were carried through reinforced holes at the hinge spars and run through the tail plane to the fuselage with those which were connected to the holes in the tail plane, as clearly shown in figure 4. After the tubes were placed, the tail was covered, care being taken to see that none of the tubes were kinked or that none of them were pierced by the needle when sewing on the fabric. After the fabric had been doped and varnished, the cloth was carefully cut through above each pressure hole so that a clean sharp edge resulted. The tail plane was then placed on the airplane and the tubes run through the fuselage to the manometer as shown in figure 5, great care being taken that none of them was restricted in any way.



PLAN OF JN4H TAIL SURFACE
Showing location of Pressure Holes.

Fig. 1.



PLAN OF SPECIAL TAIL SURFACE
Showing location of Pressure Holes.

Fig. 2.

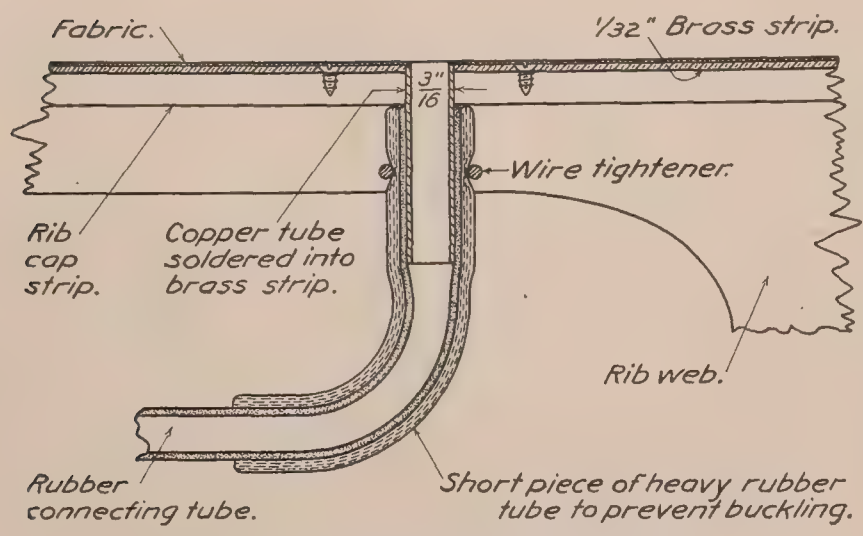


Fig. 3. METHOD OF ATTACHING PRESSURE TUBES TO SURFACE OF TAIL PLANE.

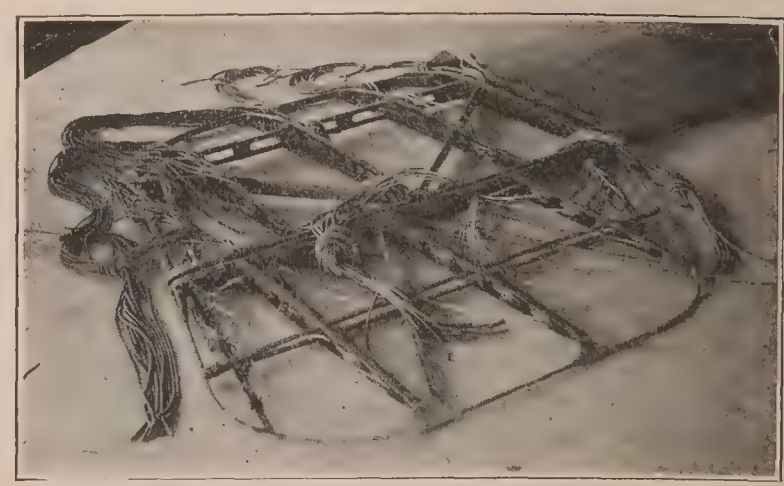


FIG. 4.—Tail plane with tubes in place, before covering.

FIG. 5.—Method of connecting the manometer to the tail surface, and showing static head on strut.

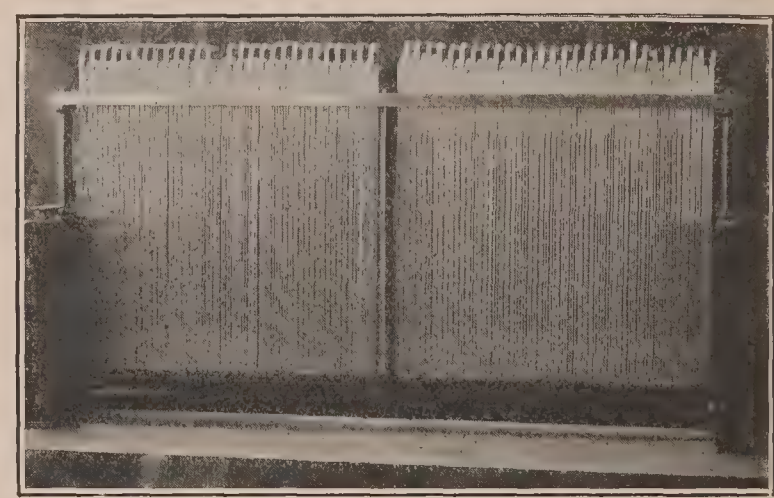


FIG. 6.—Multiple manometer with 112 tubes.

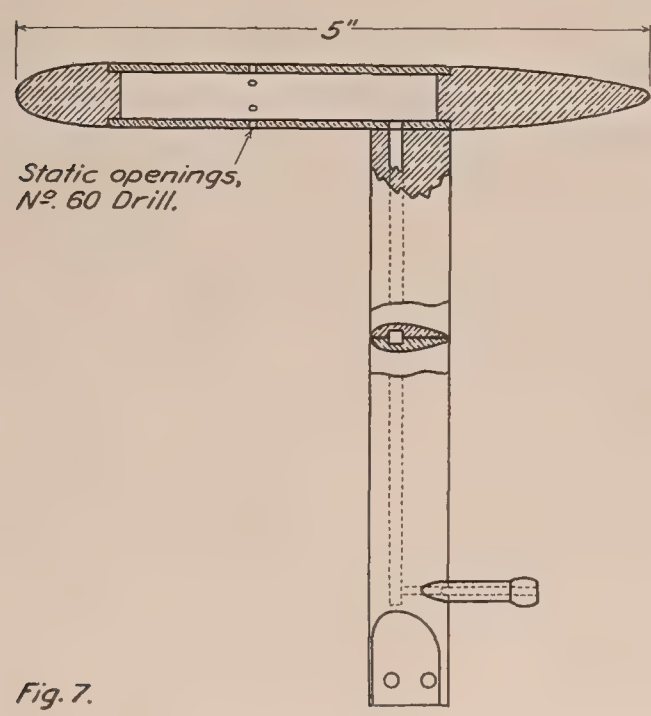


Fig. 7. STATIC HEAD.

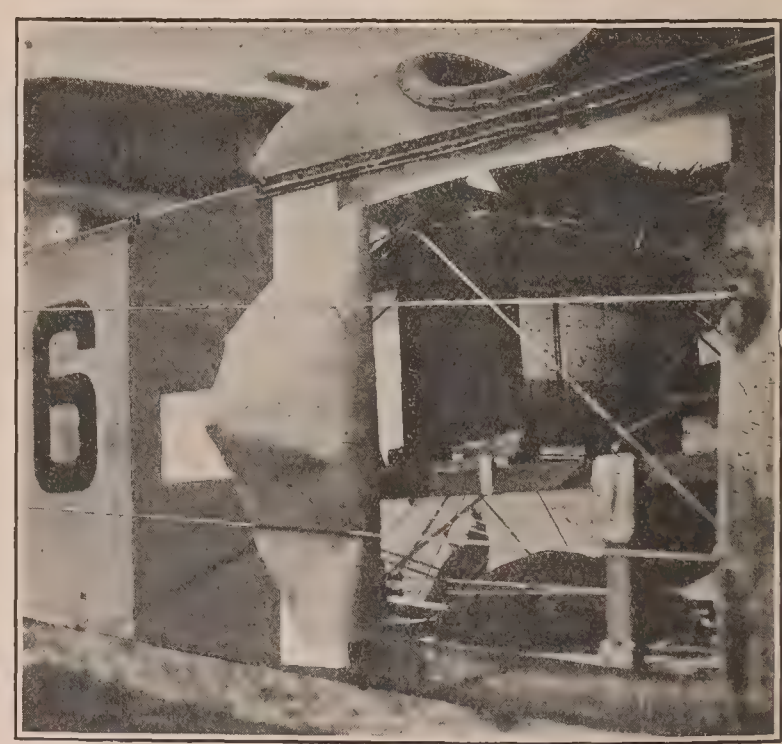


FIG. 8.—Mounting of manometer in the fuselage.

MANOMETER.

The multiple manometer, as shown in figure 6, consists essentially of two end reservoirs connected by a central brass tube out of which rise 112 one-eighth-inch glass tubes, each of their upper ends being connected to a hole in the tail plane. The top of the reservoir and the two end glass tubes are all connected to a static head on the wing strut as shown in figure 7, as it was found that the static pressure in the cockpit was quite different from the true static pressure of the air. In order to make the instrument as compact as possible the glass tubes were cemented into the reservoir at the lower end with plaster of Paris and beeswax. It was found impossible to prevent a slight leakage, but this was so small that it did not cause great inconvenience in the work, and this leakage could evidently be stopped by using some alcohol-proof cement. It was unfortunately impossible to obtain small, thin-walled glass tubing of enough uniformity of bore to avoid the necessity of making capillary corrections, and as these corrections entail a considerable amount of labor in the computation, every effort should be made to obtain tubes of uniform bore for use in future instruments of this type.

The manometer was mounted in a wooden case for protection and a piece of opal glass was placed immediately behind the tubes in order to give an evenly illuminated background against which to photograph. The top of each glass tube was connected by a short rubber tube to a brass connection plate on the top of the wooden case so that all subsequent connections could be made without danger of breaking the glass tubes. The wooden case was mounted in the airplane on the seat rails and pieces of sponge rubber were placed beneath the instrument in order to prevent excessive vibrations from reaching the instrument; however, the mounting was rigid enough so that only very slight relative motions would be allowed. It was found that a fair illumination could be obtained through the fabric of the fuselage, which gave a very even and subdued light, but in future tests it would be advisable to put translucent celluloid windows in the sides of the fuselage. A photograph of the installation of the gauge was taken (fig. 8) by cutting away a portion of the fabric of the fuselage.

CAMERA.

In order to obtain a reading at all pressures simultaneously the multiple manometer was photographed by an automatic camera installed on the seat rail in the fuselage just aft of the pilot's seat. As the space was very restricted it was necessary to use a wide-angle lens, but as this lens was of a high quality the distortion was quite negligible. The camera is shown in figure 9, and consists of a light-tight box containing two large diameter film rolls so that the movement of the film would not appreciably change the diameter of the spools and so change the spacing of the pictures. The spool initially containing the film has attached to it a friction brake, so that at all times the film is kept tight; the other spool contains a large diameter windlass around which is initially wound a cord which runs through pulleys to a smaller windlass at the pilot's seat, one turn of which will move the film along sufficiently for one exposure. In the same way the shutter is connected by wires to a trigger located conveniently for the pilot, so that by a single operation of the trigger the shutter will automatically give a one-second exposure and set itself for the next. A 10-foot length of 3-inch panoramic film was used, and this was sufficient for taking the 22 exposures which completed one run. The mounting of this camera is shown in figure 10.

An exposure of one second was taken in order to give an average reading of each pressure to eliminate the effect of the small oscillations; however, observations and the extreme sharpness of the photographs indicate that the height of the liquid in each tube was very nearly constant. Every tenth tube was painted with a black spot in order to facilitate the measuring of the tubes, and a scale was placed at the center of the gauge in order that the true magnification could be measured when the film was placed in the projection lantern. A section of one of the records is shown in figure 11. It was found necessary in order to secure satisfactory pictures to color the alcohol red and there was some difficulty in finding a coloring matter which would not be oxidized by the denatured alcohol which was used, but finally an analine dye was found which would satisfactorily retain its color.

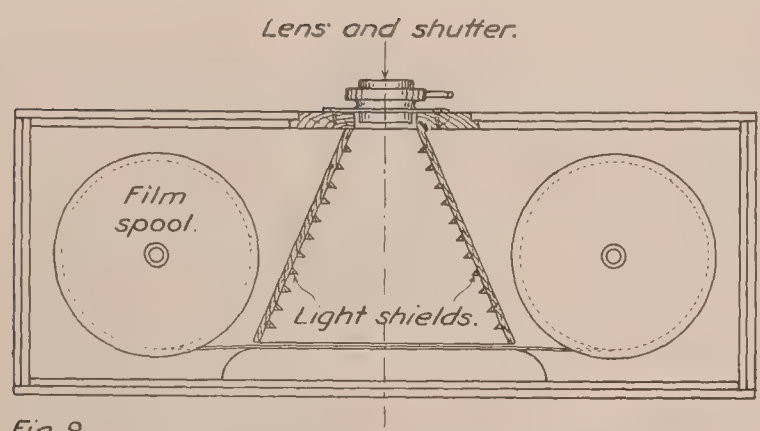


Fig. 9.

TOP VIEW OF AUTOMATIC CAMERA.

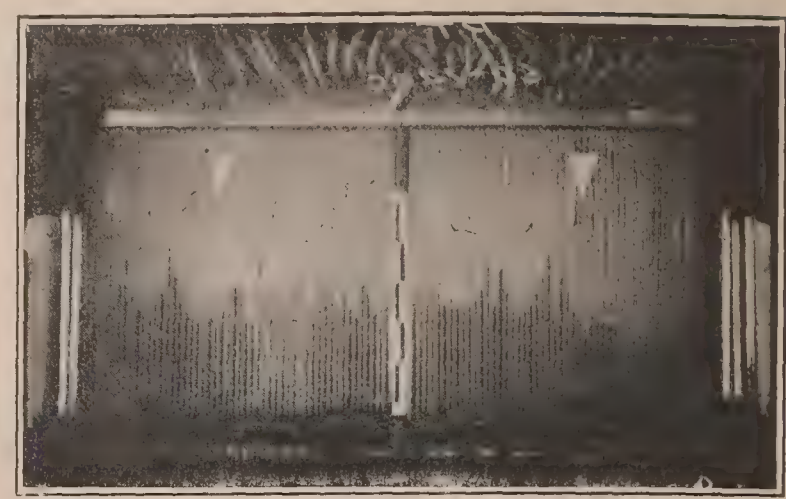


FIG. 11.—A record taken by camera

FIG. 10.—Mounting of automatic camera in airplane.

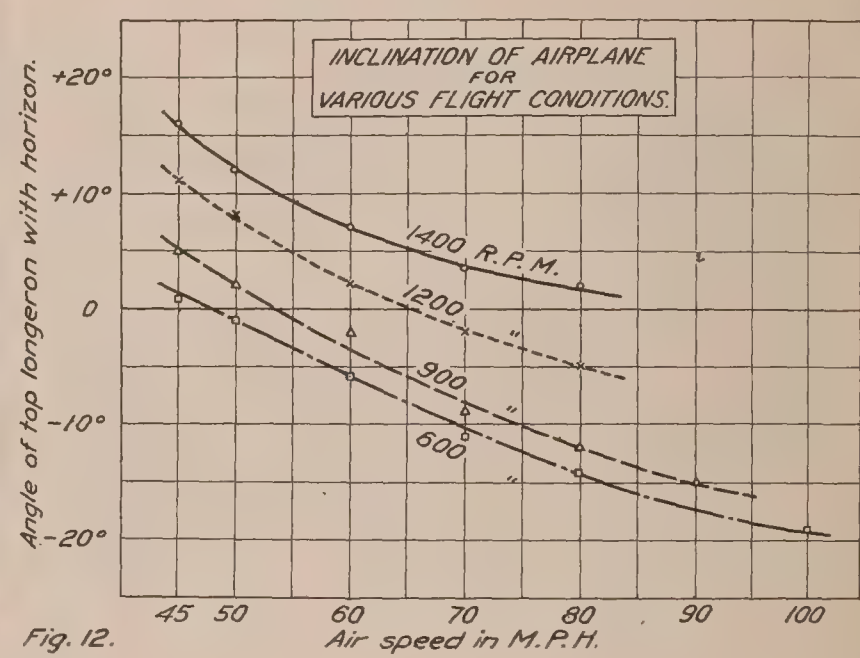


Fig. 12.

PILOT'S PROCEDURE.

All of the runs were made at an altitude corresponding to a density of 0.9 in the following manner: At the start of each flight the altimeter zero was set by the reading of a mercurial barometer so that the desired density would come at a reading of just 3,000 feet. Before each exposure the airplane was climbed considerably above the given height and the exposure was made as the airplane reached the 3,000-foot level, having previously attained a steady condition of flight. In the computations this factor 0.9 does not enter in, as the air speed was not corrected for density, for the same density effect will obviously apply to the air-speed reading as to the surface pressures. There is one exception, however—the effect of the slip stream—which will be a function of the density, and it was for this reason that all the tests were made at a constant density.

METHODS OF READING FILMS AND COMPUTATIONS.

As it was very tiring to read the height of the menisci from the photographic record with a magnifying glass, it was thought best to place the negatives in a projection lantern and throw the image onto a screen at a certain definite magnification. In this way the height of the liquid could be measured with a large scale very quickly and conveniently, using as zero a line scratched across the film between the menisci of the two static tubes. It was also necessary to determine the capillary correction in the same manner, and these corrections were in every case subtracted from the readings. The head of alcohol thus determined was then corrected by multiplying it by factors for the density of the alcohol and the inclination of the tubes due to the longitudinal position of the airplane in flight.

The longitudinal angle of the airplane was determined for each flight speed with a liquid inclinometer, and the results are shown in figure 12. It will be noticed that the angle is only 19° in a 100-mile-an-hour dive, and this would ordinarily be called a steep dive, for the angle is always overestimated in such conditions. The head of alcohol as read was then multiplied by the cosine of the angle of inclination of the tubes from the vertical.

METHODS OF PLOTTING AND INTEGRATION.

The values in true head of water were then plotted along each row of holes in the direction of the air flow, the upper and lower surface using the same base line, so that the planimetered area between them represents the total pressure along that section. A few of these curves are shown in figures 36 to 79. Because of the irregularity of the holes occasioned by structural considerations, it was necessary to cross fair these preliminary curves, thus obtaining pressure sections at right angles to the air flow. These curves are shown in figures 80 to 122, and give the sum of the air forces on upper and lower surfaces. These curves are sections of a solid representing the total load on the tail, and are revolved about their respective base lines into the plane of the tail, so that in the figures an area on the forward (leading edge) side of the base line will represent an upward or positive load. The area under each of these curves was found by planimetering, and by using the values thus obtained as ordinates for a curve drawn parallel with the air flow; the total pressure on the tail is represented by the area under this second curve. The point of action of this pressure was found in each case by integrating this area about the hinge with a mechanical integrator. These curves are shown in figures 123 to 242.

In each figure two vectors are shown, one, marked T, representing the center of pressure of the complete tail surface, and the other, marked E, representing the center of pressure on the elevator alone. In each case the vectors are shown to scale (in position, but not in magnitude), except where they are off of the paper, but the distance of the vector from the hinge is given each time in inches. In the curves representing the total pressure parallel to the leading edge the same system of vectors is used, except that they represent in all cases the force on the complete half of the tail; and the moment of that half about the center line is given in addition to the distance from the center line. The force on the elevator control was found by taking the moment of the pressure on the elevator about the hinge, but as the results were not as reliable nor as easy to obtain as direct force measurements on the stick, no results of this character are plotted in this report.

There is another method of easily obtaining the total load, or the moment about any axis of an irregular pressure surface, which may be termed the plastic method. Let us suppose that a scale model is made of the tail with adjustable pins at the points at which the pressure readings were taken. If each pin is then set at such a height as will correspond to the pressure at that point,

FIG. 13.—Model of pressure surface—Case I—600 r. p. m.; 100 m. p. h. The light-colored material represents down pressure and the dark material up pressure. The region of down load is evident at the leading edge.

FIG. 14.—Model of pressure surface—Case VI—600 r. p. m.; 100 m. p. h. This shows the very large down load at the leading edge and the sharp peaks of up load at the hinge.

a skeleton of the desired surface will be obtained. By filling the space between the pins with a plastic material—clay or plasticene—and fairing in the surface, which it is very easy to do with a little practice, a solid is obtained which represents the actual load on the given surface.

The total load or its moment may be easily found from this model solid simply by weighing it, or by balancing it about any desired axis, provided the density of the plastic material is known. It is evident that this method is applicable to an irregular spacing of points where a sectional method would be impossible or at least very laborious, and even with evenly spaced holes it is very much quicker than a two-dimensioned method. The plastic method was used in this investigation for only a few cases because of the necessity of obtaining the sectional curves for two-dimensioned reproduction. (See figs. 13 and 14.)

PRECISION OF THE RESULTS.

ERRORS IN READING FILM AND COMPUTATION.

It is possible in nearly all cases to measure the actual head of alcohol from the enlarged negative within the limits of plus or minus 0.003 of an inch. As all the calculations were carried out to the nearest 0.001 of an inch, the computed head should be correct in every case to at least 0.005 of an inch. The areas under the curves were planimetered several times in order to get a correct reading, so that all the area can be considered correct within plus or minus 0.01 of a square inch. An error somewhat larger than this is introduced, however, because of the incorrect representation of the curve between the given points, and this is especially true at the end of the tail plane where the pressure gradient is very steep. As 1 square inch under the final curve is equal to approximately 10 pounds, the total load on the tail will be given to within plus or minus 0.1 pound if the curves are assumed to be correct. The difficulty in drawing a true pressure curve through the given points will, however, considerably increase this error, so that the total load on the tail can not be assumed to be correct to better than one-half of a pound from these causes of error.

ERRORS IN FLIGHT.

The errors due to computation and reading are, however, small compared with the errors inherent in all full-flight work; that is, errors arising from irregular atmospheric conditions. For example, if the airplane happens to strike a bump in the air at the time that the exposure is taken, it may considerably alter the value of the pressure obtained, especially those pressures on the movable portion of the tail plane. For this reason the first runs were repeated in order to check the results, and it was found in most cases that the check run agreed very closely with the original run. The subsequent work was very complete and there was ample chance for checking; as so little change was made in the flight conditions from one run to another, it was not thought necessary to make check runs in every case. The excellent agreement that can be observed between the pressure curves taken under nearly identical conditions leads one to have confidence in the results, as any part that is appreciably in error will be at once shown up by its lack of resemblance to a similar curve.

It will be noted that the points calculated from the total load on the tail in order to obtain the pitching moment due to the wings and body do not lie smoothly on their representative curves and in comparison with wind tunnel results, for instance, they would look rather unsatisfactory; but it should be remembered that all results obtained in full flight are inherently irregular, due to conditions of the atmosphere which can not be overcome, and it is only possible by taking large numbers of readings and averaging them to get a satisfactory mean value. The moment curves check well with one another and the close agreement between the center of pressure travel curves for the different combinations used, all give a good indication of the accuracy of the results. For determining pitching moments, however, the pressure distribution method is not as accurate as the method used by the British where the fuselage was jointed.[1] As all of the readings and the computations have been carefully checked at every step by this method, it is felt that on the whole the results are presented as closely as it is possible to do so.

[1] British Advisory Committee, R. & M. No. 400. "The Full Scale Determination of the Pitching Moments of a Biplane."

SCOPE OF TESTS.

In order to represent as fully as possible all of the conditions that occur in flight on tail planes the following variations in the airplaine were made. It should be noted that in this report the center of gravity coefficient is the distance of the center of gravity from the nose of the mean chord in fractions of the chord. The mean chord is taken at distances from the upper and lower wings which is inversely proportional to their areas; that is, 60 per cent of the gap above the lower wing.

Case I.—On this run the airplane had a standard rigging of 17½ inches of stagger and a center of gravity coefficient of 0.381. This position of the center of gravity is about the standard position for an airplane of this type, and in order to compensate for the lack of a passenger in the rear seat a sand bag was tied to the floor boards in the rear cockpit.

Case II.—In this run the conditions were the same as before except that the sand bag was removed from the rear cockpit and some lead was placed in the nose of the airplane just aft of the radiator. This gave a center of gravity coefficient of 0.326.

Case III.—With the same position of the center of gravity as in the previous case the tail plane was given a negative angle of 1½°; that is, the flat lower surface, which in the previous run was parallel to the longerons, was raised at the rear end in order to give this surface a slope of 1½°.

Case IV.—In this run the conditions of the airplane were exactly the same as in the previous run; the only change being the sealing of the crack between the elevator and the tail plane with flat sheets of celluloid, which we screwed to the tail plane and were allowed to rest by their own weight on the movable portion of the tail surface, smoothly covering up the intervening crack.

Case V.—In this run the stagger of the airplane was reduced to 8¼ inches, the tail plane was inverted so that the flat surface was uppermost and parallel to the longerons, also enough lead was placed in the nose of the airplane to give a center of gravity coefficient of 0.244.

Case VI.—In this run the stagger was again 8½ inches, and the center of gravity coefficient, due to the heavy tail, was 0.370. The tail surface was a special one of thick section and high aspect ratio (fig. 2), with its chord parallel to the longerons.

In none of these cases was the airplane very badly out of balance and at all times it could be flown with a fair degree of ease. In the fifth case, however, it was a little difficult to get the tail down in making a landing. It was noticed when flying the airplane with the special tail plane that the longitudinal stability was greatly improved and the airplane flew hands off with both open and closed throttle. This is quite remarkable in view of the rearward position of the center of gravity, and a more complete study will be made in a subsequent report.

RANGE OF AIR AND ENGINE SPEEDS.

For the first five cases the following 22 combinations of air and engine speeds were used each time:

	Revolutions per minute.			
	600	900	1,200	1,400
Air speed.....	45 50 60 70 80 100	45 50 60 70 80 90	45 50 60 70 80	45 50 60 70 80

In Case VI only the following combinations were used, as it was thought that, taken with the preceding data, the information obtained would be sufficient.

	Revolutions per minute.	
	600	1,400
Air speed	50 60 70 80 100	45 50 60 70 80

GENERAL DISCUSSION OF RESULTS.

The discussion will be mainly confined to the consideration of the actual pressures and to their relation to the structure. Further on the change in load with flight conditions will be analyzed more carefully in respect to airplane stresses and stability. The subject embraced by the report is so wide that it is difficult to keep in mind all of the data at one time, and only by constant reference to the curves can a comprehensive view be obtained.

THE PRESSURE DISTRIBUTION WITH A STANDARD RIGGING OF THE AIRPLANE.

For this run the preliminary curves under all conditions are given in figures 36 to 79. These are given in order to show the pressure distribution as separated on the upper and lower surfaces. A suction is plotted downward and a pressure upward, the upper surface and the lower surface using the same base line. The greatest suction occurs on the lower surface of the tail plane very close to the leading edge. The suction is also large at the very tips of the tail plane, this being due to the raked tip acting in the same manner as a leading edge. There is also a tendency at high speed to have a large suction on the upper side of the elevator, especially near the outer part of the span. At no point, however, on the tail plane did the suction reach a value higher than $1\frac{1}{2}$ inches of water, and as this suction is comparatively very small there should be no fear of stripping the fabric on a well-constructed job.

It will be noticed that the pressure curve on the elevator has a rather definite wave form with a period of about 12 inches. The reason for this is probably that the discontinuity due to the joint between the elevator and the tail plane gives rise to a stationary wave which continues along the surface of the elevator. It will be noticed later that when the crack is sealed up these waves are much reduced, but they are even then in evidence. The discontinuity of pressure at the hinge of the tail plane is in no case very large; in fact, it is much smaller than the results from the model test would indicate. The line of zero pressure used as a base line in a good many cases lies above the pressures on both the upper and lower surfaces, showing that the static pressure in the slip stream is below that of the true static; but this would not, of course, affect the load on the surface which is independent of the base line. It was also noticed that the pressure in the cockpit was from an eighth to a quarter of an inch below the true static.

In order to give a better visualization of the pressures over the surface the total pressure has been plotted in figures 80 to 101 in order to show this more clearly. It is evident that the greatest load occurs on the leading edge of the right-hand side of the tail plane under practically all flight conditions. These curves show very strikingly how great a difference there is between the pressure distribution on the right and left side of the tail plane, so that a pressure distribution comprising only one-half of the tail would be of very little value in determining the total tail load. Some of this difference is undoubtedly due to the fact that the rudder and fins are not symmetrical in flight, but the greater part is due to the fact that the angle of incidence and velocity over the two halves of the tail plane are different, due to the rotation of the slip stream. It should also be noticed that a very large torsional load could be put on the fuselage by not rigging the elevators at the same angle of incidence; that is, if one elevator is several degrees higher than the other it will greatly change the pressure distribution on the tail plane, so that

great care should be taken when rigging an airplane to ascertain that the elevators are properly lined up.

In figures 123 to 144 the curves shown represent the total pressure on the tail plane plotted on a section taken parallel to the plane of symmetry of the airplane. These curves are shown, one for the load on the right-hand half of the tail plane, and one for the load on the left-hand half of the tail plane, and the third serves to represent the total load. The most important features shown by these curves are the considerable difference between the right and left half of the tail plane and the large suction at the leading edge of the tail plane.

In figures 243 to 264 are shown curves giving the total pressure on the tail plane on a section at right angles to the X-axis of the airplane. These curves show that the pressure is greatest at the tips of the tail plane and elevators. By plotting the moment of the area of this curve about its line of symmetry it is possible to compute the torque of the tail plane about the X-axis of the airplane, which will be discussed more in detail in a later portion of the report.

An attempt has been made in figure 13 to give a more realistic picture of the distribution of pressure than is conveyed by a set of curves. It is not possible to satisfactorily represent a three-dimensioned figure in two dimensions, but it is hoped that the photographs of these models will aid the reader in grasping the general trend of the tail pressures. The white surface is a plane lying in the tail surface and consequently representing zero pressure. A light-colored plastic material is used for negative pressures (down loads) and a dark material for the positive pressures, and the lighting is such as to show the relief to the best advantage.

It can be stated in general that with the airplane rigged as described above that under no condition in uniform flight are the loads set up by the pressure on the tail plane at all large, the load per square foot on the tail plane seldom exceeding one-half pound per square foot and the maximum reached amounted to only 1¼ pounds per square foot. This load is quite insignificant and amounts to little more than the weight of the tail itself, so that it may be concluded in steady flight that the stresses in, or due to, the tail plane may be considered of quite negligible value. The torque on the fuselage due to the tail plane is, however, considerably larger than has been realized before.

THE EFFECT OF CHANGING THE POSITION OF THE CENTER OF GRAVITY.

In Run II the center of gravity was moved forward and the resulting pressure distribution on the tail plane was found to be similar to those of Case I. The preliminary curves were omitted, as it was thought that they would be of little interest, for they showed no great difference from the curves obtained in the first case. As was expected, moving the center of gravity forward produces a greater negative load on the tail plane. Examination of the curves, however, shows that there is practically no difference in the distribution of the pressure contained in the two cases. A discussion of variation in total load on the tail plane, due to the change in air speed and revolutions per minute of the engine, will be discussed later in relation to their effect on the stability of the airplane; but in general it may be stated that the distribution of pressure on the tail is not appreciably affected by any change of center of gravity position.

EFFECT OF CHANGING THE STABILIZER ANGLE.

Case III is a duplicate of Case II, except that the position of the tail plane has been moved to 1½° of negative angle. Contrary to the case of altering the center of gravity position, the changing of the angle of the tail plane produces a considerable change in the distribution of pressure. This is explained by the fact that the total load on the tail must be the same as before, and as the tail plane is at a greater negative angle the load on this part of the tail plane would be more negative while the corresponding load on the elevator must be more positive, thus necessitating the lowering of the elevators. These facts are confirmed by the greater force that must be applied to the stick when the tail plane is turned to a more negative angle; that is, the ship feels as if it were more tail heavy with a negative tail-plane setting. This change of pressure distribution can be seen more clearly by comparing the curves of total pressure (figs. 145–166) in Case II with similar curves occurring for Case III (figs. 167–188). The curves in

figure 15 show this change in load between the tail plane and elevators with a change in tail-plane setting. It is clear from these curves that a negative tail-plane setting of 1½° with the same center of gravity position will shift a load of about 20 pounds from the tail plane to the elevator, and it is interesting to notice that this distance between the different curves is practically constant in amount for any condition of flight. The difference in load, however, between the tail plane and elevator load varies in inverse proportion to the air speed. It may be concluded that a change in the angle of tail-plane setting makes a considerable difference in the distribution of pressure over the tail plane and in general a more negative setting of the tail gives a more negative pressure on the tail plane and a more positive pressure on the elevator.

EFFECT OF SEALING THE HINGE CRACK BETWEEN THE ELEVATOR AND TAIL PLANE.

As model tests indicated that there was a considerable amount of leakage of air through the hinge crack of the tail plane, it was thought that some data of interest might be obtained by taking a run with the crack sealed up. This was accomplished by placing thin sheet celluloid over the crack on both the upper and the lower surfaces so that the resulting surfaces were quite smooth and continuous with any elevator position. Except for a slight tendency to smooth out the wave form of the pressure curve on the elevator, there is no difference between the run with the crack sealed and with the crack open. For this reason only the curves of total pressure are given, in figures 189 to 210, as the other curves were similar to the curves of Run III within the errors of the experiment. The fact that there is no change in the pressure distribution with the sealing of the crack does not show conclusively that sealing the crack has no effect on the efficiency of the tail, for the pressure distribution on a thin surface of this type does not indicate the drag, so that it may be that sealing the crack will considerably decrease this factor.

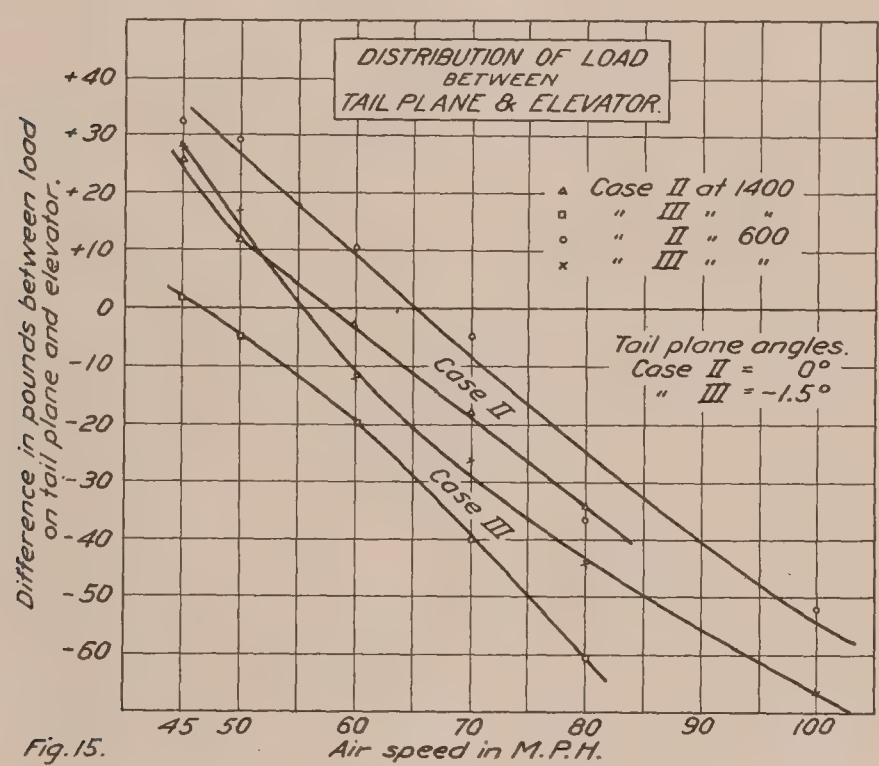


Fig. 15.

THE EFFECT OF INVERTING THE TAIL PLANE.

In this run the tail plane was inverted so that the flat surface was uppermost and parallel to the longerons. As should be expected, this change makes a marked difference in the distribution of pressure and the effect is clearly seen in the curves of pressure in figures 105 to 112. The greatest effect on the pressure distribution in this position of the tail plane is the more even distribution of pressure across the span, especially the elimination of the region of high suction near the tip of the tail plane. Curves of total pressure (figs. 211–232) show that the point of maximum suction is moved back from the leading edge a considerable amount and that the load on the elevator is also moved nearer to the trailing edge. That is, instead of being a sharp peak of suction immediately above the leading edge, as in Case II with the ordinary disposition of the tail plane, in the case where the tail plane is inverted this pressure is distributed fairly evenly over the whole tail plane, reaching its maximum about a third of the distance from the leading edge of the tail plane; so that it would seem that from a structural point of view it would be of great advantage to use this form of tail plane, as it would distribute the load in a more satisfactory manner. As will be shown later, this tail plane also gives a greater degree of stability, and its use is recommended from all points of view.

THE PRESSURE DISTRIBUTION OVER THE SPECIAL TAIL PLANE.

The tail plane shown in figure 2 is of a more modern design and has a rather thick double-cambered section and also a considerably higher aspect ratio. The pressure curves which are given in figures 113 to 122 show that the pressure over the tail plane is greatly altered from that over the ordinary tail plane. The greatest difference is the very high peaks at the leading edge and at the elevator hinge, one up and the other down, which reach very great magnitudes at the higher speeds. This introduces high local loads and a large twisting moment that may seriously stress the tail structure. The cause of this peculiar distribution of pressure may be due either to the thick section or the high aspect ratio, but the former is the more probable cause. A model of this tail at 600 revolutions per minute and 100 miles per hour is shown in figure 14. This twisting about the Y-axis may be quite serious, and the front spar should be made very stiff in thick-sectioned tail planes to prevent a deflection of this member. In one case the moment about the Y-axis was over 600 foot-pounds—an exceedingly large moment, and one that might have caused failure in a less strongly constructed tail surface.

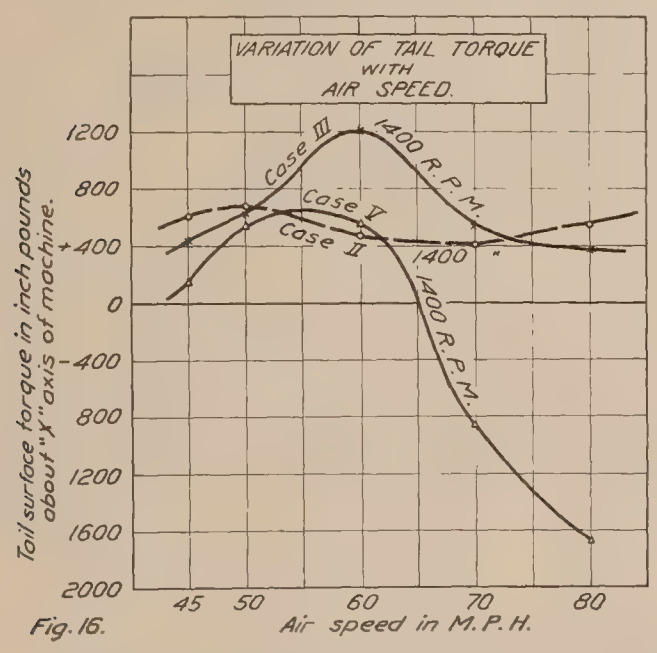


Fig. 16.

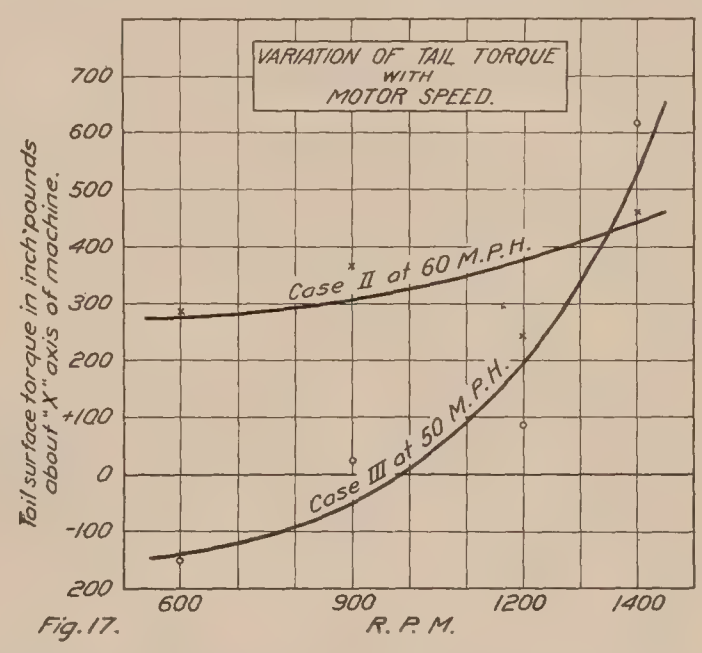


Fig. 17.

TORSIONAL EFFECT DUE TO THE UNSYMMETRICAL LOADING OF THE TAIL PLANE.

DISTRIBUTION OF LOAD ALONG THE SPAN OF THE TAIL SURFACE.

In figures 243 to 264 a few cases have been worked out showing the total pressure on the tail taken normal to the X-axis of the airplane. These curves, while they show the shape of the pressure variation quite accurately, should not be taken as an indication of the exact total pressure, as it is impossible to fair these curves exactly, due to the irregular position of the holes; and for this reason they do not check with the more accurate determination obtained from cross fairing. They do show, however, the concentration of the load on the tip of the tail plane and also the unsymmetrical loading due to the rotation of the slip stream.

All of the curves show a double maximum at the tip, the edge of the slip stream causing the inner one and the raked tips the outer. Although no torque curves were drawn for Case VI, it is very evident from an examination of the pressure curves that the torque on this tail is very small. This can be explained by the fact that the tips of the tail plane are out of the slip stream, so that there is little inducement for the building up of high pressures at these points.

VARIATION OF TORQUE WITH AIR SPEED.

In figure 16 are shown three curves representing the torque in inch-pounds about the X-axis of the airplane plotted against air speed in miles per hour for several cases, at 1,400 revolutions per minute. All of the curves increase to a positive value of the torque from 45 miles per hour to about 60. From that part on, however, the curves fall off quite rapidly, especially in Run V where the curves indicate a considerable negative value. The shape of these curves is rather peculiar and it would not be expected that a maximum of positive torque

would be reached at a speed of 60 miles an hour. An explanation may be obtained perhaps by studying the velocity diagram obtained by the Royal Aircraft Establishment on a BE2C airplane of the tractor type.[2] These results show that the slip-stream is divided by the wings and body into two separate streams, one with an inclination downward and the other with an inclination upward, and that these two streams are in various positions according to the slip of the propeller. It would be expected that the rotation of the slip stream would be proportional to the slip of the propeller; that is, at low air speeds and high engine speeds that the rotation of the slip stream would be a maximum and at high air speeds and low revolutions per minute that the slip stream might even rotate in the direction opposite to that of the propeller. Therefore, with a constant engine speed the positive torque on an airplane should be greatest at the low speeds and should increase proportionally to the air speed. The results of the British, however, as above referred to, show that the regions of high velocity do not occur at the same place at all flying conditions, but that the high-velocity regions are rotated with the propeller as the slip increases. This effect may then cause the high-velocity region of the slip stream on one side to flow either above or below the tail plane in some flying conditions, so that the torque would be in that case greatly changed as the slip stream on one side would strike that half of the tail plane and the slip stream on the other side would come above or below

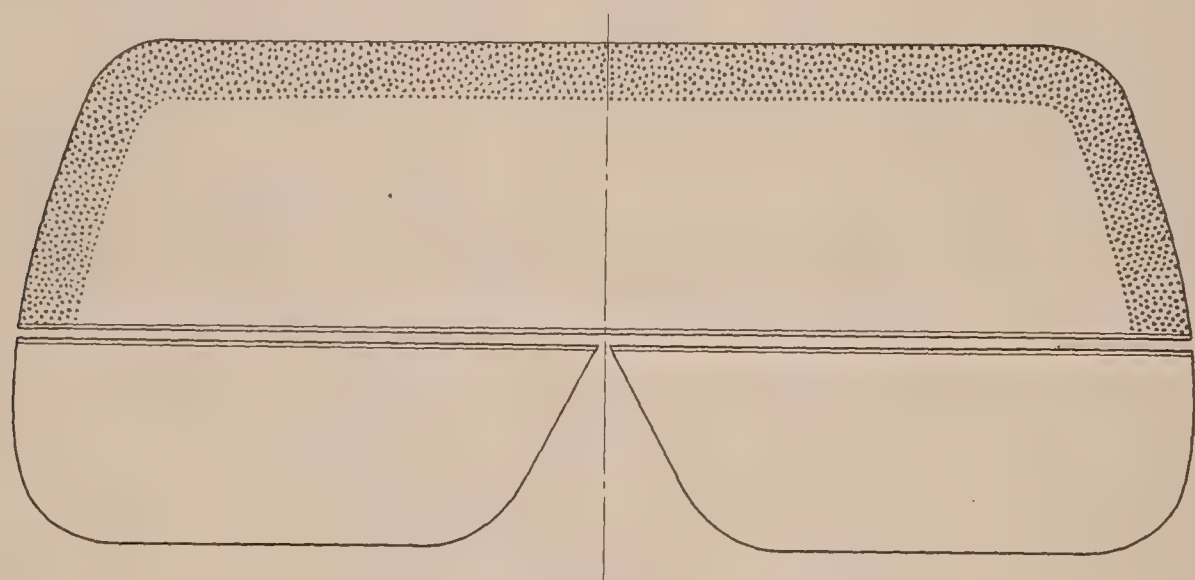


Fig. 18. METHOD OF DISTRIBUTING LOAD IN A SAND TEST.

it, thus making a large difference in the pressures on the two sides of the tail plane. That is, one-half of the tail, although it may have the higher angle of attack, due to slip stream rotation, at the same time it can have the lower load because of the greatly lowered slip-stream velocity on that side. This is undoubtedly the cause of the large negative torque that occurs on Run V at high air speeds.

VARIATION OF TORQUE WITH ENGINE SPEED.

In figure 17 are plotted curves showing the variation in tail-plane torques with a change in engine speed. These curves show, as would be expected, that the torques take a more positive value as the revolutions per minute of the engine is increased. A cause that may change the pressure distribution on the tail plane somewhat, and one that can not well be eliminated, is the effect of the rudder, as this member must be set over more to the right as the rate of engine speed is increased, in order to balance the torque of the propeller.

RECOMMENDATIONS FOR SAND LOADING TAIL SURFACES.

It is, of course, impossible to draw a final conclusion on the most logical method of sand loading until the work on tail loads in accelerated flight has been completed; however, as the tail load is usually considered to be of considerable magnitude in a rapid nose dive, the general characteristics of the regions of highest pressure may be summarized as follows:

[2] The Design of Screw Propellers for Aircraft, H. C. Watts, pp. 180–182.

(1) The most highly loaded region lies along the leading edge and along the tips of the tail plane and is of very narrow width, being about one-fifth of the chord of the whole tail surface, with the exception of Case V—where the tail plane was inverted—in which case the distribution was over the first three-fifths of the tail surface.

(2) The load on the tail (negative) is proportional to the position of the center of gravity; that is, the further forward the center of gravity the greater is the load on the tail.

(3) The distribution of load along the span of the tail plane shows regions of high pressure at the tips.

(4) In some cases, particularly at high engine speeds, there may be a large up pressure on one half of the tail plane and a large down pressure on the other half.

(5) The down load on the elevator is in all cases very small and at times the up load may be considerable, but the latter is balanced by the static weight of the elevator and so is relatively unimportant.

It should be noticed that the preceding conclusions are based only on uniform flight, with speeds up to 100 miles per hour, and the distribution, especially the down load on the elevator, may be considerably altered when this member is moved to a large angle with the tail plane, as would occur in accelerated flight. It is probable, however, that the load on the tail plane

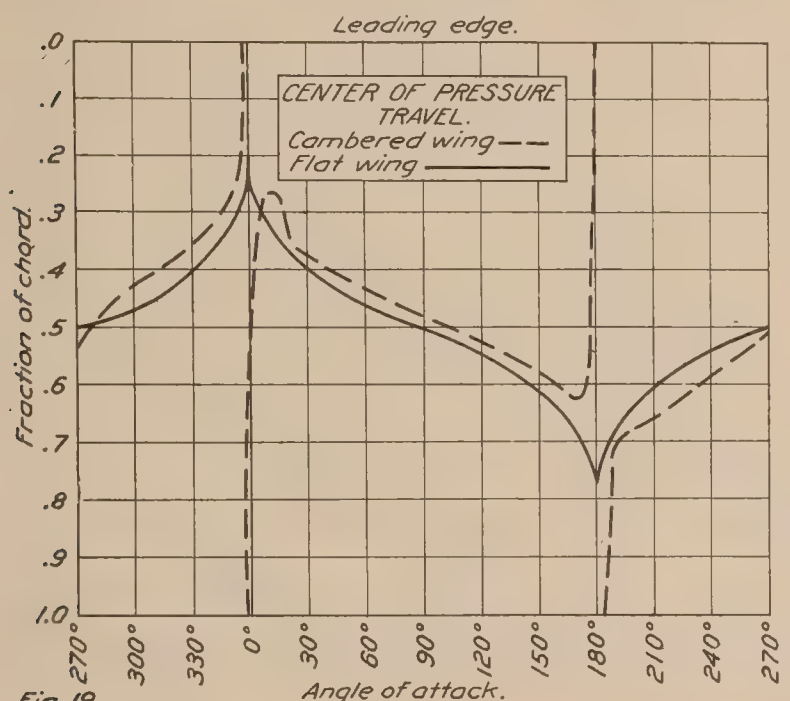


Fig. 19.

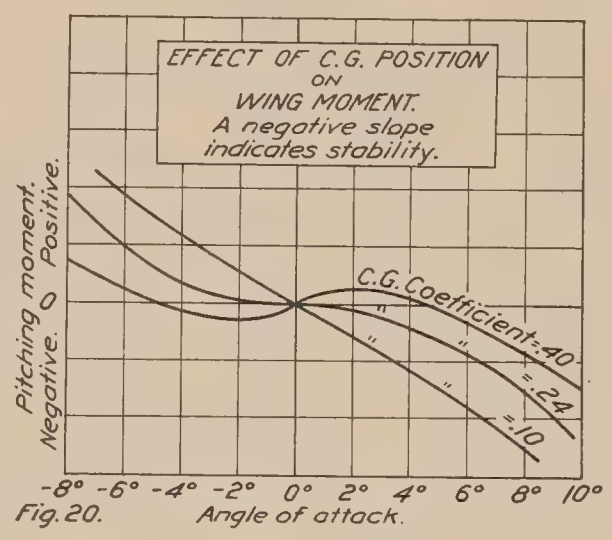


Fig. 20.

will not be much increased in stunting, as the damping effect will somewhat neutralize the load. The following recommendations for sand loading the tail surface are based on the conslusions from these tests in uniform flight:

1. A unit loading on the tail surface of 3 pounds per square foot with a center of gravity position of 0.30 will be sufficient for speeds up to 120 miles per hour. Of course, this unit loading should be multiplied by the load factor which is used on the remainder of the airplane.

2. The unit tail loading of the airplane should vary with the position of the center of gravity and it is recommended that if a center of gravity coefficient of 0.30 is taken as unity, 5 per cent of the tail load should be subtracted for every per cent that the center of gravity is back of this point (the center of gravity coefficient not to exceed 0.40), and 10 per cent should be added to the tail load for every per cent the center of gravity is forward of this point, percentage being taken on the mean wing chord.

3. The load should be distributed on the tail in a sand test around the leading edge and at the tips with a width of about one-fifth of the total tail surface chord as shown in figure 18.

4. Because of the unsymmetrical loading of the tail surface at times, one side of the tail surfaces should be loaded at a time; that is, one half should be completely loaded, the load removed, the second half completely loaded and then both halves loaded together.

5. In uniform flight the elevator load can be neglected, as the loads are upward and these are balanced in a large part by the static weight of the elevators. The elevators should have a separate test in order to withstand a pull on the control stick which will represent the maximum that it would be possible for the pilot to exert, but this load would occur only when suddenly pulling up the elevators and would not be present in uniform flight.

LONGITUDINAL STABILITY.

The subject of longitudinal stability in free flight is in a rather confused state because of the complex effects of the slip stream, which are not as yet thoroughly understood. Practically all of the stability theory is based on coefficients which are determined from model tests made in the wind tunnel, and up to the present time it has been impossible to reproduce accurately the slip stream in the wind tunnel; so that the tests made in this way are not in any way comparable with conditions in free flight when the engine is running. Only by a study of the forces on the controls and an exploration of the velocity and angle of the slip stream during various flight conditions can we come to any conclusions as to the effects on the longitudinal stability of various alterations in the airplane. It has been shown that the two causes which have the greatest effect on the longitudinal stability are the position of the center of gravity and the size, form, and position of the tail plane.

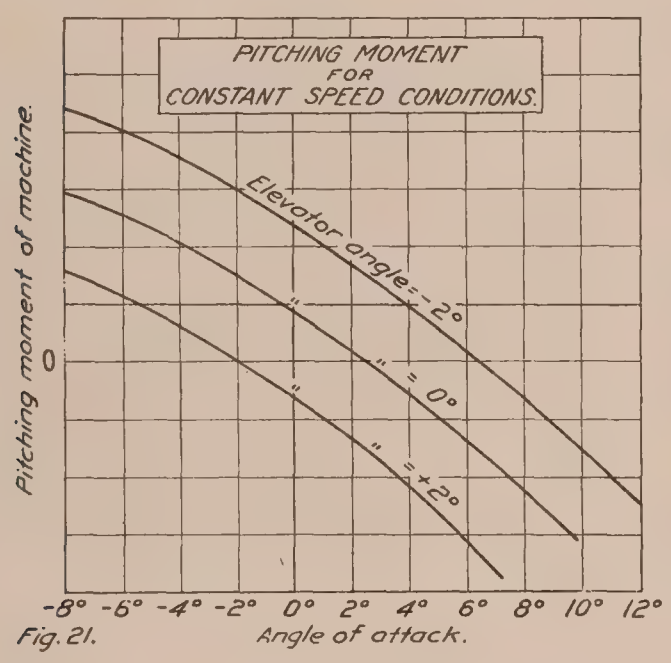


Fig. 21.

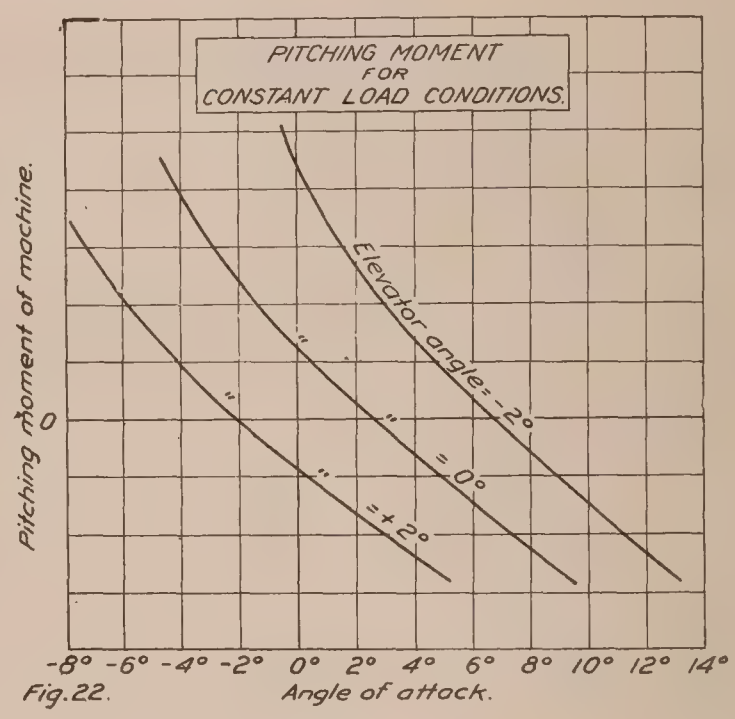


Fig. 22.

THE FUNCTION OF THE HORIZONTAL TAIL SURFACES.

The methods of obtaining longitudinal stability by means of a horizontal tail surface can be made clear by citing a few simple examples. There is, in figure 19, plotted the center of pressure travel for a flat plate and a typical aerofoil. If the flat plate is supported at its leading edge or ahead of its leading edge it will evidently be stable—in the manner of a weather vane—for any angle of incidence, although there will be a position of unstable equilibrium at 180°. If the support is now moved 0.3 of the chord back from the leading edge, the plate will be stable at an angle of +70 (where the center of pressure is at the support), and is in unstable equilibrium at 0°. This is the case of a balanced control surface which has a small unstable range about the zero position, a rather undesirable condition, especially on airships. Therefore a flat plate or a thin symmetrical surface must be supported at least as far forward as the leading edge to obtain complete stability when moving at a constant velocity.

Now, considering the stability of a cambered surface, which has an unstable center of pressure travel, it can be shown that if the air passes by the wing at constant speed we may still have stability by placing the support far enough forward. For this purpose moment curves are plotted in figure 20 for various positions of the support, and it is seen that the wing is completely stable—a curve of negative slope—when it is supported less than 0.24 of the chord from

the leading edge. It should be noted that the support need not be taken as far forward in the case of cambered wing as with a flat plate in order to get stability even though the latter has a stable center of pressure travel. It may be concluded that a surface run at constant speed may be made stable, no matter what the center of pressure travel, provided the support be taken sufficiently far forward.

For the full-sized airplane, however, we have different conditions; that is, there is a constant weight on the airplane and the speed varies in uniform flight in such a way that this weight will just be supported. Also in uniform flight the moment about the center of gravity must always be zero; that is, the moment due to the wings and body must at all times be balanced by a moment, in the opposite direction and of the same magnitude, exerted by the tail surfaces. In the case of an airplane in flight, then, the moment curve will be defined by the weight of the airplane multiplied by the distance of the center of pressure to the center of gravity. This curve will, therefore, always have the same shape no matter what the position of the center of gravity, thus giving the moment curves of the wings stability or instability according to whether the center of pressure travel is stable or unstable.

As it is impossible to make a simple surface stable at varying speeds unless the center of pressure travel is stable and there is no practical surface with this property, it is necessary to resort to a secondary surface known as the tail plane to obtain stability in a full-sized airplane. The tail plane acts in the same way as the vane on a weathercock; that is, the moment curve due to the tail is very stable because of the large distance between the tail plane and the center of gravity. In this case the tail plane does not carry a constant load, as do the wings, so that its moment curve can be very stable even though the center of pressure travel for the surface itself be unstable. Suppose a series of moment curves are plotted as in figure 21 for a model airplane in the wind tunnel at various positions of the elevator. Now, if this airplane is flying at varying speed and constant load, the moments on the model tests at any given angle of incidence would be multiplied by the square of the ratio of the corresponding full-scale speed to the speed of the model test, giving the series of curves shown in figure 22. It is seen that these new moment curves are stable at high angles of incidence and are all tangent to the constant moment curve at zero moment; but as the moment of the full scale airplane at a constant flight speed must be zero, the only part of the variable-speed curve we are interested in is where it crosses the zero moment axis. The stability is determined by the slope of the moment curve, and as the two curves—constant and varying speed—are tangent at zero moment, it is evident that the stability at varying speed is the same as at constant speed, providing the moment curves for various elevator settings are parallel; and this is the case in practically all airplanes. From the standpoint of efficiency it is not desirable to move the center of gravity far enough forward to obtain complete stability for the wings alone, but this is unnecessary if the moment curve of the tail is stable, for by combining a very stable tail with a slightly unstable wing the complete airplane will have stability.

The moments of unstable wings can be made stable at constant speed by moving the center of gravity far enough forward, but with a constant load and varying speeds, the stability of the wings can not be changed by varying the position of the center of gravity. The complete airplane, however, has the same stability for varying speeds as it has for constant speed. It should be noticed that although the vectors obtained by a model test may be stable, this does not mean that an airplane in free flight will be stable; on the other hand, an airplane may be stable in flight even though it has an unstable position of the vectors. However, if the slope of the moment curve on the model is negative the full-sized airplane will be stable, and if positive it will be unstable, of course neglecting the effect of the slip stream.

ACTION OF THE TAIL PLANE AND ELEVATOR IN PRODUCING PITCHING MOMENTS.

The horizontal tail surface consists, in almost all modern airplanes, of a fixed portion with a movable portion hinged to its rear edge. The turning of the rear part will, first, change the angle of attack of the entire tail surface, and, second, change the camber of the section, thus effectively changing the lift coefficient of the tail by the application of very little force to the con-

trols. The pressure curves indicate that the distribution of load is constant between the two parts of the tail for the small angular changes used in flight, so that it may be assumed that the force on the elevator is very nearly proportional to the force on the whole horizontal tail surface. This is confirmed by the similarity between the stick force curves and the pitching moment curves of the tail, although there is a discrepancy at very low speeds where the elevator angle is quite large.

It is desirable in every airplane to have the stick force zero at the normal flying speed, a result that may be accomplished in two ways; first, the inevitable weight of the elevator may be balanced statically by weights or springs, or, second, the weight may be balanced—as is usual—by the air force acting on that member. Now, in order to obtain a sufficient air force to balance

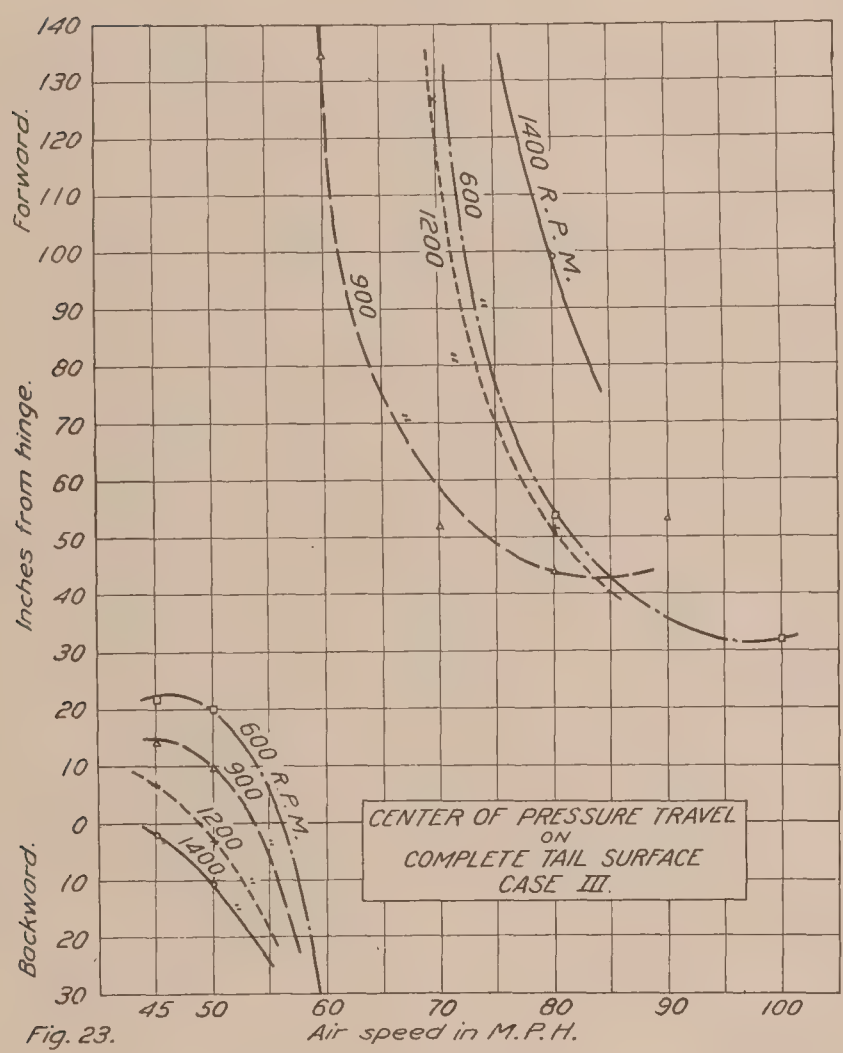


Fig. 23.

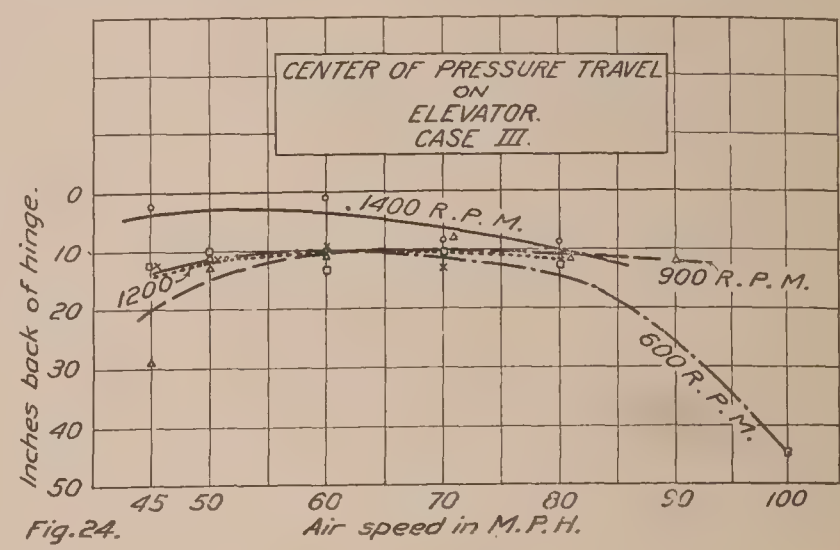


Fig. 24.

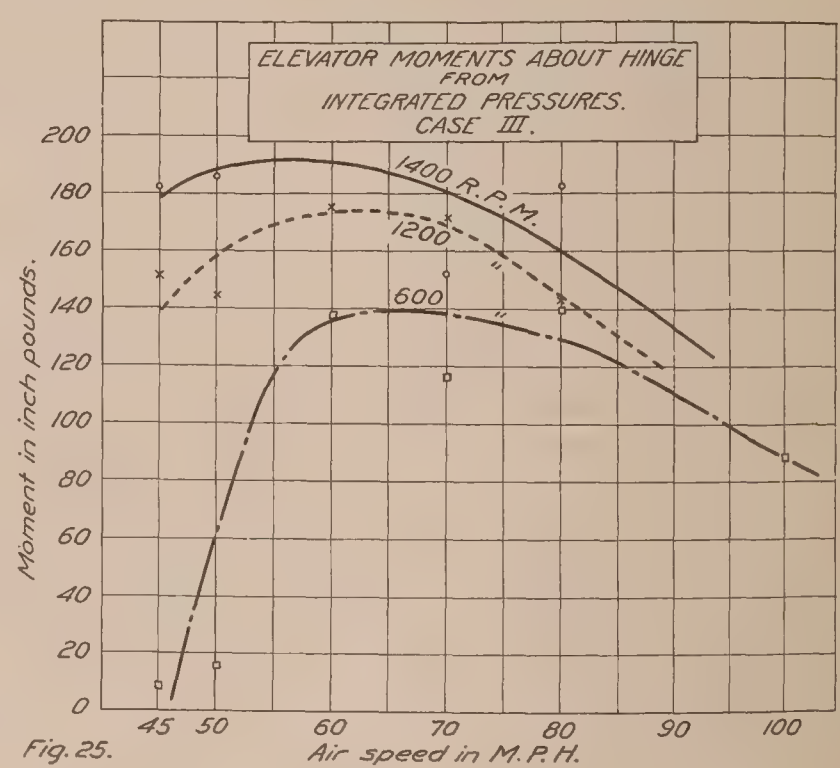


Fig. 25.

the weight of an ordinary elevator, it is necessary to make a change (backward) of the center of gravity to increase the total up load on the tail, of which the elevator will get a small share; or, better, to decrease the angular setting of the tail plane, which, as has been shown, increases the proportion of load on this part. This is the function of the adjustable tail plane which can be used to minimize the stick force at any ordinary flight condition. Even a small change in elevator weight may greatly alter the stability of the airplane because of the important changes that are made on the airplane to balance it aerodynamically. As will be shown later, these changes greatly affect the distribution of pressure on the tail surface.

The tail surface works normally at rather small angles of incidence and with a small total load; so that the center of pressure is often off of the surface, thus having little significance. The load on the tail generally reverses in sign at a certain speed making the center of pressure curve discontinuous. The center of pressure curves for Case III are plotted in figure 23 against air speed; a separate curve for each engine speed. The curves are quite regular and the vectors

are further forward with the lower engine speeds. At 65 miles per hour, where the load changes sign, the vector passes off the rear and returns at the forward edge of the tail plane.

The center of pressure curves for the elevator alone are shown in figure 24; the curves for the various engine speeds being close together, with a tendency for the center of pressure to be further forward with higher engine speeds. The movement of the center of pressure is small, and moves forward from 45 miles per hour to 60 miles per hour, but at higher speeds again moves to the rear.

In figure 25 are plotted the moment curves of the elevator about its hinge for various engine speeds and air speeds, and while it is evident that a great many of the points do not lie on the curve, because a very small error in drawing the pressure curve at the trailing edge of the elevator makes a great difference in the moment about the hinge, these curves can be considered as fairly accurate, as they check with the moment curve obtained by measuring the direct force on the control stick, and it may be considered that they are sufficiently correct for an illustration. With a center of gravity position in this run—a coefficient of 0.324—it is shown that the moments are all positive; that is, there is a tendency of the air to lift the elevator. At low air speeds this upward force on the elevator is rather small, especially at low engine speeds; but as the air speed is increased the load reaches a maximum at about 65 miles an hour. As the speed increases from this value the curves all tend to go downward; that is, the upward force on the elevator is increased about proportionally to the air speed. In order to complete the data, in figure 26 are plotted curves showing the angles of the elevator for a similar condition of flight, and it is interesting to notice from these curves the small angle the elevator moves through for greatly changing flight conditions, and for this reason it is unnecessary in this report to resolve any of the forces on the elevator into the plane of the tail plane. These curves show that the angle of the elevator is increased; that is, is more positive as the air speed increases up to about 55 miles per hour; from this point on, however, the angle of the elevator slightly decreases and reaches about a neutral angle at the higher flight speed. It also shows that the higher engine speed requires the greater positive angle of elevator setting.

ACTUAL PITCHING MOMENT ABOUT THE CENTER OF GRAVITY DUE TO THE TAIL PLANE.

The pitching moment produced by the tail plane about the center of gravity of the airplane is evidently found by the product of the distance from the center of pressure of the horizontal tail surfaces to the center of gravity and the load on the tail as found by integrating the pressure over the whole of the horizontal surface. In figures 27 to 31 the curves as determined in this way are plotted against the air speed in miles per hour for various engine speeds. As shown in the curves for Case I, the pitching moment has a more positive value as the engine speed is decreased, except at 600 revolutions per minute, which is in nearly all the other cases abnormally high. The curves all decrease from a negative to a more positive value as the air speed is increased, and within the experimental error the curves are parallel; at low speeds the curves become more nearly horizontal. It will be noted that in Case I, where the center of gravity was a large distance back on the mean chord, that the curves for different engine speeds are quite widely separated, while in Cases II, III, and IV the curves lie very closely together; but in Case V the curves are again separated at high speeds, and it is a remarkable fact that the order of the curves in this case is entirely reversed; that is, that the curve at 1,400 revolutions per minute is lowest and that at 600 revolutions per minute is highest. As the thrust line of the engine is about 3 inches below the center of gravity of the airplane, it would be expected that at high engine speeds the thrust would tend to produce a positive moment on the airplane and that this would be resisted by a pitching or negative moment on the tail plane, and this is evidently what happened in most of the cases. The reason for the abnormally high negative value of the pitching moment curve for 600 revolutions per minute is not very clear, and it is probably due to some effect of the slip stream—or lack of slip stream—on the wings or body. The complete reversal of the moment curves for Case V, however, is still more puzzling and can only be explained by the change in stagger—which was necessary in this case—producing different forces by interaction with the slip stream. As would be expected, the pitching moment

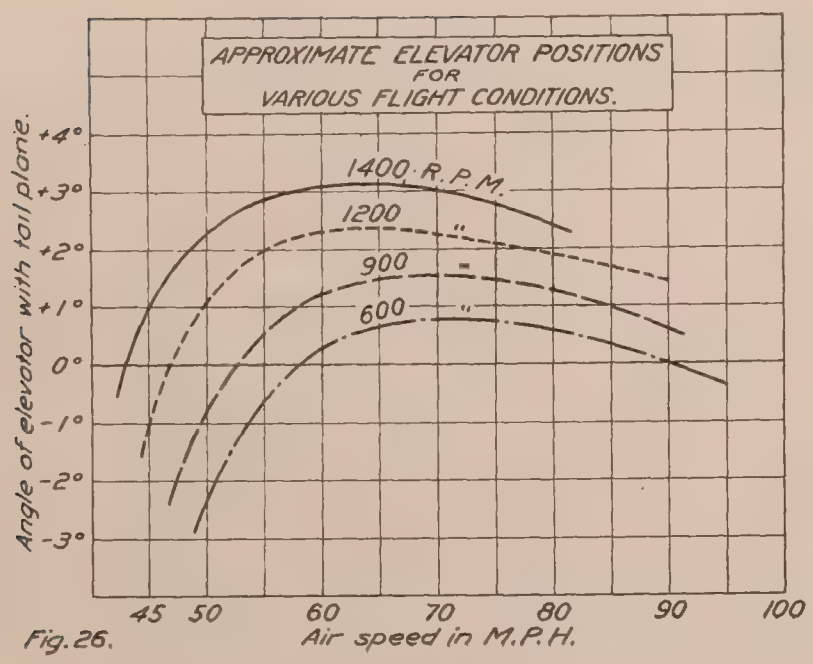


Fig. 26.

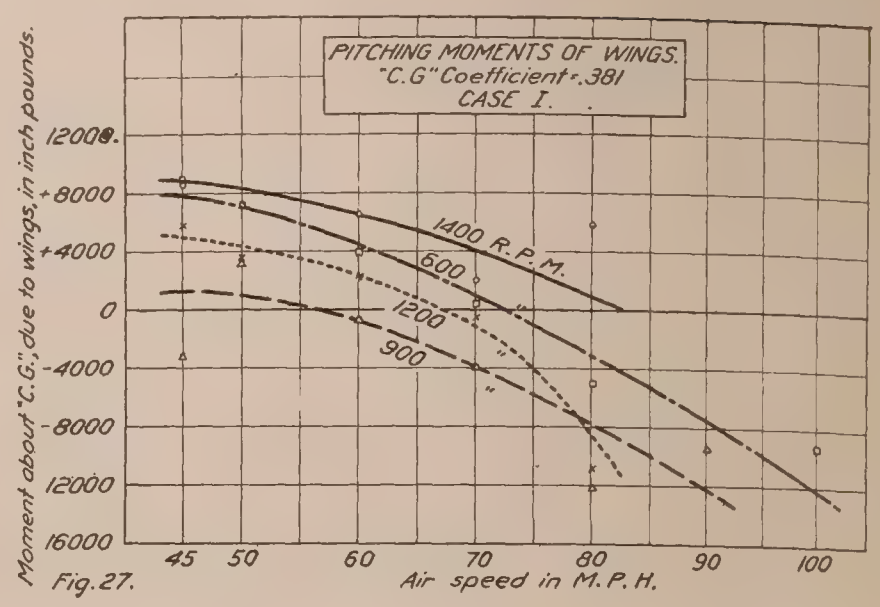


Fig. 27.

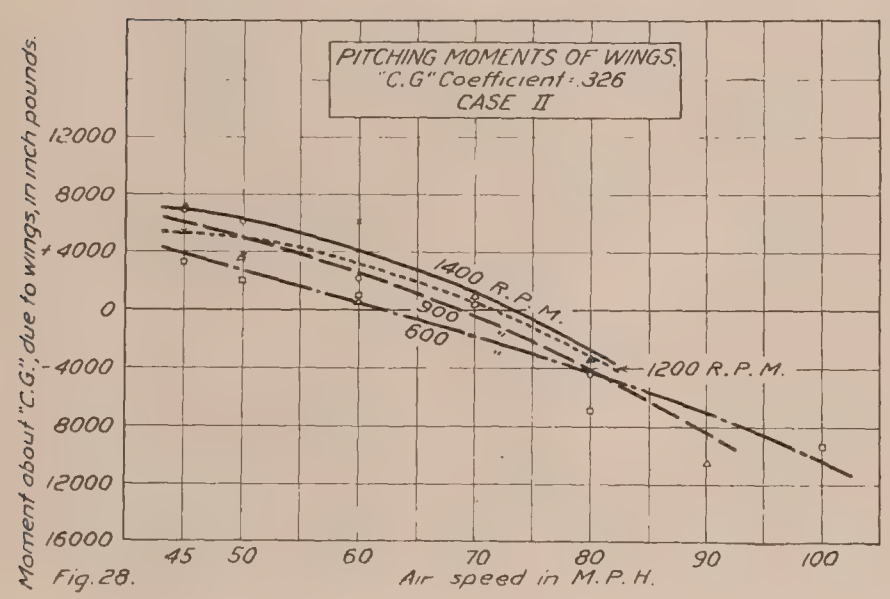


Fig. 28.

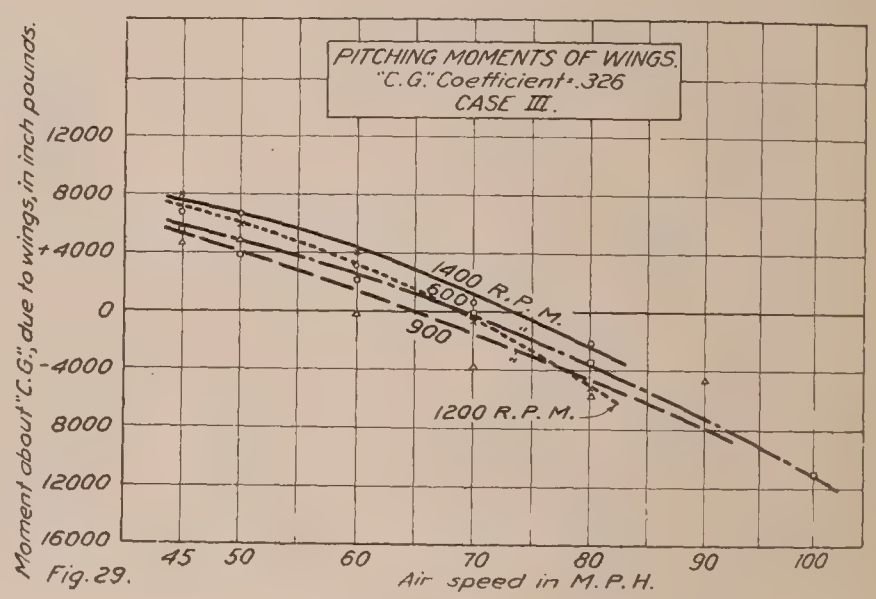


Fig. 29.

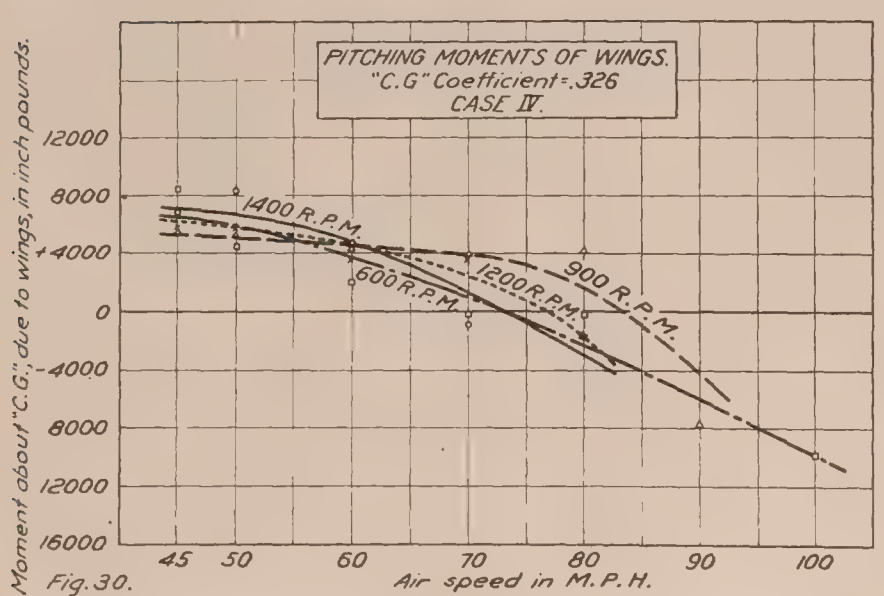


Fig. 30.

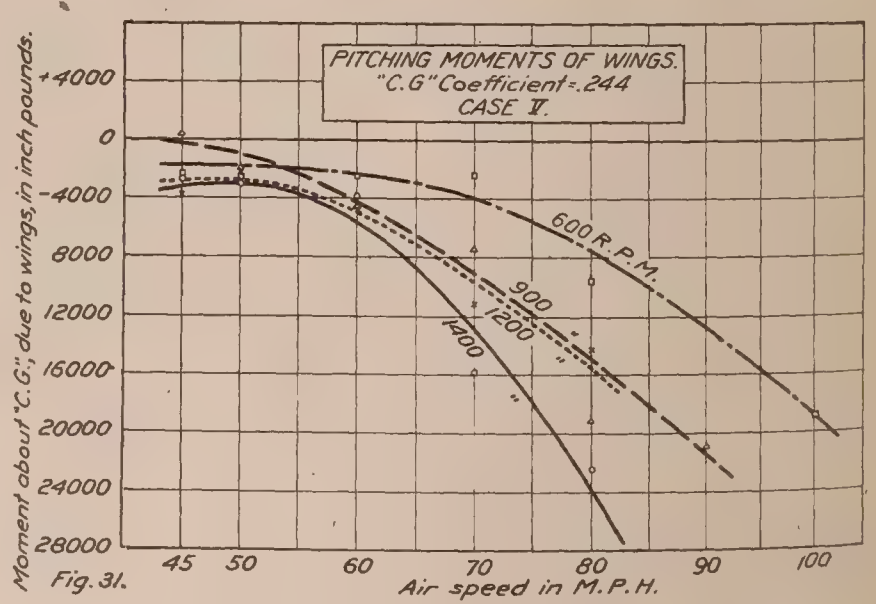


Fig. 31.

curves have a more negative value as the center of gravity is moved forward, and, as will be shown later on, the center of pressure curves from these moments show very good agreement for all center of gravity positions.

CENTER OF PRESSURE TRAVEL COMPUTED FROM MOMENT CURVES.

The distance of the center of pressure from the center of gravity of the airplane is evidently equal to the moment about the center of gravity due to the wings, divided by the weight of the airplane, which is constant; so that in all cases the center of pressure curve will be parallel to the moment curve no matter what position the center of gravity has. In figure 32 is shown the center of pressure curves for all cases and also for the case of a wing tested in the wind tunnel having the same section as that of the full-sized wing. In order to obtain the true pitching moment of the wings alone, the moment of the chassis and body, although small, must be subtracted; so a model body and chassis were tested in the wind tunnel in order to obtain their true pitching moment about the center of gravity in each case (fig. 33). These values, corrected for scale, were then subtracted from the full-scale pitching moment due to the tail, thus giving the moment for the wings alone. It will be seen that the results of the several cases check very closely, and it may be concluded that this center of pressure travel is very close to that actually occurring in flight. The full-flight center of pressure travel is, however, considerably farther forward than it is in the model, and this is the same conclusion that was

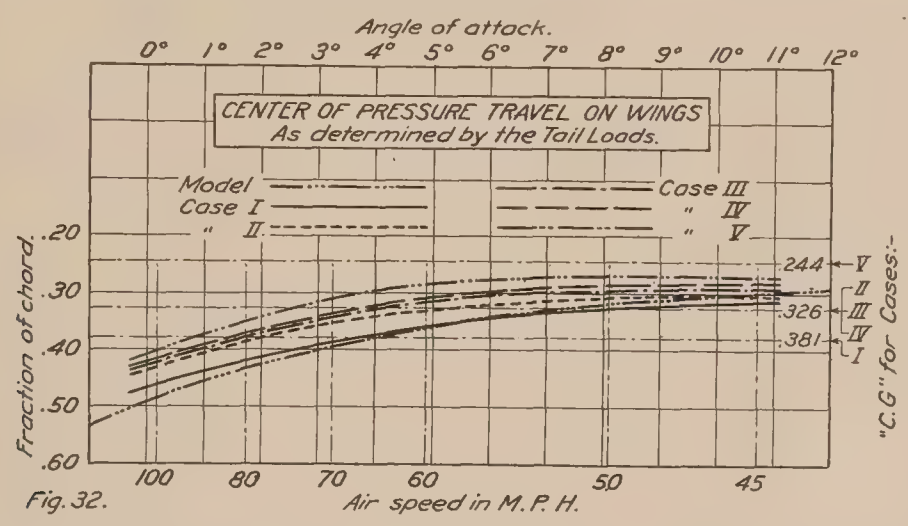


Fig. 32.

reached by the British in their tests as noted previously. The difference is not great, but still it is enough to account for a considerable lack of balance in the full-size airplane if the center of gravity position is computed strictly from wind-tunnel results. The total travel of the center pressure on the full-sized airplane is less than it is on the model; so that the full-sized airplane might be expected to be more stable than the model under similar conditions.

THE RELATION OF THE STABILITY TO THE PRESSURE DISTRIBUTION ON THE TAIL SURFACES.

The stability of the airplane is dependent upon two factors; the first is the moment curve of the wings alone, which in most cases is unstable, and the second the moment curve of the tail, which is usually very stable. The first is mainly affected by the wing section used, by the amount of stagger, and, most important, by the position of the center of gravity; the second is affected almost entirely by the length of the body and by the aspect ratio of the tail plane, for the moment curve will evidently be more stable the more rapidly the lift of the tail plane is changed for a given change in the angle of attack, and this change in slope of the lift curve is mainly affected by the aspect ratio of this surface. The stability of the airplane, then, can be increased in two ways; first, by making the moment curve of the wings more stable, or, second, by increasing the efficiency of the tail surface.

The stability of an airplane, as shown in Report No. 96, may be measured in two different ways; one with fixed controls and the other with free controls, and, as is generally the case, the stability with fixed control evidently depends on the change in the position of the elevator for

different flight conditions, while the stability with free controls depends entirely on the force on the elevator for various flight conditions. It is this last condition which can be studied quite readily from the distribution of pressure on the tail surfaces, and in no other way can the exact process of the stabilizing effect on the tail plane be visualized.

In order to study more clearly the effect on the pressure distribution on the horizontal tail surfaces of various changes in the airplane, the pressure distribution curves are shown in figure 34 for the different cases at 600 revolutions per minute and at the various air speeds investigated. These curves are not drawn as accurately as the originals and are only intended to bring together in one place the various curves, so that they may be more easily compared; but their accuracy is quite sufficient for use as an illustration.

In Case I there is a marked upward pressure at the leading edge of the tail plane at the lowest speeds, but as the speed increases this region of upward pressure moves backward, and at about 60 miles per hour a region of downward pressure appears at the lower side, increasing to larger and larger values as the speed increases. It will be noticed that as the air speed drops from 60 to 50 miles per hour the upload on the elevator increases (unstable), due to increased suction near the hinge. From 50 to 45 miles per hour the upload increases (stable), due to the elimination of the downward pressure at the trailing edge of the elevator caused by its more negative

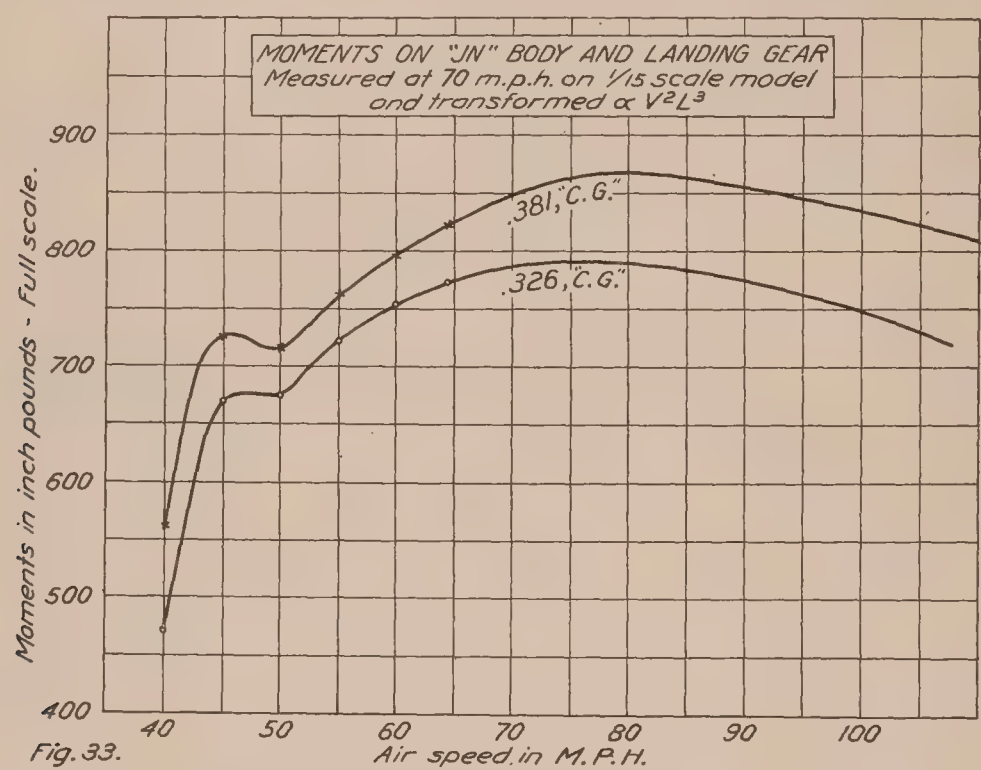


Fig. 33.

angle. On the other hand, as the speed increases from 60 to 100 miles per hour the upload is decreased (unstable) until at the highest speed the load becomes downward, due to a negative pressure region at the rear of the elevator caused by the positive elevator angle. At the high speed not only the weight of the elevator itself but the downward air force must be held by the pilot, making it necessary for him to exert a considerable pull on the control stick; and if the control in this position were released the airplane would immediately nose over onto its back, a thing which has actually happened with this type of airplane in actual flight.

It might be well to define here the terms "tail heavy" and "nose heavy" as used by the pilot. Generally speaking, the pilot considers an airplane nose heavy when there is for the greater portion of the flying range a pull on the stick; inversely, it is tail heavy if there is an undue push on the stick. This force on the stick may be due, however, to two causes: The first is the weight of the elevator itself, which is no unimportant part of total stick pull, and the second is the air load on the elevator, which, it will be seen, varies with the different flight conditions. For a balanced condition of the control it is necessary, then, to have an upward air force on the elevator which is equivalent to the weight of that member, and in some instances the elevator is so heavy that this necessitates a large and usually detrimental change in the airplane to affect this condition.

In Case II the only change that has been made from the preceding is the movement of the center of gravity from a coefficient of 0.326 to 0.381. It will be seen that there is no great change in the distribution of pressure; a general increase on the upload of the tail surface is all that is seen, as would be expected from a change in the center of gravity. The general characteristics of the pressure on the elevator are the same as before, except that the load on the elevator is more downward at low speeds and of about the same value at high speeds, so that the airplane is on the whole more nose heavy; still the stability has been improved, this improvement being due primarily, of course, to the more stable moment curve of the wings, and the tail load is balanced by this moment.

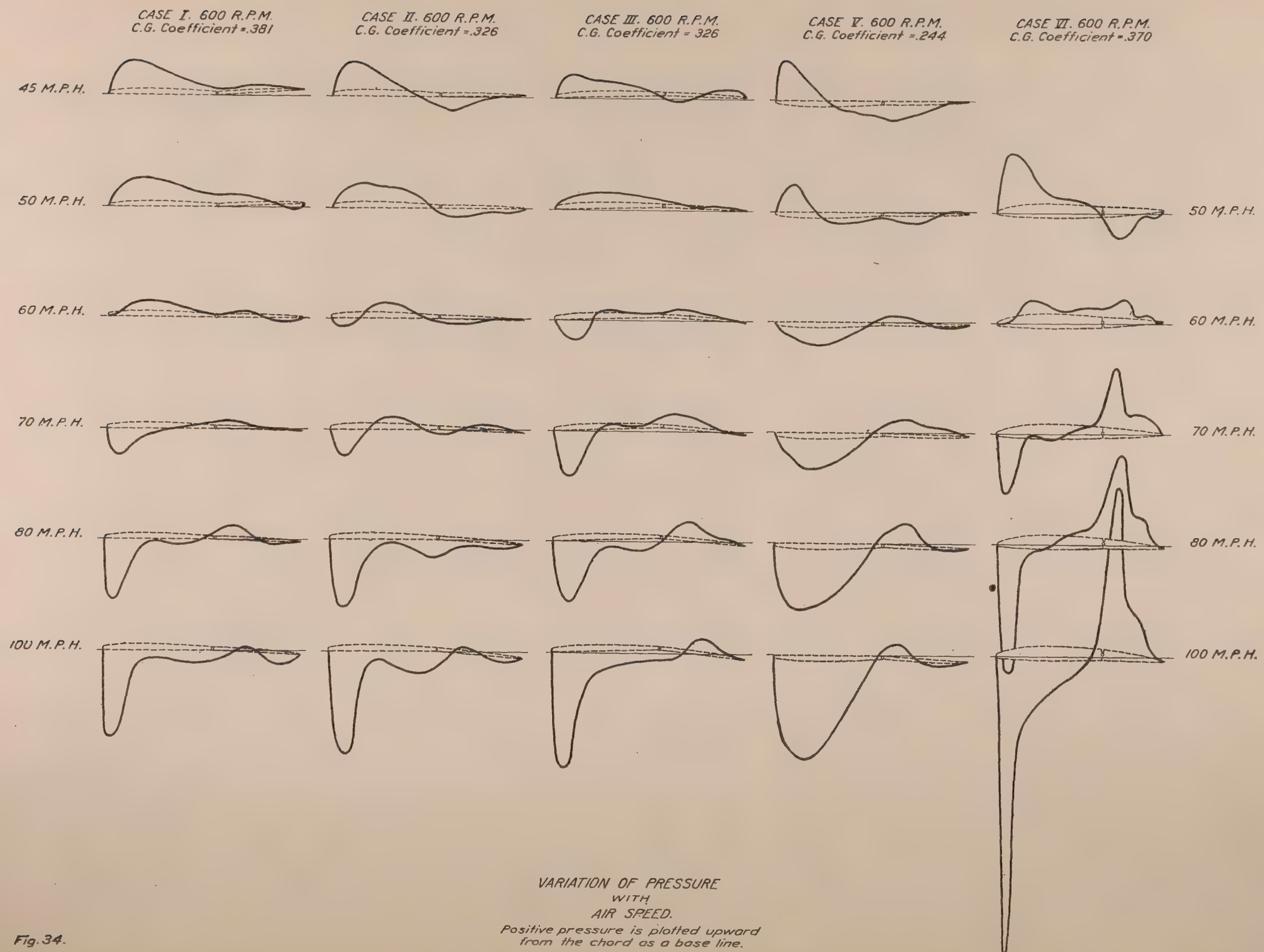
In Case III the only change was the more negative setting of the tail plane; the pressure distribution, however, suffers a considerable alteration. In the first place, there is at the leading edge a small region of upward pressure at low speeds and a greater region at low speeds than occurred in the preceding case, and, as was shown before, the distribution of pressure between the tail plane and elevator is such as to increase the positive load on the elevator and decrease that on the tail plane. The elevator has a considerable upload in all cases due to the elimination of the negative pressure area at its trailing edge, and this is confirmed by the decreased nose heaviness in flight. The function of the adjustable tail plane, therefore, is not to alter the total load on the tail plane, as this must be constant for any one flight speed and center of gravity position, but to vary the distribution of load between the movable and fixed portion of the tail surface, so that the moment about the elevator hinge may be made as small as desired. It is evident by the pressure distribution curve that the stability is increased by a negative change in the stabilizing angle; that is, there is less down load on the elevator at the higher flying speeds.

In Case V where the tail plane is inverted—that is, the flat surface uppermost—a very considerable change in the pressure distribution is shown. The negative region at the leading edge at low speeds is concentrated more nearly at the front, but at higher speeds the pressure reverses and the positive region is more to the rear; so that the region of maximum pressure is moved a considerable distance back on the tail plane, and altogether there is a greater load on the tail plane than in any other case, as would be expected with a more forward position of center of gravity. The pressure conditions on the elevator are quite different in this case, due chiefly to the very forward position of the center of gravity, but undoubtedly the more efficient action of the tail plane in this position increases the high speed stability as shown in Report No. 96. There is a down load on the elevator at low speeds which changes to an up load as the speed increases, producing considerable stability, although the airplane is at all times rather nose heavy.

In Case VI the center of gravity coefficient was 0.370 and the special tail plane was used having a greater span and a thicker section. The distribution of pressure over the tail plane is very different from the other cases, and while the pressure curves for similar conditions have somewhat the same shape, the down pressure at the nose and the up pressure at the tail were very much exaggerated, producing exceedingly large values at high speed; and these higher pressure regions are concentrated on a very small portion of the chord in each case. It is also evident from the curves how great is the stability of the airplane with this tail plane even though the center of gravity is far back—as the downward pressure on the elevator at low speeds rapidly increases with the air speed to rather great upward values.

THE EFFECT OF THE SLIP STREAM ON STABILITY.

It has been clearly shown in Report No. 96 that the longitudinal stability of the tail plane is considerably decreased by the effect of the slip stream on the tail surface, because if these members are in constant or nearly constant flow of air it is much more difficult to obtain stability than if they are in an air flow which varies as the speed of flight. The efficiency of the tail surface then will be proportional to the amount of area which is outside of the slip stream, so that a tail plane of high aspect ratio is of great advantage in increasing the stability, not only because it increases the steepness of the lift curve but because it has a considerable amount of area which is outside of the slip stream.



VARIATION OF PRESSURE
WITH
AIR SPEED.
Positive pressure is plotted upward
from the chord as a base line.

Fig. 34.

The effect on the stability of an airplane from the variation in the slip stream caused by changing the engine speed is very complex and little actual data have been obtained on it. For the purpose of showing more clearly the effect on the pressure distribution over the tail surfaces with the variation in engine speeds, there are shown in figure 35 the various curves of pressure distribution for the four engine speeds tried, and for the various conditions of the airplane. As in figure 34, these curves are not intended to represent accurately the pressures, but are only drawn in a general way to serve as an illustration; Case IV was omitted for the reason that it showed no difference from Case III, and in Case VI there were no runs taken at 900 and 1,200 revolutions per minute.

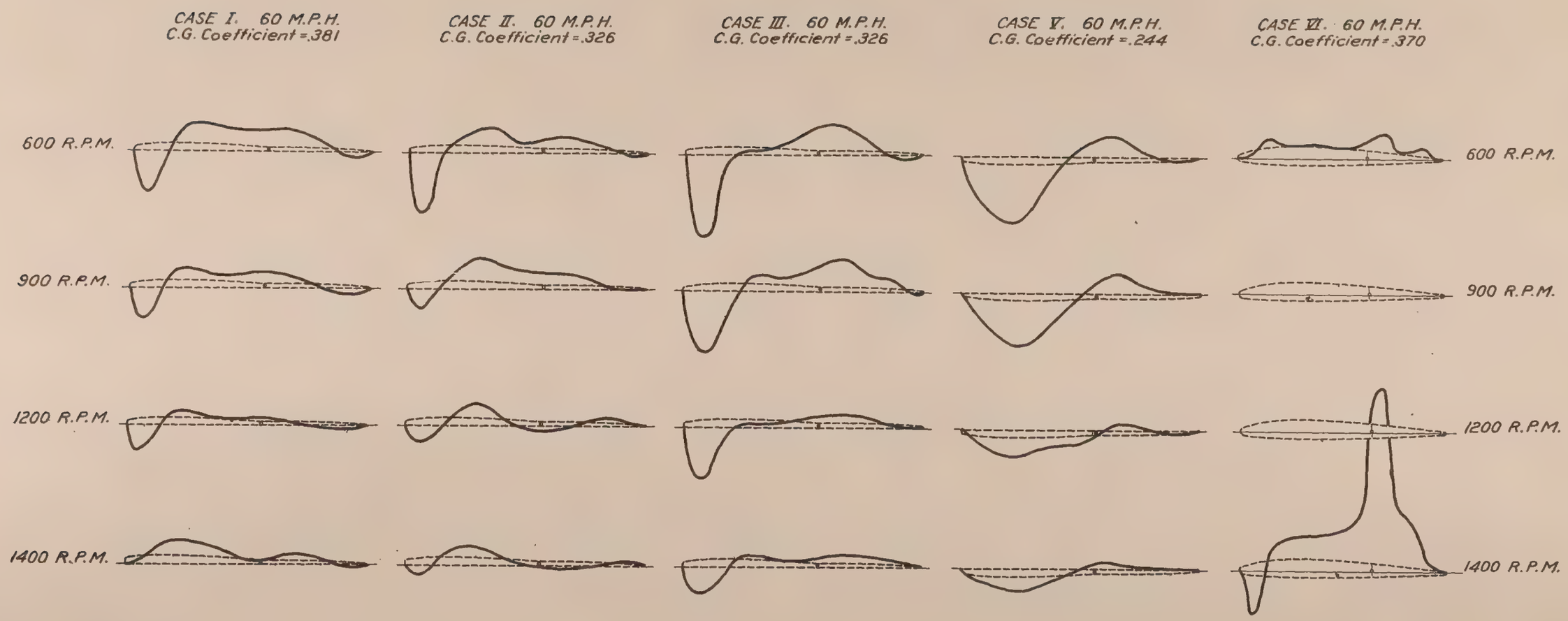
The free control stability as determined for this airplane in Report No. 96 shows that in general there is a pull on the stick, and the down load on the elevator is greatest at low engine speeds. This effect is shown in all the cases, although it is not as evident visually as might be expected because the downward pressure at the low engine speeds occurs at the trailing edge of the elevator, and even though the total load on the elevator does not decrease with the air speed, the moment about the hinge will be more negative with the lower air speeds. In all cases the lower engine speed produces a greater down load at the nose of the tail plane and at the same time a higher region over the hinge, and as the engine speed decreases there is a greater region of down pressure at the rear of the elevator.

How far the effect of the slip stream changes the distribution of pressure over the tail surfaces by a change in velocity or by a change in direction can not be determined until more complete data are obtained on the slip-stream direction and velocity around the tail surfaces. It is necessary, however, to distinguish between cause and effect; that is, there is a pitching moment about the center of gravity due to the wings, the body, and the moment of the propeller thrust about the center of gravity; all of which must be balanced by the load on the tail, and in steady flight the elevator must be placed in such a position that this load produces a moment about the center of gravity which is exactly equal to the total moment produced from the other causes. Therefore, the change in total load on the tail surfaces at a given air speed must be due to a change in moment about the center of gravity, caused either by the moment of the propeller thrust about the center of gravity or by the slip-stream effect on the wings and body; on the other hand, the distribution of pressure over the tail surfaces and the change in pressure on the elevator in particular are determined by the direction and velocity of the air passing over these surfaces, and this changes (at constant flight speed) only with a variation in the speed of the engine. With a constant setting of the elevator in an airplane which is flying, trimmed at, say, 600 revolutions per minute, when the engine is opened up to 1,400 revolutions per minute the average airplane will tend to climb. This is due not primarily to the fact that the thrust line is slightly below the center of gravity but is mainly due to the fact that the velocity over the tail is greater and in a more downward direction—that is, the negative load on the tail is greater—which tends to raise the nose of the airplane to such an angle that the moment of the tail plane is balanced by the moment of the wings about the center of gravity; therefore, the more stable the airplane the less variation in air speed will there be between various throttle settings for equilibrium conditions.

SUGGESTIONS FOR FURTHER RESEARCH.

As the value of a research is not only in answering questions but also in finding questions to answer, it is believed that a short discussion of the difficulties encountered in this investigation and the problems for which a satisfactory solution has not been arrived at will be of value in guiding future work of this kind. It might be argued that unsolved problems would be of little value to the average person, but the recognition of problems that are worth solving is a step ahead in any branch of research and their suggestion may lead to new lines of thought.

One of the most important problems, and one on which there has been only a little light shed, is the section of the tail plane. It is seen from the tests in this report that the tail plane with a thick section gives very high local pressures at the leading edge and at the elevator hinge, but at the same time it also gives to the airplane an excellent degree of stability; and the ques-



VARIATION OF PRESSURE
WITH
MOTOR SPEED.
Positive pressure is plotted upward
from the chord as a base line.

Fig. 35.

tion that has not been answered is whether the even distribution of pressure over the tail surfaces is incompatible with a stable tail moment curve. It is suggested in Report No. 96 that the point of maximum stability depended upon the relation of the camber on the upper or lower surface of the tail plane, and that a high upper camber gives stability at low speeds, while a high lower camber gives stability at high speeds. Also the results in this report show conclusively that a tail plane with a flat upper surface and a considerable camber below gives a much more even distribution of pressure than the usual type of tail where the flat surface is the lower. The whole problem, then, consists in finding that section which will give the greatest stability with the most even distribution of load over the tail surface. Of course in some airplanes a high degree of stability is not required or even desirable, in which case it is still best to use the most efficient tail section, and if the stability is to be reduced, to cut down the area of the surfaces.

Another unsolved problem which is of the greatest importance in the design of tail surfaces is the investigation of the causes for the vibrations which occur in some tail planes. It is believed that this is the reason for the failure of some tail planes which are structurally quite strong enough to withstand the static load which is imposed upon them. The data from this investigation show that there is in some instances a very great torsional load on the tail plane about the Y-axis which produces a considerable change in the angle of setting of this member due to deflection, and this deflection in turn would cause a still greater change in the pitching moment; so it is quite conceivable that pressure conditions could arise in which an oscillation would be set up about the Y-axis which might become dangerously large for certain speeds.

In regard to the changes in the methods and instruments which would be recommended for another test of this kind, the first would be the recording on the same film with the pressures, as at present, the value of the air speed, the engine speed, and the angle of the elevator. This could be done very easily by measuring the air speed by the height of the liquid in one of the tubes in the same way that the pressures are measured, and the revolutions of the engine could be recorded in the same way with a liquid column raised by a centrifugal pump as in the Veeder liquid tachometer. The angle of the elevator could be easily recorded by placing a scale in the center of the gauge and having a small pointer running over it on a flexible wire connected to the control system. In this way a better check could be obtained upon the accuracy of flying done by the pilot, and it would also give additional data which would be of value in studying the results of the pressure readings. As mentioned before, the necessity for making capillary corrections for each one of these tubes in the gauge was very laborious, and in another instrument every effort would be made to obtain tubes of such uniform bore that these corrections could be neglected.

CONCLUSIONS.

STRUCTURAL CONSIDERATIONS.

The most interesting results obtained from these tests are, perhaps, the very low average load per square foot on the usual type of tail plane during steady flight; so small in fact that it could not in any conceivable way cause failure, even on the weakest tail plane. With the tail plane of thick section, however, the conditions were quite different, even though the average load was of course the same as before. A very high local down load occurred at the leading edge and conversely a very high up load occurred at the hinge, which consequently produced a very large torsional moment about the Y-axis of the airplane, so that this type of loading might prove structurally unsafe unless the front spar was made exceptionally stiff. It is also shown that there may be considerable danger in the use of an adjustable tail plane which can reduce the load on the controls to a very small amount, yet the load on the tail plane itself may be very large, in some cases dangerously so, without giving warning to the pilot. The advantages of an inverted tail plane—that is, a section with the greatest camber on the lower surface—are made evident from the pressure curves, which show a much more even distribution of pressure over the tail surfaces with this arrangement. The use of a tail plane of high aspect ratio is in some cases a structural advantage because it brings the tips of the tail surface outside of

the slip stream, and it is the tip of the tail plane which carries a considerable proportion of the load; and if the tip is in the slip stream, as is the case with a great many tail planes, this load is considerably augmented. In this case the load may be upward on one side of the tail plane and down on the other, due to the rotation of the slip stream, producing a very large torque about the X-axis of the airplane, while with the high aspect ratio plane this torque is considerably reduced.

STABILITY.

The conclusions reached on stability may be summed up briefly by stating that the efficiency of the tail plane is increased by an increase in aspect ratio far more than by any other change, and this is proved not only by pressure distribution but by theory and by tests on the stability of the airplane from measurements of the elevator force and position. The center of pressure travel for the wings as determined from the integrated pressure on the tail surfaces give curves that are fairly consistent with themselves and with results obtained in another way by the British, but the center of pressure is slightly further forward on the full size airplane than it is on the model.

TABLE OF RESULTS.

	Revolutions per minute.	Air speed.	Center of pressure on whole tail, inches ahead of hinge.	Center of pressure on elevator, inches back of hinge.	Load on tail plane, in pounds per square foot.	Load on elevator, in pounds per square foot.	Total load, in pounds per square foot.	Maximum local load in pounds per square foot.	Moment about center line of tail, in inch-pounds.
CASE I	1,400	45	11	10	1.1	0.5	0.8	+ 7.1	.....
		50	3	12	.7	.6	.7	+ 8.1	.....
		60	3	7	.7	.5	.6	+ 7.7	.....
		70	− 57	− 2	.0	.3	.2	+ 0.8	.....
		80	− 37	12	.0	1.1	.5	+ 1.2	.....
	1,200	45	26	11	1.0	.1	.6	− 2.8	.....
		50	11	1	.4	.2	.4	+ 2.8	.....
		60	2	−23	.3	.1	.2	+ 5.2	.....
		70	60	67	.0	.1	.0	+ 7.0	.....
		80	18	18	1.4	.6	1.1	+ 9.9	.....
	900	45	− 34	19	.2	.8	.2	+ 9.1	.....
		50	27	30	.6	.0	.3	− 2.1	.....
		60	20	30	.0	.1	.1	+ 3.6	.....
		70	26	40	.6	.1	.4	+ 5.6	.....
		80	22	19	1.9	.4	1.2	+10.7	.....
		90	16	20	1.2	.6	.9	+ 8.1	.....
	600	45	6	15	1.2	.4	.9	− 3.7	.....
		50	22	− 3	1.2	.2	.8	− 2.9	.....
		60	21	− 4	.6	.1	.4	− 2.6	.....
		70	− ∞	10	.2	.3	.0	− 4.2	.....
		80	41	− 4	.8	.2	.6	+ 8.8	.....
		100	23	25	1.4	.3	1.0	+ 8.8	.....
CASE II	1,400	45	17	1	1.0	.0	.7	− 4.2	613
		50	7	11	.7	.4	.6	+ 6.5	668
		60	− 30	8	.1	.3	.2	+ 7.8	460
		70	−298	10	.3	.5	.0	+10.8	406
		80	47	− 4	1.0	.2	.5	+13.6	549
	1,200	45	13	20	.7	.3	.5	− 2.9	.....
		50	10	18	.5	.2	.3	− 2.9	.....
		60	11	8	.7	.3	.6	+ 4.7	242
		70	− 14	16	.0	.1	.0	+ 6.5	.....
		80	38	−80	.7	.0	.4	+ 9.4	.....
	900	45	10	−13	1.0	.1	.5	− 3.1	.....
		50	27	8	.7	.1	.4	− 2.6	.....
		60	− 48	33	.0	.1	.0	+ 3.1	367
		70	− 20	27	.1	.0	.1	+ 3.1	.....
		80	39	8	.7	.1	.4	+ 6.5	.....
		90	28	16	1.8	.2	1.1	+ 9.9	.....
	600	45	43	3	.9	.3	.4	− 3.1	.....
		50	55	14	.7	.4	.3	− 1.8	.....
		60	37	4	.3	.1	.1	+ 2.3	284
		70	−216	23	.0	.1	.0	+ 3.1	.....
		80	25	12	1.1	.2	.7	+ 6.8	.....
		100	29	160	1.7	.0	1.0	+11.5	.....
CASE III	1,400	45	− 2	12	.6	.7	.6	+ 6.5	432
		50	− 11	11	.4	.8	.6	+ 7.8	619
		60	∞	0.5	.4	.4	.0	+ 9.6	1,216
		70	356	9	−.6	.4	.1	+12.3	534
		80	99	9	1.3	1	.4	+14.6	382
	1,200	45	8	12	.9	.6	.8	+ 2.9	.....
		50	− 3	11	.4	.6	.5	+ 4.4	87
		60	− 40	10	.0	.8	.3	+13	.....
		70	131	13	.7	.6	.0	+ 9.1	.....
		80	51	11	1.5	.6	.6	+11.5	.....
	900	45	14	29	.6	.2	.4	− 2.1	.....
		50	10	13	.6	.1	.5	− 1.8	27
		60	140	11	.3	.1	.0	+ 3.7	.....
		70	53	8	1.2	.5	.5	+ 7.8	.....
		80	44	11	1.5	.5	.7	+ 8.3	.....
		90	55	12	1.5	.6	.6	+ 8.3	.....

+ Down loading. − Up loading.

TABLE OF RESULTS—Continued.

	Revolutions per minute.	Air speed.	Center of pressure on whole tail, inches ahead of hinge.	Center of pressure on elevator, inches back of hinge.	Load on tail plane, in pounds per square foot.	Load on elevator, in pounds per square foot.	Total load, in pounds per square foot.	Maximum local load in pounds per square foot.	Moment about center line of tail, in inch-pounds.
CASE III	600	45	21	12	0.9	10	0.6	− 3.1	.....
		50	21	10	.6	10	.4	− 1.6	152
		60	− 64	14	.1	.5	.1	+ 1.8	.....
		70	195	10	.5	.5	.1	+ 5.2	.....
		80	53	13	1.1	.5	.4	+ 6.8	.....
		100	32	45	2.1	.1	1.2	+12.3	.....
CASE IV	1,400	45	4	11	.7	.5	.6	+ 6.5	.....
		50	5	12	.6	.9	.7	.....	.....
		60	− 29	12	.1	.7	.4	.....	.....
		70	124	7	.8	.7	.2	.....	.....
		80	123	10	1.3	1.0	.3	.....	839
	1,200	45	7	10	.7	.4	.5	.....	.....
		50	− 5	11	.5	.6	.5	.....	.....
		60	− 31	10	.0	.6	.6	+ 6.0	.....
		70	− 53	12	.1	.8	.3	.....	.....
		80	84	13	.7	.4	.3	.....	.....
	900	45	19	21	.9	.2	.5	.....	.....
		50	13	12	.6	.3	.5	.....	.....
		60	− 1	11	.4	.4	.4	.....	.....
		70	− 26	15	.0	.6	.4	.....	.....
		80	− 45	10	.0	.8	.3	+ 6.8	.....
		90	34	− 16	1.5	.1	.8	.....	.....
	600	45	16	22	1.2	.4	.8	.....	.....
		50	16	9	.6	.2	.4	.....	.....
		60	1	17	.2	.1	.2	.....	.....
		70	145	8	.2	.2	.0	.....	.....
		80	−176	13	.4	.8	.1	.....	.....
		100	43	12	2.5	.7	1.1	+15.6	.....
CASE V	1,400	45	23	−180	.5	.0	.3	+ 3.1	157
		50	59	11	1.1	.7	.4	+ 5.2	545
		60	57	11	1.3	.7	.5	+ 6.8	566
		70	25	176	2.9	.0	1.6	+ 6.8	845
		80	24	38	3.8	.2	2.3	+ 9.4	1,673
	1,200	45	.....	.....	.6	.1	.4	− 3.7	.....
		50	.....	.....	.7	.3	.3	− 3.1	.....
		60	43	11	1.3	.5	.5	+ 3.9	.....
		70	.....	.....	2.2	.2	1.2	+ 6.5	.....
		80	.....	.....	2.9	.4	1.5	+ 8.3	.....
	900	45	64	70	.0	.0	.0	− 2.8	.....
		50	5	16	.2	.1	.2	− 2 8	.....
		60	21	45	.8	.0	.5	+ 2.1	.....
		70	32	2	1.6	.3	.8	+ 4.2	.....
		80	25	6	3.5	.1	2.0	+ 7.0	.....
		90	23	− 2	3.9	.2	2.1	+10.8	.....
	600	45	− 65	9	.1	.6	.2	− 4.2	.....
		50	− 7	11	.2	.2	.2	− 2.6	.....
		60	35	6	.5	.1	.3	+ 1.5	.....
		70	69	13	1.1	.7	.3	+ 4.4	.....
		80	31	− 4	1.9	.2	1.0	+ 6.2	.....
		100	29	1	3.6	.2	2.0	+ 8.3	.....
CASE VI	1,400	45	.....	.....	.....	.....	.....	− 8.6	.....
		50	.....	.....	.....	.....	.....	− 8.3	.....
		60	.....	.....	.....	.....	.....	− 9.1	.....
		70	.....	.....	.....	.....	.....	−12.6	.....
		80	.....	.....	.....	.....	.....	−14.6	.....
	600	50	.....	.....	.....	.....	.....	− 4.7	.....
		60	.....	.....	.....	.....	.....	− 3.9	.....
		70	.....	.....	.....	.....	.....	+ 6.5	.....
		80	.....	.....	.....	.....	.....	+10.8	.....
		100	.....	.....	.....	.....	.....	+15.6	

+ Down loading. − Up loading.

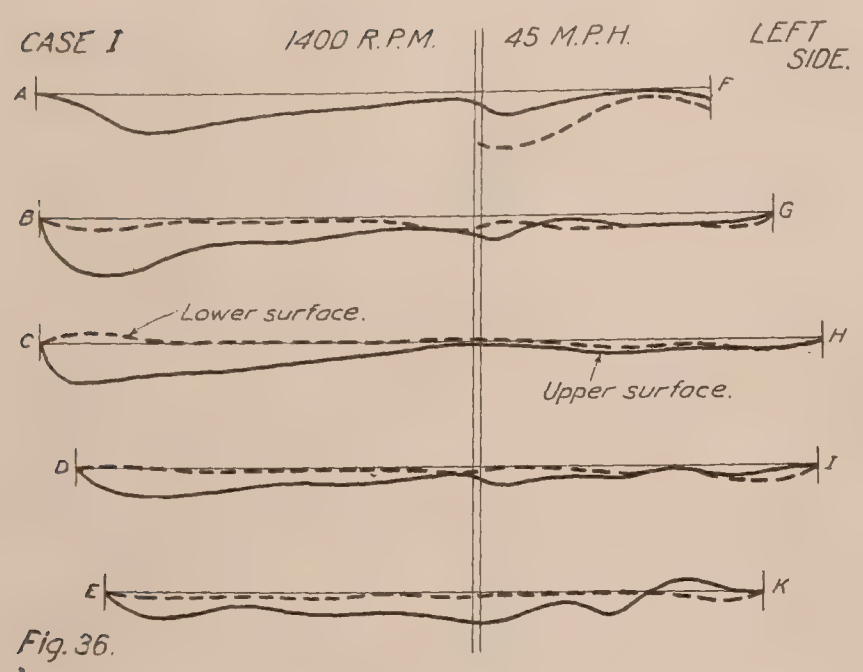


Fig. 36.

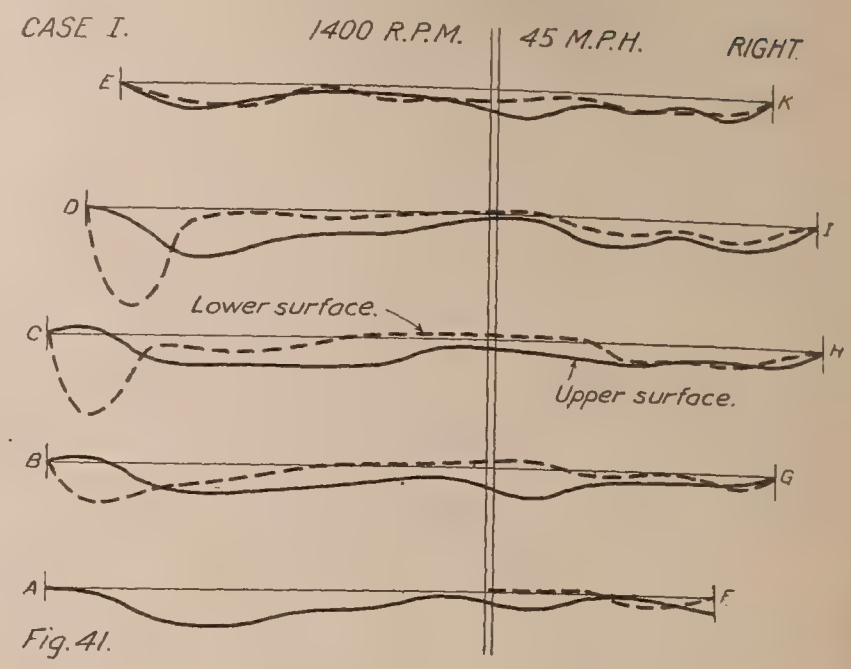


Fig. 41.

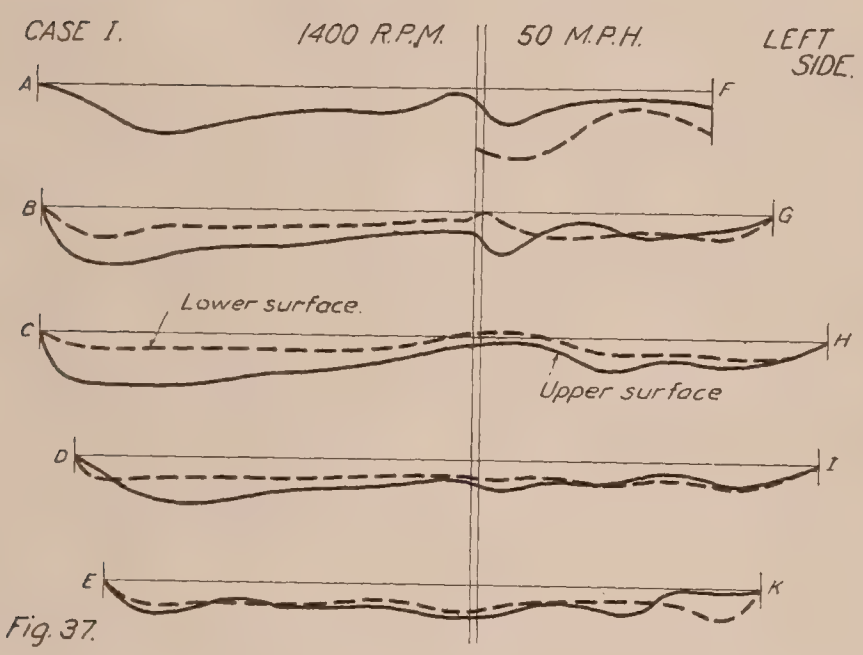


Fig. 37.

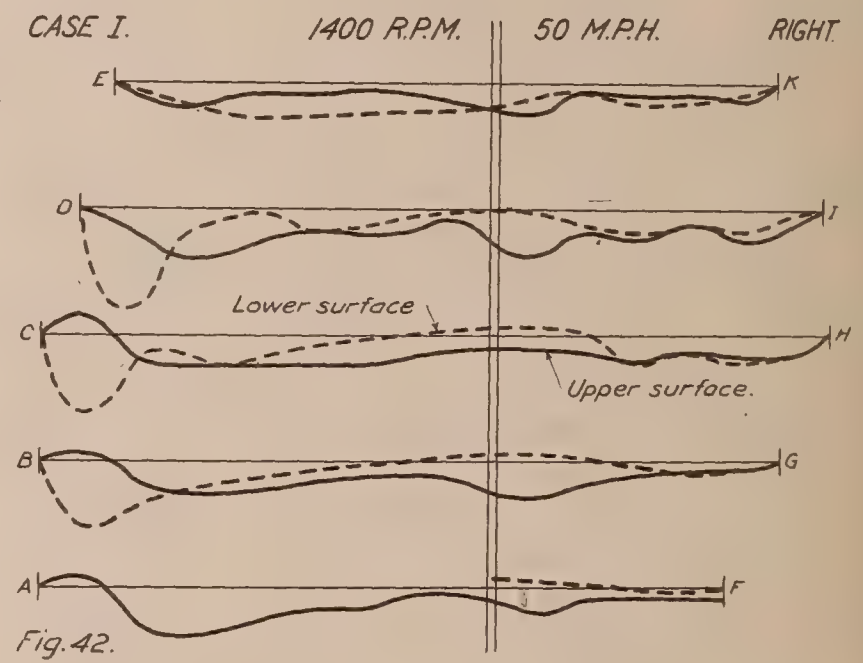
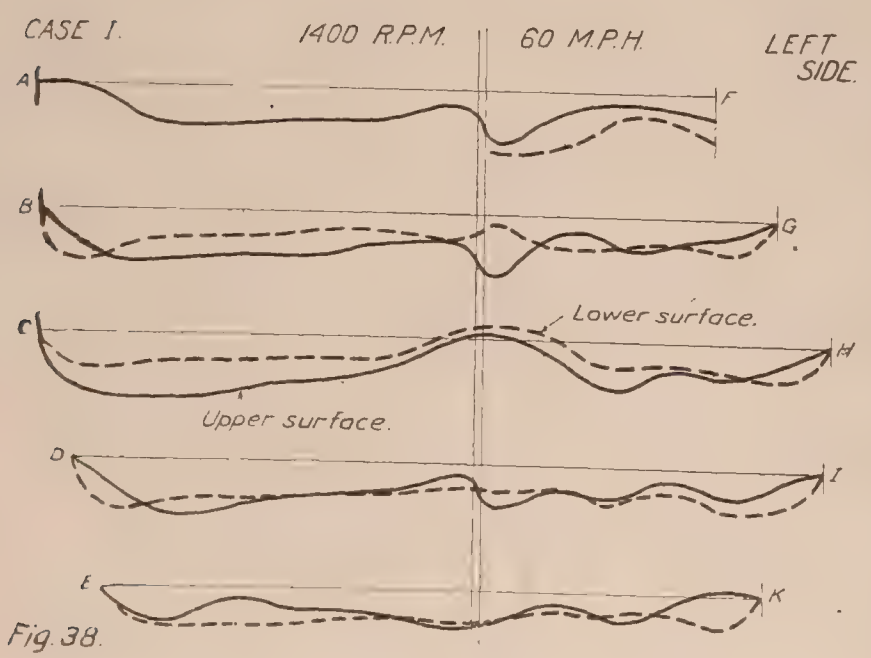


Fig. 42.



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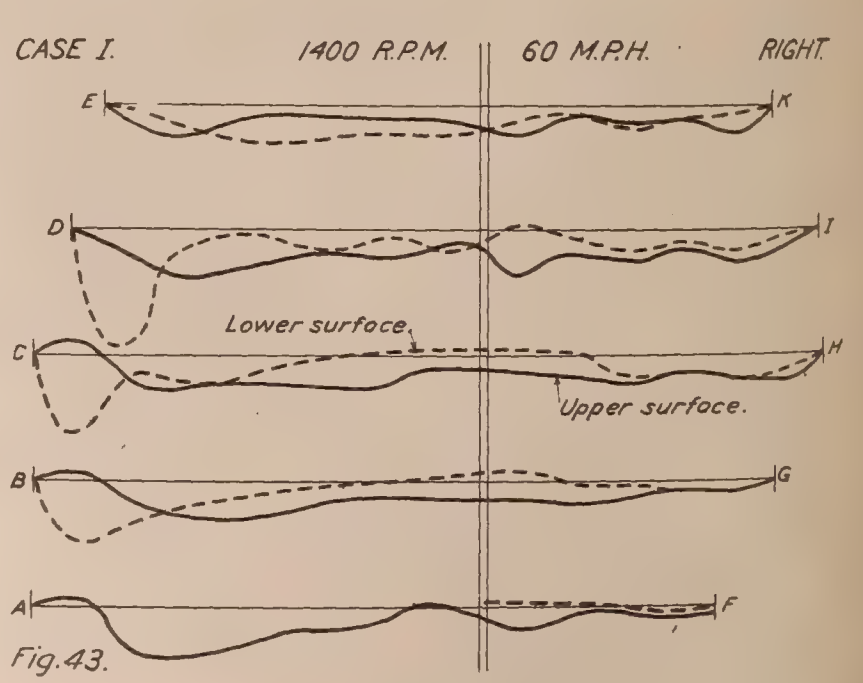


Fig. 43.

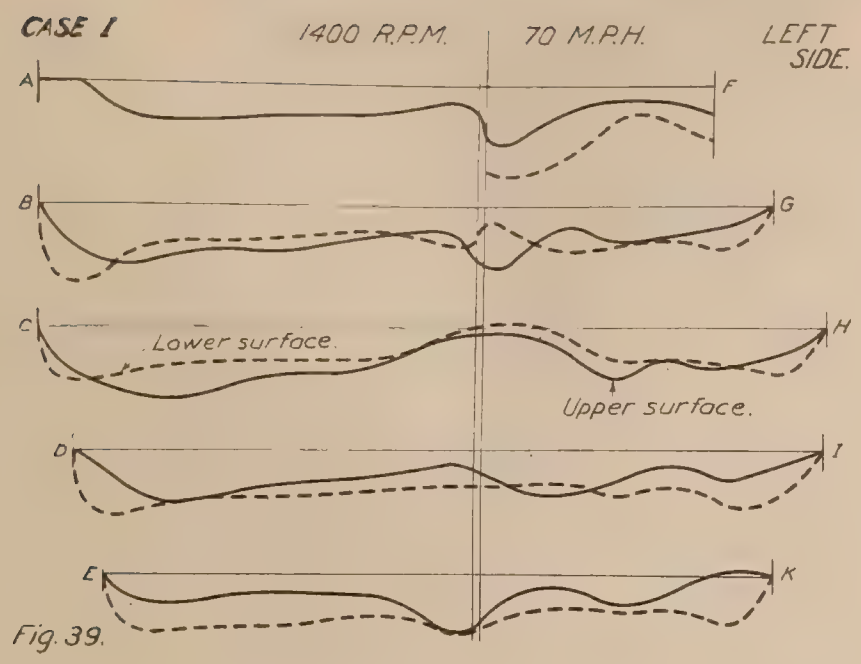


Fig. 39.

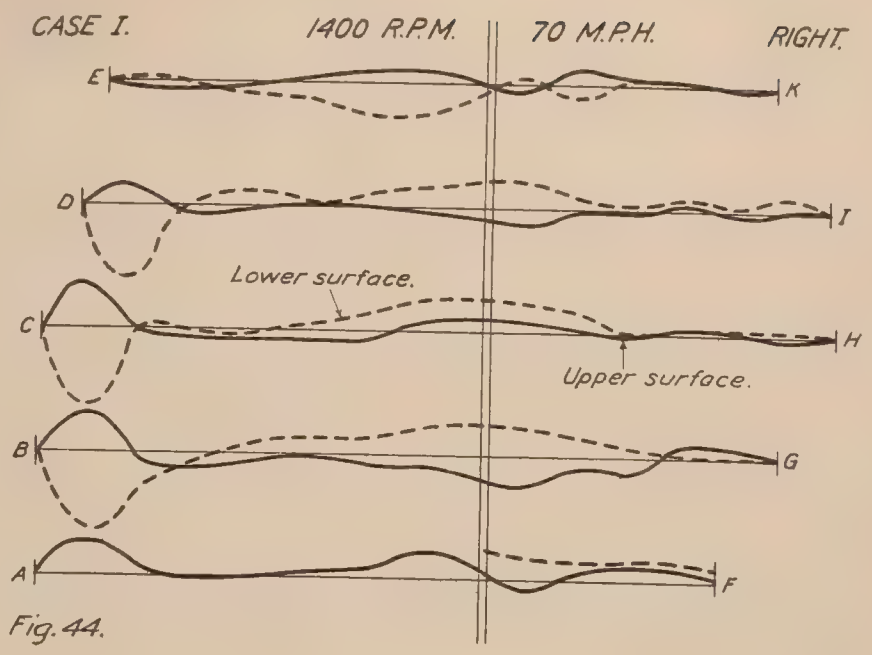


Fig. 44.

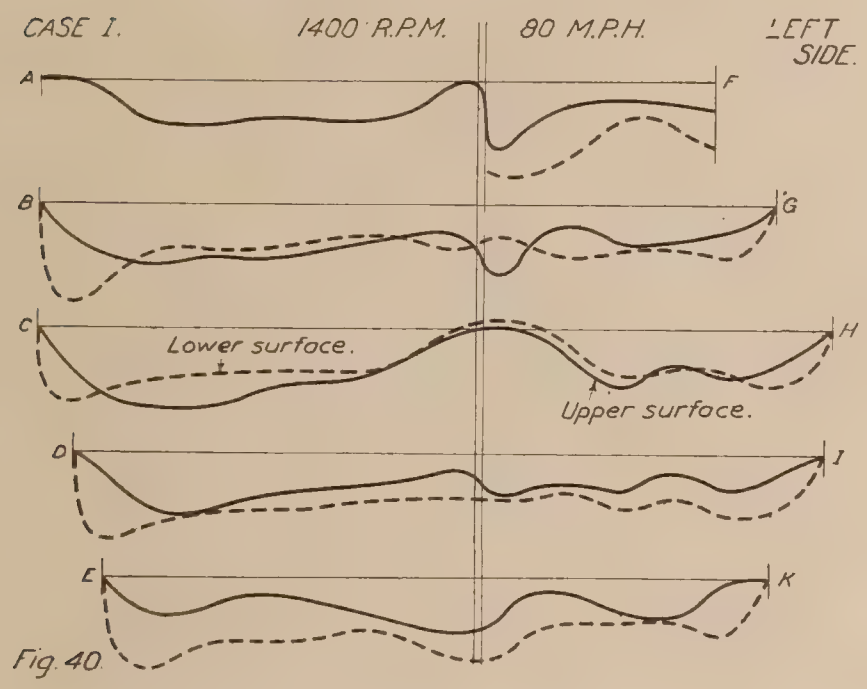


Fig. 40.

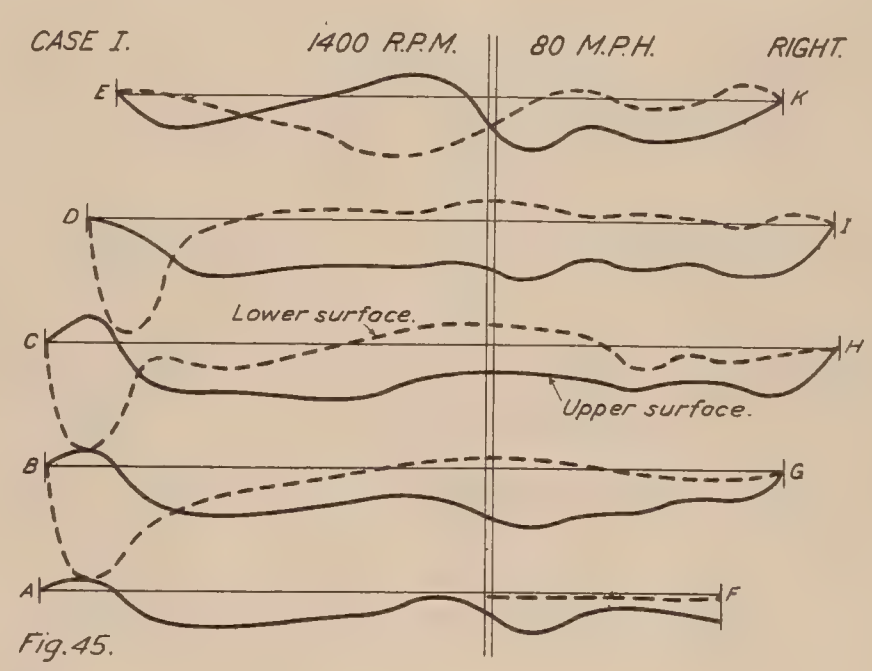
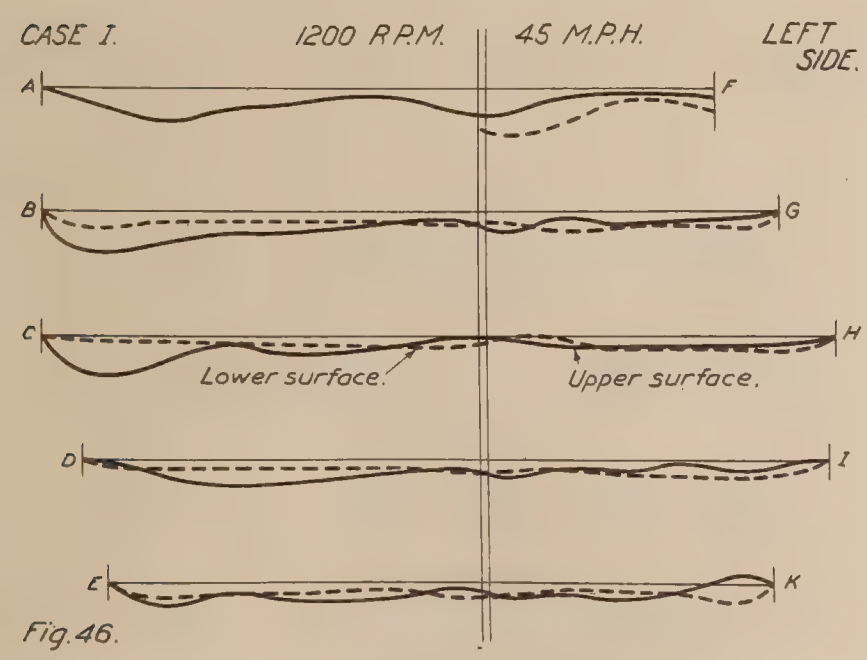


Fig. 45.



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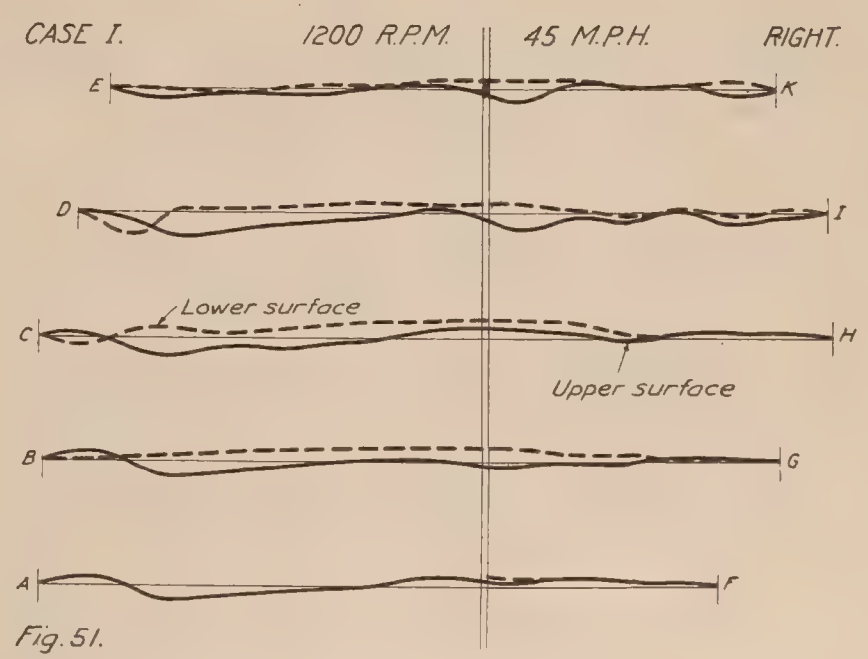


Fig. 51.

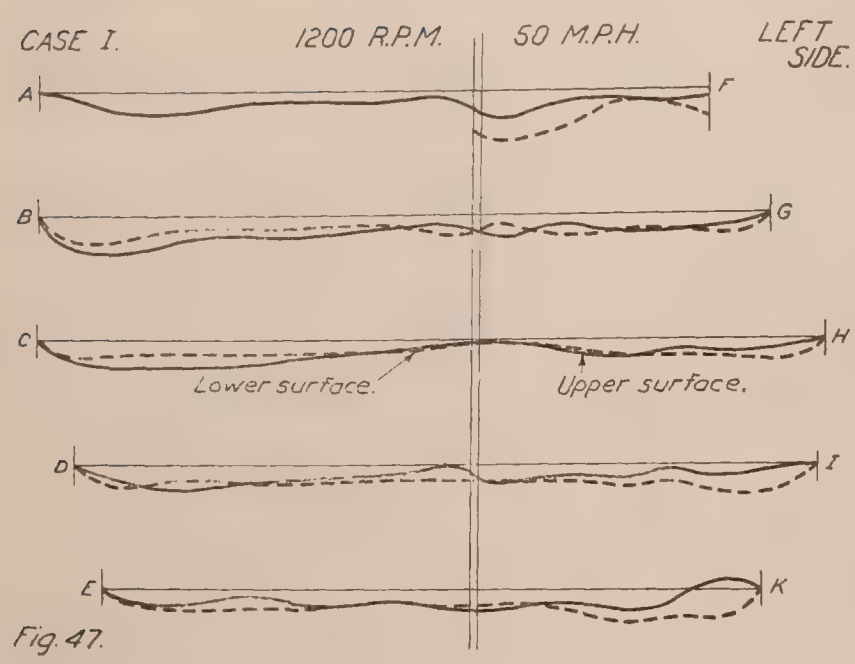
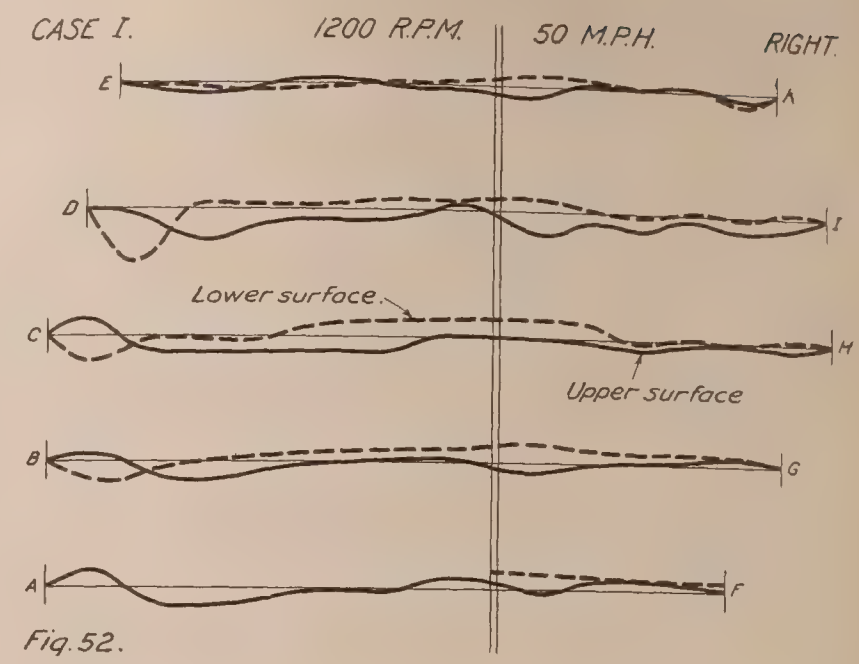


Fig. 47.



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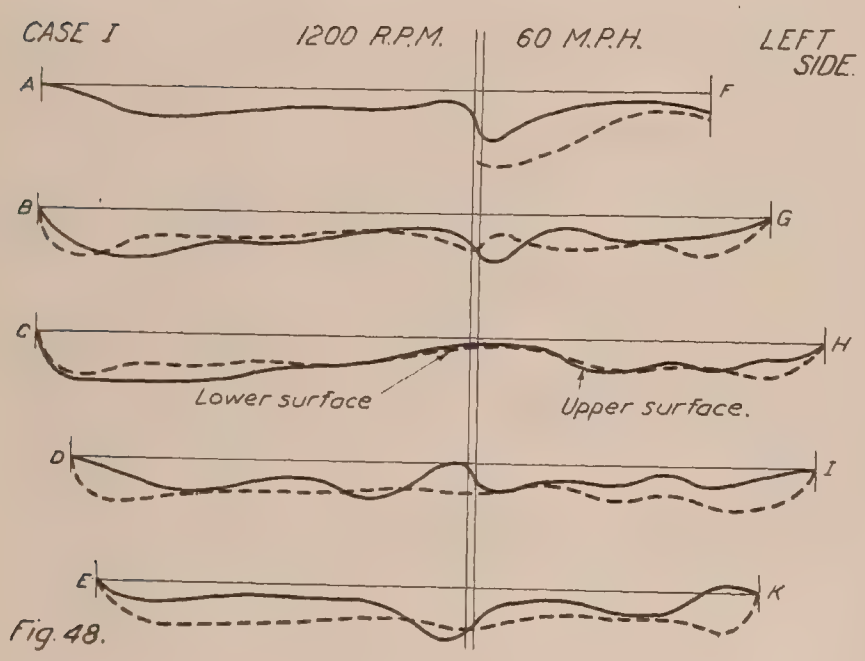


Fig. 48.

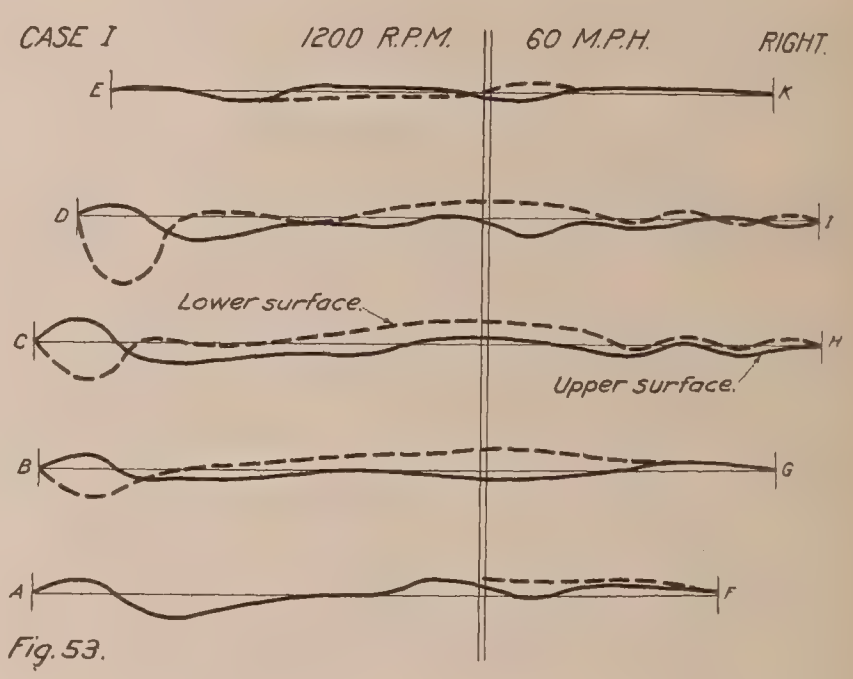


Fig. 53.

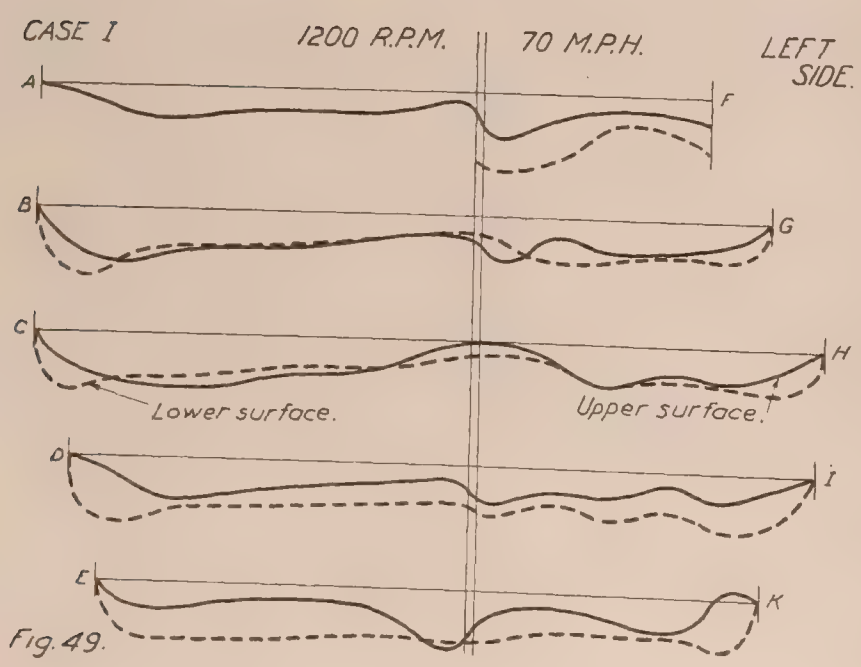
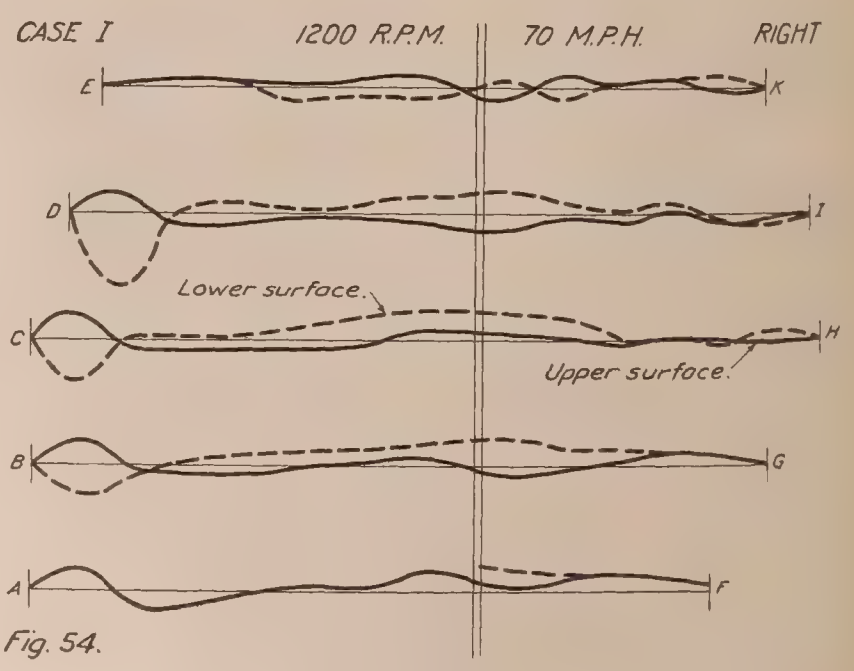


Fig. 49.



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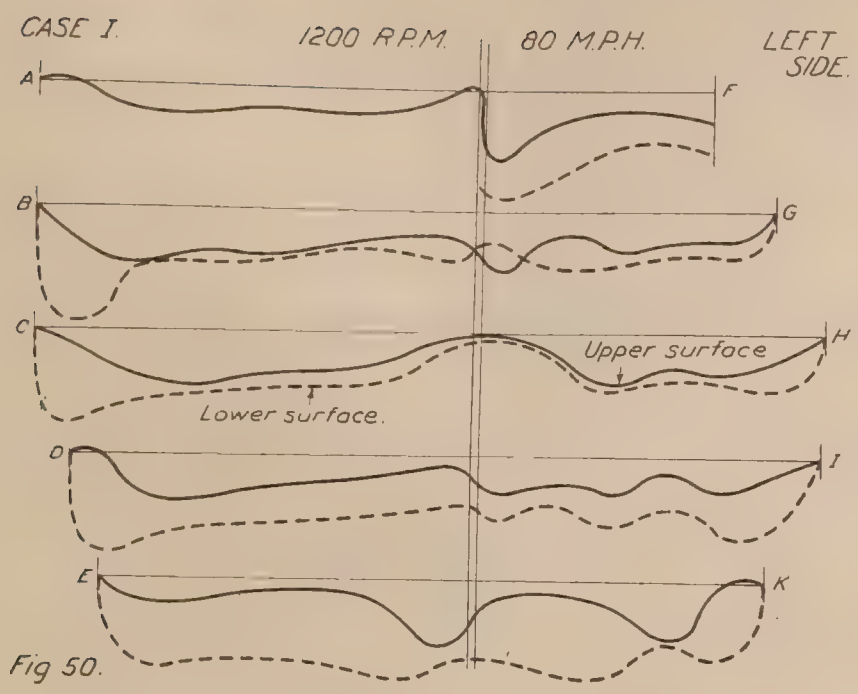

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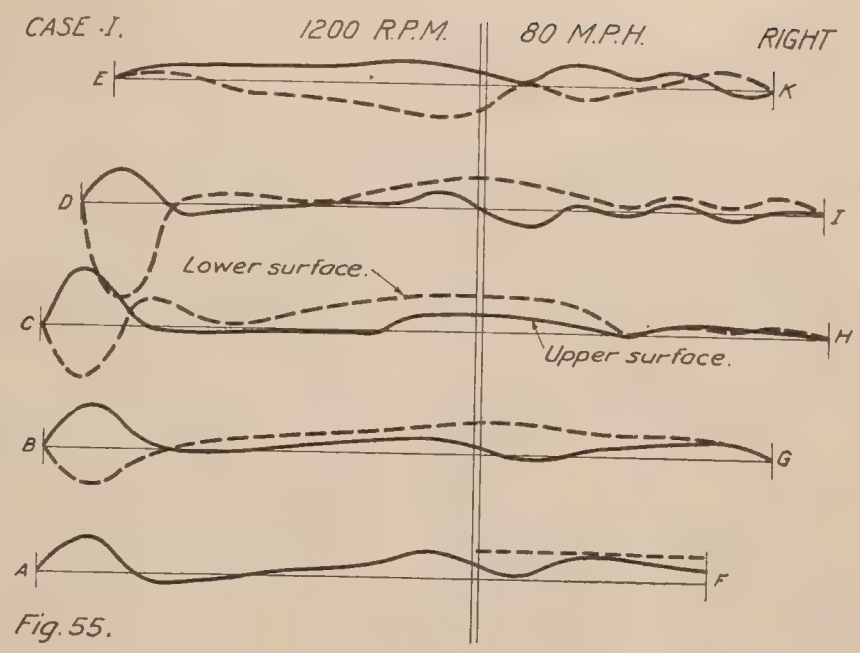
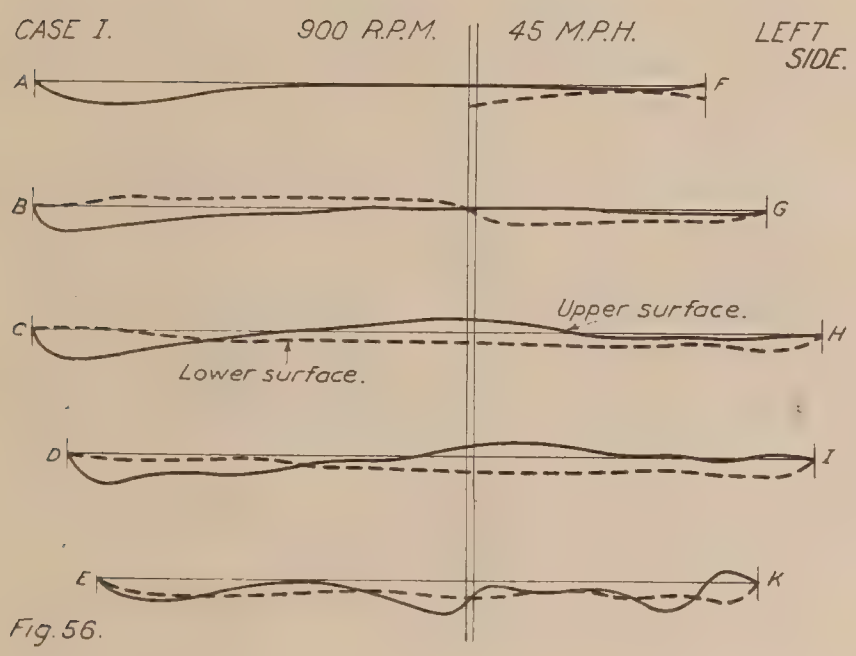


Fig. 55.



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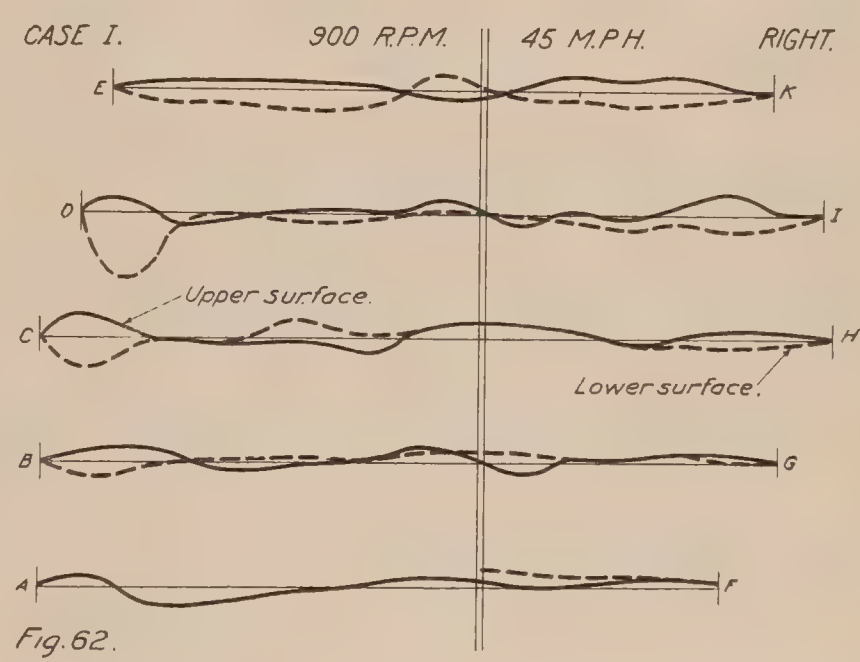


Fig. 62.

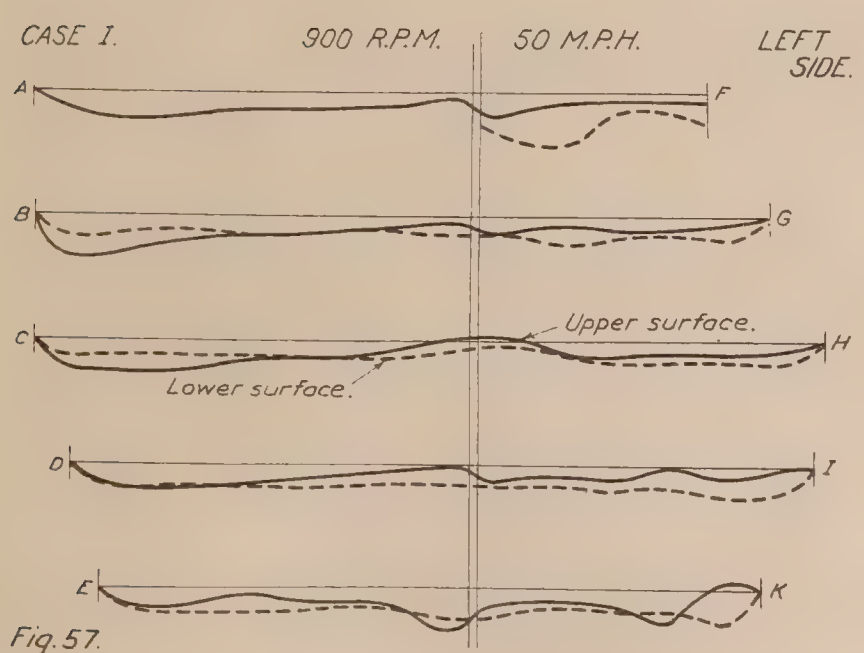


Fig. 57.

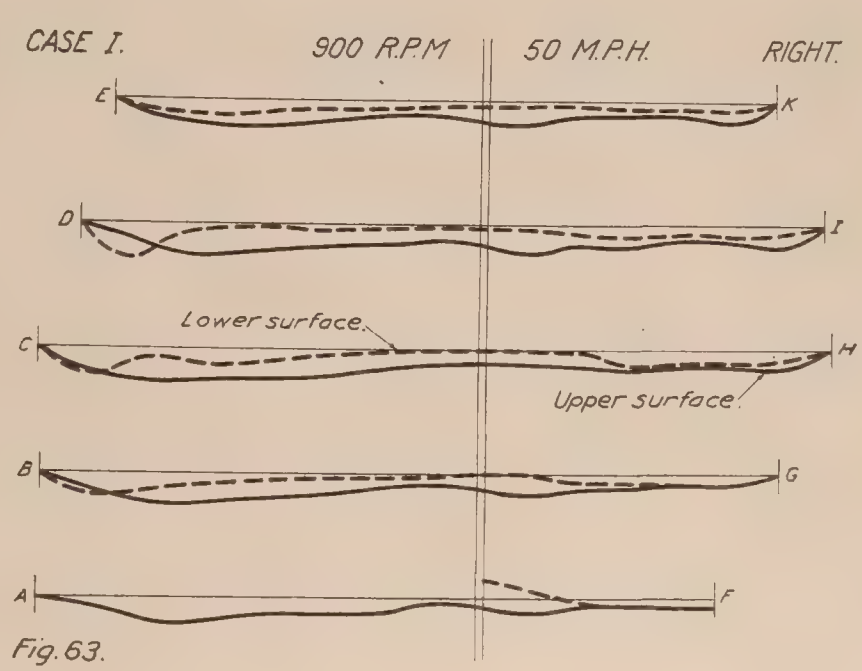


Fig. 63.

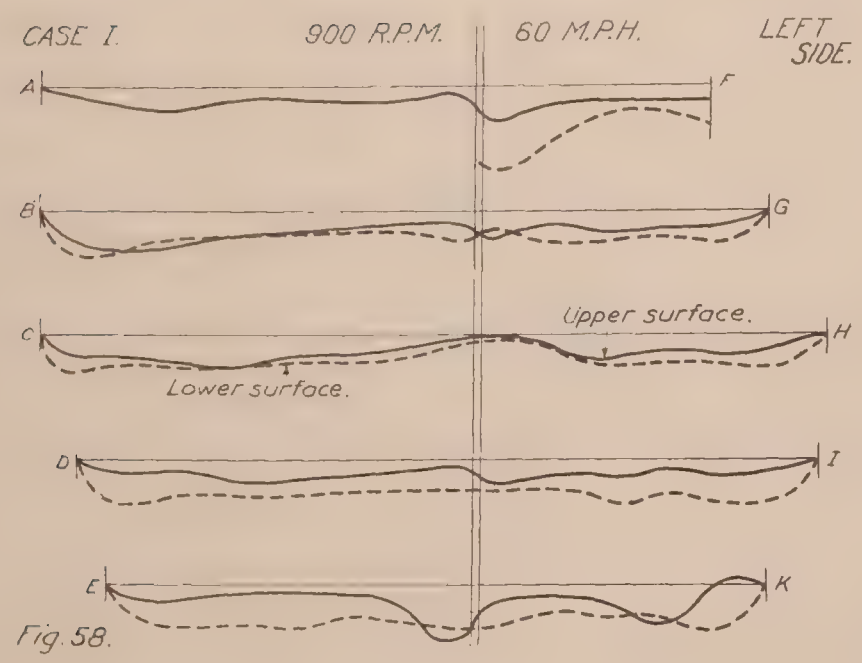


Fig. 58.

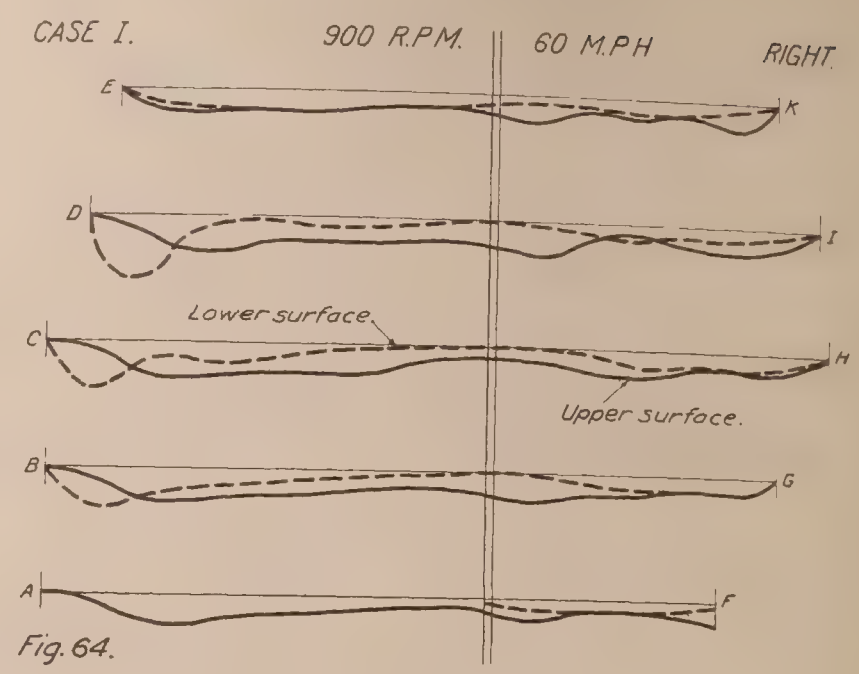


Fig. 64.

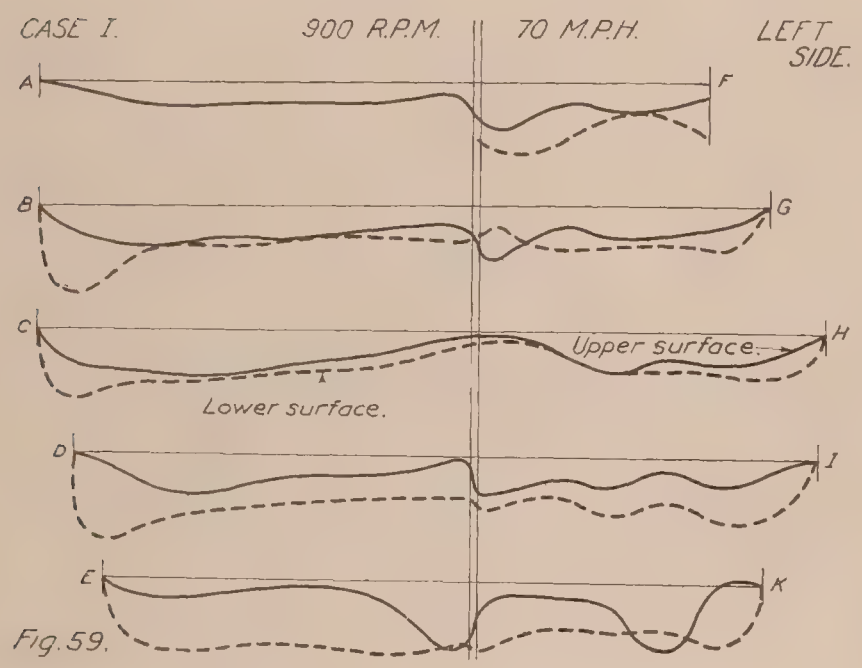


Fig. 59.

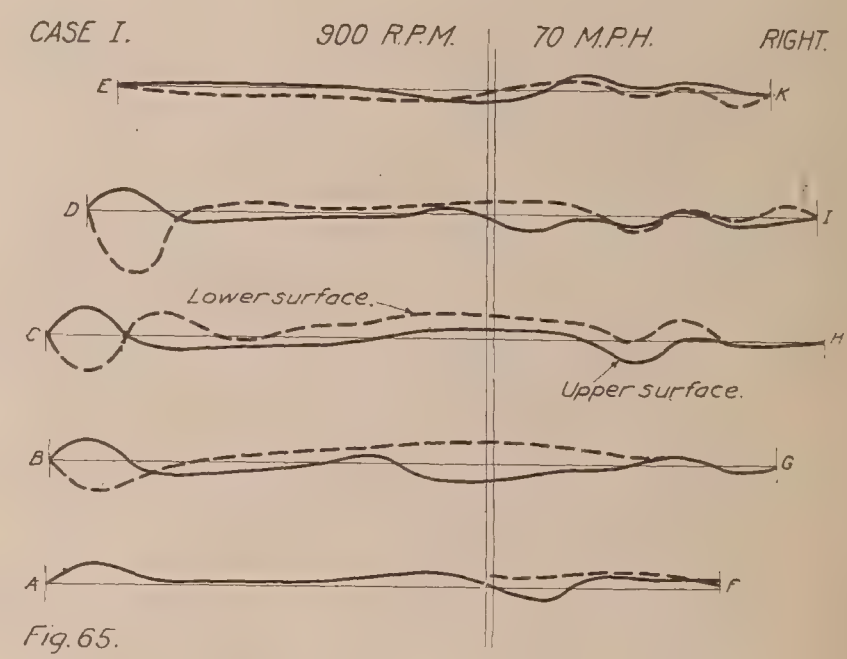


Fig. 65.

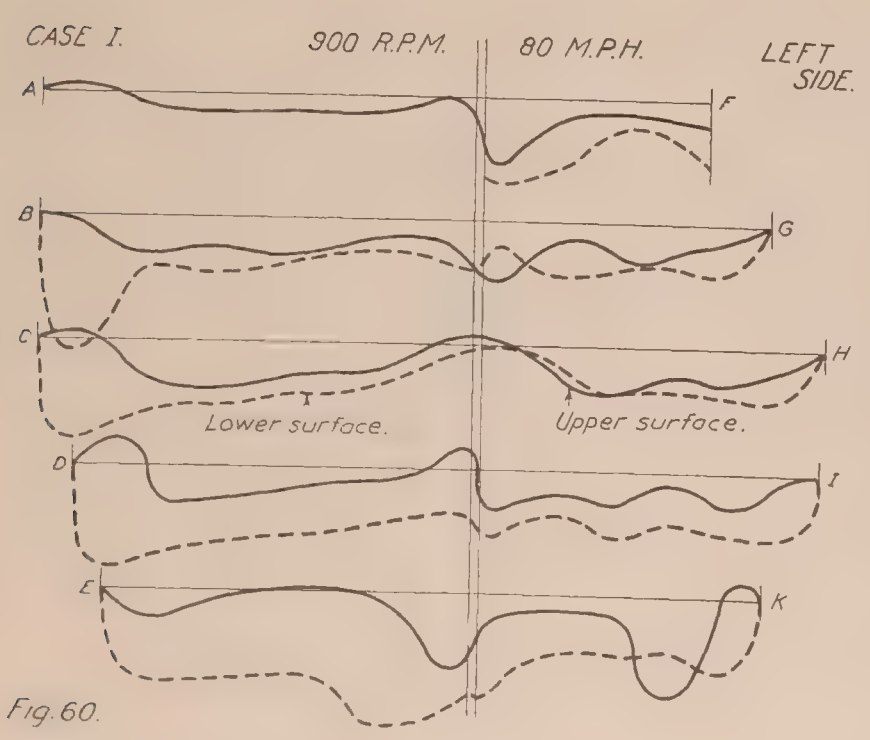


Fig. 60.

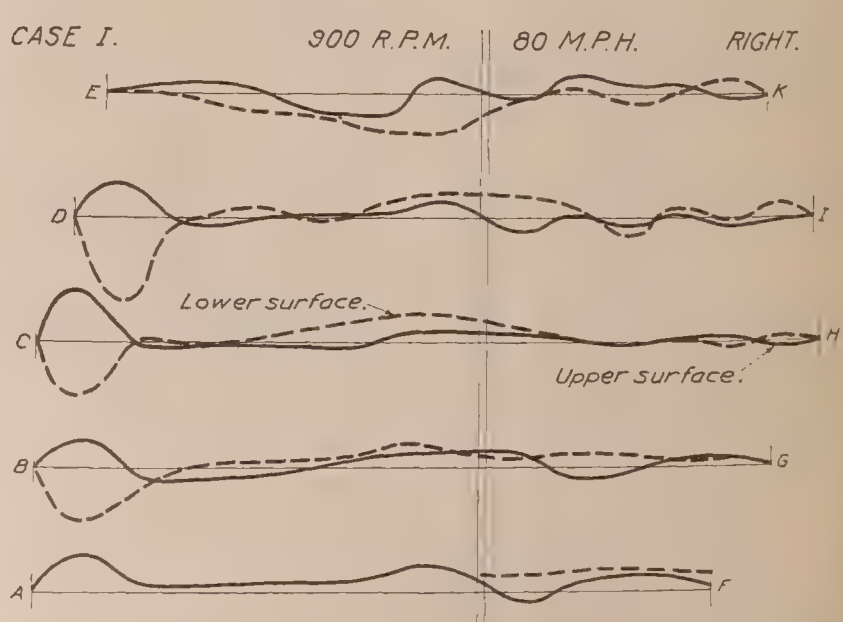


Fig. 66.

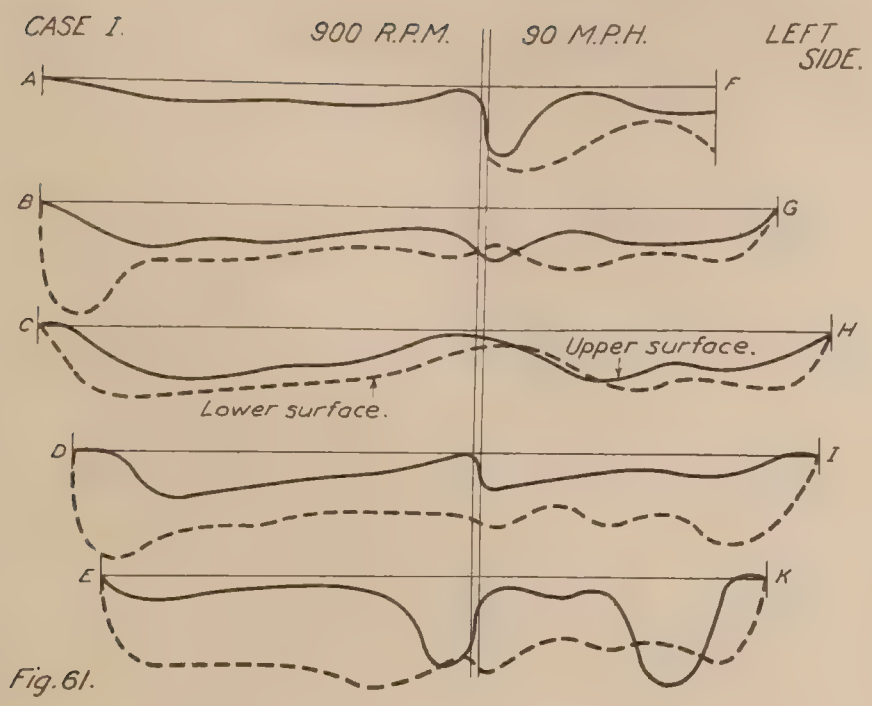


Fig. 61.

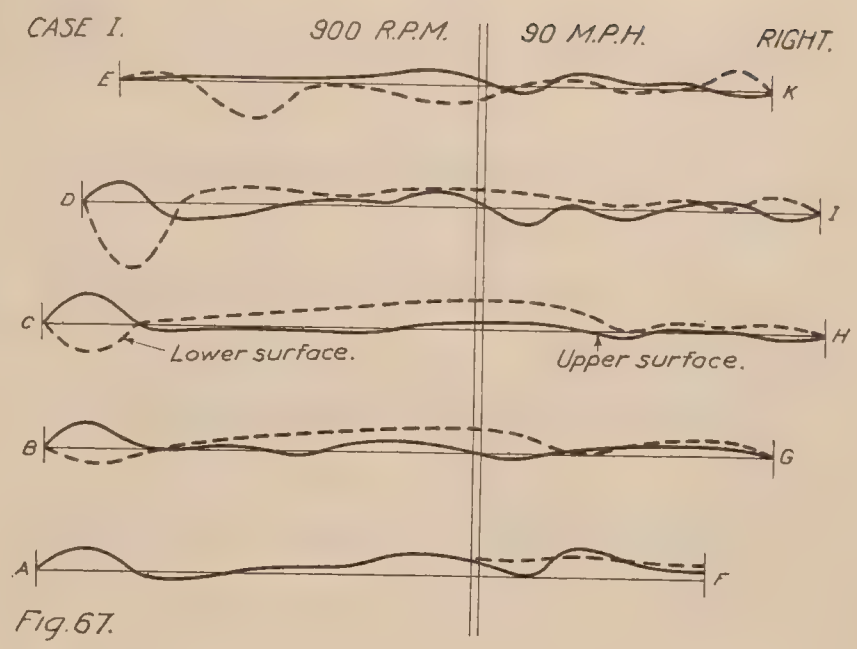


Fig. 67.

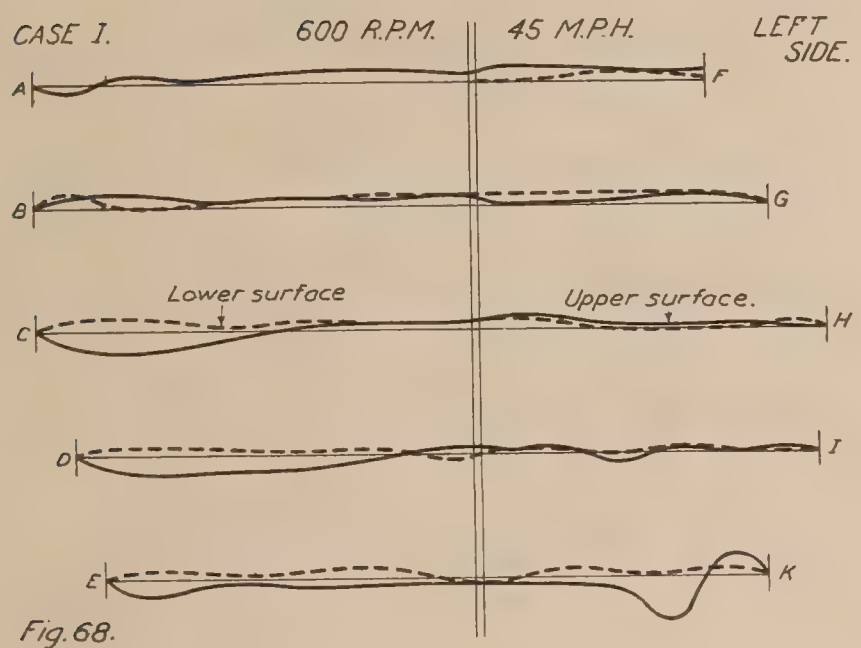


Fig. 68.

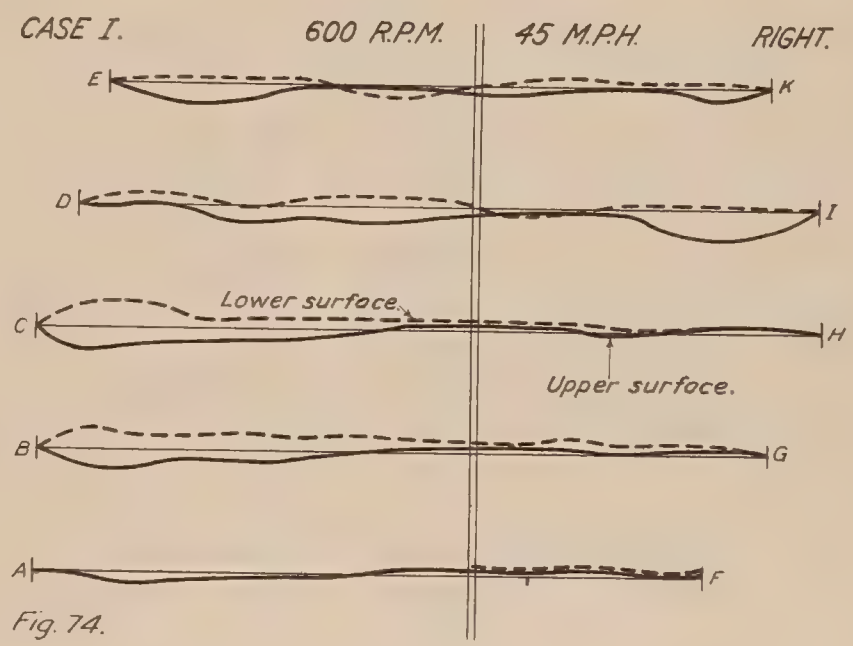


Fig. 74.

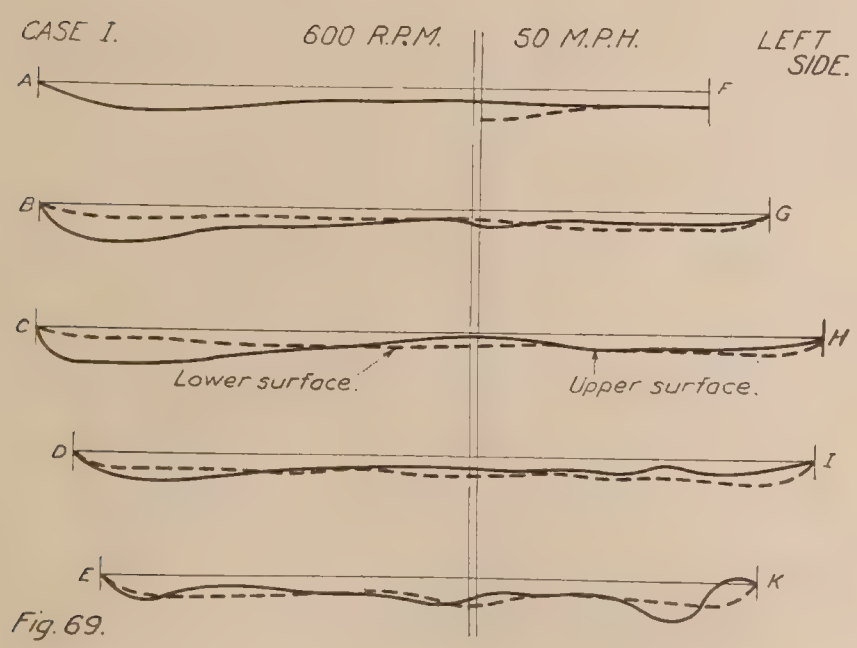


Fig. 69.

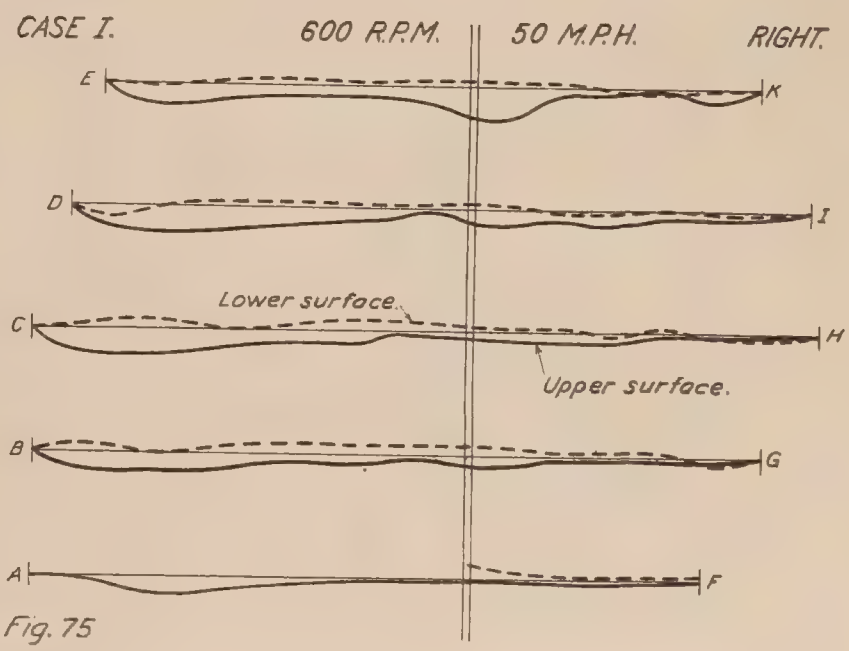


Fig. 75

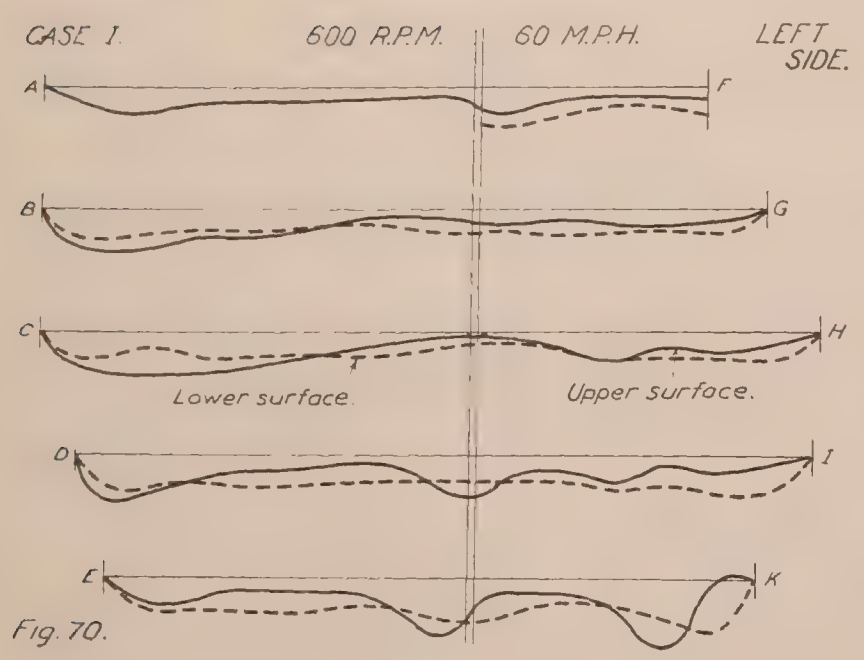


Fig. 70.

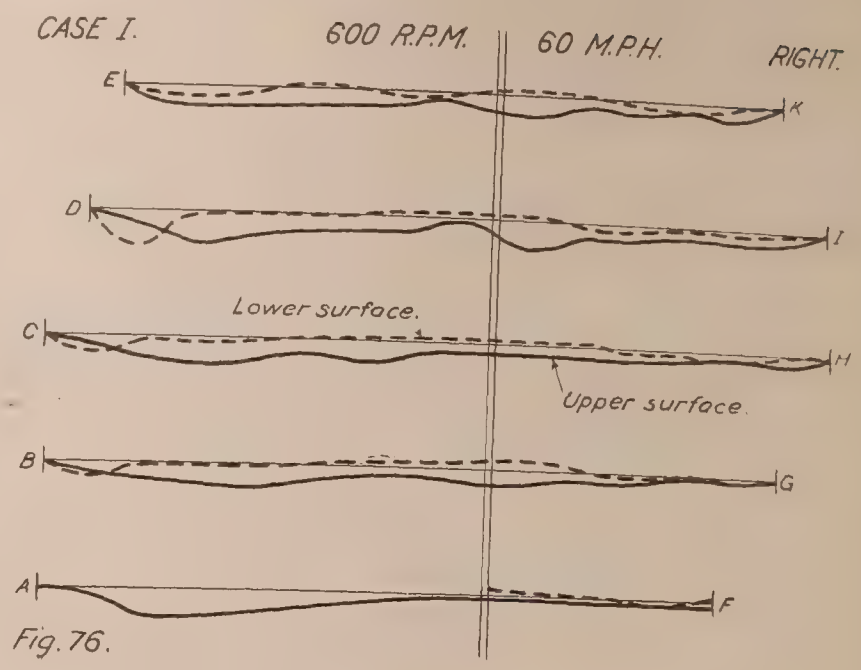


Fig. 76.

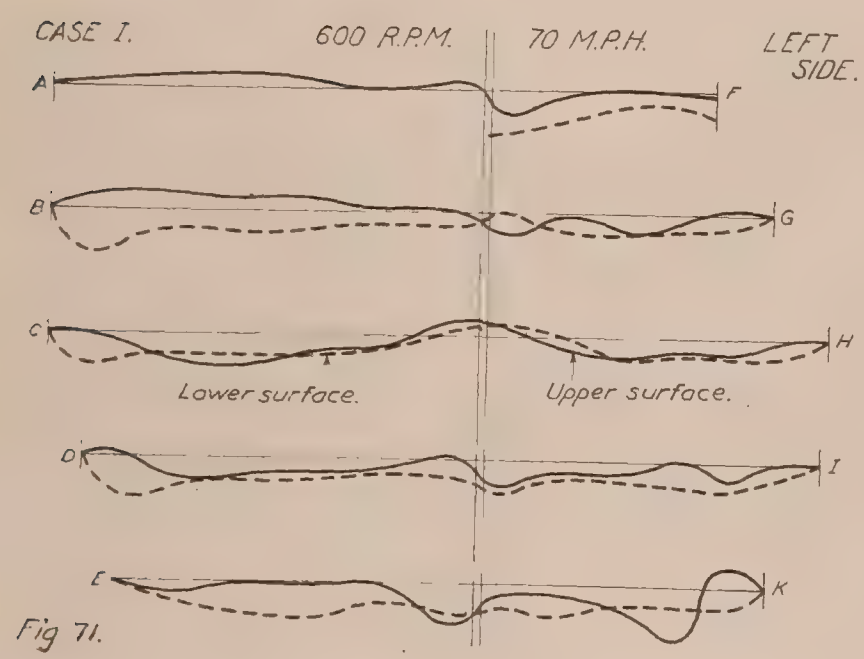


Fig 71.

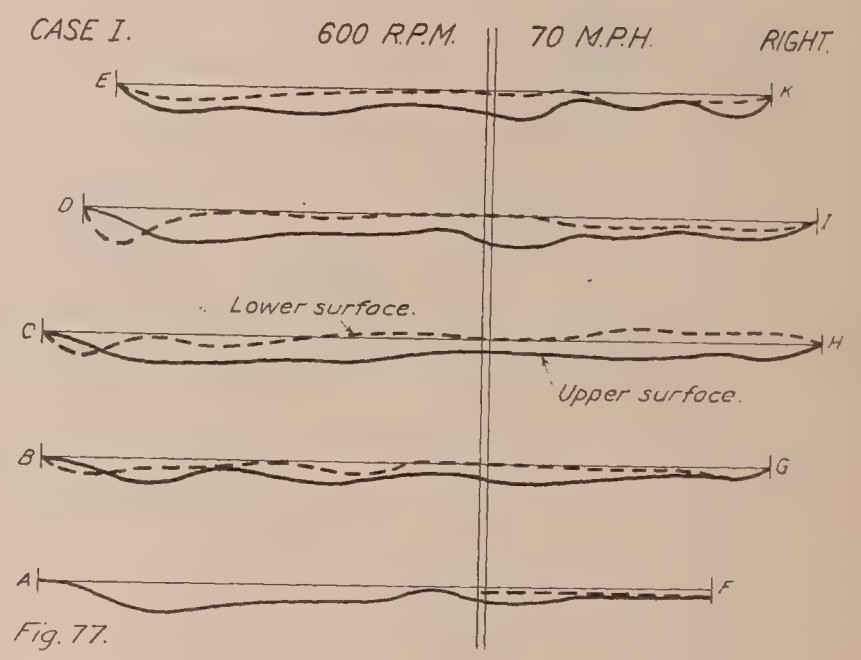


Fig. 77.

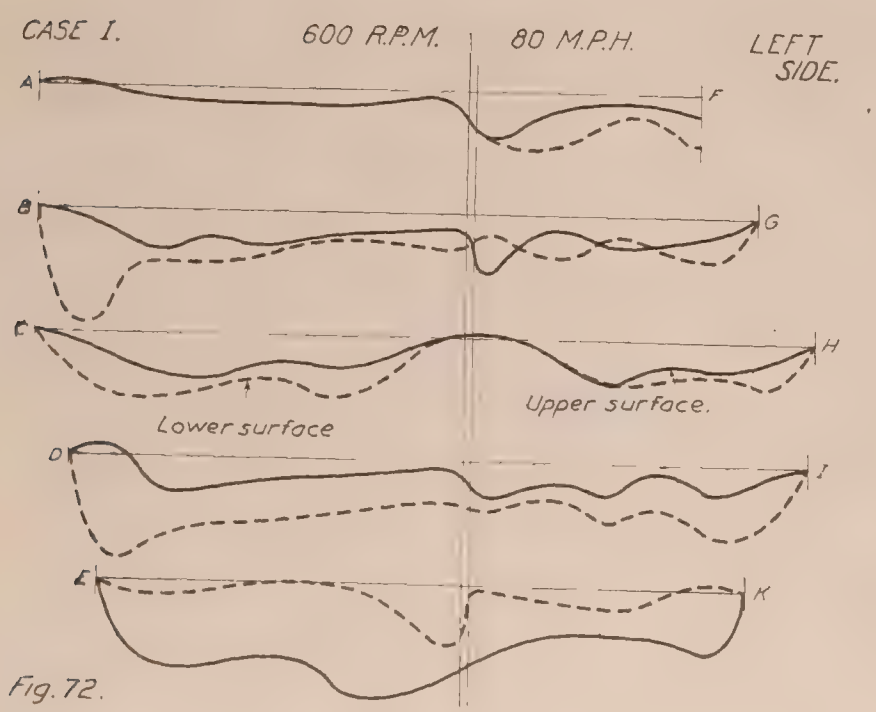


Fig. 72.

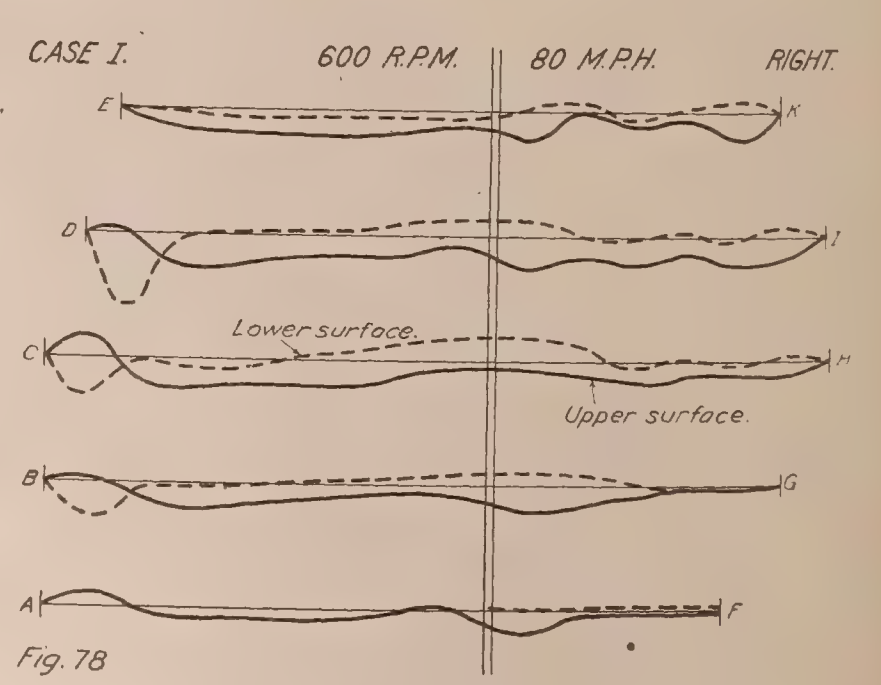


Fig. 78

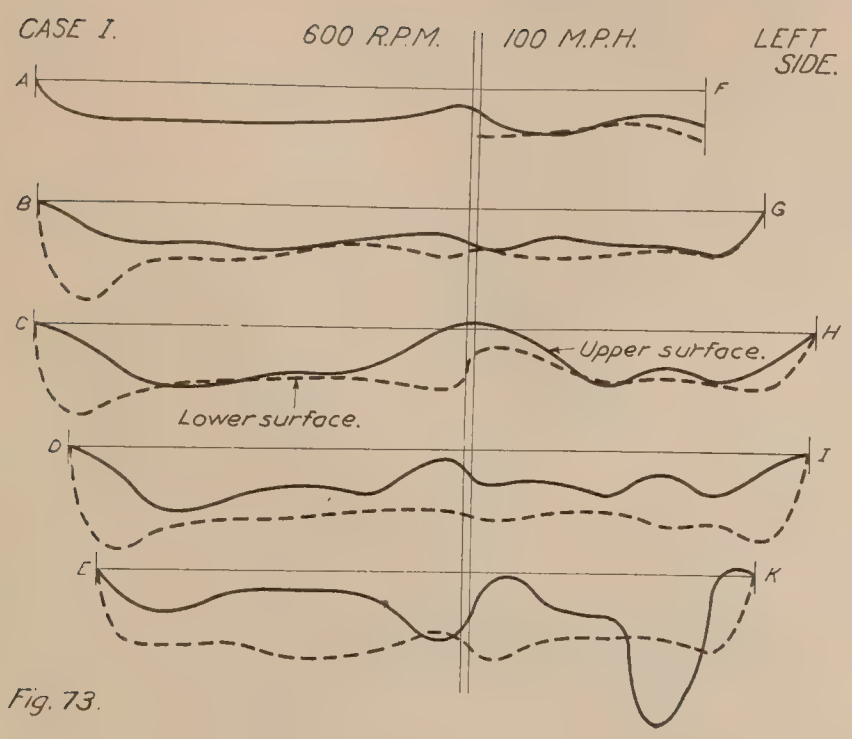

CASE I.
600 R.P.M.
100 M.P.H.
LEFT SIDE.
A
F
B
G
C
H
Upper surface.
Lower surface.
D
I
E
K

Fig. 73.

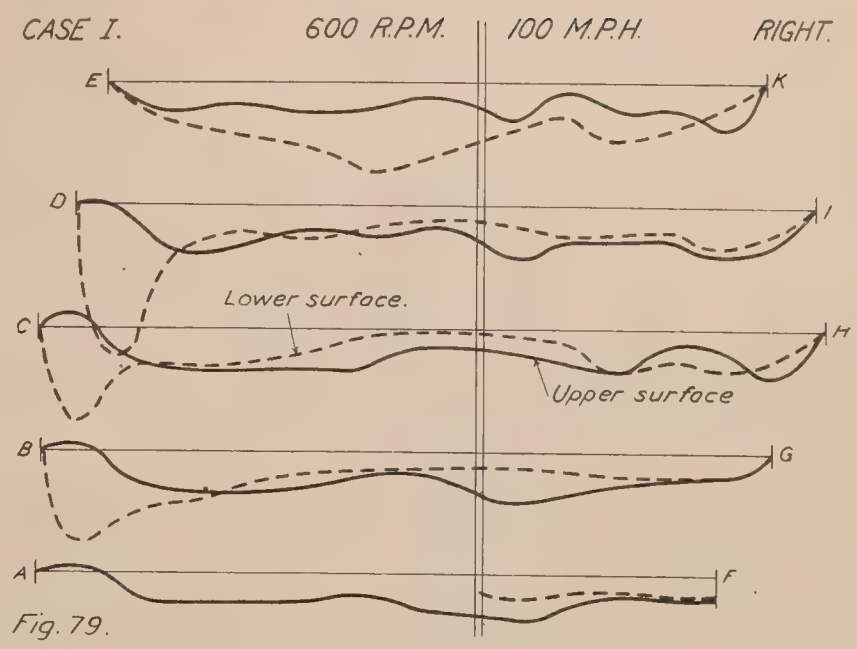

CASE I.
600 R.P.M.
100 M.P.H.
RIGHT.
E
K
D
I
Lower surface.
C
H
Upper surface
B
G
A
F

Fig. 79.

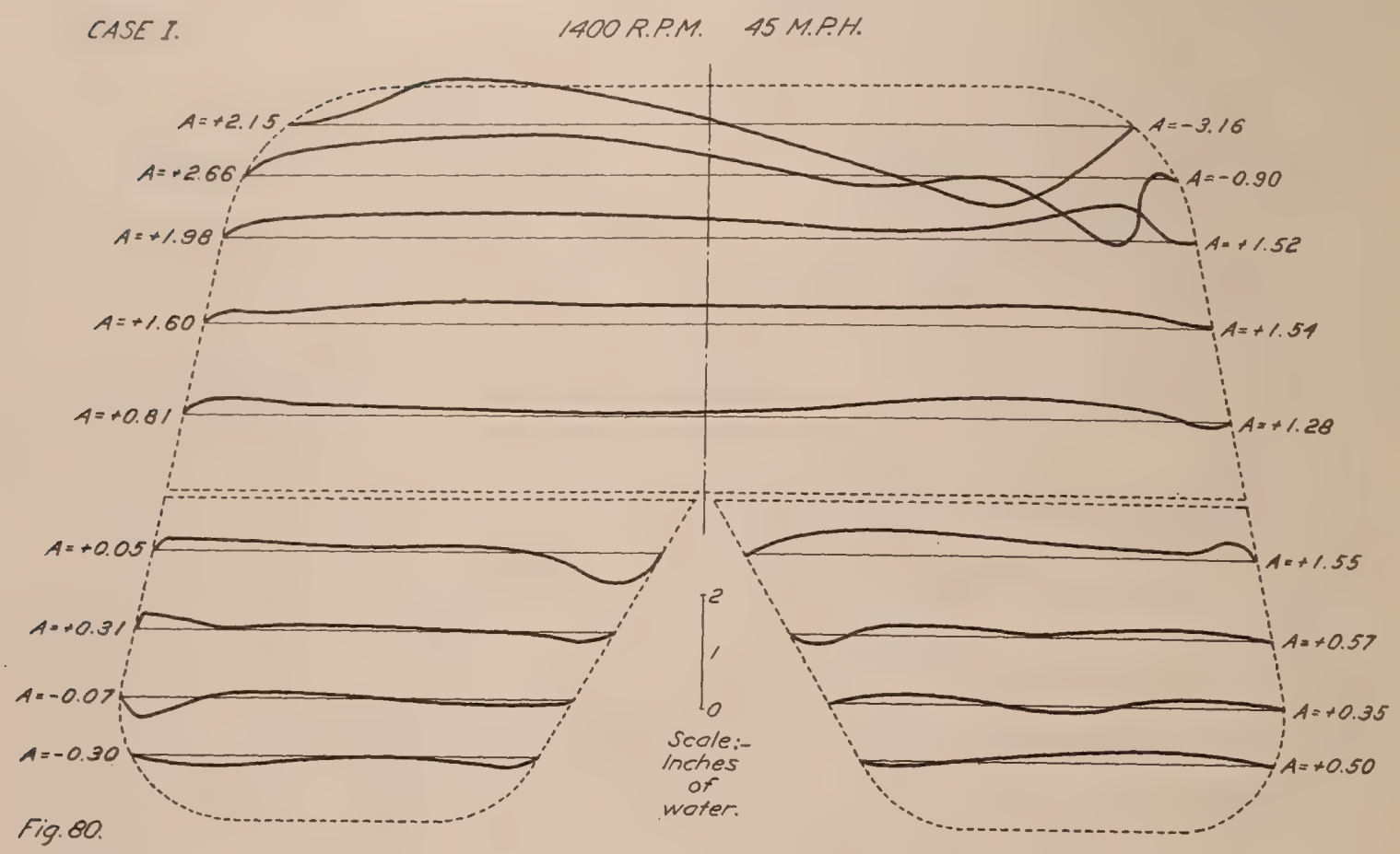

CASE I.
1400 R.P.M. 45 M.P.H.
A=+2.15
A=+2.66
A=+1.98
A=+1.60
A=+0.81
A=+0.05
A=+0.31
A=-0.07
A=-0.30
A=-3.16
A=-0.90
A=+1.52
A=+1.54
A=+1.28
A=+1.55
A=+0.57
A=+0.35
A=+0.50
2
1
0
Scale:-
Inches
of
water.

Fig. 80.

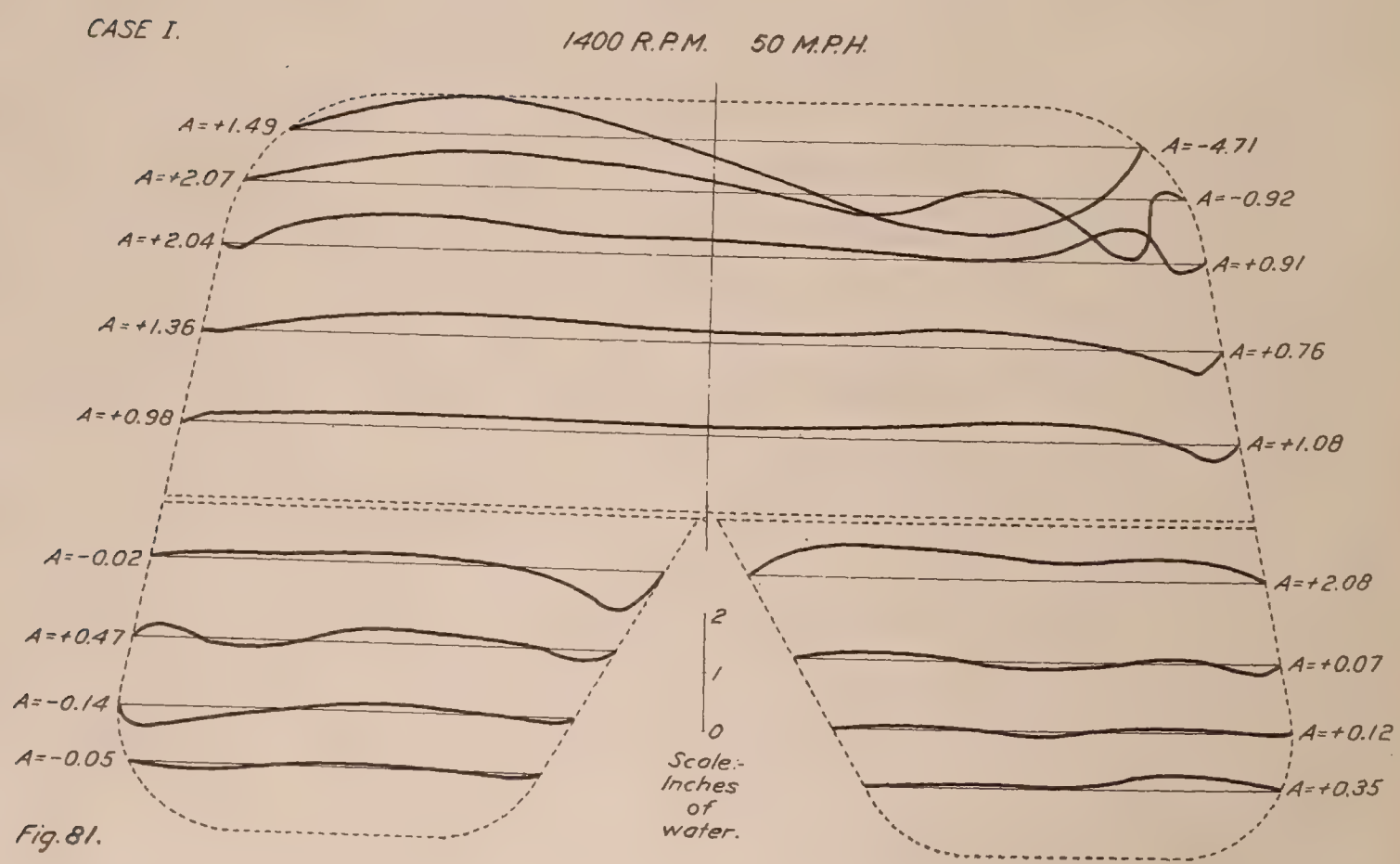

CASE I.
1400 R.P.M. 50 M.P.H.
A=+1.49
A=+2.07
A=+2.04
A=+1.36
A=+0.98
A=-0.02
A=+0.47
A=-0.14
A=-0.05
A=-4.71
A=-0.92
A=+0.91
A=+0.76
A=+1.08
A=+2.08
A=+0.07
A=+0.12
A=+0.35
2
1
0
Scale:-
Inches
of
water.

Fig. 81.

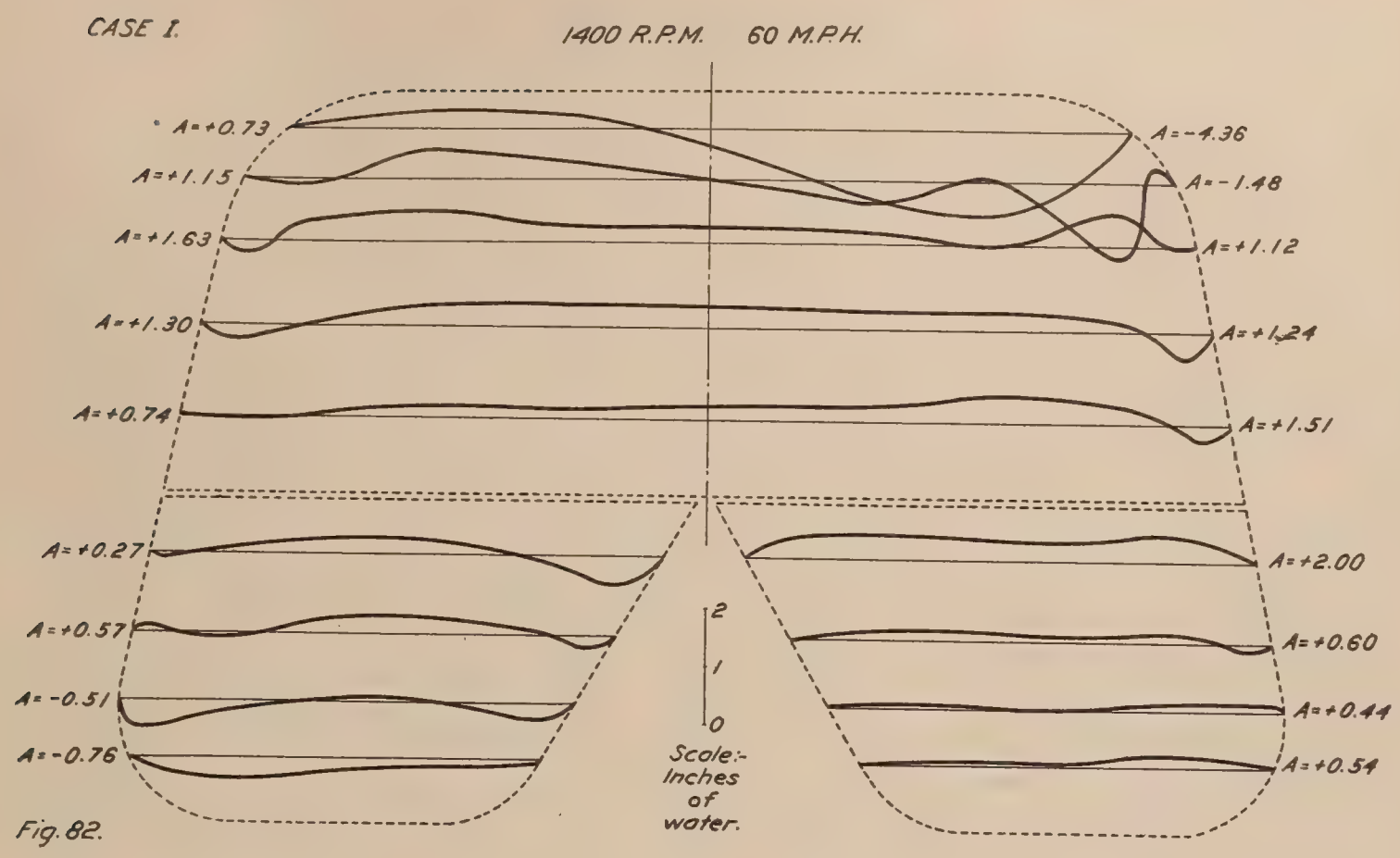

CASE I.
1400 R.P.M. 60 M.P.H.
A=+0.73
A=+1.15
A=+1.63
A=+1.30
A=+0.74
A=+0.27
A=+0.57
A=-0.51
A=-0.76
A=-4.36
A=-1.48
A=+1.12
A=+1.24
A=+1.51
A=+2.00
A=+0.60
A=+0.44
A=+0.54
2
1
0
Scale:-
Inches
of
water.

Fig. 82.

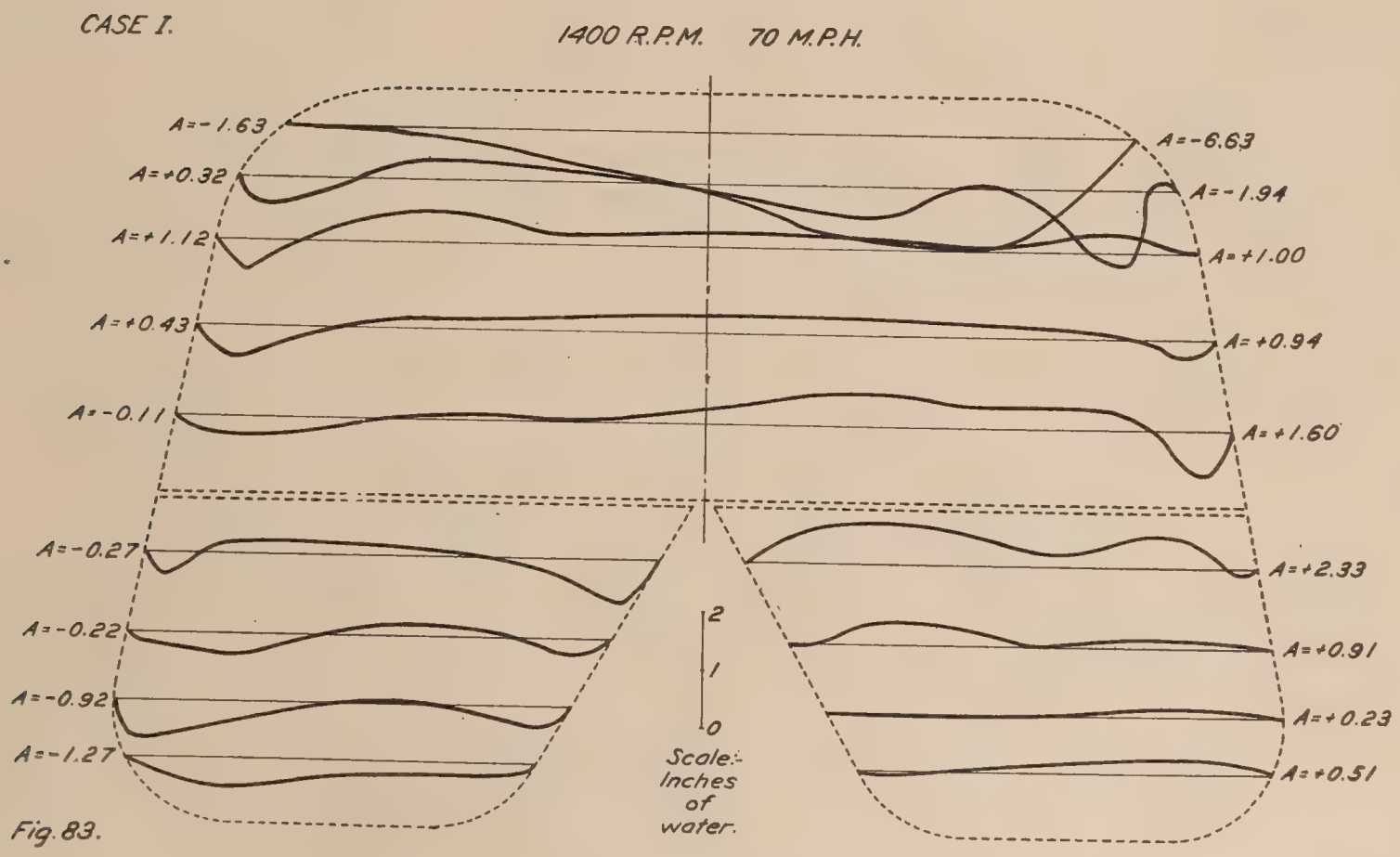

CASE I.
1400 R.P.M. 70 M.P.H.
A=-1.63
A=+0.32
A=+1.12
A=+0.43
A=-0.11
A=-0.27
A=-0.22
A=-0.92
A=-1.27
A=-6.63
A=-1.94
A=+1.00
A=+0.94
A=+1.60
A=+2.33
A=+0.91
A=+0.23
A=+0.51
2
1
0
Scale:-
Inches
of
water.

Fig. 83.

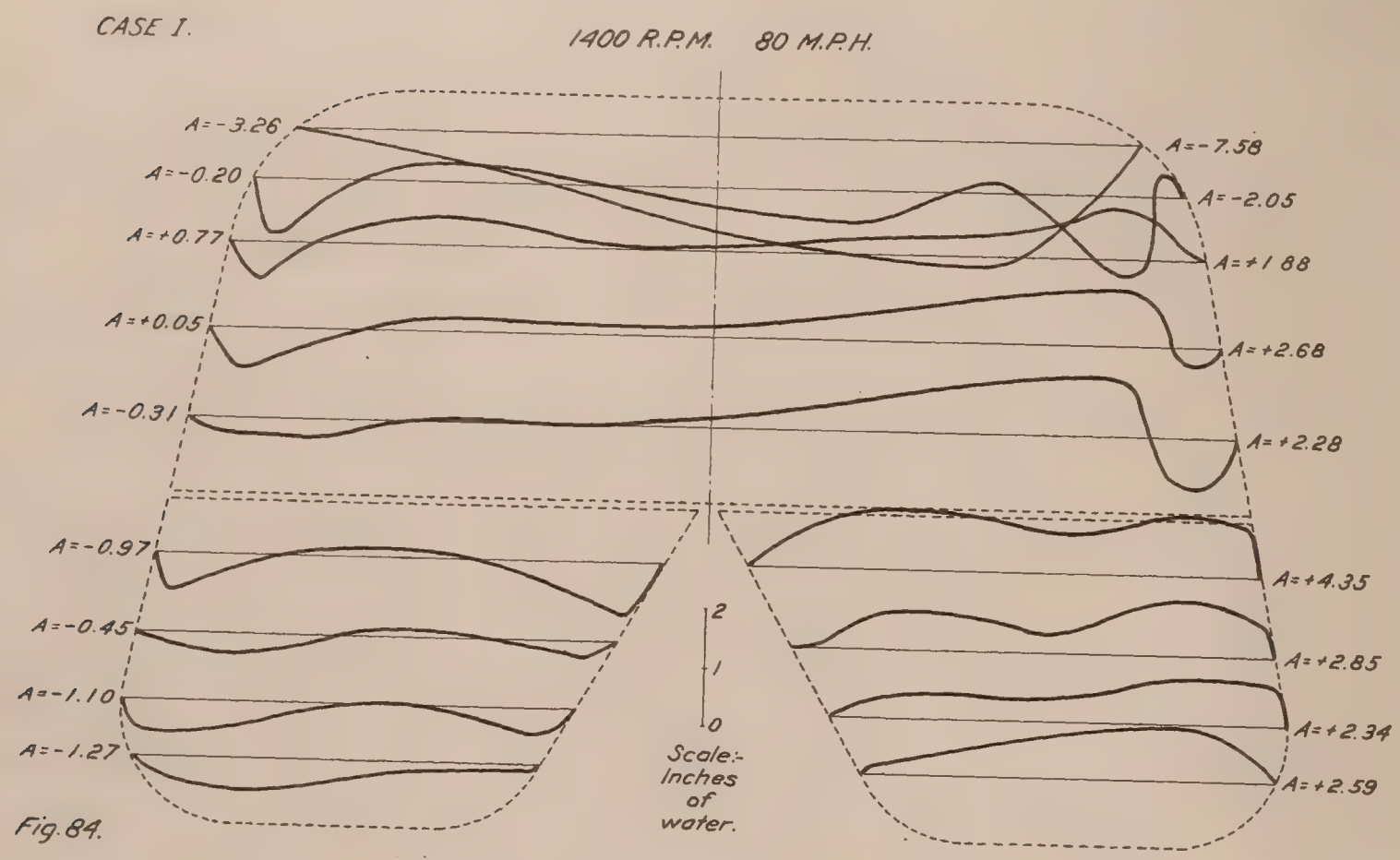

CASE I.
1400 R.P.M. 80 M.P.H.
A=-3.26
A=-0.20
A=+0.77
A=+0.05
A=-0.31
A=-0.97
A=-0.45
A=-1.10
A=-1.27
A=-7.58
A=-2.05
A=+1.88
A=+2.68
A=+2.28
A=+4.35
A=+2.85
A=+2.34
A=+2.59
2
1
0
Scale:- Inches of water.

Fig. 84.

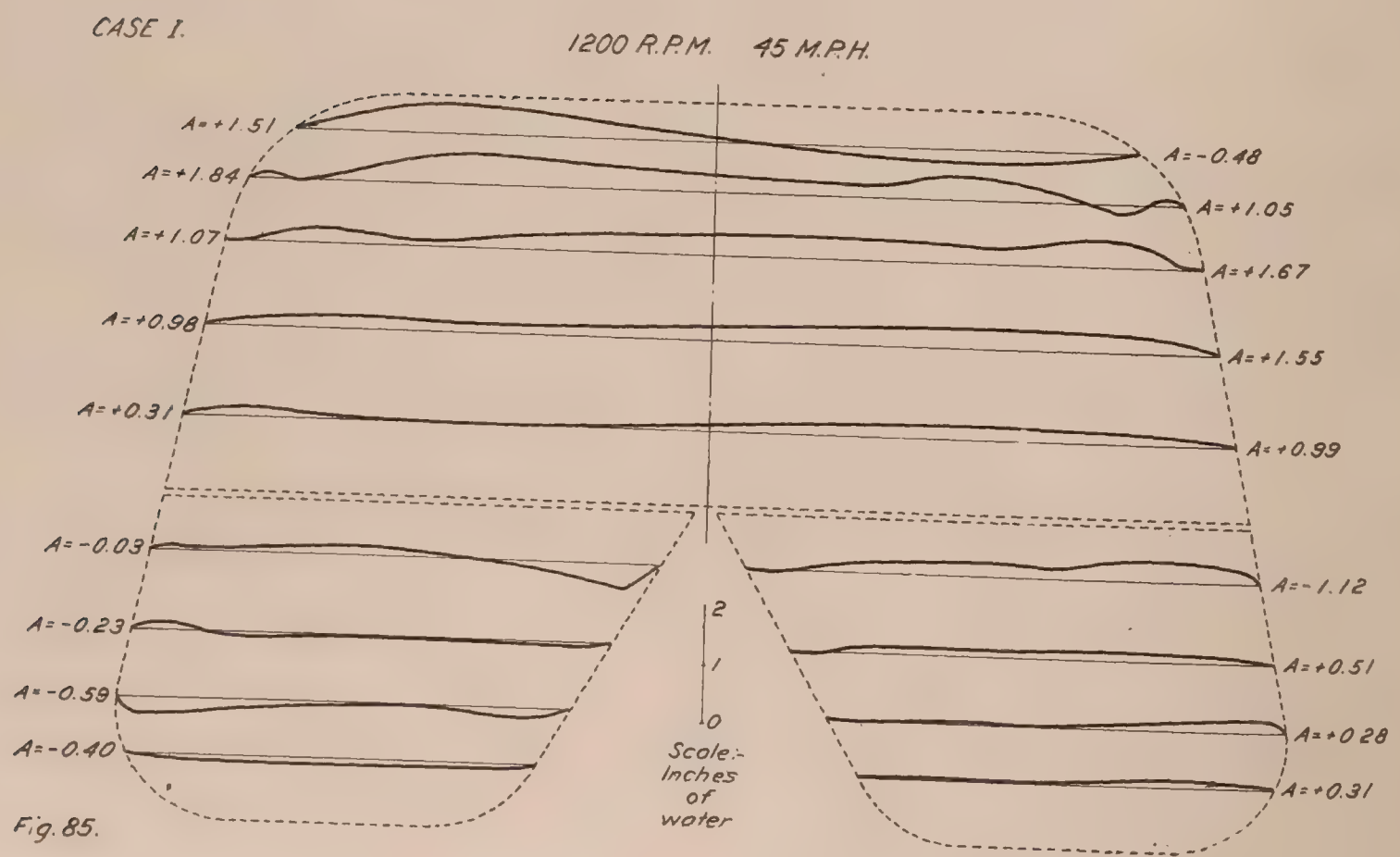

CASE I.
1200 R.P.M. 45 M.P.H.
A=+1.51
A=+1.84
A=+1.07
A=+0.98
A=+0.31
A=-0.03
A=-0.23
A=-0.59
A=-0.40
A=-0.48
A=+1.05
A=+1.67
A=+1.55
A=+0.99
A=-1.12
A=+0.51
A=+0.28
A=+0.31
2
1
0
Scale:- Inches of water

Fig. 85.

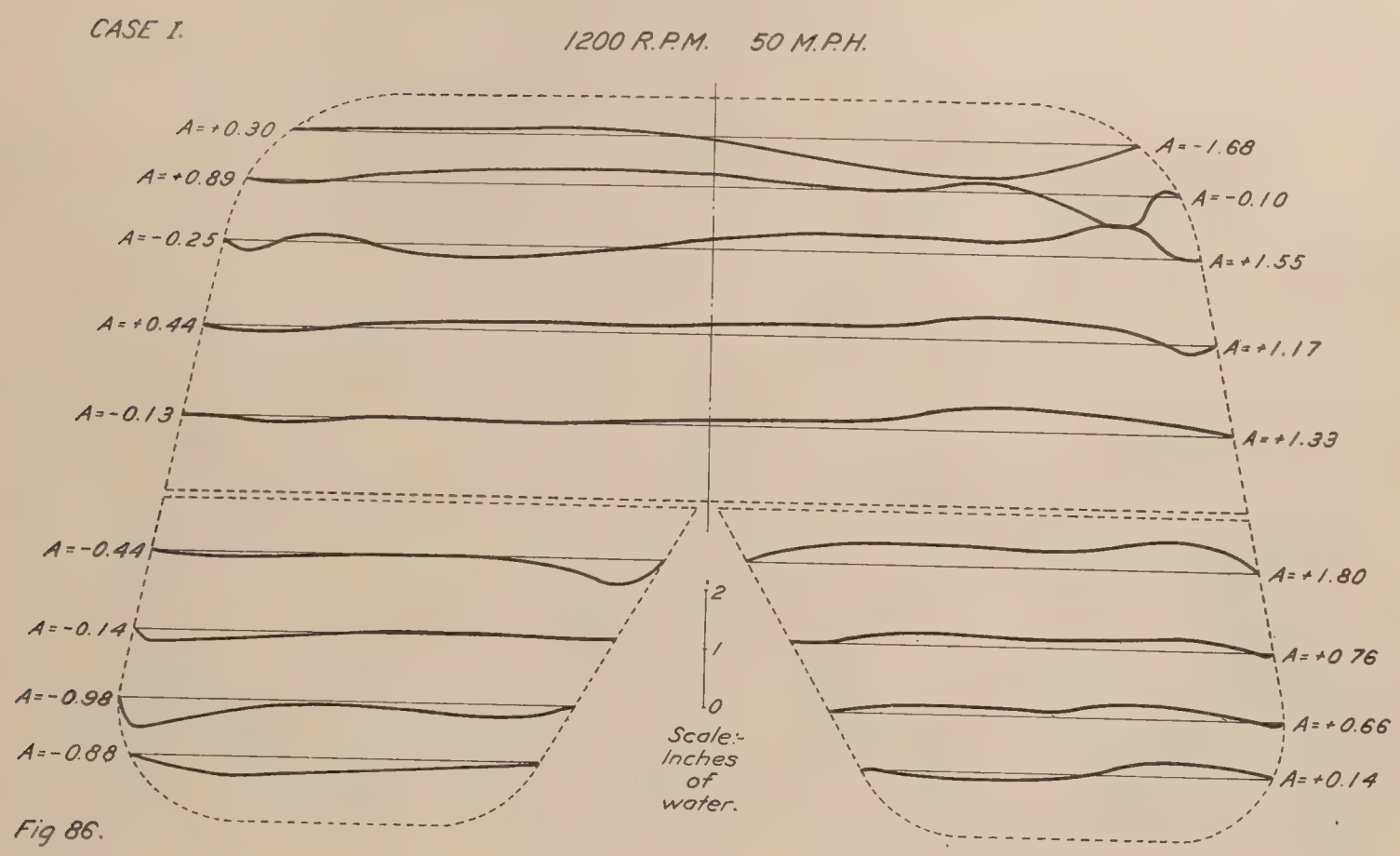

CASE I.
1200 R.P.M. 50 M.P.H.
A=+0.30
A=+0.89
A=-0.25
A=+0.44
A=-0.13
A=-0.44
A=-0.14
A=-0.98
A=-0.88
A=-1.68
A=-0.10
A=+1.55
A=+1.17
A=+1.33
A=+1.80
A=+0.76
A=+0.66
A=+0.14
2
1
0
Scale:-
Inches
of
water.

Fig 86.

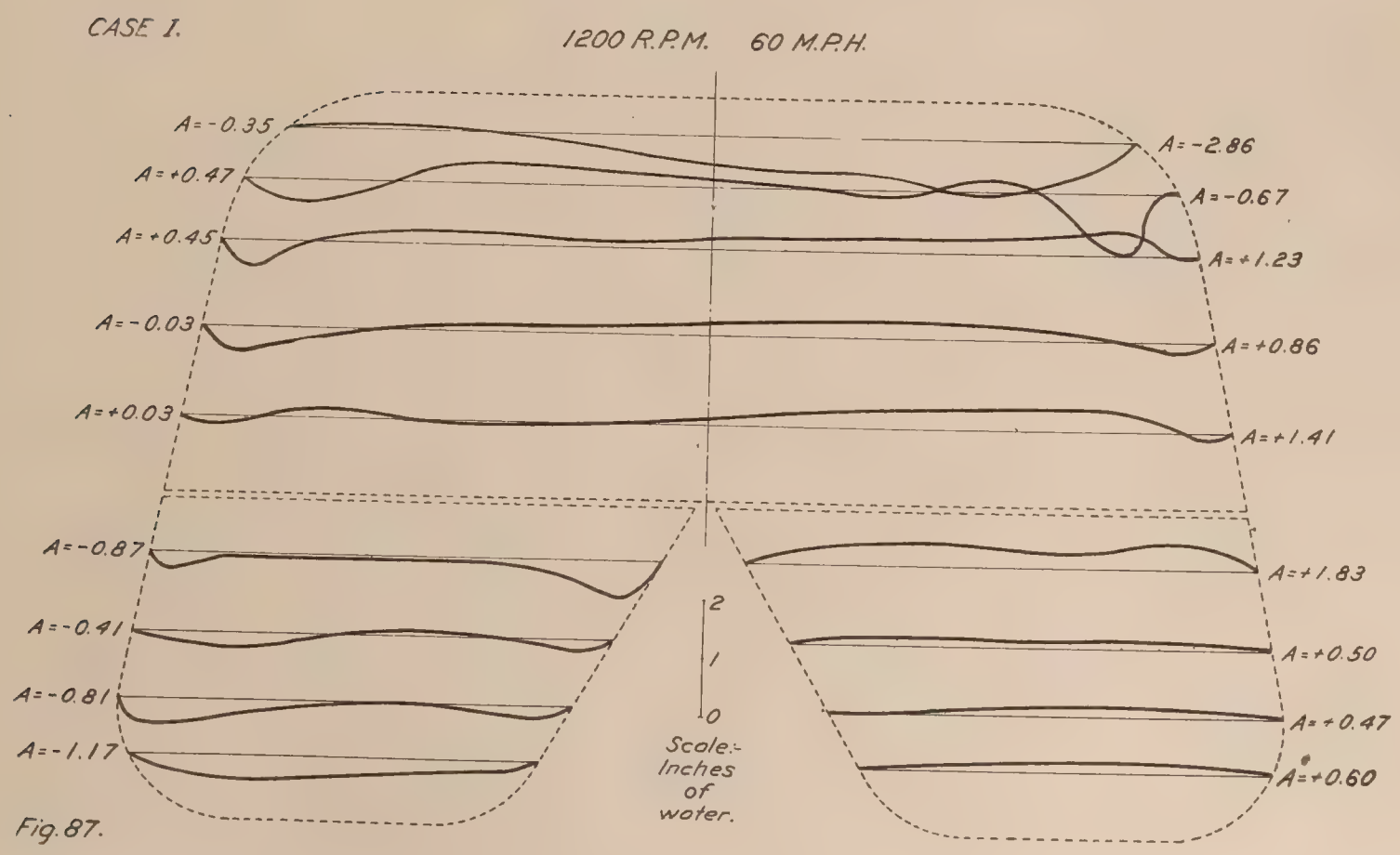

CASE I.
1200 R.P.M. 60 M.P.H.
A=-0.35
A=+0.47
A=+0.45
A=-0.03
A=+0.03
A=-0.87
A=-0.41
A=-0.81
A=-1.17
A=-2.86
A=-0.67
A=+1.23
A=+0.86
A=+1.41
A=+1.83
A=+0.50
A=+0.47
A=+0.60
2
1
0
Scale:-
Inches
of
water.

Fig. 87.

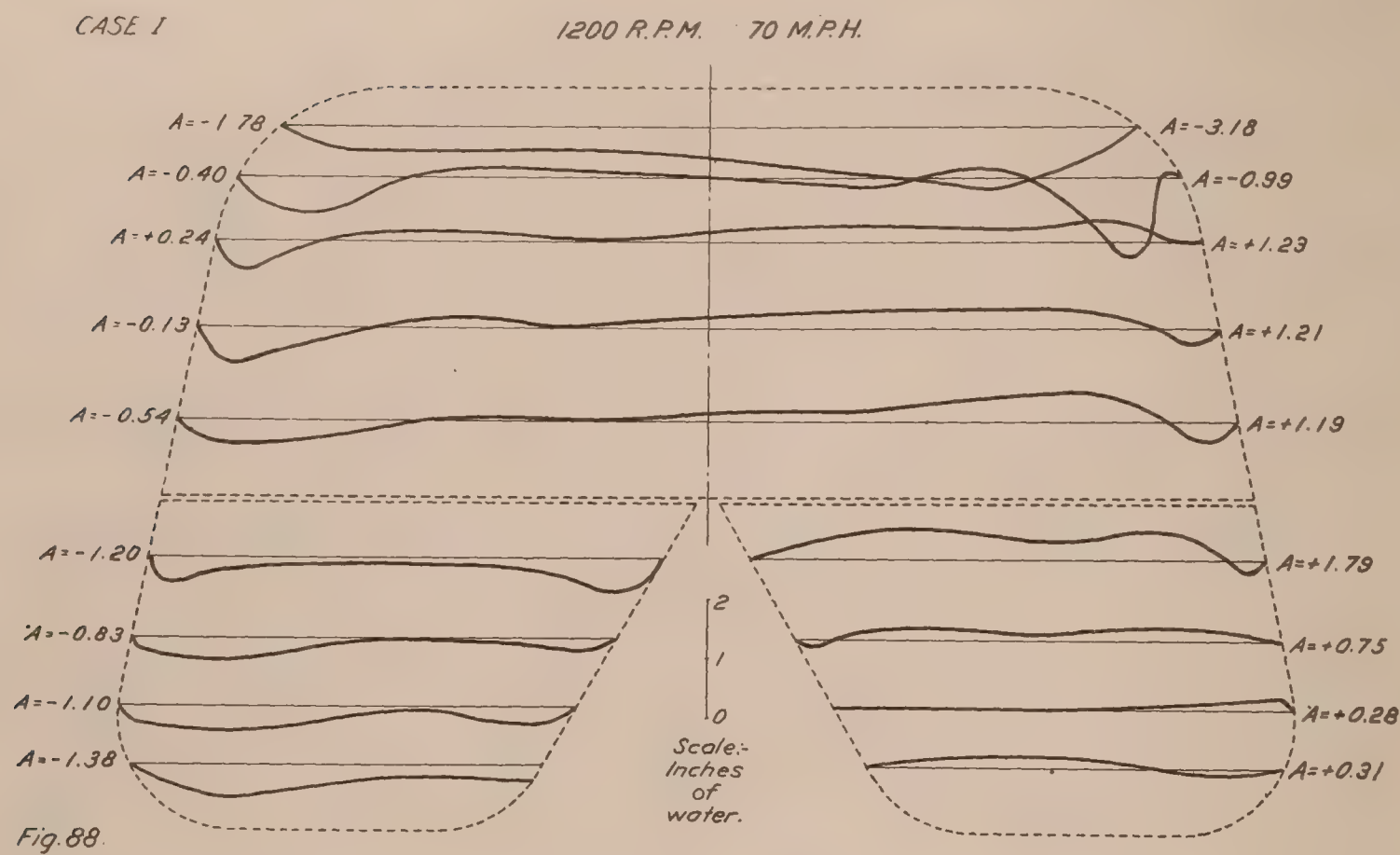

CASE I
1200 R.P.M. 70 M.P.H.
A=-1.78
A=-0.40
A=+0.24
A=-0.13
A=-0.54
A=-1.20
A=-0.83
A=-1.10
A=-1.38
A=-3.18
A=-0.99
A=+1.23
A=+1.21
A=+1.19
A=+1.79
A=+0.75
A=+0.28
A=+0.31
2
1
0
Scale:- Inches of water.

Fig. 88.

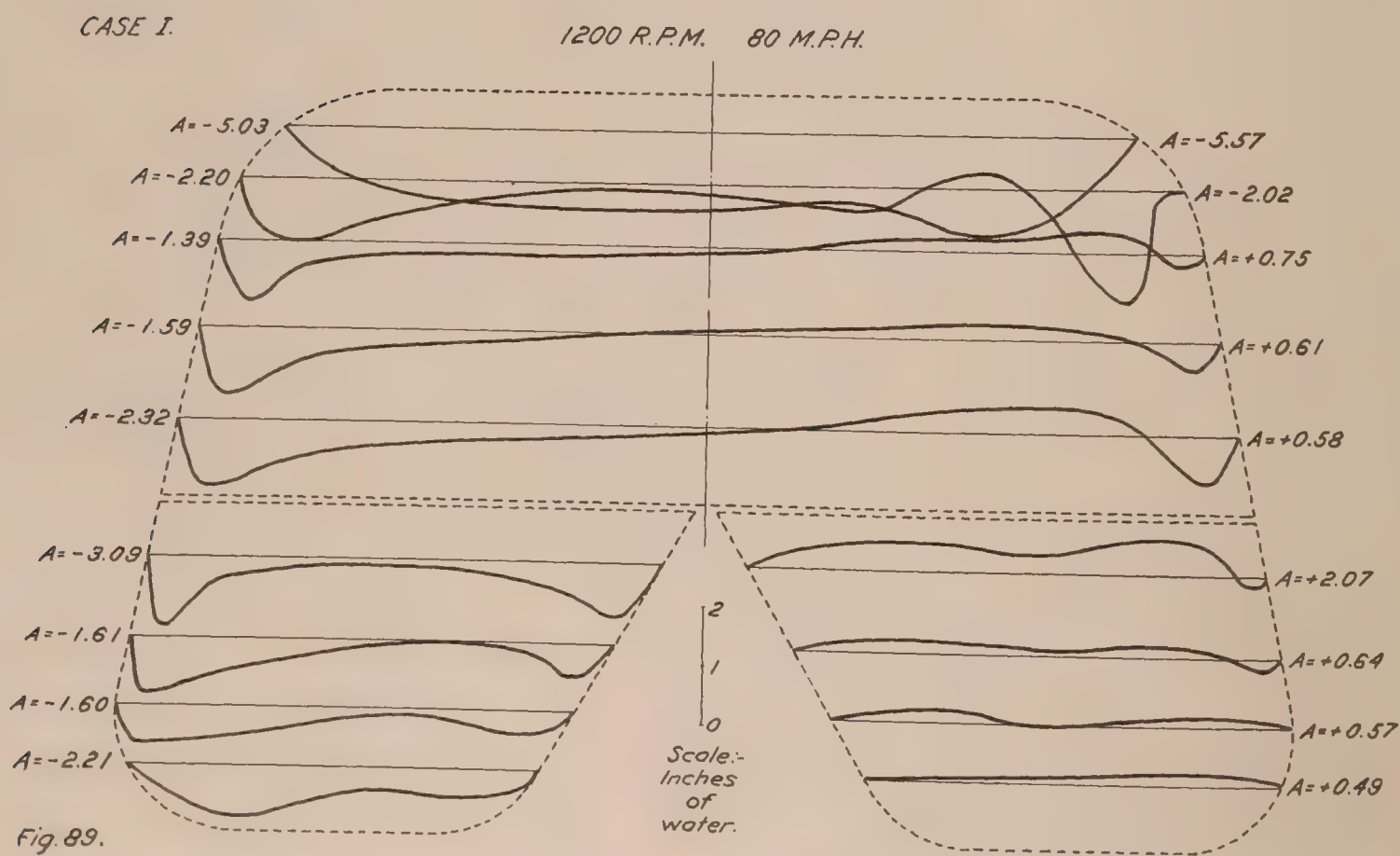

CASE I.
1200 R.P.M. 80 M.P.H.
A=-5.03
A=-2.20
A=-1.39
A=-1.59
A=-2.32
A=-3.09
A=-1.61
A=-1.60
A=-2.21
A=-5.57
A=-2.02
A=+0.75
A=+0.61
A=+0.58
A=+2.07
A=+0.64
A=+0.57
A=+0.49
2
1
0
Scale:- Inches of water.

Fig. 89.

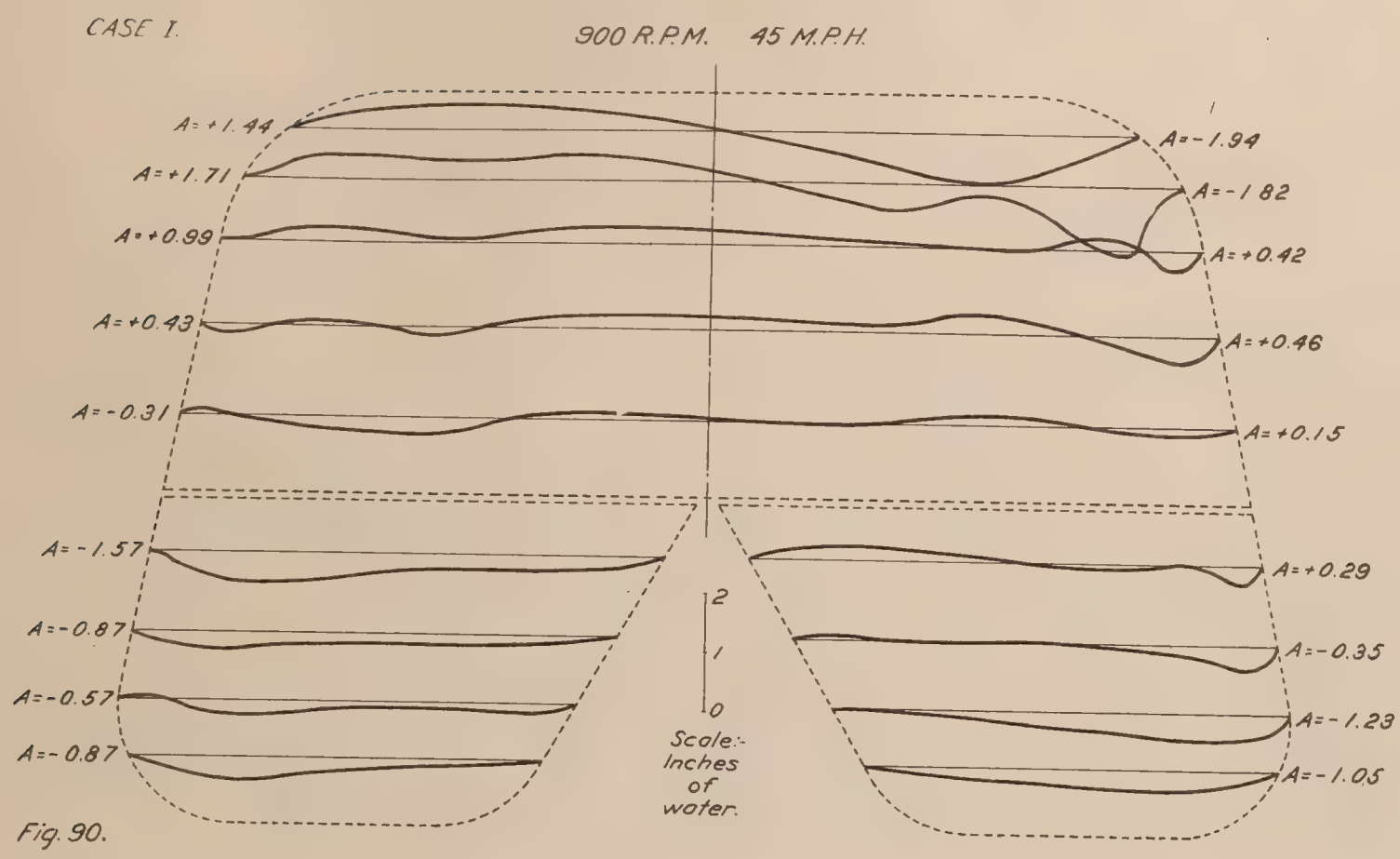

CASE I.
900 R.P.M. 45 M.P.H.
A=+1.44
A=+1.71
A=+0.99
A=+0.43
A=-0.31
A=-1.57
A=-0.87
A=-0.57
A=-0.87
A=-1.94
A=-1.82
A=+0.42
A=+0.46
A=+0.15
A=+0.29
A=-0.35
A=-1.23
A=-1.05
2
1
0
Scale:-
Inches
of
water.

Fig. 90.

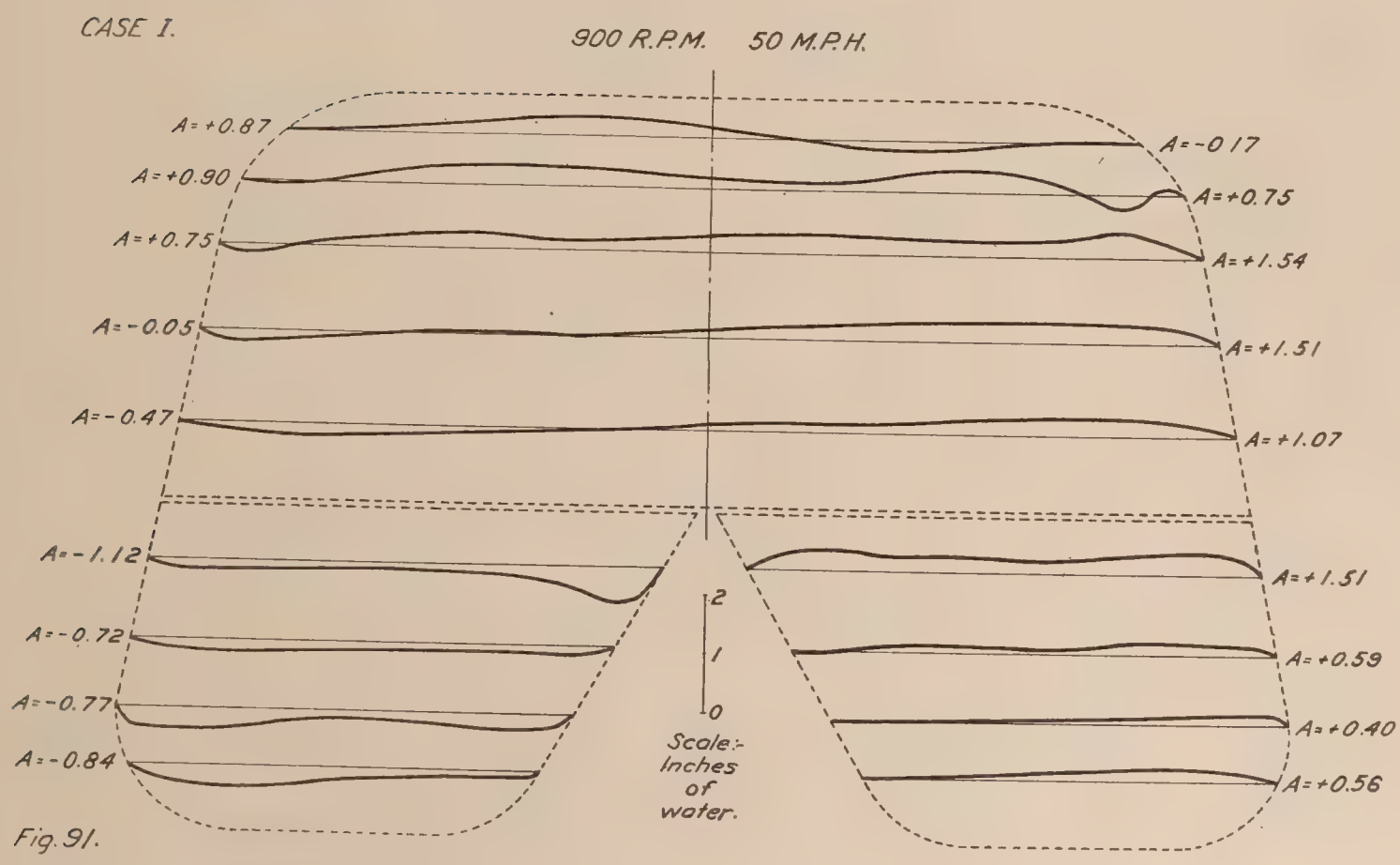

CASE I.
900 R.P.M. 50 M.P.H.
A=+0.87
A=+0.90
A=+0.75
A=-0.05
A=-0.47
A=-1.12
A=-0.72
A=-0.77
A=-0.84
A=-0.17
A=+0.75
A=+1.54
A=+1.51
A=+1.07
A=+1.51
A=+0.59
A=+0.40
A=+0.56
2
1
0
Scale:-
Inches
of
water.

Fig. 91.

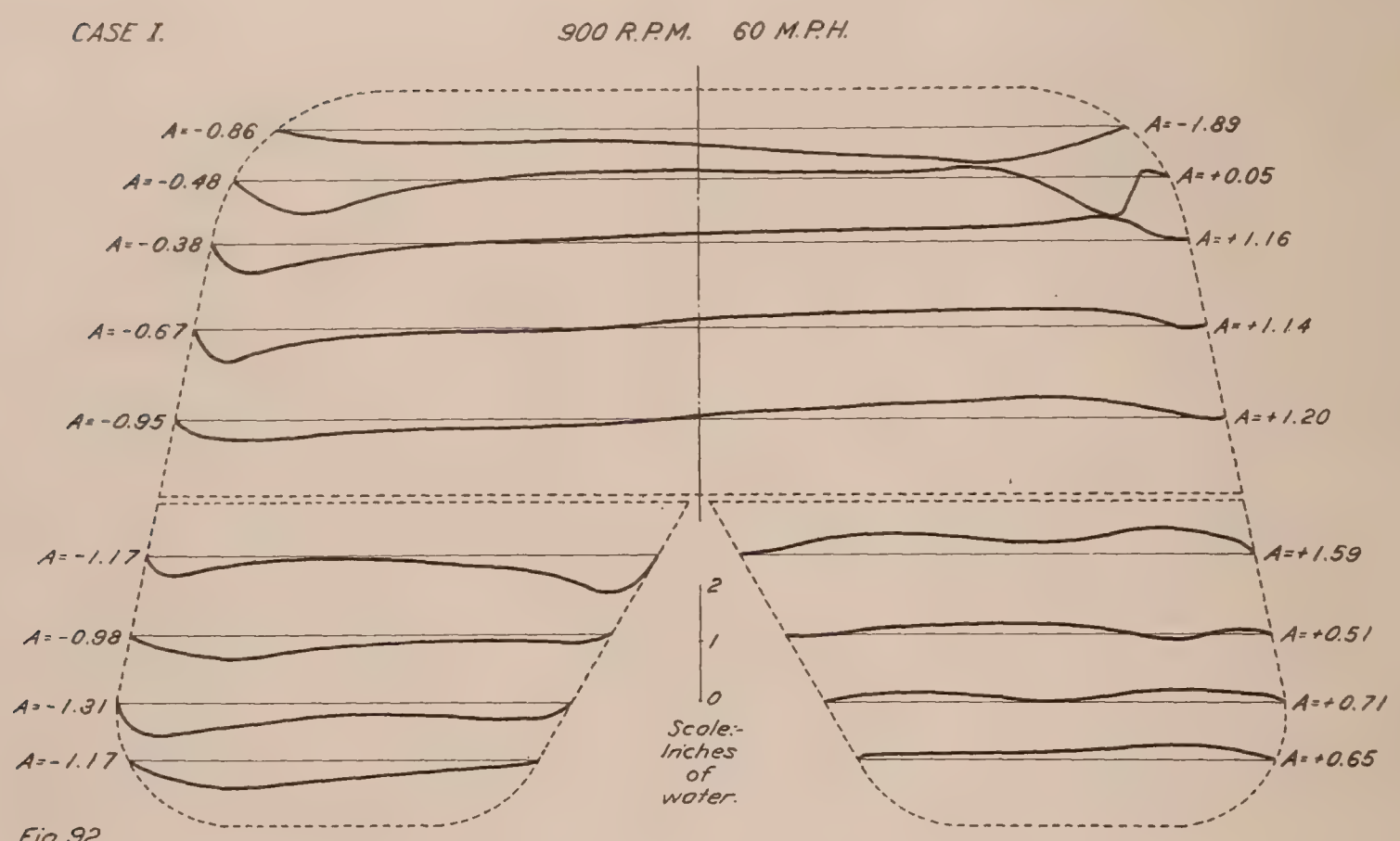

CASE I.
900 R.P.M. 60 M.P.H.
A=-0.86
A=-0.48
A=-0.38
A=-0.67
A=-0.95
A=-1.17
A=-0.98
A=-1.31
A=-1.17
A=-1.89
A=+0.05
A=+1.16
A=+1.14
A=+1.20
A=+1.59
A=+0.51
A=+0.71
A=+0.65
2
1
0
Scale:-
Inches
of
water.

Fig. 92.

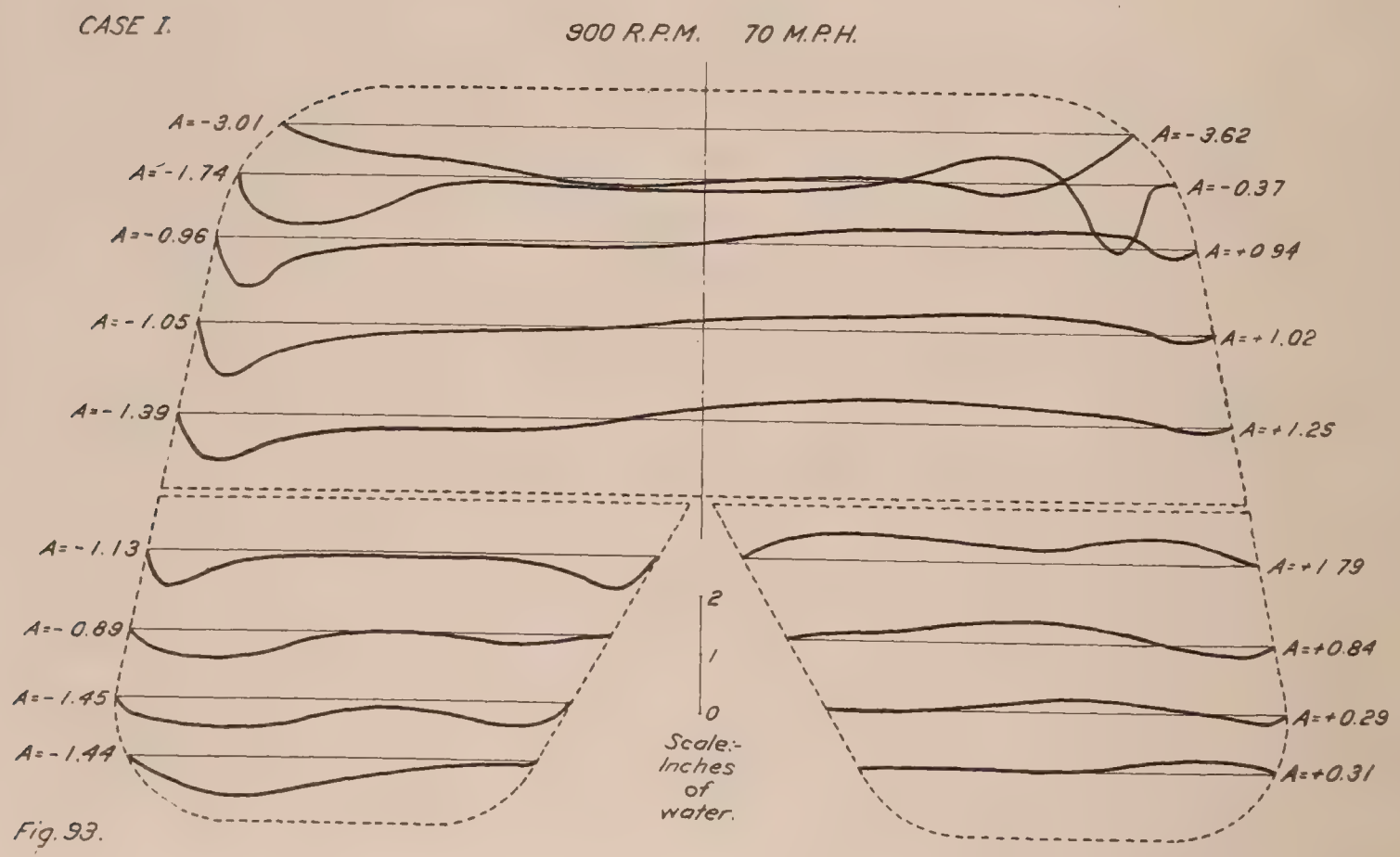

CASE I.
900 R.P.M. 70 M.P.H.
A=-3.01
A=-1.74
A=-0.96
A=-1.05
A=-1.39
A=-1.13
A=-0.89
A=-1.45
A=-1.44
A=-3.62
A=-0.37
A=+0.94
A=+1.02
A=+1.25
A=+1.79
A=+0.84
A=+0.29
A=+0.31
2
1
0
Scale:-
Inches
of
water.

Fig. 93.

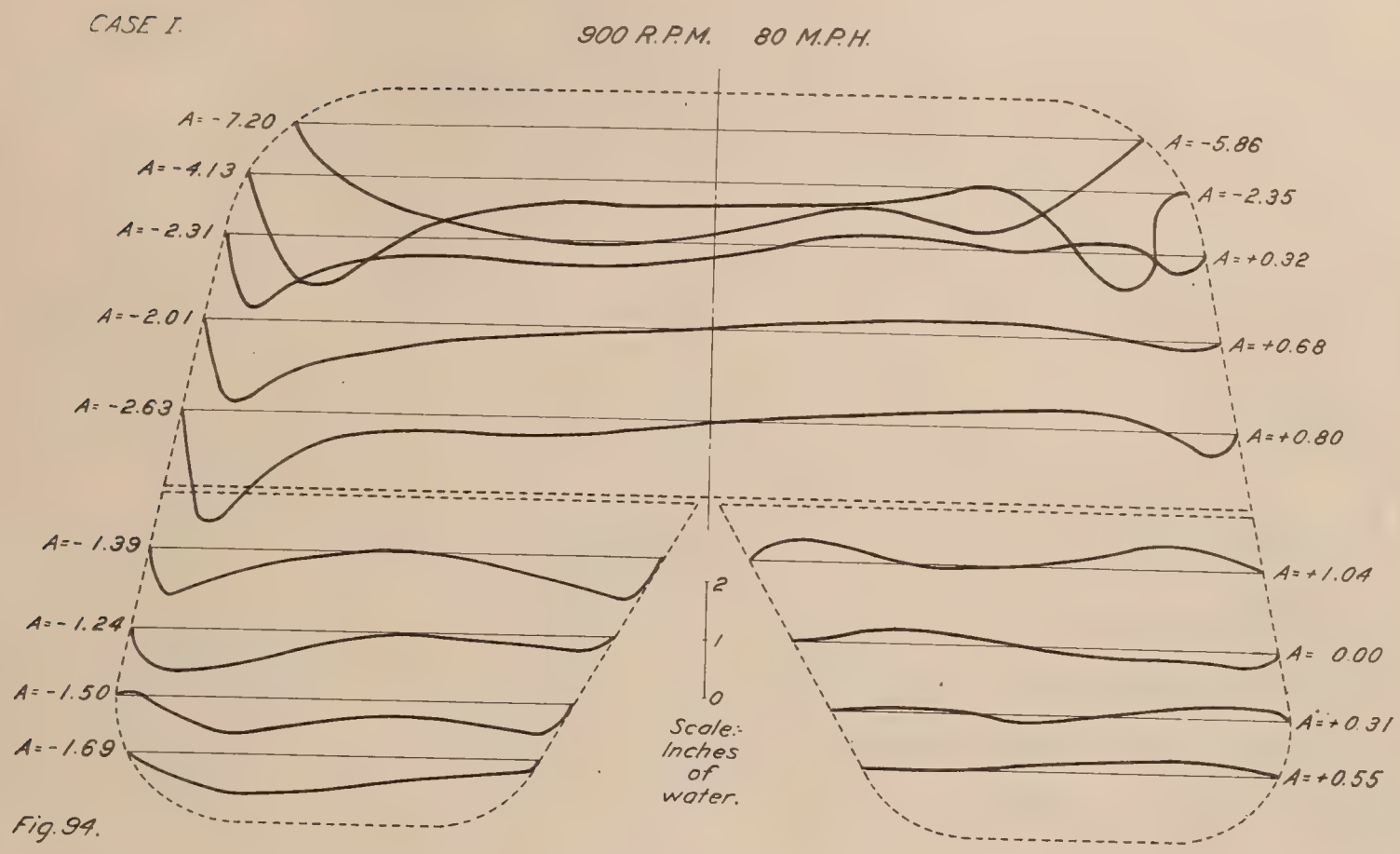

CASE I.
900 R.P.M. 80 M.P.H.
A=-7.20
A=-4.13
A=-2.31
A=-2.01
A=-2.63
A=-1.39
A=-1.24
A=-1.50
A=-1.69
A=-5.86
A=-2.35
A=+0.32
A=+0.68
A=+0.80
A=+1.04
A= 0.00
A=+0.31
A=+0.55
2
1
0
Scale:-
Inches
of
water.

Fig. 94.

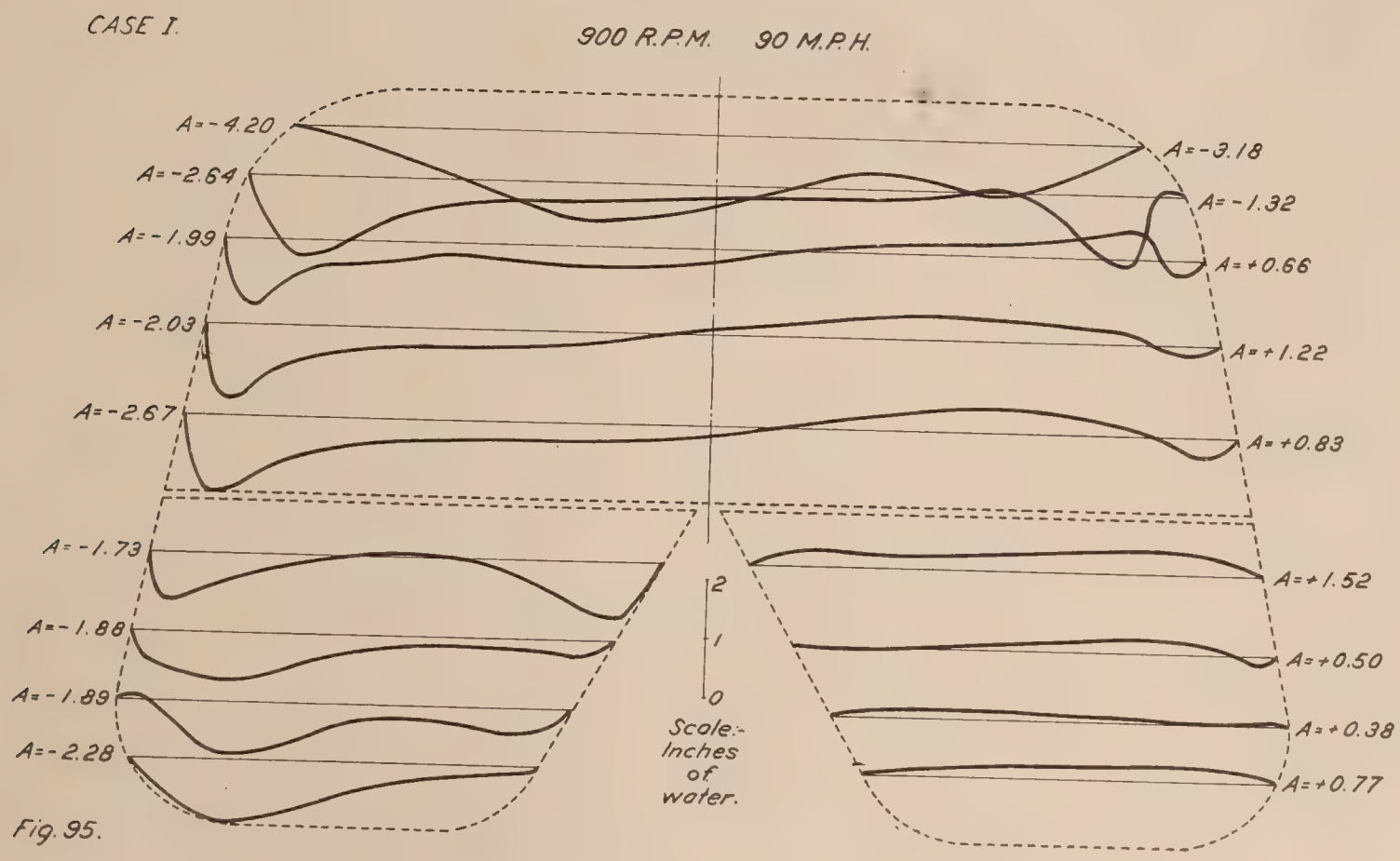

CASE I.
900 R.P.M. 90 M.P.H.
A=-4.20
A=-2.64
A=-1.99
A=-2.03
A=-2.67
A=-1.73
A=-1.88
A=-1.89
A=-2.28
A=-3.18
A=-1.32
A=+0.66
A=+1.22
A=+0.83
A=+1.52
A=+0.50
A=+0.38
A=+0.77
2
1
0
Scale:-
Inches
of
water.

Fig. 95.

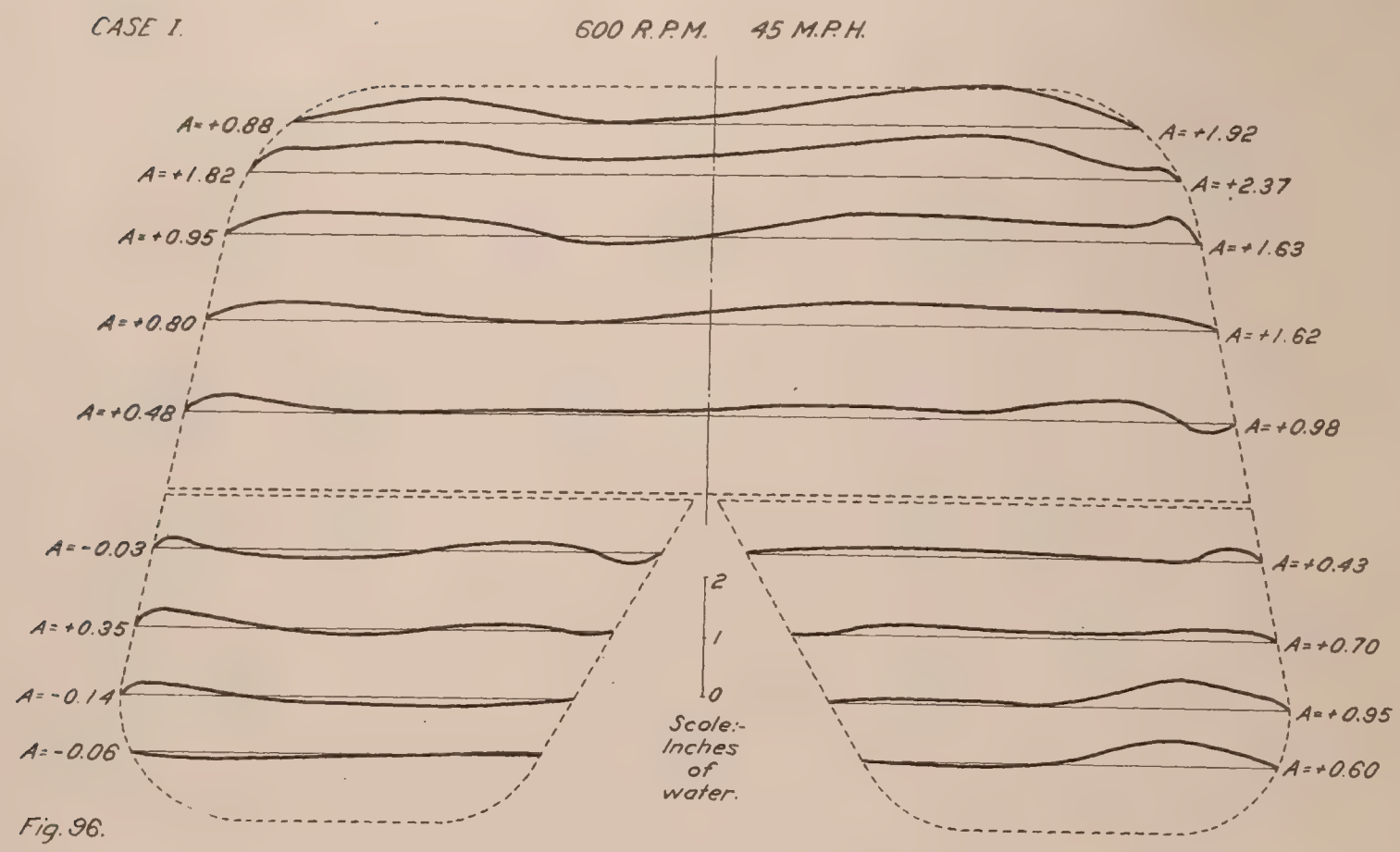

CASE I.
600 R.P.M. 45 M.P.H.
A=+0.88
A=+1.82
A=+0.95
A=+0.80
A=+0.48
A=-0.03
A=+0.35
A=-0.14
A=-0.06
A=+1.92
A=+2.37
A=+1.63
A=+1.62
A=+0.98
A=+0.43
A=+0.70
A=+0.95
A=+0.60
2
1
0
Scale:-
Inches
of
water.

Fig. 96.

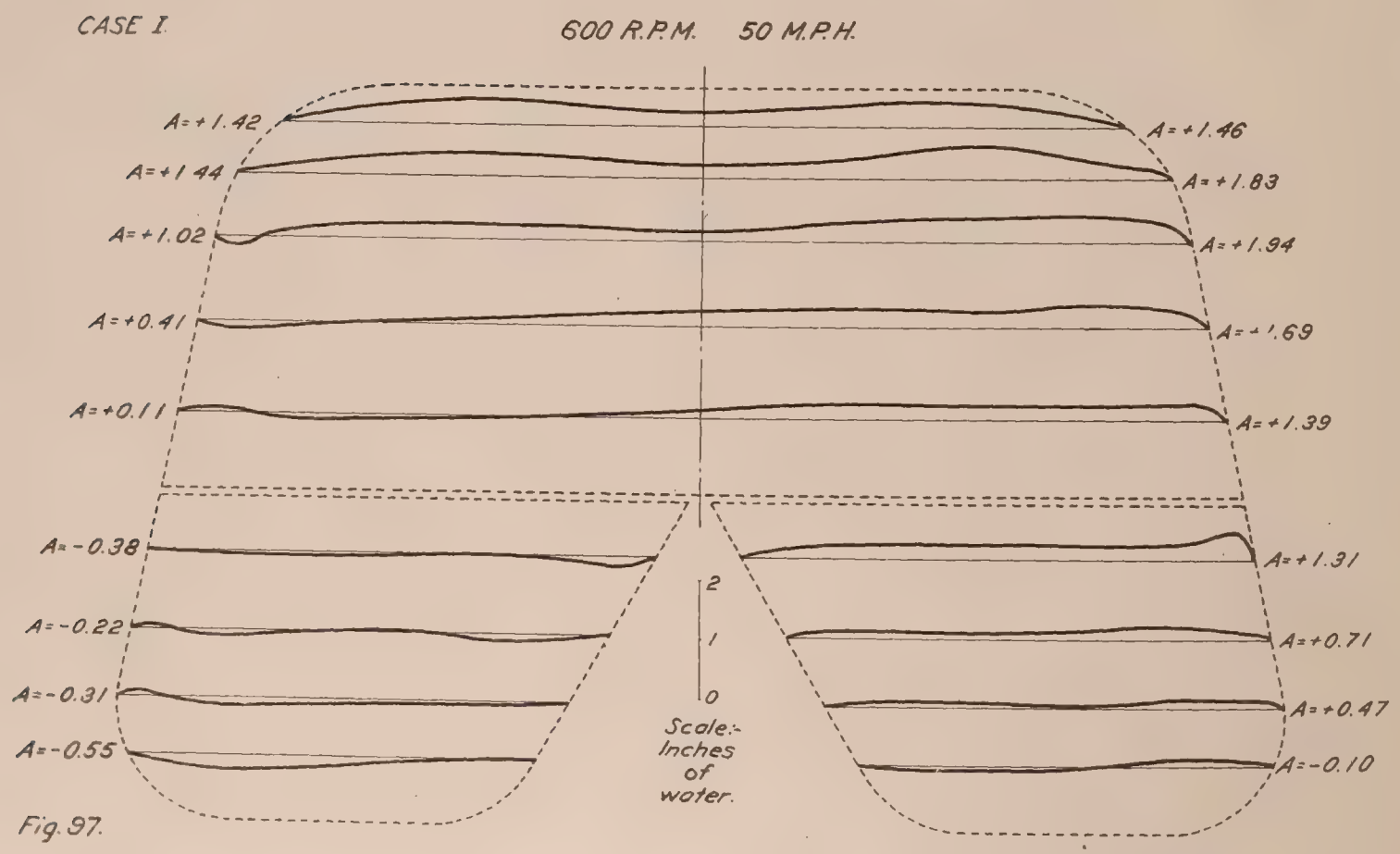

CASE I.
600 R.P.M. 50 M.P.H.
A=+1.42
A=+1.44
A=+1.02
A=+0.41
A=+0.11
A=-0.38
A=-0.22
A=-0.31
A=-0.55
A=+1.46
A=+1.83
A=+1.94
A=+1.69
A=+1.39
A=+1.31
A=+0.71
A=+0.47
A=-0.10
2
1
0
Scale:-
Inches
of
water.

Fig. 97.

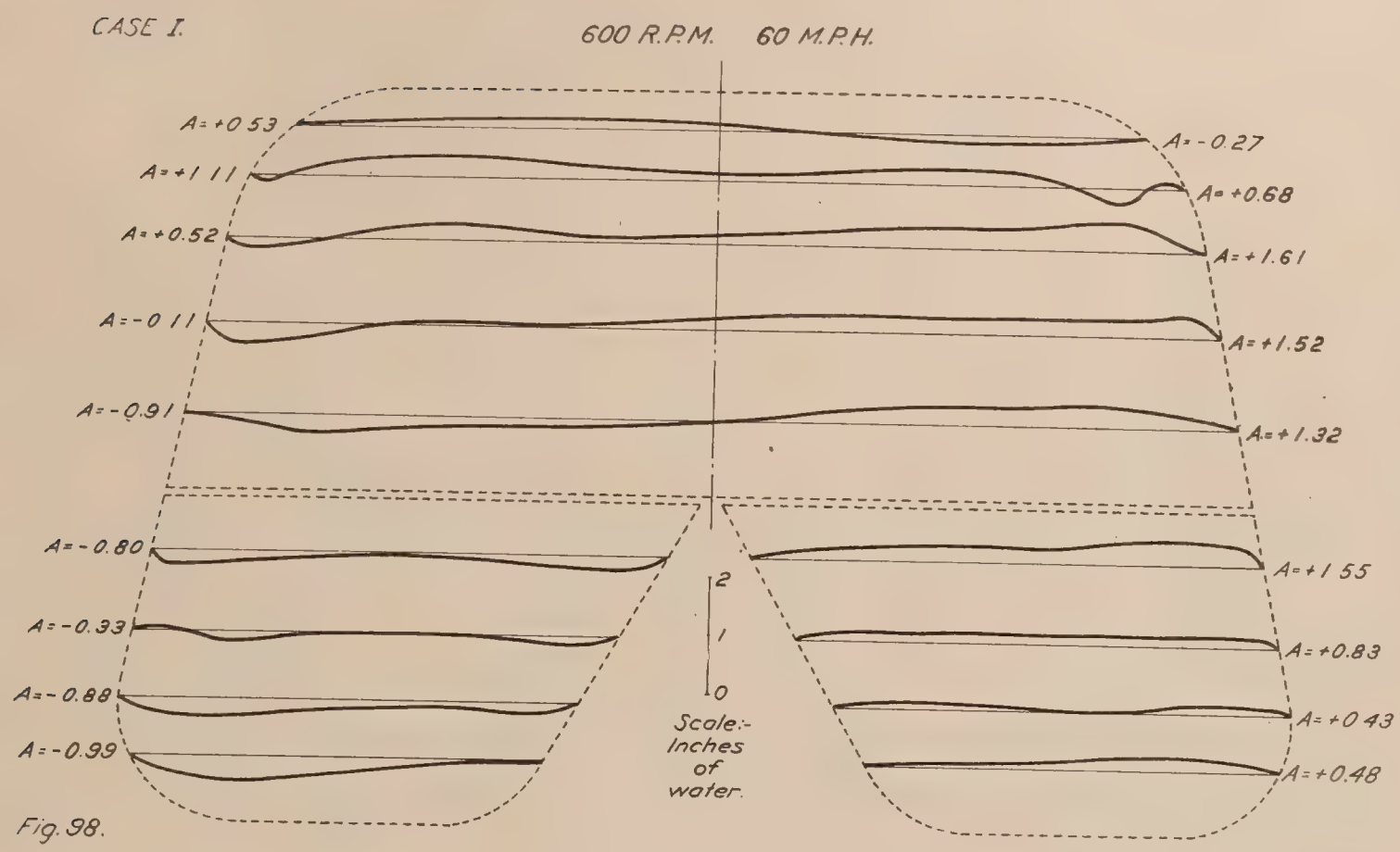

CASE I.
600 R.P.M. 60 M.P.H.
A=+0.53
A=+1.11
A=+0.52
A=-0.11
A=-0.91
A=-0.80
A=-0.33
A=-0.88
A=-0.99
A=-0.27
A=+0.68
A=+1.61
A=+1.52
A=+1.32
A=+1.55
A=+0.83
A=+0.43
A=+0.48
2
1
0
Scale:-
Inches
of
water.

Fig. 98.

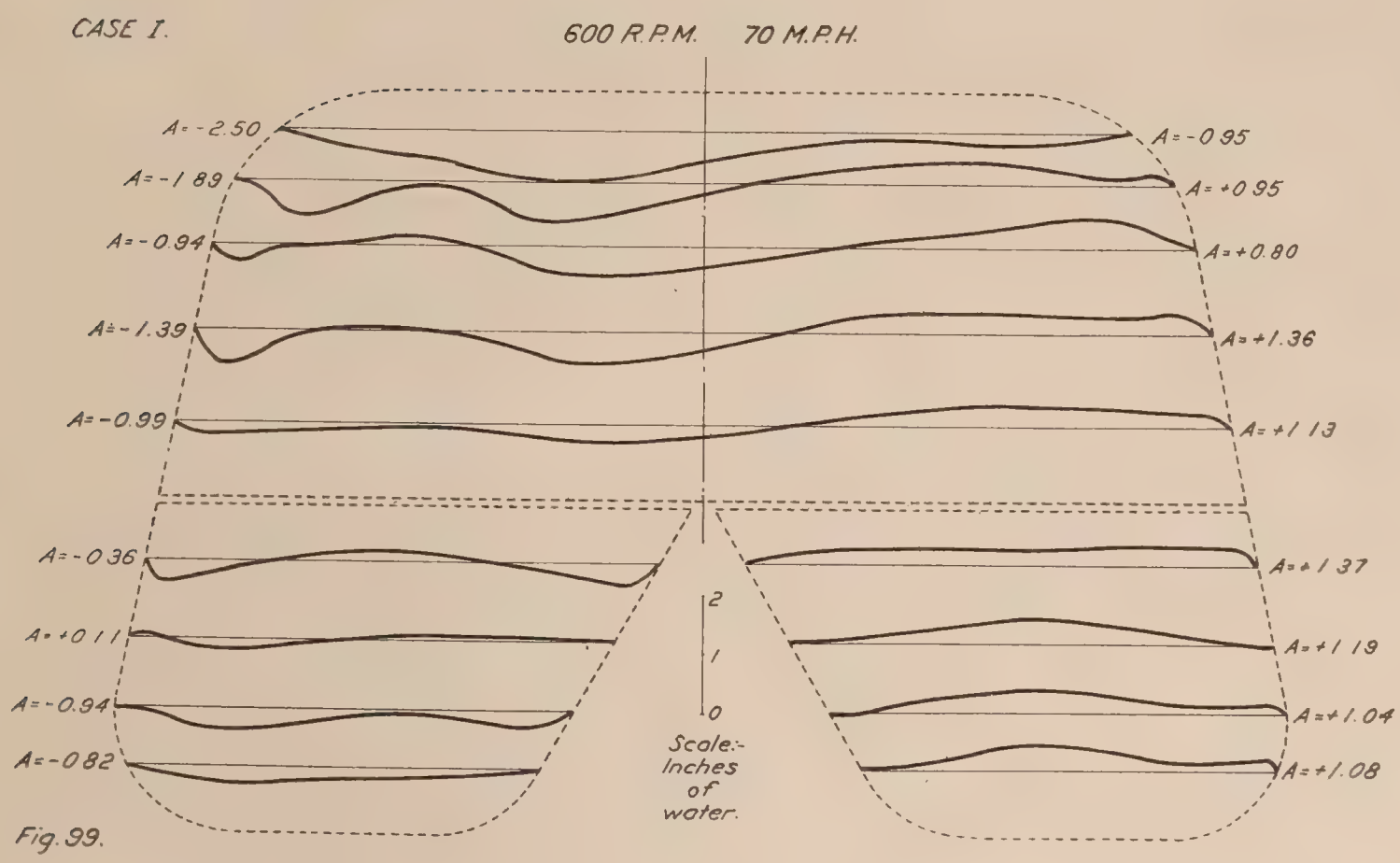

CASE I.
600 R.P.M. 70 M.P.H.
A=-2.50
A=-1.89
A=-0.94
A=-1.39
A=-0.99
A=-0.36
A=+0.11
A=-0.94
A=-0.82
A=-0.95
A=+0.95
A=+0.80
A=+1.36
A=+1.13
A=+1.37
A=+1.19
A=+1.04
A=+1.08
2
1
0
Scale:-
Inches
of
water.

Fig. 99.

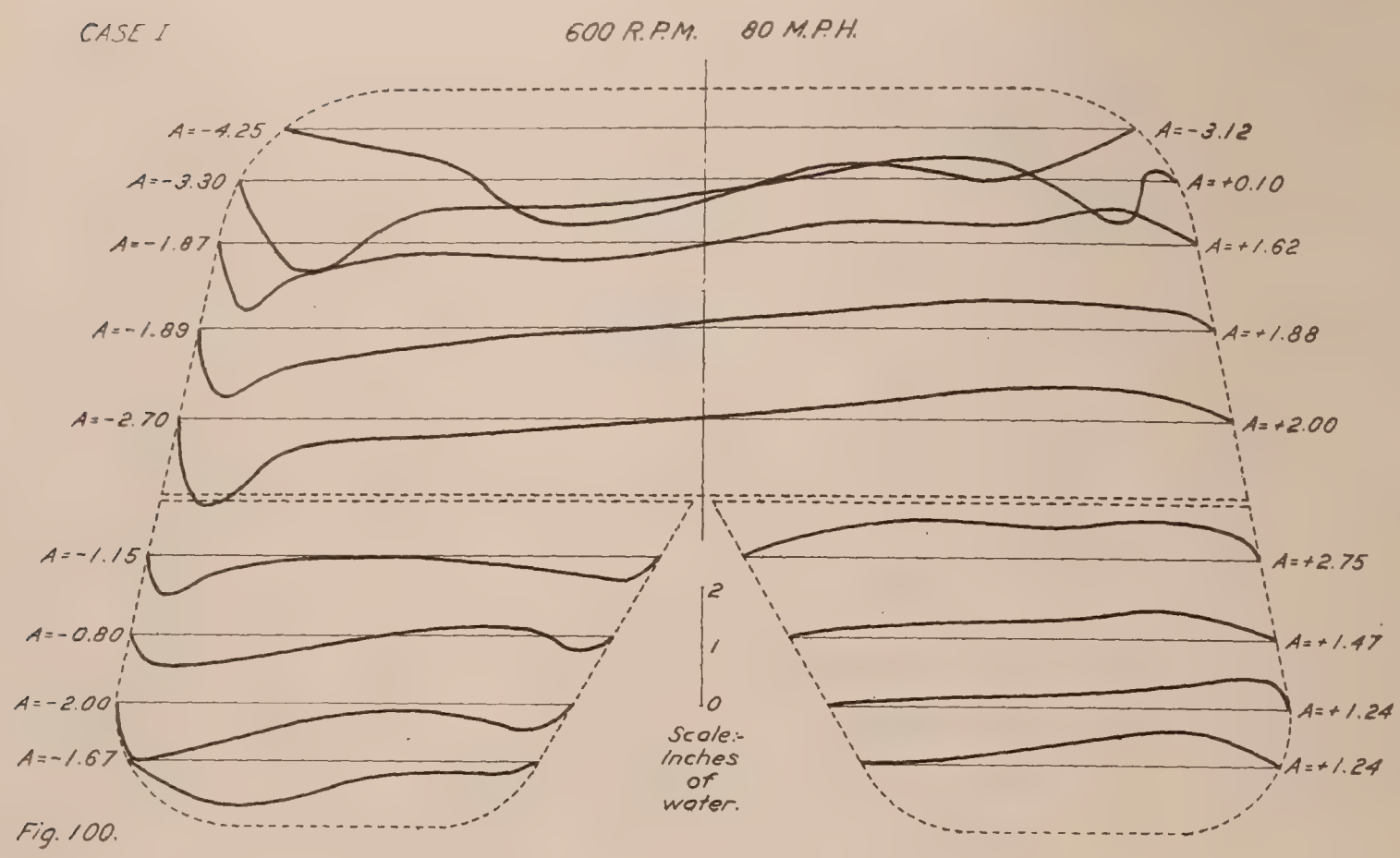


Fig. 100.

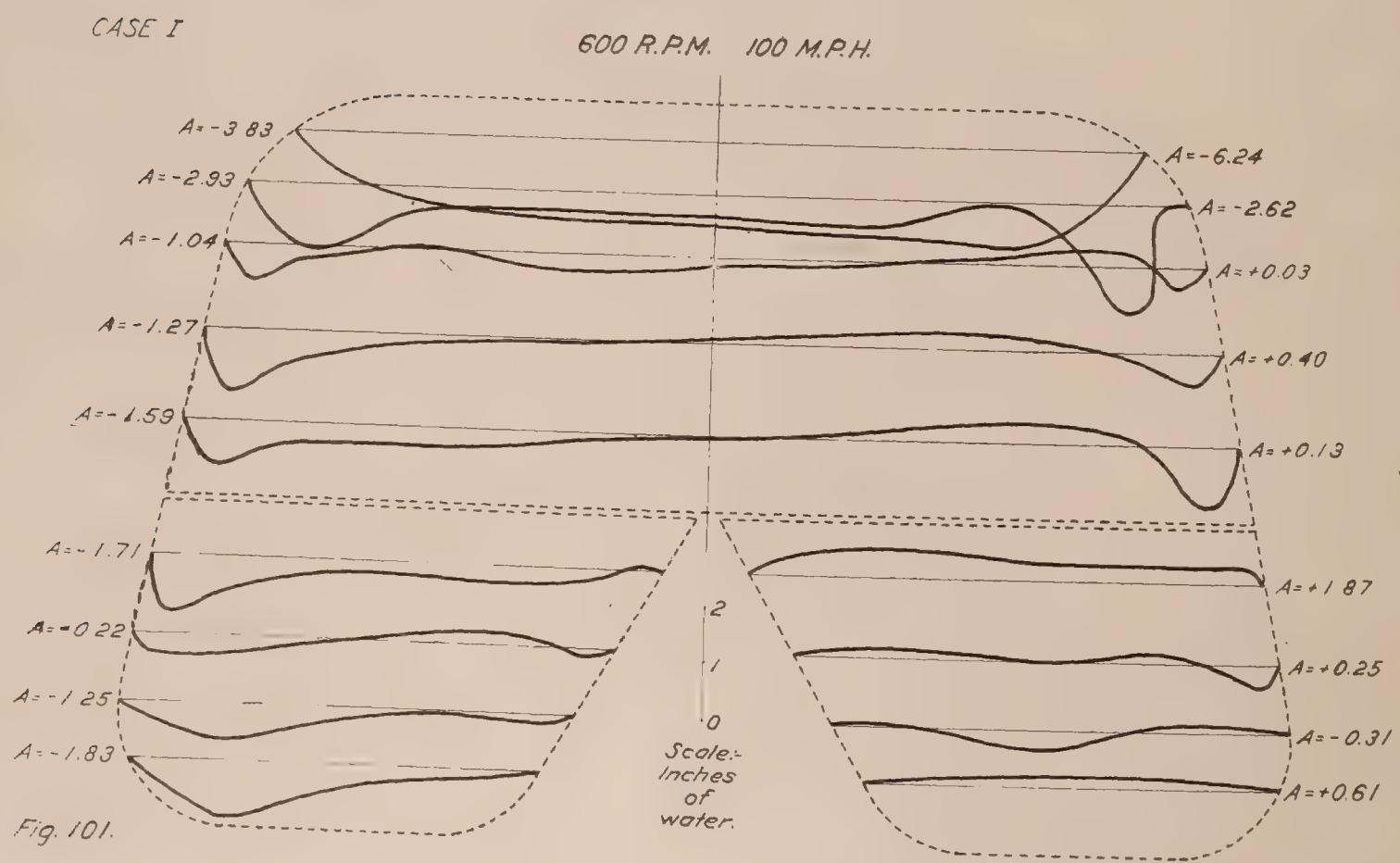


Fig. 101.

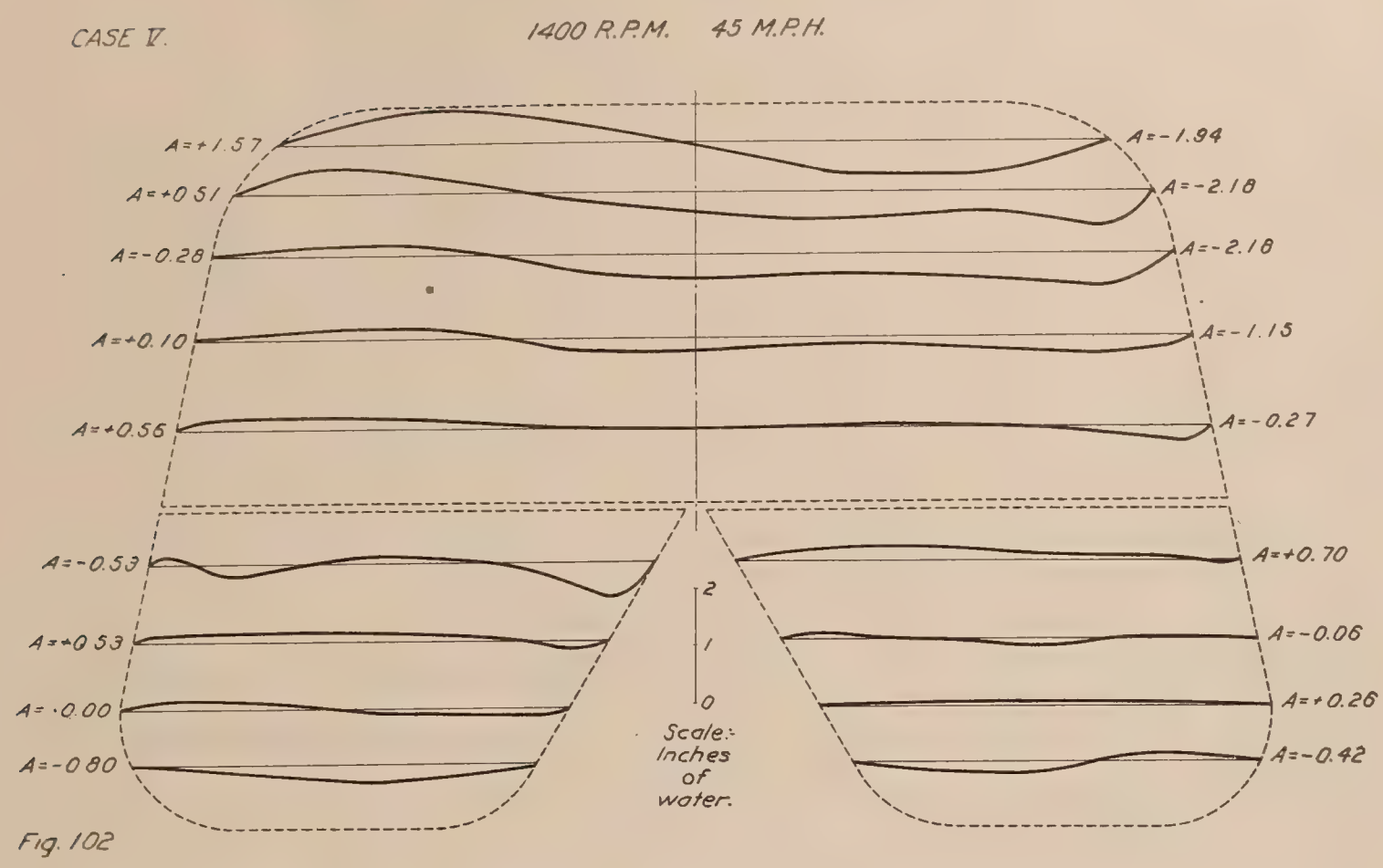


Fig. 102

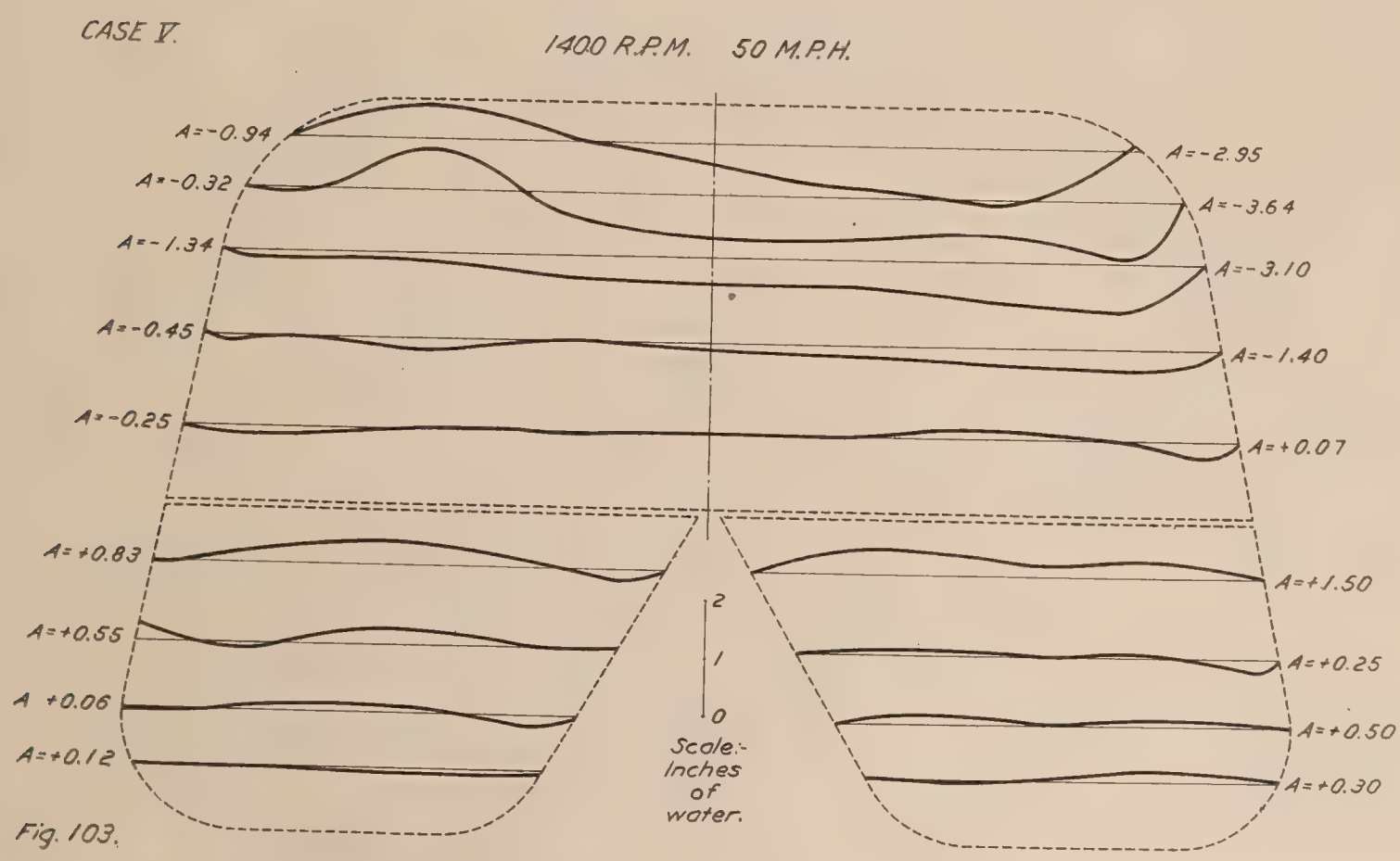


Fig. 103.

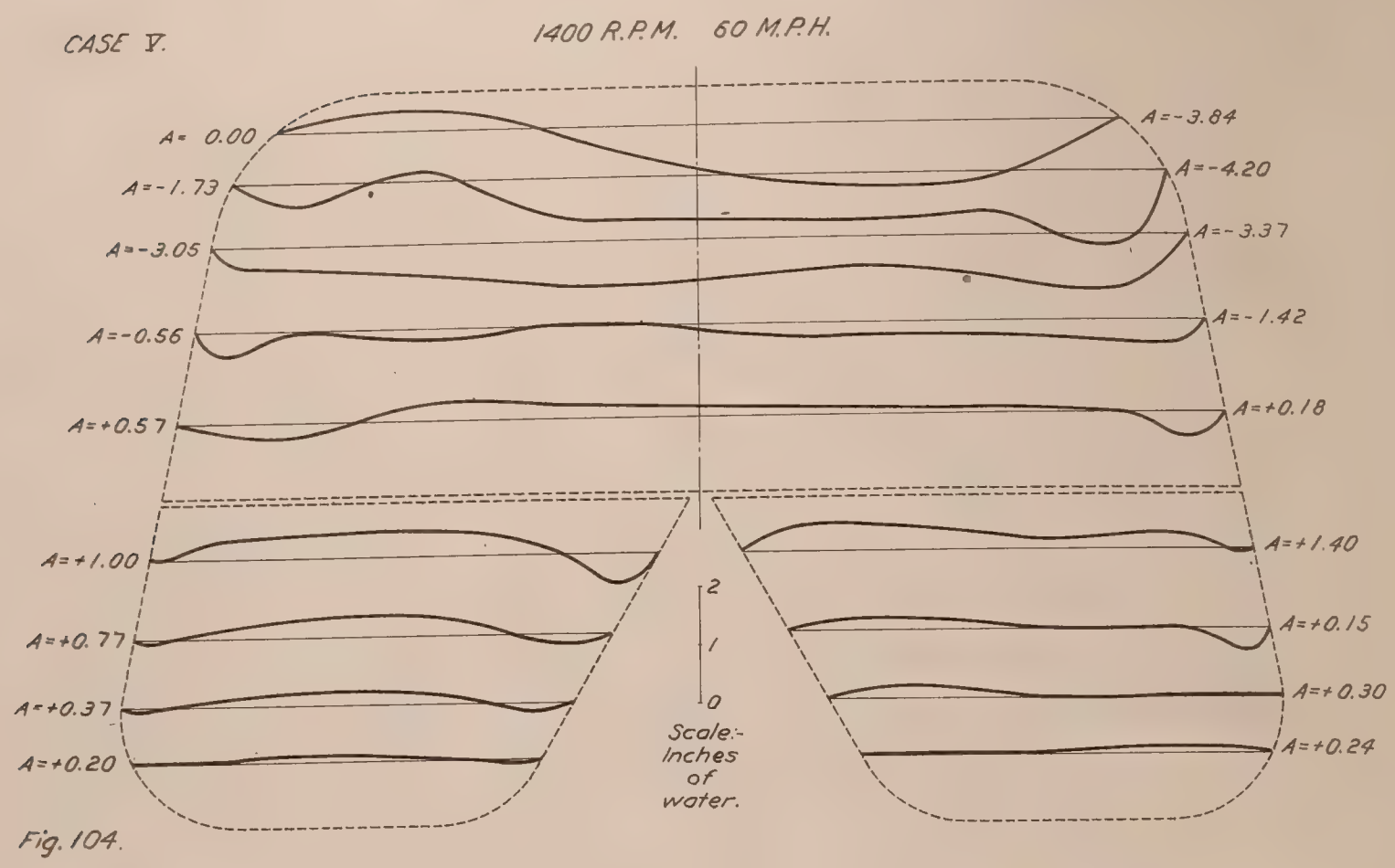


Fig. 104.

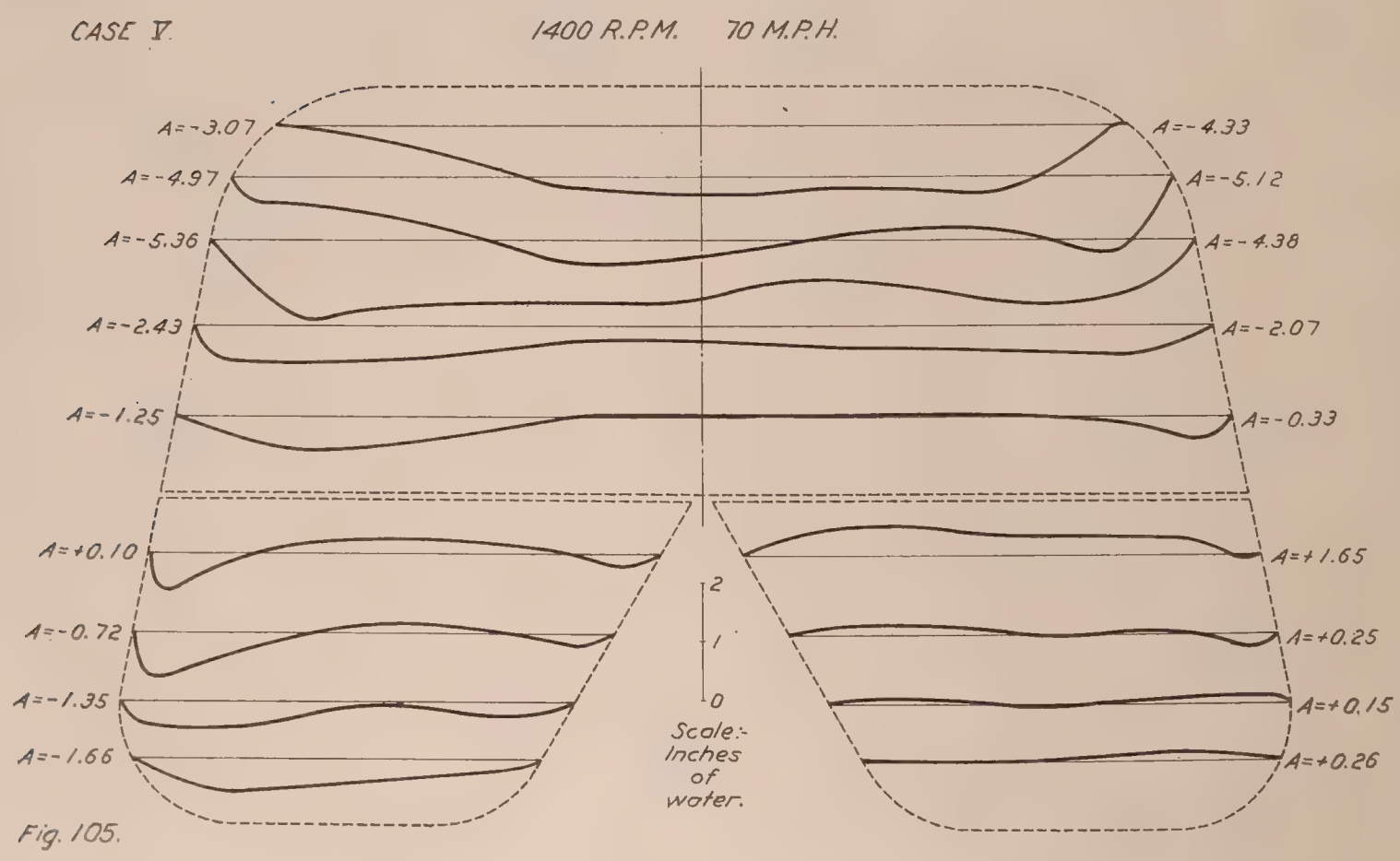


Fig. 105.

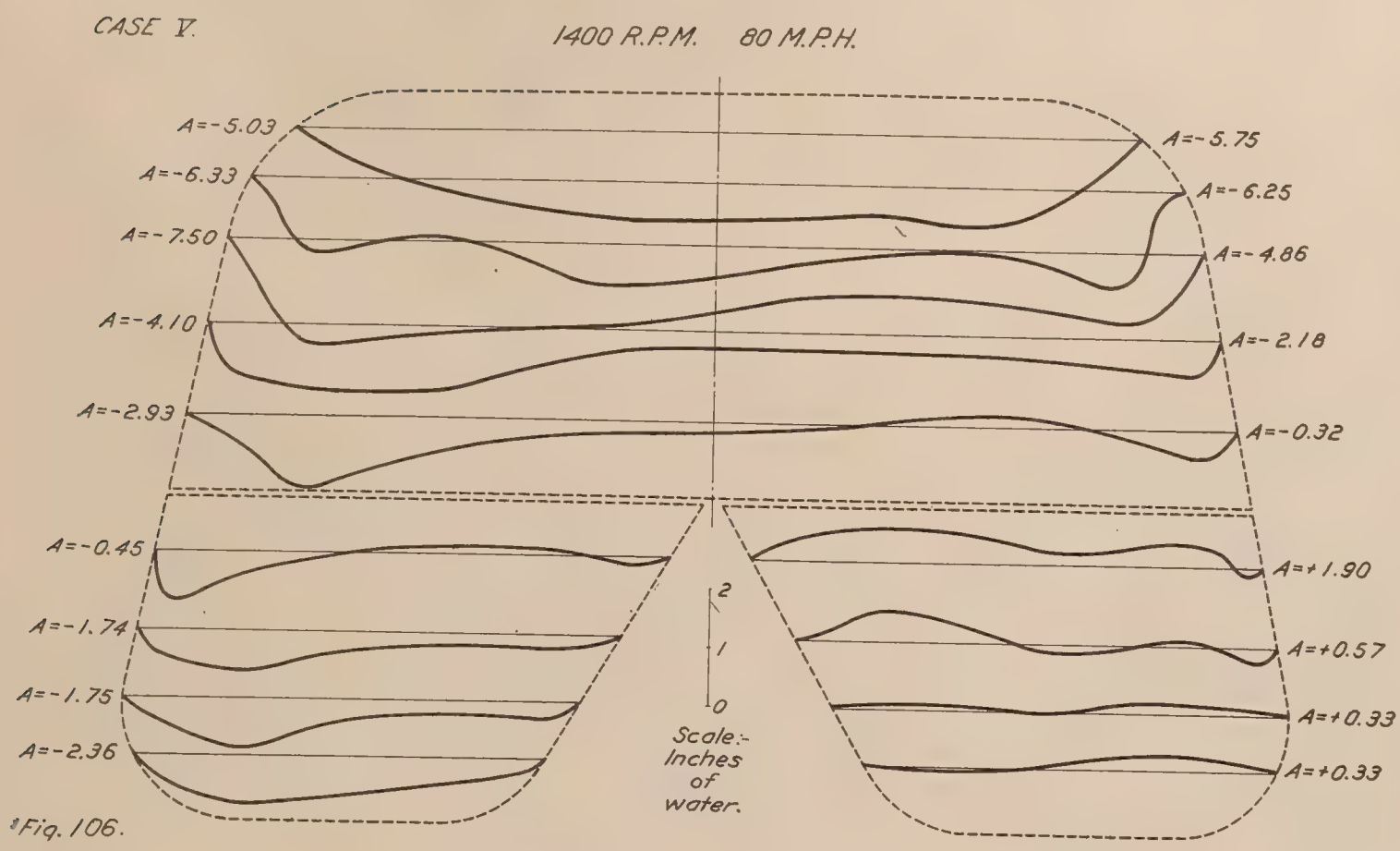


Fig. 106.

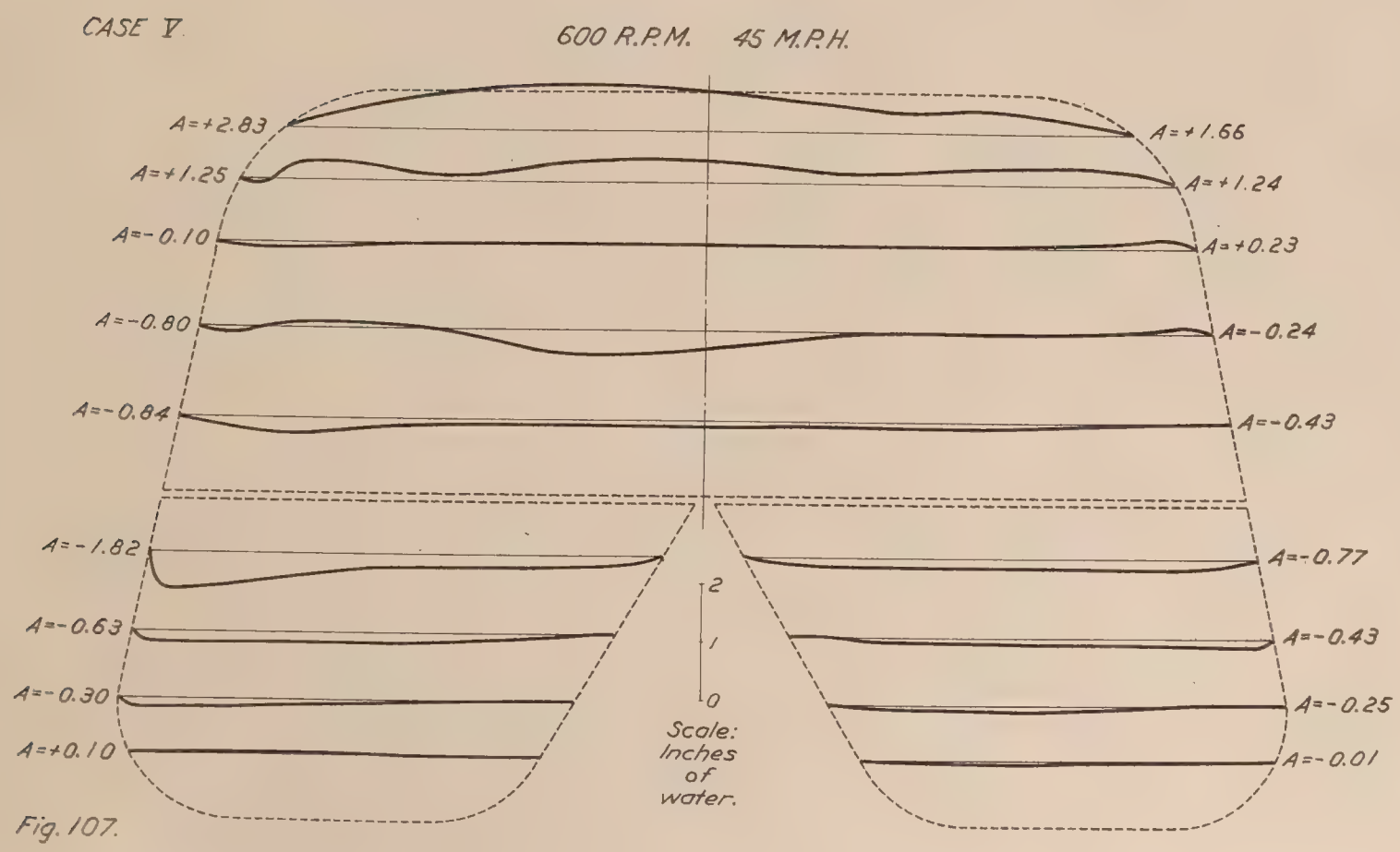


Fig. 107.

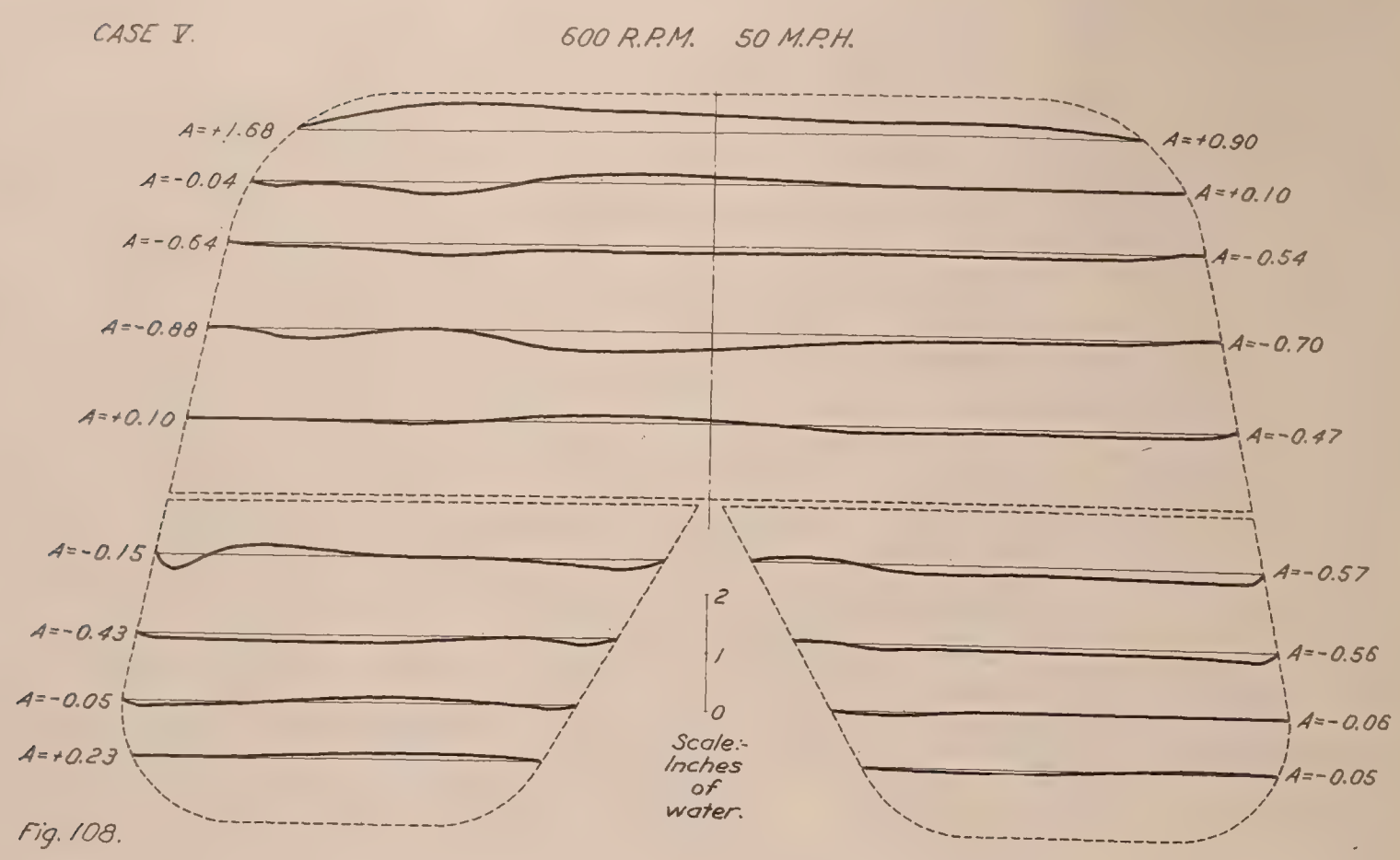


Fig. 108.

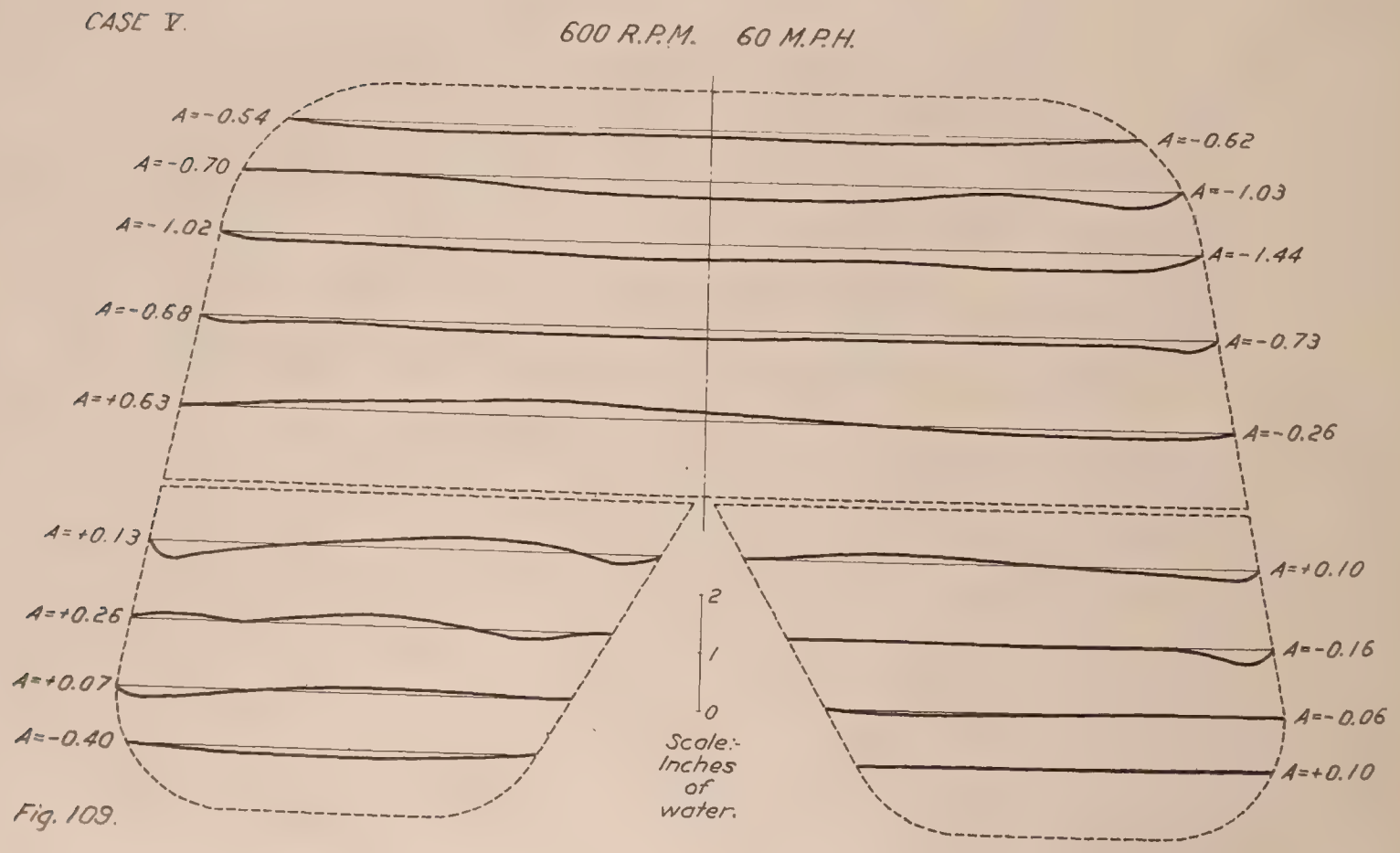


Fig. 109.

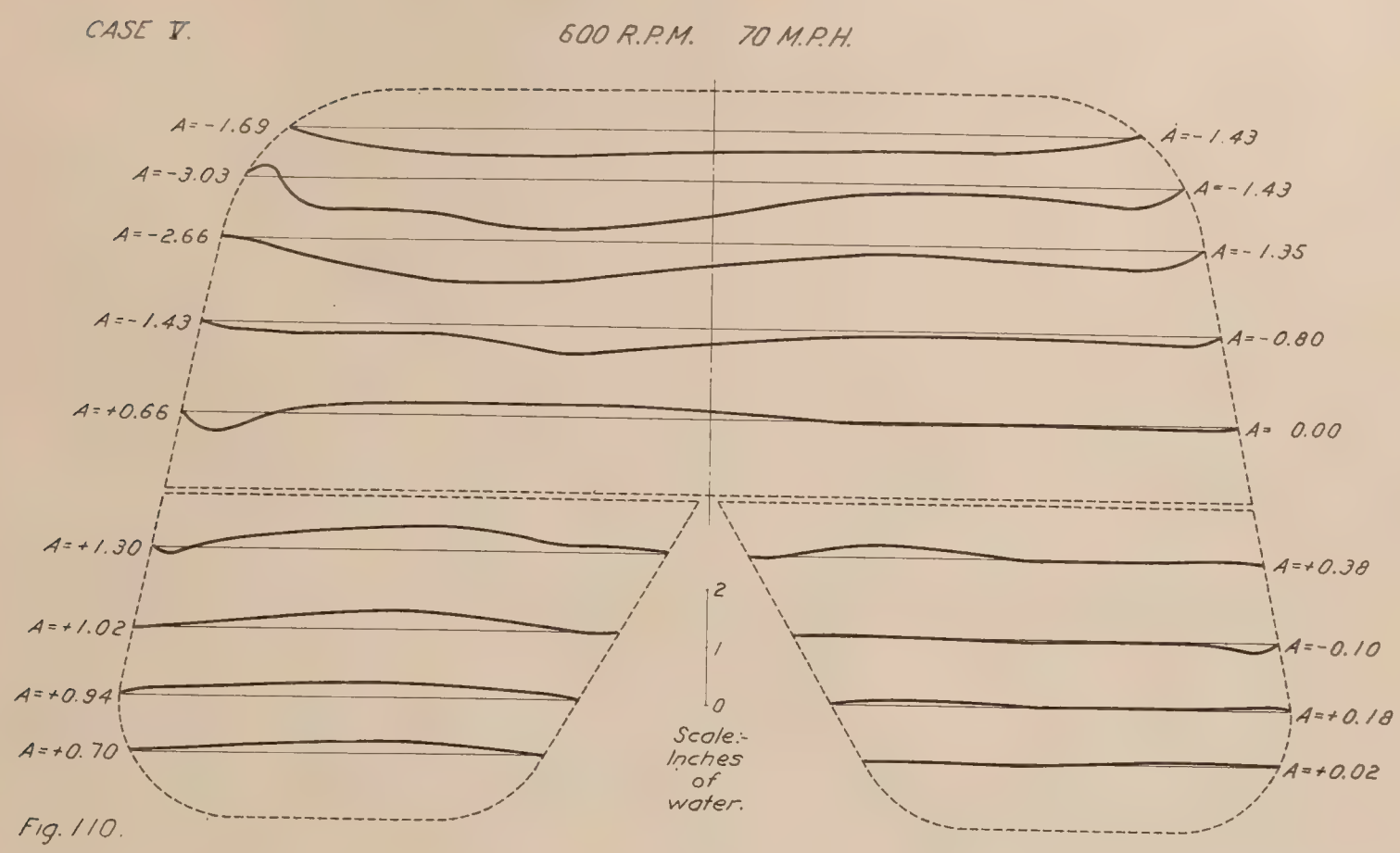


Fig. 110.

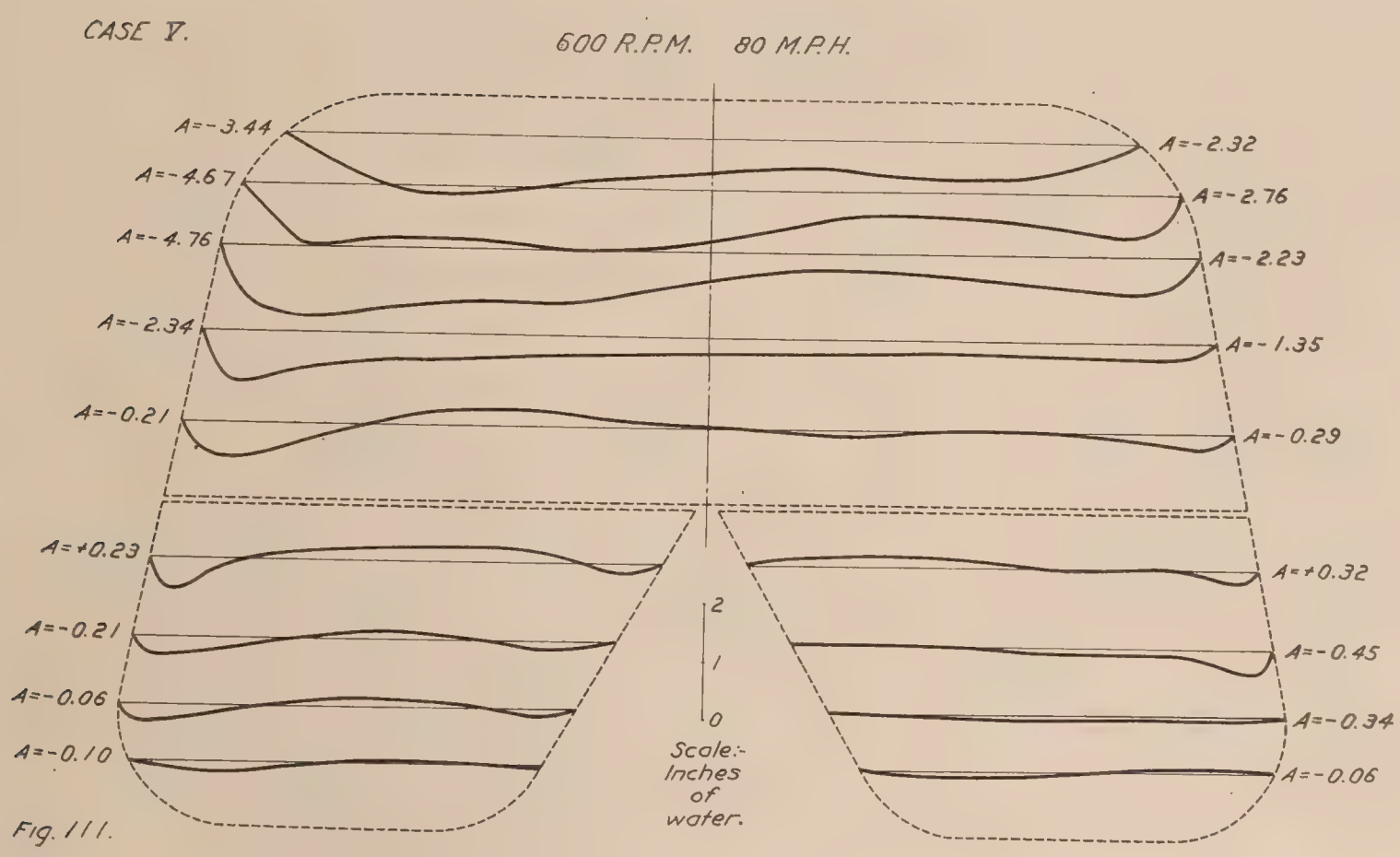


Fig. 111.

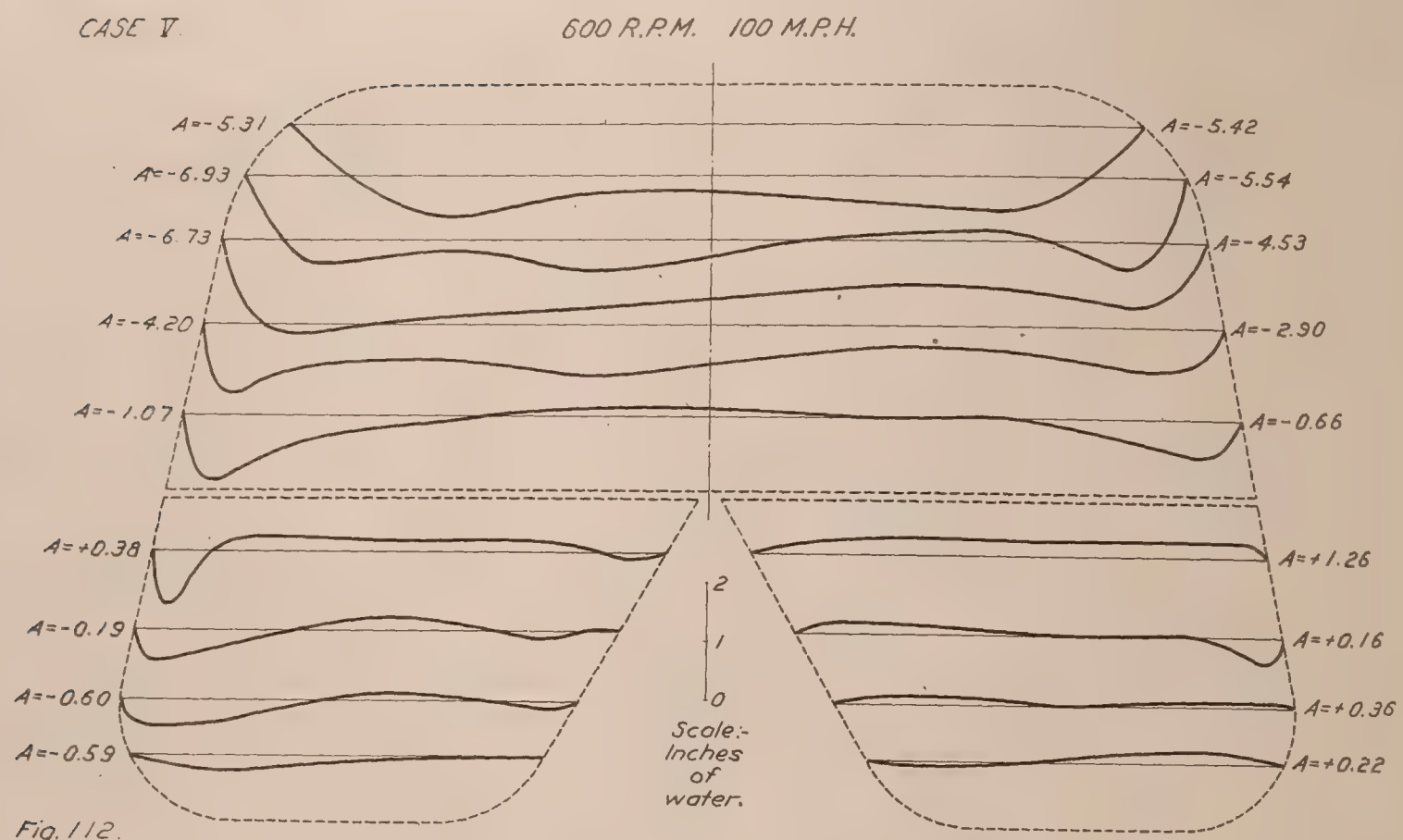


Fig. 112.

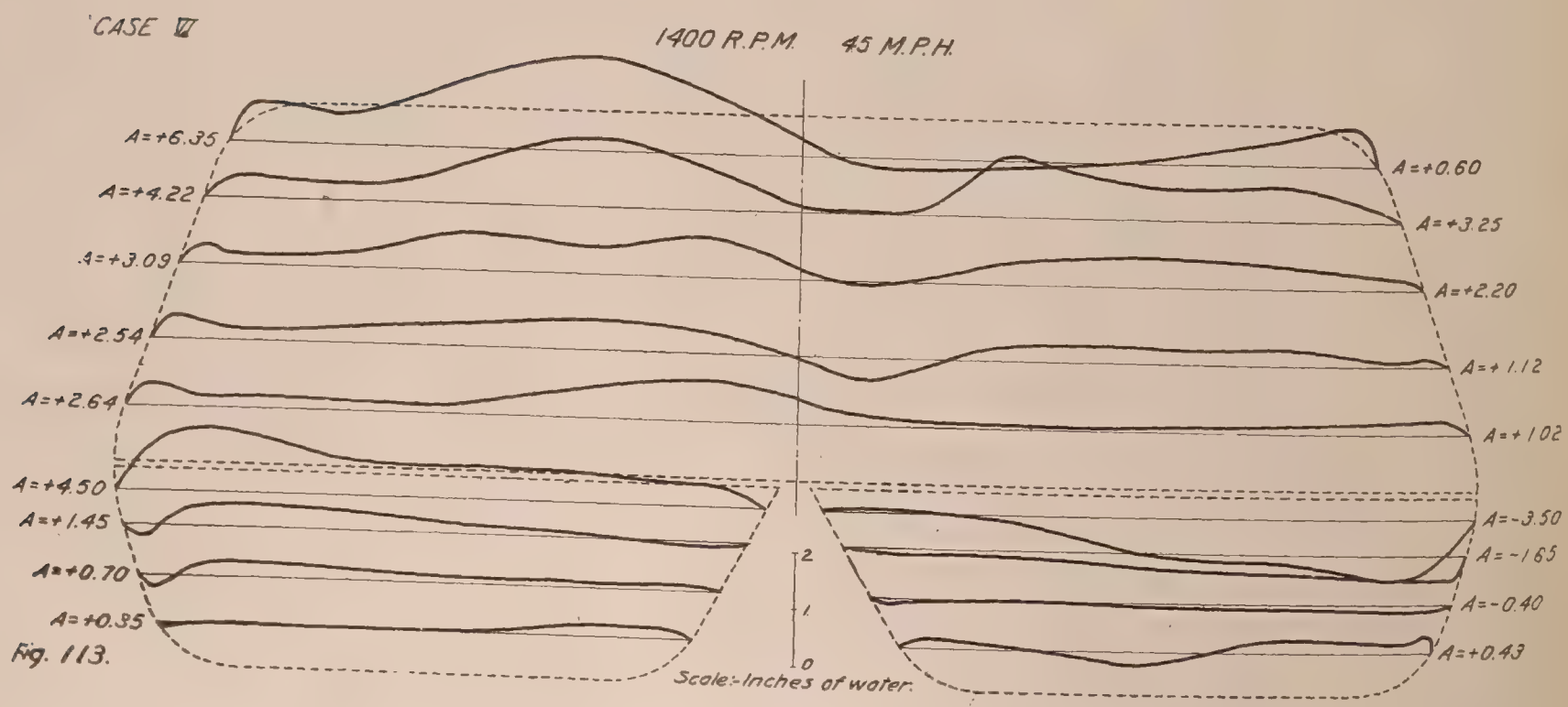


Fig. 113.

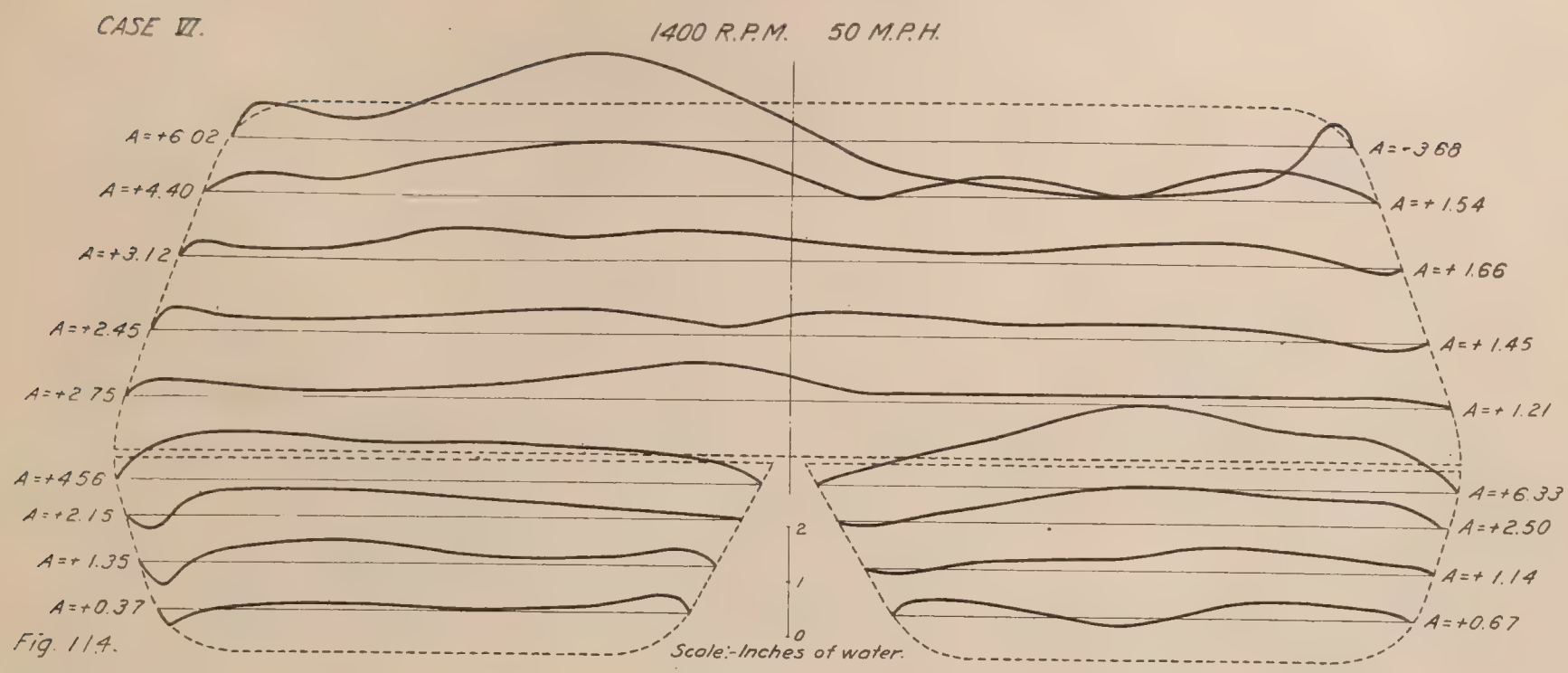


Fig. 114.

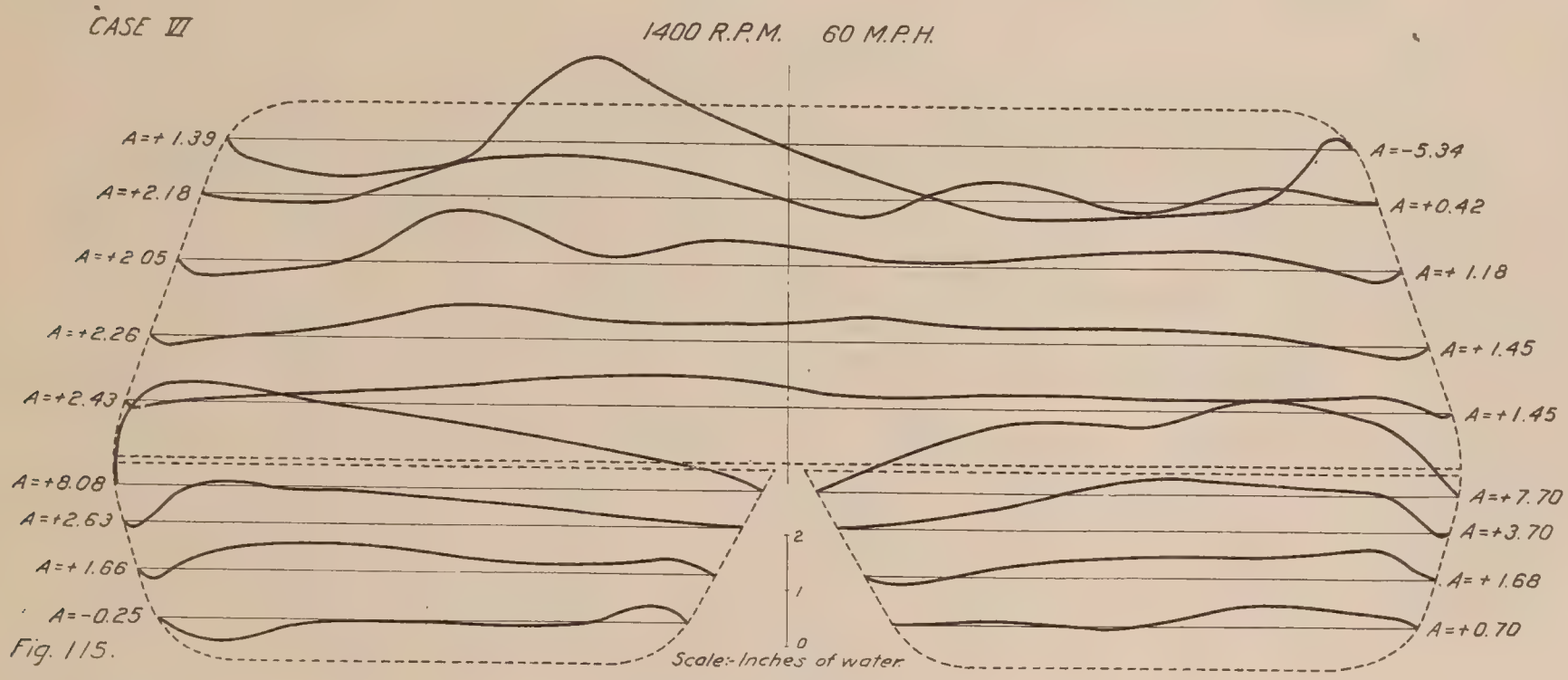


Fig. 115.

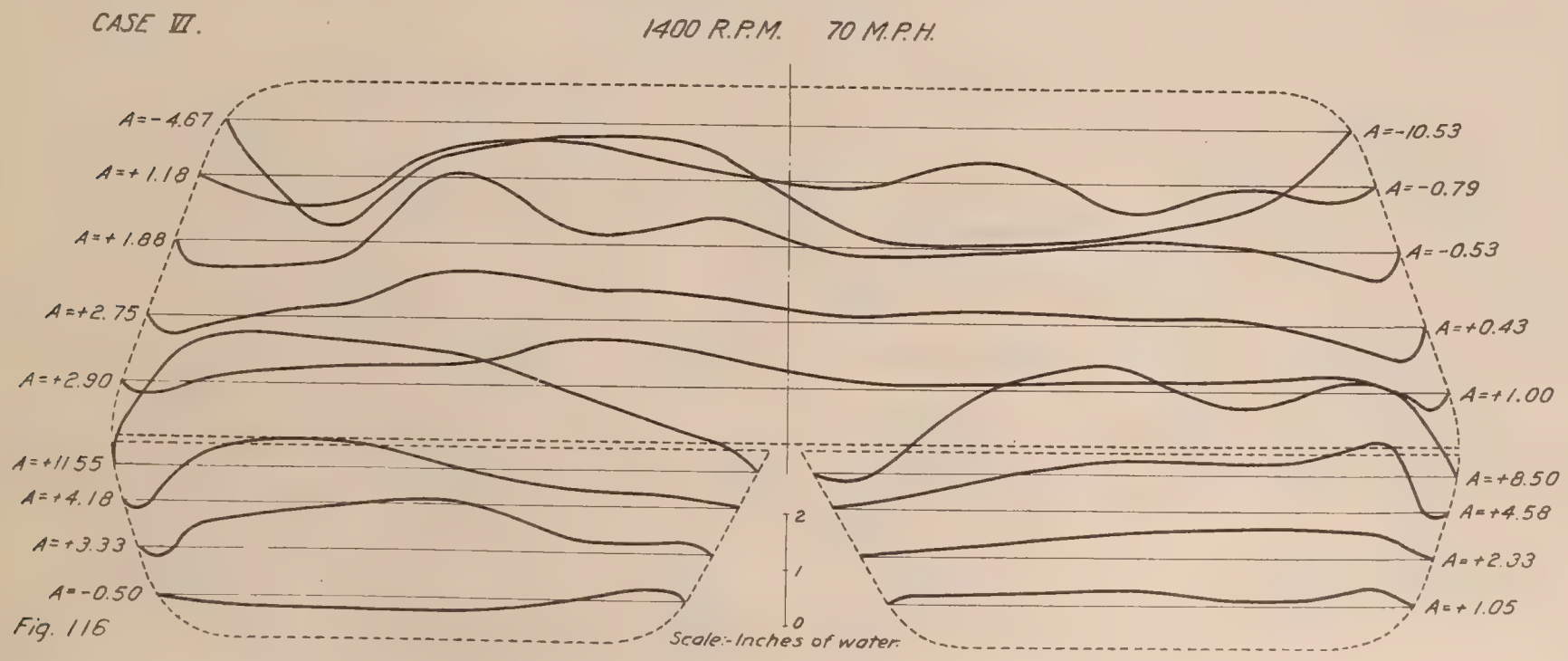


Fig. 116

CASE VI. 1400 R.P.M. 80 M.P.H.

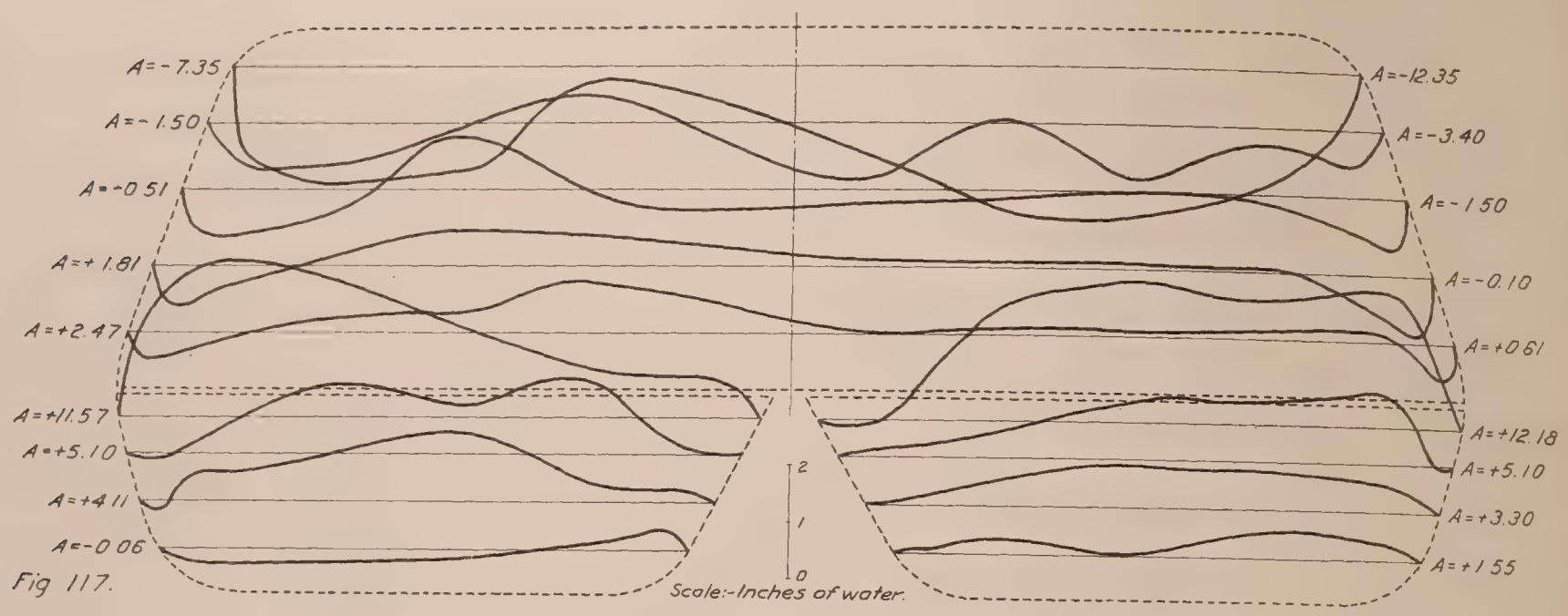


Fig. 117.

CASE VI 600 R.P.M. 50 M.P.H.

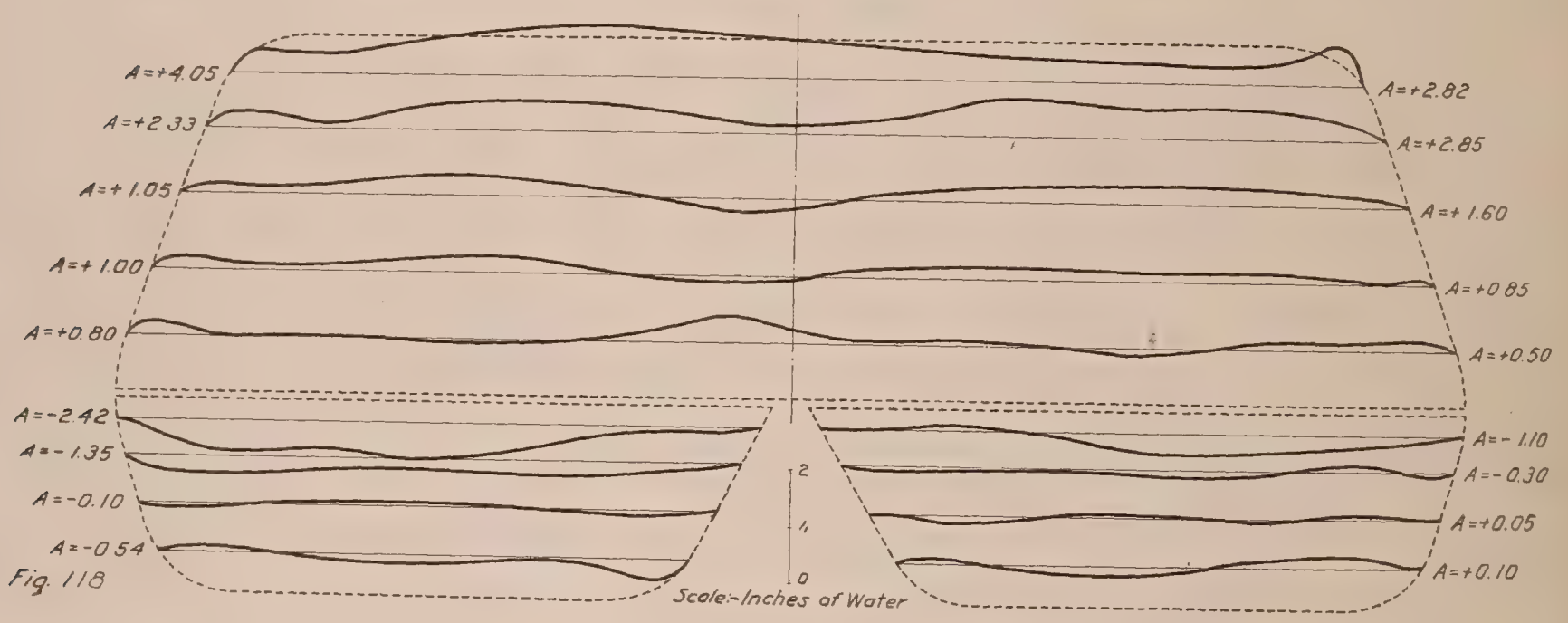


Fig. 118

CASE VI. 600 R.P.M. 60 M.P.H.

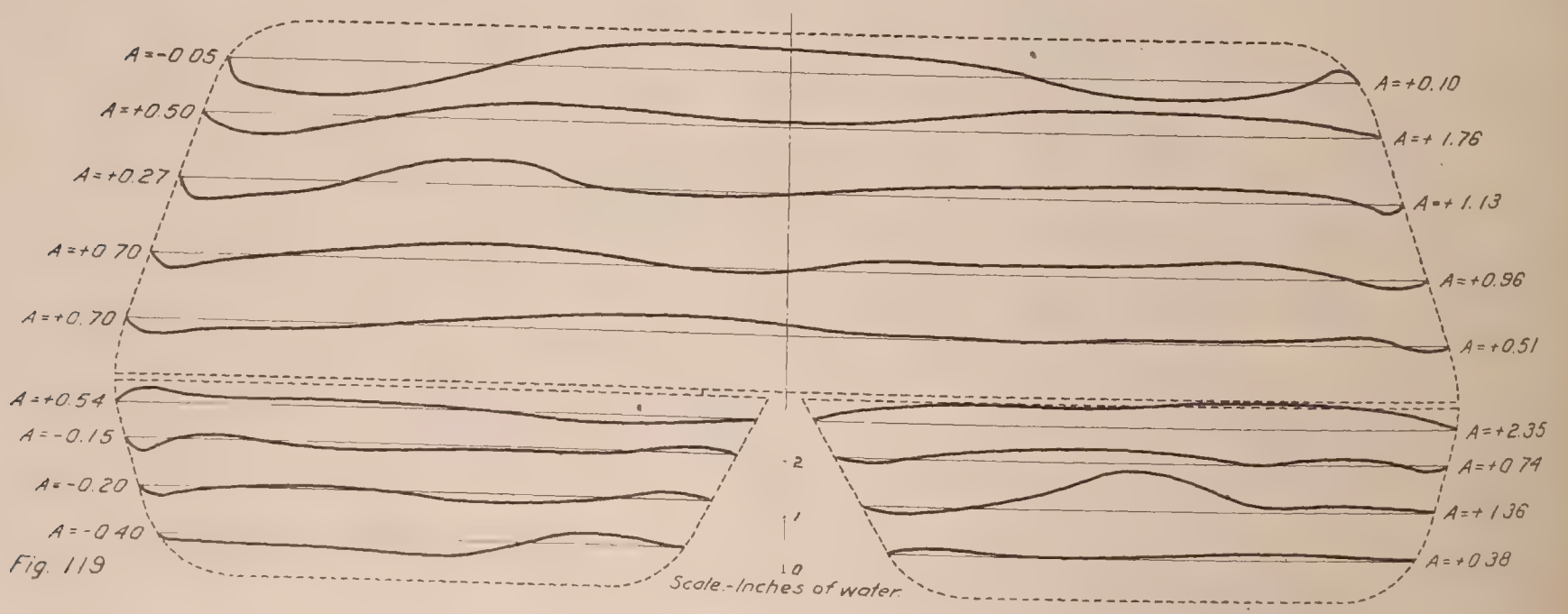


Fig. 119

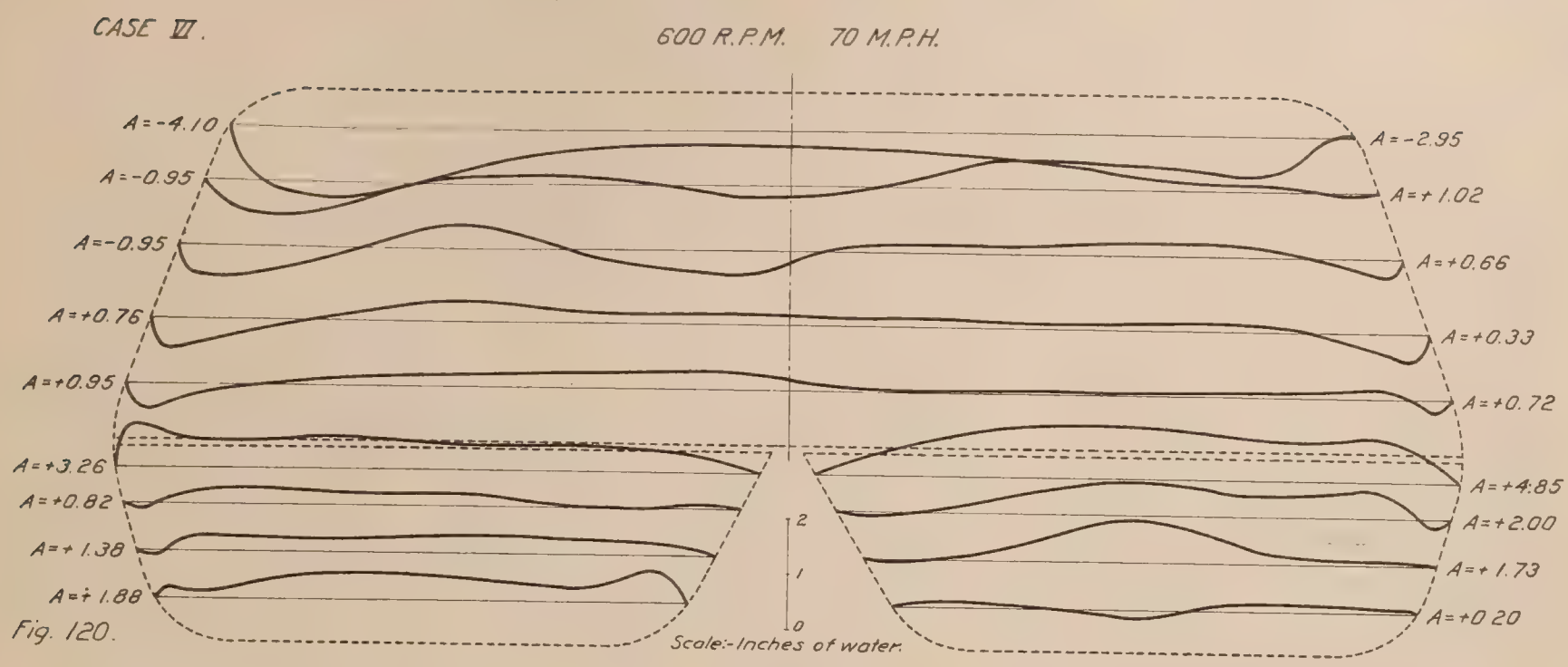


Fig. 120.

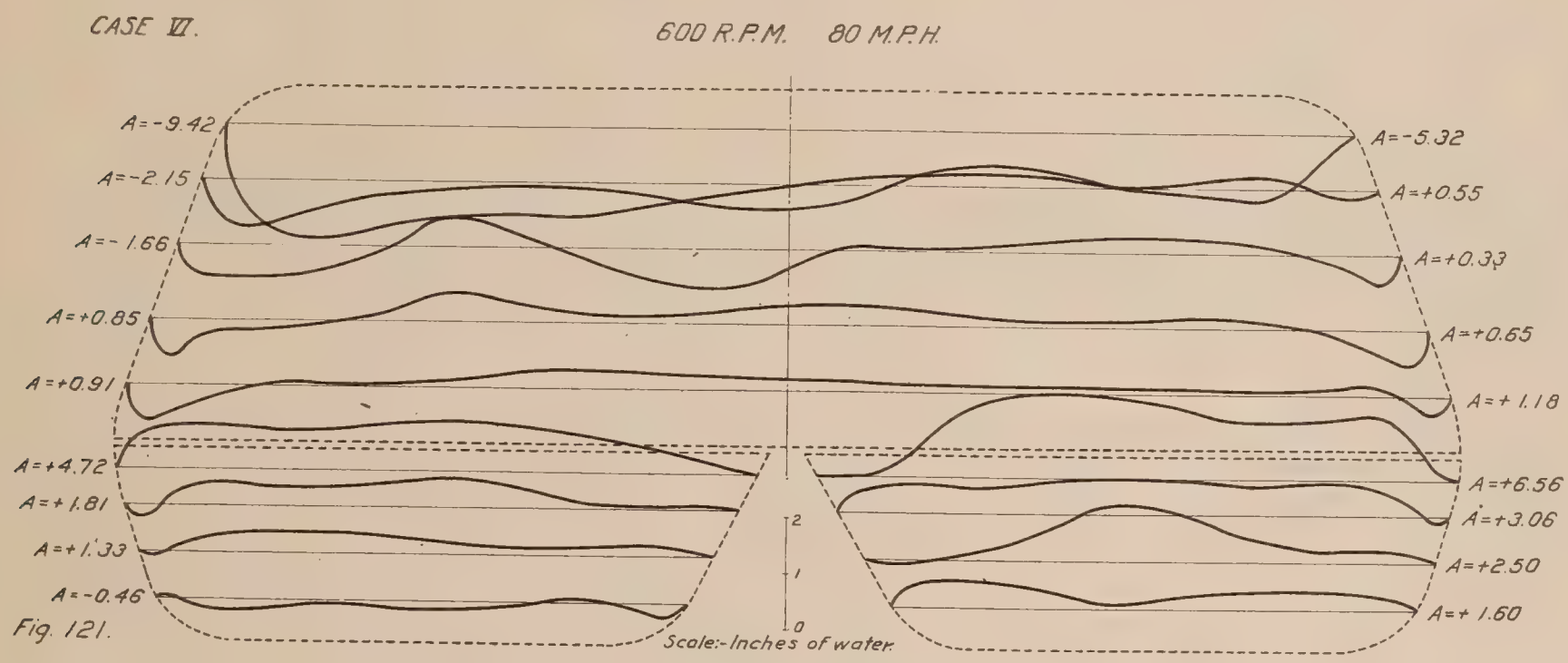


Fig. 121.

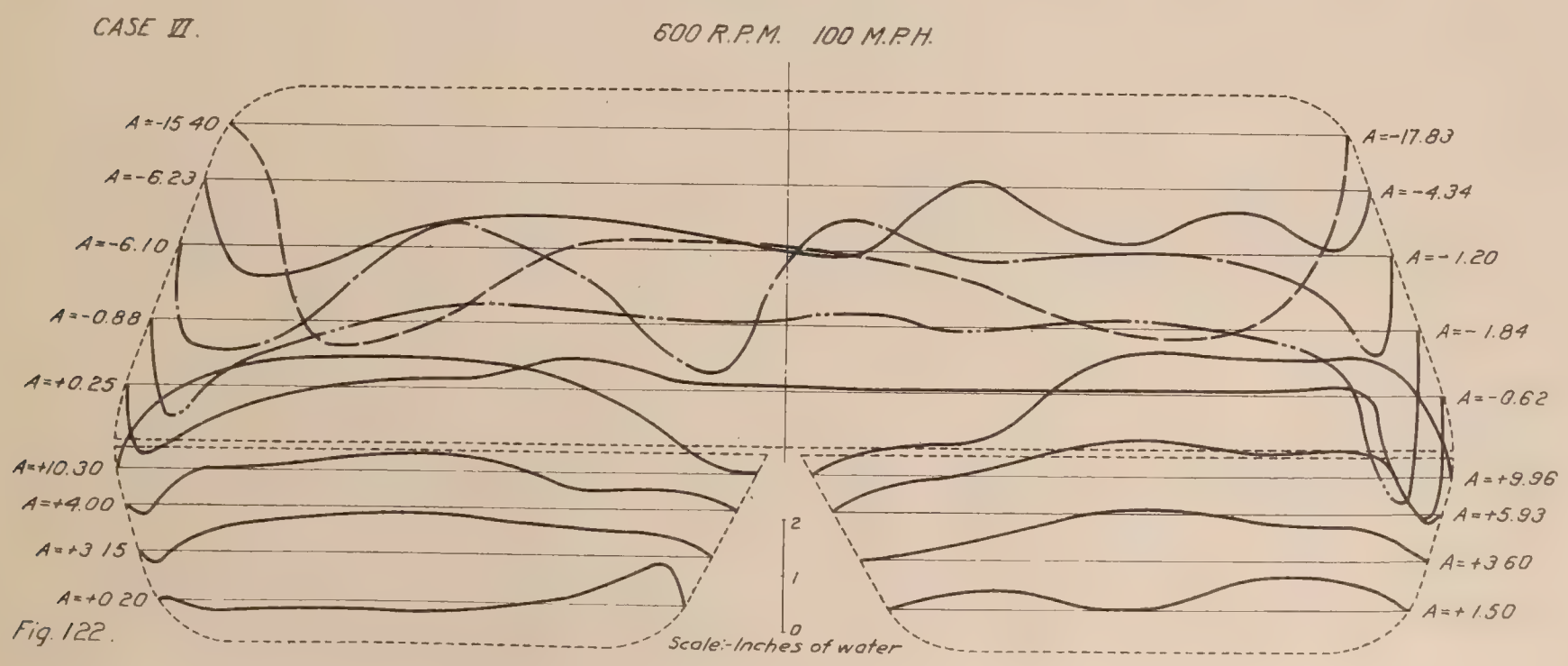
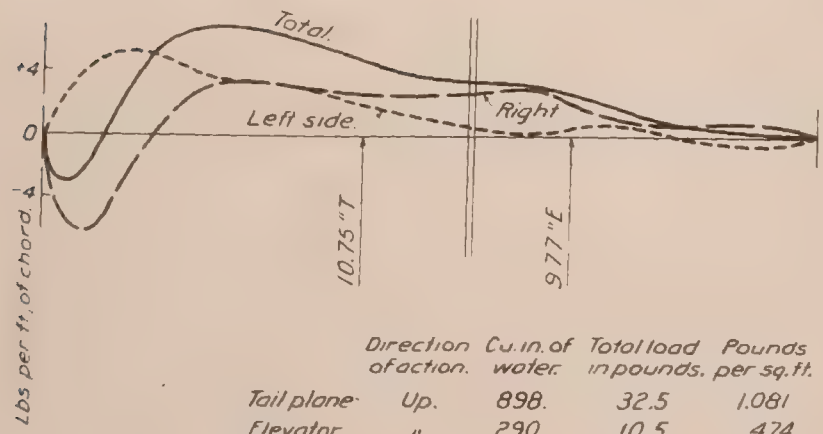


Fig. 122.

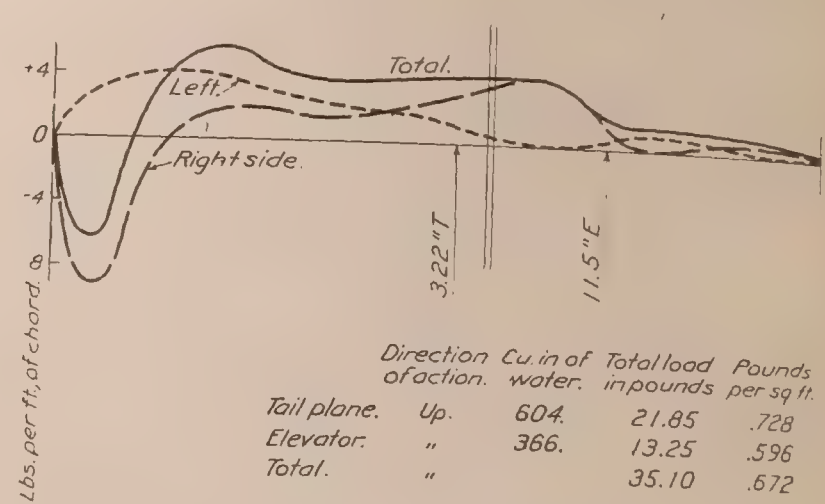
CASE I. 1400 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	898.	32.5	1.081
Elevator	"	290	10.5	.474
Total	"		43.0	.824

Fig. 123. Elevator moment about hinge = 102.6 in. lbs.

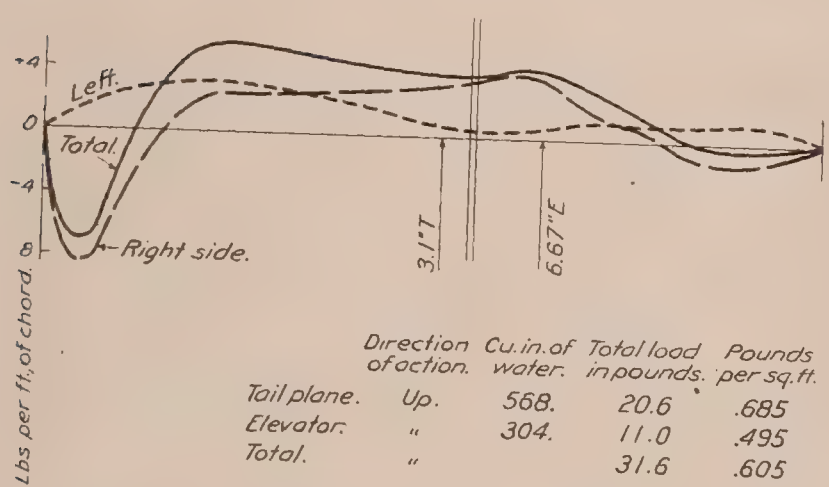
CASE I. 1400 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in of water.	Total load in pounds	Pounds per sq ft.
Tail plane.	Up.	604.	21.85	.728
Elevator.	"	366.	13.25	.596
Total.	"		35.10	.672

Fig. 124. Elevator moment about hinge = 152. in. lbs.

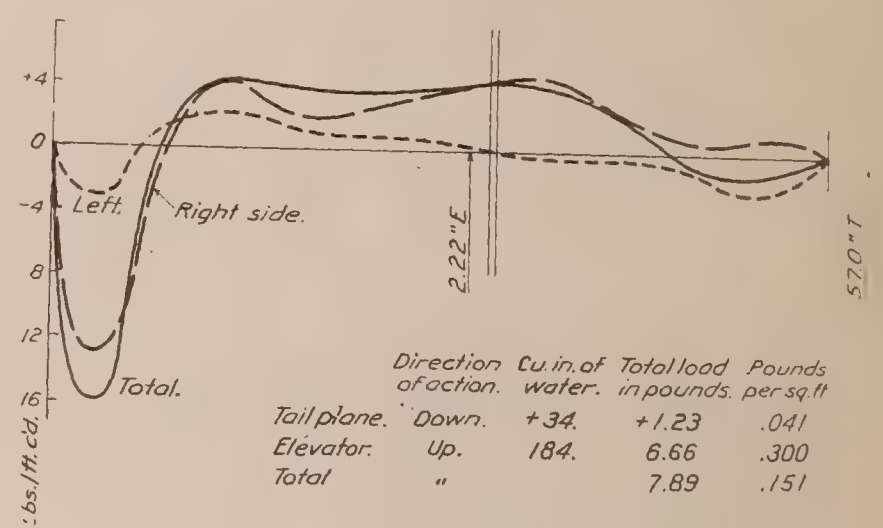
CASE I. 1400 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	568.	20.6	.685
Elevator.	"	304.	11.0	.495
Total.	"		31.6	.605

Fig. 125. Elevator moment about hinge = 73.5 in. lbs.

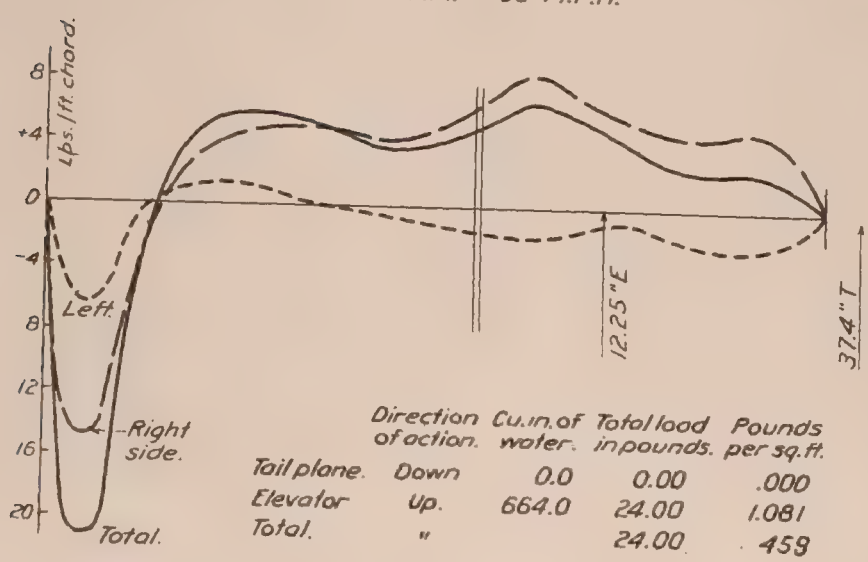
CASE I. 1400 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft
Tail plane.	Down.	+34.	+1.23	.041
Elevator.	Up.	184.	6.66	.300
Total	"		7.89	.151

Fig. 126. Elevator moment about hinge = -14.8 in. lbs.

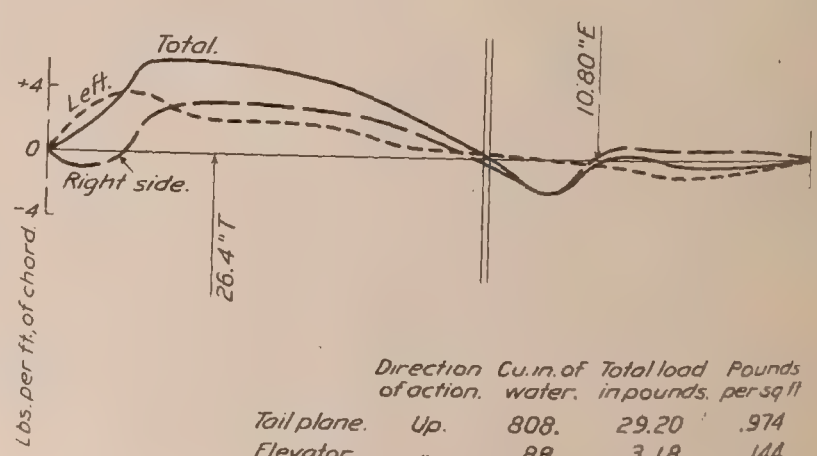
CASE I. 1400 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down	0.0	0.00	.000
Elevator	Up.	664.0	24.00	1.081
Total.	"		24.00	.458

Fig. 127. Elevator moment about hinge = 294 in. lbs.

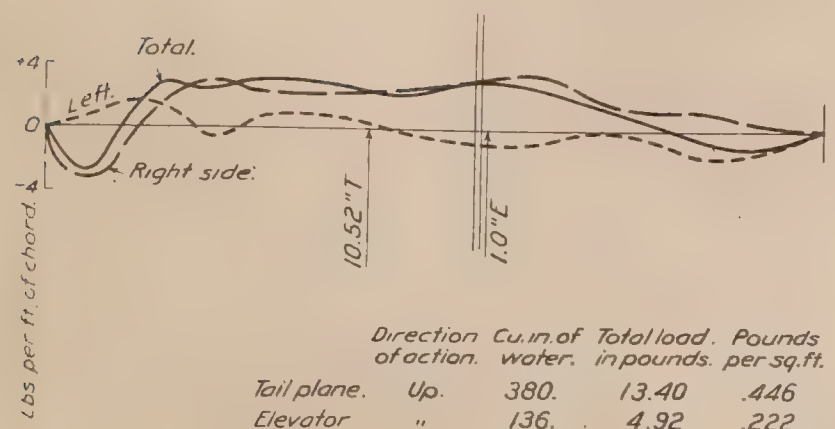
CASE I. 1200 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq ft
Tail plane.	Up.	808.	29.20	.974
Elevator.	"	88.	3.18	.144
Total.	"		32.38	.628

Fig. 128. Elevator moment about hinge = 34.4 in. lbs.

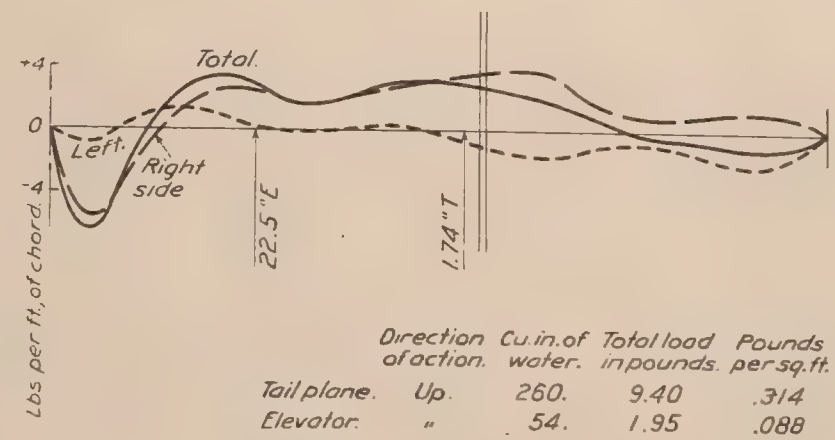
CASE I. 1200 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	380.	13.40	.446
Elevator	"	136.	4.92	.222
Total.	"		18.32	.351

Fig. 129. Elevator moment about hinge = 4.92 in. lbs.

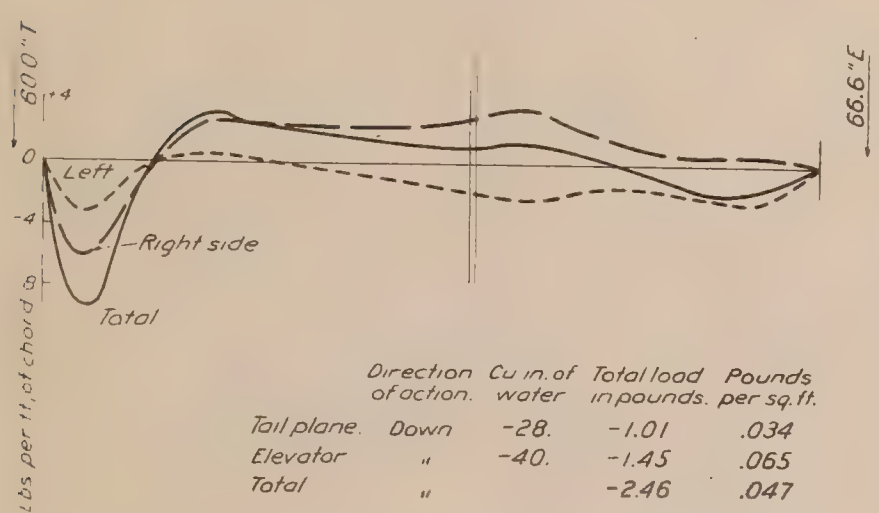
CASE I. 1200 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	260.	9.40	.314
Elevator.	"	54.	1.95	.088
Total	"		11 35	.217

Fig. 130. Elevator moment about hinge = -43.9 in. lbs.

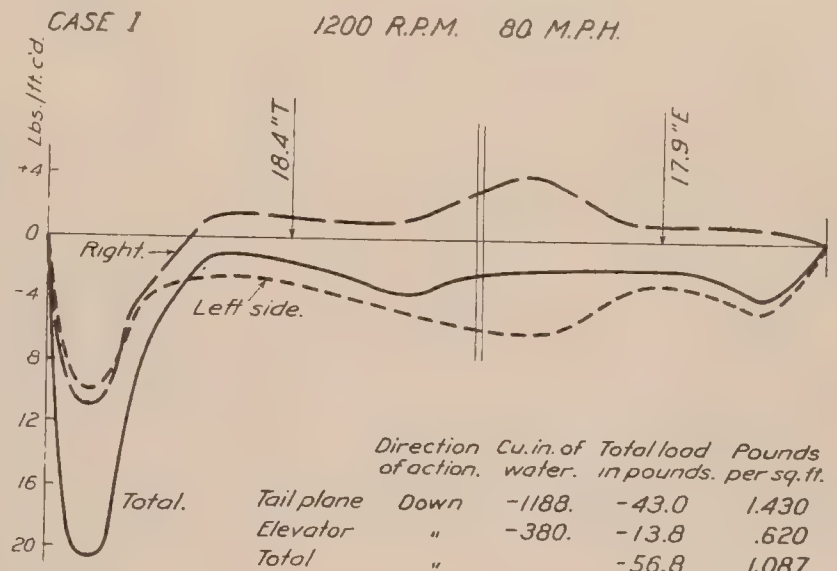
CASE I 1200 R.P.M. 70 M.P.H



	Direction of action.	Cu. in. of water	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down	-28.	-1.01	.034
Elevator	"	-40.	-1.45	.065
Total	"		-2.46	.047

Fig. 131. Elevator moment about hinge = -96.7 in. lbs.

CASE I 1200 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down	-1188.	-43.0	1.430
Elevator	"	-380.	-13.8	.620
Total	"		-56.8	1.087

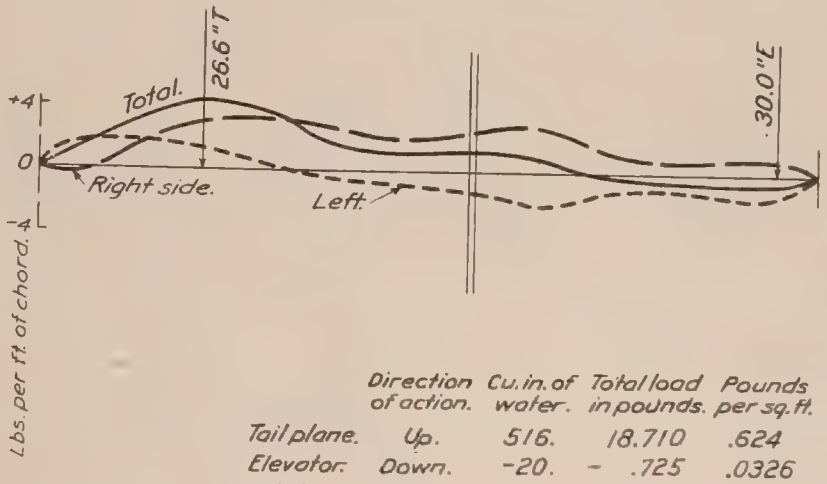
Fig. 132 Elevator moment about hinge = -147.0 in. lbs.

CASE I. 900 R.P.M. 45 M.P.H.

	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	122.	4.42	.147
Elevator.	Down.	-462.	-16.70	.753
Total.	"		-12.28	.235

Fig. 133. Elevator moment about hinge = -309.0 in. lbs.

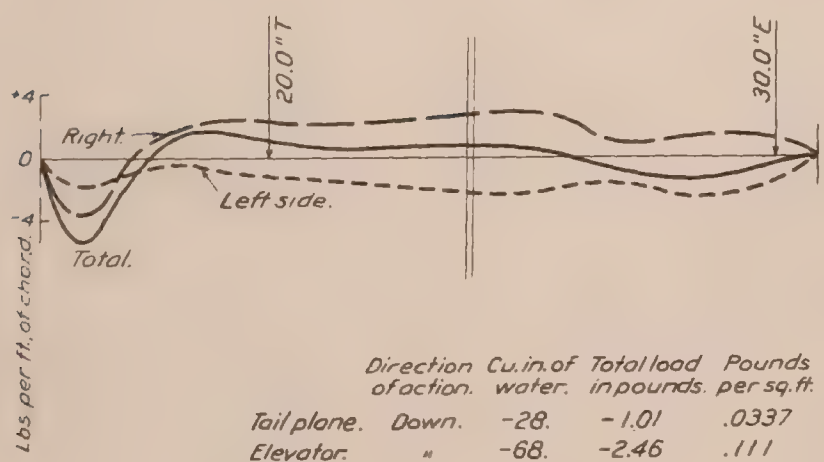
CASE I. 900 R.P.M. 50 M.P.H



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	516.	18.710	.624
Elevator.	Down.	-20.	- .725	.0326
Total.	Up.		17.985	.344

Fig. 134. Elevator moment about hinge = -2.175 in. lbs.

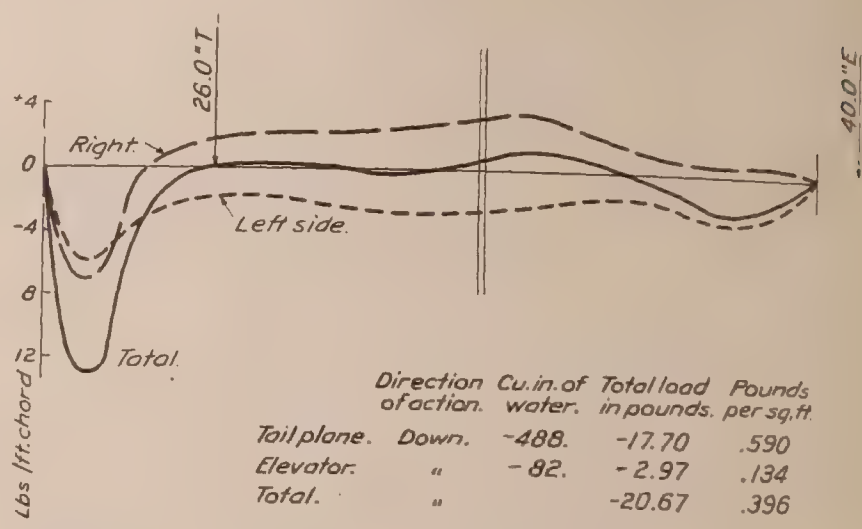
CASE I 900 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-28.	-1.01	.0337
Elevator.	"	-68.	-2.46	.111
Total.	"		-3.47	.0665

Fig. 135. Elevator moment about hinge = -73.8 in. lbs.

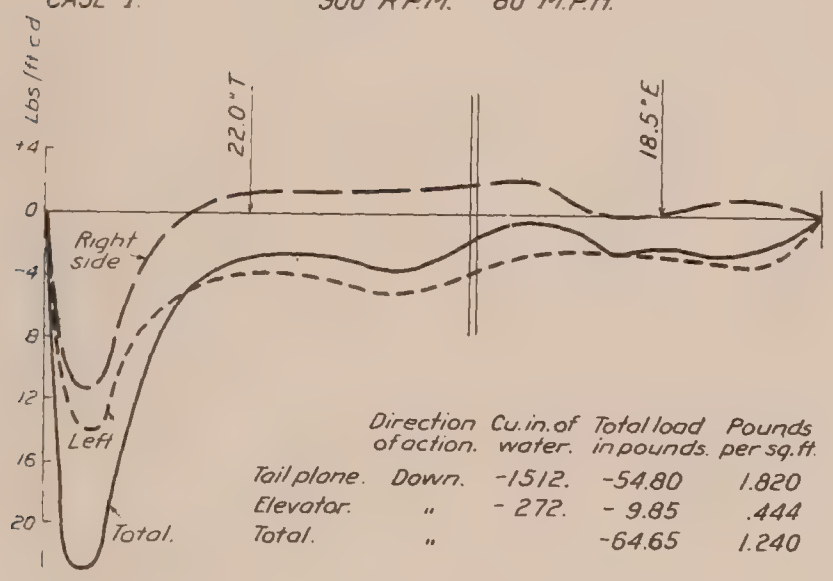
CASE I 900 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-488.	-17.70	.590
Elevator.	"	-82.	-2.97	.134
Total.	"		-20.67	.396

Fig. 136. Elevator moment about hinge = -119. in. lbs.

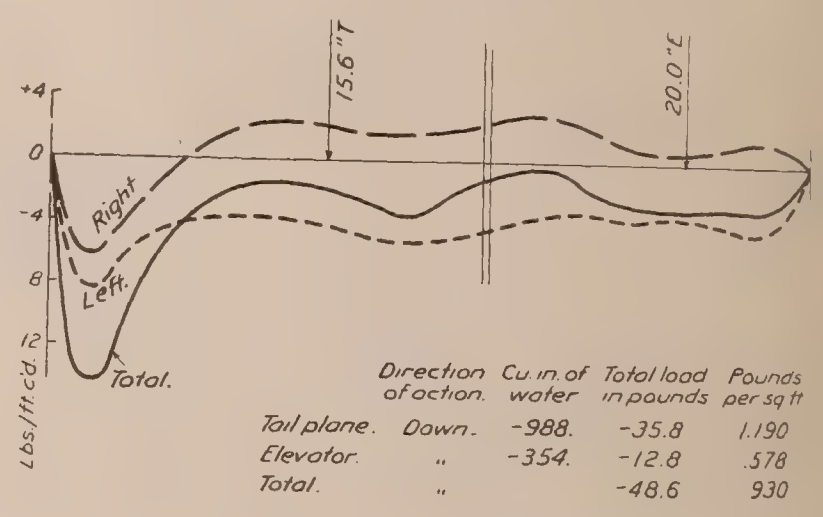
CASE I. 900 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1512.	-54.80	1.820
Elevator.	"	-272.	-9.85	.444
Total.	"		-64.65	1.240

Fig 137 Elevator moment about hinge = -182. in. lbs.

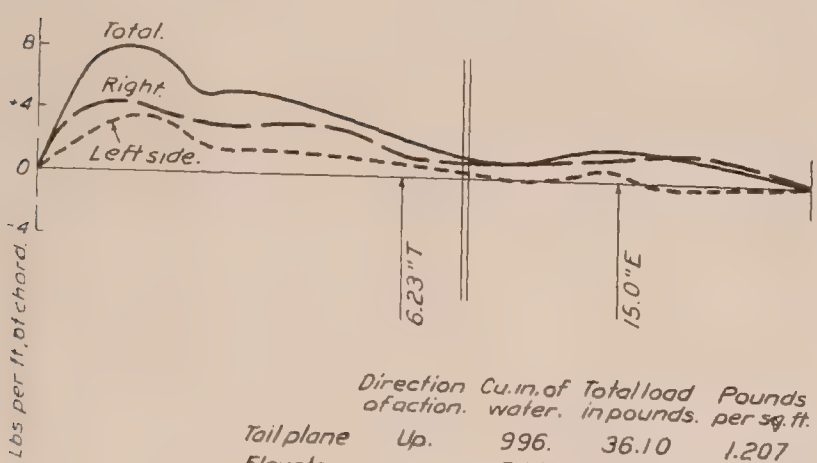
CASE I. 900 R.P.M. 90 M.P.H.



	Direction of action.	Cu. in. of water	Total load in pounds	Pounds per sq ft
Tail plane.	Down.	-988.	-35.8	1.190
Elevator.	"	-354.	-12.8	.578
Total.	"		-48.6	930

Fig. 138 Elevator moment about hinge = -256. in. lbs.

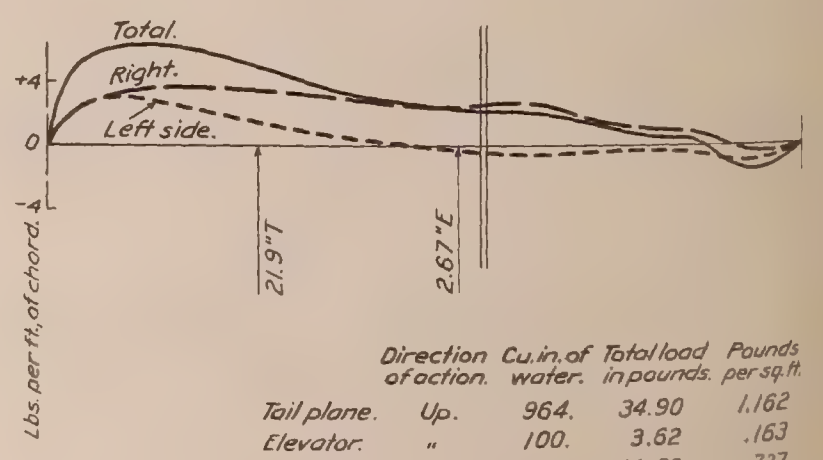
CASE I 600 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	996.	36.10	1.207
Elevator.	"	244.	8.84	.398
Total.	"		44.94	.860

Fig. 139. Elevator moment about hinge = 132.6 in. lbs.

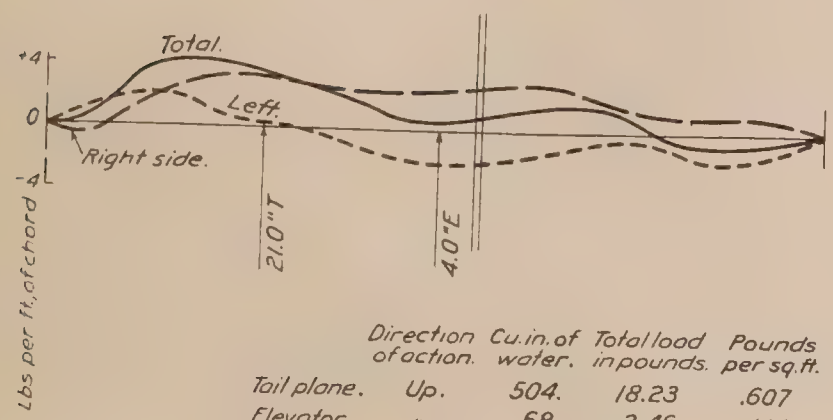
CASE I. 600 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	964.	34.90	1.162
Elevator.	"	100.	3.62	.163
Total.	"		38.52	.737

Fig. 140. Elevator moment about hinge = -9.66 in. lbs.

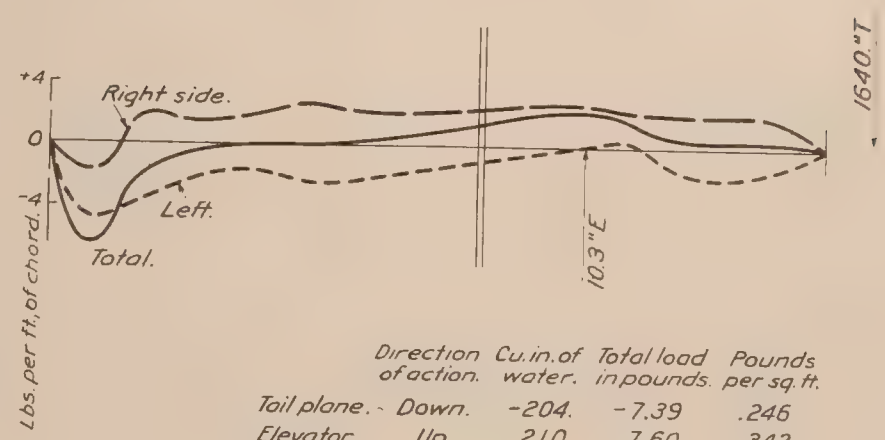
CASE I. 600 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	504.	18.23	.607
Elevator.	"	68.	2.46	.111
Total.	"		20.69	.396

Fig. 141. Elevator moment about hinge = -9.85 in. lbs.

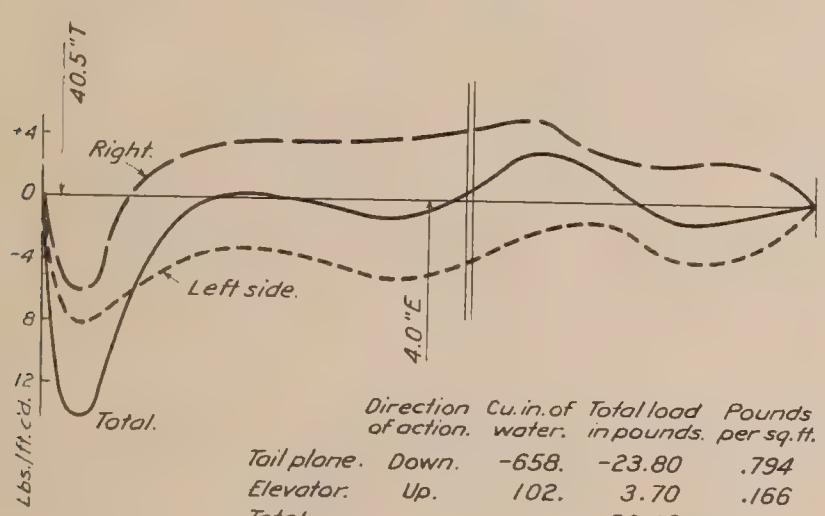
CASE I. 600 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-204.	-7.39	.246
Elevator.	Up.	210.	7.60	.342
Total.	"		.21	.0040

Fig. 142. Elevator moment about hinge = 78.3 in. lbs.

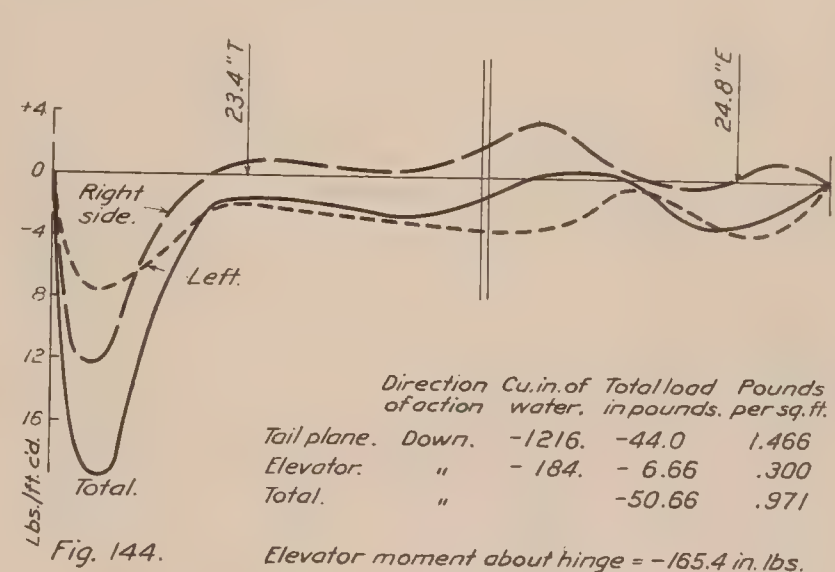
CASE I. 600 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-658.	-23.80	.794
Elevator.	Up.	102.	3.70	.166
Total.	"		-20.10	.576

Fig. 143. Elevator moment about hinge = -14.8 in. lbs.

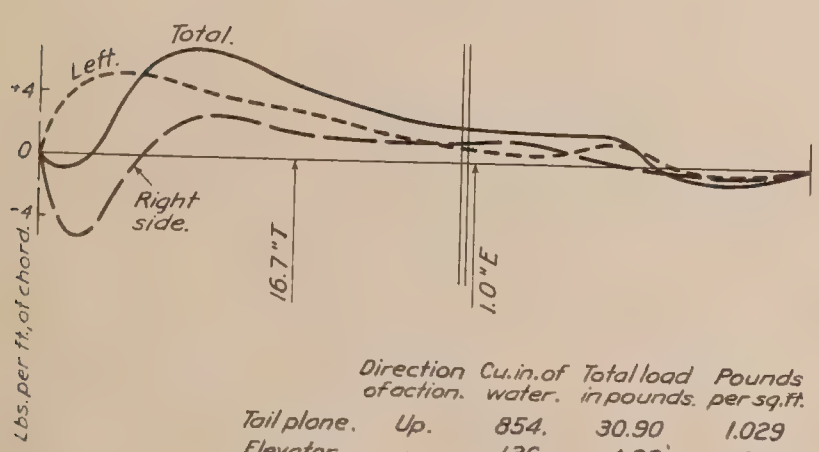
CASE I. 600 R.P.M. 100 M.P.H.



	Direction of action	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1216.	-44.0	1.466
Elevator.	"	- 184.	- 6.66	.300
Total.	"		-50.66	.971

Fig. 144. Elevator moment about hinge = -165.4 in. lbs.

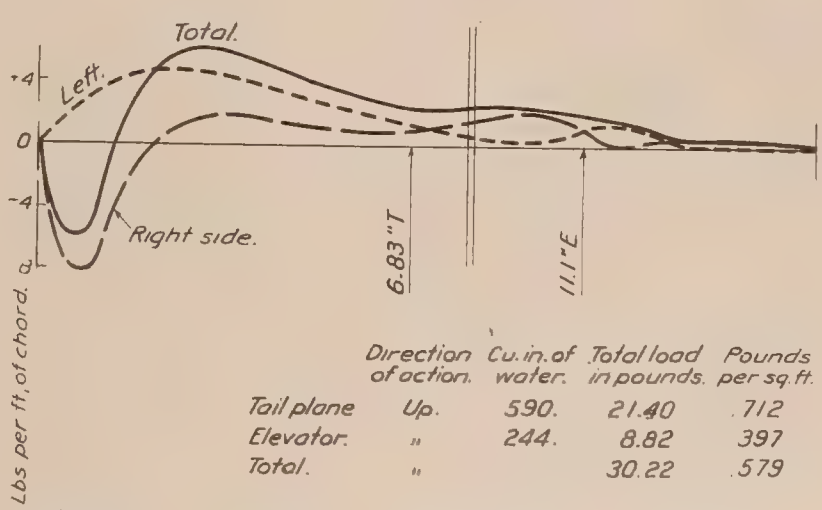
CASE II. 1400 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	854.	30.90	1.029
Elevator.	"	136.	4.92	.0221
Total.	"		35.82	.686

Fig. 145. Elevator moment about hinge = 4.92 in. lbs.

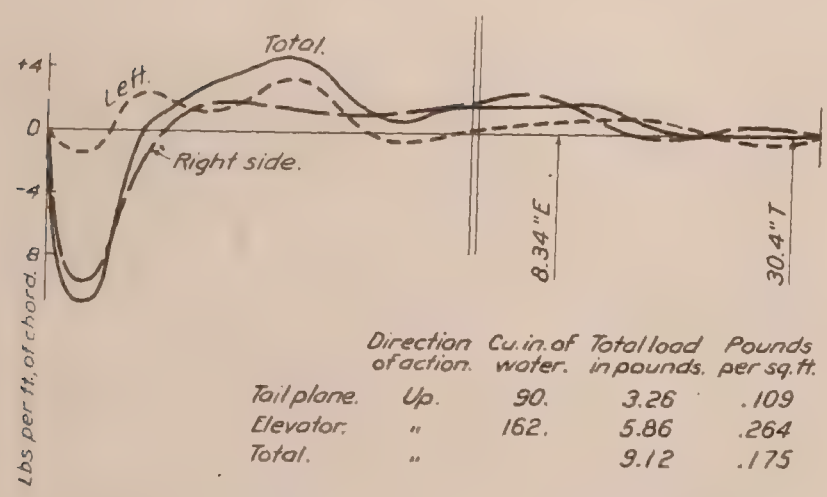
CASE II. 1400 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	590.	21.40	.712
Elevator.	"	244.	8.82	.397
Total.	"		30.22	.579

Fig. 146. Elevator moment about hinge = 98. in. lbs.

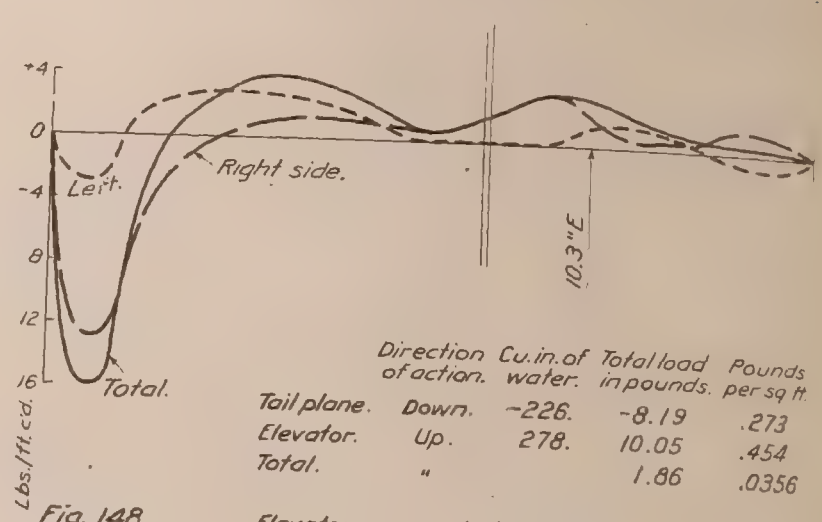
CASE II. 1400 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	90.	3.26	.109
Elevator.	"	162.	5.86	.264
Total.	"		9.12	.175

Fig. 147. Elevator moment about hinge = 48.9 in. lbs.

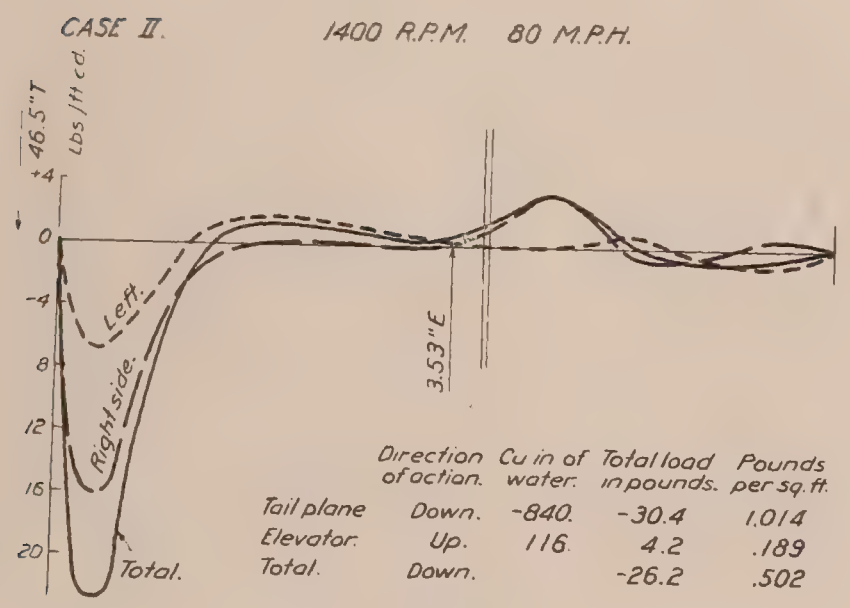
CASE II. 1400 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-226.	-8.19	.273
Elevator.	Up.	278.	10.05	.454
Total.	"		1.86	.0356

Fig. 148. Elevator moment about hinge = 103.6 in. lbs.

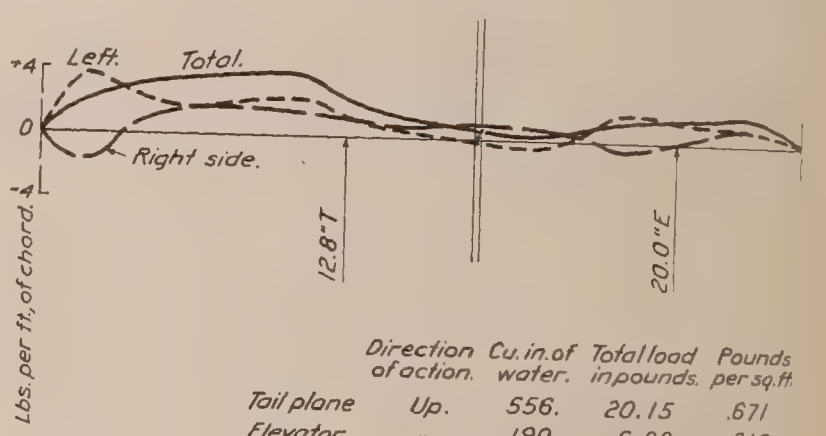
CASE II. 1400 R.P.M. 80 M.P.H.



	Direction of action.	Cu in of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down.	-840.	-30.4	1.014
Elevator.	Up.	116.	4.2	.189
Total.	Down.		-26.2	.502

Fig. 149. Elevator moment about hinge = -14.8 in. lbs.

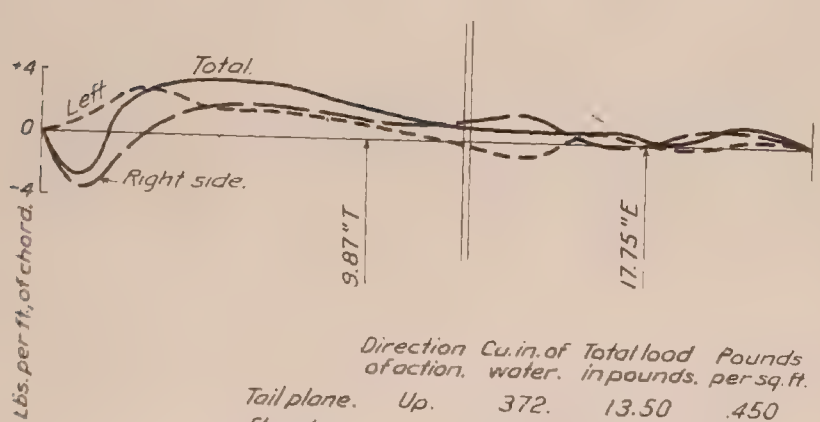
CASE II. 1200 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	556.	20.15	.671
Elevator.	"	190.	6.88	.310
Total.	"		27.03	.517

Fig. 150. Elevator moment about hinge = 137.8 in. lbs.

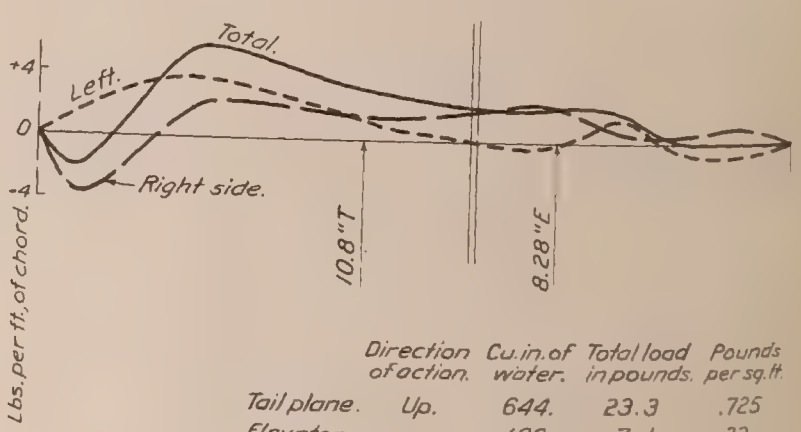
CASE II. 1200 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	372.	13.50	.450
Elevator.	"	122.	4.42	.199
Total.	"		17.92	.344

Fig. 151. Elevator moment about hinge = 78.6 in. lbs.

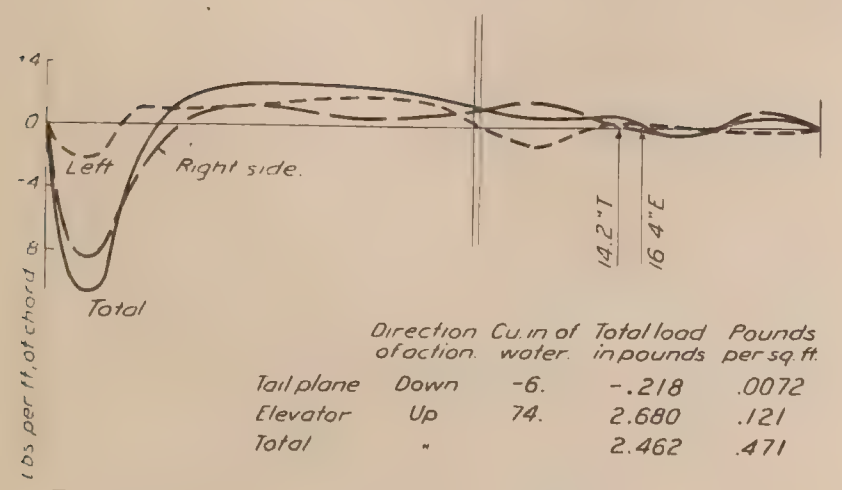
CASE II. 1200 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	644.	23.3	.725
Elevator.	"	196.	7.1	.32
Total.	"		30.4	.582

Fig. 152. Elevator moment about hinge = 58.8 in. lbs.

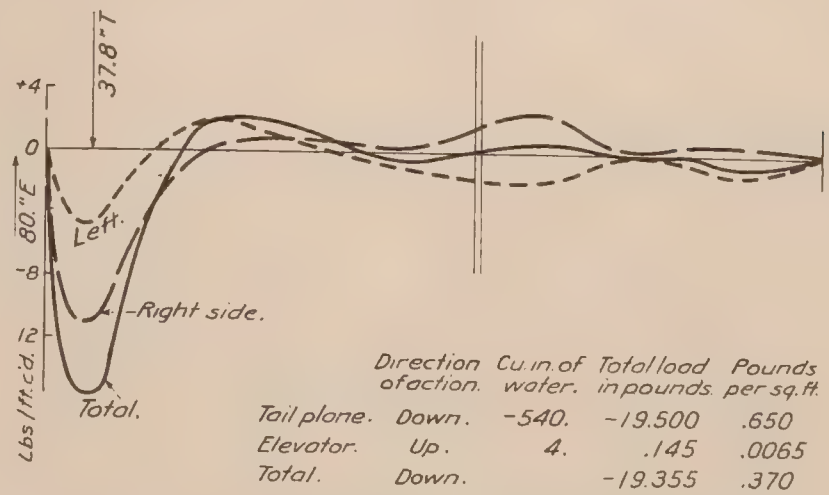
CASE II 1200 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in of water.	Total load in pounds	Pounds per sq. ft.
Tail plane	Down	-6.	-.218	.0072
Elevator	Up	74.	2.680	.121
Total	"		2.462	.471

Fig. 153. Elevator moment about hinge = 40.4 in. lbs.

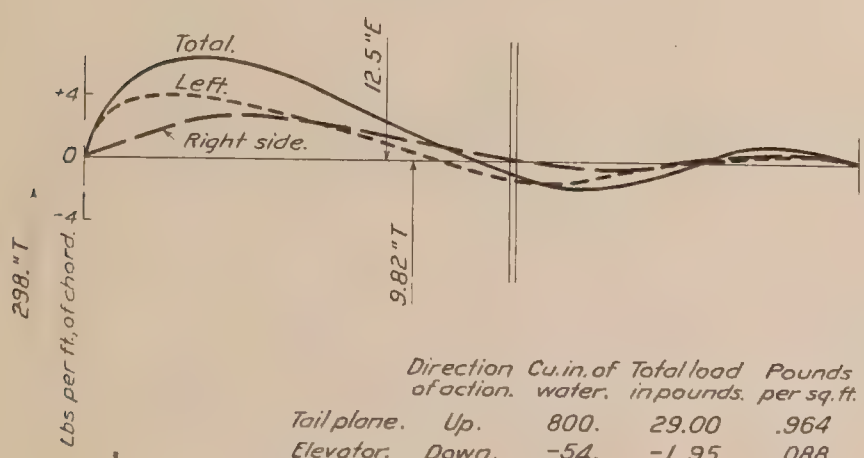
CASE II. 1200 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-540.	-19.500	.650
Elevator.	Up.	4.	.145	.0065
Total.	Down.		-19.355	.370

Fig. 154. Elevator moment about hinge = 11.6 in lbs

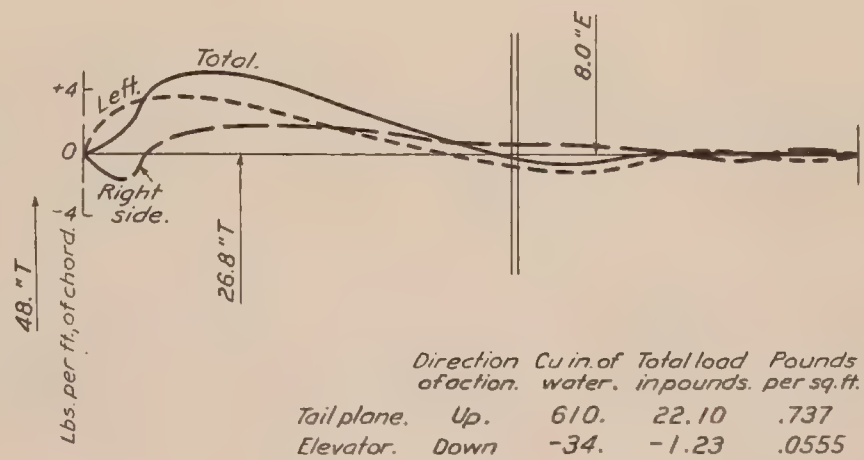
CASE II. 900 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	800.	29.00	.964
Elevator.	Down.	-54.	-1.95	.088
Total.	Up.		27.05	.518

Fig. 155. Elevator moment about hinge = -24.4 in. lbs.

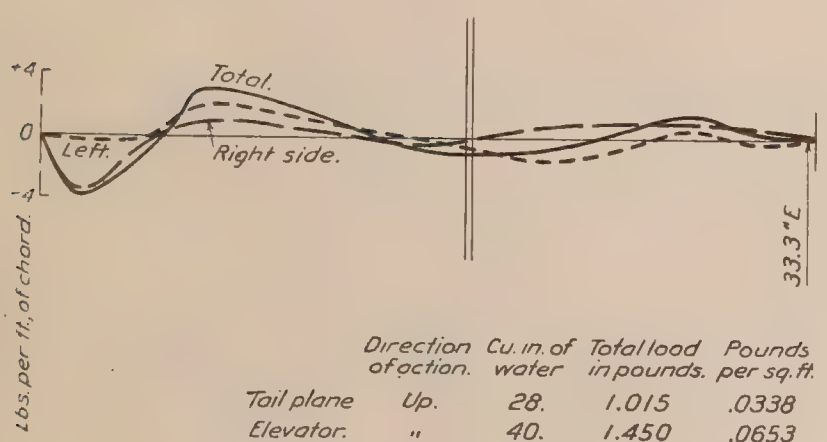
CASE II. 900 R.P.M. 50 M.P.H.



	Direction of action.	Cu in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	610.	22.10	.737
Elevator.	Down	-34.	-1.23	.0555
Total.	Up.		20.87	.400

Fig. 156. Elevator moment about hinge = -9.85 in. lbs.

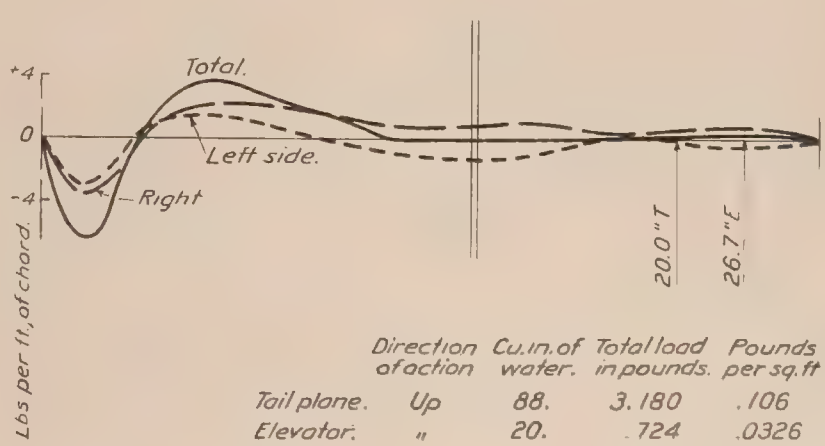
CASE II. 900 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	28.	1.015	.0338
Elevator.	"	40.	1.450	.0653
Total.	"		2.465	0471

Fig. 157. Elevator moment about hinge = 48.4 in. lbs

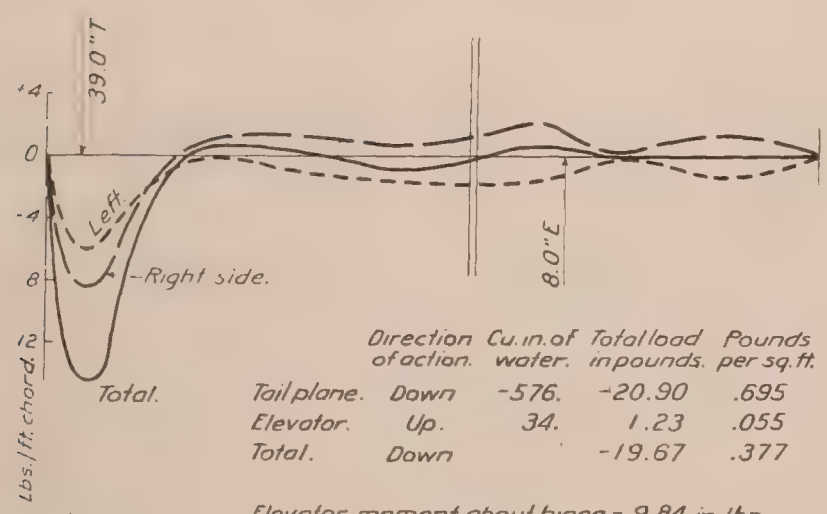
CASE II. 900 R.P.M. 70 M.P.H.



	Direction of action	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft
Tail plane.	Up	88.	3.180	.106
Elevator.	"	20.	.724	.0326
Total.	"		3.904	0746

Fig. 158. Elevator moment about hinge = 19.3 in. lbs.

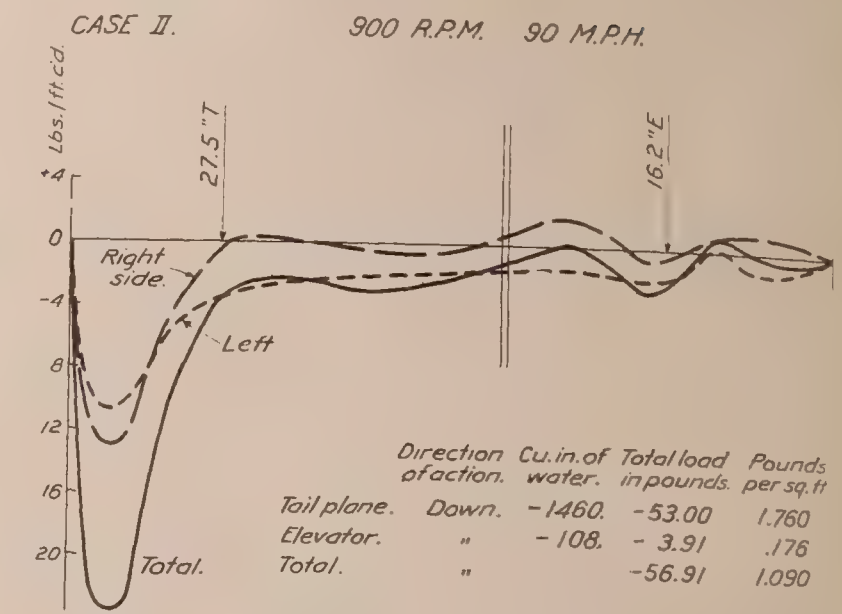
CASE II 900 R.P.M. 80 M.P.H



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down	-576.	-20.90	.695
Elevator.	Up.	34.	1.23	.055
Total.	Down		-19.67	.377

Fig. 159. Elevator moment about hinge = 9.84 in. lbs.

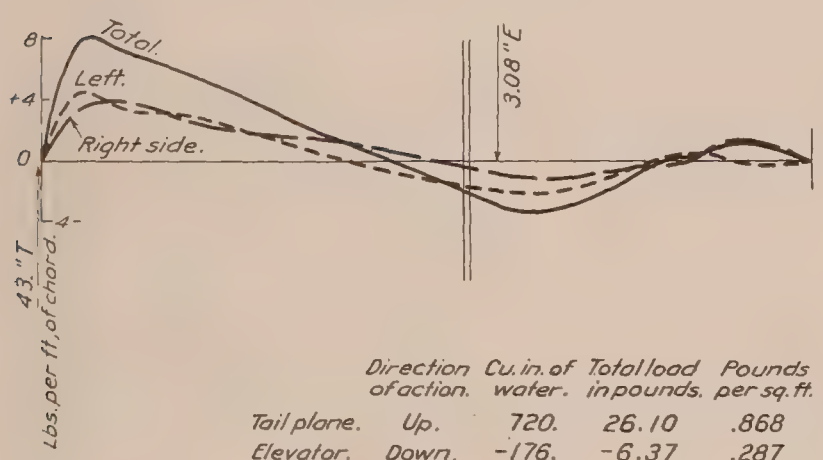
CASE II. 900 R.P.M. 90 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft
Tail plane.	Down.	-1460.	-53.00	1.760
Elevator.	"	-108.	-3.91	.176
Total.	"		-56.91	1.090

Fig. 160. Elevator moment about hinge = -63.4 in. lbs.

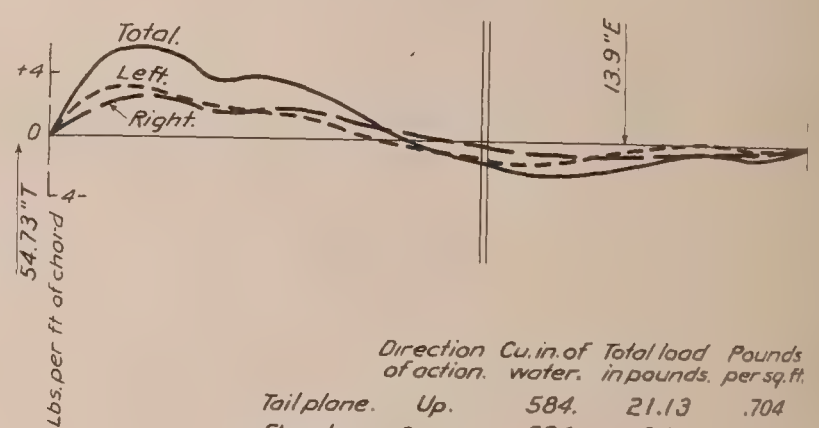
CASE II. 600 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	720.	26.10	.868
Elevator.	Down.	-176.	-6.37	.287
Total.	Up.		19.73	.378

Fig. 161. Elevator moment about hinge = 19.6 in. lbs.

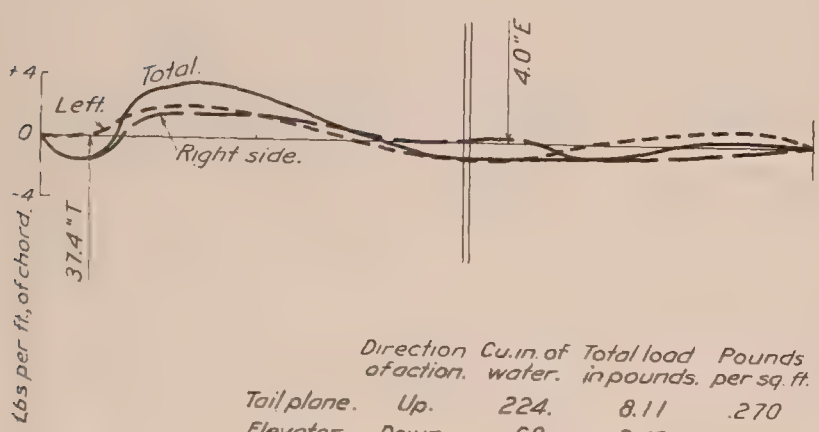
CASE II. 600 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	584.	21.13	.704
Elevator.	Down.	-224.	-8.11	.365
Total.	Up.		13.02	.249

Fig. 162. Elevator moment about hinge = 113.0 in. lbs.

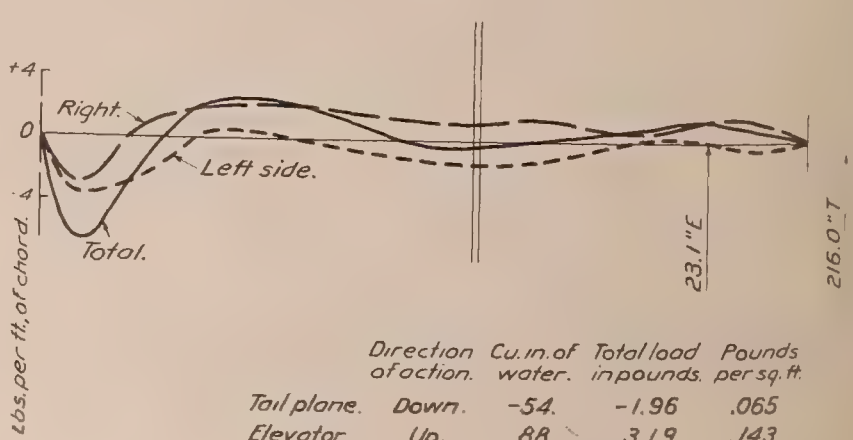
CASE II. 600 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	224.	8.11	.270
Elevator.	Down.	-68.	-2.46	.111
Total.	Up		5.65	.108

Fig. 163. Elevator moment about hinge = -9.85 in. lbs.

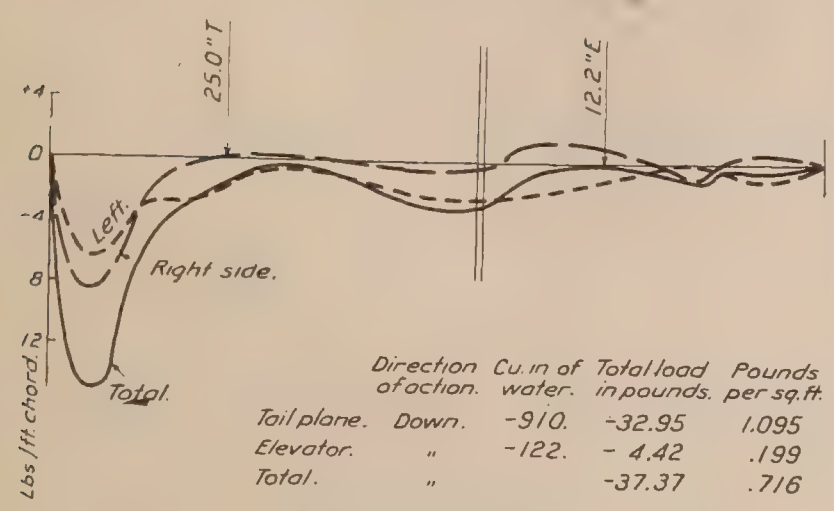
CASE II. 600 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-54.	-1.96	.065
Elevator	Up.	88.	3.19	.143
Total.	"		1.23	.0236

Fig. 164. Elevator moment about hinge = 73.7 in. lbs.

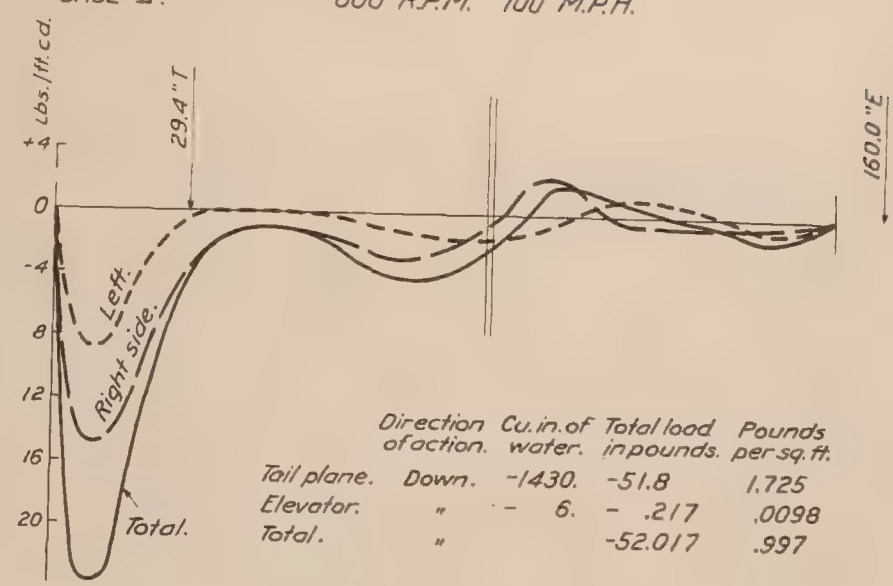
CASE II 600 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-910.	-32.95	1.095
Elevator.	"	-122.	- 4.42	.199
Total.	"		-37.37	.716

Fig. 165. Elevator moment about hinge = -540 in. lbs.

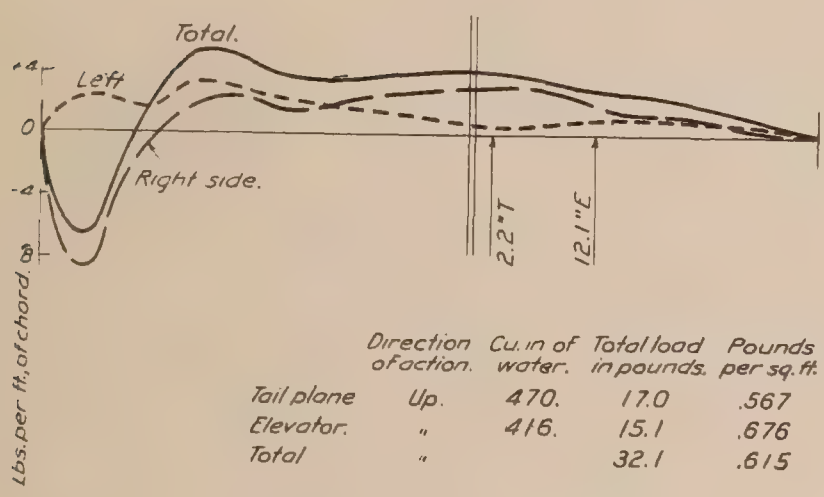
CASE II. 600 R.P.M. 100 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1430.	-51.8	1.725
Elevator.	"	- 6.	- .217	.0098
Total.	"		-52.017	.997

Fig. 166. Elevator moment about hinge = -34.75 in. lbs.

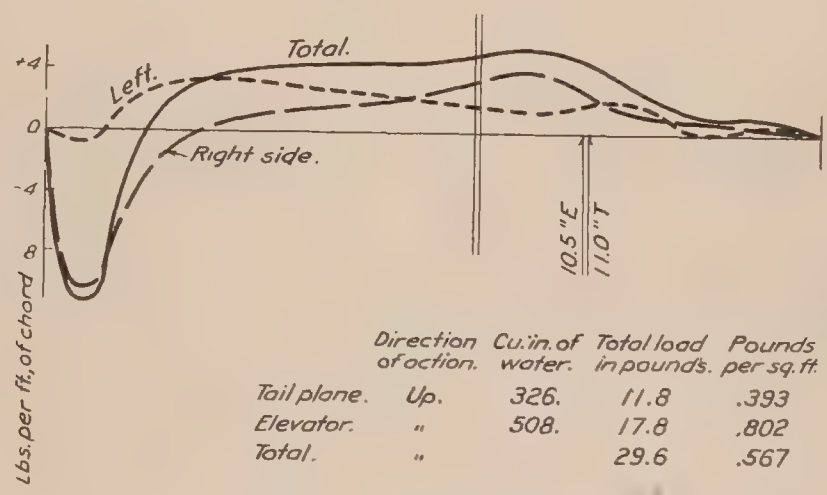
CASE III 1400 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	470.	17.0	.567
Elevator.	"	416.	15.1	.676
Total	"		32.1	.615

Fig. 167. Elevator moment about hinge = 182.7 in. lbs.

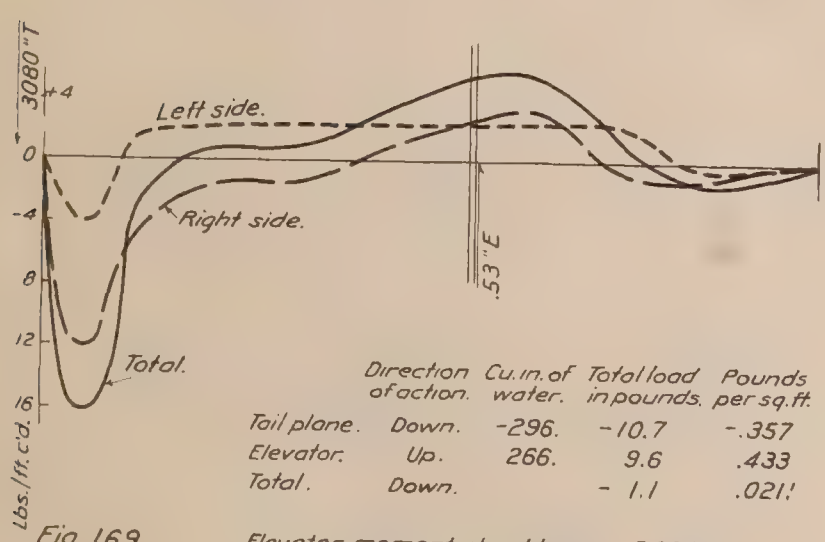
CASE III. 1400 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	326.	11.8	.393
Elevator.	"	508.	17.8	.802
Total.	"		29.6	.567

Fig. 168. Elevator moment about hinge = 187 in. lbs.

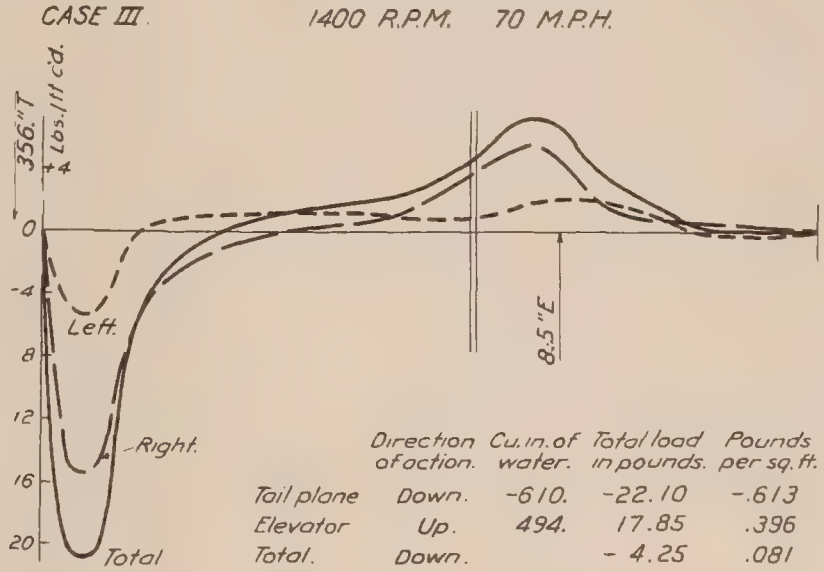
CASE III. 1400 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-296.	-10.7	-.357
Elevator.	Up.	266.	9.6	.433
Total.	Down.		- 1.1	.021

Fig. 169. Elevator moment about hinge = 5.08 in. lbs.

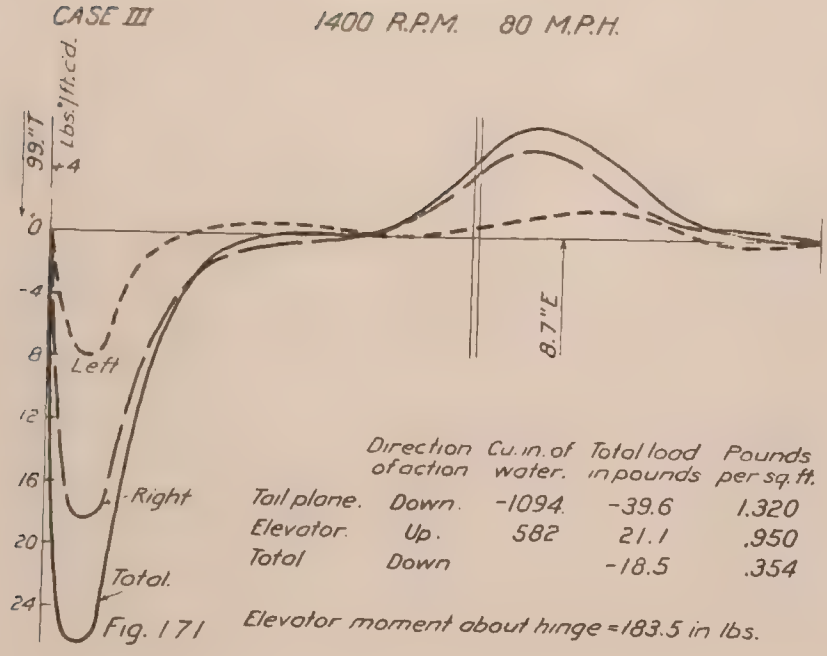
CASE III. 1400 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down.	-610.	-22.10	-.613
Elevator	Up.	494.	17.85	.396
Total.	Down.		- 4.25	.081

Fig. 170 Elevator moment about hinge = 152. in. lbs.

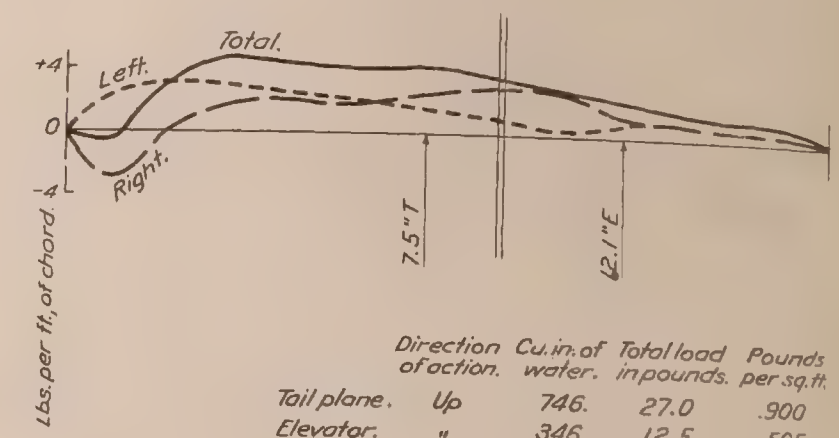
CASE III 1400 R.P.M. 80 M.P.H.



	Direction of action	Cu. in. of water.	Total load in pounds	Pounds per sq. ft.
Tail plane.	Down.	-1094.	-39.6	1.320
Elevator.	Up.	582	21.1	.950
Total	Down		-18.5	.354

Fig. 171 Elevator moment about hinge = 183.5 in lbs.

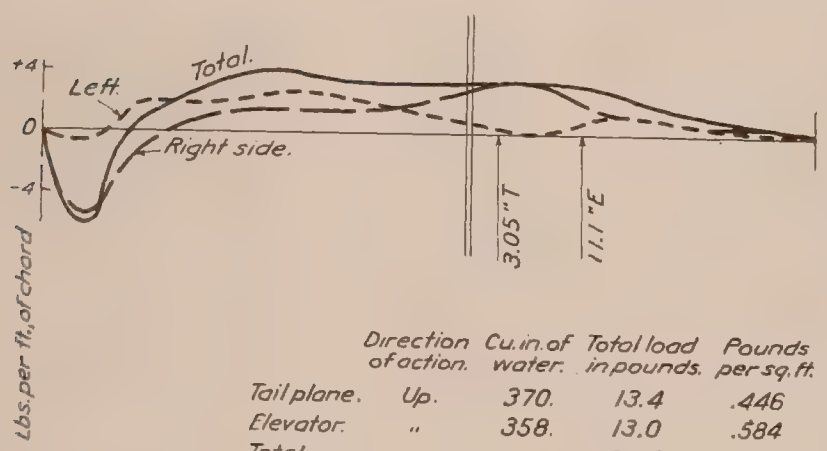
CASE III. 1200 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up	746.	27.0	.900
Elevator.	"	346.	12.5	.565
Total.	"		39.5	.756

Fig. 172. Elevator moment about hinge = 151.3 in. lbs.

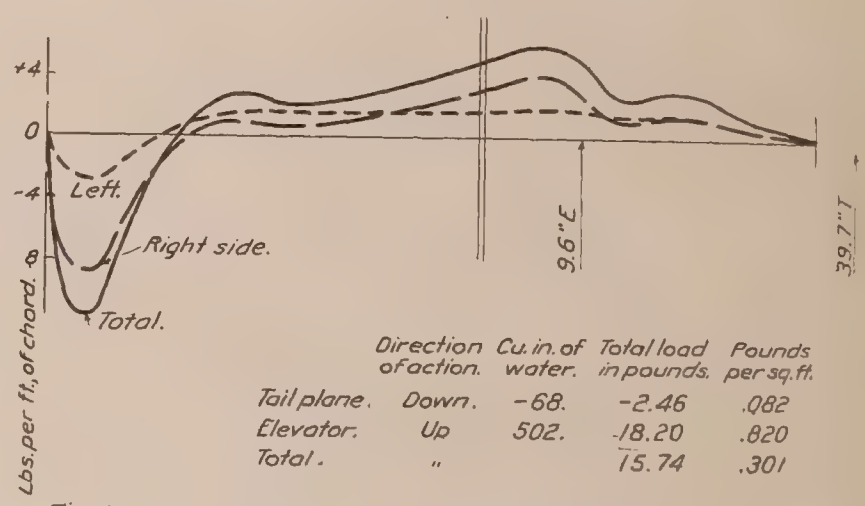
CASE III. 1200 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	370.	13.4	.446
Elevator.	"	358.	13.0	.584
Total.	"		26.4	.505

Fig. 173. Elevator moment about hinge = 144.3 in. lbs.

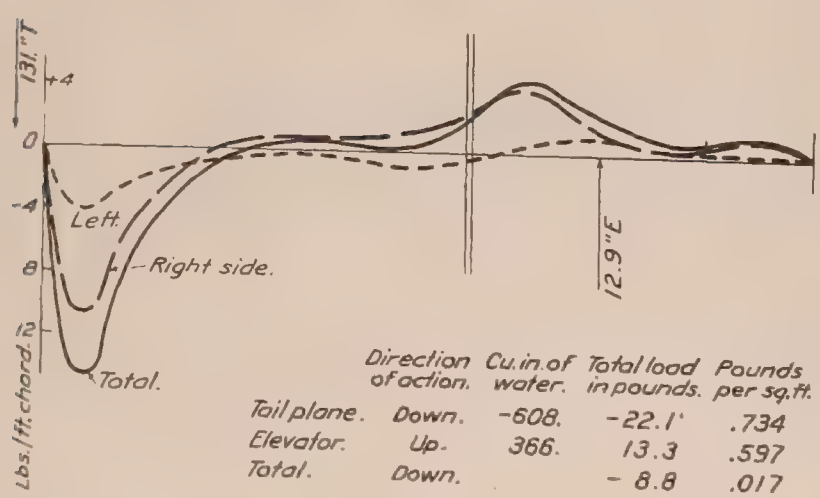
CASE III. 1200 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-68.	-2.46	.082
Elevator.	Up	502.	-18.20	.820
Total.	"		15.74	.301

Fig. 174. Elevator moment about hinge = 175.0 in. lbs.

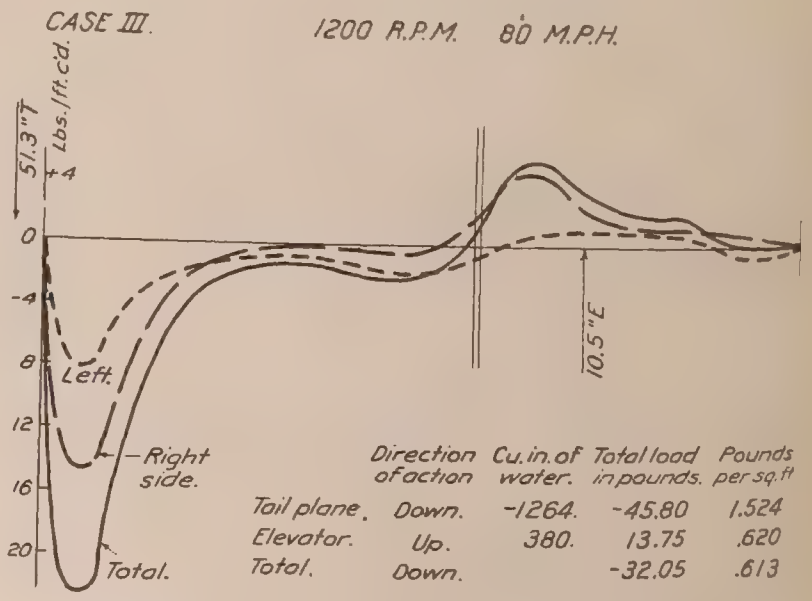
CASE III 1200 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-608.	-22.1	.734
Elevator.	Up.	366.	13.3	.597
Total.	Down.		- 8.8	.017

Fig. 175. Elevator moment about hinge = 171.6 in. lbs.

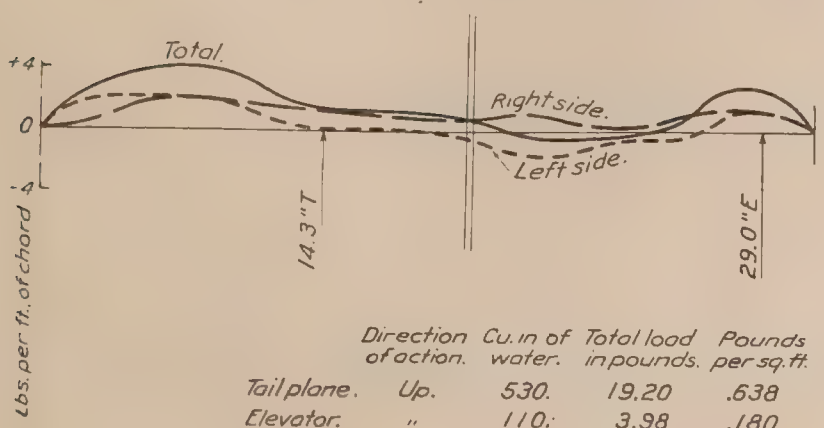
CASE III. 1200 R.P.M. 80 M.P.H.



	Direction of action	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft
Tail plane.	Down.	-1264.	-45.80	1.524
Elevator.	Up.	380.	13.75	.620
Total.	Down.		-32.05	.613

Fig. 176. Elevator moment about hinge = 144.4 in. lbs.

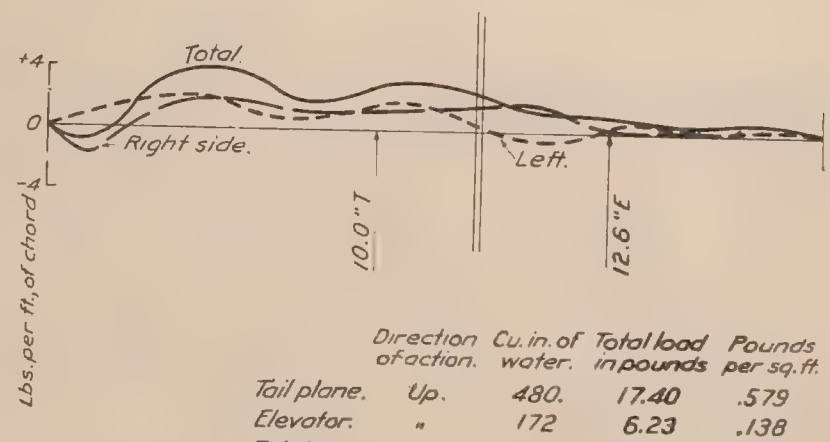
CASE III 900 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	530.	19.20	.638
Elevator.	"	110.	3.98	.180
Total.	"		23.18	.444

Fig. 177. Elevator moment about hinge = 116.0 in. lbs.

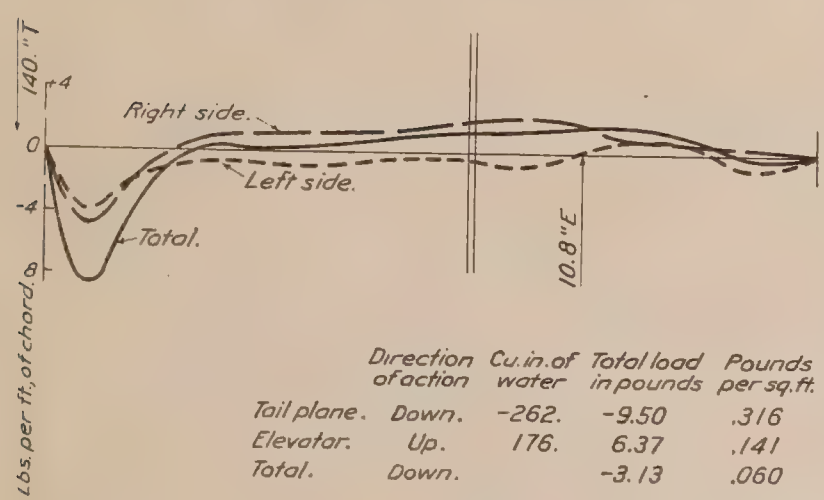
CASE III. 900 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds	Pounds per sq. ft.
Tail plane.	Up.	480.	17.40	.579
Elevator.	"	172	6.23	.138
Total.	"		23.63	.452

Fig. 178. Elevator moment about hinge = 78.5 in. lbs.

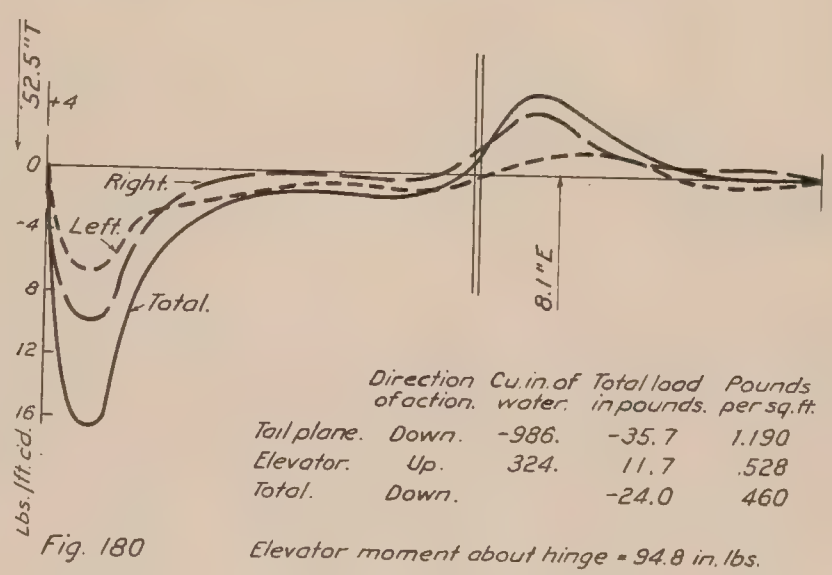
CASE III. 900 R.P.M. 60 M.P.H.



	Direction of action	Cu. in. of water	Total load in pounds	Pounds per sq. ft.
Tail plane.	Down.	-262.	-9.50	.316
Elevator.	Up.	176.	6.37	.141
Total.	Down.		-3.13	.060

Fig. 179. Elevator moment about hinge = 63.6 in. lbs.

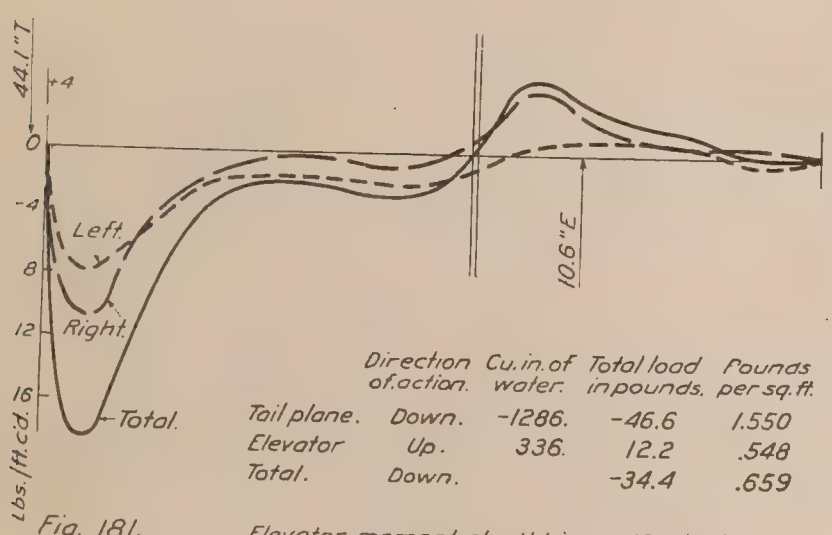
CASE III. 900 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-986.	-35.7	1.190
Elevator.	Up.	324.	11.7	.528
Total.	Down.		-24.0	460

Fig. 180 Elevator moment about hinge = 94.8 in. lbs.

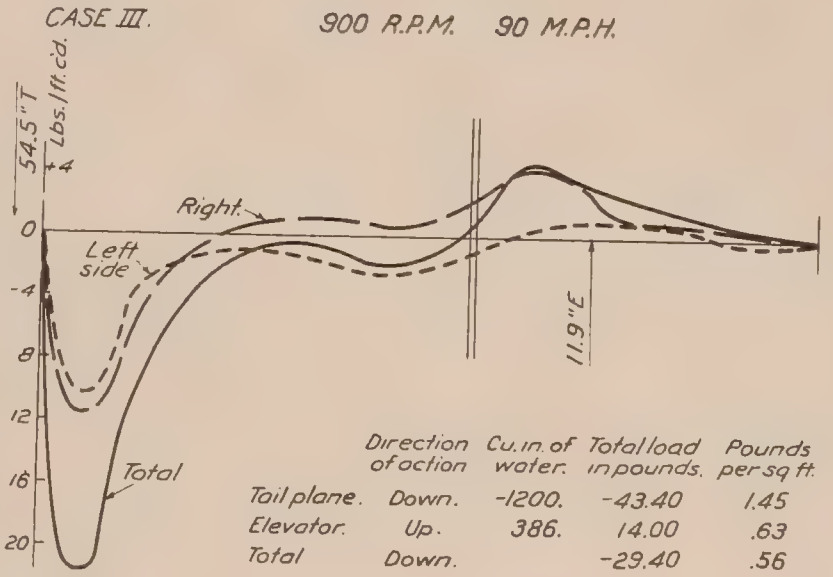
CASE III. 900 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1286.	-46.6	1.550
Elevator	Up.	336.	12.2	.548
Total.	Down.		-34.4	.659

Fig. 181. Elevator moment about hinge = 129. in. lbs.

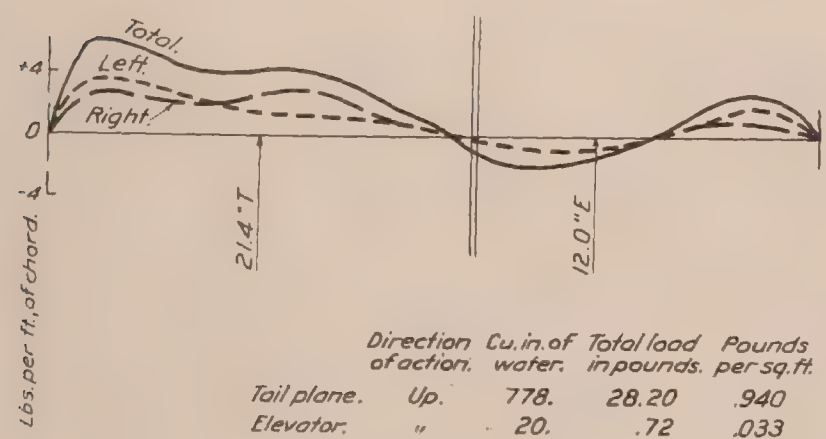
CASE III. 900 R.P.M. 90 M.P.H.



	Direction of action	Cu. in. of water.	Total load in pounds.	Pounds per sq ft.
Tail plane.	Down.	-1200.	-43.40	1.45
Elevator.	Up.	386.	14.00	.63
Total	Down.		-29.40	.56

Fig. 182. Elevator moment about hinge = 166.0 in lbs.

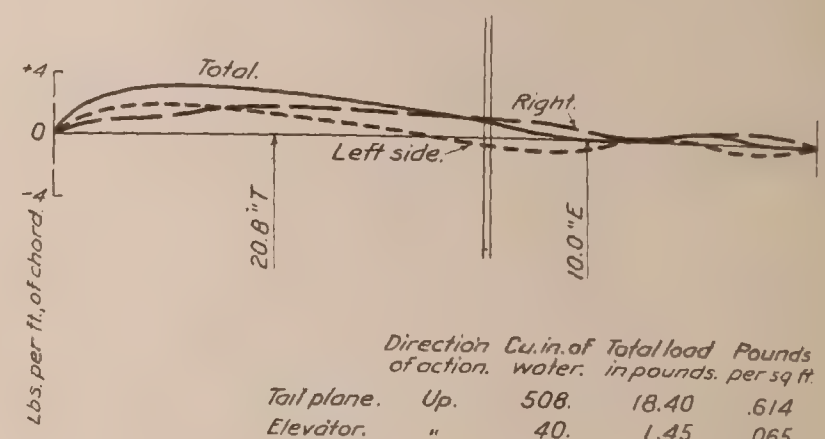
CASE III. 600 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	778.	28.20	.940
Elevator.	"	20.	.72	.033
Total.	"		28.92	.554

Fig. 183. Elevator moment about hinge = 8.64 in. lbs.

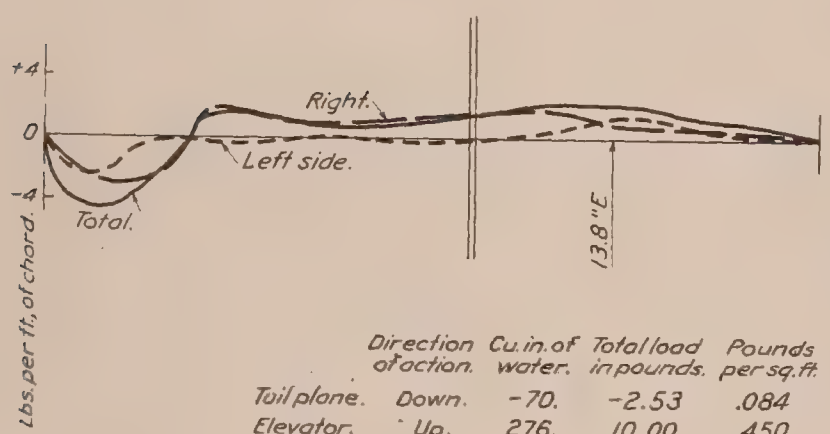
CASE III. 600 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	508.	18.40	.614
Elevator.	"	40.	1.45	.065
Total.	"		19.85	.380

Fig. 184. Elevator moment about hinge = 14.5 in. lbs.

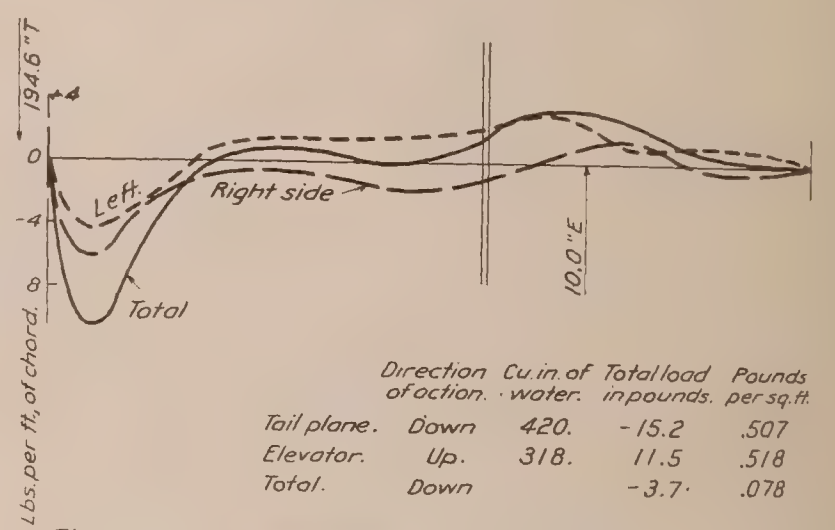
CASE III. 600 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-70.	-2.53	.084
Elevator.	Up.	276.	10.00	.450
Total	"		7.47	.143

Fig. 185. Elevator moment about hinge = 138. in. lbs.

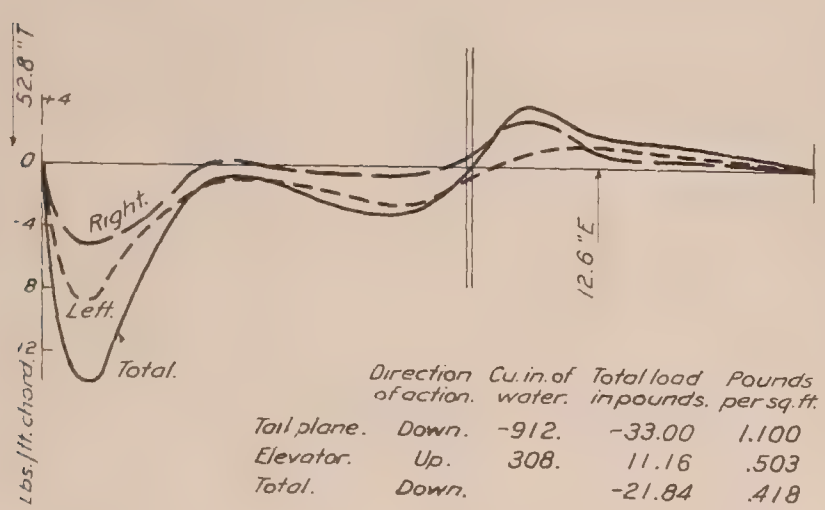
CASE III. 600 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down	420.	-15.2	.507
Elevator.	Up.	318.	11.5	.518
Total.	Down		-3.7	.078

Fig. 186. Elevator moment about hinge = 115. in. lbs.

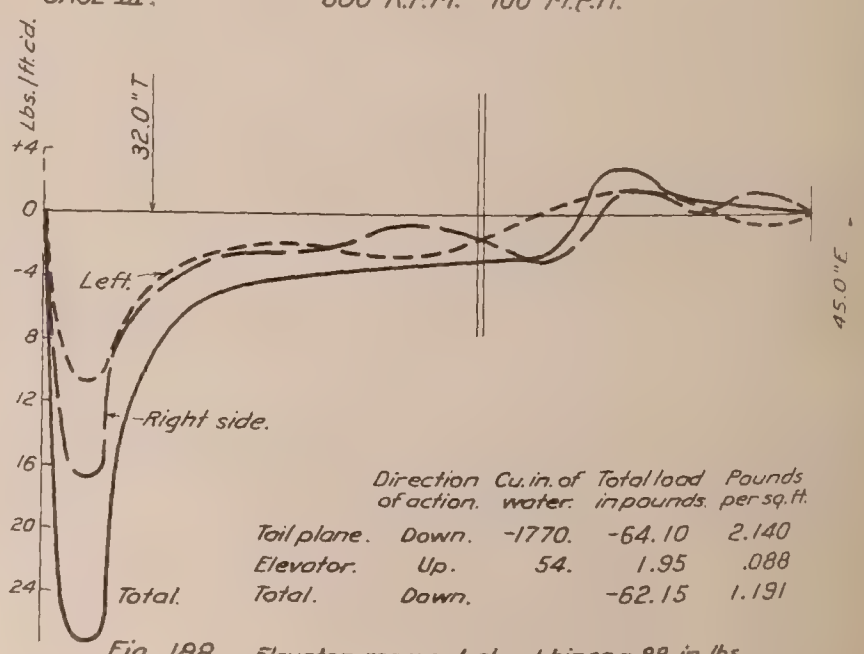
CASE III. 600 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-912.	-33.00	1.100
Elevator.	Up.	308.	11.16	.503
Total.	Down.		-21.84	.418

Fig. 187. Elevator moment about hinge = 140.6 in lbs.

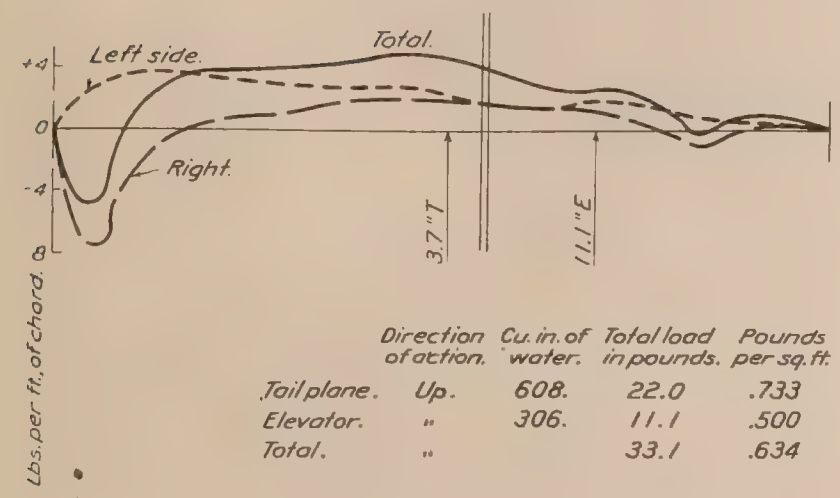
CASE III. 600 R.P.M. 100 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1770.	-64.10	2.140
Elevator.	Up.	54.	1.95	.088
Total.	Down.		-62.15	1.191

Fig. 188. Elevator moment about hinge = 88. in. lbs.

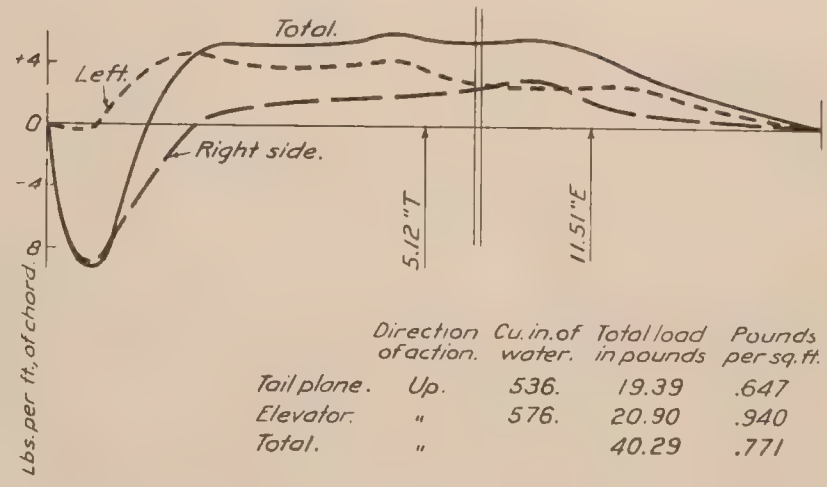
CASE IV. 1400 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	608.	22.0	.733
Elevator.	"	306.	11.1	.500
Total.	"		33.1	.634

Fig. 189. Elevator moment about hinge = 123.2 in. lbs.

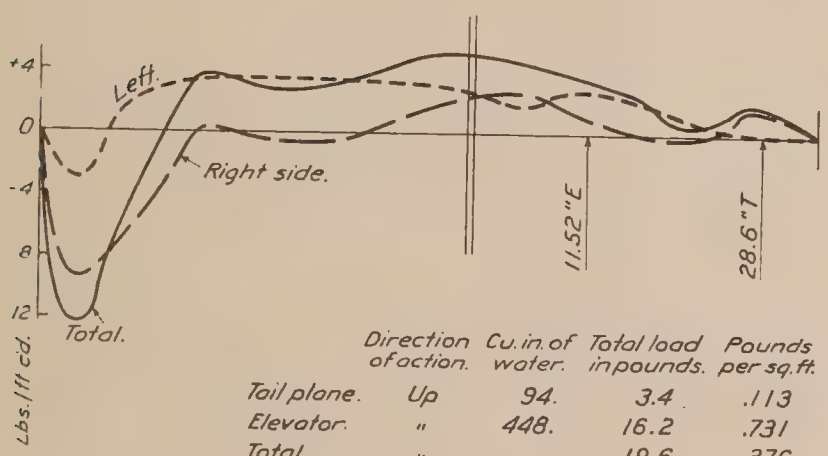
CASE IV. 1400 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds	Pounds per sq. ft.
Tail plane.	Up.	536.	19.39	.647
Elevator.	"	576.	20.90	.940
Total.	"		40.29	.771

Fig. 190. Elevator moment about hinge = 241.0 in. lbs.

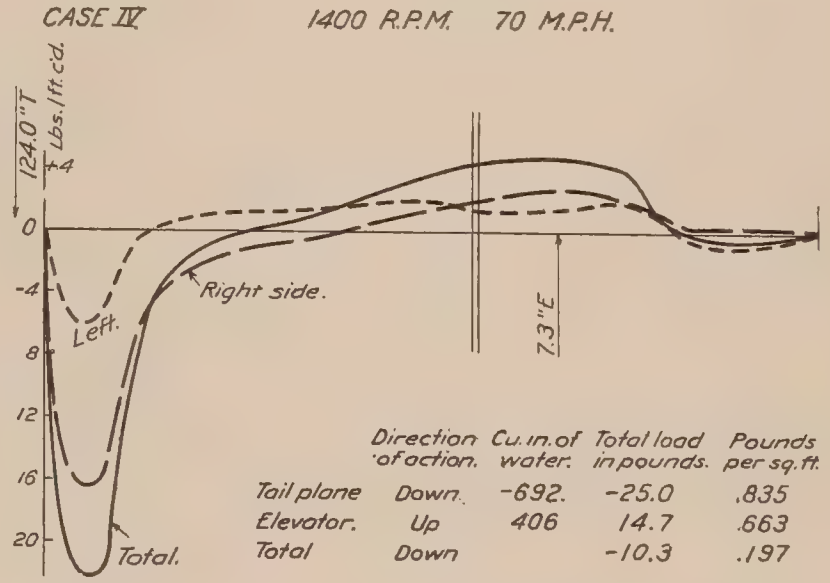
CASE IV. 1400 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up	94.	3.4	.113
Elevator.	"	448.	16.2	.731
Total.	"		19.6	.376

Fig. 191 Elevator moment about hinge = 187.0 in. lbs.

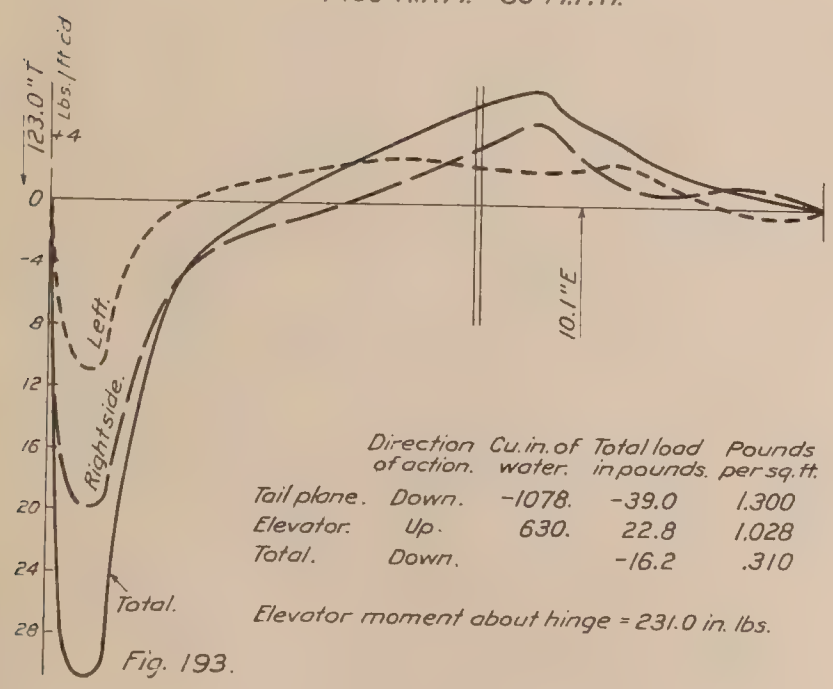
CASE IV. 1400 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down.	-692.	-25.0	.835
Elevator.	Up	406	14.7	.663
Total	Down		-10.3	.197

Fig. 192. Elevator moment about hinge = 107.5 in. lbs.

CASE IV. 1400 R.P.M. 80 M.P.H.

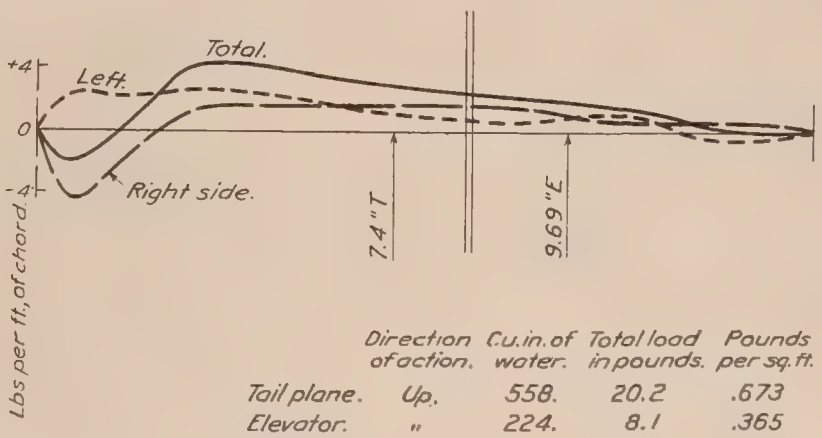


	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1078.	-39.0	1.300
Elevator.	Up.	630.	22.8	1.028
Total.	Down.		-16.2	.310

Elevator moment about hinge = 231.0 in. lbs.

Fig. 193.

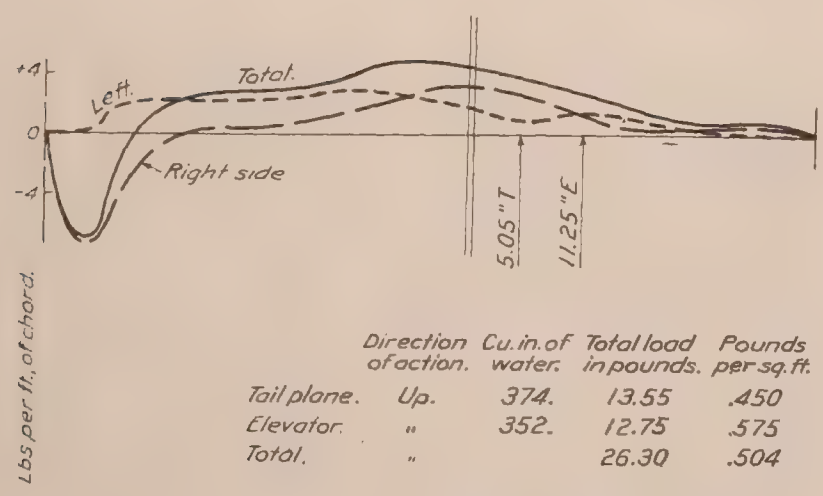
CASE IV. 1200 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	558.	20.2	.673
Elevator.	"	224.	8.1	.365
Total.	"		28.3	.542

Fig. 194. Elevator moment about hinge = 78.5 in. lbs.

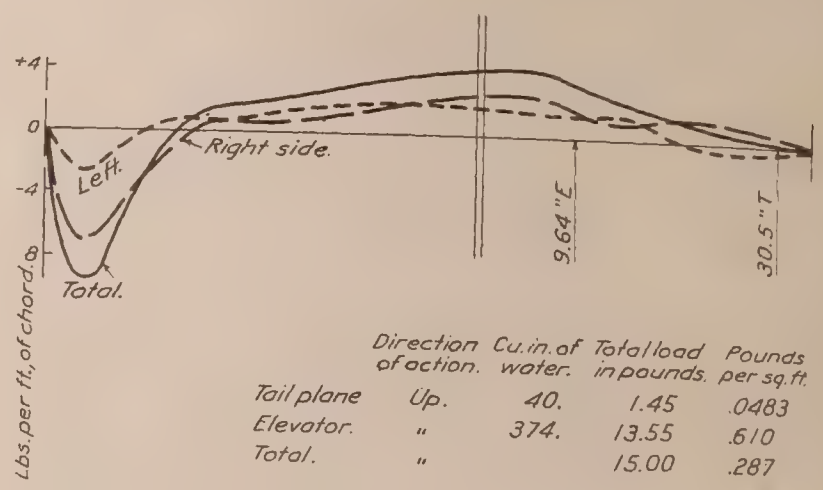
CASE IV. 1200 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	374.	13.55	.450
Elevator.	"	352.	12.75	.575
Total.	"		26.30	.504

Fig. 195. Elevator moment about hinge = 143.5 in. lbs.

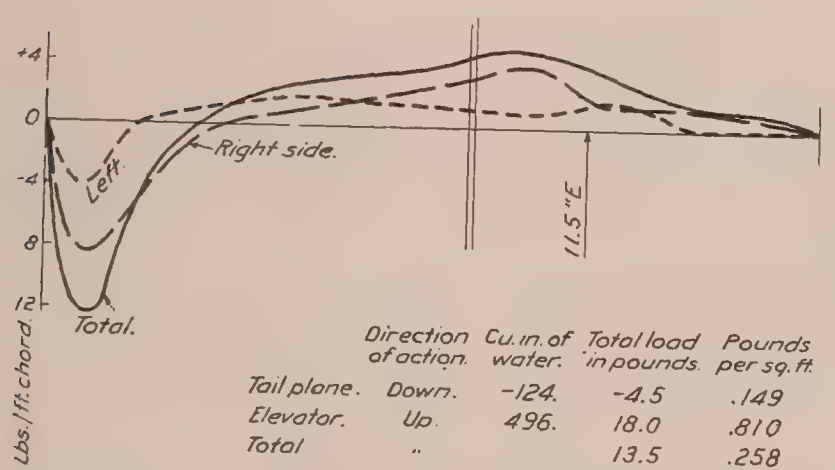
CASE IV. 1200 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	40.	1.45	.0483
Elevator.	"	374.	13.55	.610
Total.	"		15.00	.287

Fig. 196 Elevator moment about hinge = 130.5 in. lbs.

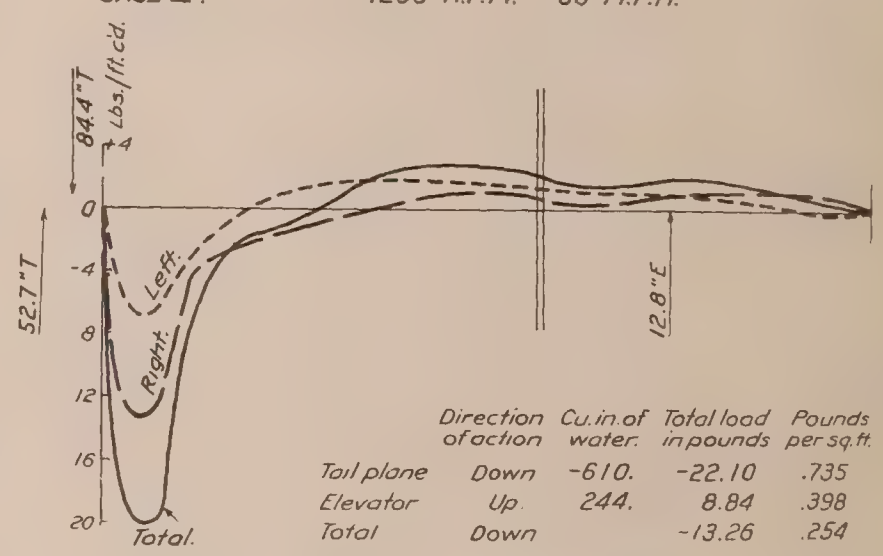
CASE IV. 1200 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-124.	-4.5	.149
Elevator.	Up.	496.	18.0	.810
Total	"		13.5	.258

Fig. 197. Elevator moment about hinge = 207. in. lbs.

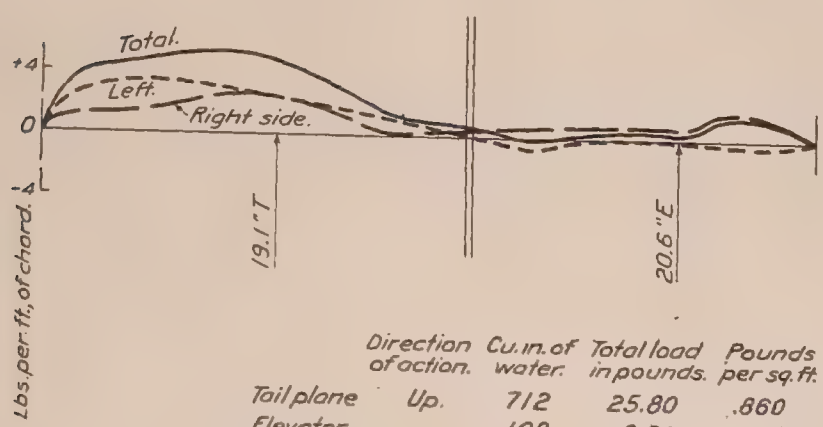
CASE IV. 1200 R.P.M. 80 M.P.H.



	Direction of action	Cu. in. of water.	Total load in pounds	Pounds per sq. ft.
Tail plane	Down	-610.	-22.10	.735
Elevator	Up.	244.	8.84	.398
Total	Down		-13.26	.254

Fig 198 Elevator moment about hinge = 113. in. lbs.

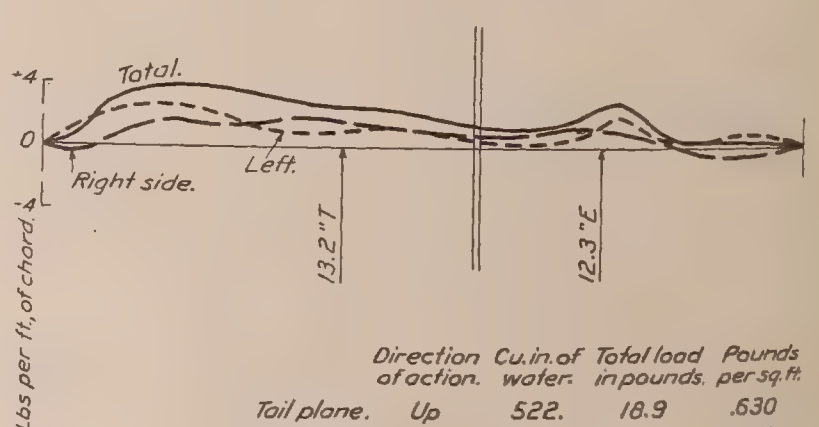
CASE IV. 900 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Up.	712	25.80	.860
Elevator.	"	108.	3.91	.176
Total.	"		29.71	.519

Fig. 199. Elevator moment about hinge = 80.5 in. lbs.

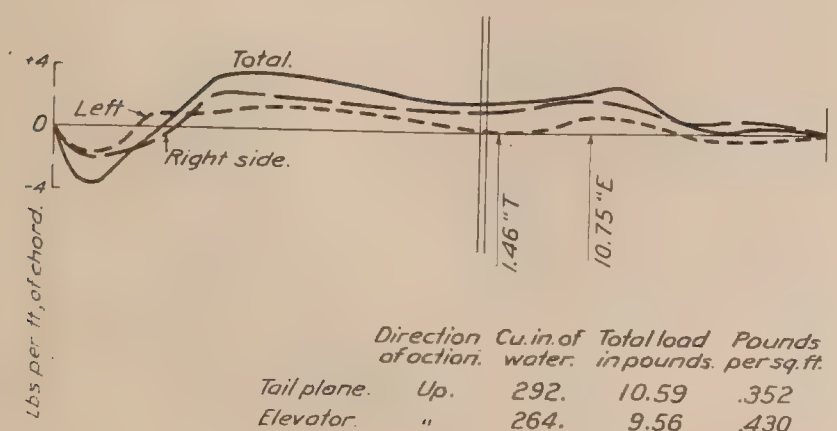
CASE IV 900 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up	522.	18.9	.630
Elevator.	"	210.	7.6	.342
Total.	"		26.5	.508

Fig. 200. Elevator moment about hinge = 93.5 in. lbs.

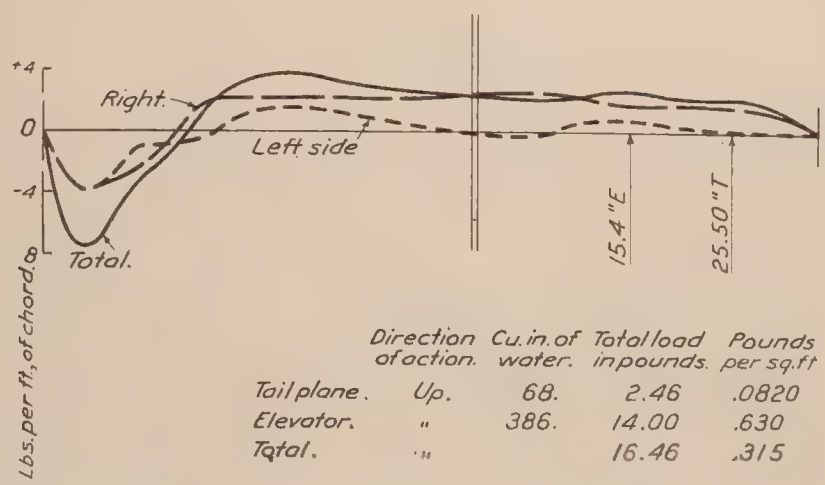
CASE IV 900 R.P.M. 60 M.P.H



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	292.	10.59	.352
Elevator.	"	264.	9.56	.430
Total.	"		20.15	.386

Fig. 201 Elevator moment about hinge = 102.5 in. lbs.

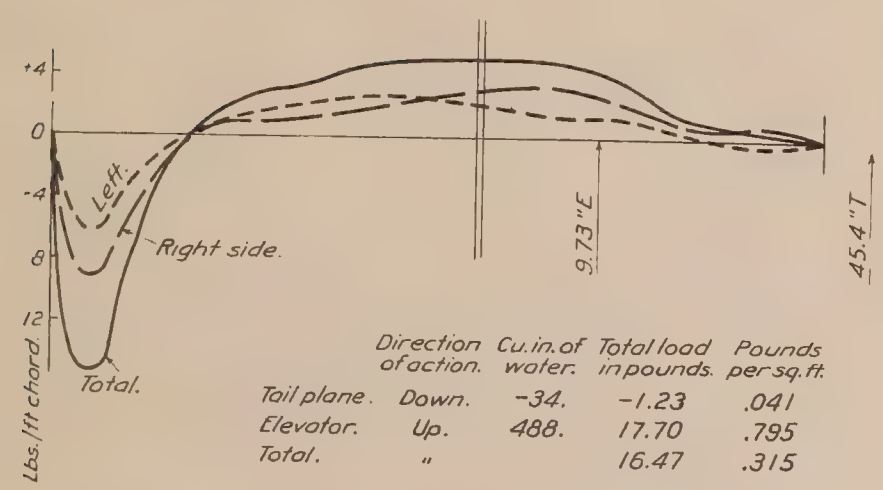
CASE IV. 900 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft
Tail plane.	Up.	68.	2.46	.0820
Elevator.	"	386.	14.00	.630
Total.	"		16.46	.315

Fig. 202. Elevator moment about hinge = 216.0 in. lbs.

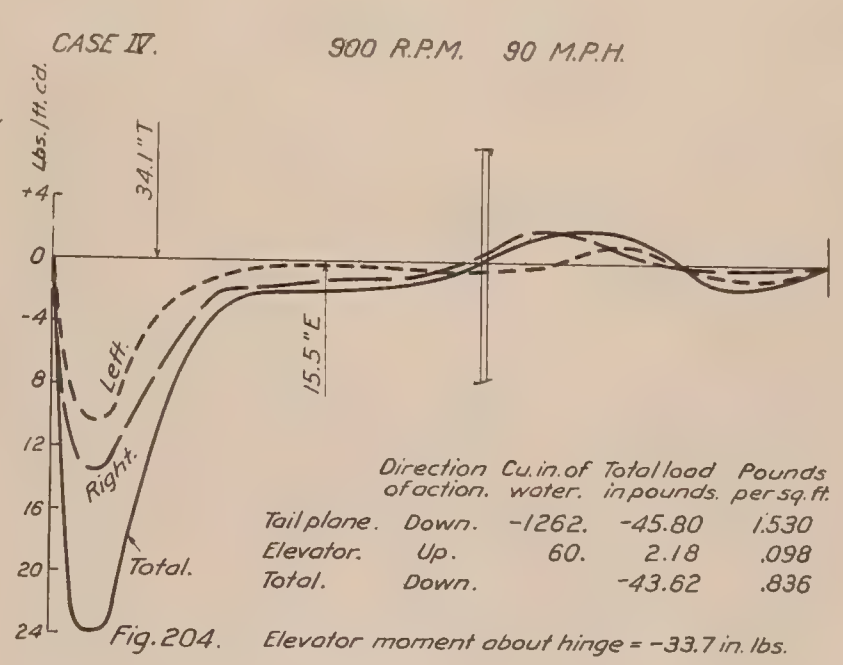
CASE IV. 900 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-34.	-1.23	.041
Elevator.	Up.	488.	17.70	.795
Total.	"		16.47	.315

Fig. 203. Elevator moment about hinge = 172.0 in. lbs.

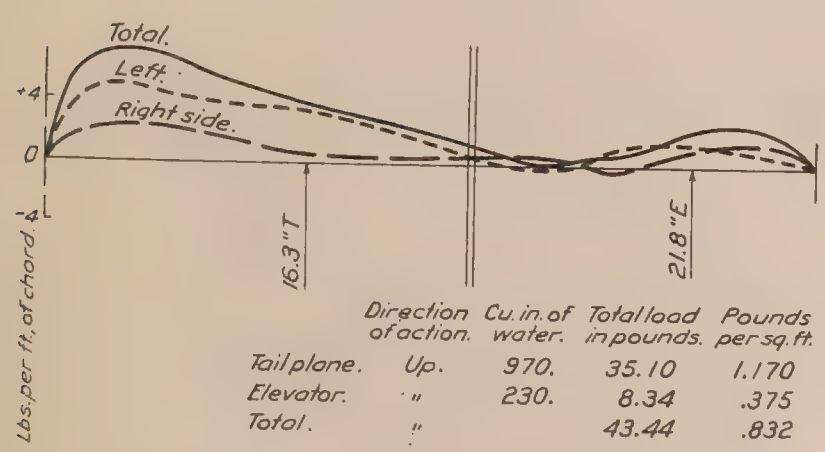
CASE IV. 900 R.P.M. 90 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1262.	-45.80	1.530
Elevator.	Up.	60.	2.18	.098
Total.	Down.		-43.62	.836

Fig. 204. Elevator moment about hinge = -33.7 in. lbs.

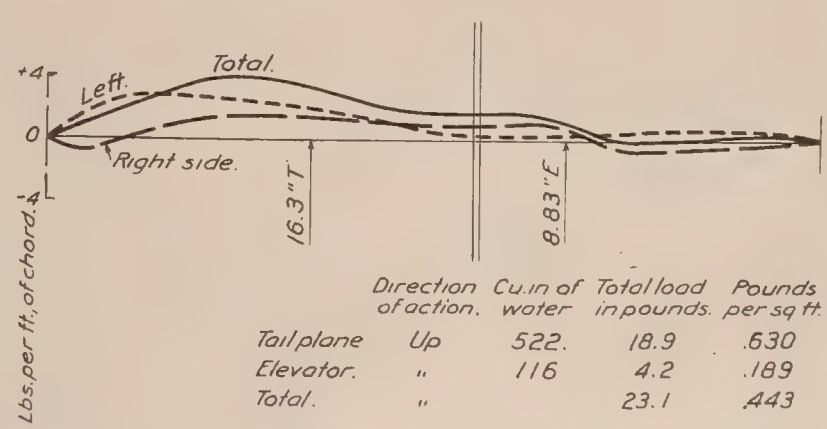
CASE IV. 600 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	970.	35.10	1.170
Elevator.	"	230.	8.34	.375
Total.	"		43.44	.832

Fig. 205. Elevator moment about hinge = 182.0 in. lbs.

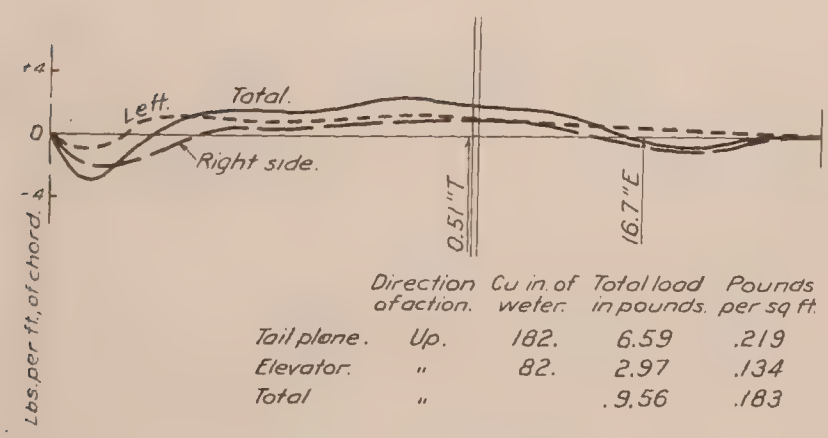
CASE IV. 600 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in of water	Total load in pounds.	Pounds per sq ft.
Tail plane	Up	522.	18.9	.630
Elevator.	"	116	4.2	.189
Total.	"		23.1	.443

Fig. 206. Elevator moment about hinge = 37.1 in. lbs

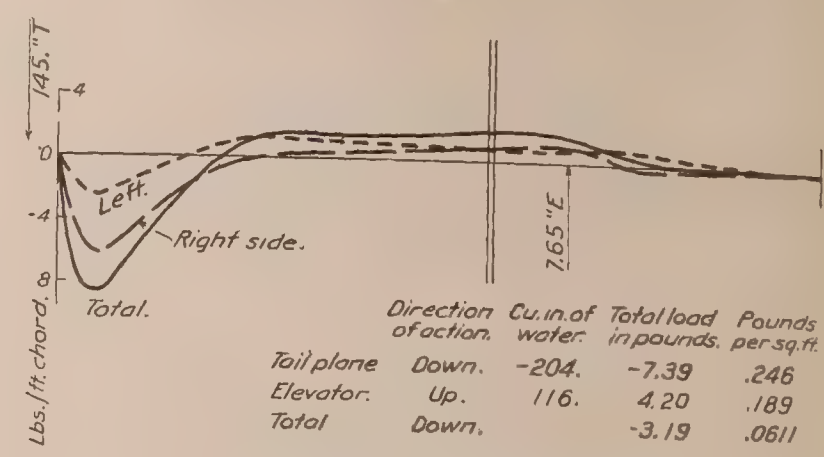
CASE IV. 600 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up.	182.	6.59	.219
Elevator.	"	82.	2.97	.134
Total	"		9.56	.183

Fig. 207. Elevator moment about hinge = 49.6 in. lbs

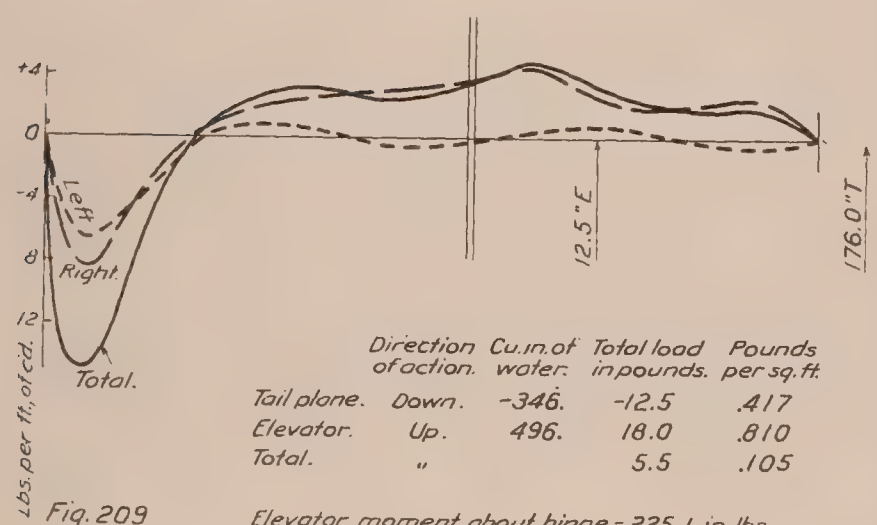
CASE IV 600 R.P.M 70 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down.	-204.	-7.39	.246
Elevator.	Up.	116.	4.20	.189
Total	Down.		-3.19	.0611

Fig. 208 Elevator moment about hinge = 32.1 in. lbs.

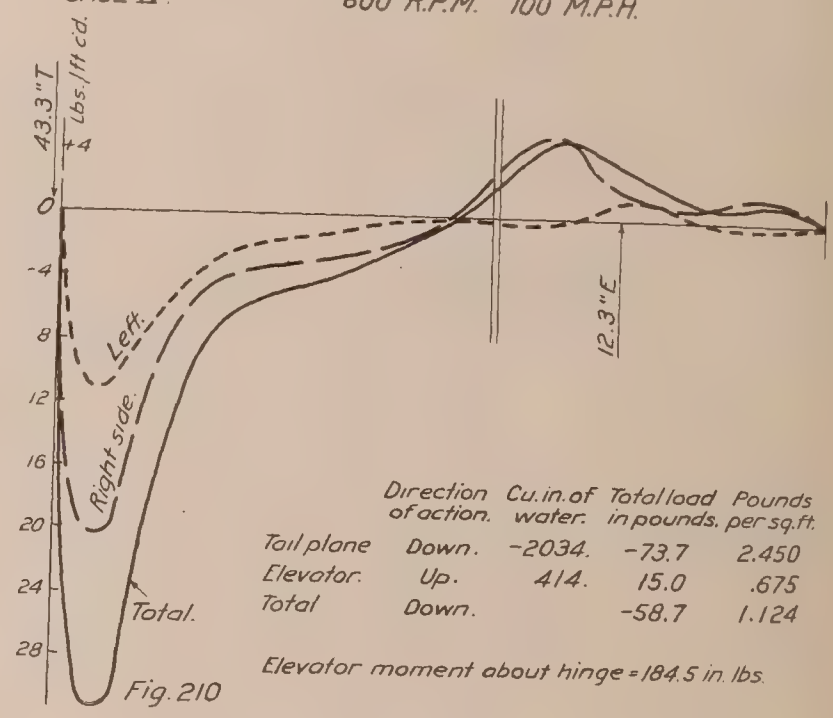
CASE IV. 600 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-346.	-12.5	.417
Elevator.	Up.	496.	18.0	.810
Total.	"		5.5	.105

Fig. 209 Elevator moment about hinge = 225.1 in. lbs.

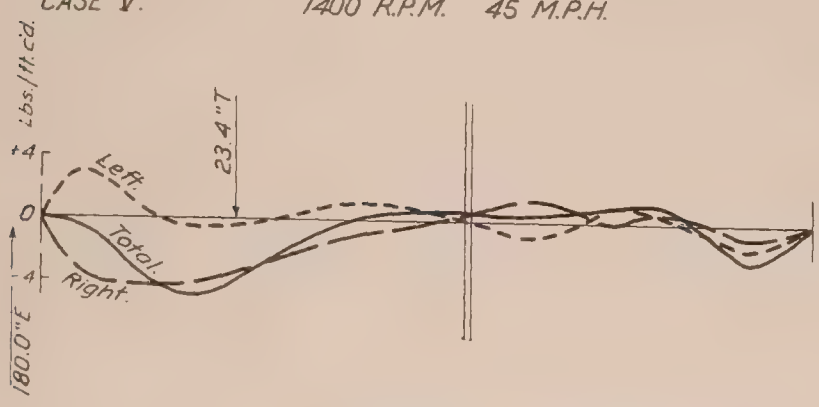
CASE IV. 600 R.P.M. 100 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down.	-2034.	-73.7	2.450
Elevator.	Up.	414.	15.0	.675
Total	Down.		-58.7	1.124

Fig. 210 Elevator moment about hinge = 184.5 in. lbs.

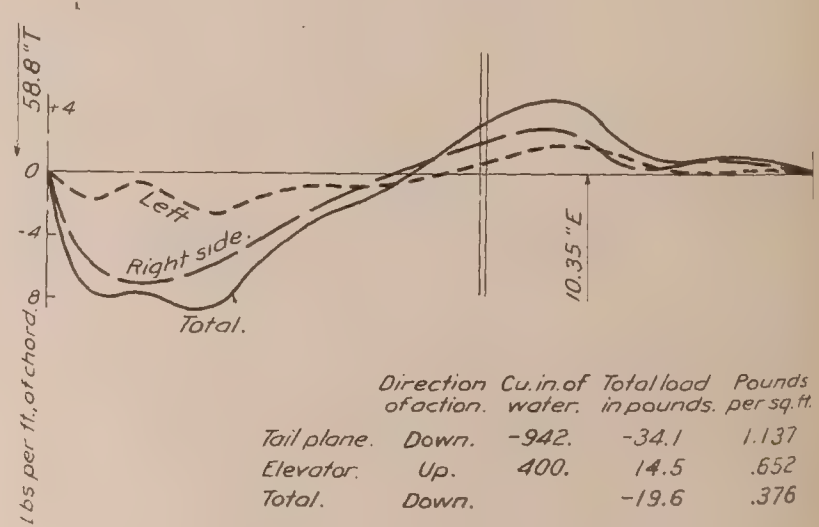
CASE V. 1400 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-386.	-14.000	.466
Elevator.	Up.	6.	.217	0098
Total.	Down.		-13.783	264

Fig. 211. Elevator moment about hinge = -39.1 in. lbs.

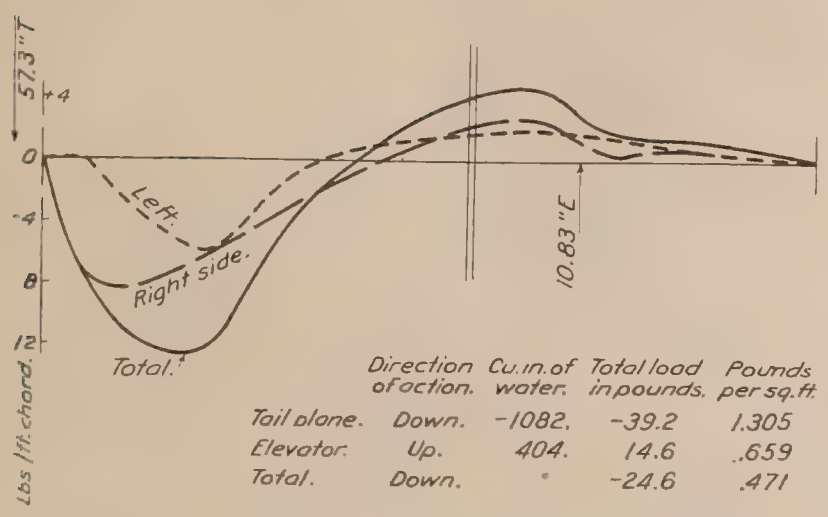
CASE V. 1400 R.P.M. 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-942.	-34.1	1.137
Elevator.	Up.	400.	14.5	.652
Total.	Down.		-19.6	.376

Fig. 212 Elevator moment about hinge = 150.0 in. lbs.

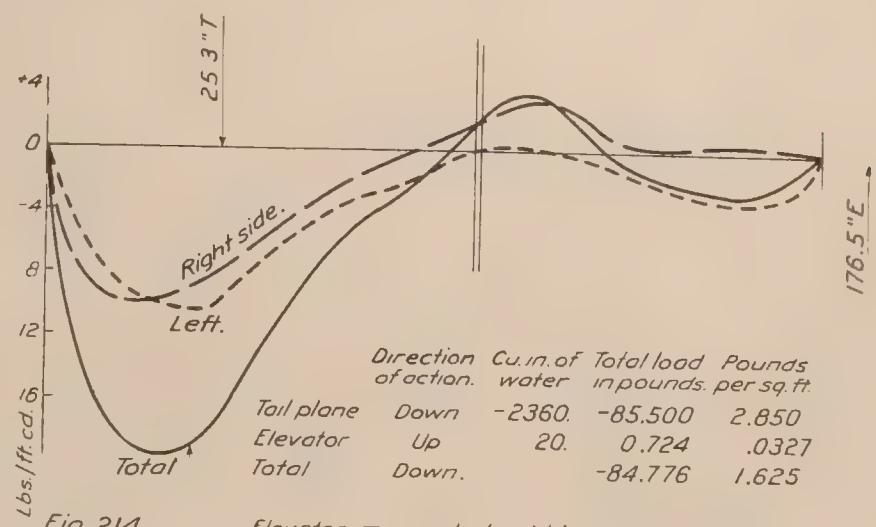
CASE V 1400 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-1082.	-39.2	1.305
Elevator.	Up.	404.	14.6	.659
Total.	Down.		-24.6	.471

Fig. 213. Elevator moment about hinge = 158.5 in. lbs.

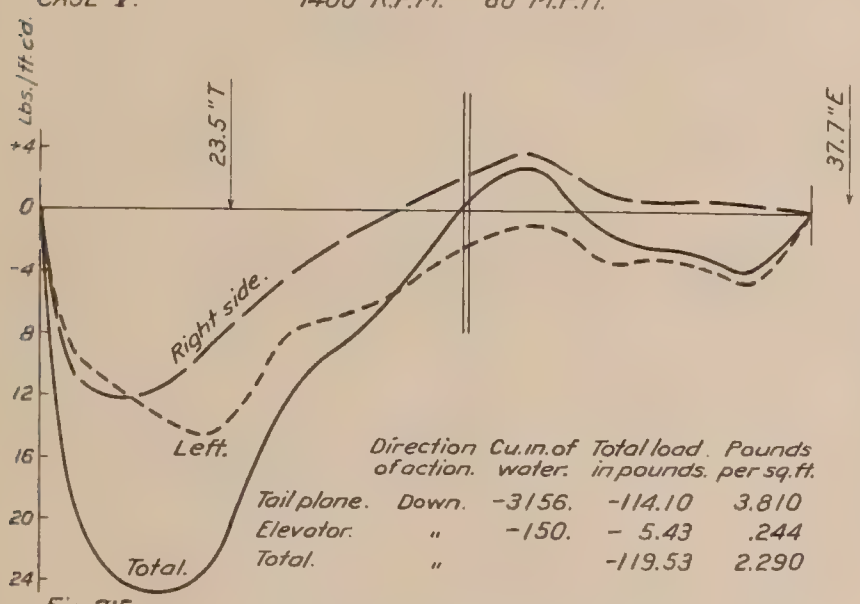
CASE V 1400 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in. of water	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down	-2360.	-85.500	2.850
Elevator	Up	20.	0.724	.0327
Total	Down.		-84.776	1.625

Fig. 214 Elevator moment about hinge = 127.5 in lbs

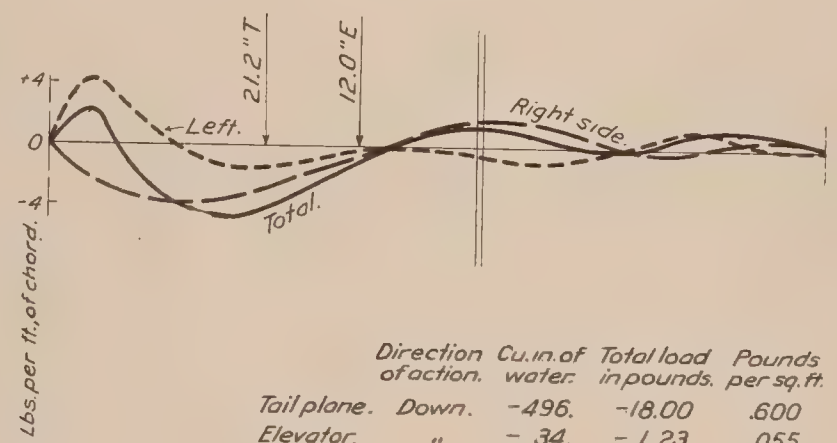
CASE V. 1400 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-3156.	-114.10	3.810
Elevator.	"	-150.	- 5.43	.244
Total.	"		-119.53	2.290

Fig. 215. Elevator moment about hinge = -205.0 in. lbs.

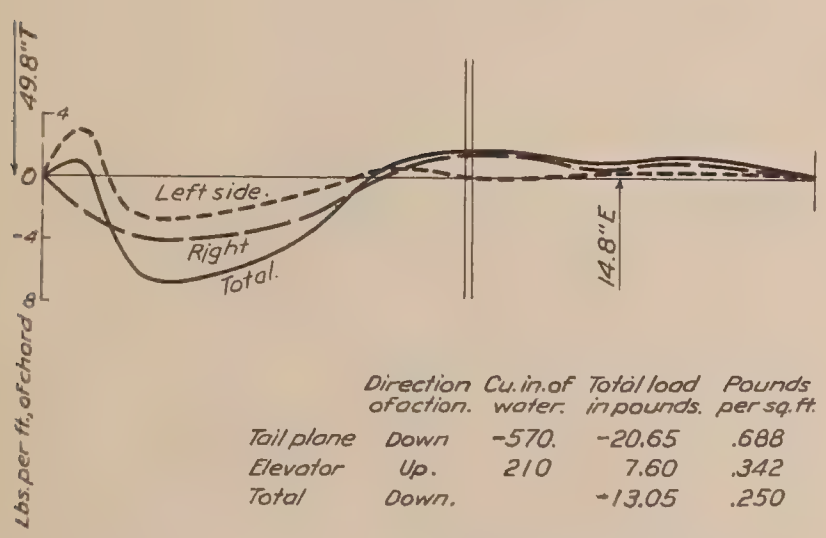
CASE V. 1200 R.P.M. 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-496.	-18.00	.600
Elevator.	"	- 34.	- 1.23	.055
Total	"		-19.23	.369

Fig. 216. Elevator moment about hinge = 14.8 in. lbs.

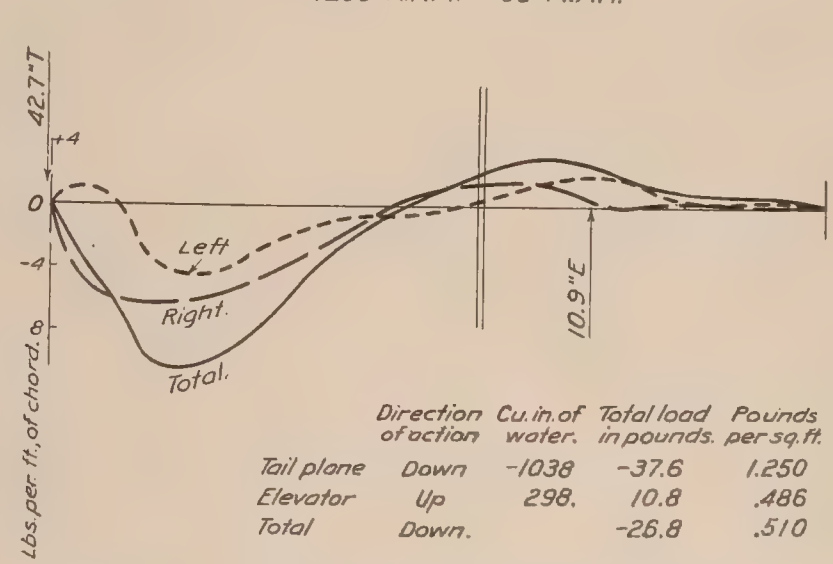
CASE V. 1200 R.P.M 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down	-570.	-20.65	.688
Elevator	Up.	210	7.60	.342
Total	Down.		-13.05	.250

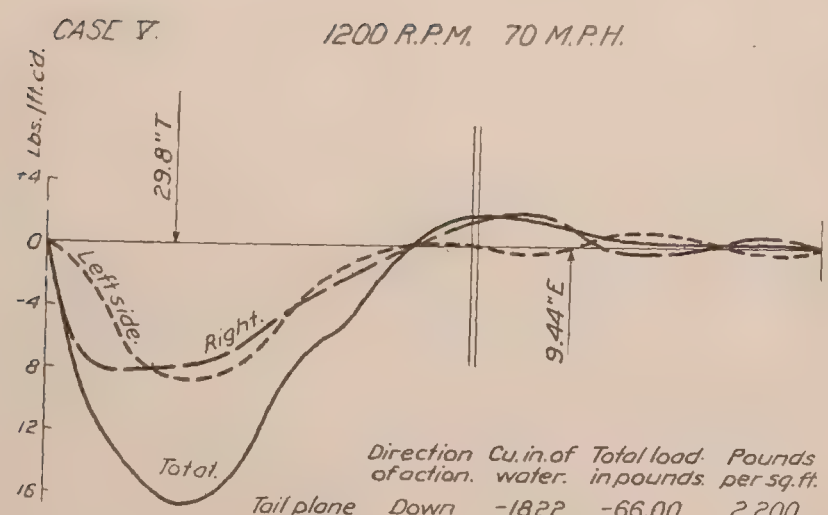
Fig. 217 Elevator moment about hinge = 112.5 in. lbs.

CASE V 1200 R.P.M. 60 M.P.H.



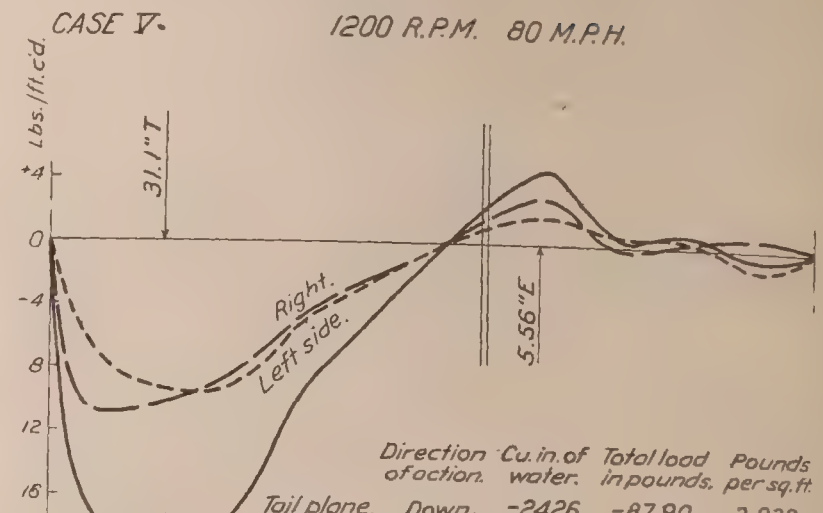
	Direction of action	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down	-1038	-37.6	1.250
Elevator	Up	298.	10.8	.486
Total	Down.		-26.8	.510

Fig. 218 Elevator moment about hinge = 118.0 in. lbs



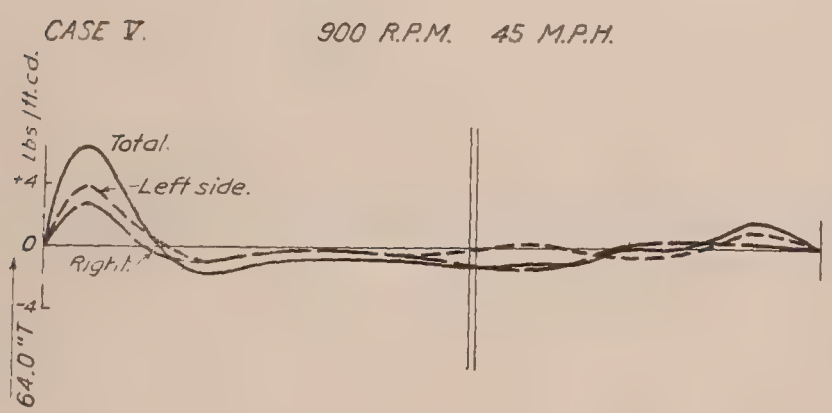
	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane	Down	-1822.	-66.00	2.200
Elevator	Up	122	4.42	.199
Total	Down		-61.58	1.180

Fig. 219 Elevator moment about hinge = 41.75 in. lbs.



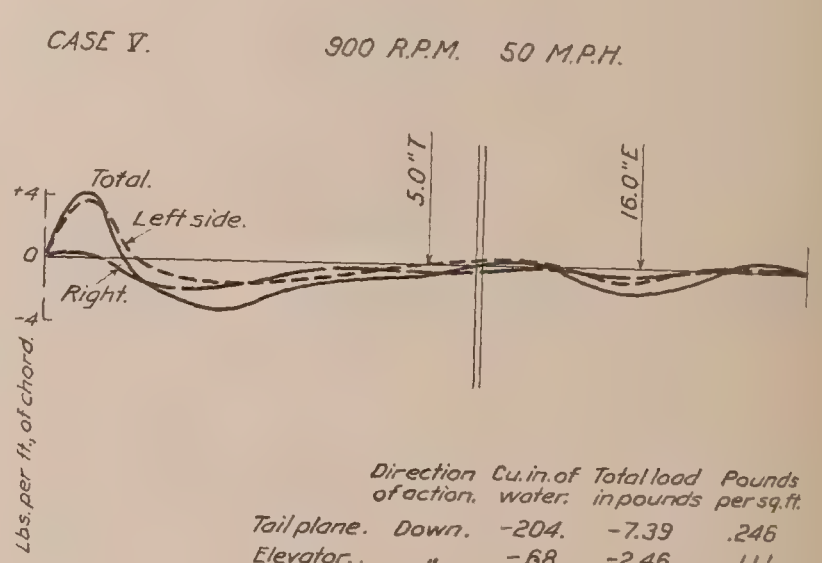
	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-2426.	-87.90	2.930
Elevator.	Up.	244.	8.83	.398
Total	Down.		-79.07	1.515

Fig. 220. Elevator moment about hinge = 49.1 in. lbs.



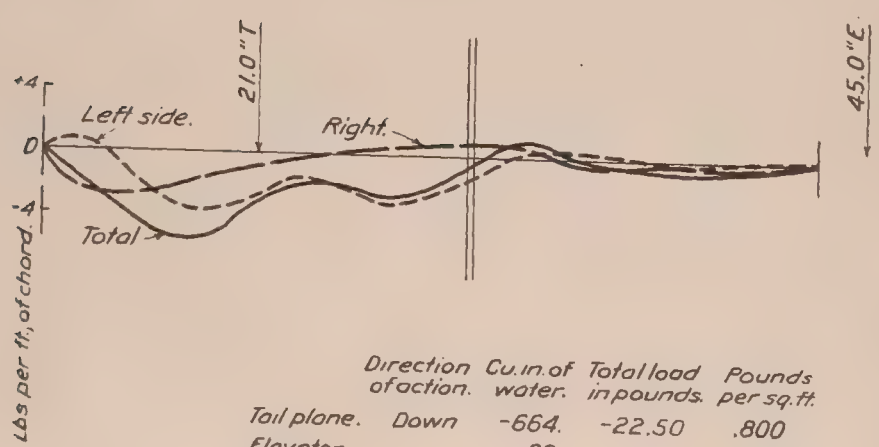
	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Up	48.	1.740	.0580
Elevator.	"	20.	.725	.0326
Total.	"		2.465	.0469

Fig. 221. Elevator moment about hinge = 50.7 in. lbs.



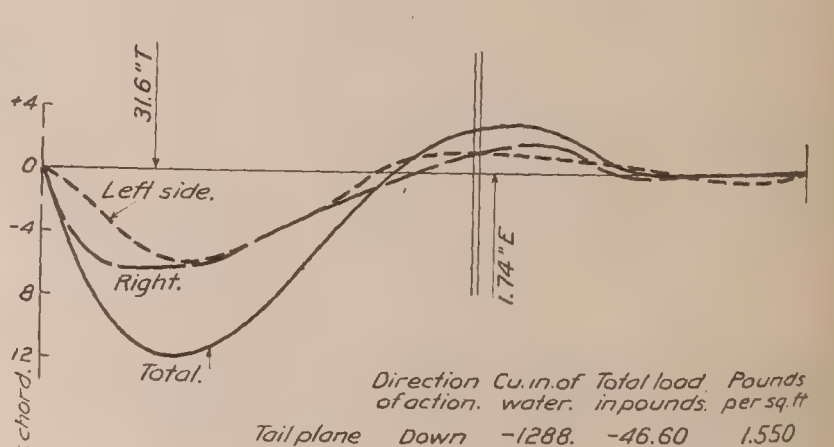
	Direction of action.	Cu. in. of water.	Total load in pounds	Pounds per sq. ft.
Tail plane.	Down.	-204.	-7.39	.246
Elevator.	"	-68.	-2.46	.111
Total.	"		-9.85	.189

Fig. 222. Elevator moment about hinge = -39.4 in. lbs.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down	-664.	-22.50	.800
Elevator.	"	-28.	.95	.0456
Total.	"		-23.45	.450

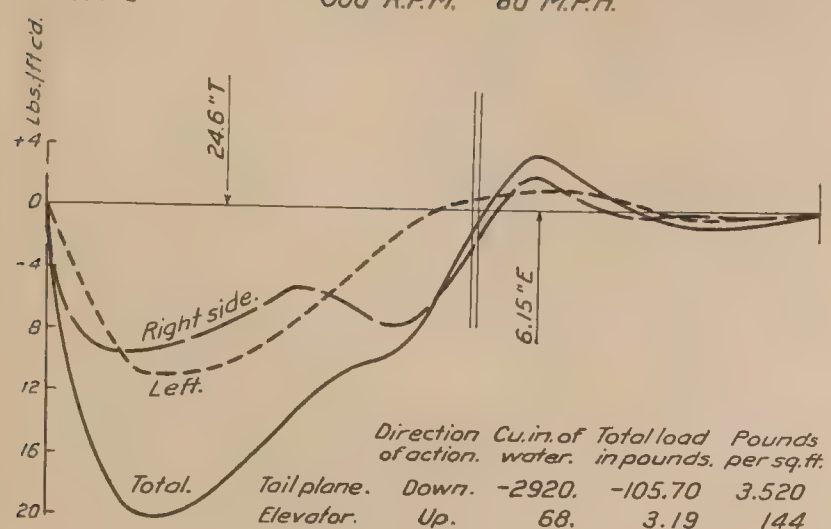
Fig. 223 Elevator moment about hinge = -42.8 in lbs.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft
Tail plane	Down	-1288.	-46.60	1.550
Elevator.	Up.	156.	5.65	.254
Total	Down.		-40.95	.784

Fig. 224. Elevator moment about hinge = 9.84 in. lbs.

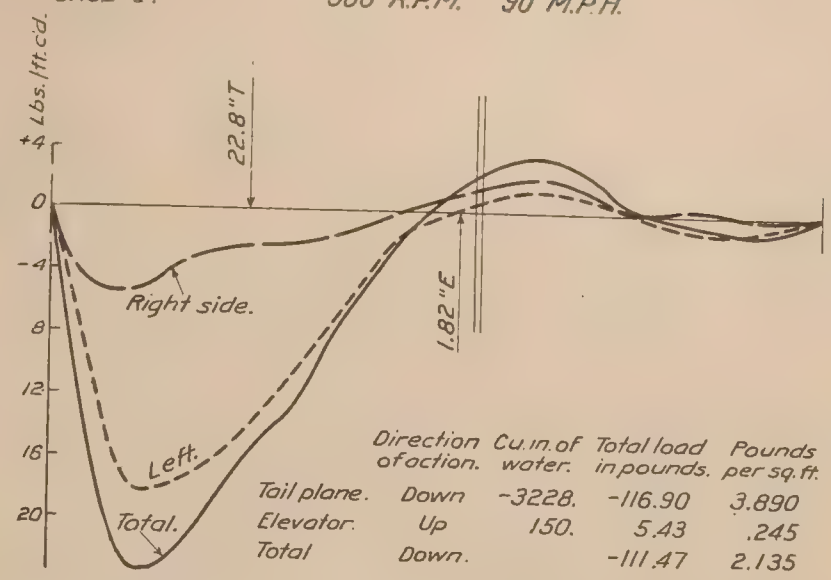
CASE V 900 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-2920.	-105.70	3.520
Elevator.	Up.	68.	3.19	144
Total.	Down.		-102.51	1.964

Fig. 225. Elevator moment about hinge = 19.63 in. lbs.

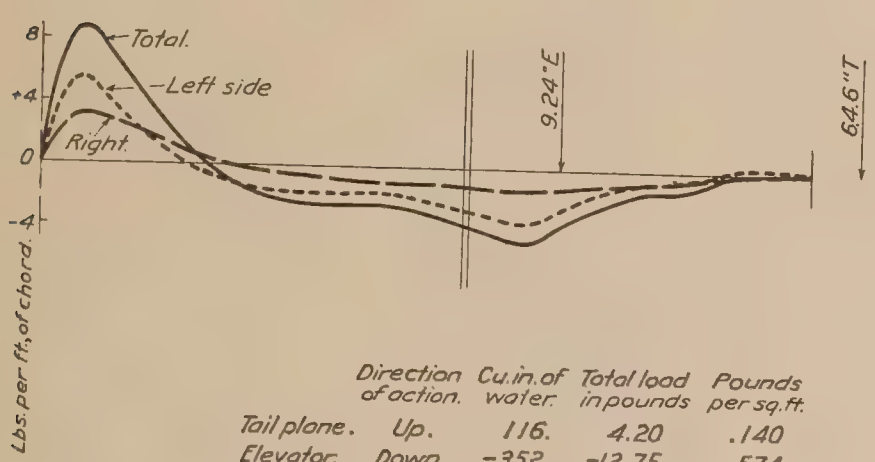
CASE V. 900 R.P.M. 90 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down	-3228.	-116.90	3.890
Elevator.	Up	150.	5.43	.245
Total	Down.		-111.47	2.135

Fig. 226. Elevator moment about hinge = -9.89 in. lbs.

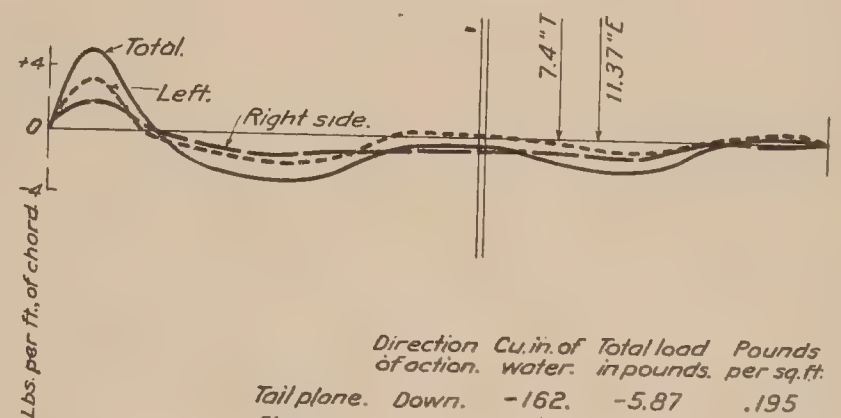
CASE V. 600 R.P.M 45 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds	Pounds per sq. ft.
Tail plane.	Up.	116.	4.20	.140
Elevator.	Down.	-352.	-12.75	.574
Total.	"		-8.55	.164

Fig. 227. Elevator moment about hinge = 117.9 in. lbs.

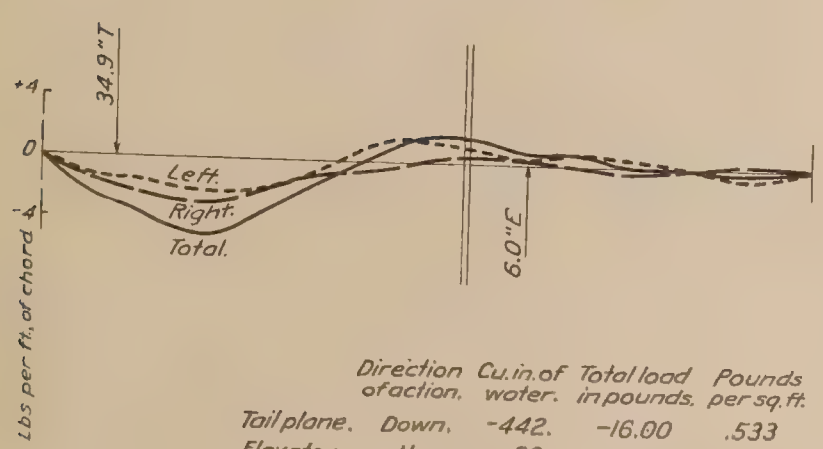
CASE V. 600 R.P.M 50 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-162.	-5.87	.195
Elevator.	"	-150.	-5.44	.244
Total.	"		-11.31	.217

Fig. 228. Elevator moment about hinge = -61.8 in. lbs.

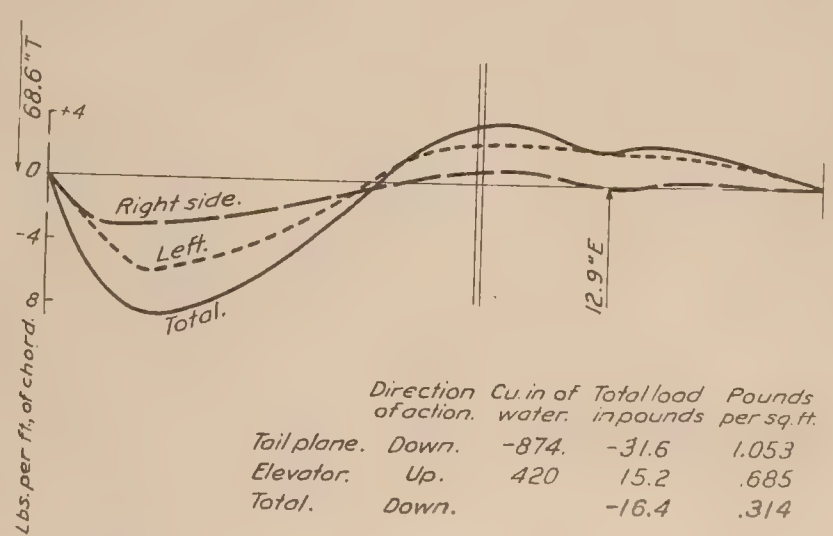
CASE V. 600 R.P.M. 60 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-442.	-16.00	.533
Elevator.	Up.	68.	2.46	.111
Total.	Down.		-13.54	.259

Fig. 229. Elevator moment about hinge = 14.75 in. lbs.

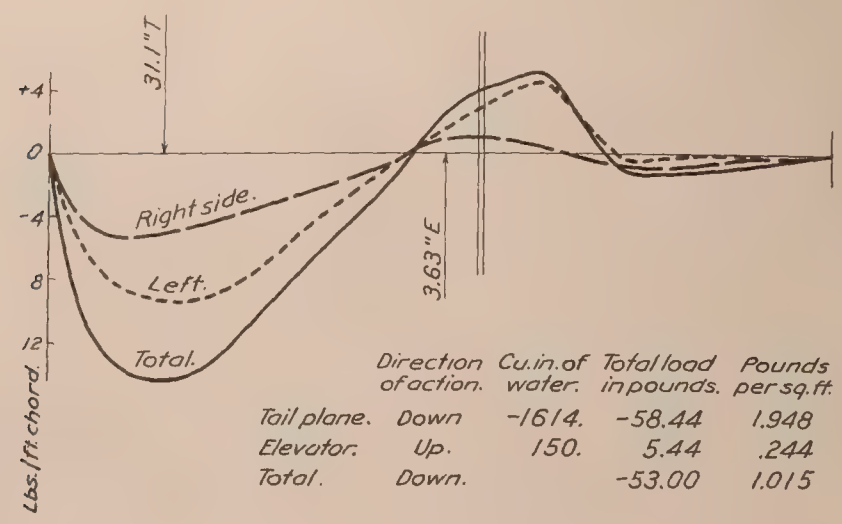
CASE V. 600 R.P.M. 70 M.P.H.



	Direction of action.	Cu. in of water.	Total load in pounds	Pounds per sq. ft.
Tail plane.	Down.	-874.	-31.6	1.053
Elevator.	Up.	420	15.2	.685
Total.	Down.		-16.4	.314

Fig. 230. Elevator moment about hinge = 196.1 in. lbs.

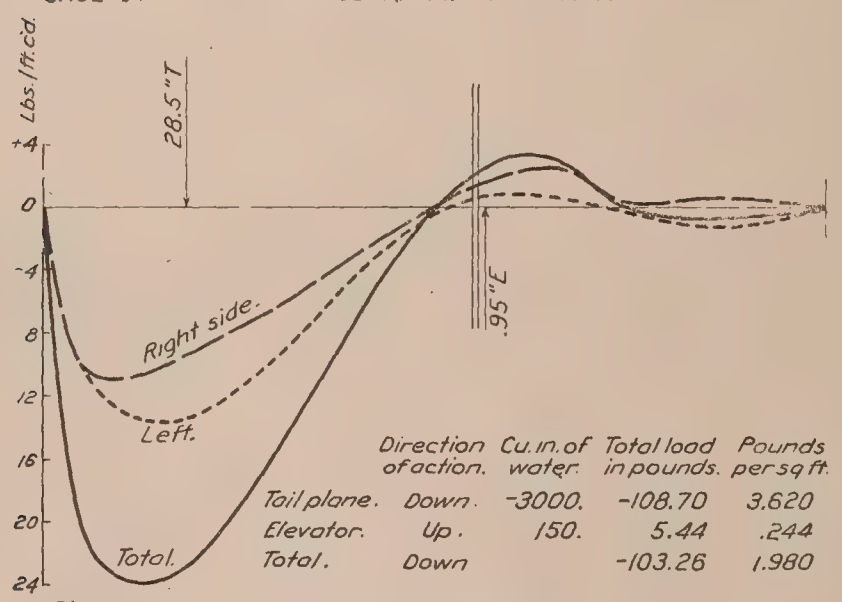
CASE V. 600 R.P.M. 80 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down	-1614.	-58.44	1.948
Elevator.	Up.	150.	5.44	.244
Total.	Down.		-53.00	1.015

Fig. 231. Elevator moment about hinge = -19.75 in. lbs.

CASE V. 600 R.P.M. 100 M.P.H.



	Direction of action.	Cu. in. of water.	Total load in pounds.	Pounds per sq. ft.
Tail plane.	Down.	-3000.	-108.70	3.620
Elevator.	Up.	150.	5.44	.244
Total.	Down		-103.26	1.980

Fig. 232. Elevator moment about hinge = 5.16 in. lbs.

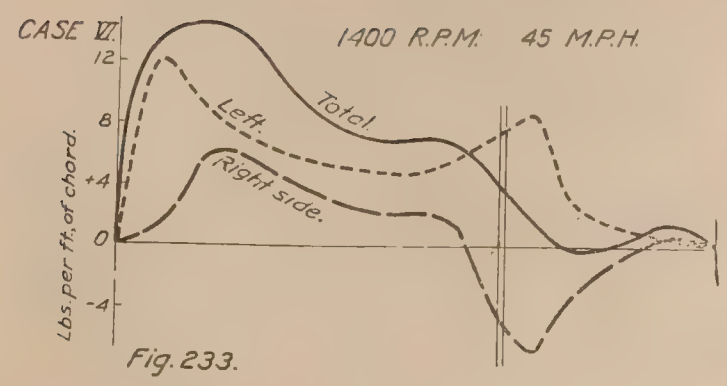


Fig. 233.

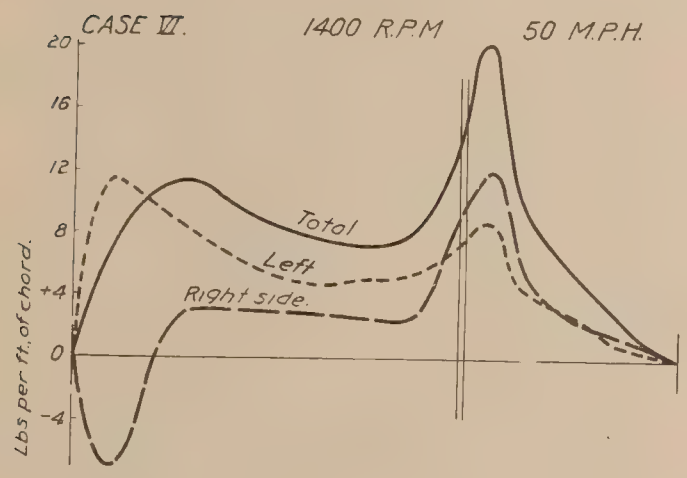


Fig 234.

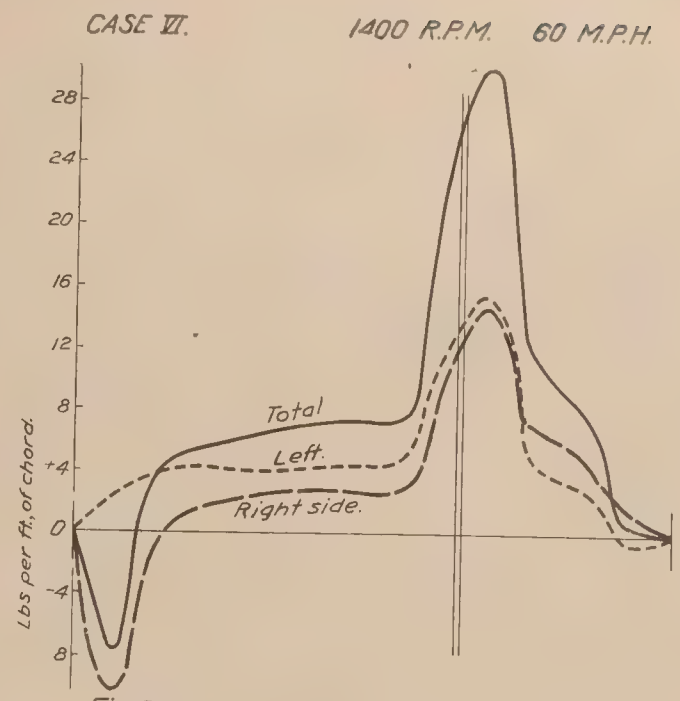


Fig. 235.

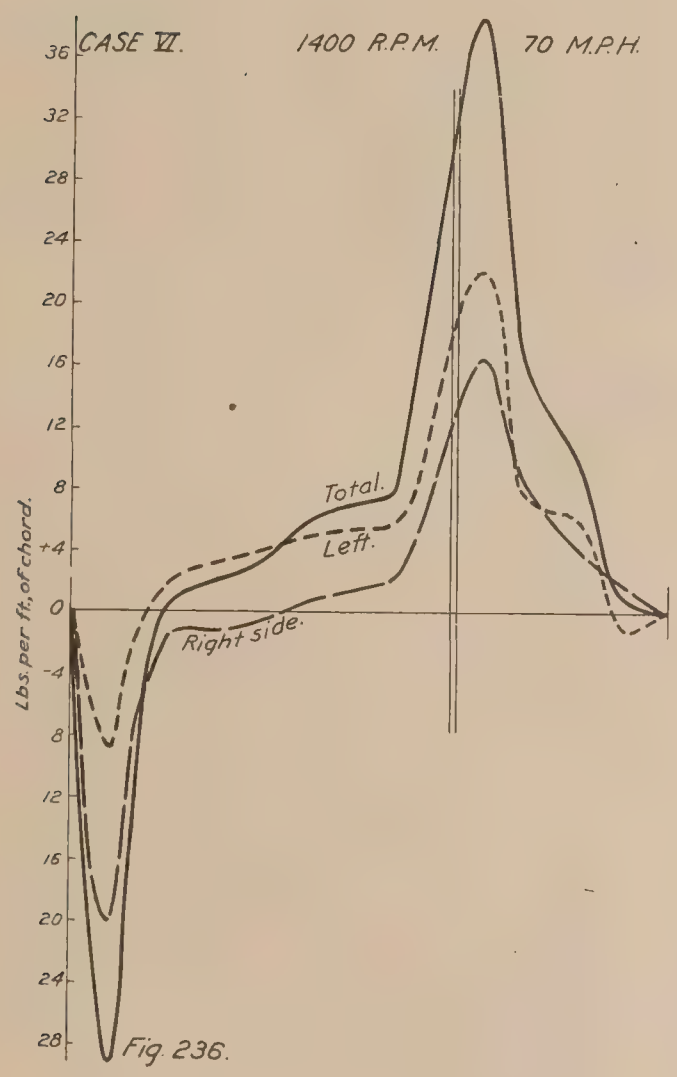


Fig. 236.

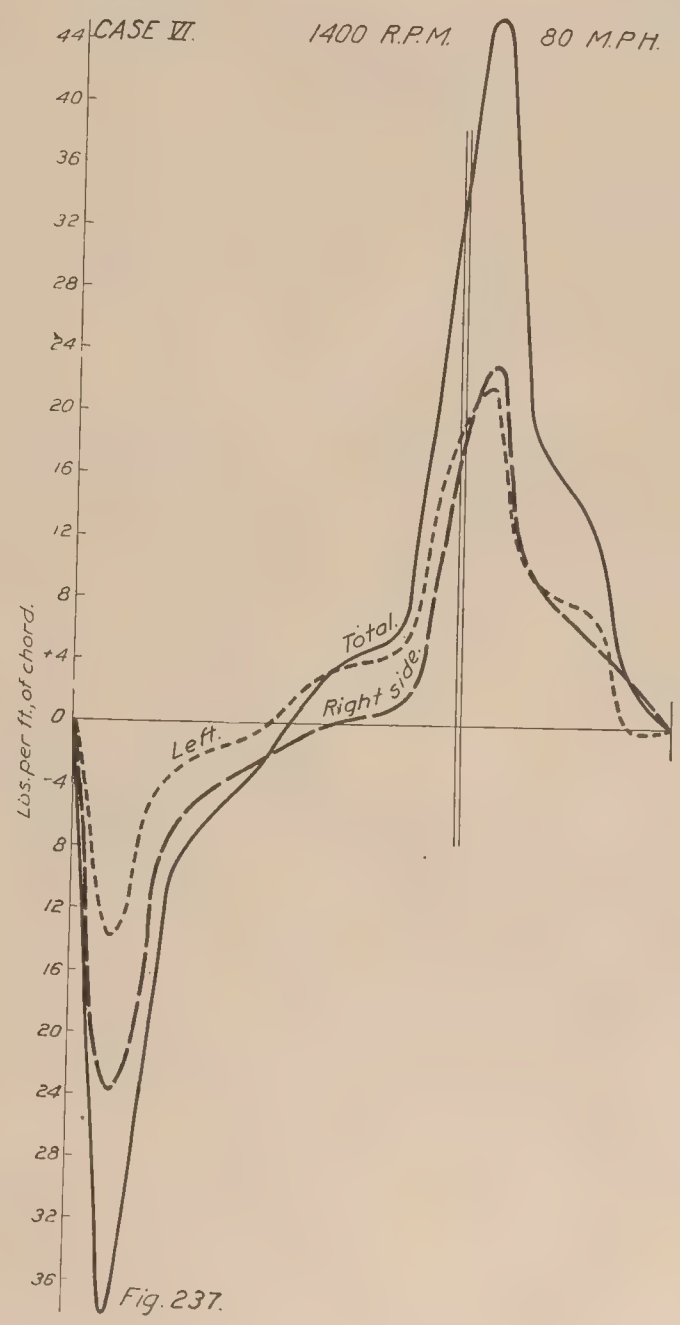


Fig. 237.

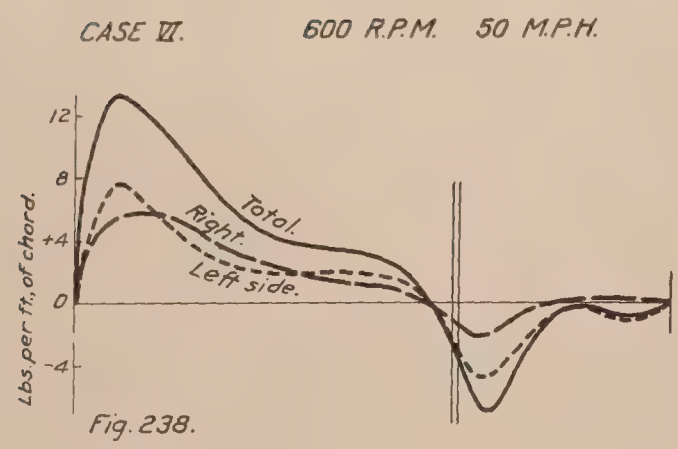


Fig. 238.

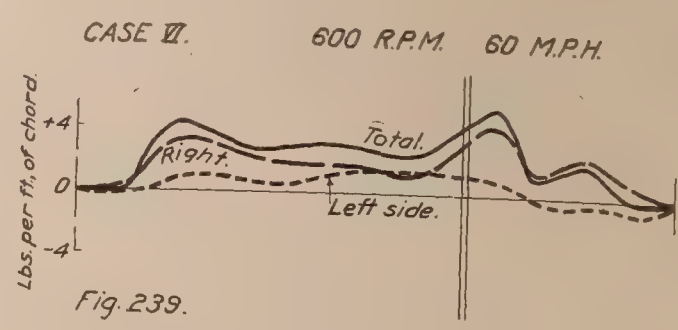


Fig. 239.

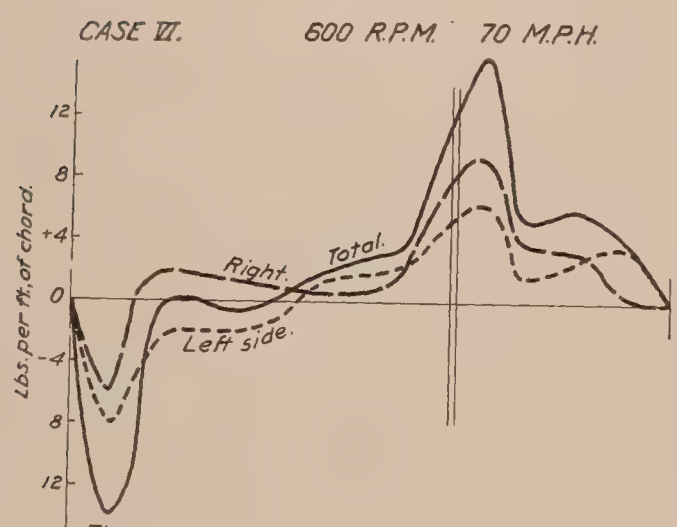


Fig. 240.

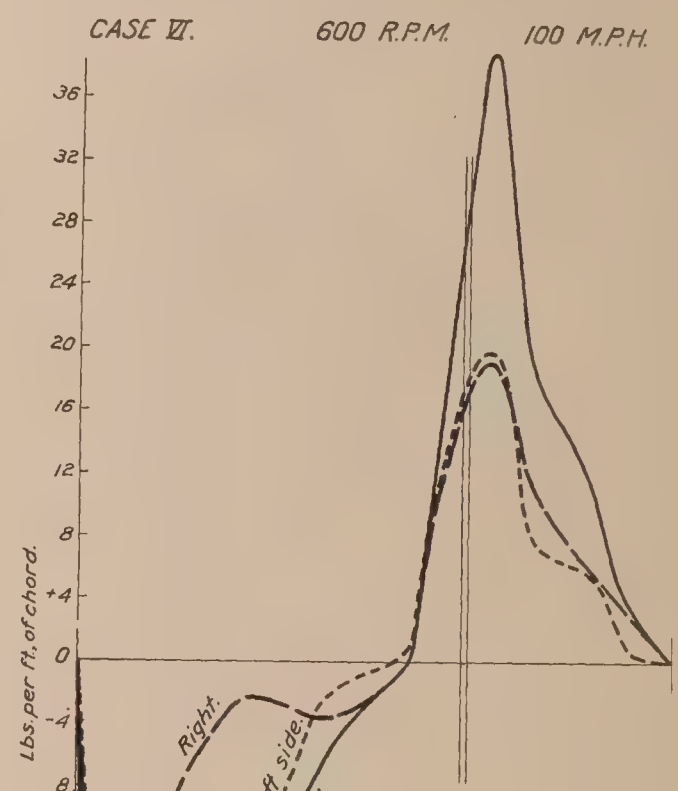


Fig. 242.

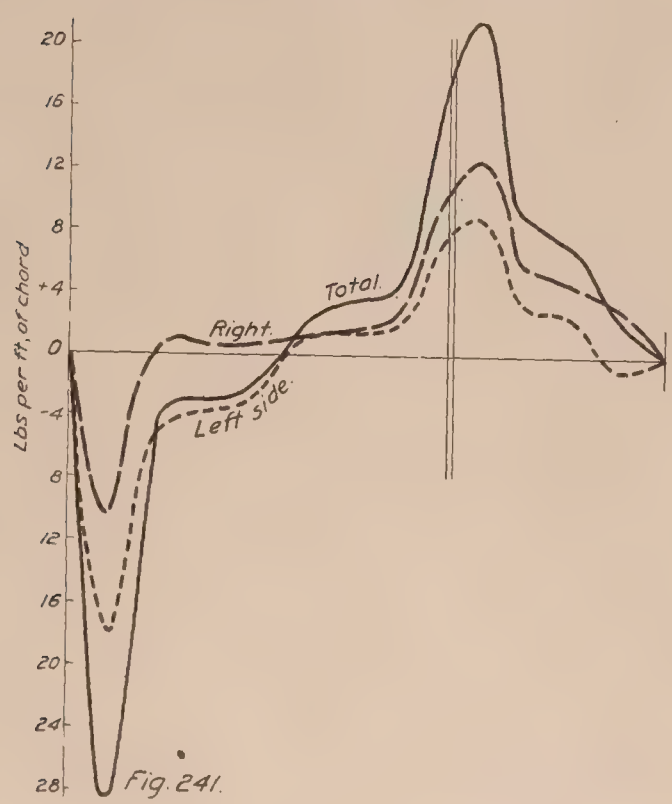


Fig. 241.

CASE II

1400 R.P.M. 45 M.P.H.

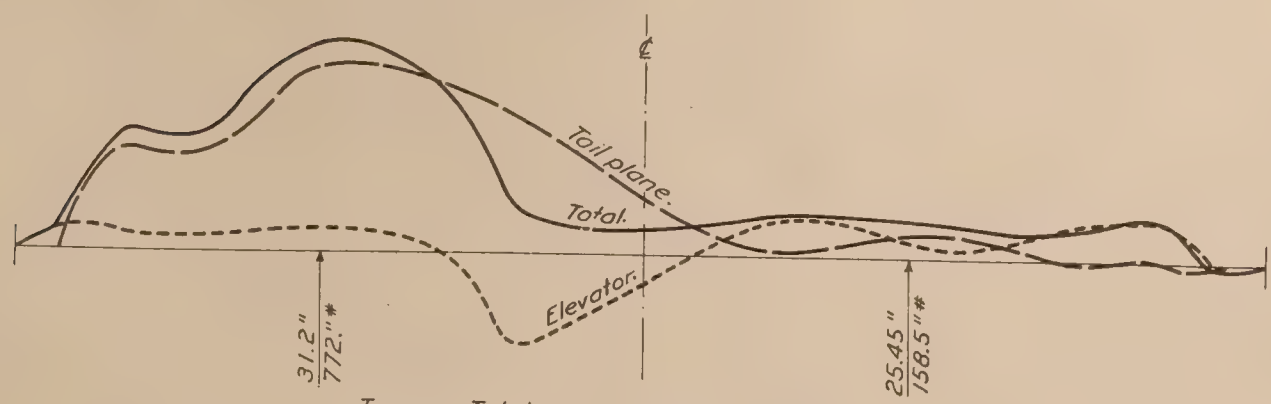


Fig. 243. Torque = Total moment = 613"# about ℄

CASE II.

1400 R.P.M. 50 M.P.H.

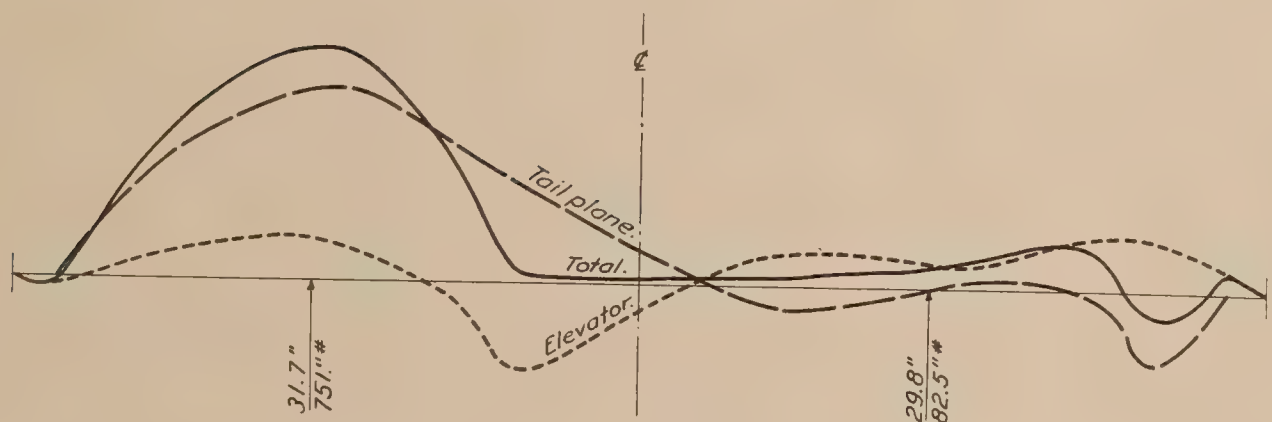


Fig. 244 Torque = Total moment = 668."# about ℄

CASE II.

1400 R.P.M. 60 M.P.H

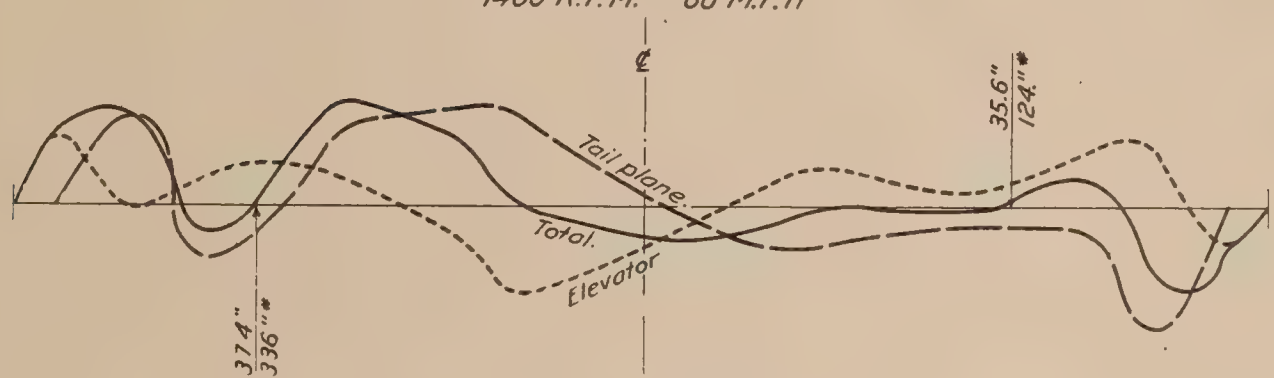


Fig. 245 Torque = Total moment = 460"# about ℄

CASE II.

1400 R.P.M. 70 M.P.H.

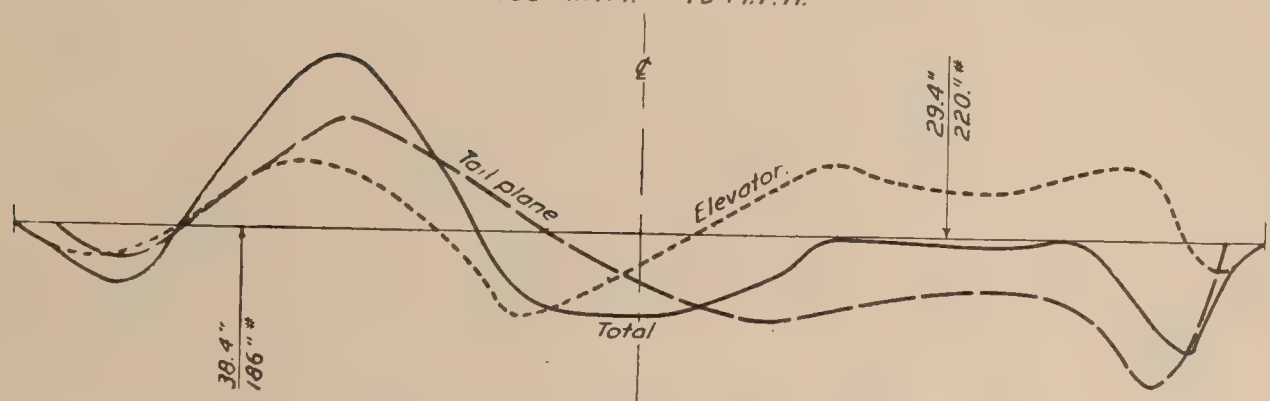


Fig. 246. Torque = Total moment = 406"# about ℄

Views of tail from rear.

CASE II.

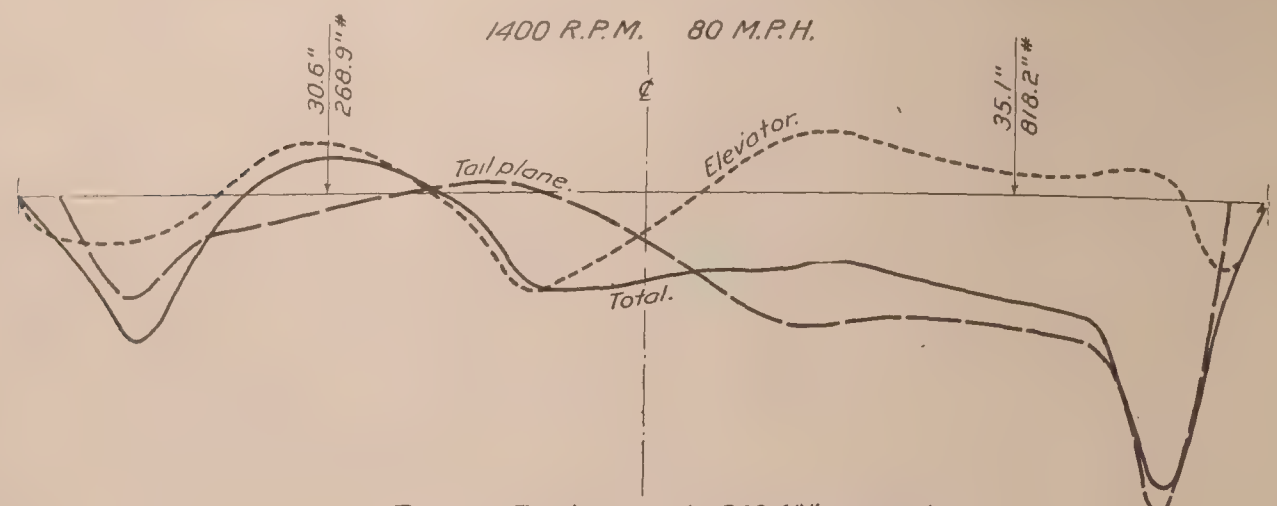


Fig. 247. Torque = Total moment = 549.4"# about ℄

CASE II.

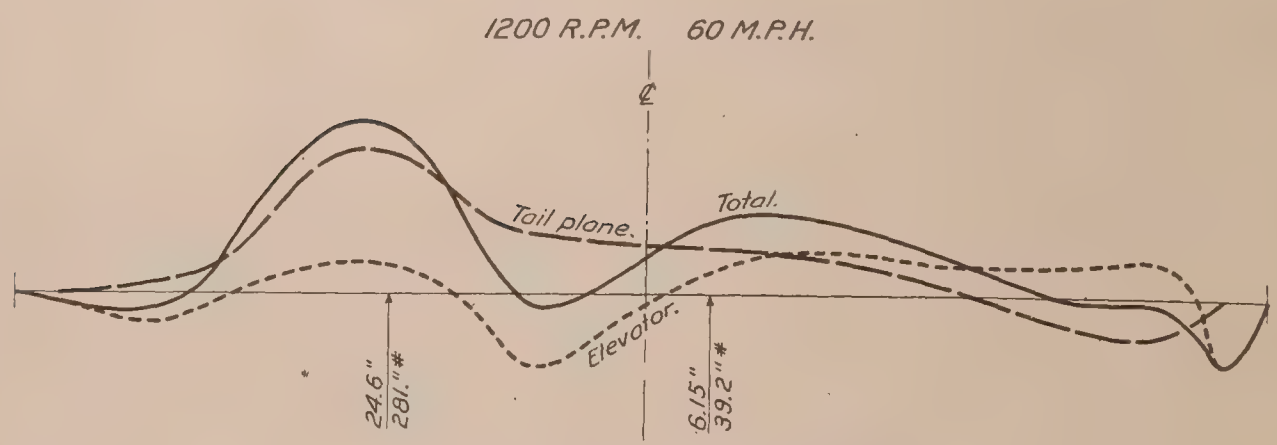


Fig. 248 Torque = Total moment = 242."# about ℄

CASE II.

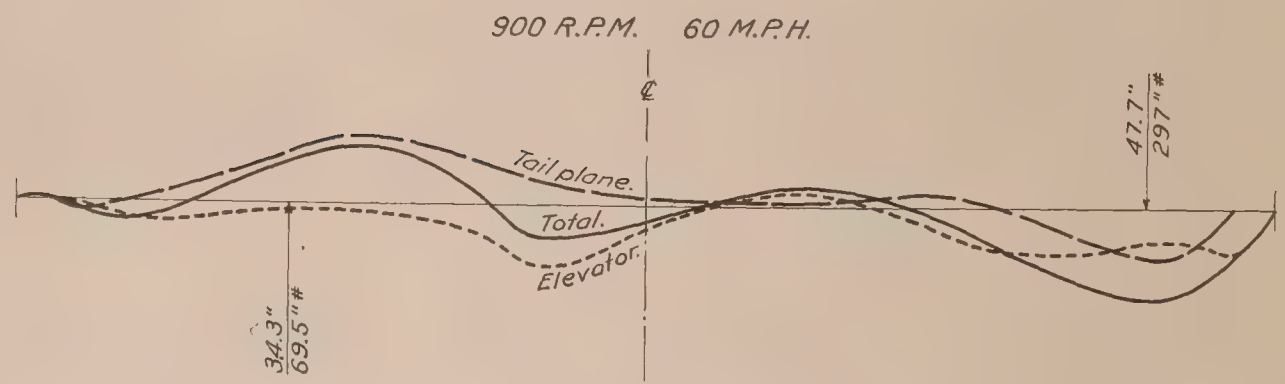


Fig. 249. Torque = Total moment = 366.5"# about ℄

CASE II

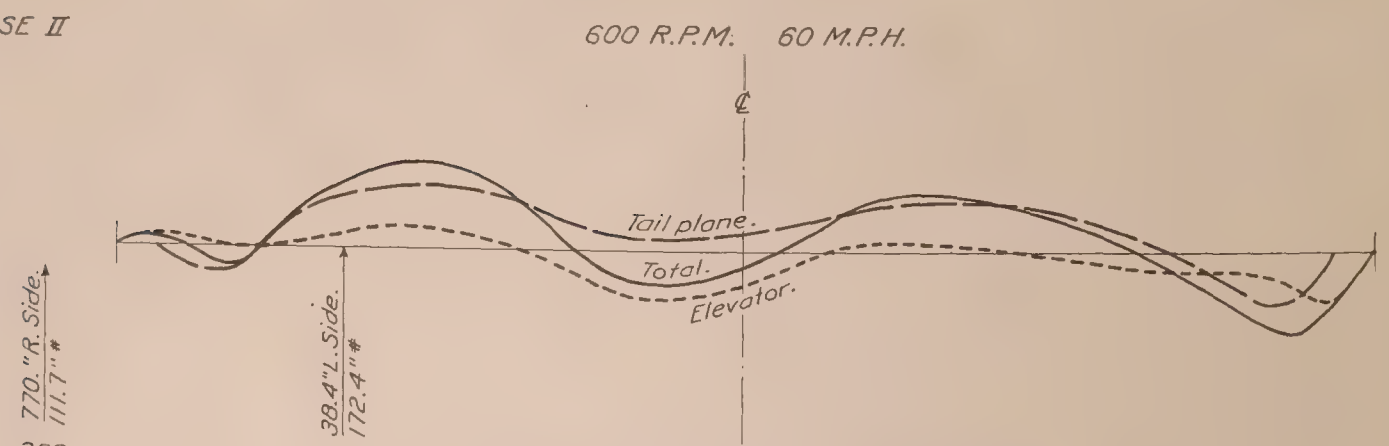


Fig. 250. Torque = Total moment = 284.1"# about ℄

Views of tail from rear.

CASE III.

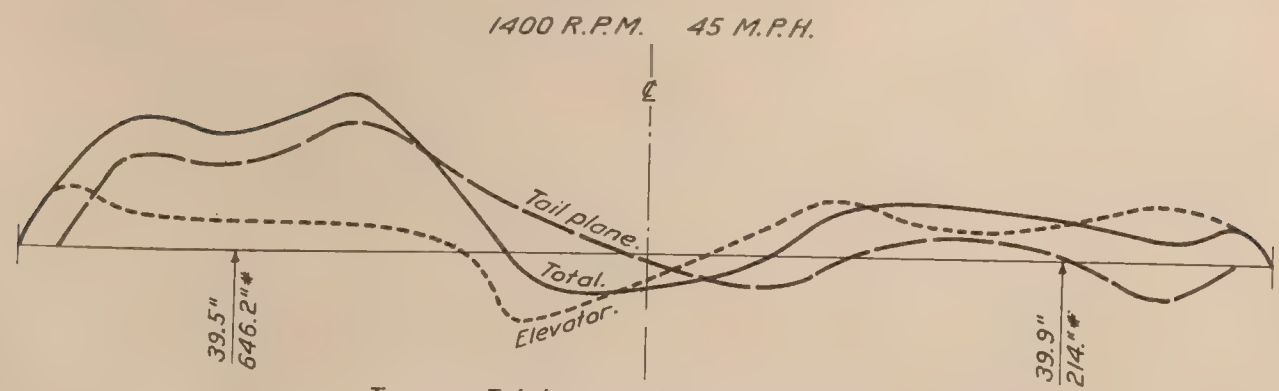


Fig. 251.

Torque = Total moment = 432.2"# about ℄

CASE III

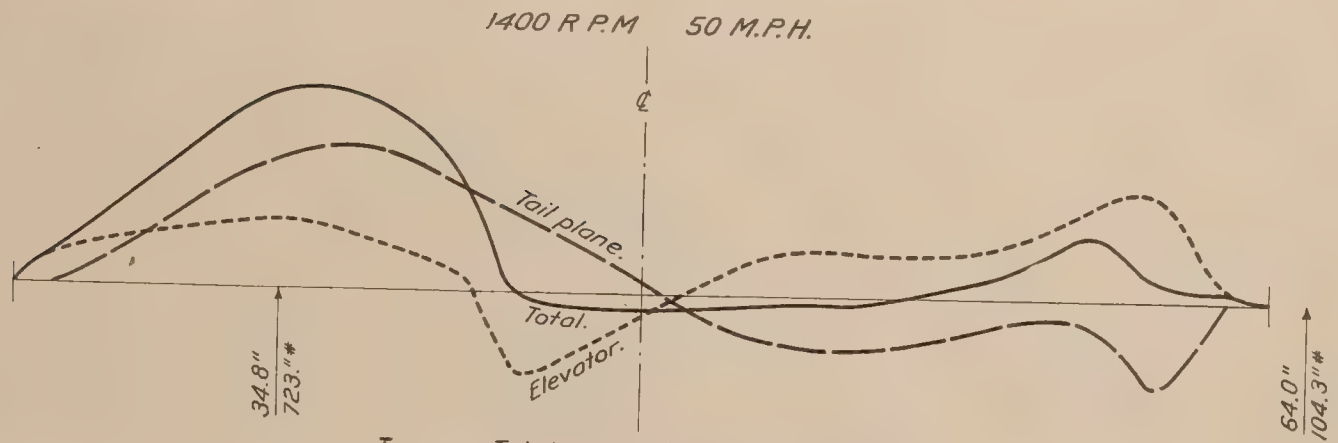


Fig. 252.

Torque = Total moment = 619."# about ℄

CASE III.

Fig. 253.

Torque = Total moment = 1216"# about ℄

CASE III.

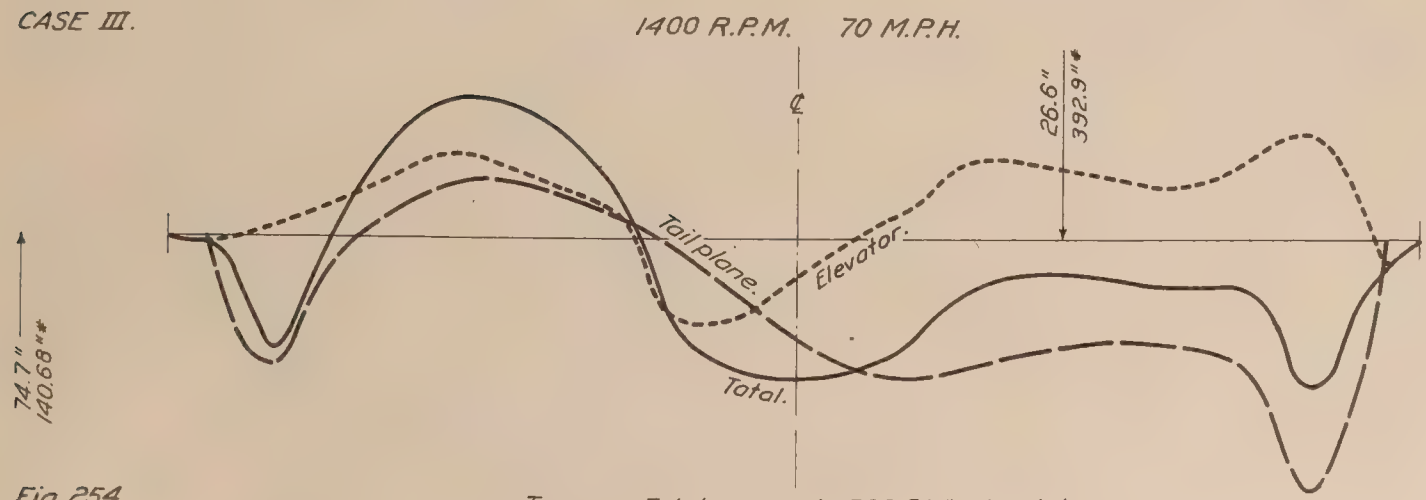


Fig. 254.

Torque = Total moment = 533.5"# about ℄

Views of tail from rear.

CASE III.

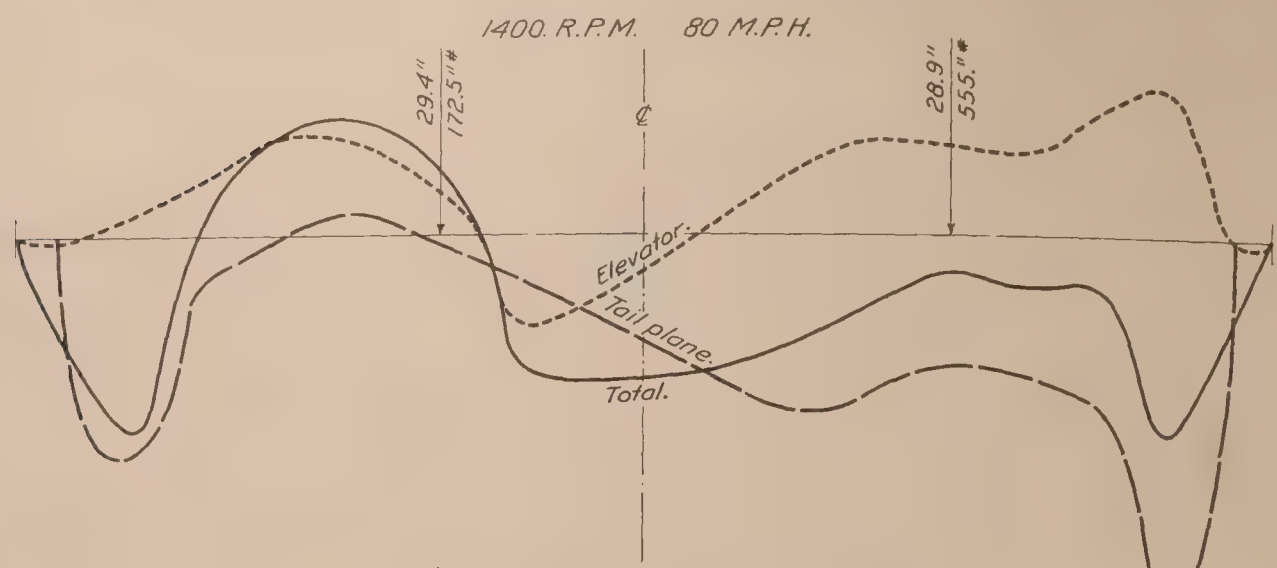


Fig. 255. Torque = Total moment = 382."# about ℄

CASE III.

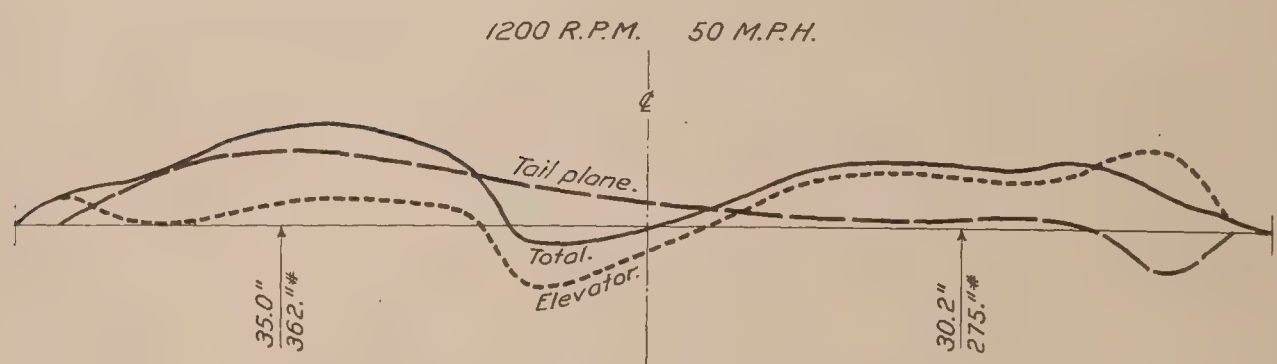


Fig. 256. Torque = Total moment = 87"# about ℄

CASE III.

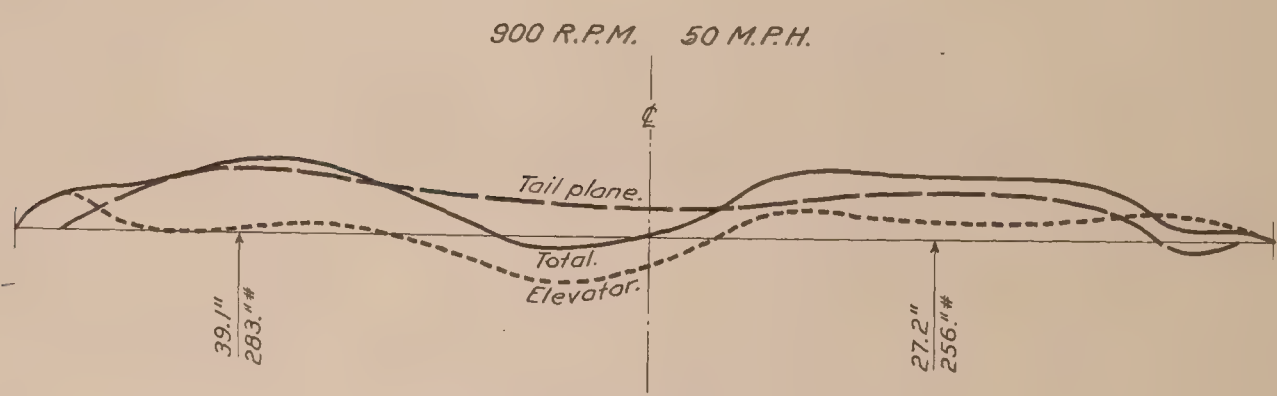


Fig. 257. Torque = Total moment = 27"# about ℄

CASE III.

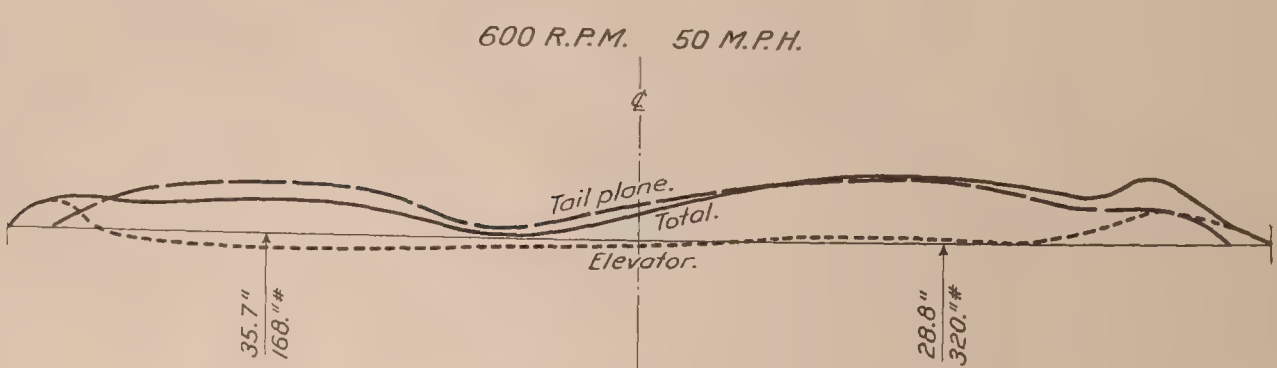


Fig. 258. Torque = Total moment = -152."# about ℄

Views of tail from rear.

CASE V.

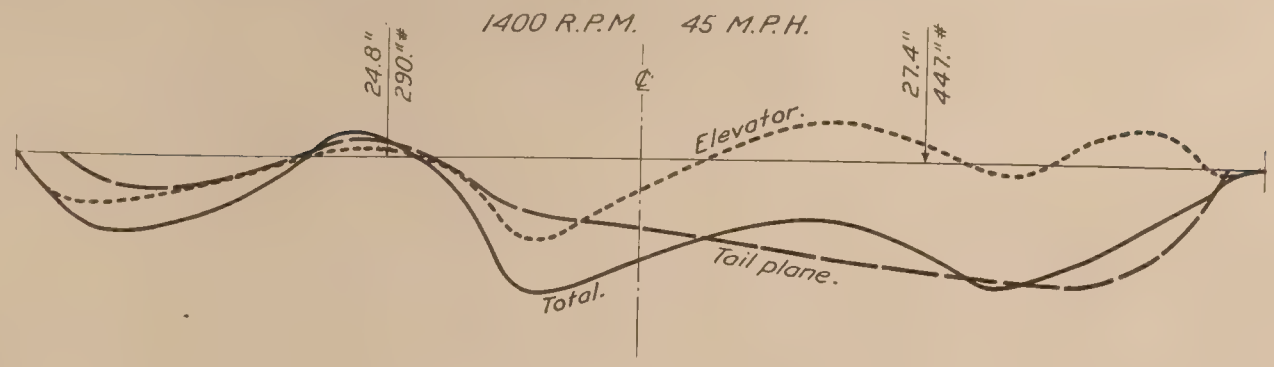


Fig. 259. Torque = Total moment = 157"# about ℄

CASE V.

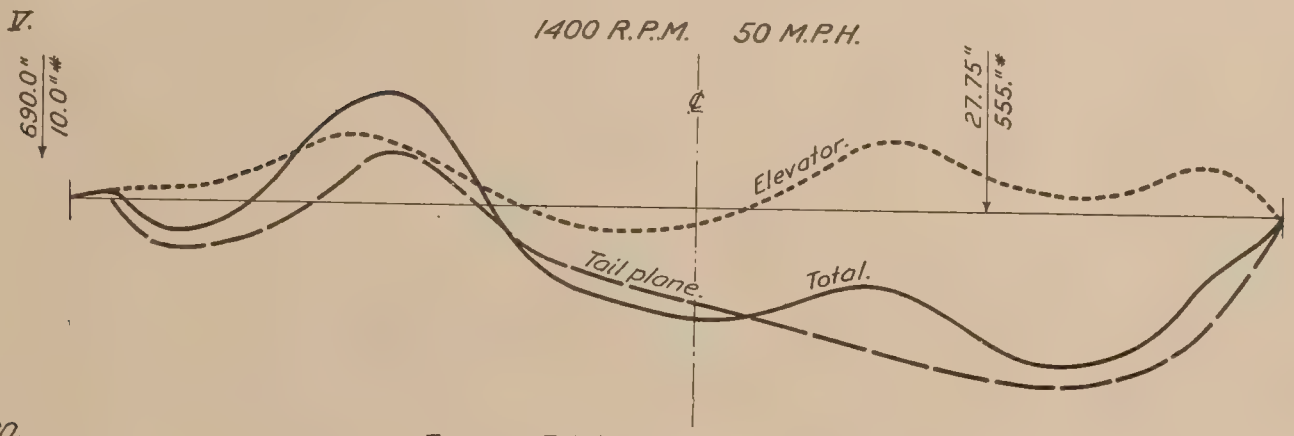


Fig. 260. Torque = Total moment = 545."# about ℄

CASE V.

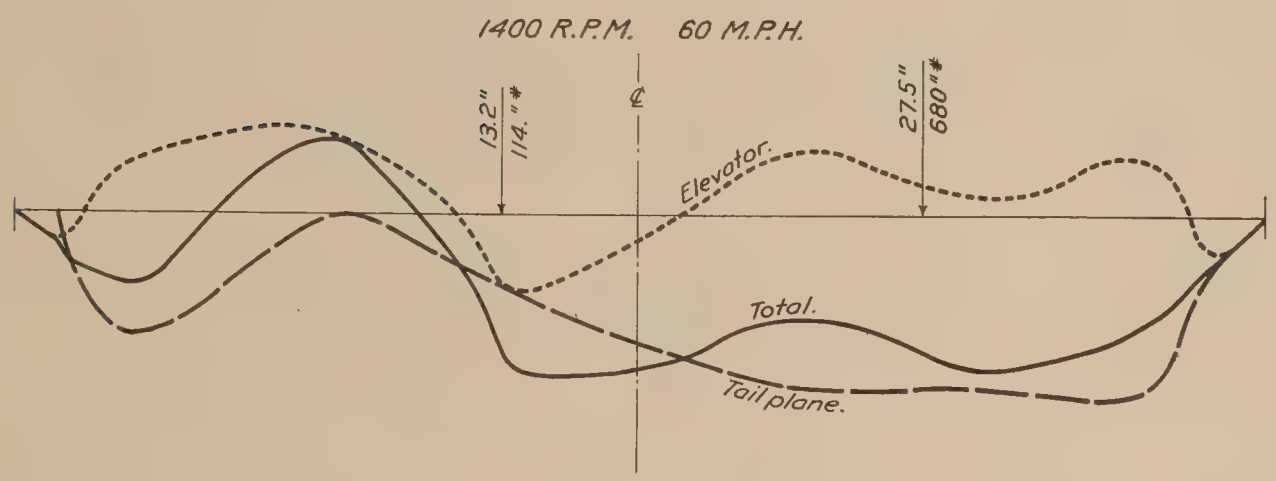


Fig. 261. Torque = Total moment = 566."# about ℄

CASE V.

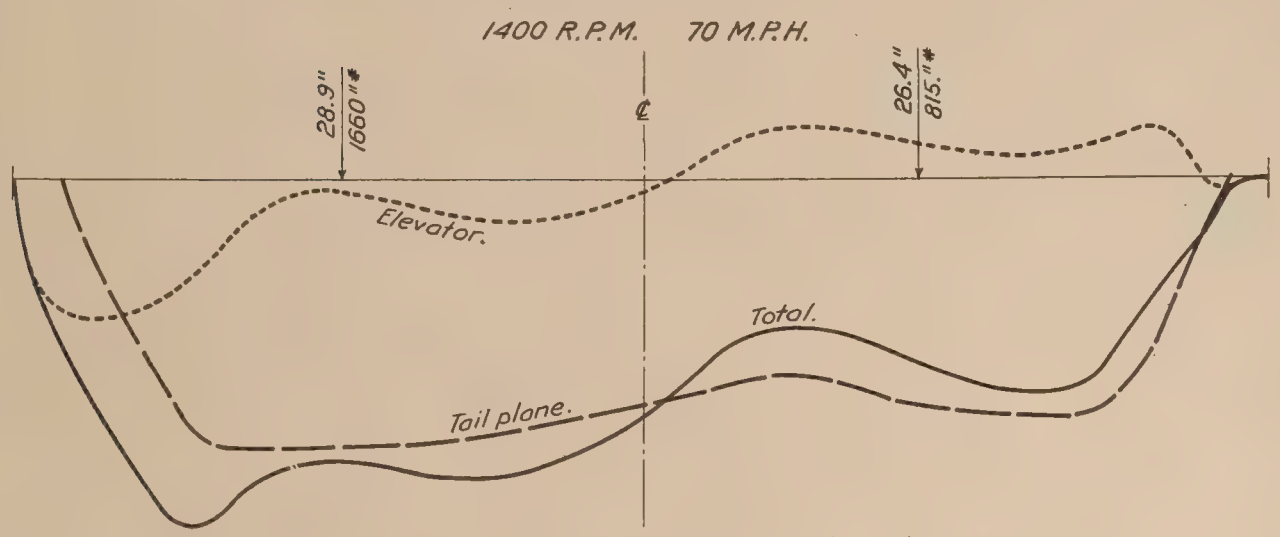


Fig. 262 Torque = Total moment = −845"# about ℄

Views of tail from rear.

CASE V.

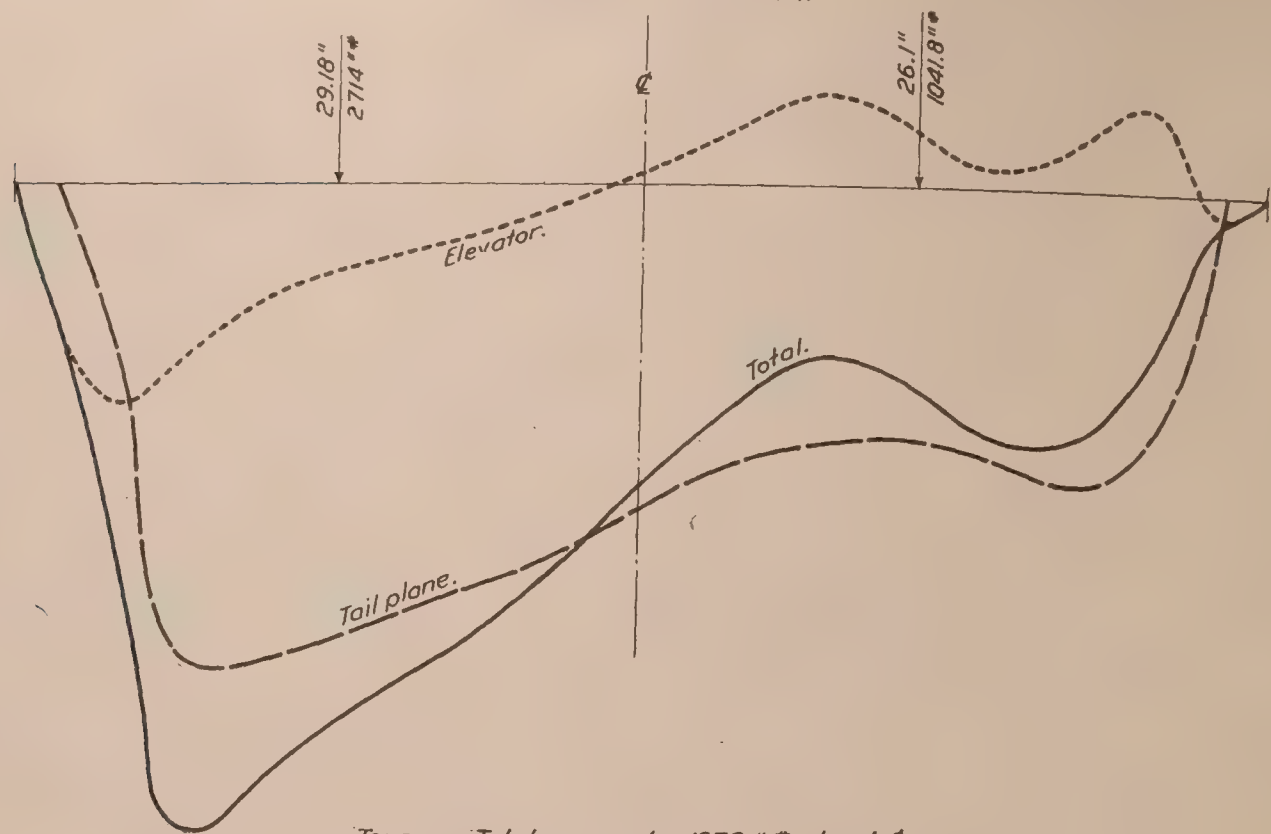


Fig. 263.

Torque = Total moment = -1673."# about ℄

CASE IV.

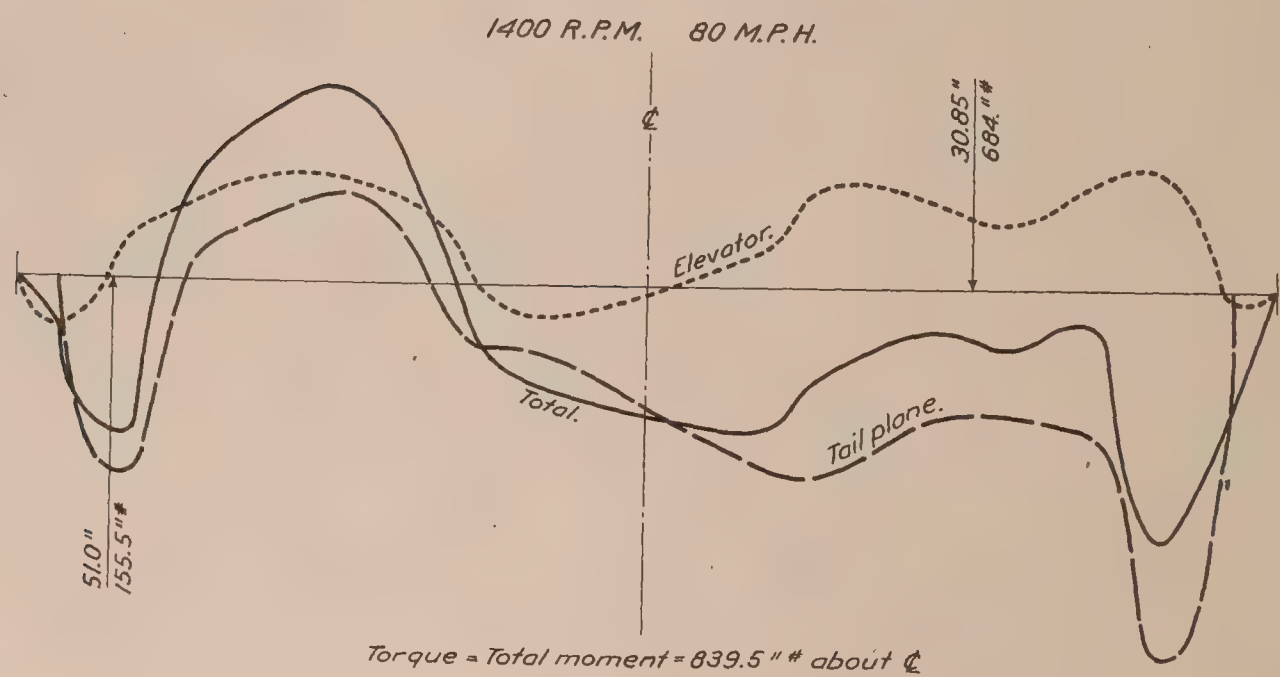


Fig. 264.

Torque = Total moment = 839.5"# about ℄

Views of tail from rear.

REPORT No. 119

THE PRESSURE DISTRIBUTION OVER THE HORIZONTAL TAIL SURFACES OF AN AIRPLANE—II

By F. H. NORTON and D. L. BACON

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

REPORT No. 119.

THE PRESSURE DISTRIBUTION OVER THE HORIZONTAL TAIL SURFACES OF AN AIRPLANE—II.

By F. H. Norton and D. L. Bacon.

SUMMARY.

This investigation was undertaken by the aerodynamic staff of the National Advisory Committee for Aeronautics at Langley Field in order to determine whether the results obtained upon model tail surfaces can be used to accurately predict loads upon the full-sized tail; and also to find the distribution of load when large elevator angles are used, as the loads from such angles can not be obtained readily in free flight. The method consisted in using a metal horizontal tail surface inside of which small air passages, connecting with a series of holes in the surface, led the pressure off from the tail in rubber tubes. In this way the pressure at each of these holes was measured by a manometer at several angles of attack and several elevator settings. The results show that the model tests give a loading which is equivalent to the loading under similar conditions in the full-sized airplane and that the manner of distribution is quite similar in the two cases when there is no slip stream.

INTRODUCTION.

The only work which has previously been done on the pressure distribution on model tail surfaces is given in the first two references below, the remaining references applying only to the forces on model tails. The scope of the present investigation comprises the determination of the distribution of the pressure over two separate tail planes, models of the tail planes used in National Advisory Committee for Aeronautics Report No. 118. The tail planes were tested on a fifteenth scale model of the JN4H, and runs were made at various angles of attack and elevator angles to cover all conditions that might occur in flight.

References:

"The Distribution of Pressure Over a Tail Plane." Bulletin of the Airplane Engineering Division, December, 1919.

"Experiments on the Effect of Altering the Position of the Hinges of the Elevators for the B.E.2C Aeroplane." British Advisory R. & M. No. 254.

"Pressure Distribution Over the Tail Plane of B2C, Part I," R. & M. No. 661.

"Flossendruckmessungen." Technische Berichte, vol. 1, No. 6, October, 1917.

"Systematishe Versuche an Leitwerkmodellen." Technische Berichte, vol. 1, No. 5, August, 1917.

APPARATUS AND METHODS.

A model was constructed in the usual manner with aluminum wings and wooden fuselage, the scale being one-fifteenth in order to make the size of the tail plane sufficient to allow large air passages. This model was supported in the center of the tunnel on a spindle from the N. P. L. balance so that it could be easily rotated to any angle of attack. As the forces on the model were considerable, the upper wing tip was firmly supported by three wires running to the side of the tunnel, but still allowing free rotation.

The tail planes, figures 1 and 2, were made of brass by Mr. Carl Selig, instrument maker at the Massachusetts Institute of Technology. The sections were one-fifteenth that given for the tail surfaces in Part I within 0.003 inch. Figure 3 shows their method of construction, which consisted of soldering together two plates which had already been grooved for the air passages. In order to prevent this solder from closing up the air passages aluminum wires were placed in them during the soldering. This soldering was so well done that there was

no leakage between the adjacent passages in any case. In order to save time it was desired to take a number of readings simultaneously, so that a separate connection was made to each air passage by means of fine rubber tubes about 1 millimeter in diameter leading to the multiple manometer as shown in figure 4.

The multiple manometer used for reading the pressures in this investigation is shown in figure 5 and consists simply of a reservoir of large area which is adjustable vertically, and a series of inclined tubes each having its corresponding scale. The two outside tubes are connected back to the reservoir so that they will read at all times the level in it, thus giving a pair of zero readings with which to compare the various heads.

When ready to make a test all of the holes in the tail plane were covered by strips of tissue paper which had been coated with paraffin. These strips of paper were applied over each row of holes with a hot iron, making a very smooth and tight joint without danger of plugging the air passages, a trouble that was encountered in previous tests. The whole system was then tested out for tightness by applying a small pressure to the reservoir in the gauge, and if this pressure was held it was proof that there were no leaks. After this test the strip of paper was removed from a single row of holes and the tunnel was run at a steady speed of 40 miles per hour. The pressures on the tail were compared with the static pressure in the tunnel by connecting a static head, which was placed near the model in the tunnel, to the reservoir of the manometer.

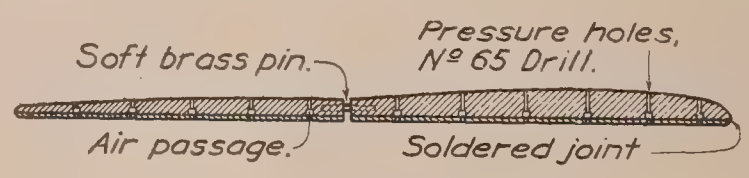


Fig. 3. METHOD OF CONSTRUCTING TAIL SURFACES.

The readings of pressure obtained were converted into vertical inches of water, and the plotting of the results carried out in exactly the same way as for Part I, excepting that the scale in this case was somewhat increased because the pressures read were considerably smaller than those obtained on the full-sized tail plane. It was not thought worth while to determine the position of the center of pressure on the model tail plane, as it was easier to find the moment about the center of gravity of the machine mechanically than it was to find it by the integrated pressures.

PRECISION OF RESULTS.

The heads of water as computed may be considered precise to 0.005 of an inch and the areas measured with a precision of 0.01 of a square inch, which is the same as for the full-flight tests. These model tests, however, do not have the errors which were inevitable in the full-flight tests due to varying atmospheric conditions, so that the evenness of the results from the model tests may be considered quite superior to those obtained in free flight, and this is evident by the smoothness and regularity of the load curves.

SCOPE OF THE TESTS.

Two horizontal tail surfaces were used in this investigation, being one-fifteenth scale models of those used in Part I. Each tail was tested at the following angles of attack of the tail plane of the airplane: $-2°$, $4°$, $10°$, $16°$, $20°$. At each angle of attack the elevator was placed at $-15°$, $-5°$, $0°$, $+5°$, $+15°$, $+25°$, all in respect to the tail plane. It should be noted that in this report a positive elevator angle is taken when the trailing edge is pulled up. In order to determine the effect on the model of closing the crack between the elevator and tail plane the pressure distribution was taken along one row of holes. In the same way the effect of giving the tail plane a more positive angle of 2° was determined.

DISCUSSION OF RESULTS.

In figures 6 to 65 are shown the distribution of pressure over the two tail planes under various conditions of angle of attack and elevator setting. The most prominent characteristic evident on the JN4H tail is the region of marked down load at the outer leading edge for small angles of

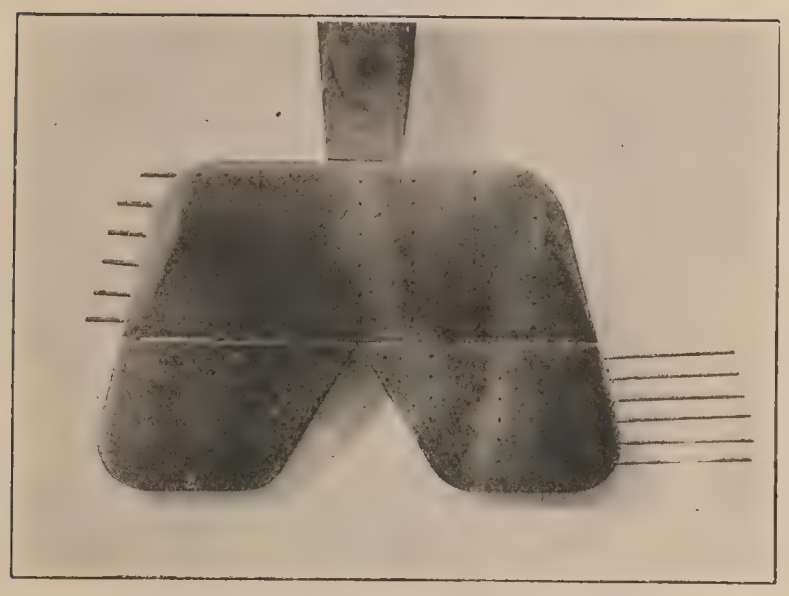


FIG. 1.—MODEL OF JN4H TAIL SURFACE.

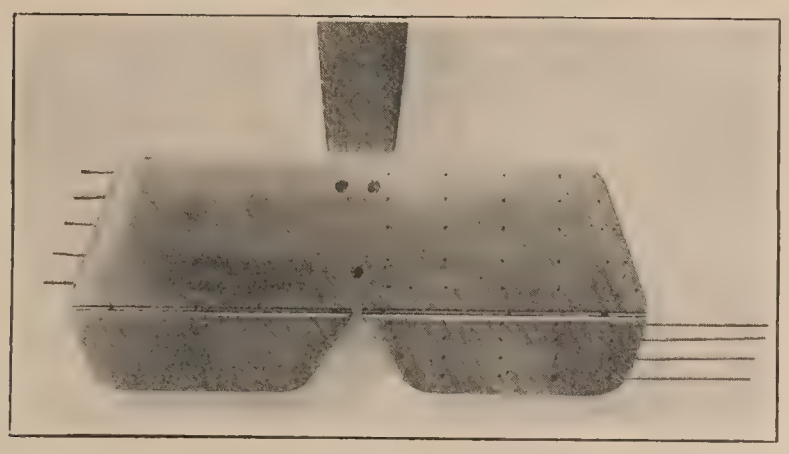


FIG. 2—MODEL OF SPECIAL TAIL SURFACE.

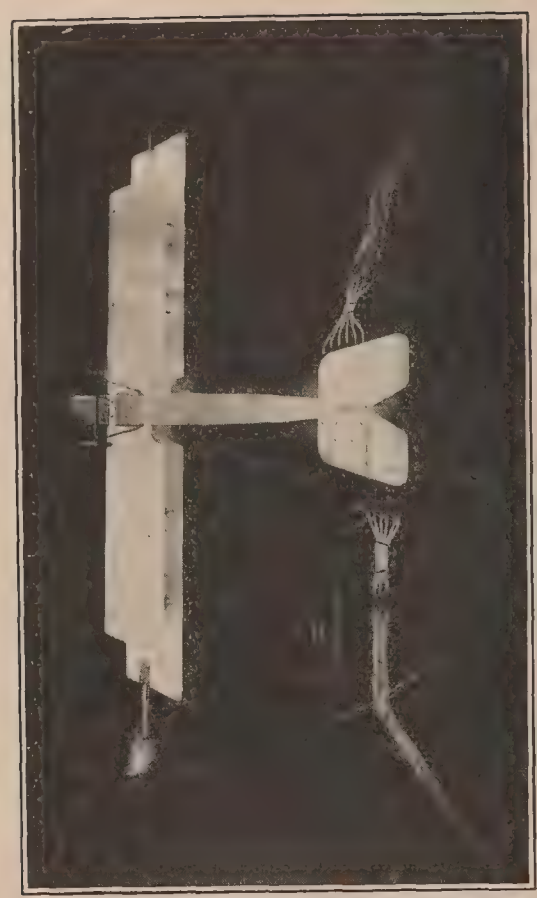


FIG. 4.—PRESSURE DISTRIBUTION MODEL SET UP IN THE TUNNEL.

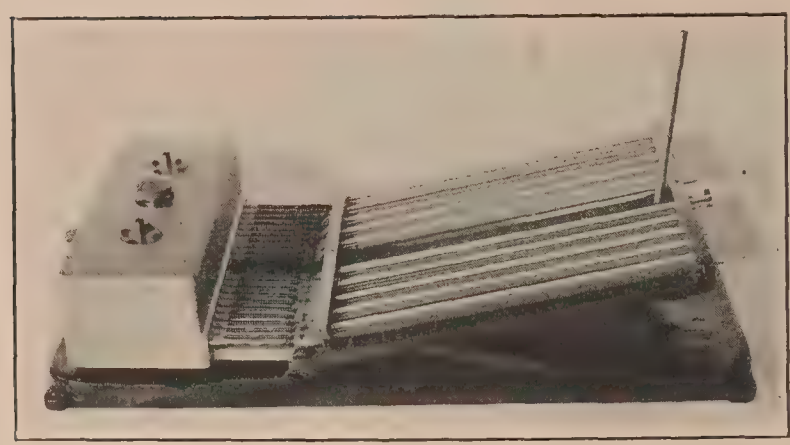


FIG. 5.—MULTIPLE MANOMETER FOR MEASURING PRESSURES ON THE TAIL.

attack and especially when the elevator is pulled up, a region of down load at the outer edge of the elevator due to the positive rake of that member. On the other hand, when the elevator is pulled down there is a sharp up load at the outer edge of the whole tail plane. In contrast to this, the special tail plane does not show a very high loading at the outer tip and the general distribution is more even along the span.

Turning now to figures 66 to 77, which show the load on the tail in cubic inches of water along the chord, it is seen that there are two distinct regions of high load, the first region being at the leading edge and the second just behind the hinge. It will be seen that the region at the leading edge varies mainly with the angle of attack while the region on the elevator varies mainly with the elevator setting, although each one is slightly affected by the other. The region of high loading on the elevator is much more concentrated and pronounced on the special tail plane than it is on the other.

By making the assumption that an airplane will retain its velocity for an instant after pulling up the elevator sharply, we may compute the loads that will arise on the full-scale tail plane in accelerated flight. For example, from figure 81 the load on the model tail at 0° attack and 0° elevator is 1 cubic inch. As this corresponds to 100 miles per hour full scale, the load on the tail will be 51 pounds, or about 1 pound per square foot, which checks very well with the value actually obtained in full flight of 0.97 pound. If the elevator is pulled up to 25° while the same speed is maintained the load, from the model curves, is increased nine times or to 459 pounds, which gives a loading of 9 pounds per square foot. The load encountered here is not large, even for the severity of the conditions, and it is interesting to compare it with the load factor of 5.5 for the wings as determined under identical conditions.[1]

A run was made on the JN4H horizontal tail surface with the crack between the elevator and tail plane closed up with wax, and another run was made on the same tail surface with the tail plane angle changed by 2°. As these changes, however, made no appreciable effect upon the distribution of pressure it was not thought worth while to include the curves of the results.

STABILIZING AND CONTROLLING PROPERTIES OF THE TAIL SURFACES.

In figures 78 and 79 there are plotted the loads on the tail plane of the two tail surfaces for various angles of attack and for various elevator settings. An evident fact is that the force on the special tail plane when the elevator is set at +5° crosses the curve for zero elevator at high angles and the force with the elevator down −5° is also very close to the force when the elevator is at zero. This all confirms what has been found in full flight, that the controlling effect of this tail surface is very poor. This is probably due to the fact that the tail is so thick and the elevator so small that a small movement of the elevator does not influence the flow passing over the forward tail plane surfaces. This leads to the conclusion that a tail should not be made of such a thick section, although more tests should be made before this statement can be considered conclusive.

In figure 80 there is plotted the force on the elevator alone for various angles of attack and elevator settings. It is evident that the force on the special tail surface varies slightly with the angle of attack; the force, however, seems to be almost exactly proportional to the angle of the elevator. On the JN4H tail, however, the force changes quite markedly with the angle of attack and changes even more than for the special tail when the elevator angle is changed.

In figures 81 and 82 there are plotted the curves for the total load on the tail surface, which is simply the summation of the previous curves. It will be noted that the slope of the curves for the special tail is considerably greater than for the JN4H tail, the former giving, as would be expected from its greater aspect ratio, about 13 per cent greater force than the other for a given change in angle of attack. A change in angle of attack of the whole surface of 1° has about the same effect on the total loading as a change of 1½° of the elevator.

In figure 83 there are plotted the pitching moments of the complete airplane for each tail plane and also for the wing cell, and the wing cell plus the fuselage and landing gear. All of these moments were taken about a center of gravity position which was 38 per cent back on the

[1] Accelerations in Flight. N. A. C. A. Report No. 99.

mean chord, which is about the standard position for the full-sized airplane. It will be seen that the moment curves for the wing cell alone are unstable up to about 16°, beyond which they are stable, which is typical of a biplane of this type. The curve of moments for the wing cell with the fuselage and chassis attached lies remarkably close to that of the wing cell alone, showing that these members have an inappreciable effect on the total moment of the airplane. Turning to the moment curves for the complete airplane it at first seems rather surprising that they all show a region of instability at angles lower than 4°. If it is remembered, however, that the models were tested under conditions of locked controls, it will be seen that the stability is almost identical with the full-flight tests. This brings up the question of why an airlpane is more stable with free than with locked controls under certain conditions. This does not seem reasonable, as in the latter case the effective tail area is much greater. A possible explanation is that the weight of the elevator itself changes the configuration of the tail-plane section. At any rate, all wind-tunnel models should be tested with both free and locked elevators, the free elevators having a weight proportional to the full-scale ones.

It was found by the British [2] in model and free-flight tests on elevators that the efficiency of the latter was greatly increased by the use of a negative rake. The reason for this, which was not clear at that time, is shown by a study of the pressure distribution on the two tails. The JN4H tail (positive rake) shows a very high loading along its outer edge due to its action as a leading edge. The other tail, however, with a negative rake shows that this load is transferred to the tail plaine, so for a given controlling effect there is much less load on the movable portion.

COMPARISON OF FULL-FLIGHT TESTS WITH MODEL TESTS.

The principal reason for conducting these model tests was to determine whether the loads on the full-sized tail plane could be accurately predicted from the results obtained on the model. Unfortunately, however, there are few cases where a direct comparison may be made. First, because of the slip stream on the full-sized airplane, and, second, because the elevator angle is not known to better than 1° at high speeds, and perhaps not better than 3° at the very low speeds on the full-sized airplane. In figures 84 and 85 there are plotted for comparison the model and full-flight tests to equivalent scales; that is, the model tests have been multiplied by the ratio of the speeds squared. These conditions were chosen because there was no slip stream in the free-flight case. A careful examination of these figures will show that the agreement between the two cases is quite disappointing. While the tendencies of the curves are the same in a great many cases it is quite evident that the assymetry of the full-flight tests even when there is no appreciable slip stream is quite large. The lack of agreement of the pressures on the elevator may be due to the fact that the elevator setting was not exactly the same in both cases.

Probably a better comparison can be had between the model and full scale by comparing the total load, as has been done for the two tail planes in figure 86. The full-scale results were taken when there was no slip stream. The corresponding curves show a fairly good agreement except at low speeds with the JN4H tail, where the elevator was turned to a large angle. The discrepancies that do occur can readily be accounted for by the lack of agreement between the elevator setting in corresponding cases. As the normal movement of the elevator in flight is only a few degrees, an error of one degree will greatly affect the loading.

CONCLUSIONS.

Within the error of comparison, the model tests will give the tail load experienced on the full-sized airplane in steady flight and without slip stream. The stability of the model compares closely with that of the full-sized airplane if care is taken to make the conditions identical. The results emphasize the desirability of using a tail surface of high aspect ratio, of not too thick a section, and with negatively raked tips. If the comparison is assumed to hold good between model and full size for accelerated flight, the tail load on a JN4H with standard tail would amount to 9 pounds per square foot when suddenly pulling out of a 100 mile per hour dive.

[2] British Advisory Committee. R. & M. 409, T. 825, T. 1115.

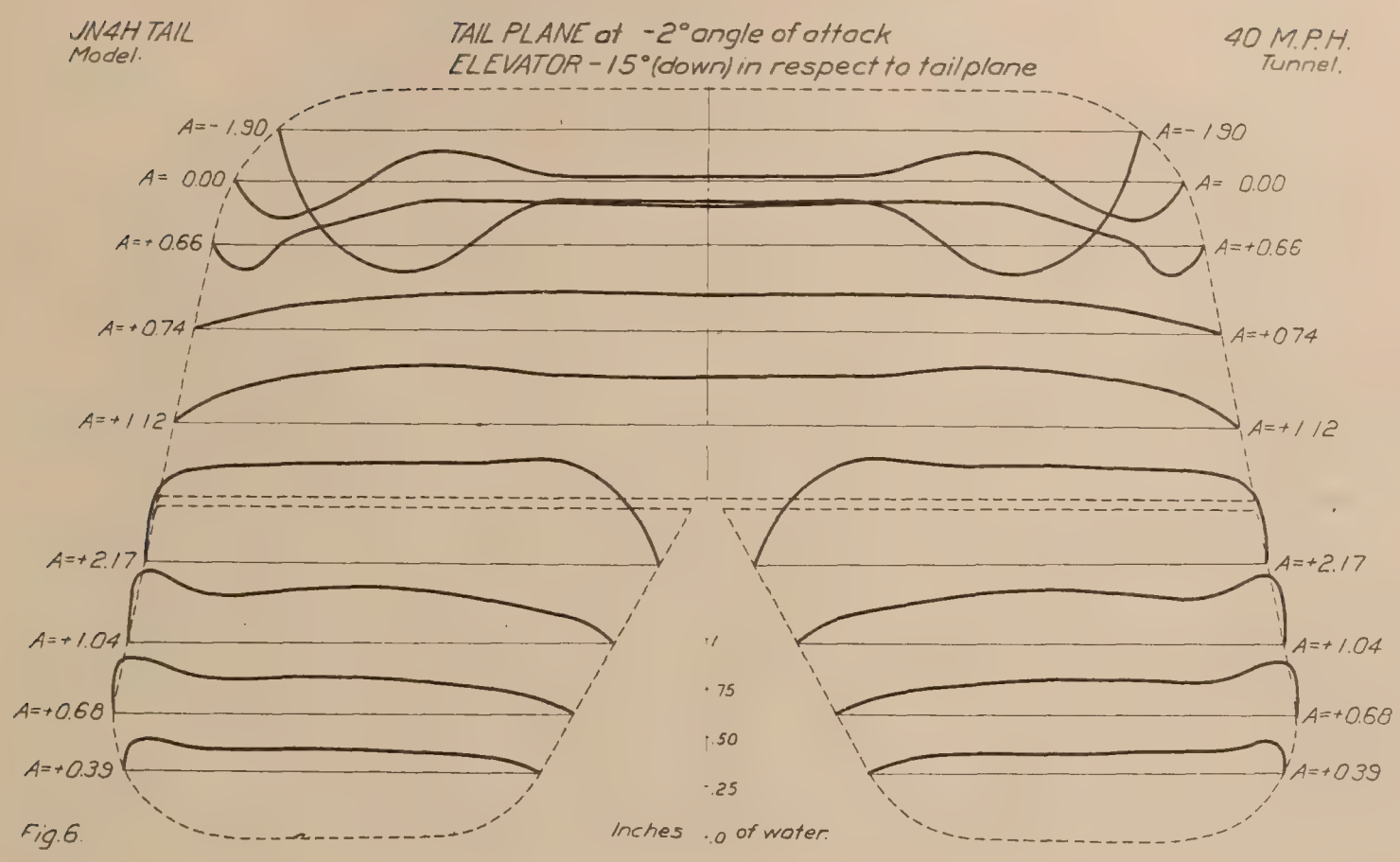

JN4H TAIL
Model.
TAIL PLANE at -2° angle of attack
ELEVATOR - 15° (down) in respect to tail plane
40 M.P.H.
Tunnel.
A=-1.90
A= 0.00
A=+0.66
A=+0.74
A=+1.12
A=+2.17
A=+1.04
A=+0.68
A=+0.39
1
.75
.50
.25
.0
Inches of water.

Fig. 6.

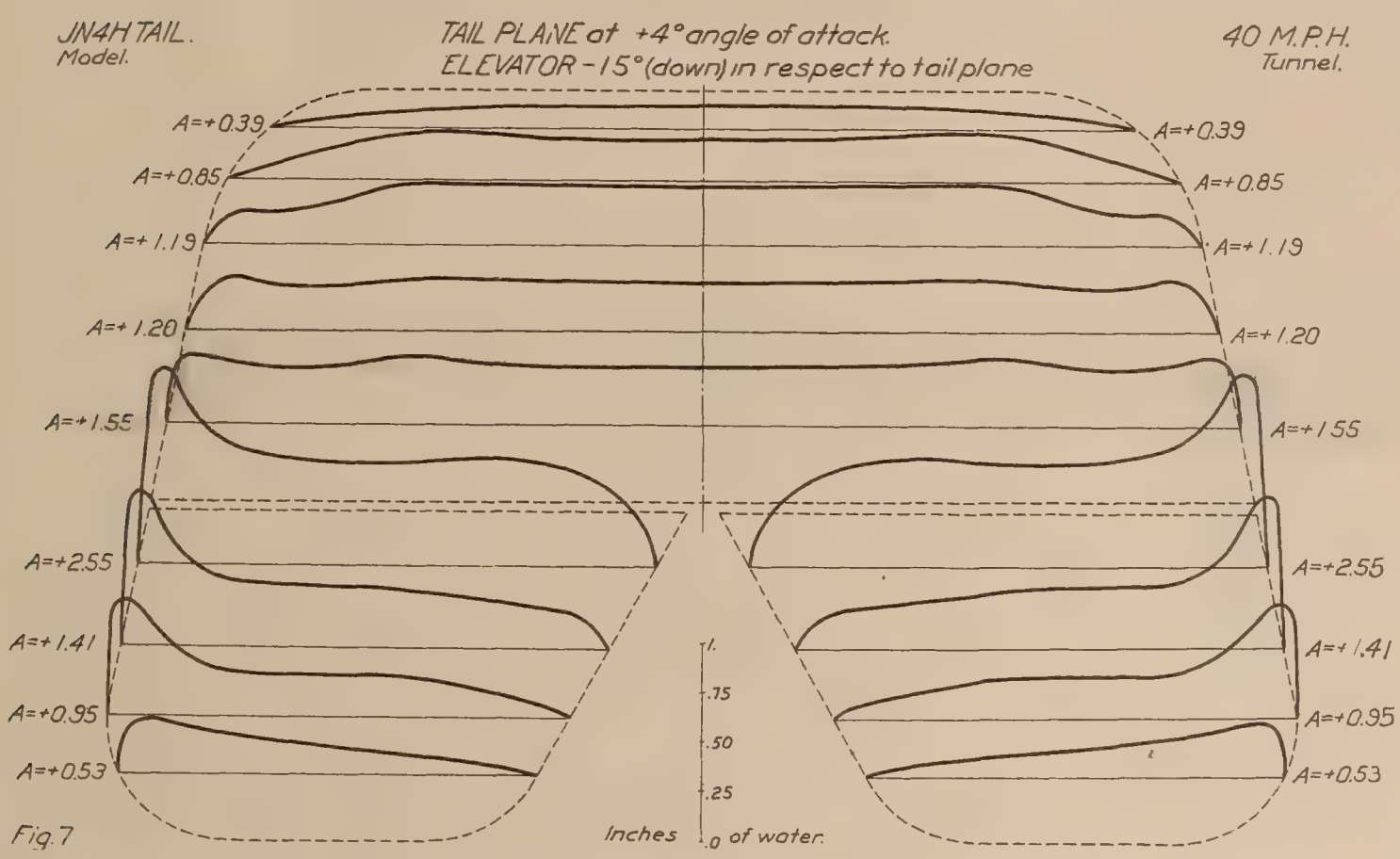

JN4H TAIL.
Model.
TAIL PLANE at +4° angle of attack.
ELEVATOR - 15° (down) in respect to tail plane
40 M.P.H.
Tunnel.
A=+0.39
A=+0.85
A=+1.19
A=+1.20
A=+1.55
A=+2.55
A=+1.41
A=+0.95
A=+0.53
1.
.75
.50
.25
.0
Inches of water.

Fig. 7

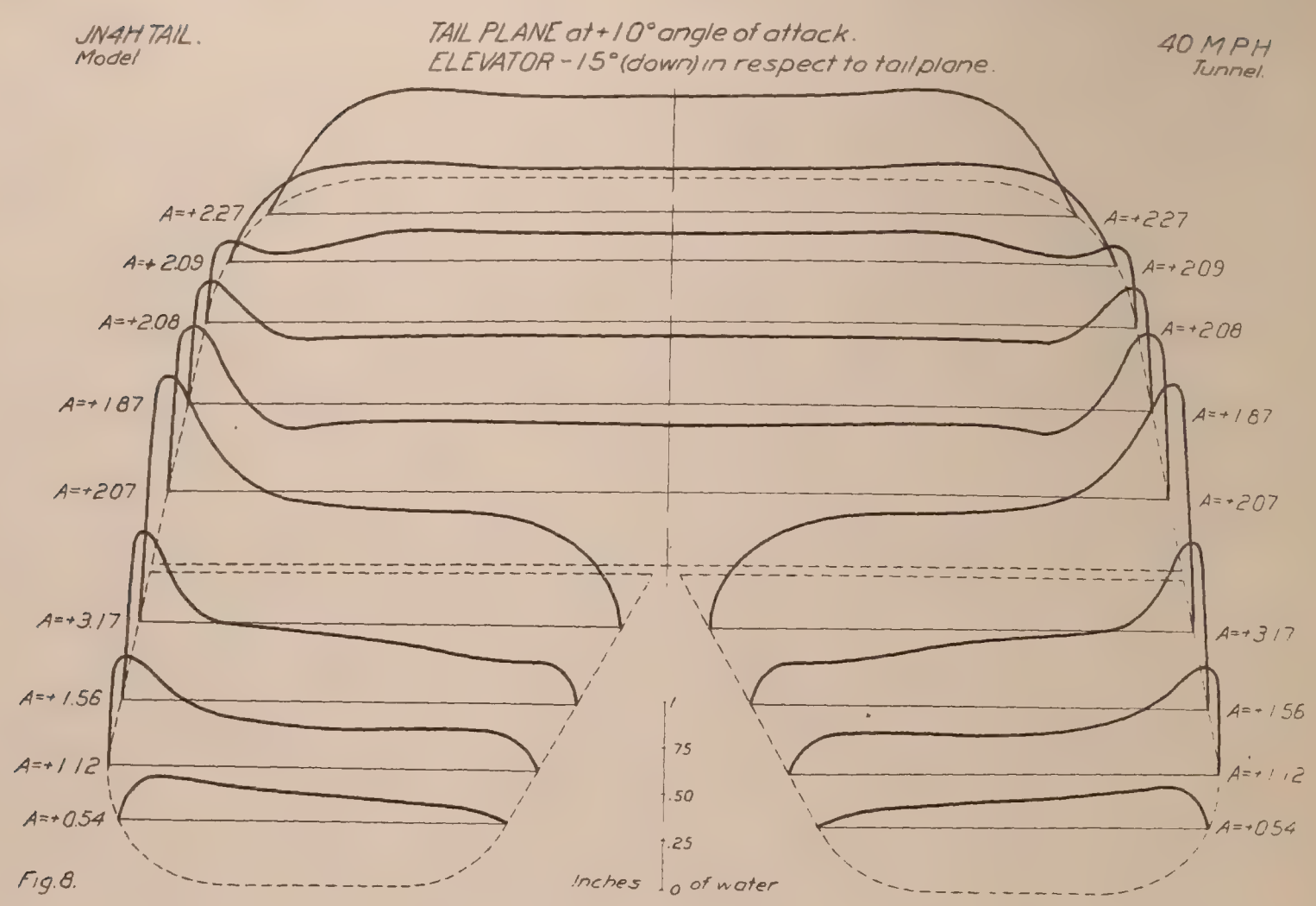

JN4H TAIL.
Model
TAIL PLANE at +10° angle of attack.
ELEVATOR -15° (down) in respect to tail plane.
40 MPH
Tunnel.
A=+2.27
A=+2.09
A=+2.08
A=+1.87
A=+2.07
A=+3.17
A=+1.56
A=+1.12
A=+0.54
1
.75
.50
.25
0
Inches of water

Fig. 8.

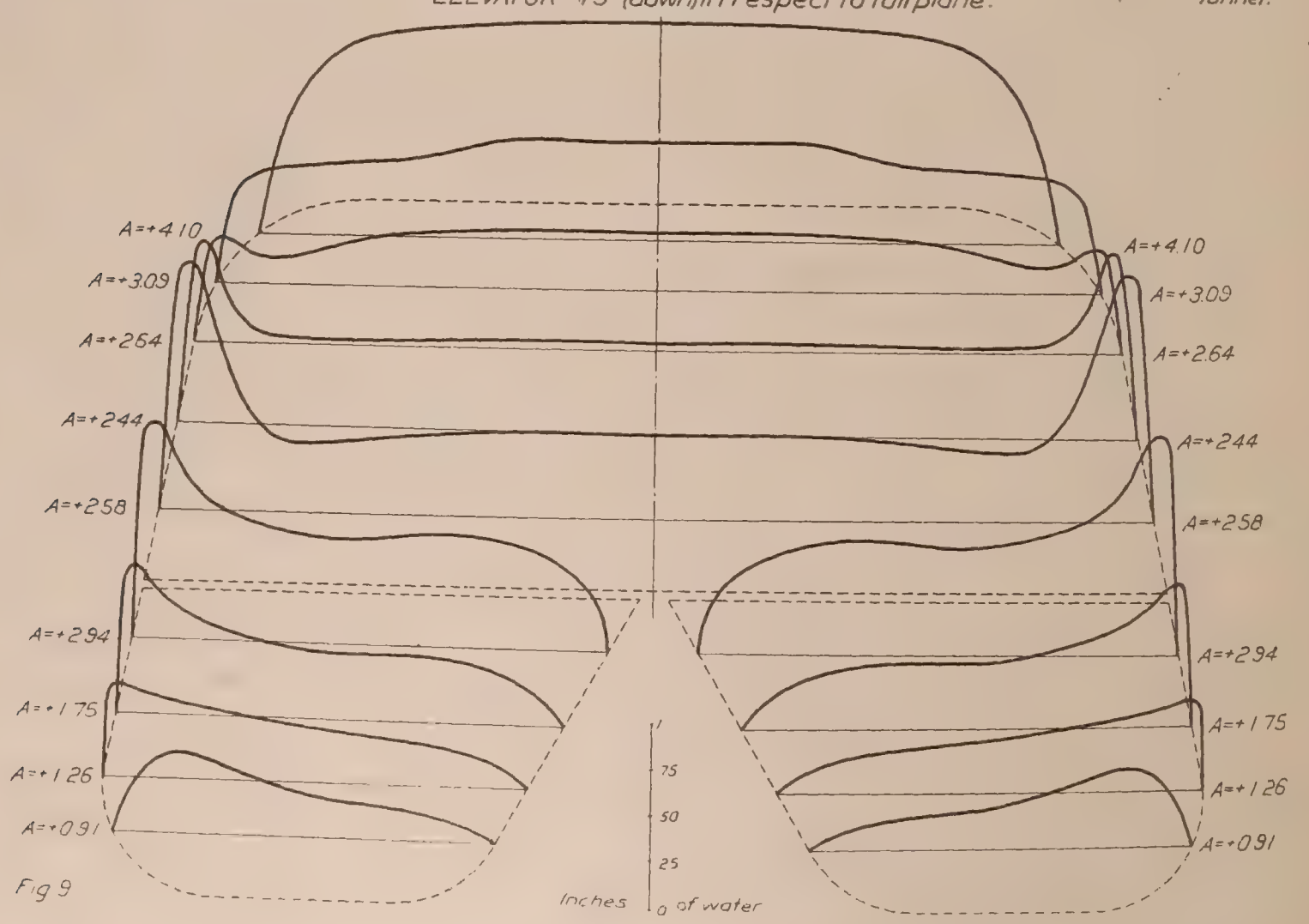

JN4H TAIL
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR -15° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+4.10
A=+3.09
A=+2.64
A=+2.44
A=+2.58
A=+2.94
A=+1.75
A=+1.26
A=+0.91
1
.75
.50
.25
0
Inches of water

Fig 9

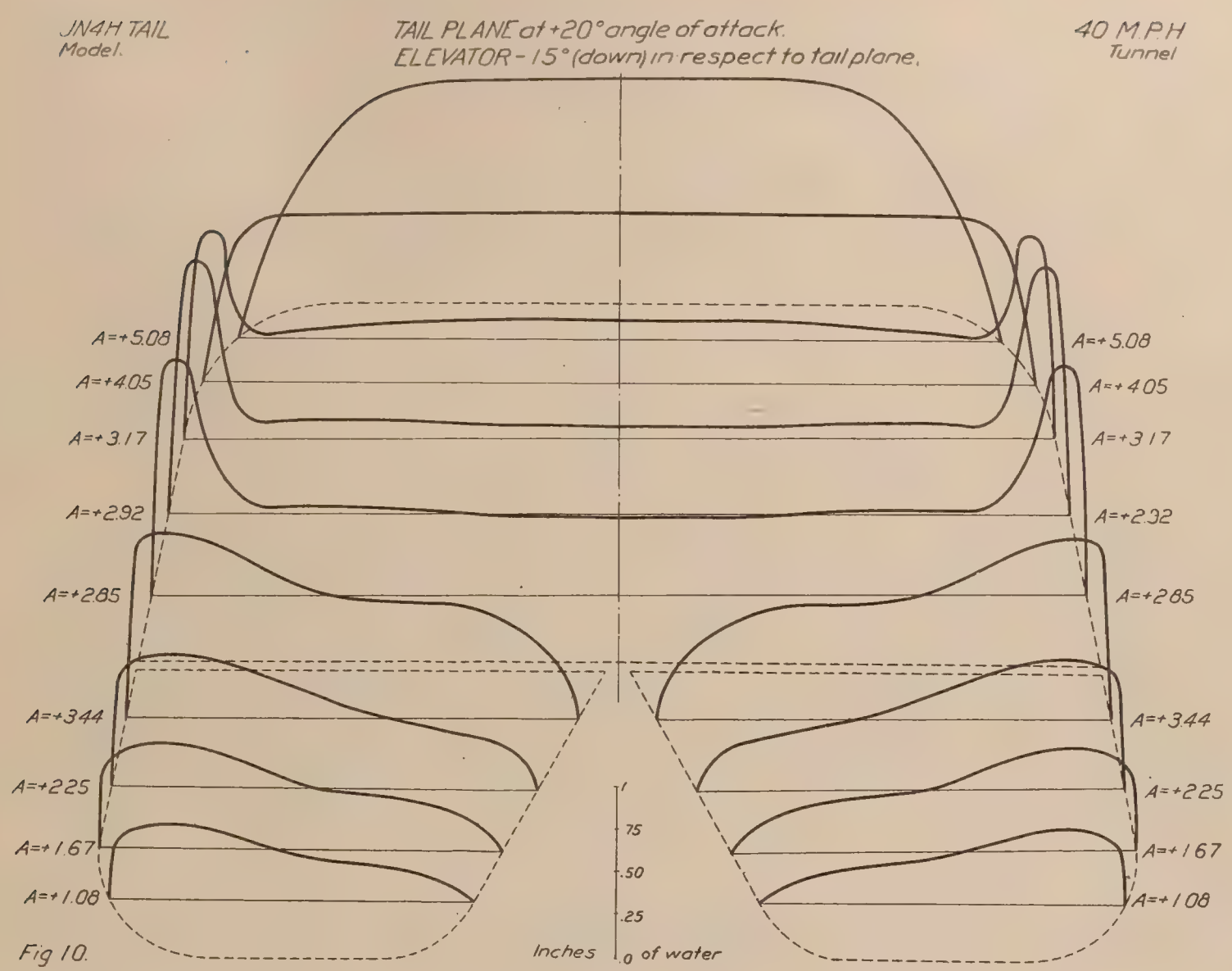

JN4H TAIL
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR - 15° (down) in respect to tail plane.
40 M.P.H
Tunnel
A=+5.08
A=+4.05
A=+3.17
A=+2.92
A=+2.85
A=+3.44
A=+2.25
A=+1.67
A=+1.08
1
.75
.50
.25
0
Inches of water

Fig 10.

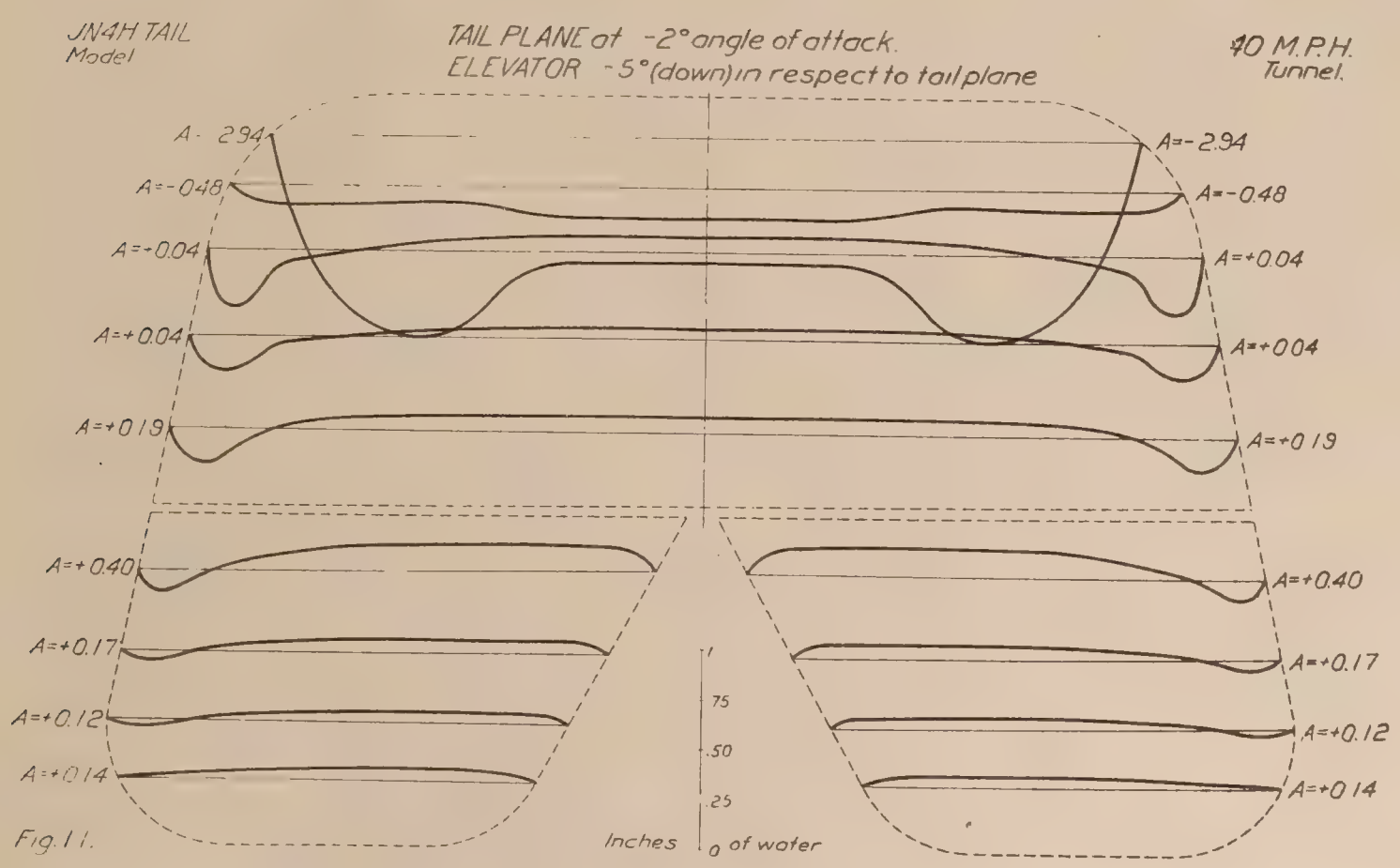

JN4H TAIL
Model
TAIL PLANE at -2° angle of attack.
ELEVATOR -5° (down) in respect to tail plane
40 M.P.H.
Tunnel.
A=-2.94
A=-0.48
A=+0.04
A=+0.04
A=+0.19
A=+0.40
A=+0.17
A=+0.12
A=+0.14
1
.75
.50
.25
0
Inches of water

Fig. 11.

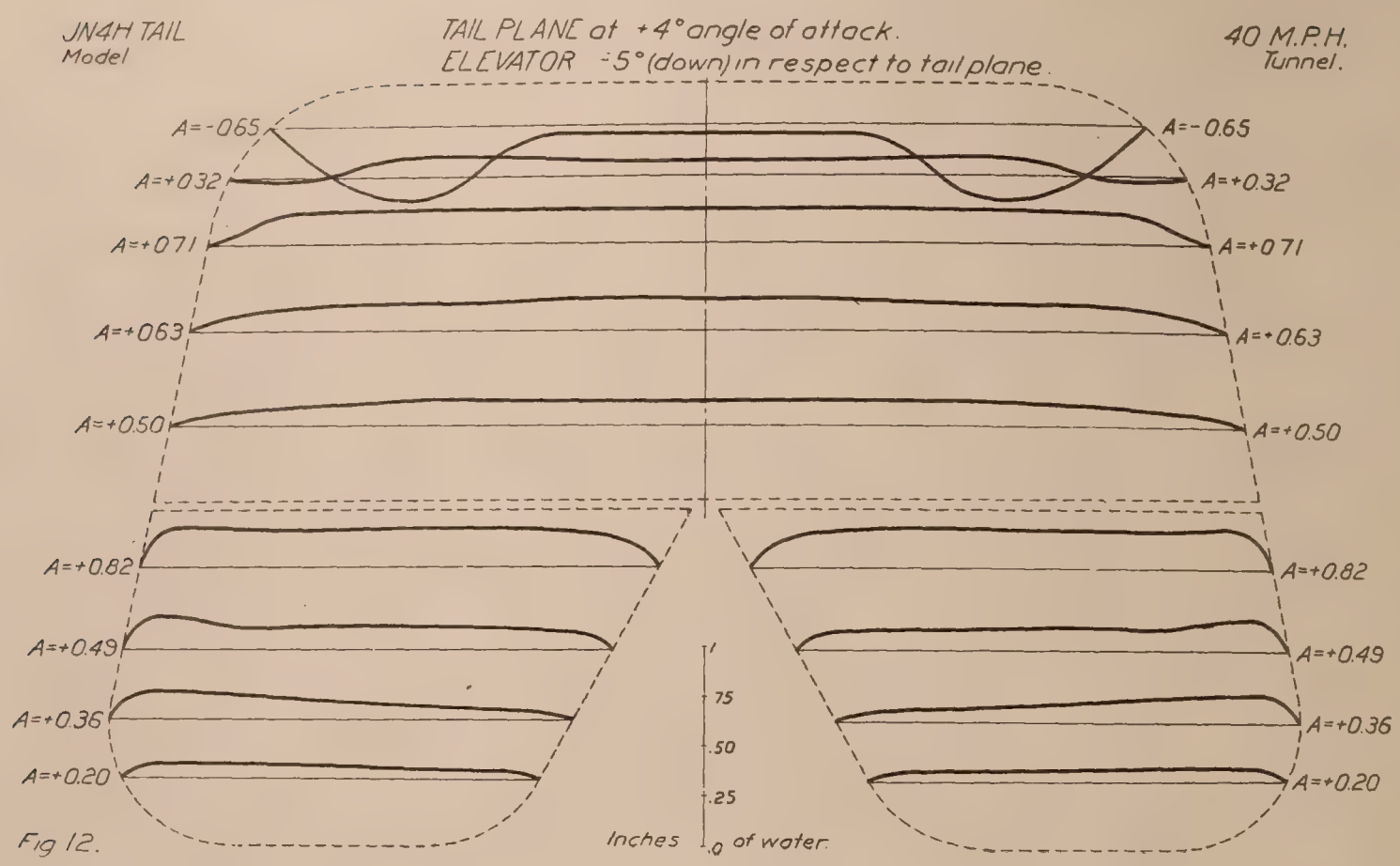

JN4H TAIL
Model
TAIL PLANE at +4° angle of attack.
ELEVATOR -5° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=-0.65
A=+0.32
A=+0.71
A=+0.63
A=+0.50
A=+0.82
A=+0.49
A=+0.36
A=+0.20
1
.75
.50
.25
.0
Inches of water.

Fig 12.

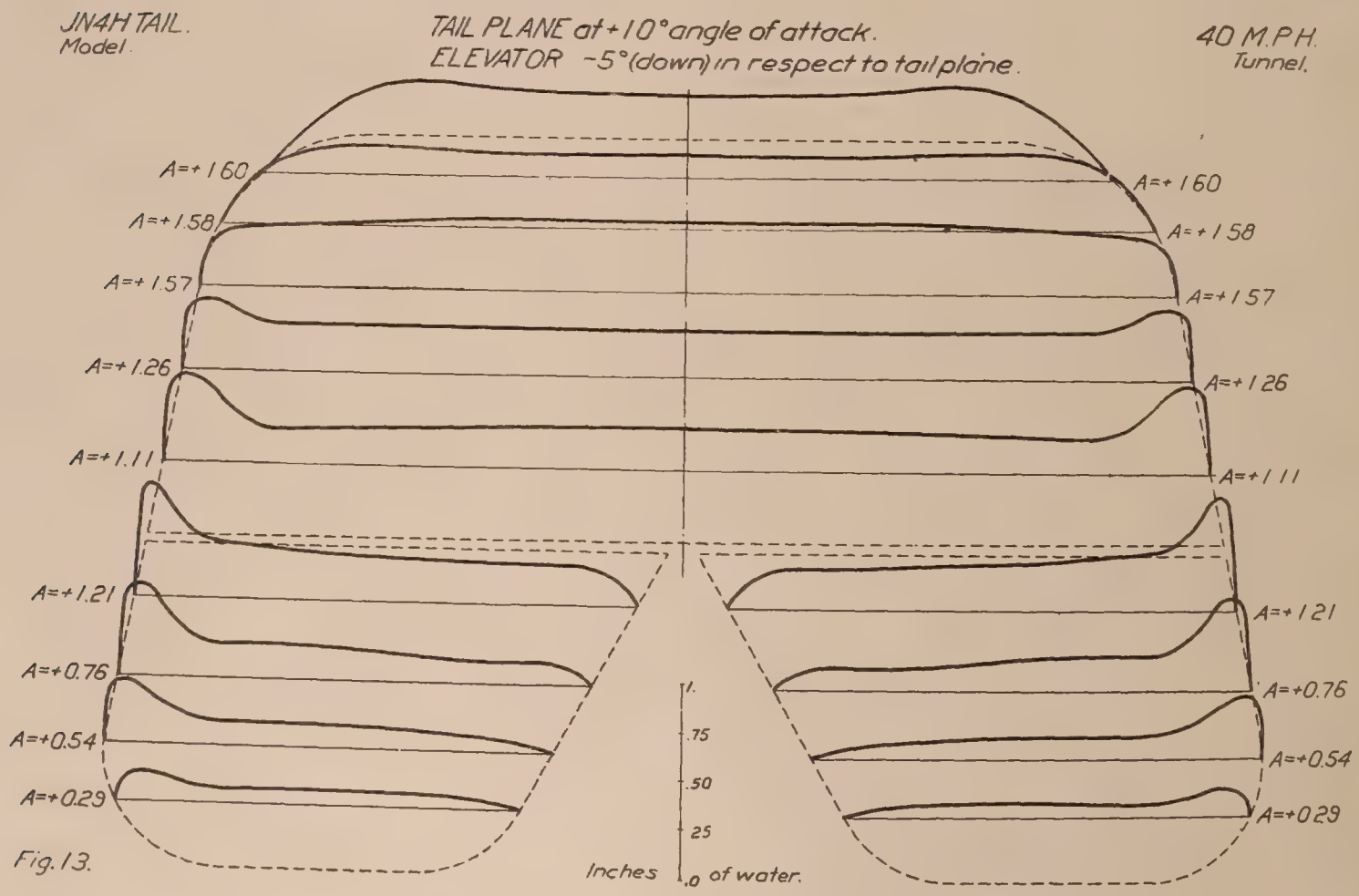

JN4H TAIL.
Model.
TAIL PLANE at +10° angle of attack.
ELEVATOR -5° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+1.60
A=+1.58
A=+1.57
A=+1.26
A=+1.11
A=+1.21
A=+0.76
A=+0.54
A=+0.29
1.
.75
.50
.25
.0
Inches of water.

Fig. 13.

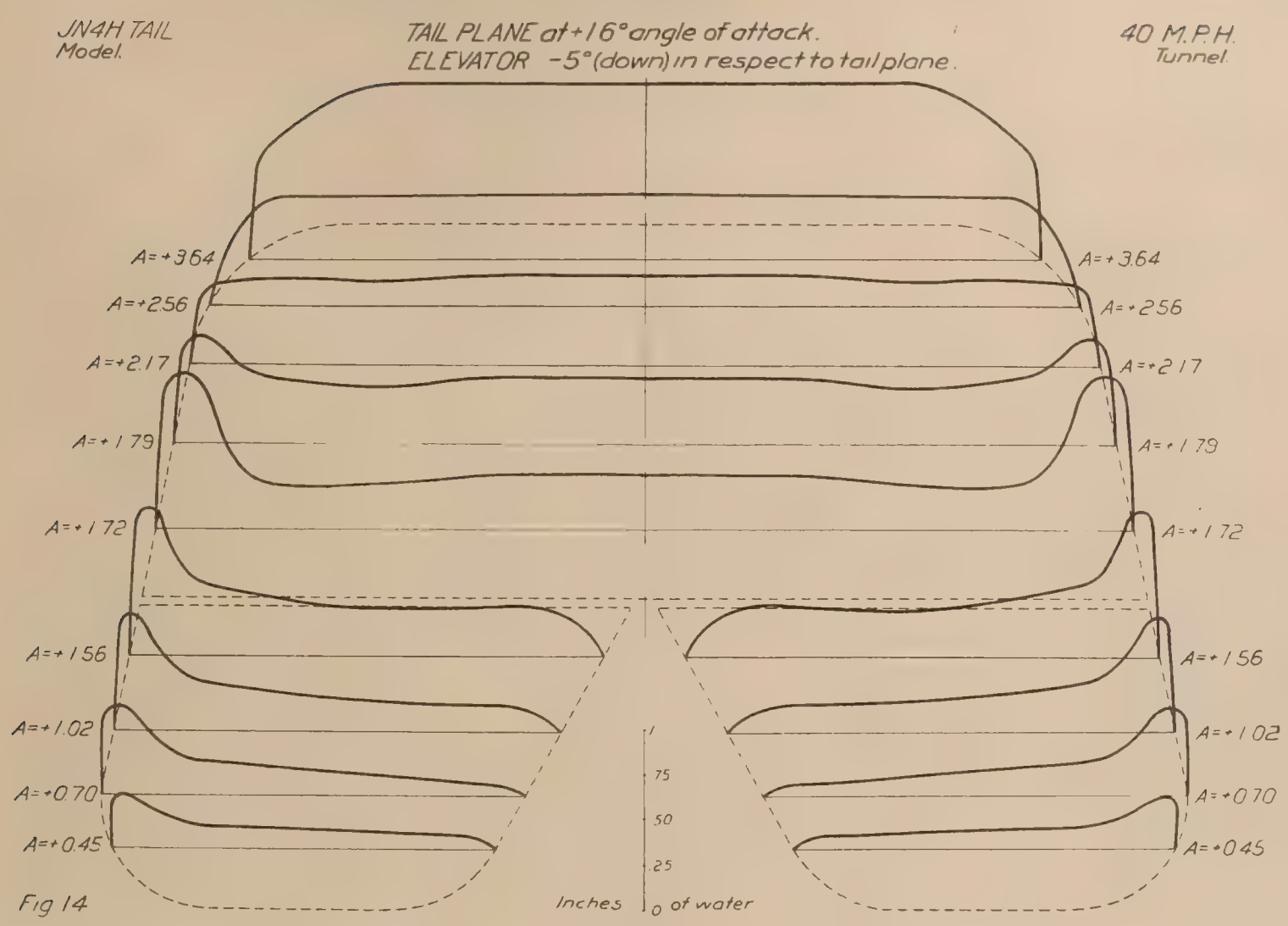

JN4H TAIL
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR -5° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+3.64
A=+2.56
A=+2.17
A=+1.79
A=+1.72
A=+1.56
A=+1.02
A=+0.70
A=+0.45
1
.75
.50
.25
0
Inches of water

Fig 14

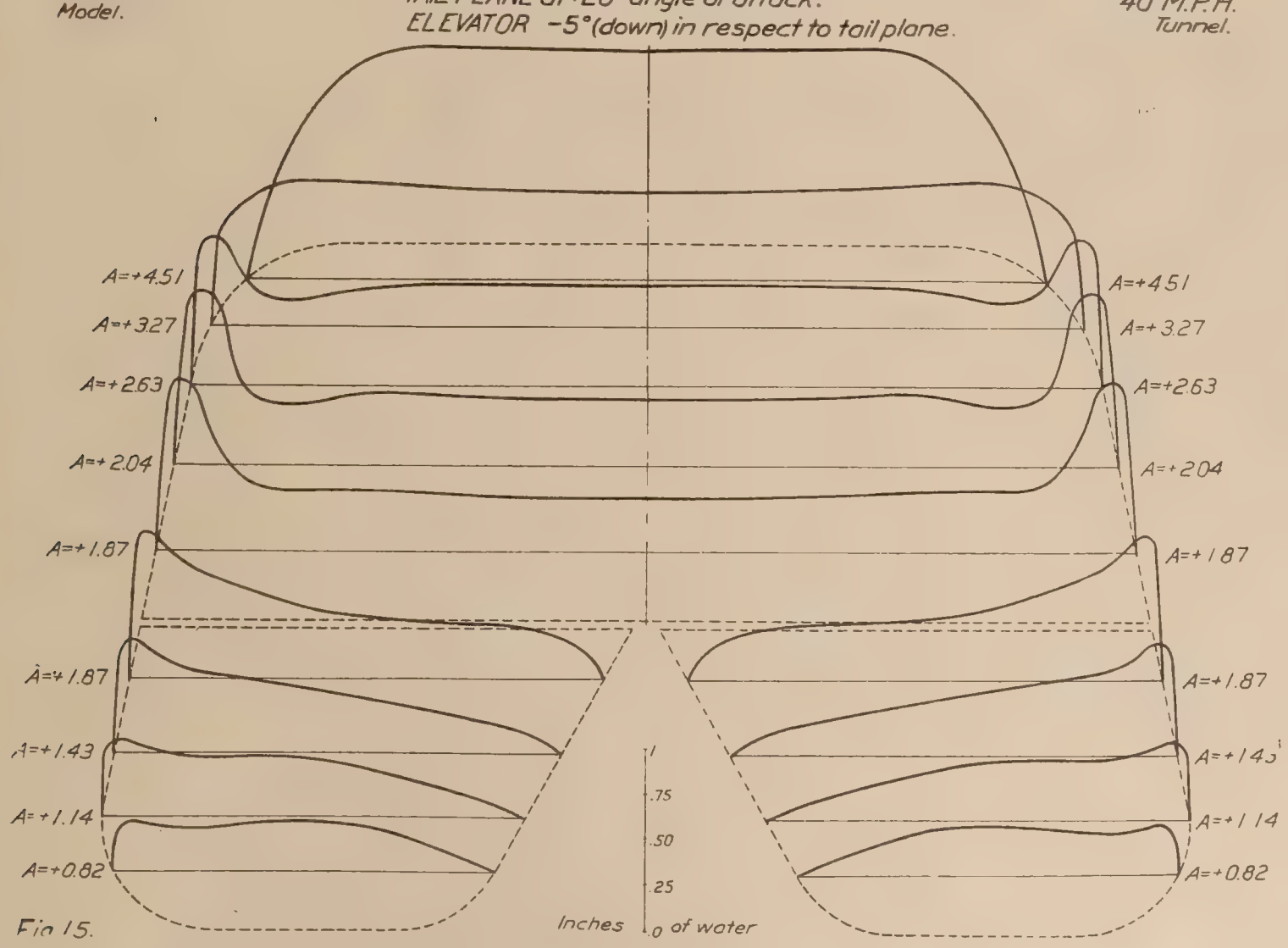

JN4H TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR -5° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+4.51
A=+3.27
A=+2.63
A=+2.04
A=+1.87
A=+1.43
A=+1.14
A=+0.82
1
.75
.50
.25
0
Inches of water

Fig 15.

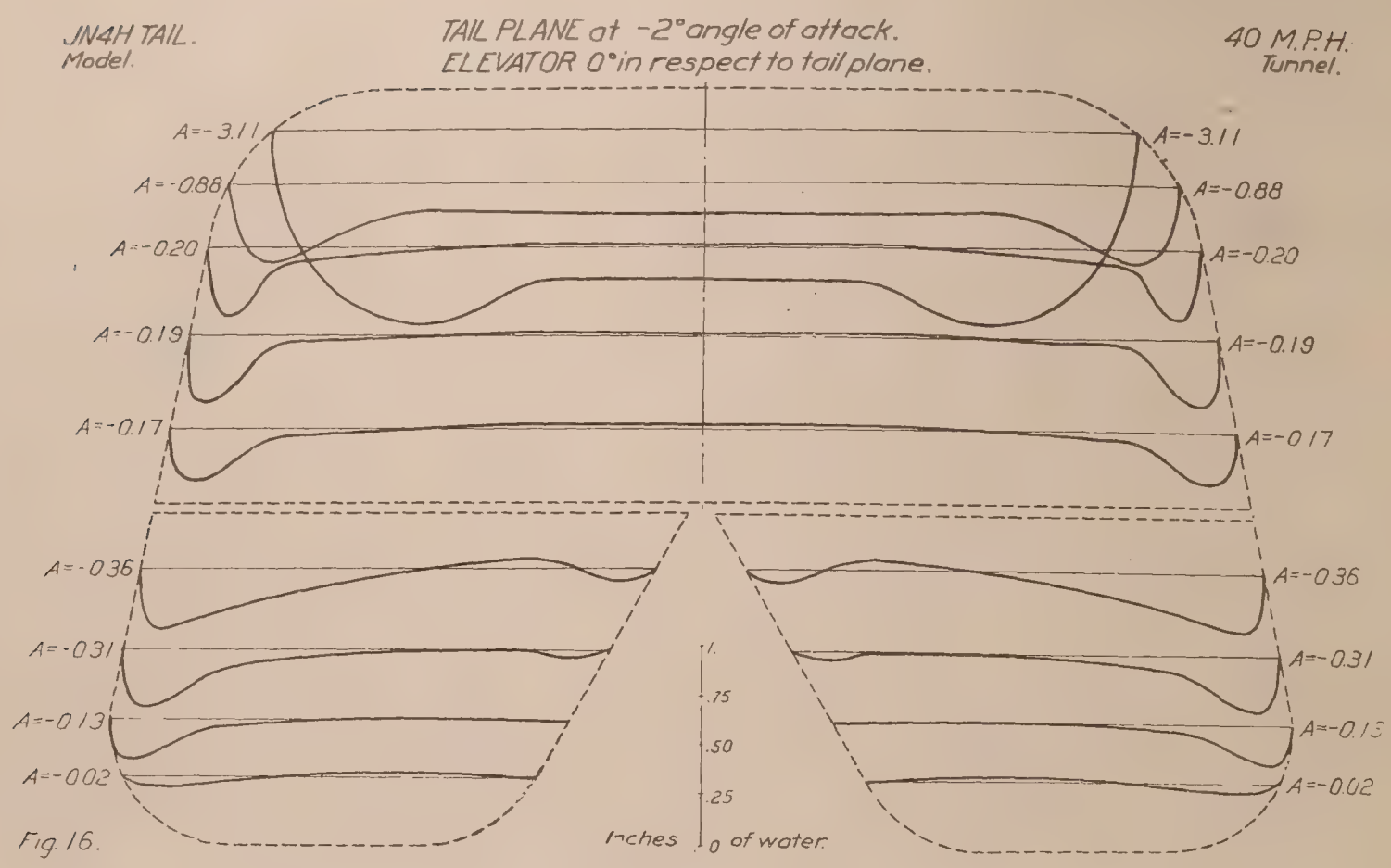

JN4H TAIL.
Model.
TAIL PLANE at -2° angle of attack.
ELEVATOR 0° in respect to tail plane.
40 M.P.H.
Tunnel.
A=-3.11
A=-0.88
A=-0.20
A=-0.19
A=-0.17
A=-0.36
A=-0.31
A=-0.13
A=-0.02
1.
.75
.50
.25
0
Inches of water.

Fig. 16.

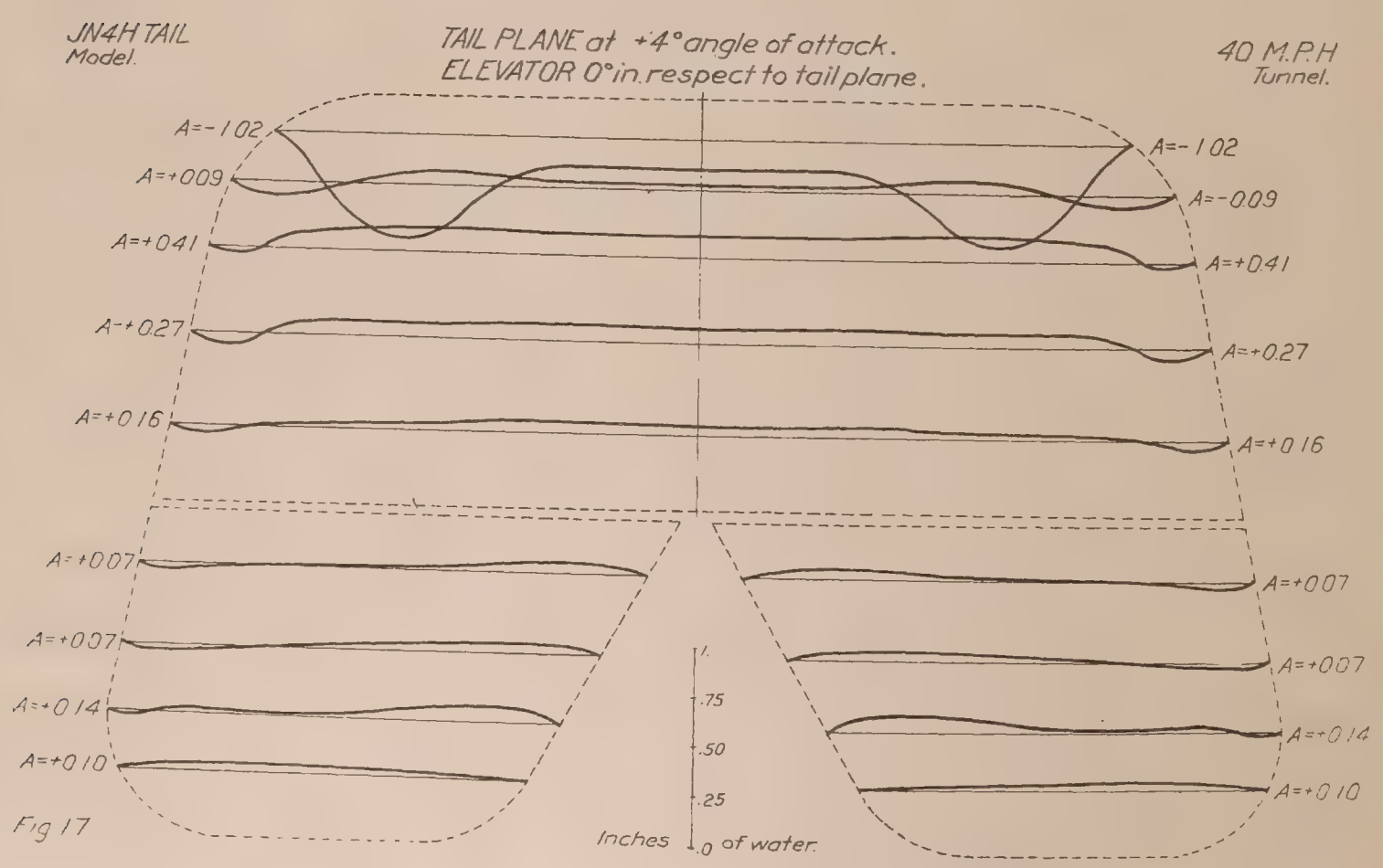

JN4H TAIL
Model.
TAIL PLANE at +4° angle of attack.
ELEVATOR 0° in respect to tail plane.
40 M.P.H
Tunnel.
A=-1.02
A=+0.09
A=-0.09
A=+0.41
A=+0.27
A=+0.16
A=+0.07
A=+0.14
A=+0.10
1.
.75
.50
.25
0
Inches of water.

Fig 17

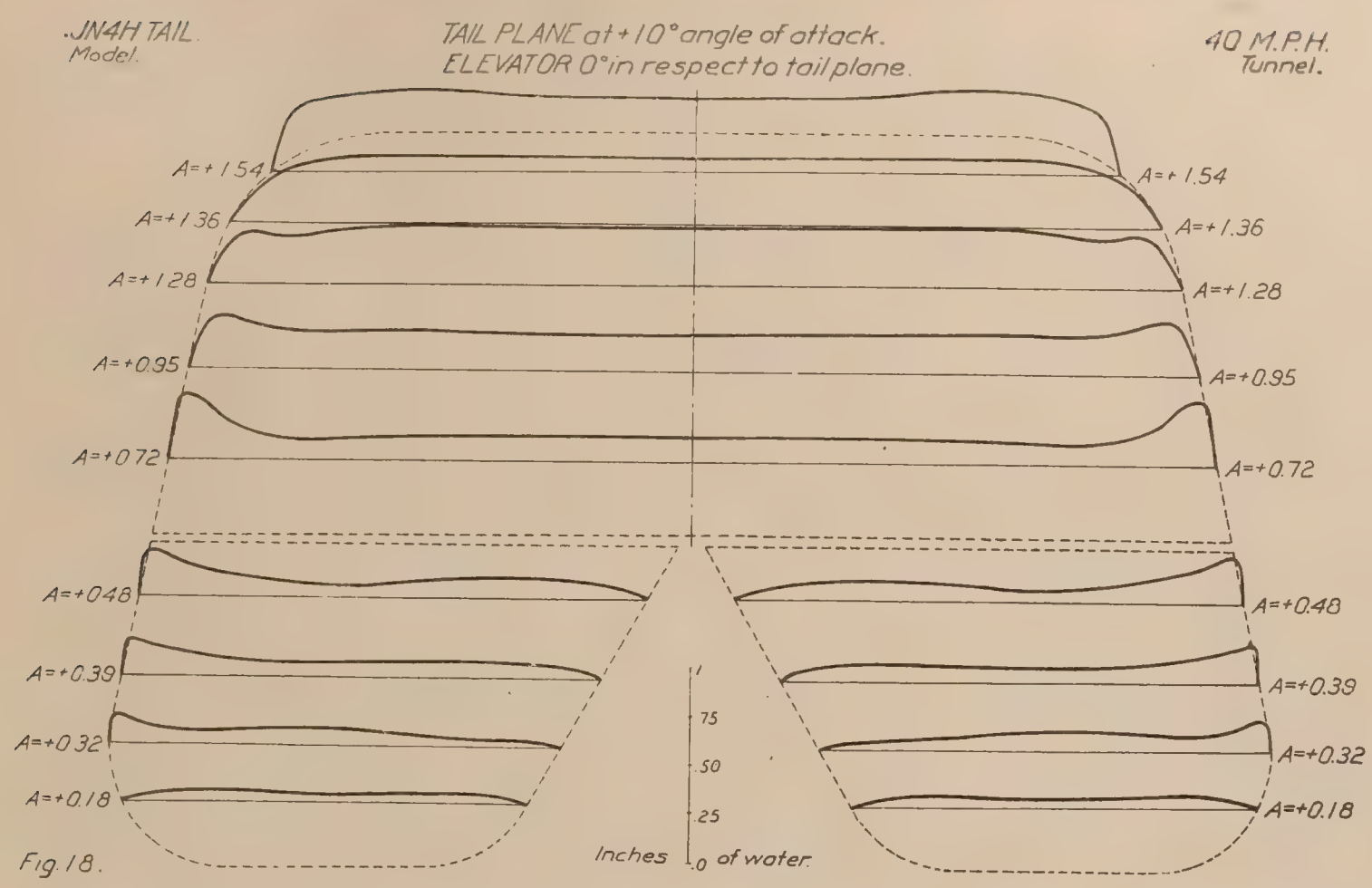

JN4H TAIL.
Model.
TAIL PLANE at +10° angle of attack.
ELEVATOR 0° in respect to tail plane.
40 M.P.H.
Tunnel.
A=+1.54
A=+1.36
A=+1.28
A=+0.95
A=+0.72
A=+0.48
A=+0.39
A=+0.32
A=+0.18
1
.75
.50
.25
.0
Inches of water.

Fig. 18.

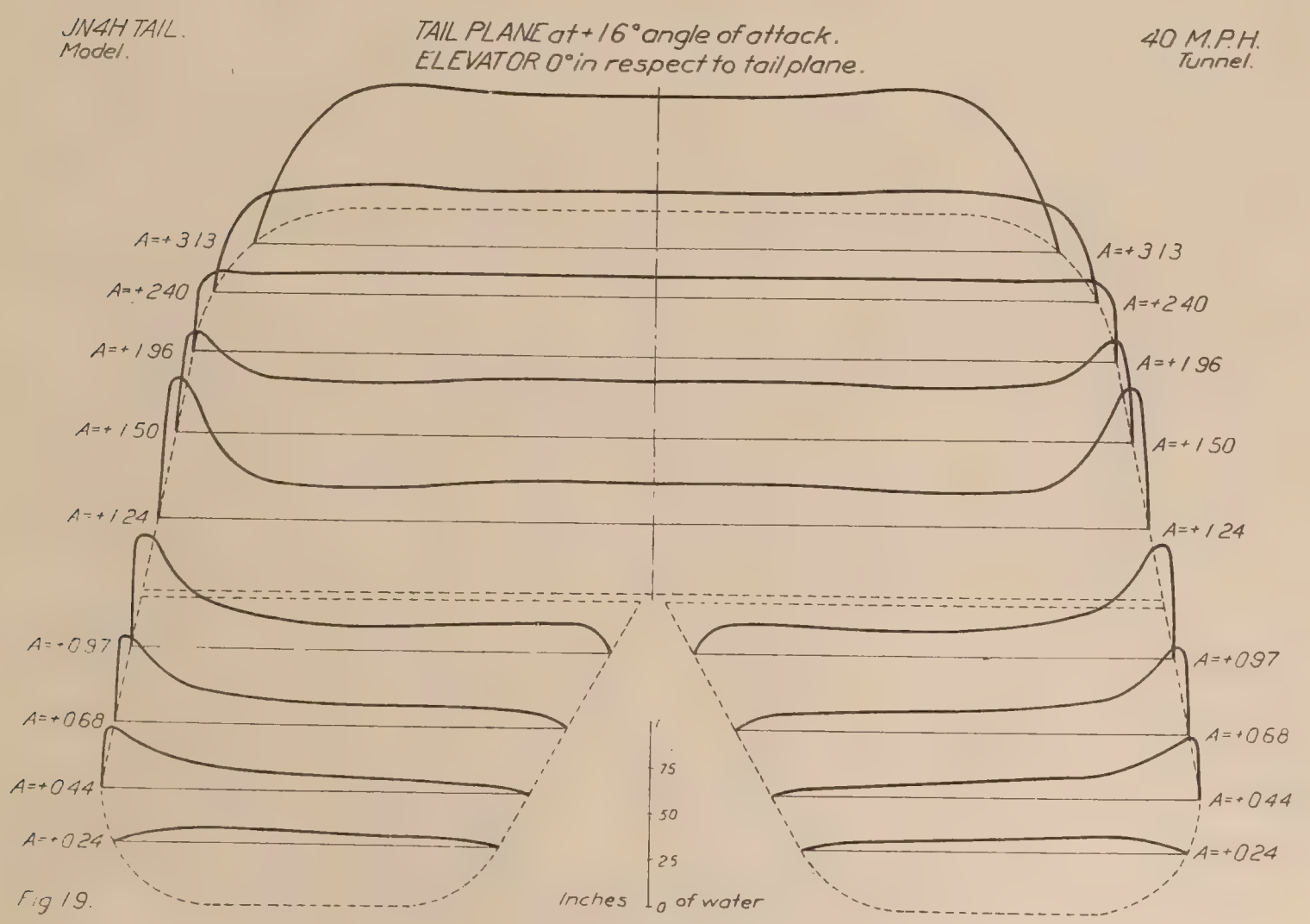

JN4H TAIL.
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR 0° in respect to tail plane.
40 M.P.H.
Tunnel.
A=+3.13
A=+2.40
A=+1.96
A=+1.50
A=+1.24
A=+0.97
A=+0.68
A=+0.44
A=+0.24
1
.75
.50
.25
.0
Inches of water

Fig. 19.

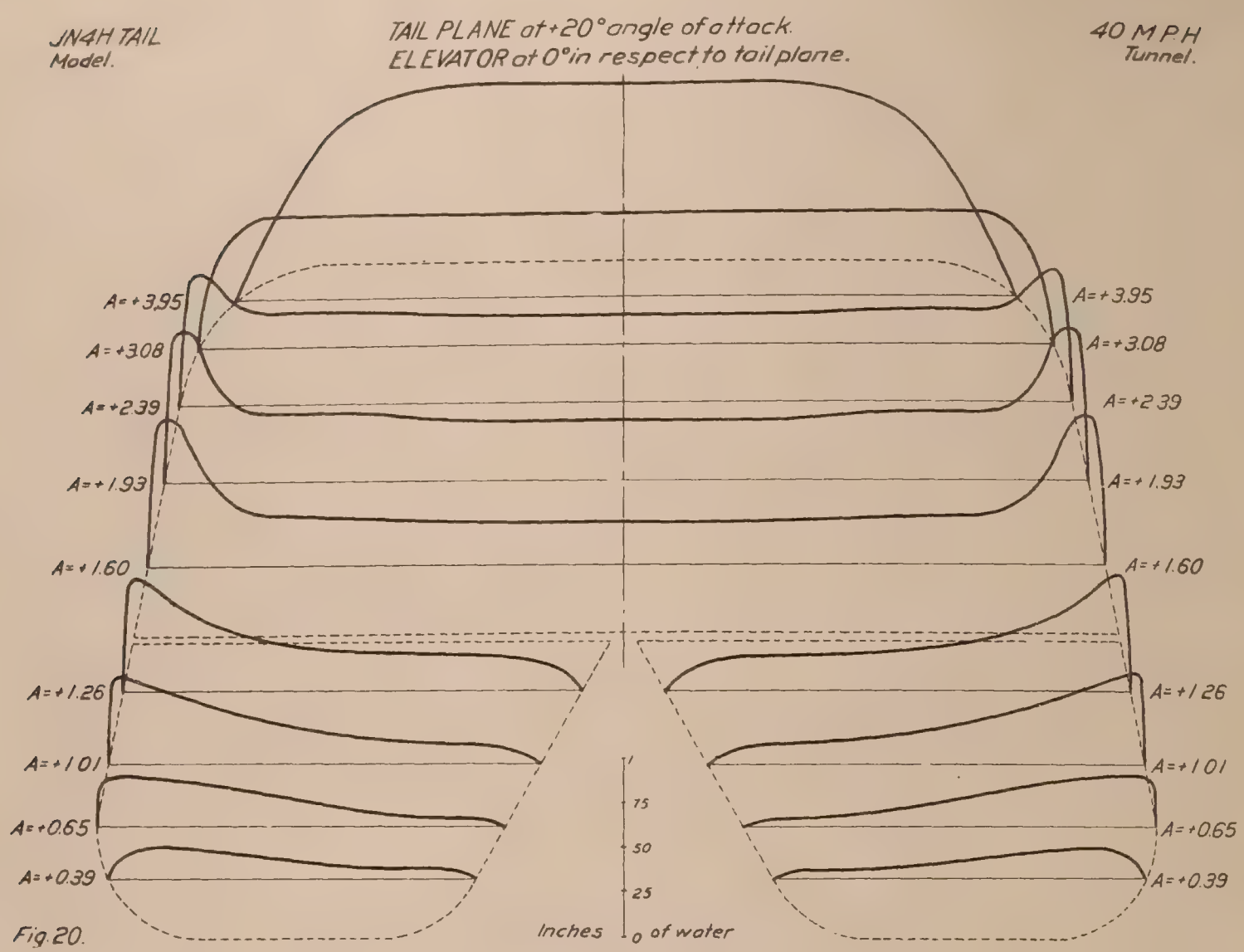

JN4H TAIL
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR at 0° in respect to tail plane.
40 M P H
Tunnel.
A=+3.95
A=+3.08
A=+2.39
A=+1.93
A=+1.60
A=+1.26
A=+1.01
A=+0.65
A=+0.39
1
.75
.50
.25
0
Inches of water

Fig. 20.

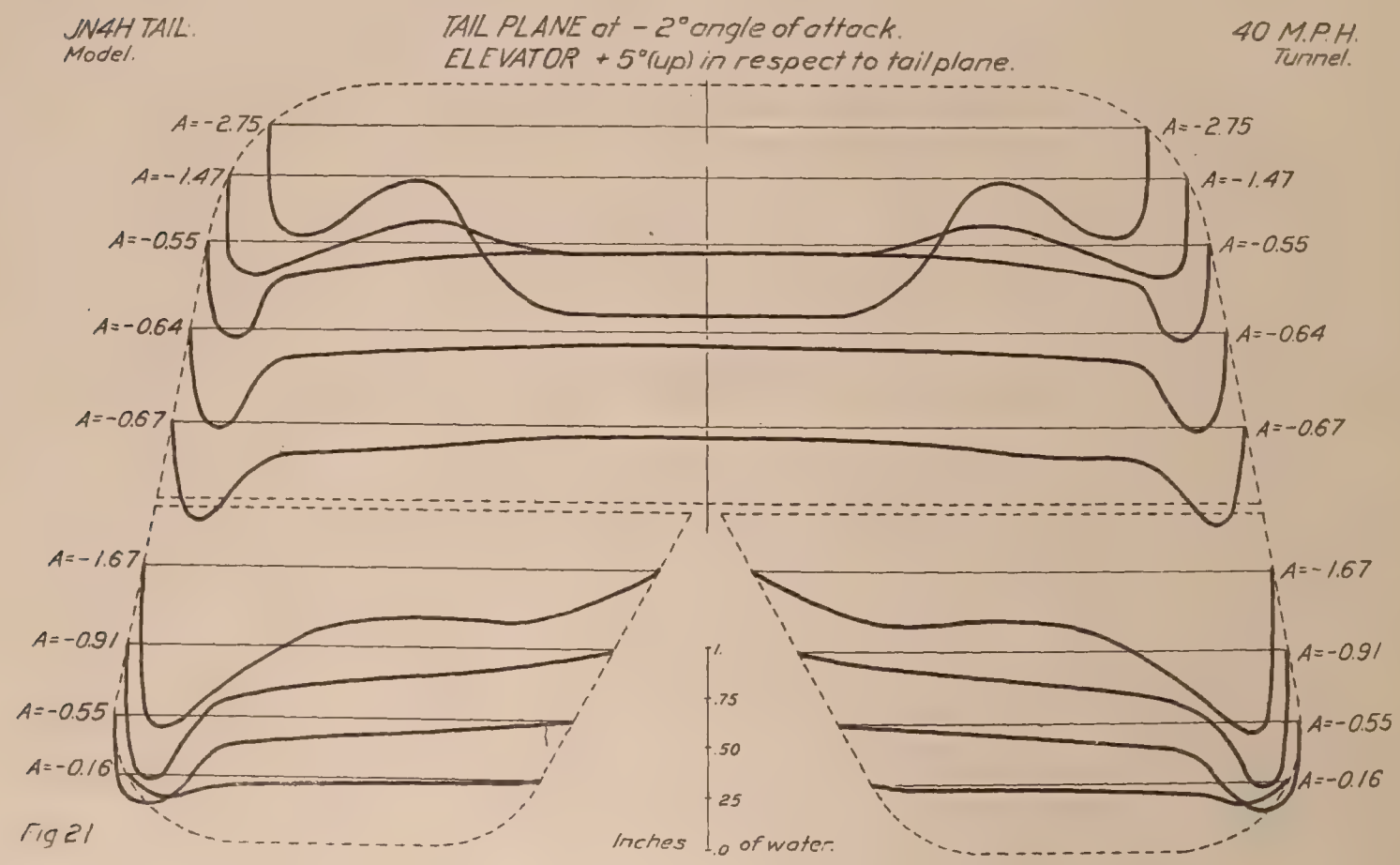

JN4H TAIL.
Model.
TAIL PLANE at −2° angle of attack.
ELEVATOR +5° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=-2.75
A=-1.47
A=-0.55
A=-0.64
A=-0.67
A=-1.67
A=-0.91
A=-0.55
A=-0.16
1.
.75
.50
.25
.0
Inches of water.

Fig 21

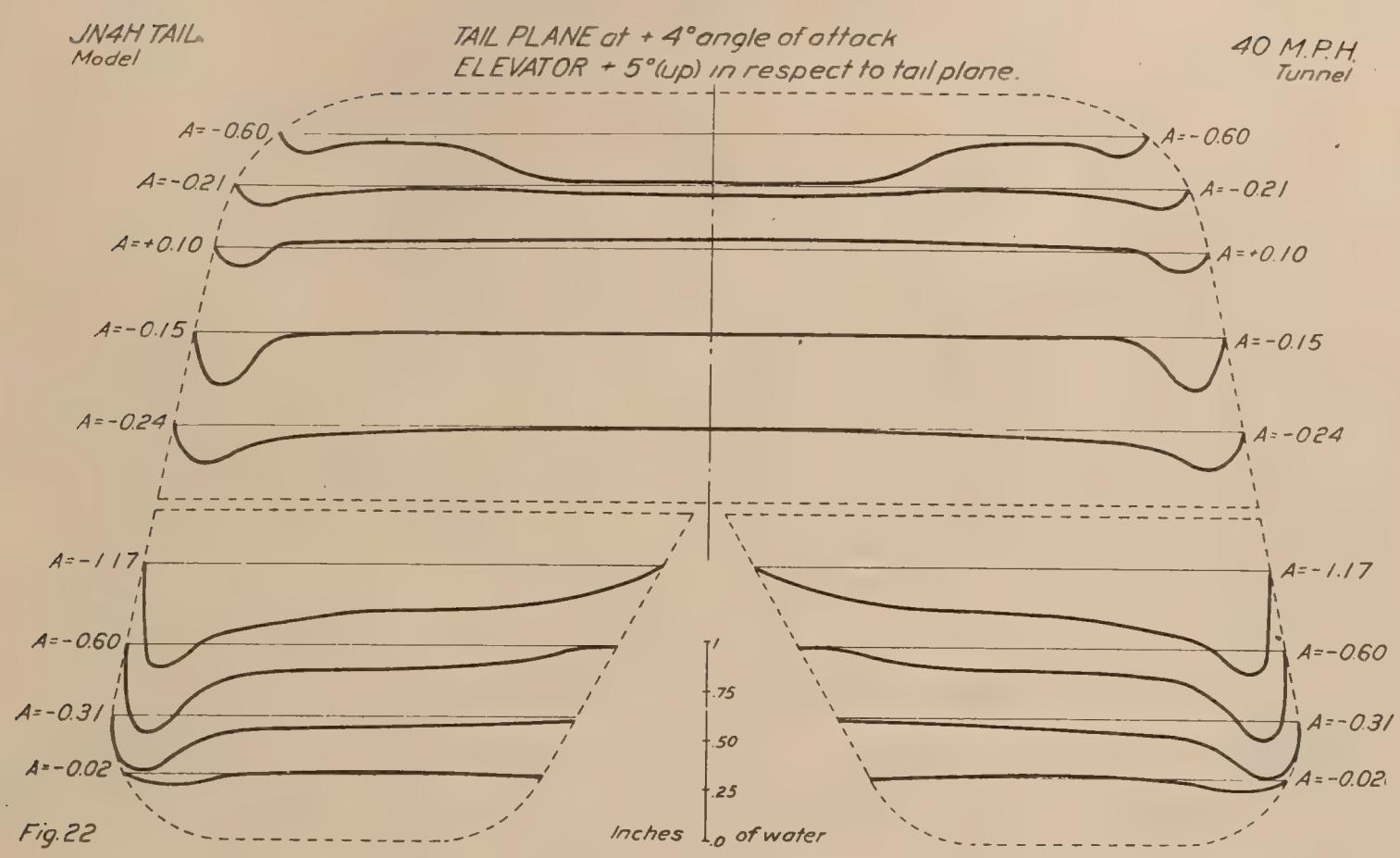

JN4H TAIL
Model
TAIL PLANE at + 4° angle of attack
ELEVATOR + 5° (up) in respect to tail plane.
40 M.P.H.
Tunnel
A=-0.60
A=-0.21
A=+0.10
A=-0.15
A=-0.24
A=-1.17
A=-0.60
A=-0.31
A=-0.02
1
.75
.50
.25
.0
Inches of water

Fig. 22

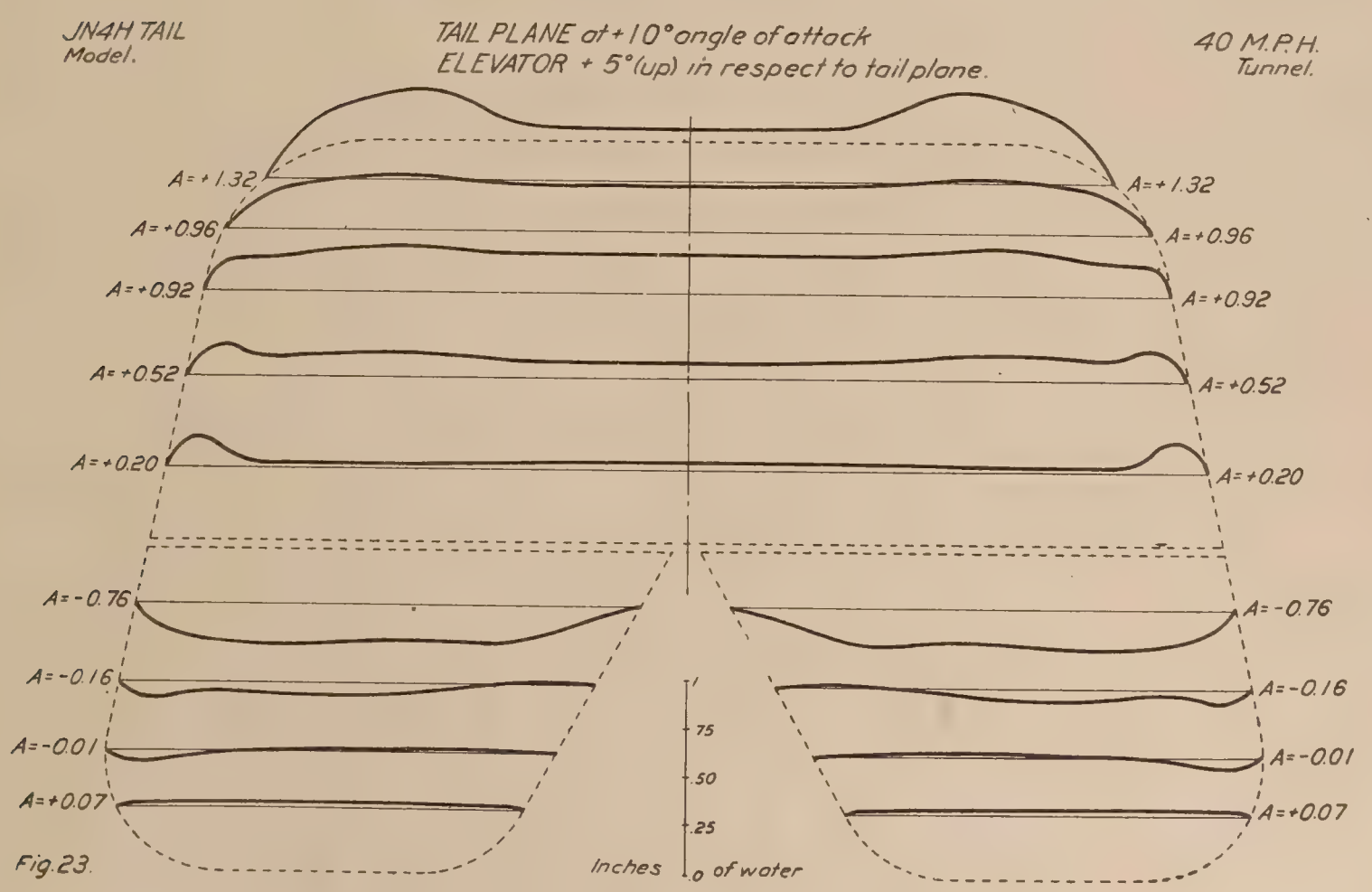

JN4H TAIL
Model.
TAIL PLANE at + 10° angle of attack
ELEVATOR + 5° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+1.32
A=+0.96
A=+0.92
A=+0.52
A=+0.20
A=-0.76
A=-0.16
A=-0.01
A=+0.07
1
.75
.50
.25
.0
Inches of water

Fig. 23.

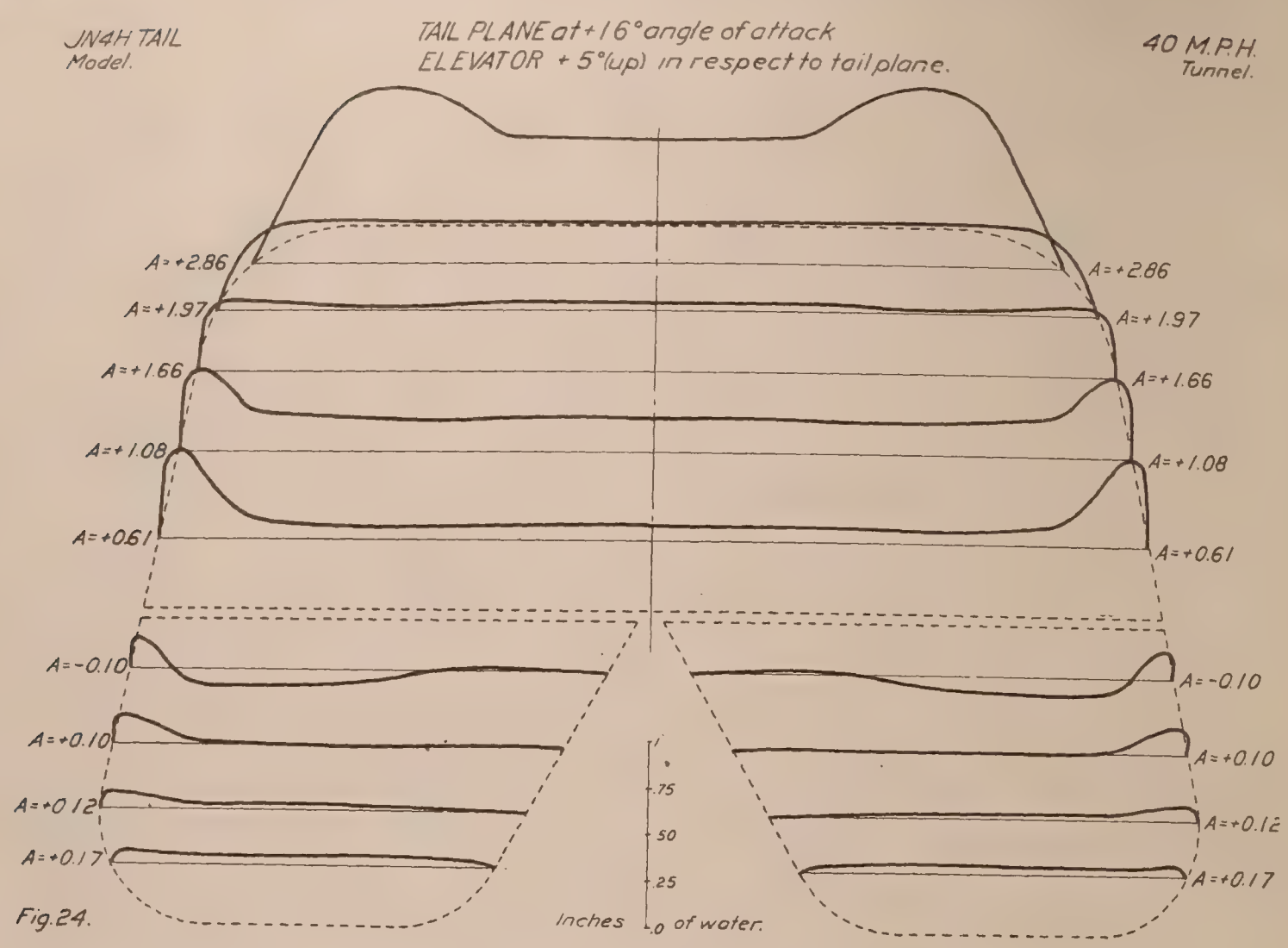

JN4H TAIL
Model.
TAIL PLANE at +16° angle of attack
ELEVATOR + 5° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+2.86
A=+1.97
A=+1.66
A=+1.08
A=+0.61
A=-0.10
A=+0.10
A=+0.12
A=+0.17
1
.75
.50
.25
0
Inches of water.

Fig. 24.

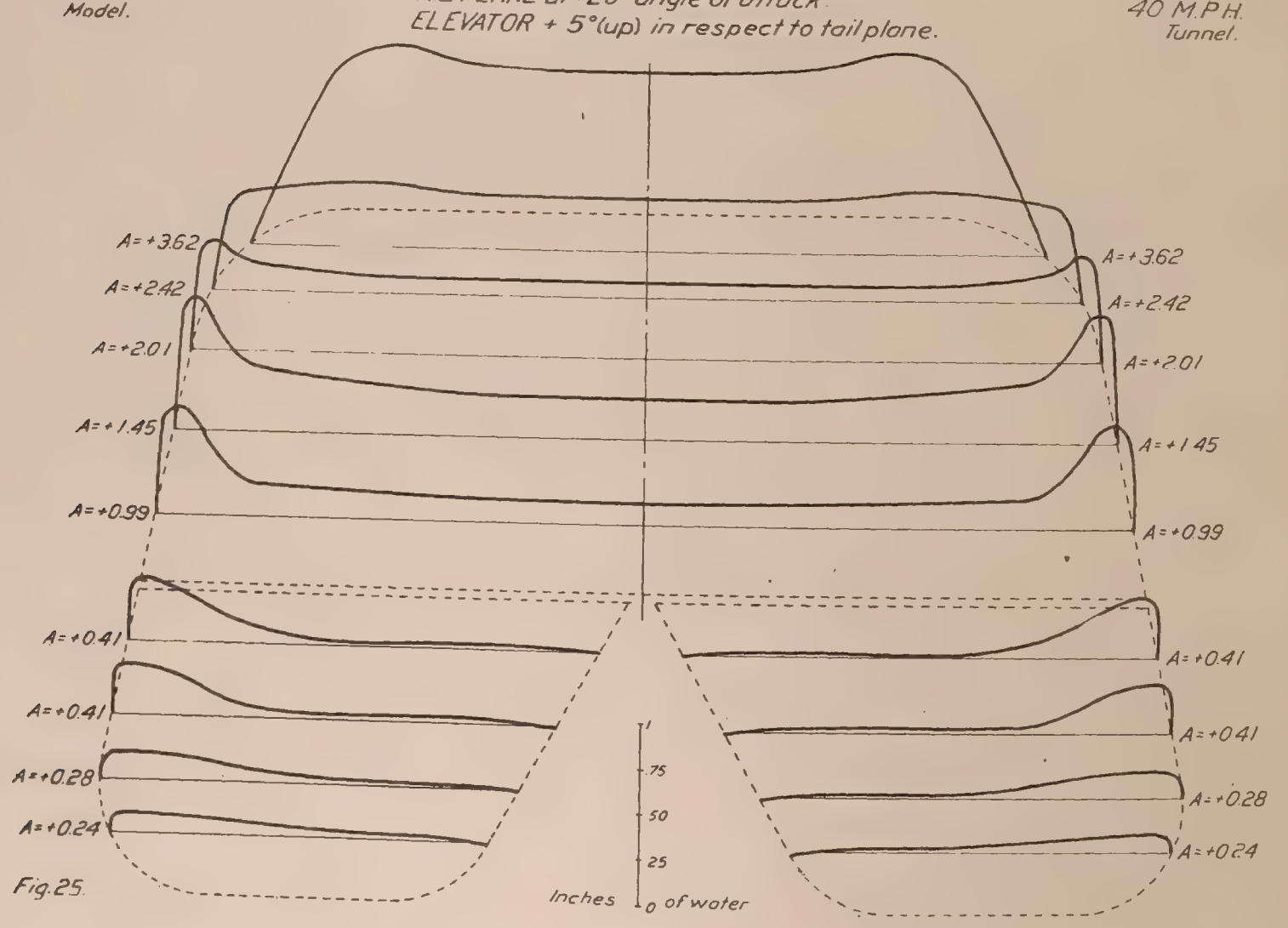

JN4H TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR + 5° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+3.62
A=+2.42
A=+2.01
A=+1.45
A=+0.99
A=+0.41
A=+0.41
A=+0.28
A=+0.24
1
.75
.50
.25
0
Inches of water

Fig. 25.

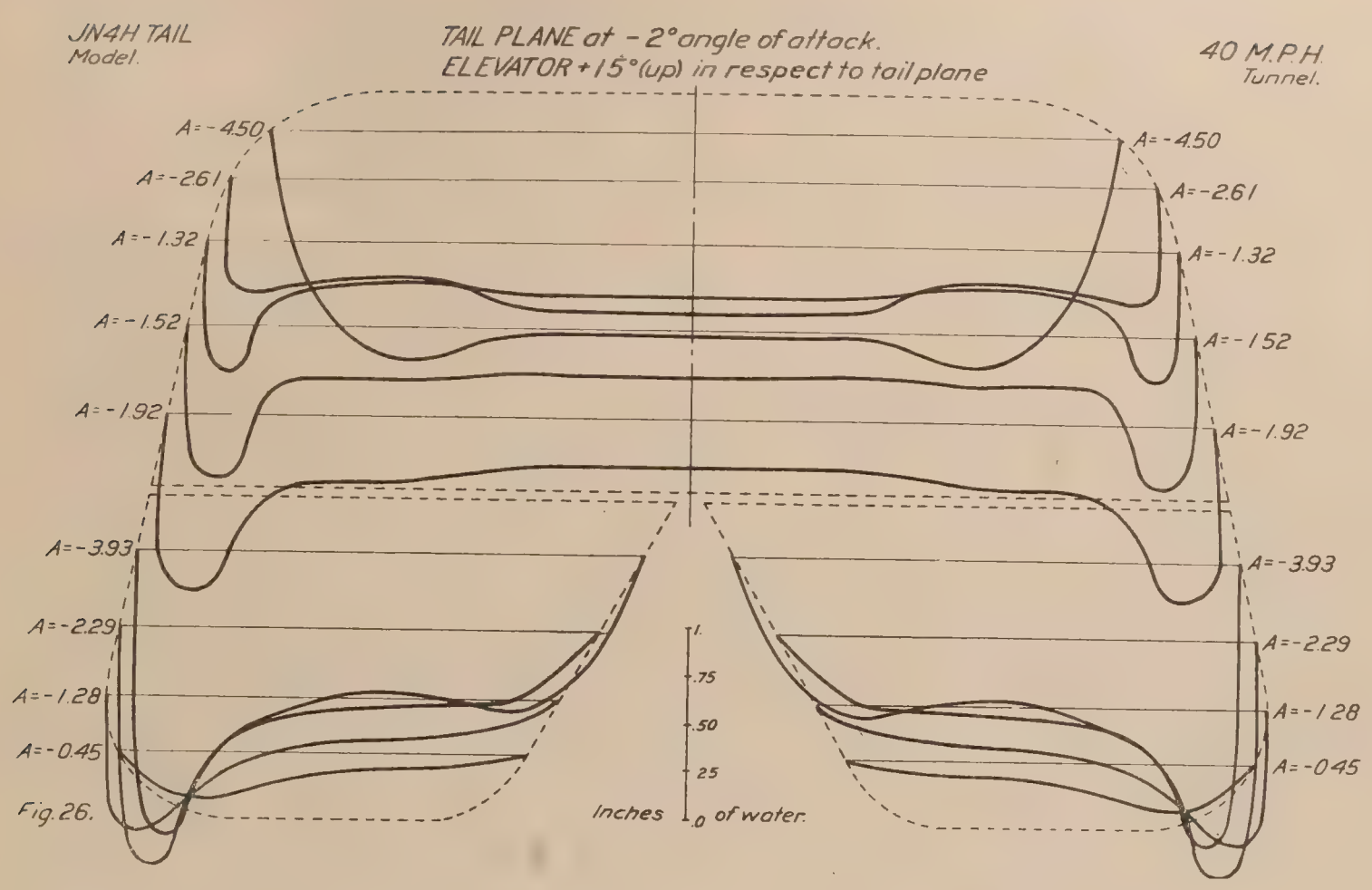

JN4H TAIL
Model.
TAIL PLANE at -2° angle of attack.
ELEVATOR +15° (up) in respect to tail plane
40 M.P.H.
Tunnel.
A=-4.50
A=-2.61
A=-1.32
A=-1.52
A=-1.92
A=-3.93
A=-2.29
A=-1.28
A=-0.45
1.
.75
.50
.25
0
Inches of water.

Fig. 26.

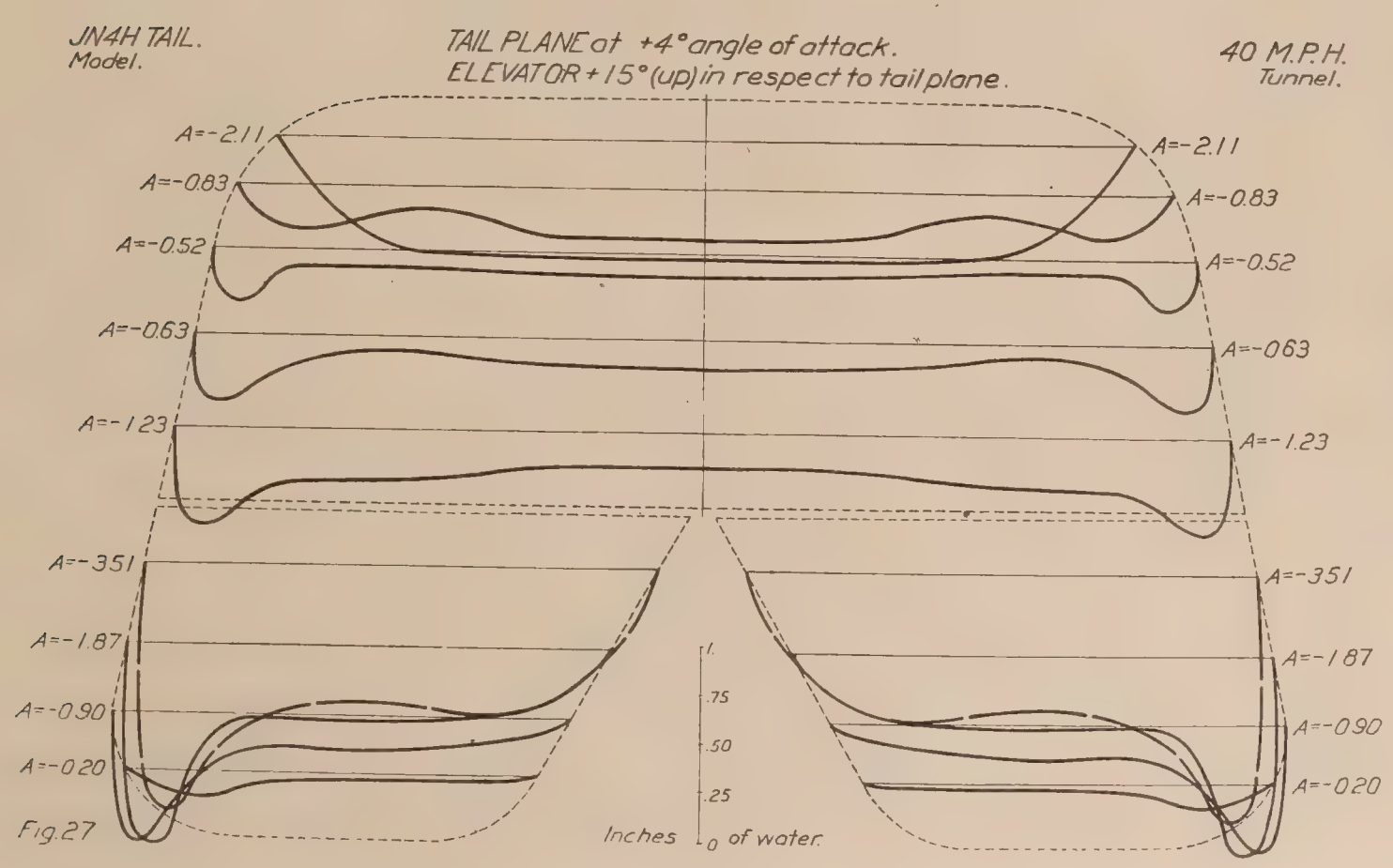

JN4H TAIL.
Model.
TAIL PLANE at +4° angle of attack.
ELEVATOR +15° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=-2.11
A=-0.83
A=-0.52
A=-0.63
A=-1.23
A=-3.51
A=-1.87
A=-0.90
A=-0.20
1.
.75
.50
.25
0
Inches of water.

Fig. 27

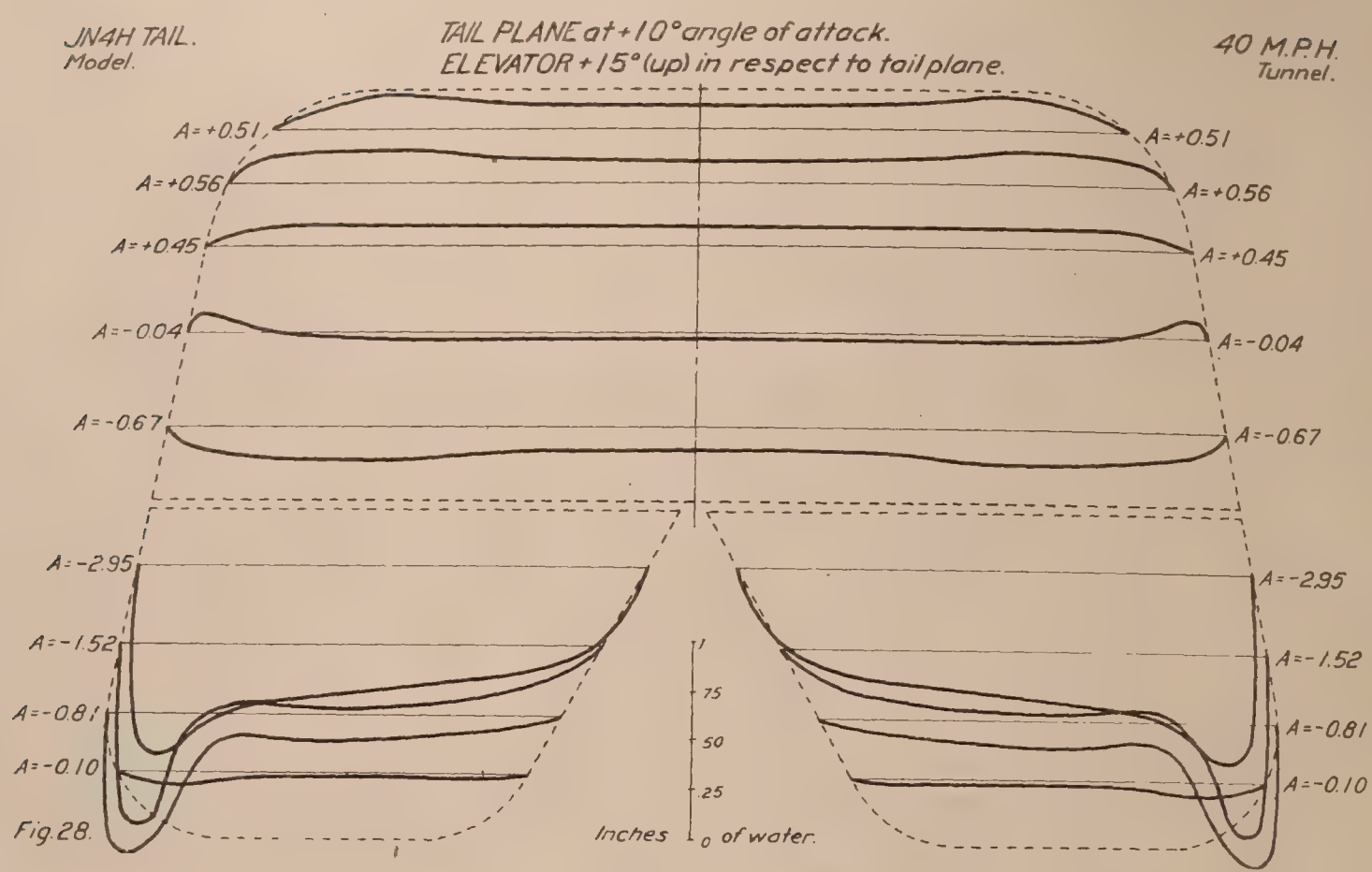

JN4H TAIL.
Model.
TAIL PLANE at +10° angle of attack.
ELEVATOR +15° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+0.51
A=+0.56
A=+0.45
A=-0.04
A=-0.67
A=-2.95
A=-1.52
A=-0.81
A=-0.10
1
.75
.50
.25
0
Inches of water.

Fig. 28.

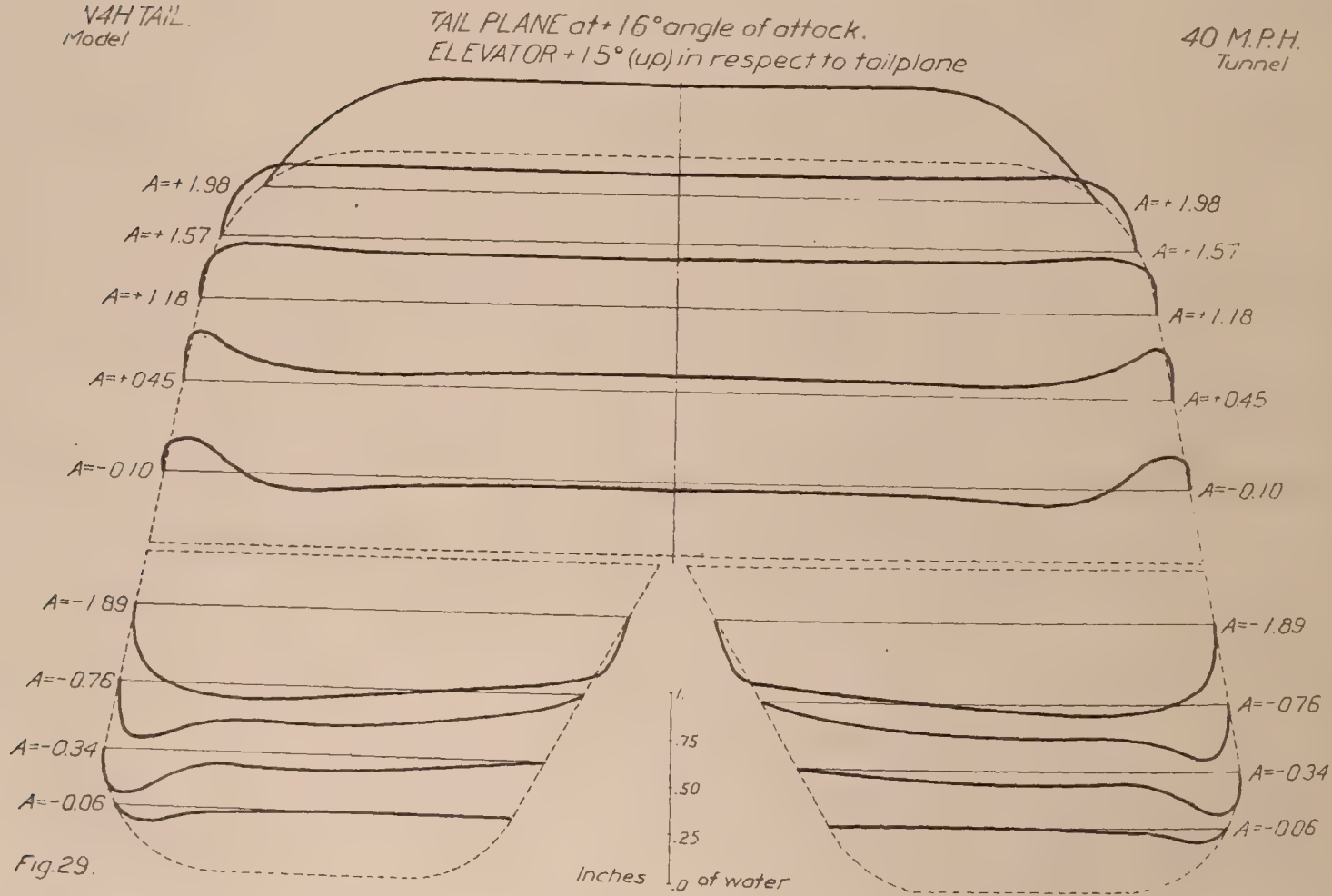

V4H TAIL.
Model
TAIL PLANE at +16° angle of attack.
ELEVATOR +15° (up) in respect to tailplane
40 M.P.H.
Tunnel
A=+1.98
A=+1.57
A=+1.18
A=+0.45
A=-0.10
A=-1.89
A=-0.76
A=-0.34
A=-0.06
1.
.75
.50
.25
.0
Inches of water

Fig. 29.

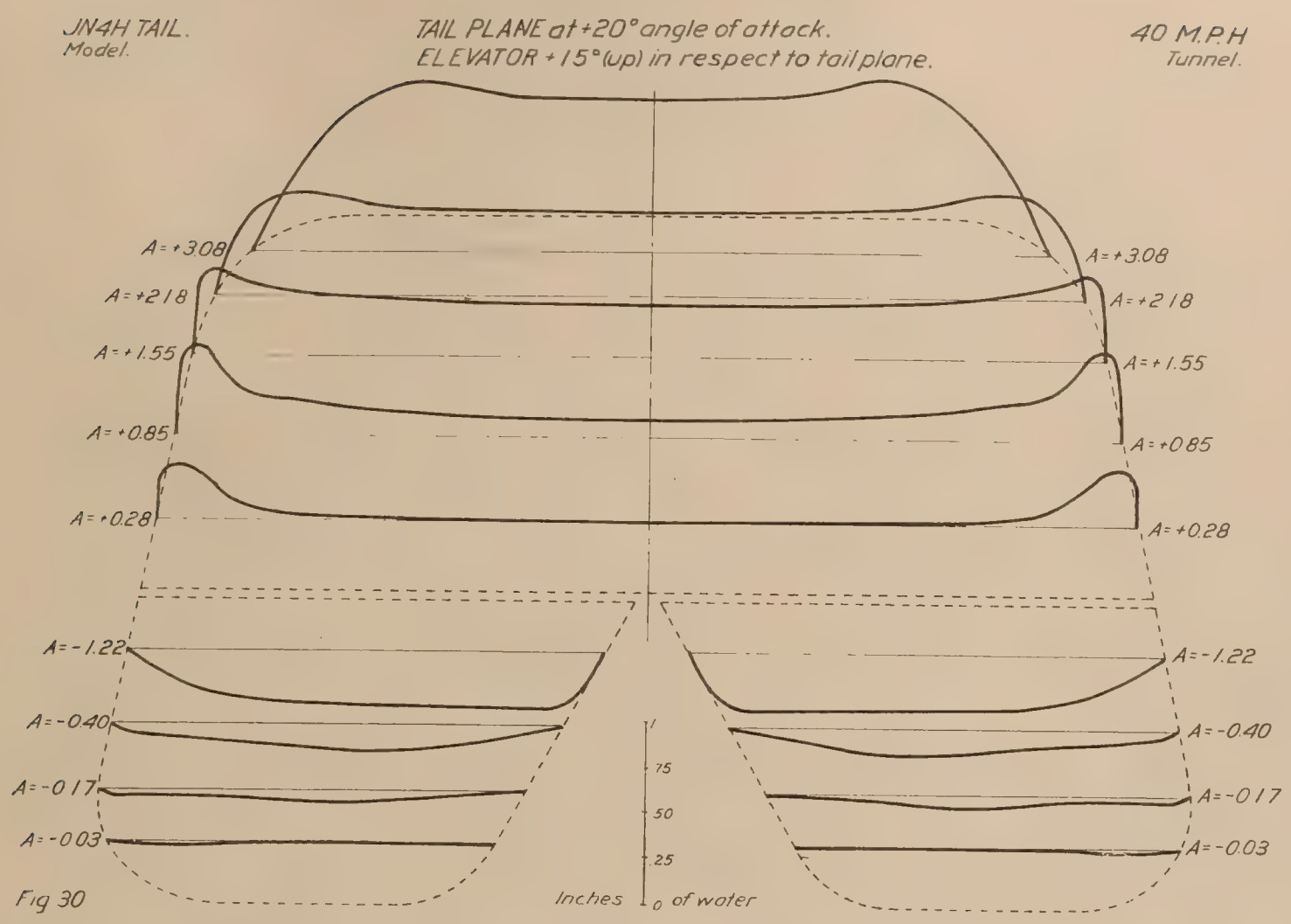

JN4H TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR +15° (up) in respect to tail plane.
40 M.P.H
Tunnel.
A=+3.08
A=+2.18
A=+1.55
A=+0.85
A=+0.28
A=-1.22
A=-0.40
A=-0.17
A=-0.03
1
.75
.50
.25
0
Inches of water

Fig 30

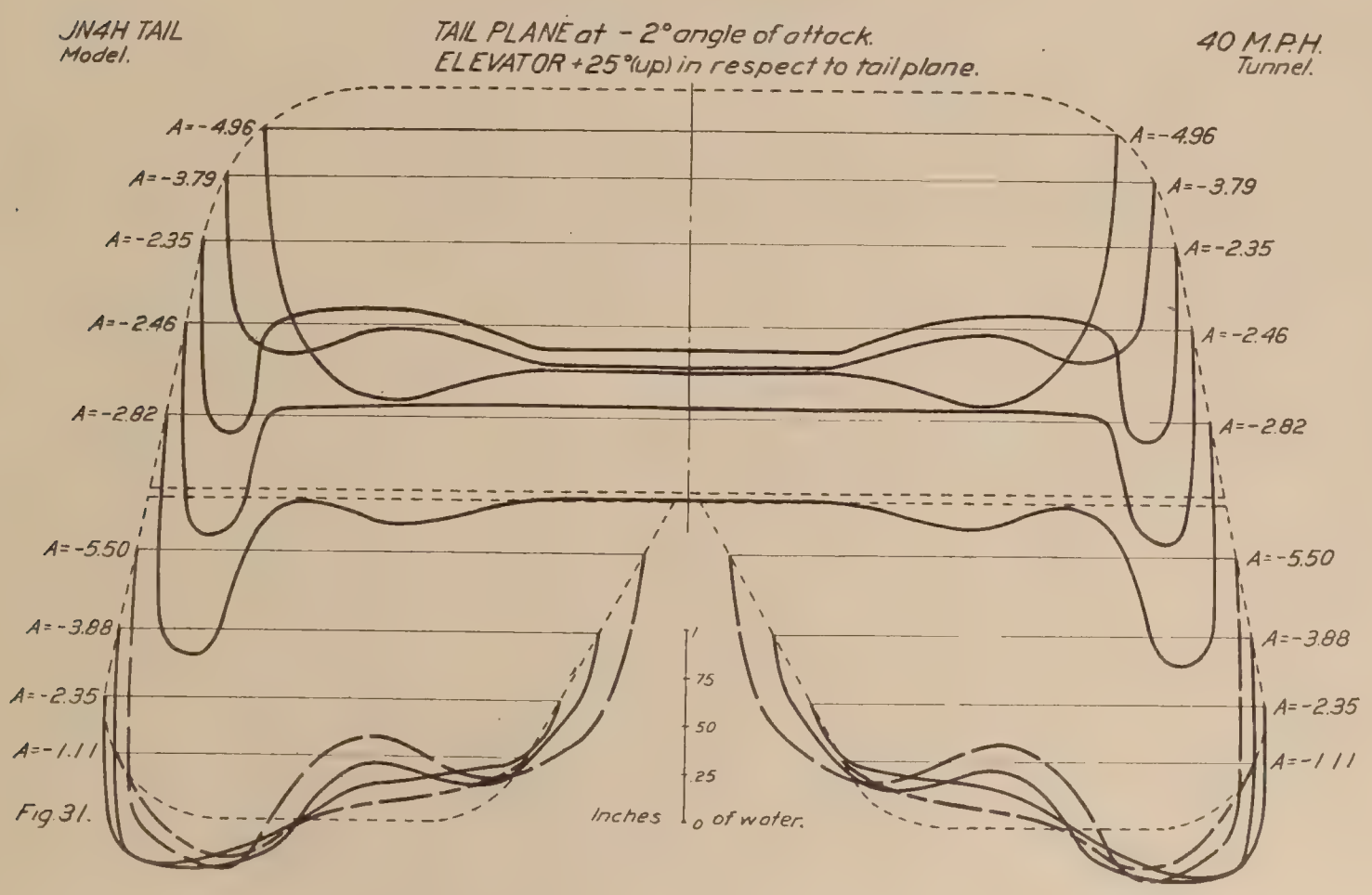

JN4H TAIL
Model.
TAIL PLANE at -2° angle of attack.
ELEVATOR +25° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=-4.96
A=-3.79
A=-2.35
A=-2.46
A=-2.82
A=-5.50
A=-3.88
A=-2.35
A=-1.11
1
.75
.50
.25
0
Inches of water.

Fig. 31.

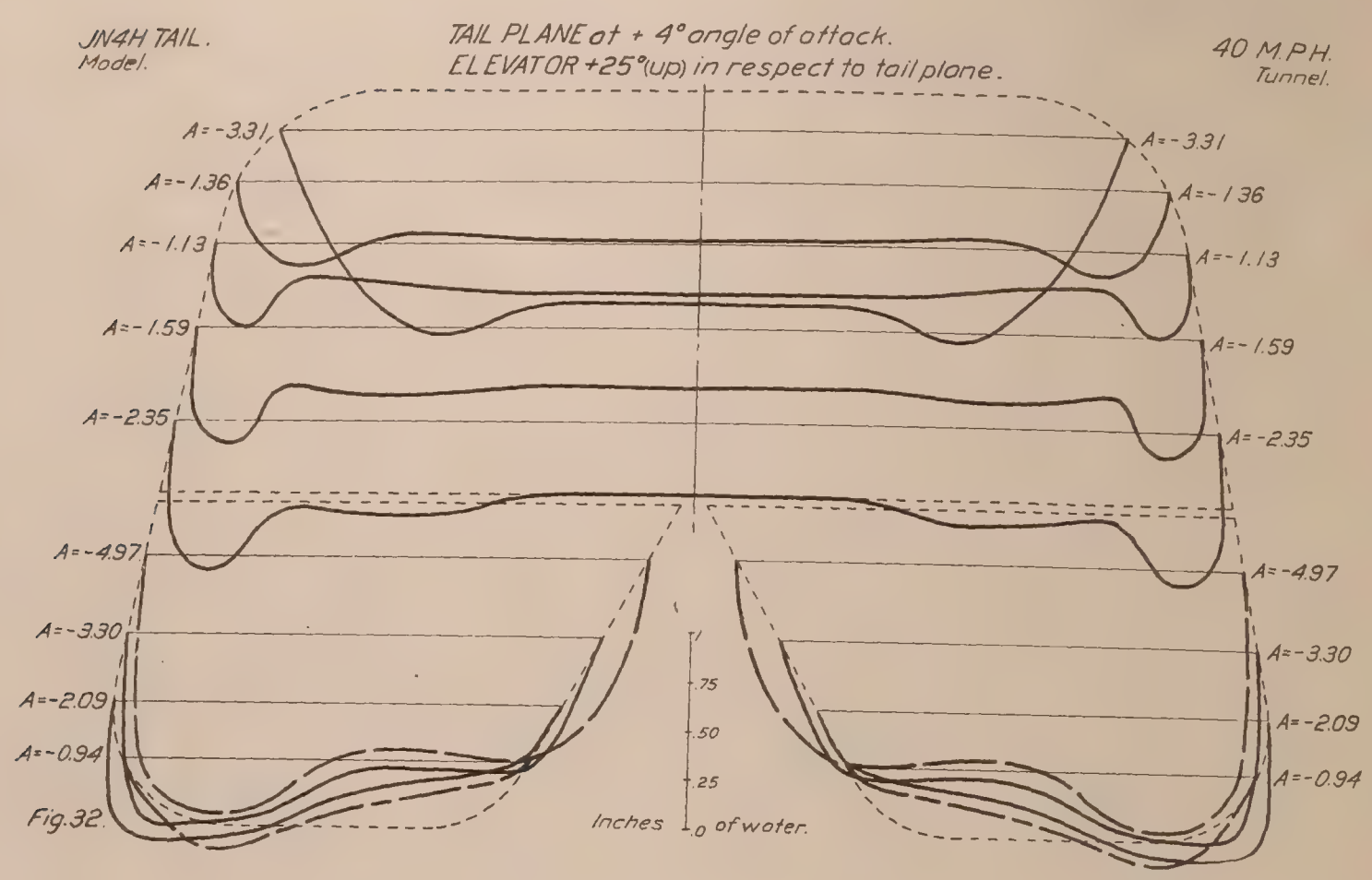

JN4H TAIL.
Model.
TAIL PLANE at + 4° angle of attack.
ELEVATOR +25° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=-3.31
A=-1.36
A=-1.13
A=-1.59
A=-2.35
A=-4.97
A=-3.30
A=-2.09
A=-0.94
.75
.50
.25
.0
Inches of water.

Fig. 32.

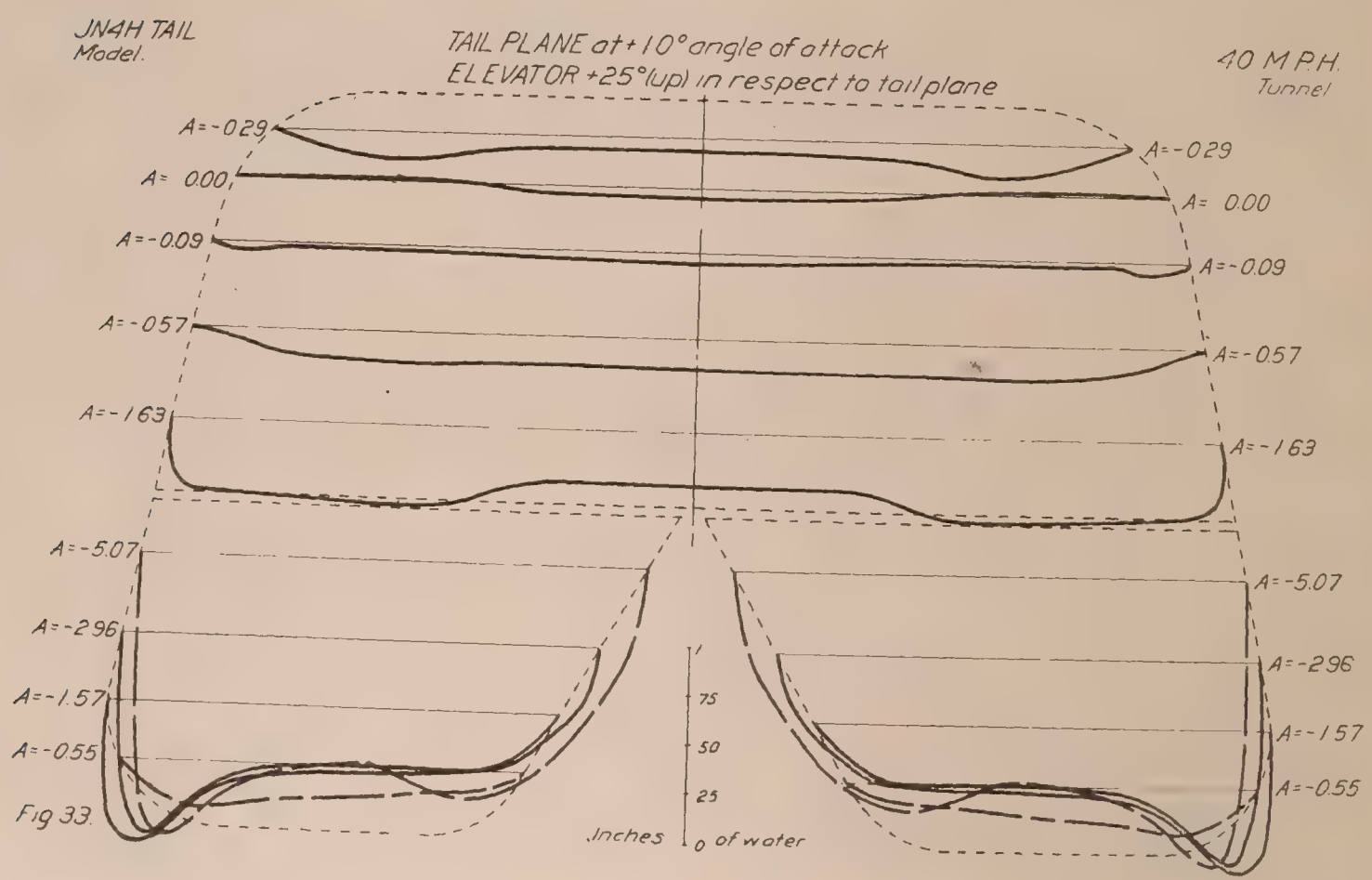

JN4H TAIL
Model.
TAIL PLANE at + 10° angle of attack
ELEVATOR +25° (up) in respect to tail plane
40 M.P.H.
Tunnel
A=-0.29
A= 0.00
A=-0.09
A=-0.57
A=-1.63
A=-5.07
A=-2.96
A=-1.57
A=-0.55
.75
.50
.25
.0
Inches of water

Fig. 33.

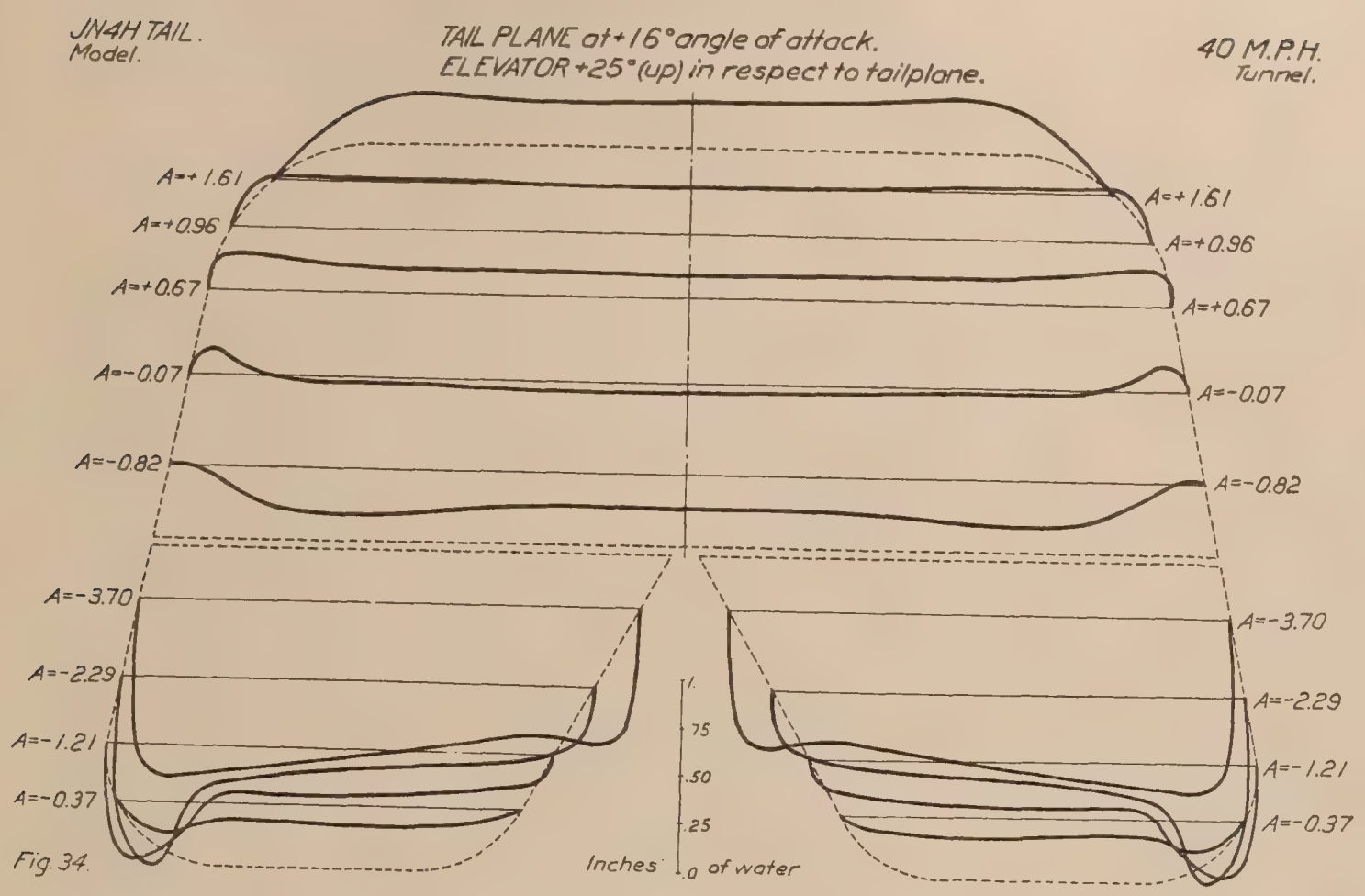

JN4H TAIL.
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR +25° (up) in respect to tailplane.
40 M.P.H.
Tunnel.
A=+1.61
A=+0.96
A=+0.67
A=-0.07
A=-0.82
A=-3.70
A=-2.29
A=-1.21
A=-0.37
1.
.75
.50
.25
.0
Inches of water

Fig. 34.

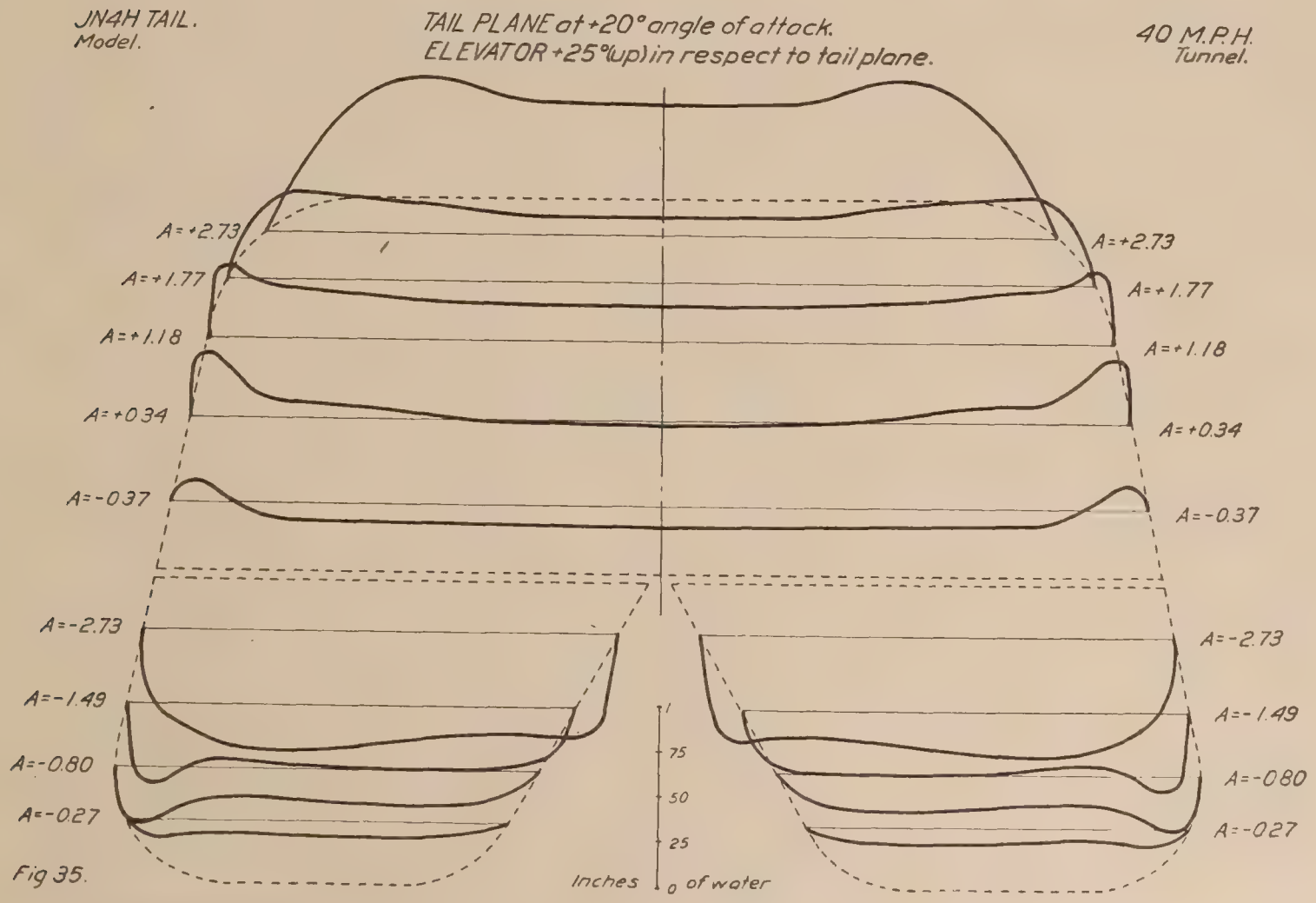

JN4H TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR +25° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+2.73
A=+1.77
A=+1.18
A=+0.34
A=-0.37
A=-2.73
A=-1.49
A=-0.80
A=-0.27
1
.75
.50
.25
.0
Inches of water

Fig 35.

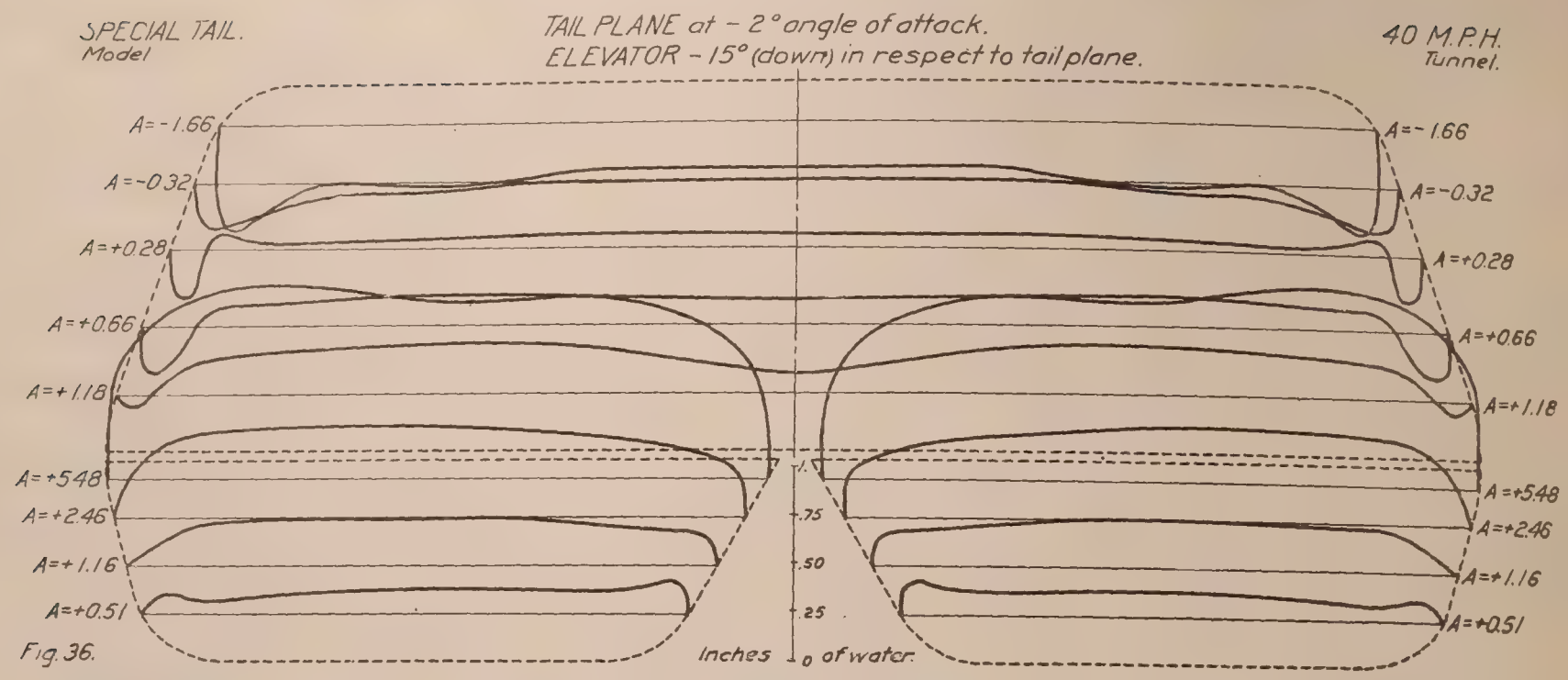


Fig. 36.

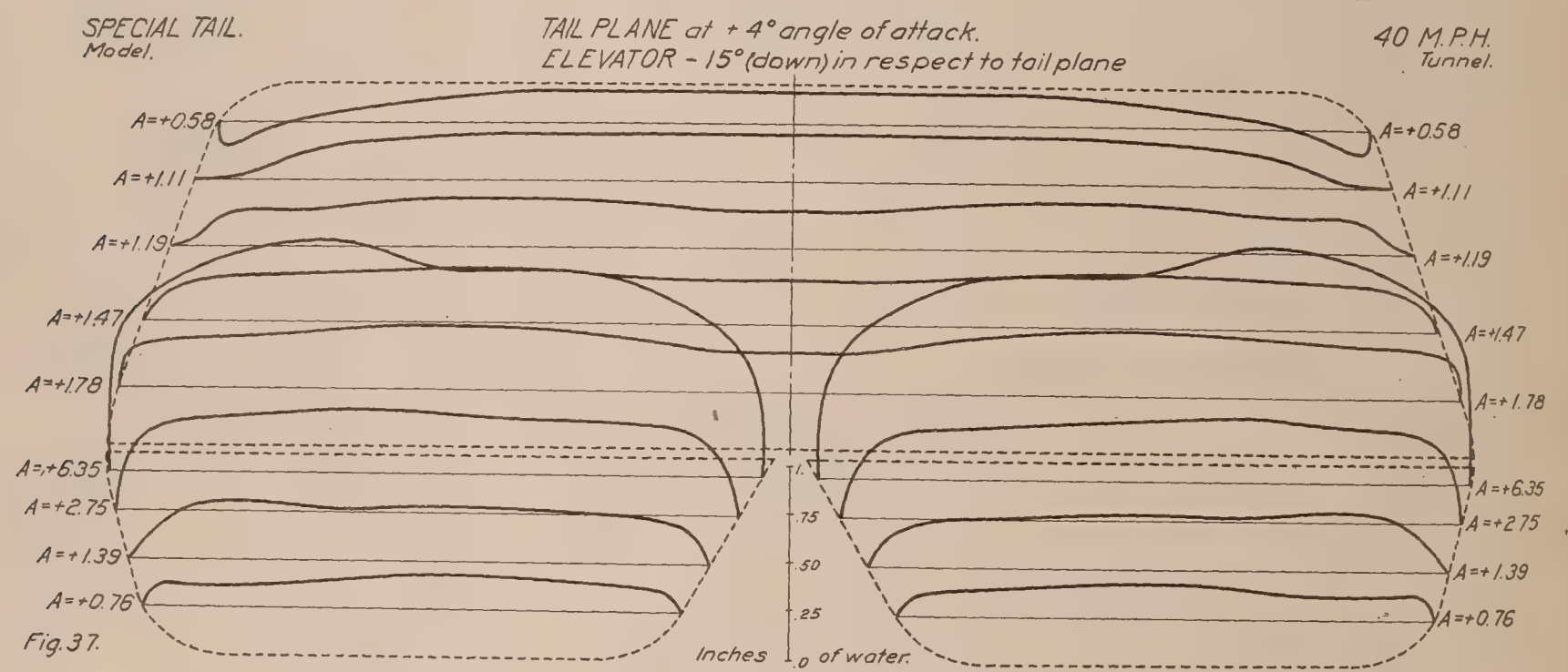


Fig. 37.

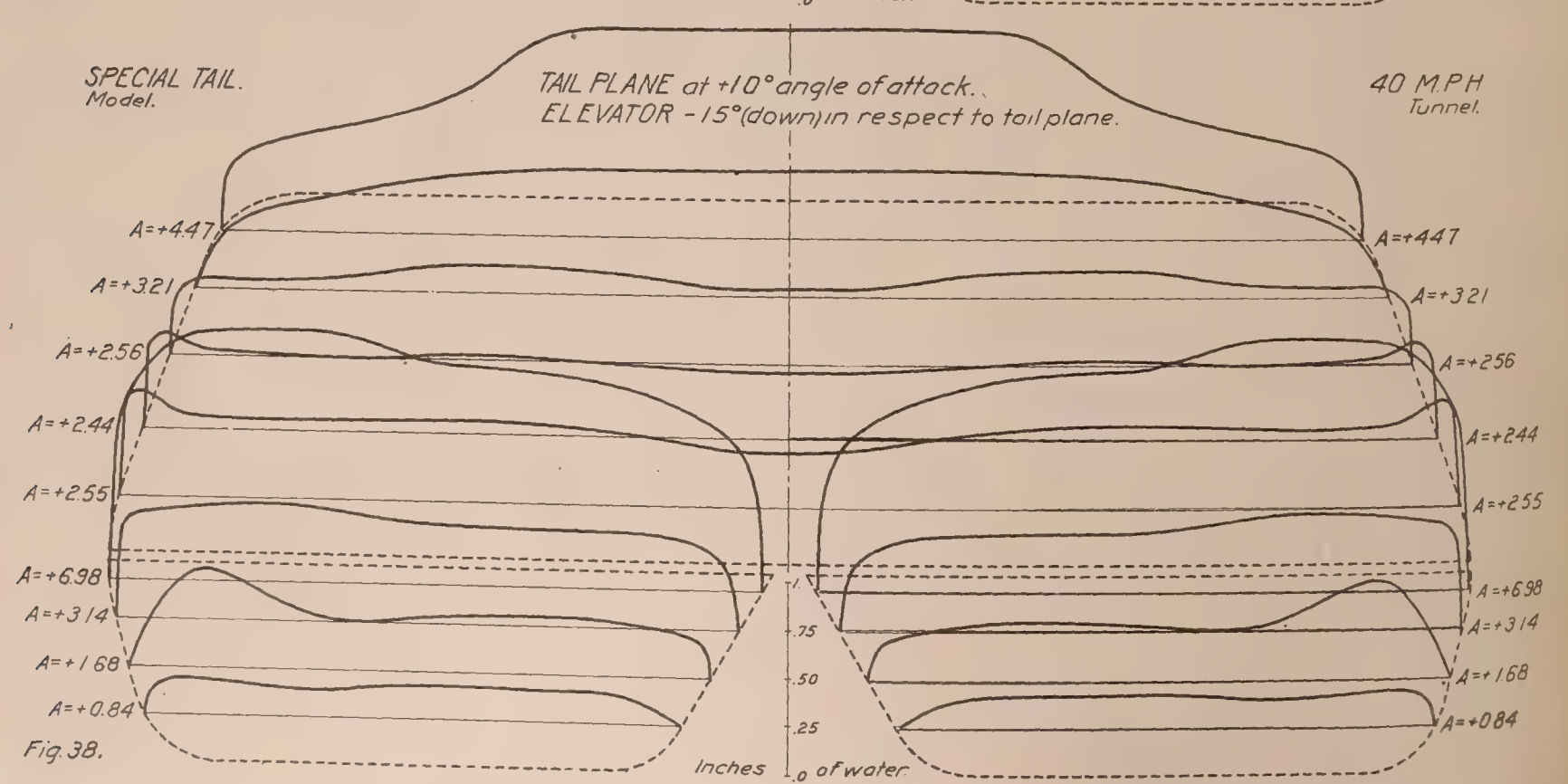


Fig. 38.

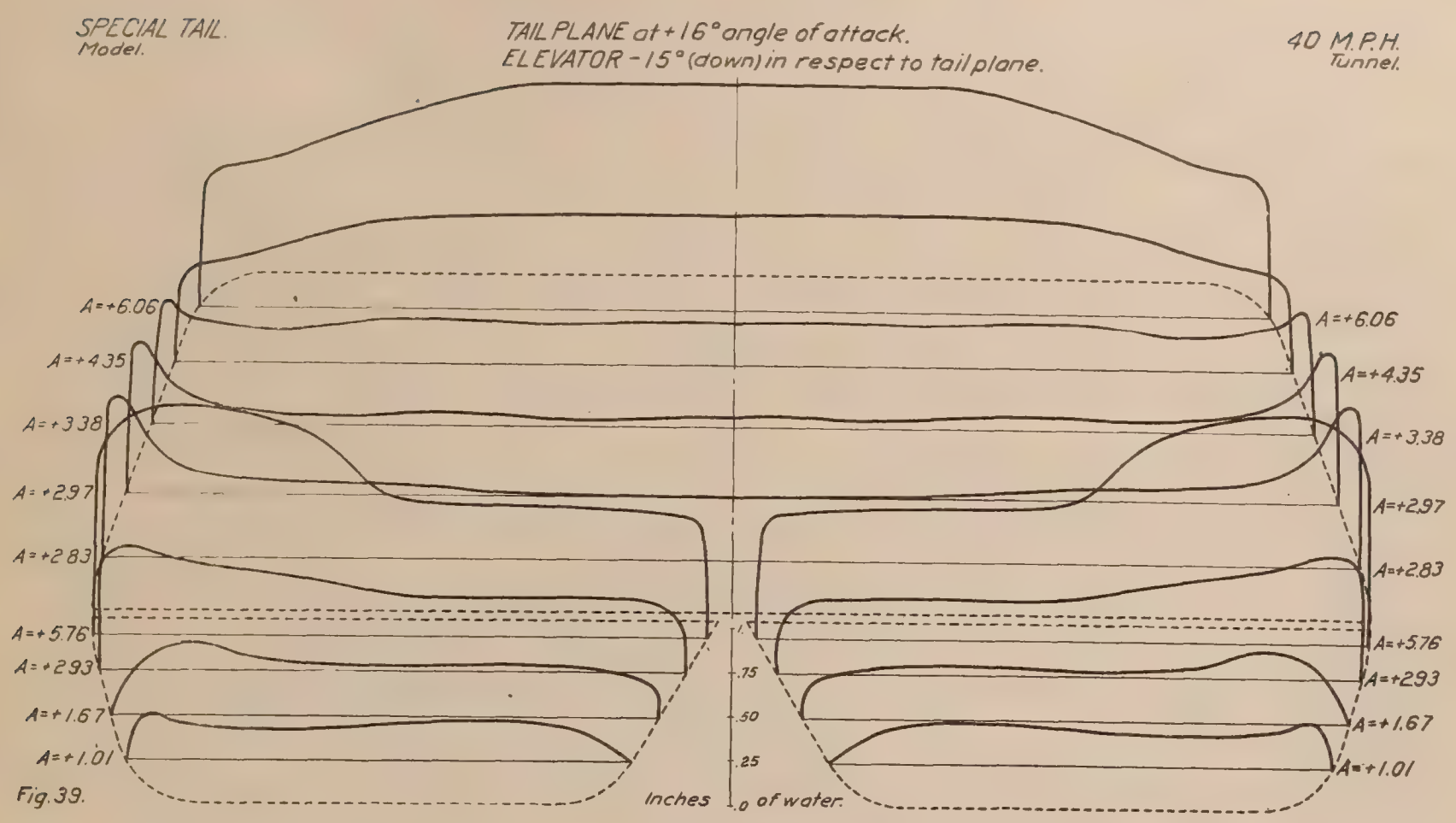

SPECIAL TAIL.
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR -15° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+6.06
A=+4.35
A=+3.38
A=+2.97
A=+2.83
A=+5.76
A=+2.93
A=+1.67
A=+1.01
1.
.75
.50
.25
.0
Inches of water.

Fig. 39.

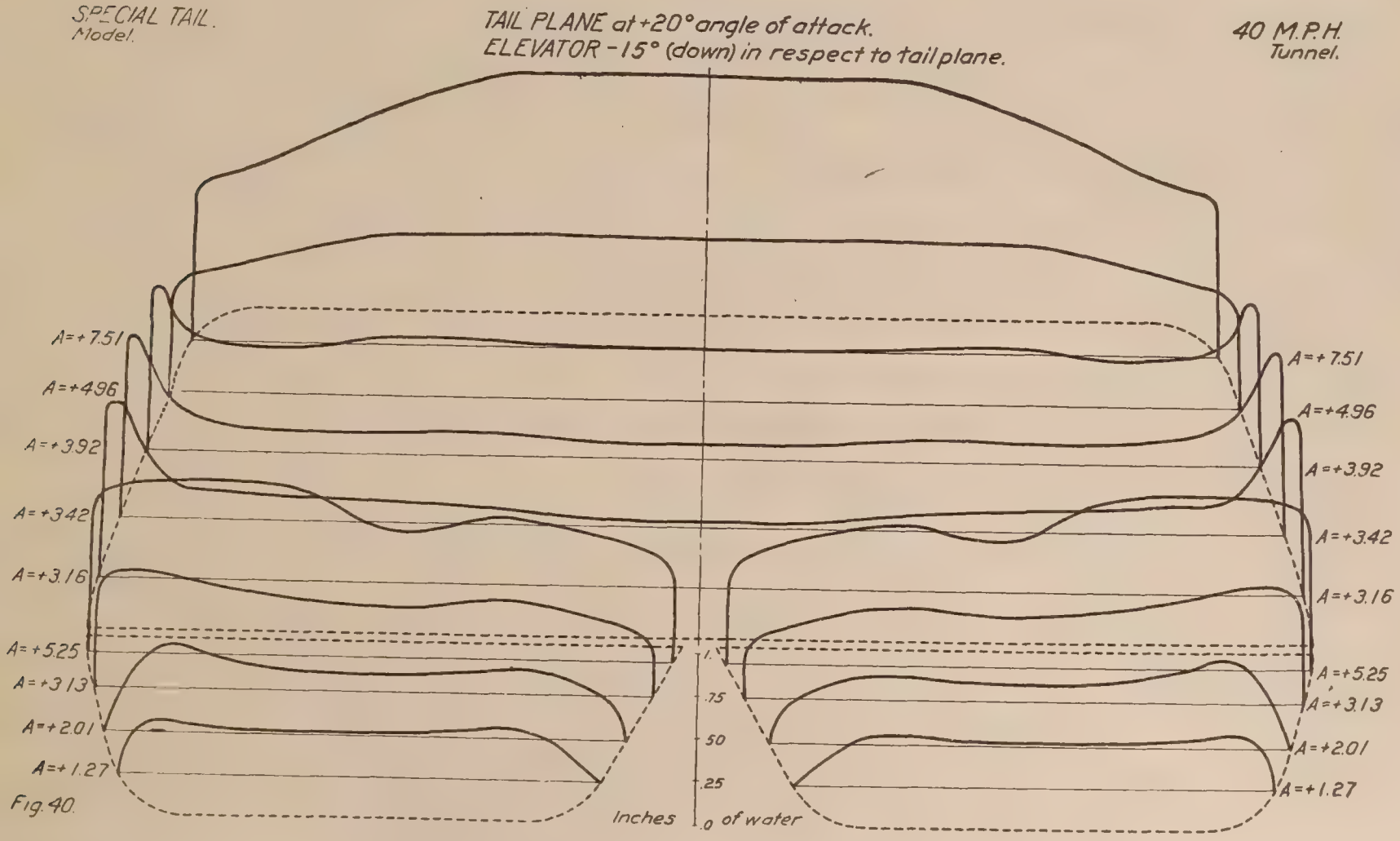

SPECIAL TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR -15° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+7.51
A=+4.96
A=+3.92
A=+3.42
A=+3.16
A=+5.25
A=+3.13
A=+2.01
A=+1.27
1.
.75
.50
.25
.0
Inches of water

Fig. 40.

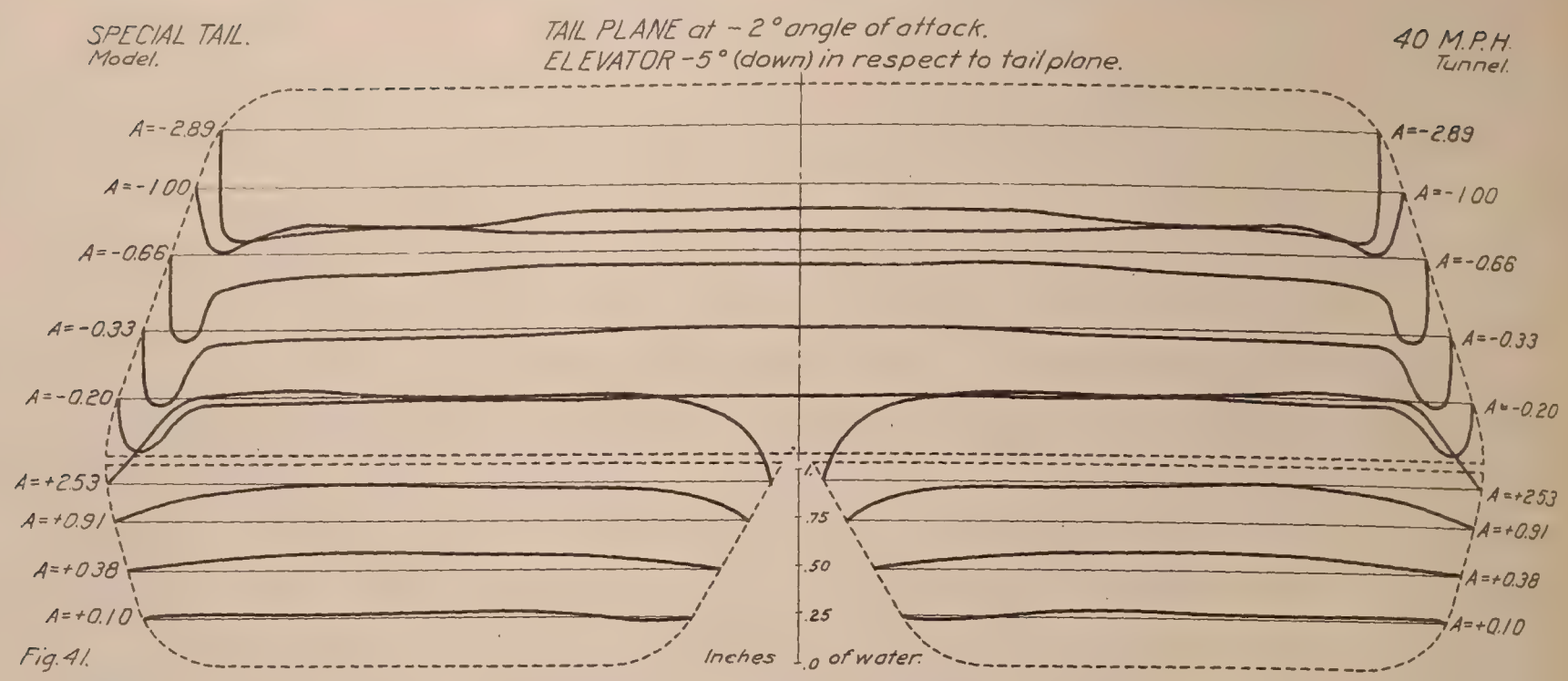


Fig. 41.

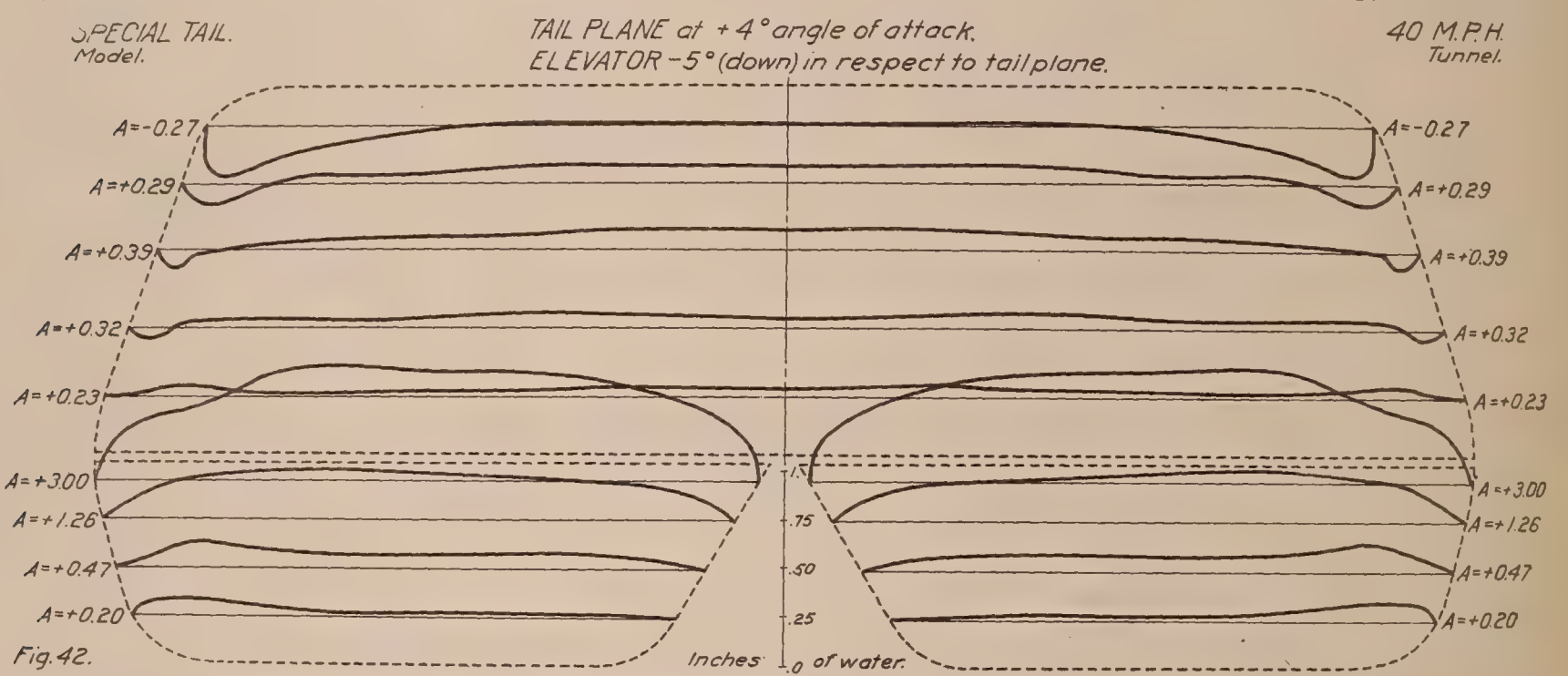


Fig. 42.

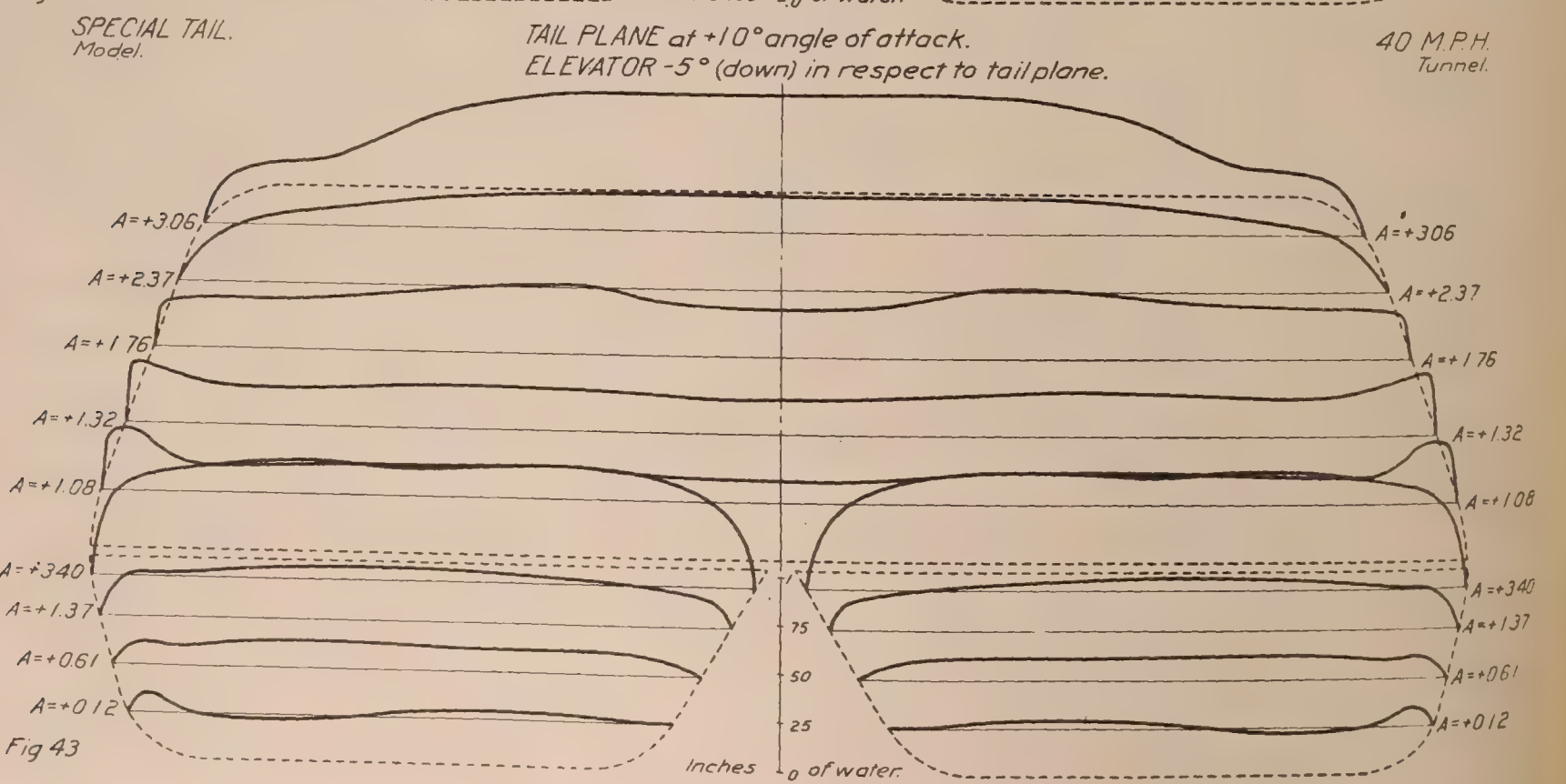


Fig 43

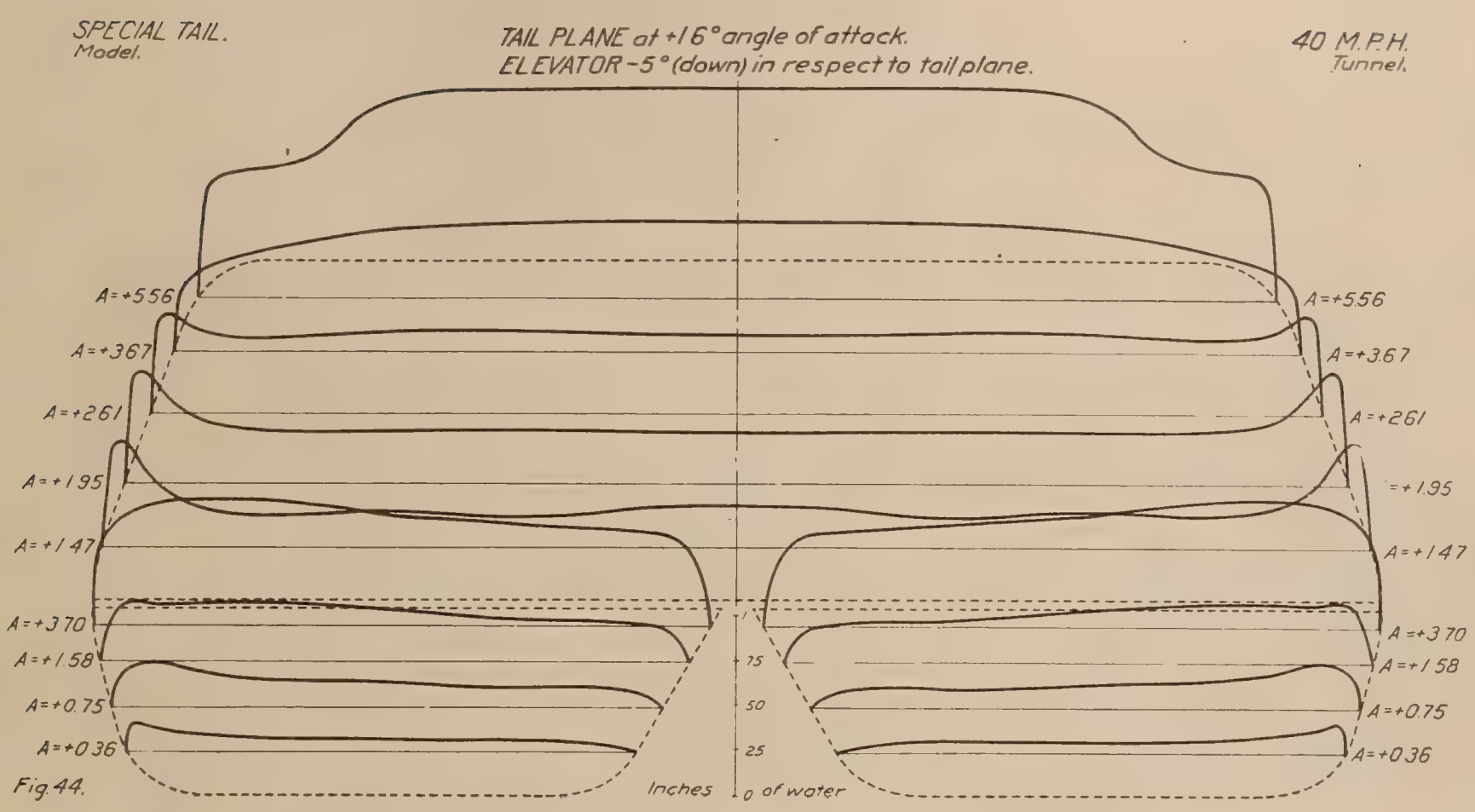

SPECIAL TAIL.
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR -5° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+5.56
A=+3.67
A=+2.61
A=+1.95
A=+1.47
A=+3.70
A=+1.58
A=+0.75
A=+0.36
1
.75
.50
.25
0
Inches of water

Fig. 44.

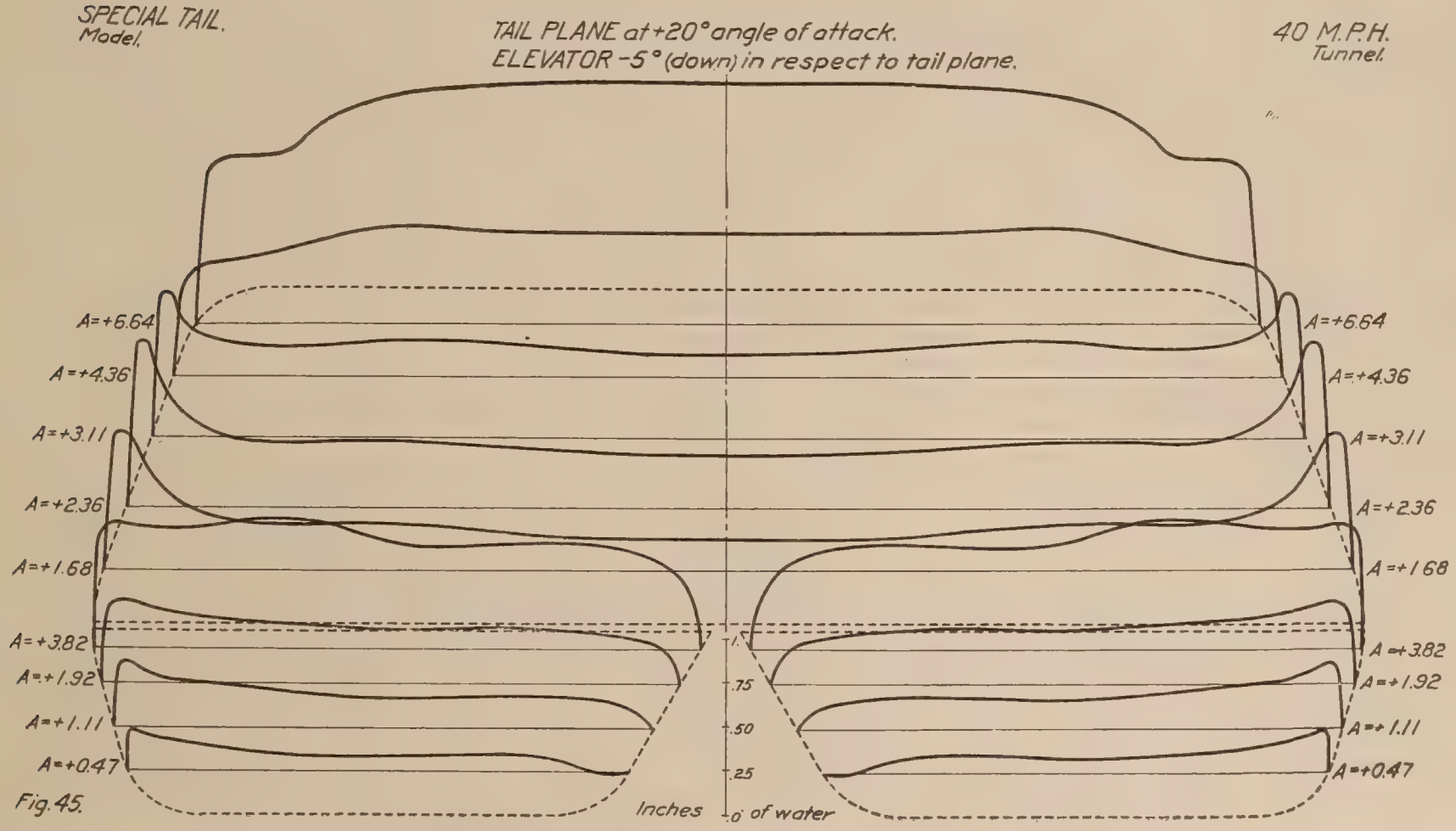

SPECIAL TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR -5° (down) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+6.64
A=+4.36
A=+3.11
A=+2.36
A=+1.68
A=+3.82
A=+1.92
A=+1.11
A=+0.47
1.
.75
.50
.25
0
Inches of water

Fig. 45.

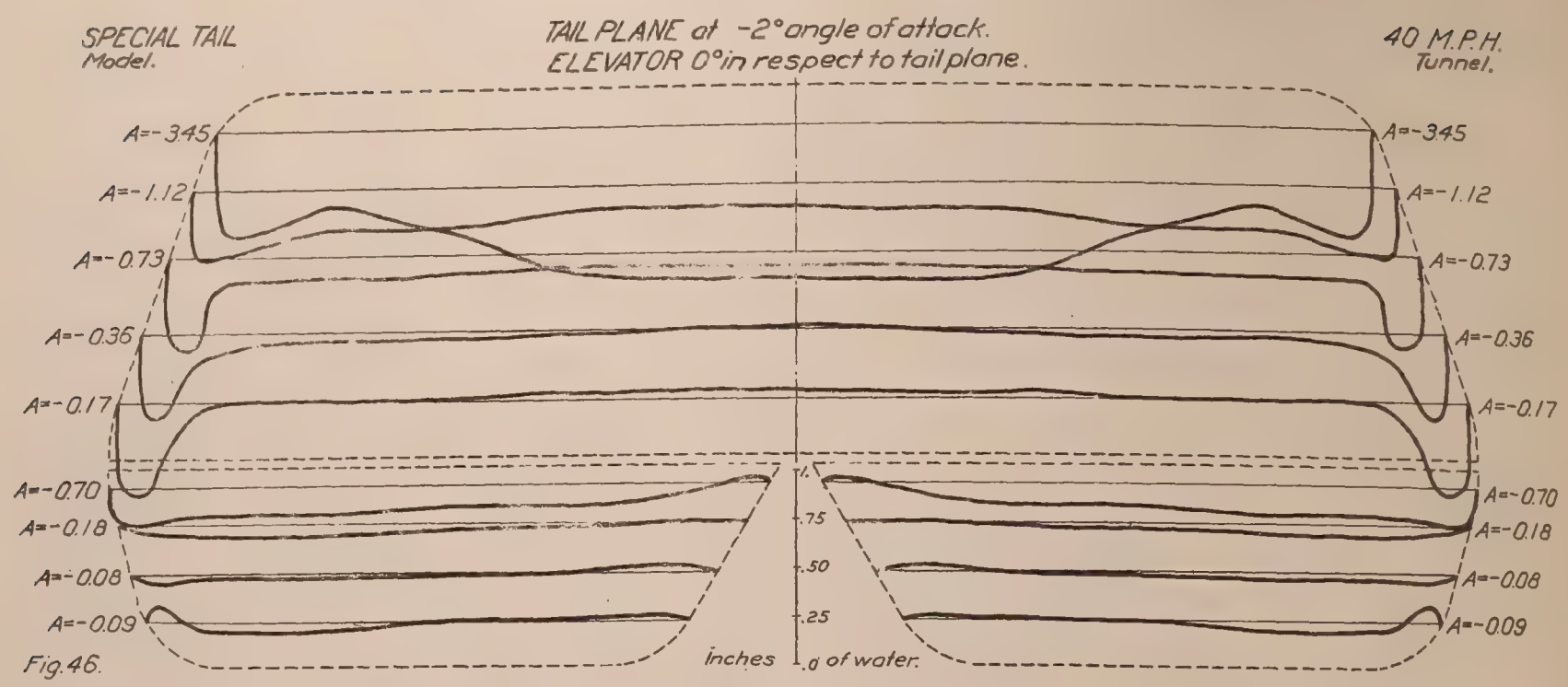


Fig. 46.

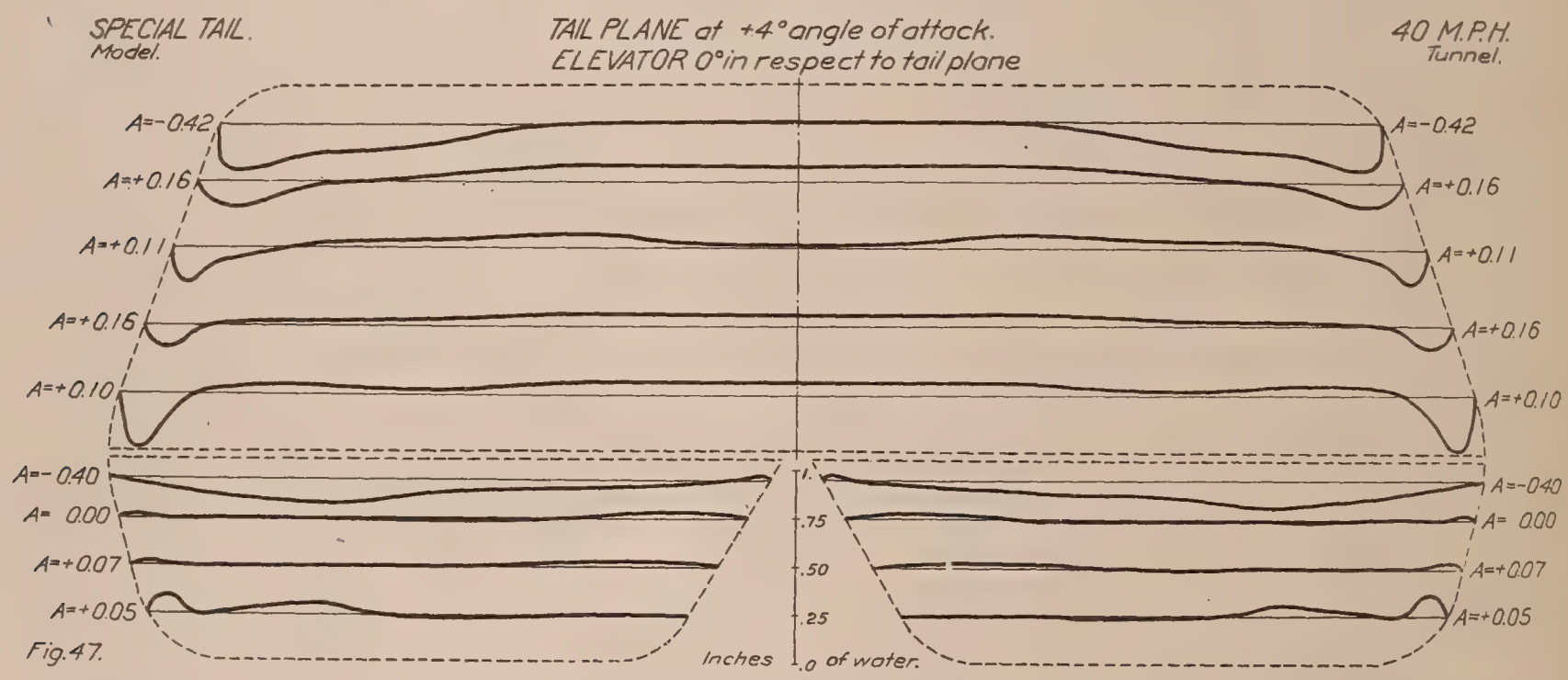


Fig. 47.

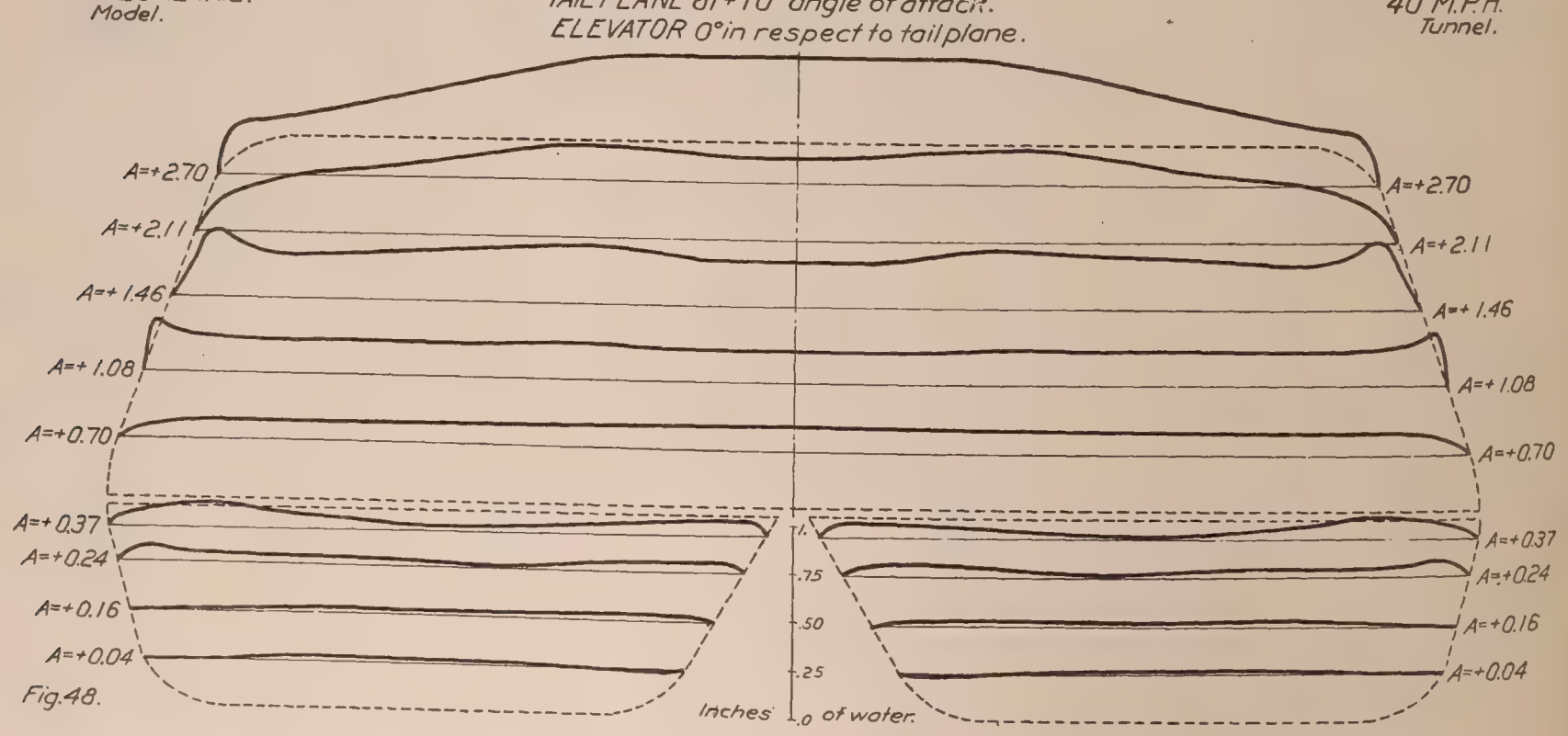


Fig. 48.

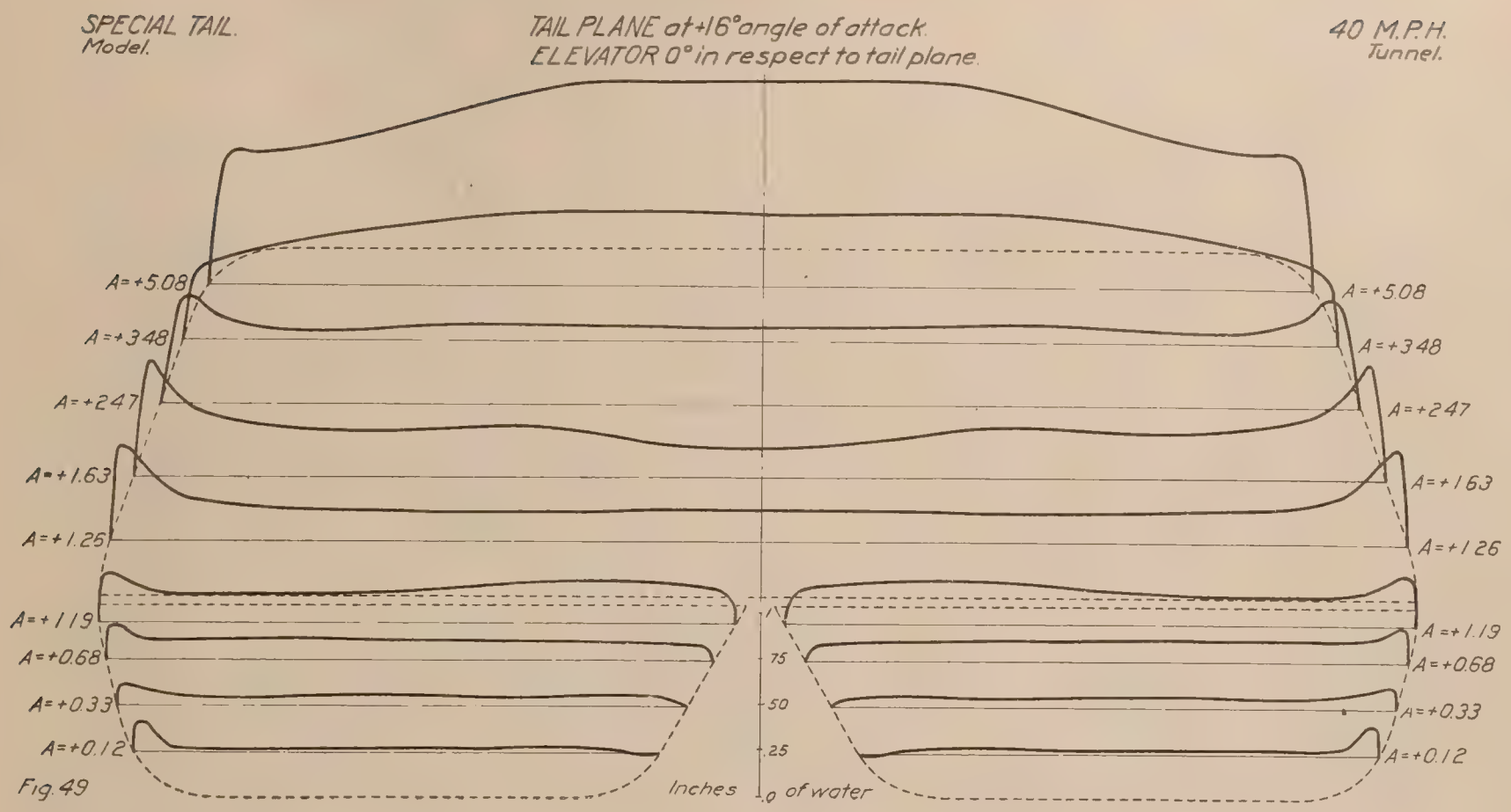

SPECIAL TAIL.
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR 0° in respect to tail plane.
40 M.P.H.
Tunnel.
A=+5.08
A=+3.48
A=+2.47
A=+1.63
A=+1.25
A=+1.19
A=+0.68
A=+0.33
A=+0.12
1
.75
.50
.25
0
Inches of water

Fig. 49

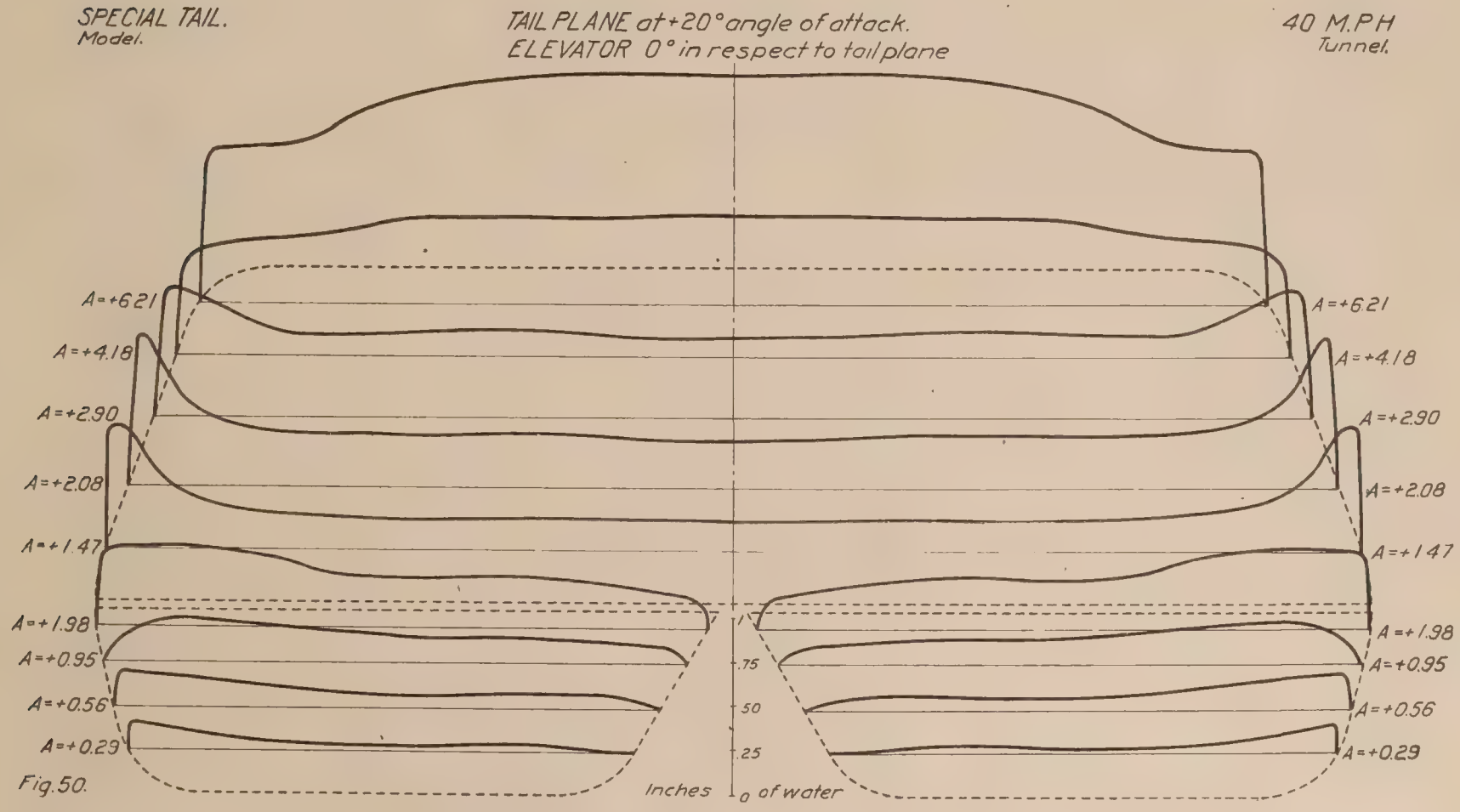

SPECIAL TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR 0° in respect to tail plane
40 M.P.H
Tunnel.
A=+6.21
A=+4.18
A=+2.90
A=+2.08
A=+1.47
A=+1.98
A=+0.95
A=+0.56
A=+0.29
1
.75
.50
.25
0
Inches of water

Fig. 50.

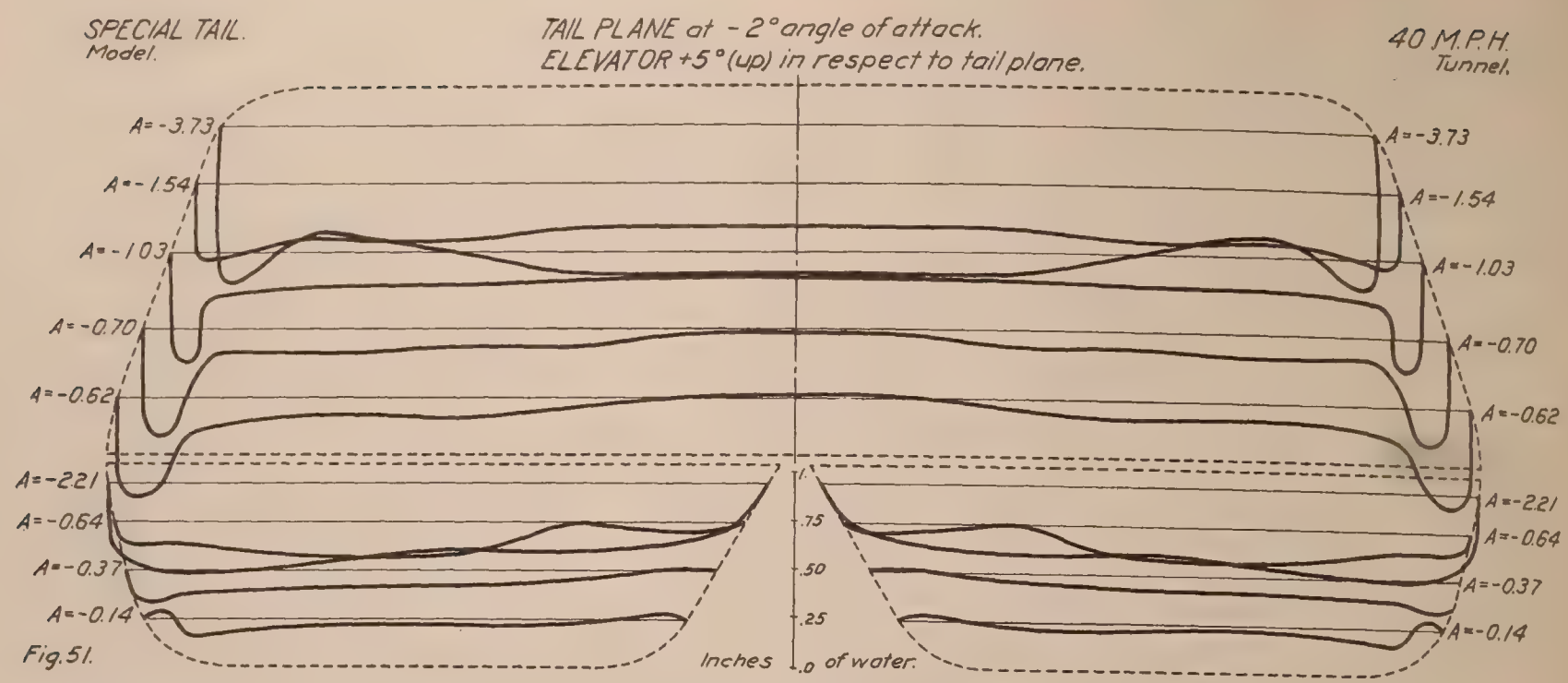


Fig. 51.

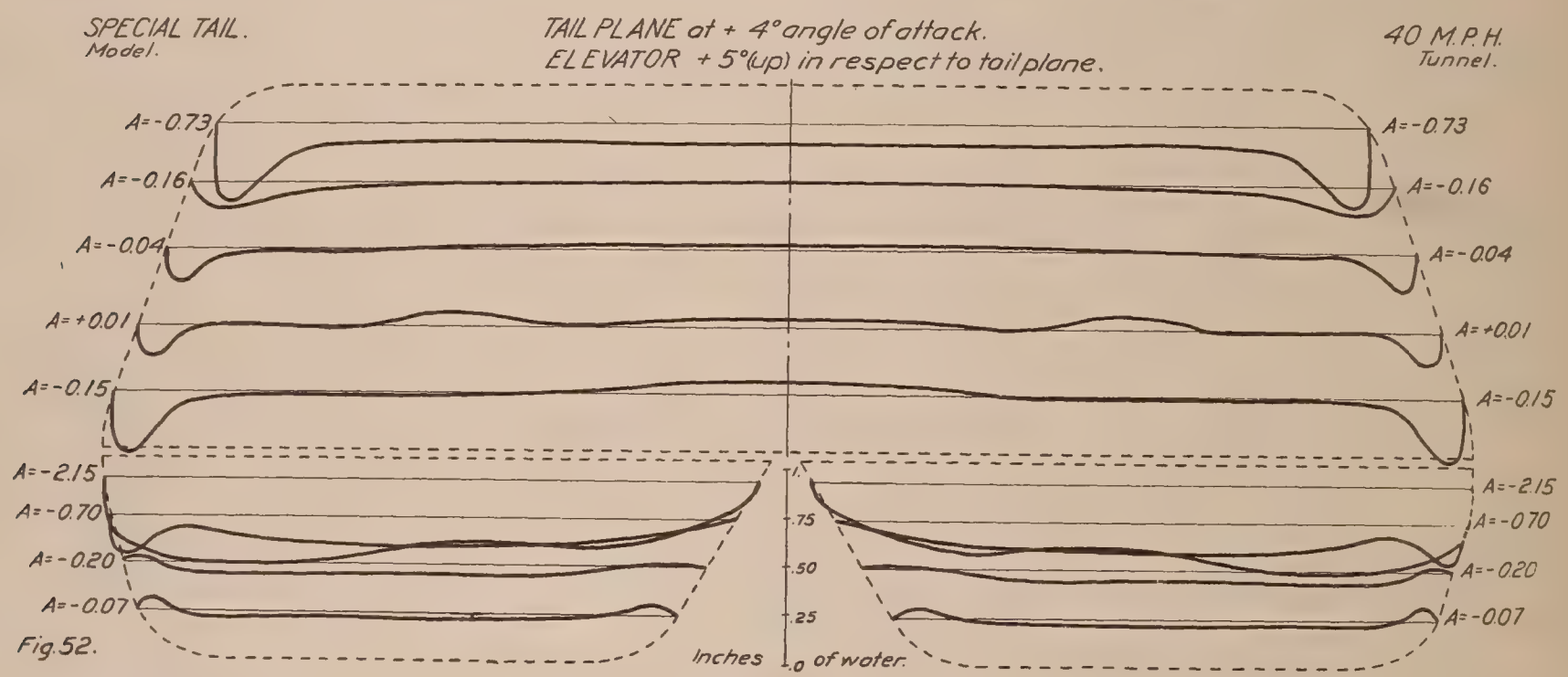


Fig. 52.

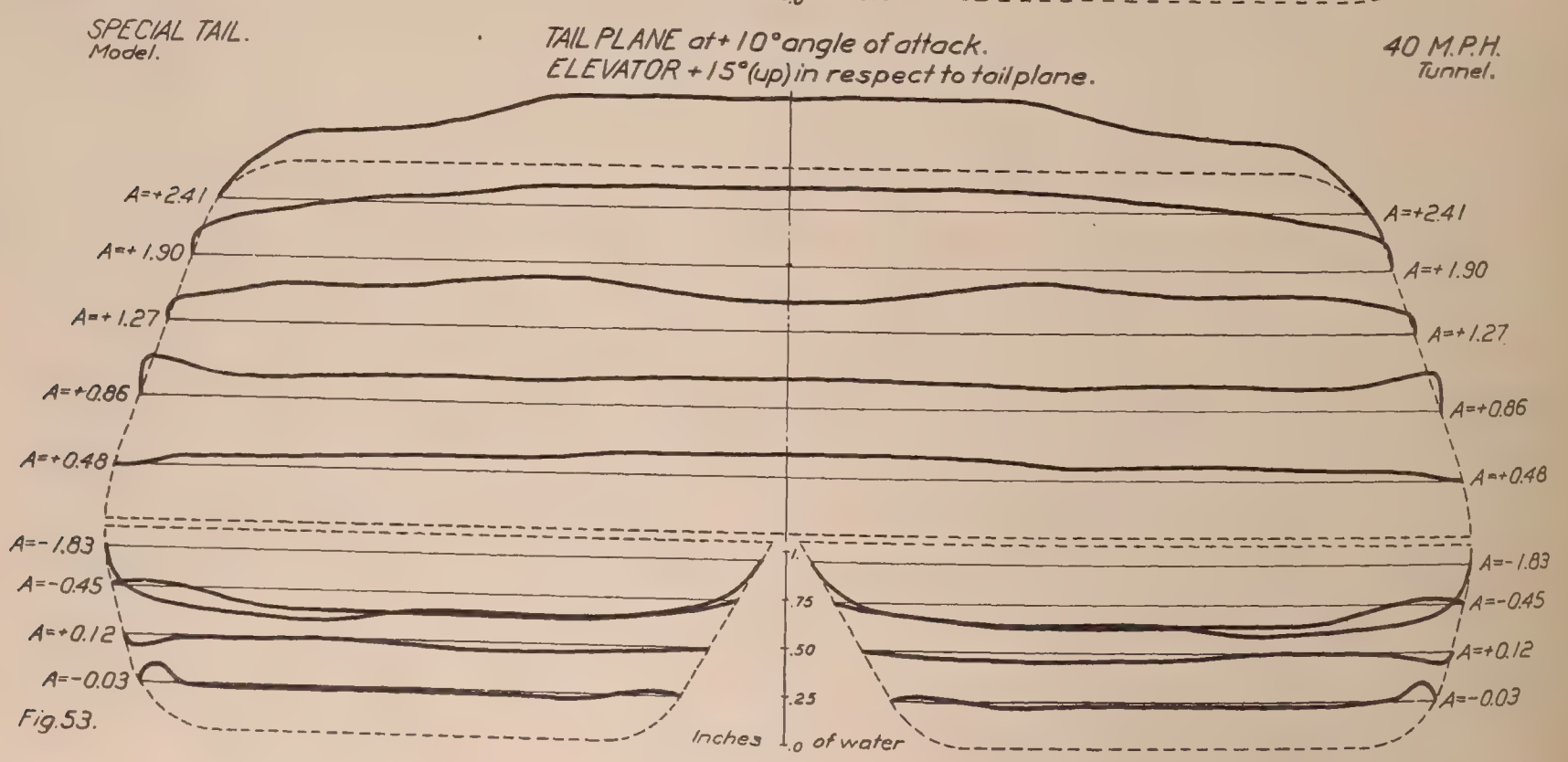


Fig. 53.

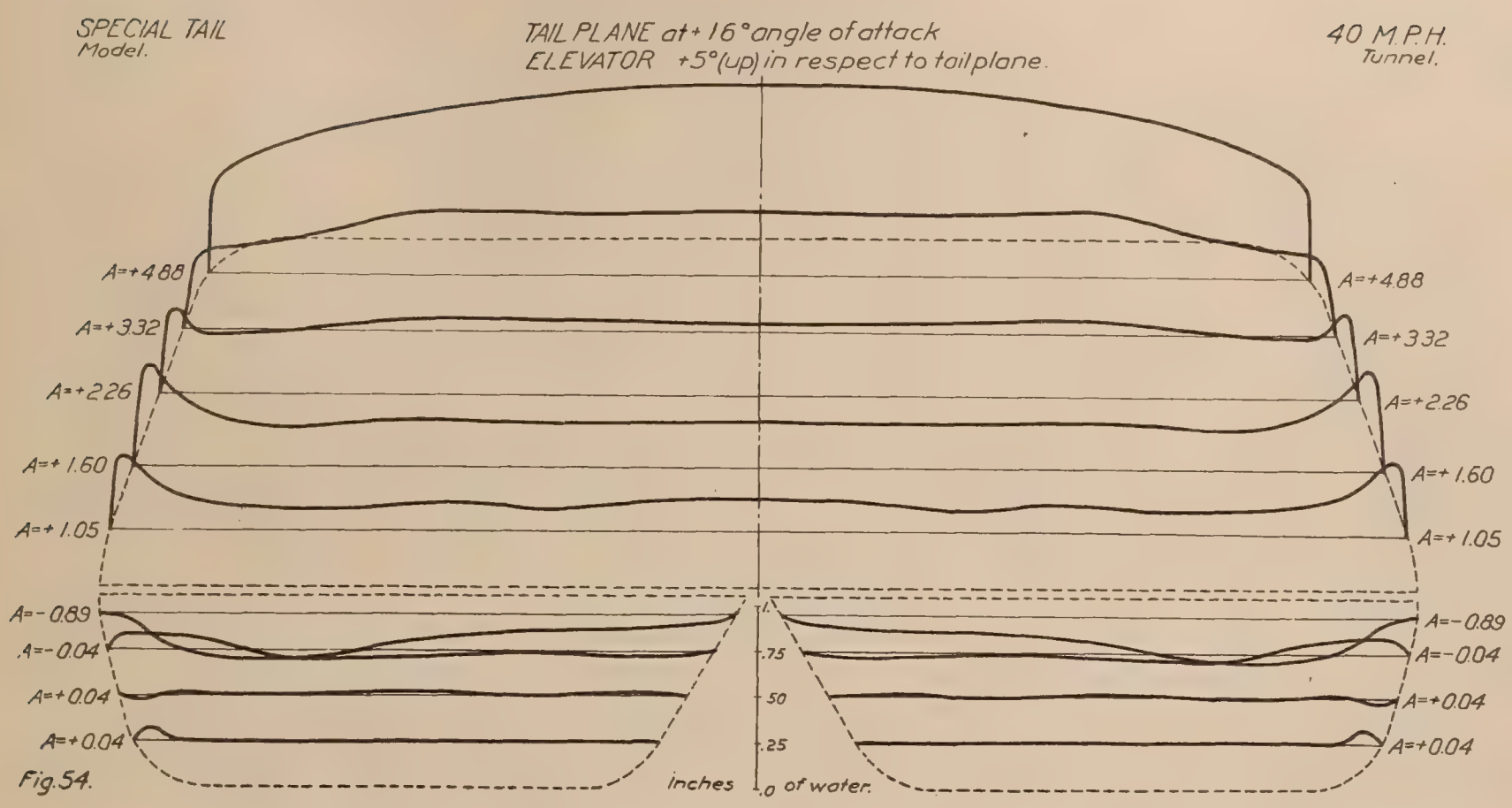

SPECIAL TAIL
Model.
TAIL PLANE at + 16° angle of attack
ELEVATOR +5° (up) in respect to tailplane.
40 M.P.H.
Tunnel.
A=+4.88
A=+3.32
A=+2.26
A=+1.60
A=+1.05
A=-0.89
A=-0.04
A=+0.04
A=+0.04
1.
.75
.50
.25
.0
inches of water.

Fig. 54.

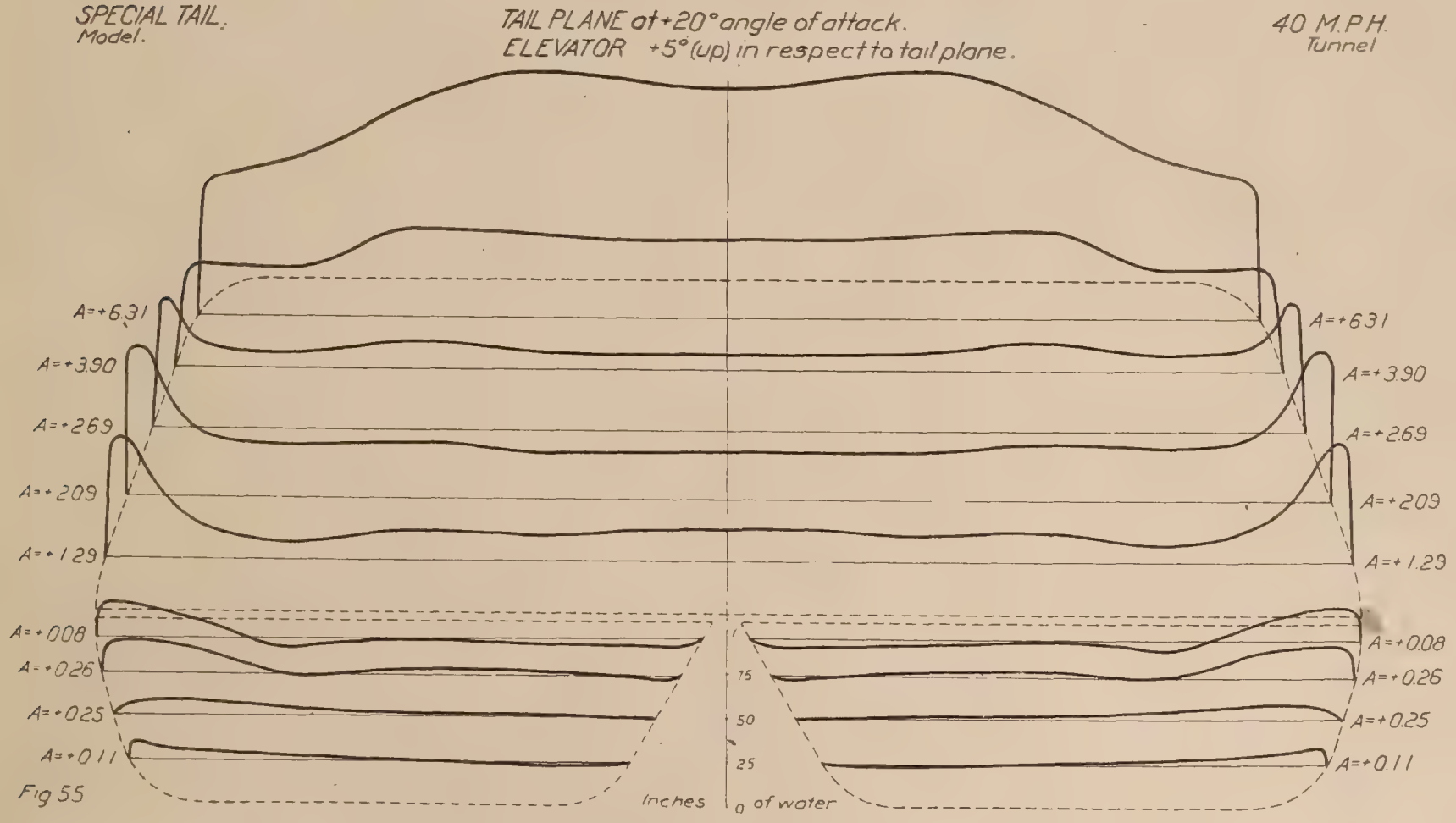

SPECIAL TAIL.
Model.
TAIL PLANE at + 20° angle of attack.
ELEVATOR +5° (up) in respect to tail plane.
40 M.P.H.
Tunnel
A=+6.31
A=+3.90
A=+2.69
A=+2.09
A=+1.29
A=+0.08
A=+0.26
A=+0.25
A=+0.11
1
.75
.50
.25
0
inches of water

Fig 55

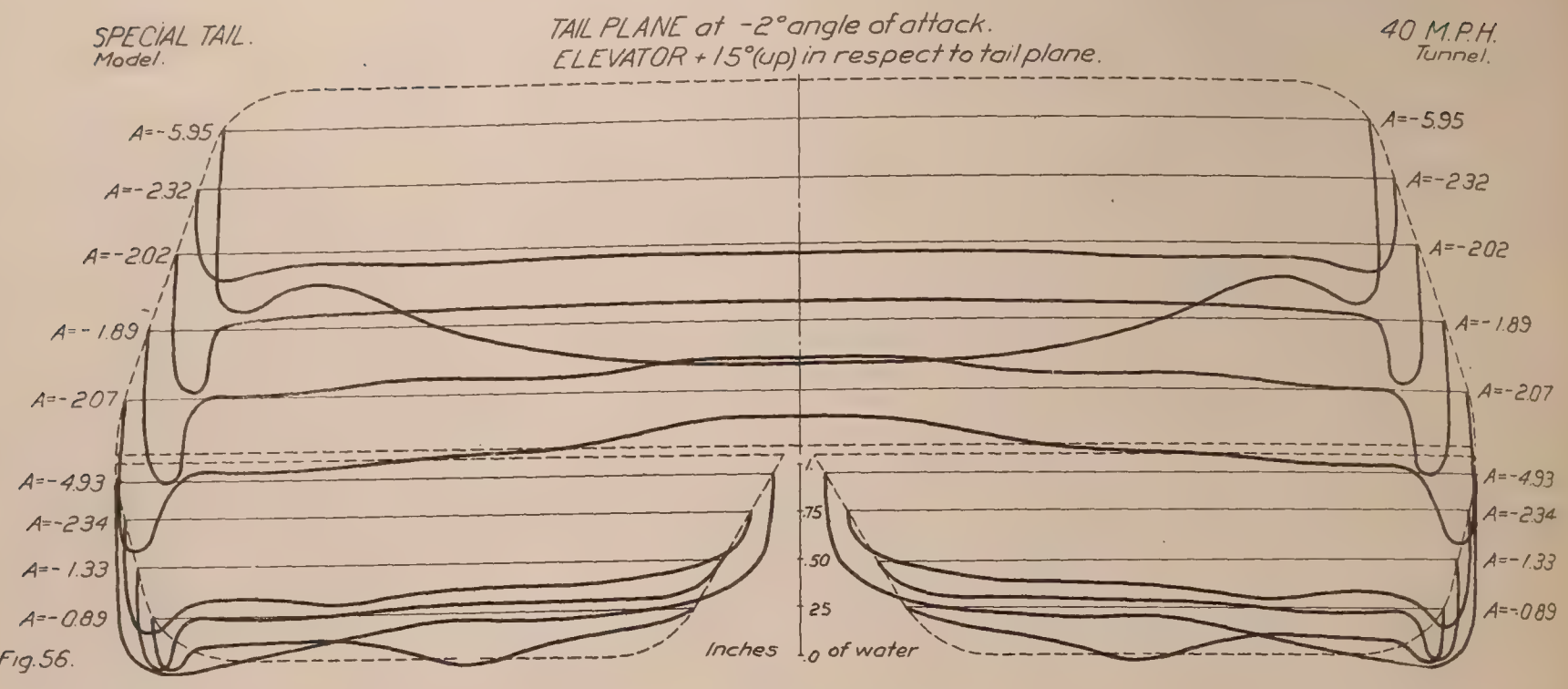


Fig. 56.

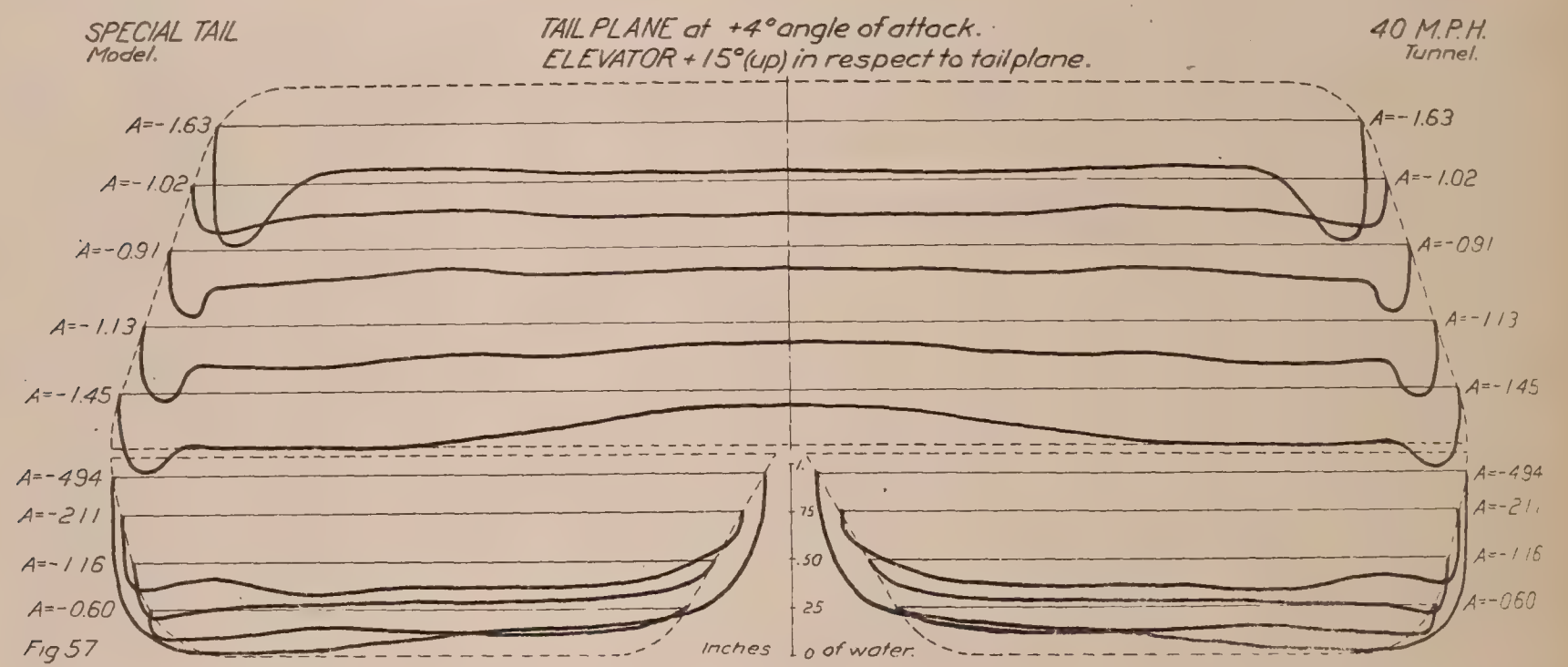


Fig 57

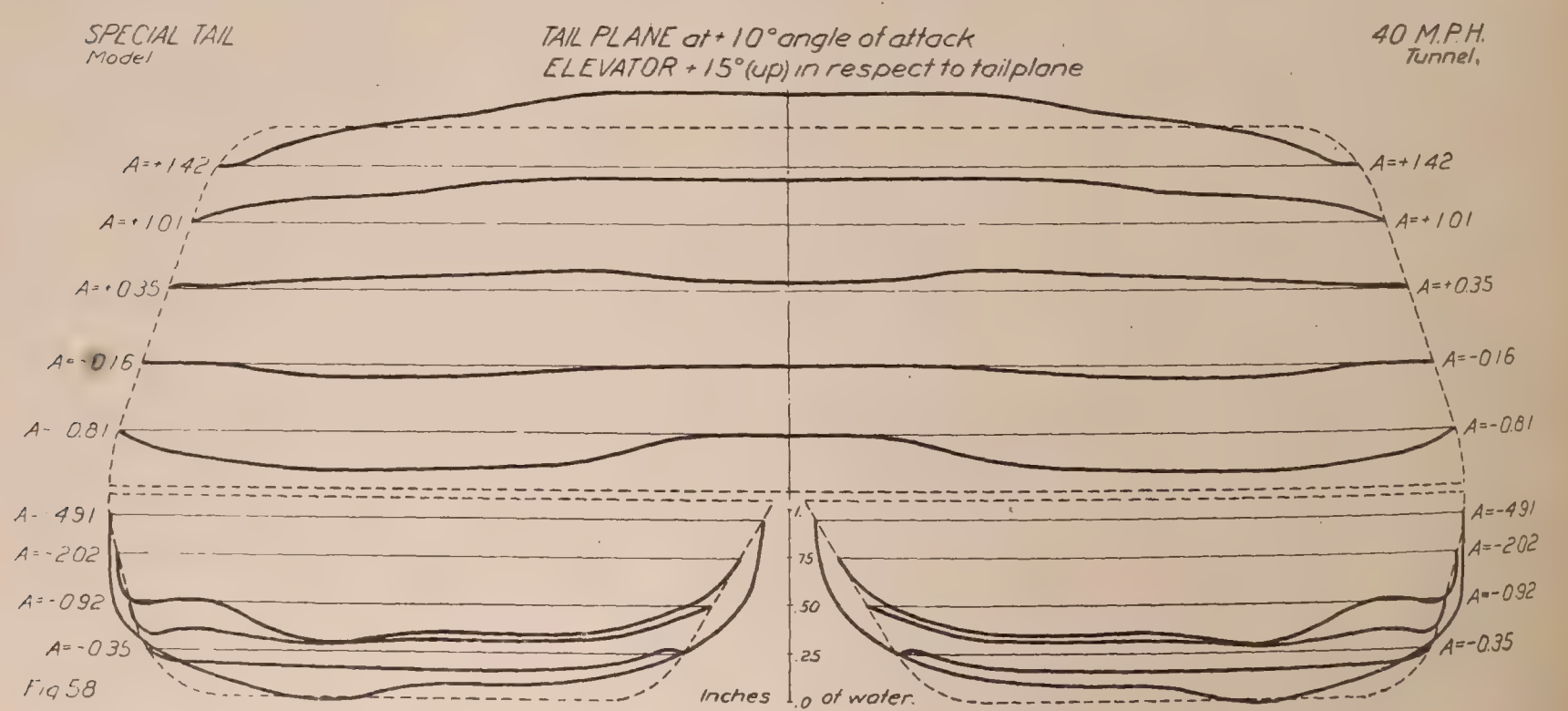


Fig 58

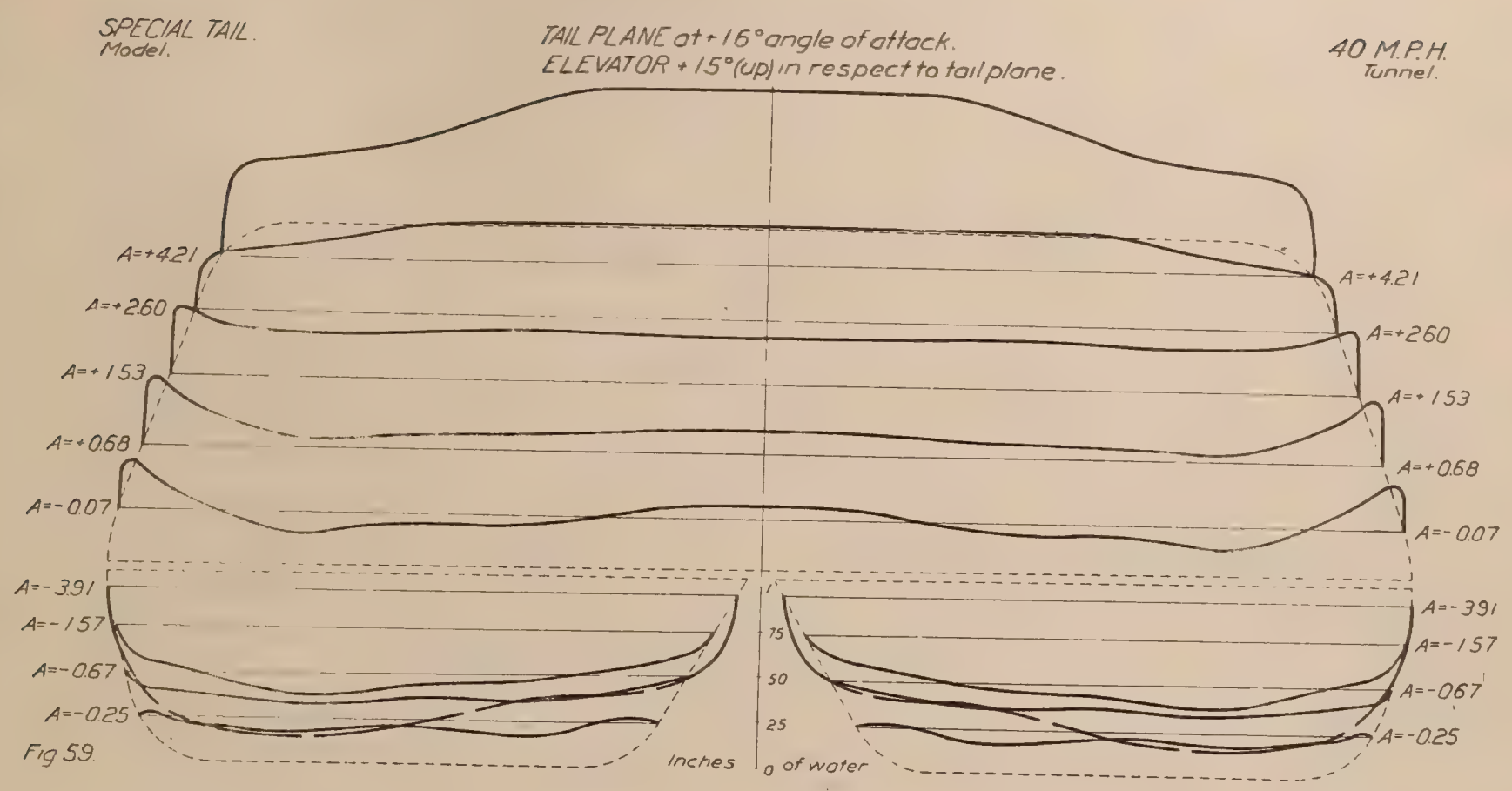

SPECIAL TAIL.
Model.
TAIL PLANE at + 16° angle of attack.
ELEVATOR + 15° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+4.21
A=+2.60
A=+1.53
A=+0.68
A=-0.07
A=-3.91
A=-1.57
A=-0.67
A=-0.25
1
.75
.50
.25
0
Inches of water

Fig 59.

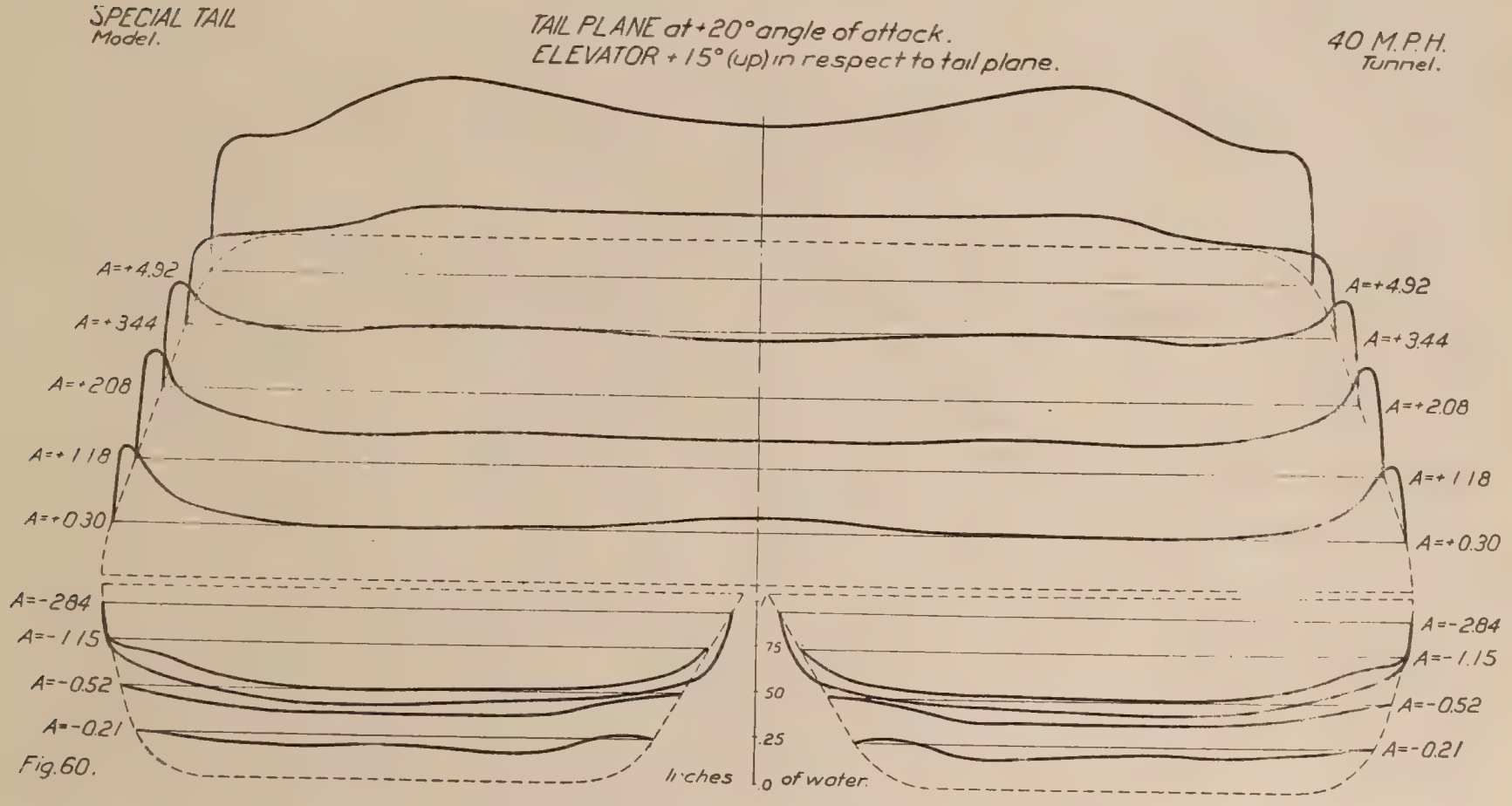

SPECIAL TAIL
Model.
TAIL PLANE at + 20° angle of attack.
ELEVATOR + 15° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+4.92
A=+3.44
A=+2.08
A=+1.18
A=+0.30
A=-2.84
A=-1.15
A=-0.52
A=-0.21
1
.75
.50
.25
0
Inches of water.

Fig. 60.

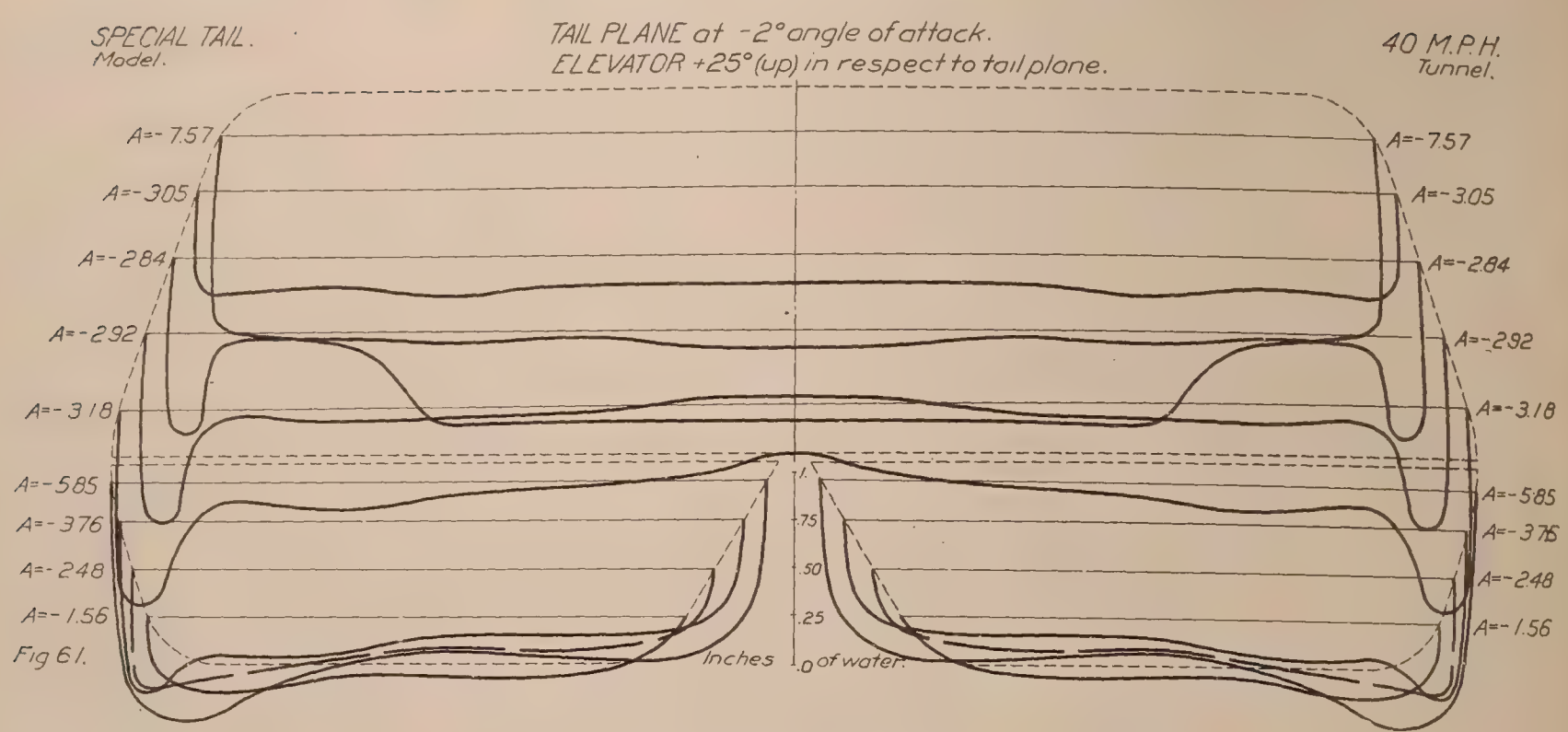


Fig 61.

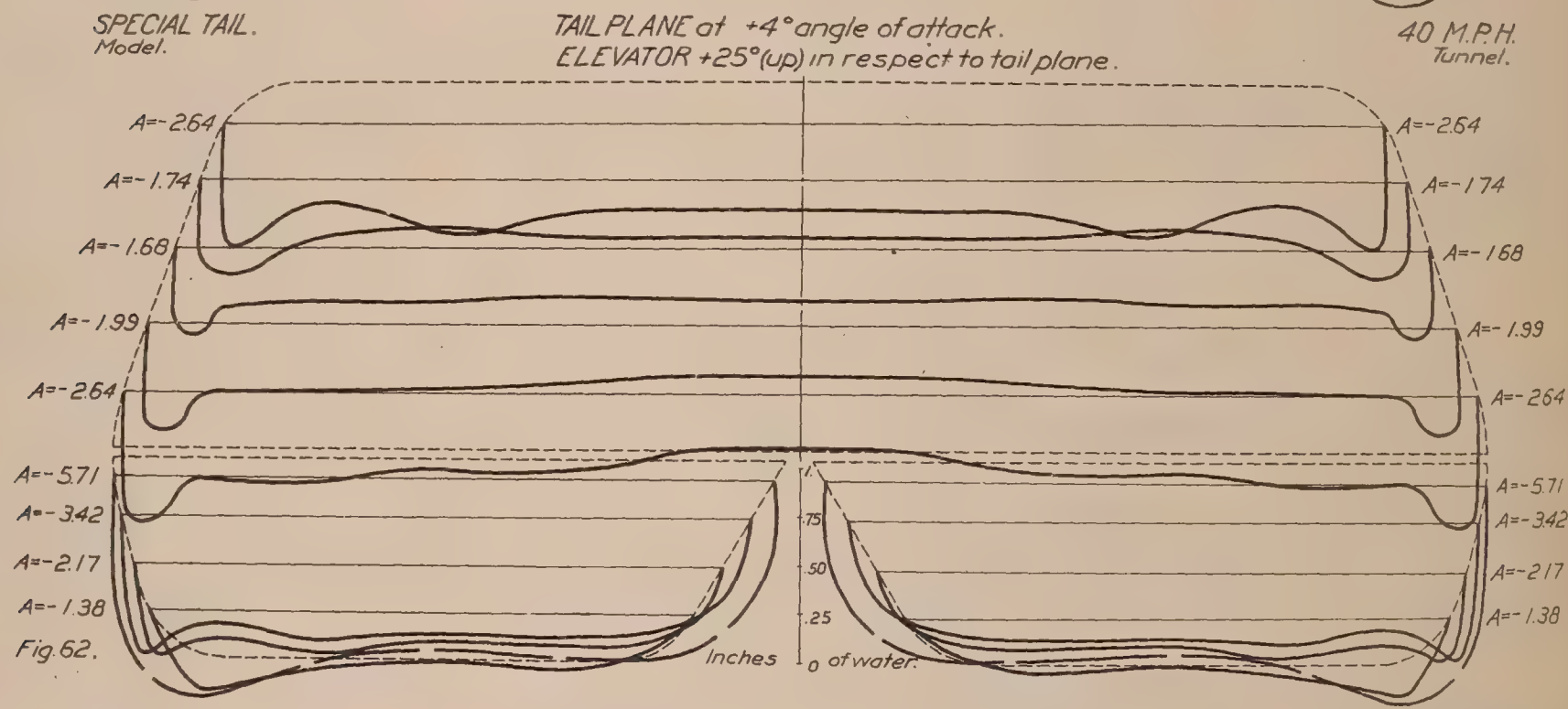


Fig. 62.

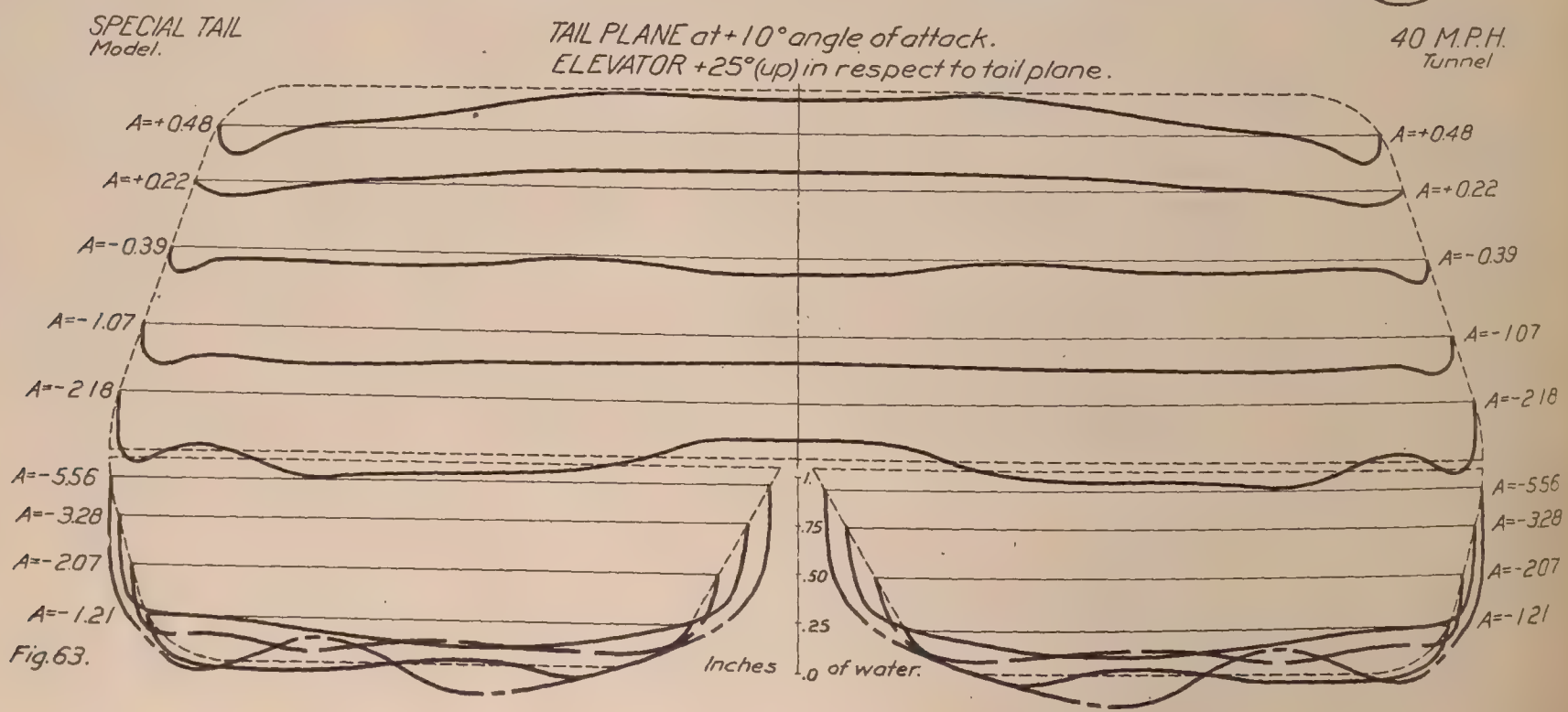


Fig. 63.

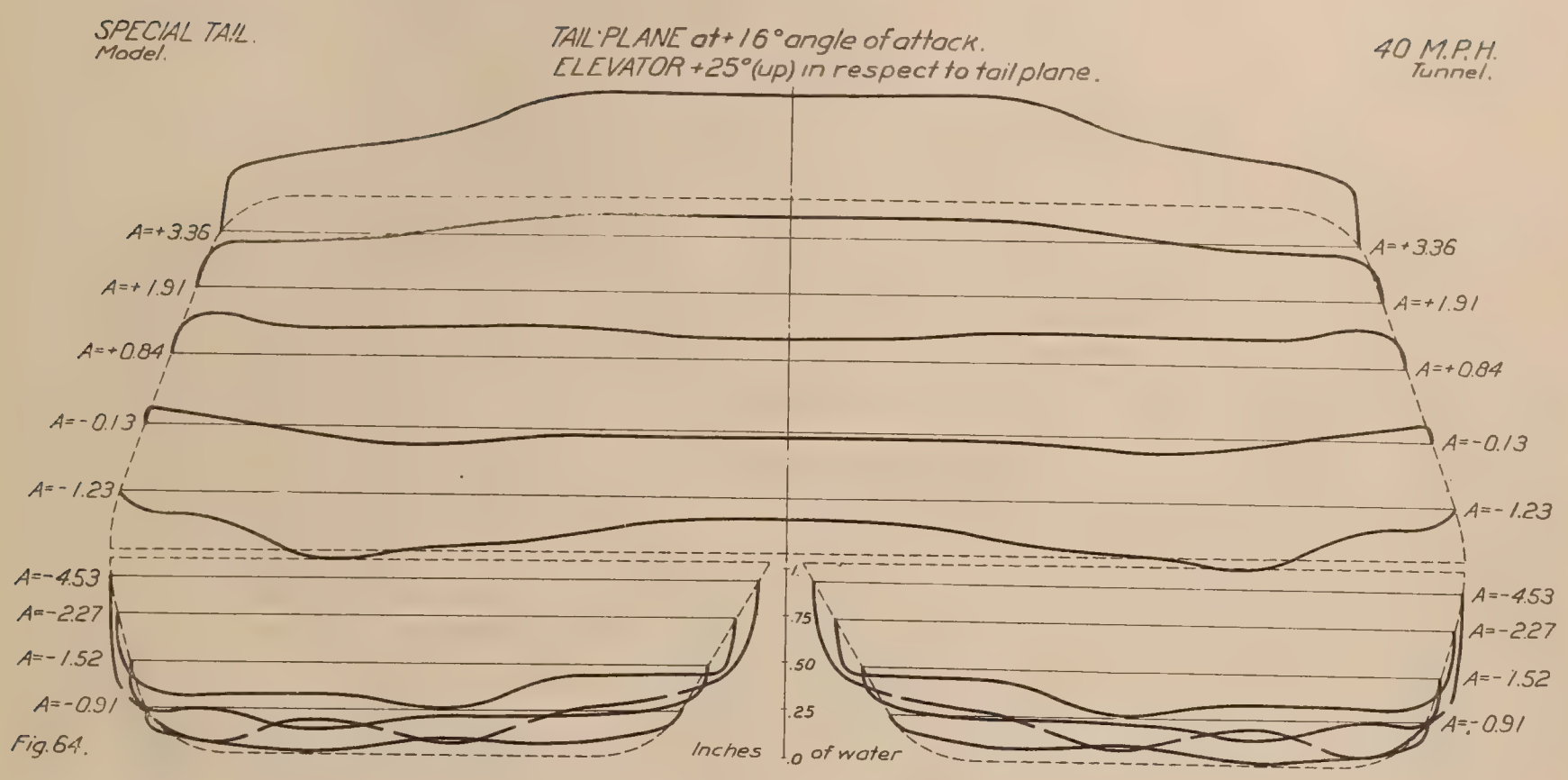

SPECIAL TAIL.
Model.
TAIL PLANE at +16° angle of attack.
ELEVATOR +25° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+3.36
A=+1.91
A=+0.84
A=-0.13
A=-1.23
A=-4.53
A=-2.27
A=-1.52
A=-0.91
1.
.75
.50
.25
.0
Inches of water

Fig. 64.

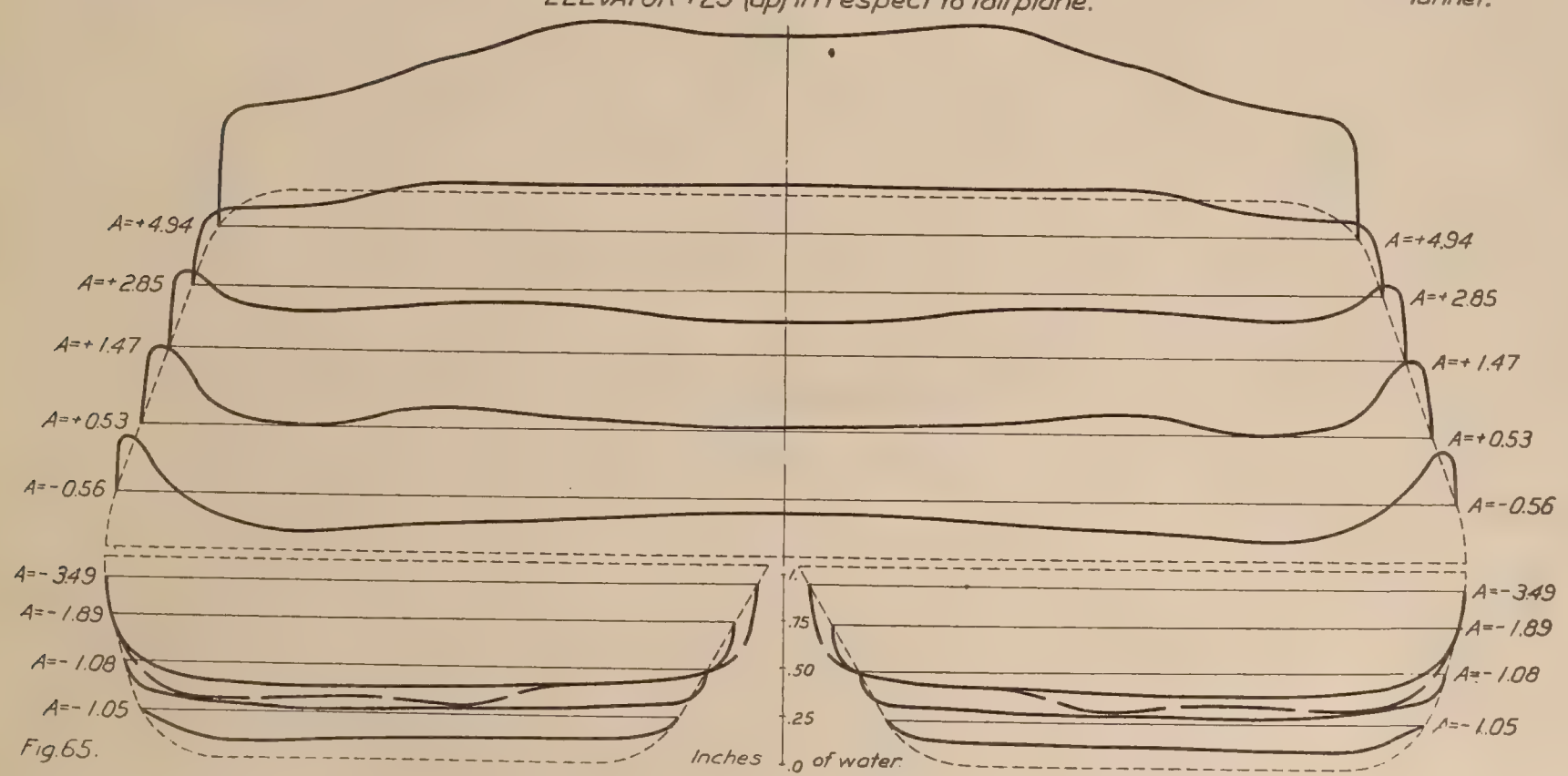
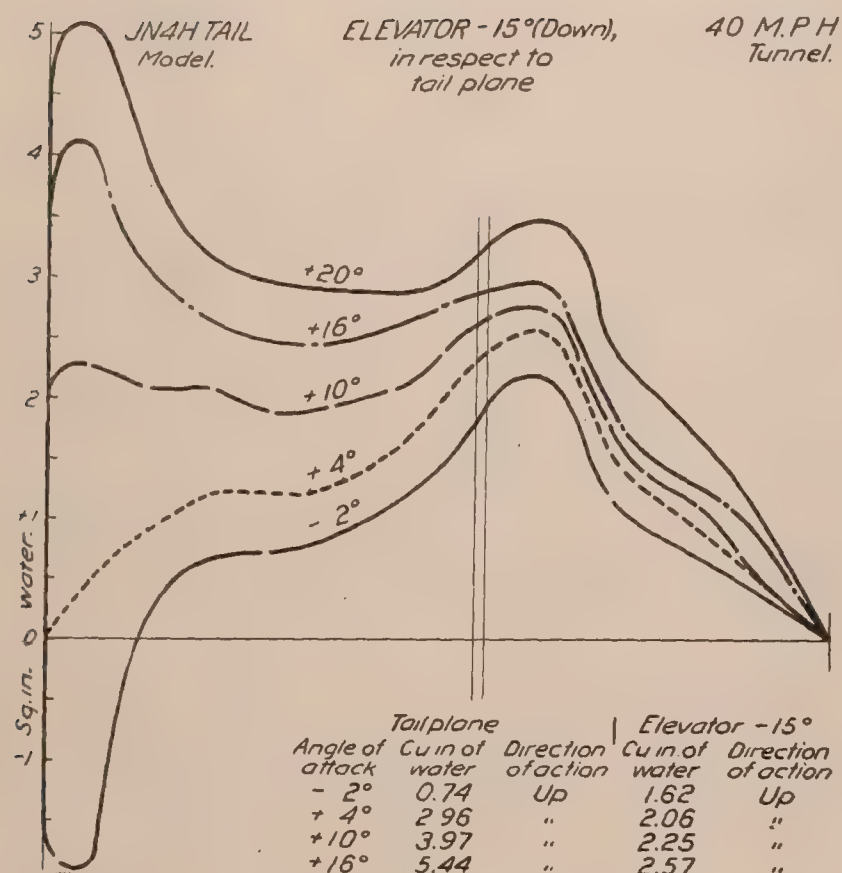
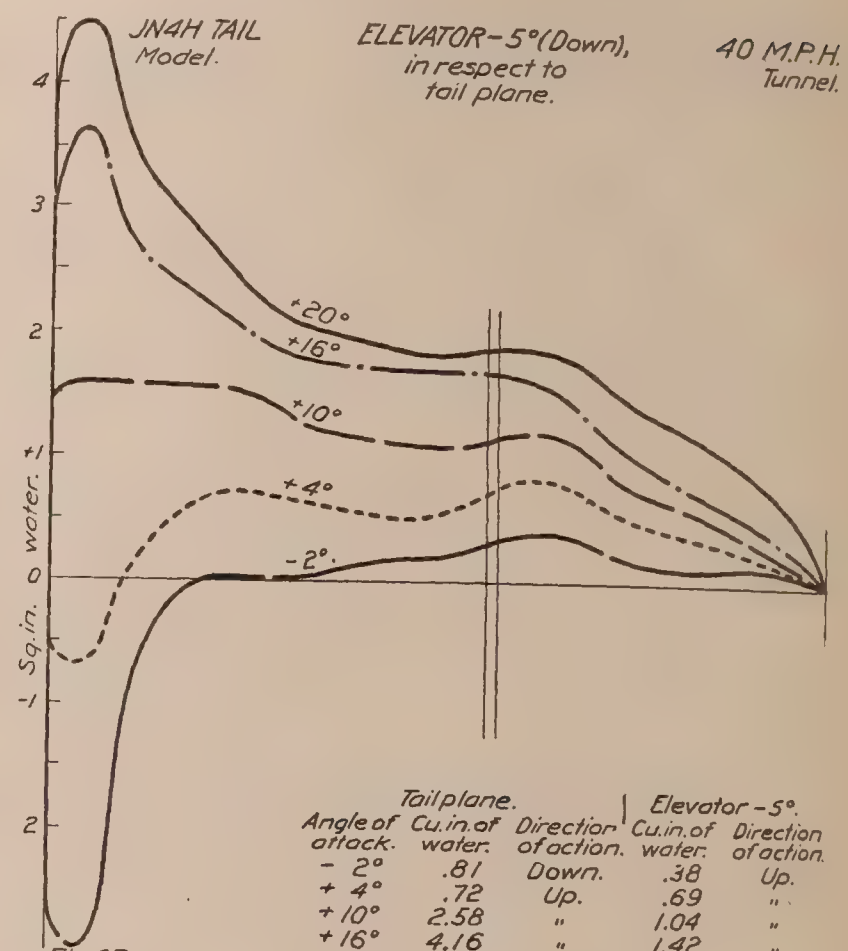

SPECIAL TAIL.
Model.
TAIL PLANE at +20° angle of attack.
ELEVATOR +25° (up) in respect to tail plane.
40 M.P.H.
Tunnel.
A=+4.94
A=+2.85
A=+1.47
A=+0.53
A=-0.56
A=-3.49
A=-1.89
A=-1.08
A=-1.05
1.
.75
.50
.25
.0
Inches of water

Fig. 65.



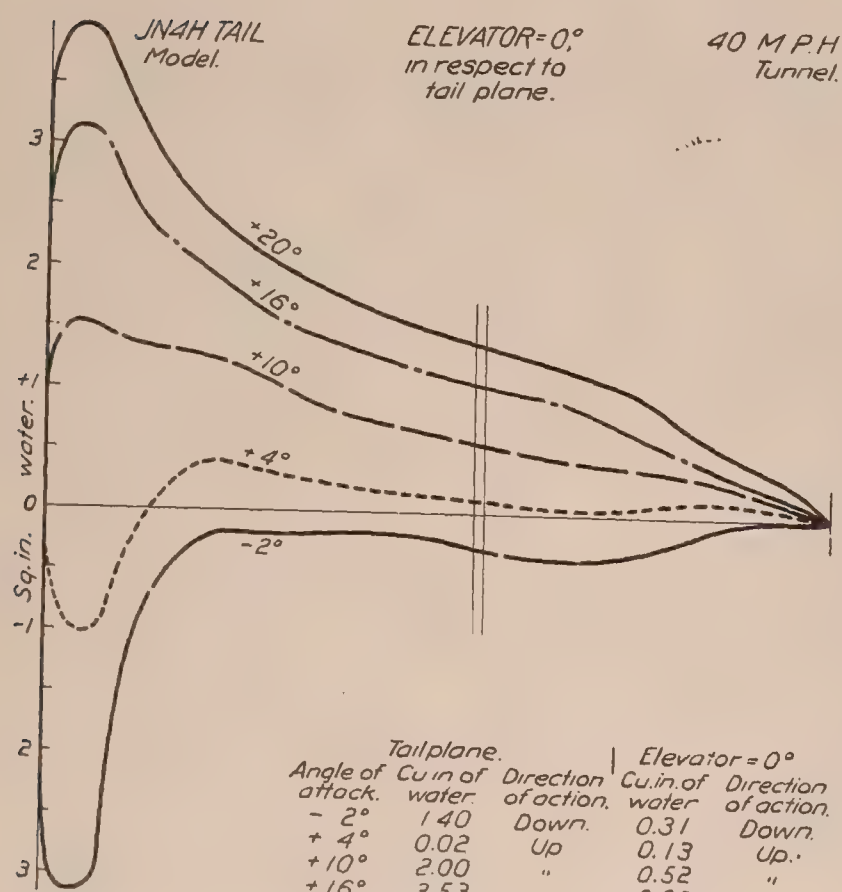
Angle of attack	Tailplane Cu in of water	Direction of action	Elevator -15° Cu in of water	Direction of action
- 2°	0.74	Up	1.62	Up
+ 4°	2.96	"	2.06	"
+10°	3.97	"	2.25	"
+16°	5.44	"	2.57	"
+20°	6.53	"	3.16	"

Fig.66.



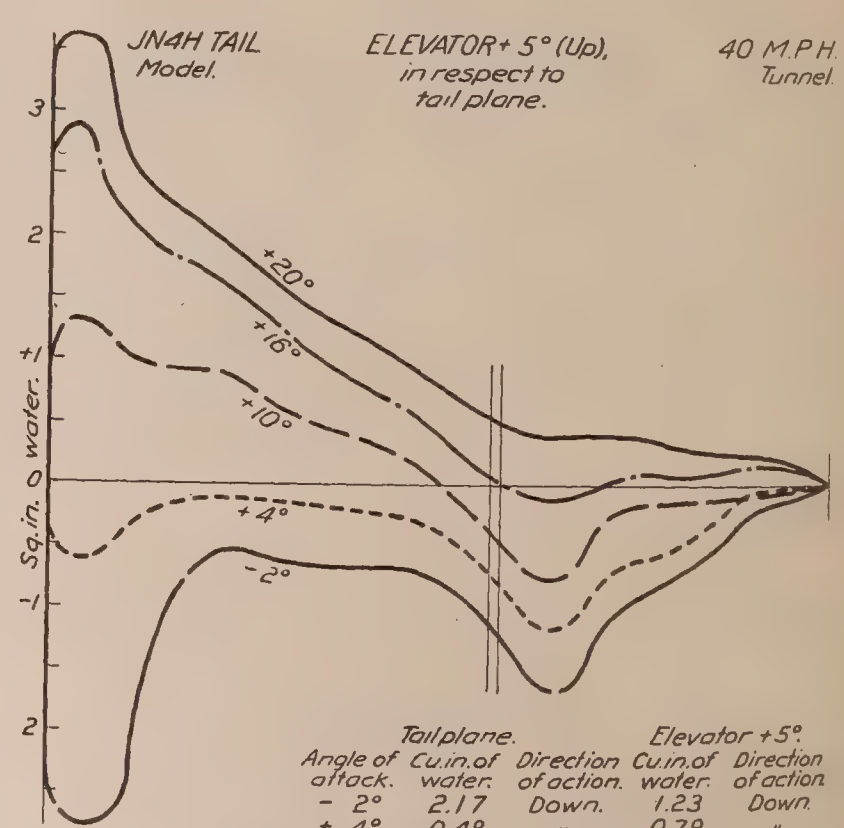
Angle of attack.	Tailplane. Cu.in.of water.	Direction of action.	Elevator -5°. Cu.in.of water.	Direction of action.
- 2°	.81	Down.	.38	Up.
+ 4°	.72	Up.	.69	"
+10°	2.58	"	1.04	"
+16°	4.16	"	1.42	"
+20°	5.01	"	1.95	"

Fig.67.



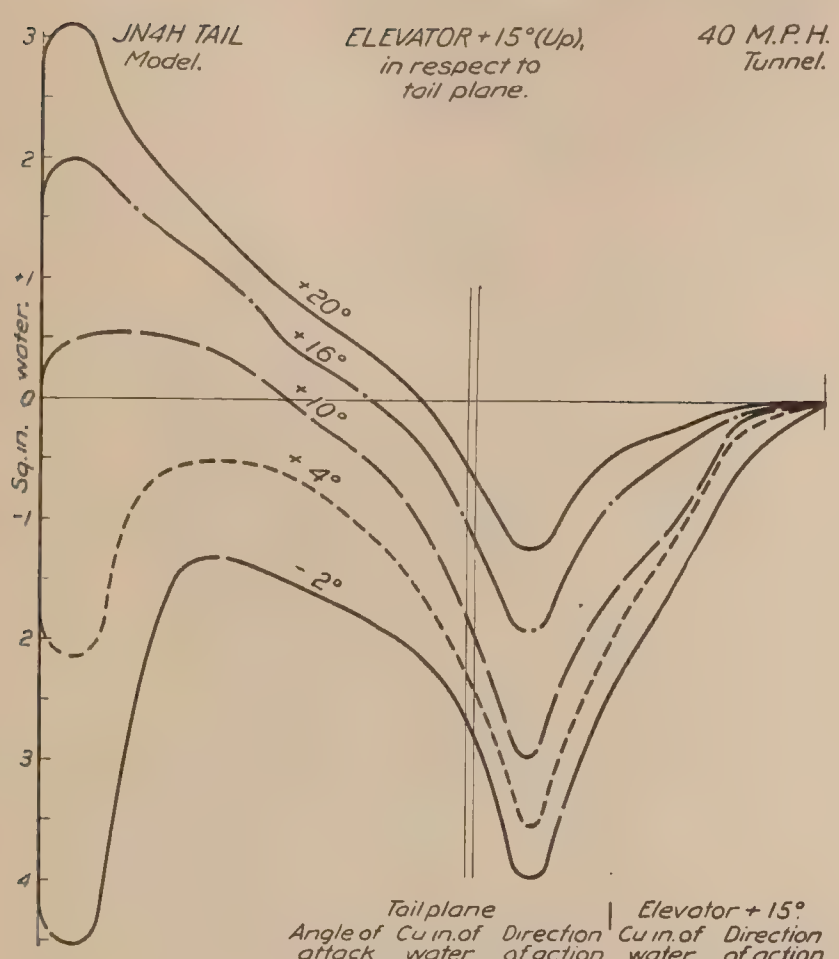
Angle of attack.	Tailplane. Cu in of water.	Direction of action.	Elevator = 0° Cu.in.of water	Direction of action.
- 2°	1.40	Down.	0.31	Down.
+ 4°	0.02	Up	0.13	Up.
+10°	2.00	"	0.52	"
+16°	3.53	"	0.89	"
+20°	4.50	"	1.27	"

Fig.68.



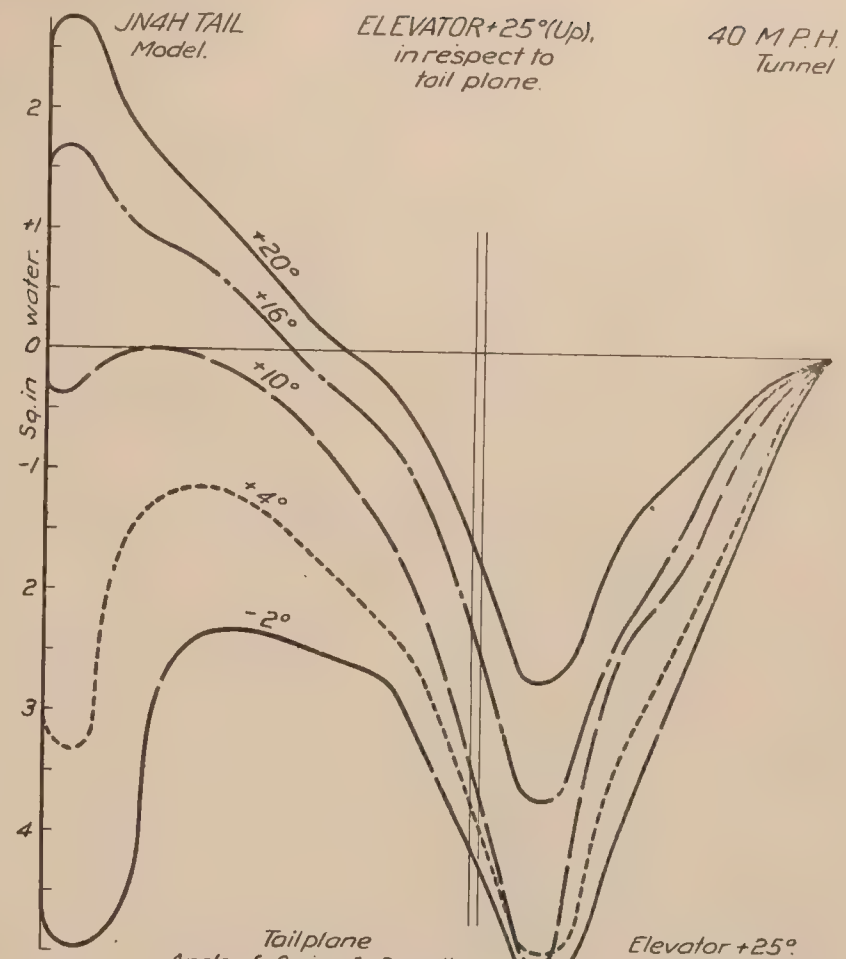
Angle of attack.	Tailplane. Cu.in.of water.	Direction of action.	Elevator +5°. Cu.in.of water.	Direction of action.
- 2°	2.17	Down.	1.23	Down.
+ 4°	0.48	"	0.79	"
+10°	1.21	Up.	0.41	"
+16°	2.62	"	0.09	Up.
+20°	3.49	"	0.50	"

Fig.69.



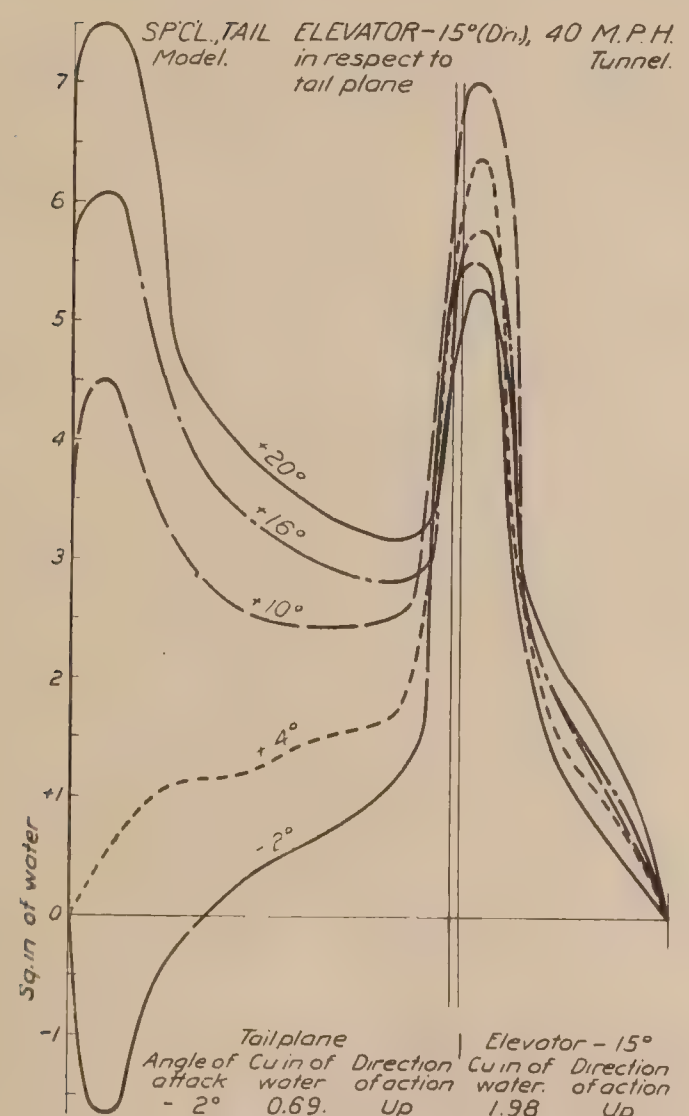
Angle of attack	Tailplane Cu in. of water.	Direction of action.	Elevator + 15° Cu in. of water.	Direction of action.
− 2°	4.29	Down.	2.93	Down.
+ 4°	2.16	"	2.37	"
+ 10°	0.13	"	1.95	"
+ 16°	1.41	Up.	1.13	"
+ 20°	2.38	"	0.67	"

Fig. 70.



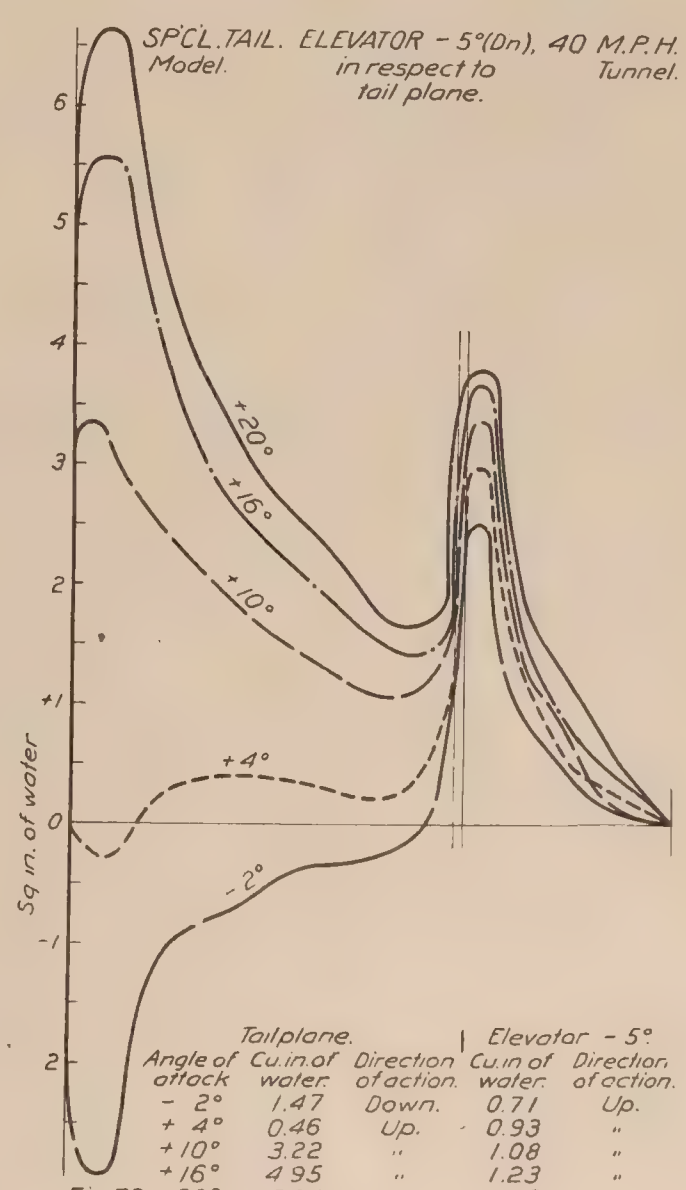
Angle of attack.	Tailplane Cu. in. of water.	Direction of action.	Elevator +25° Cu. in. of water	Direction of action.
− 2°	6.12	Down	4.64	Down
+ 4°	3.87	"	4.20	"
+ 10°	1.55	"	3.61	"
+ 16°	0.23	Up	2.81	"
+ 20°	1.00	"	1.99	"

Fig. 71.



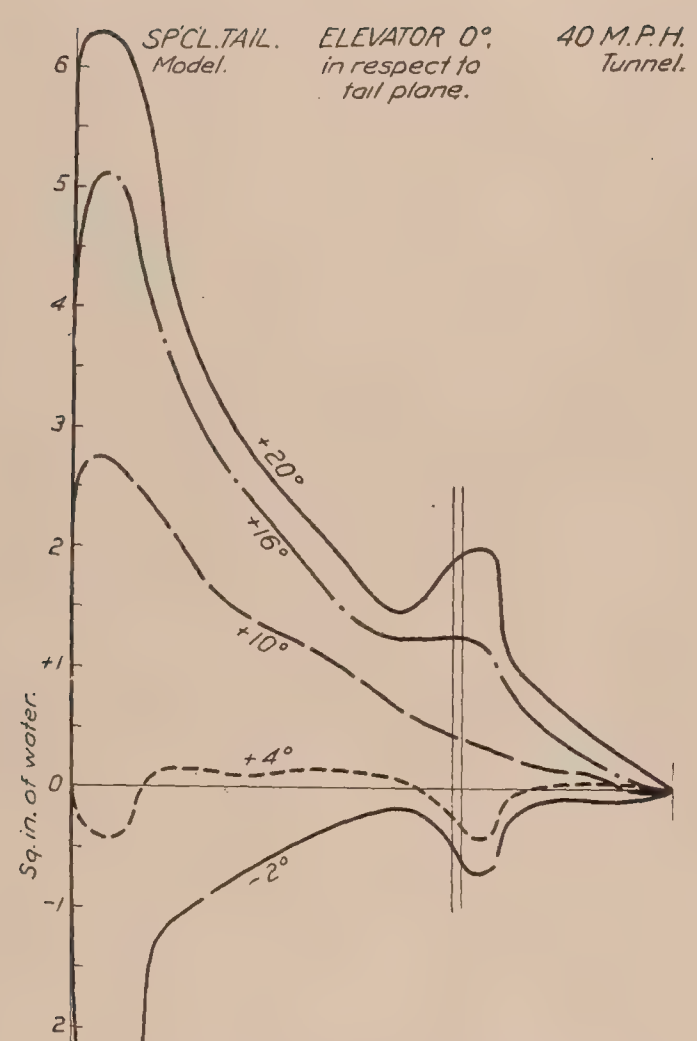
Angle of attack	Tailplane Cu in of water	Direction of action	Elevator − 15° Cu in of water.	Direction of action
− 2°	0.69.	Up	1.98	Up
+ 4°	2.54	"	2.32	"
+ 10°	5.37	"	2.72	"
+ 16°	6.62	"	2.36	"
+ 20°	7.63	"	2.66	"

Fig. 72



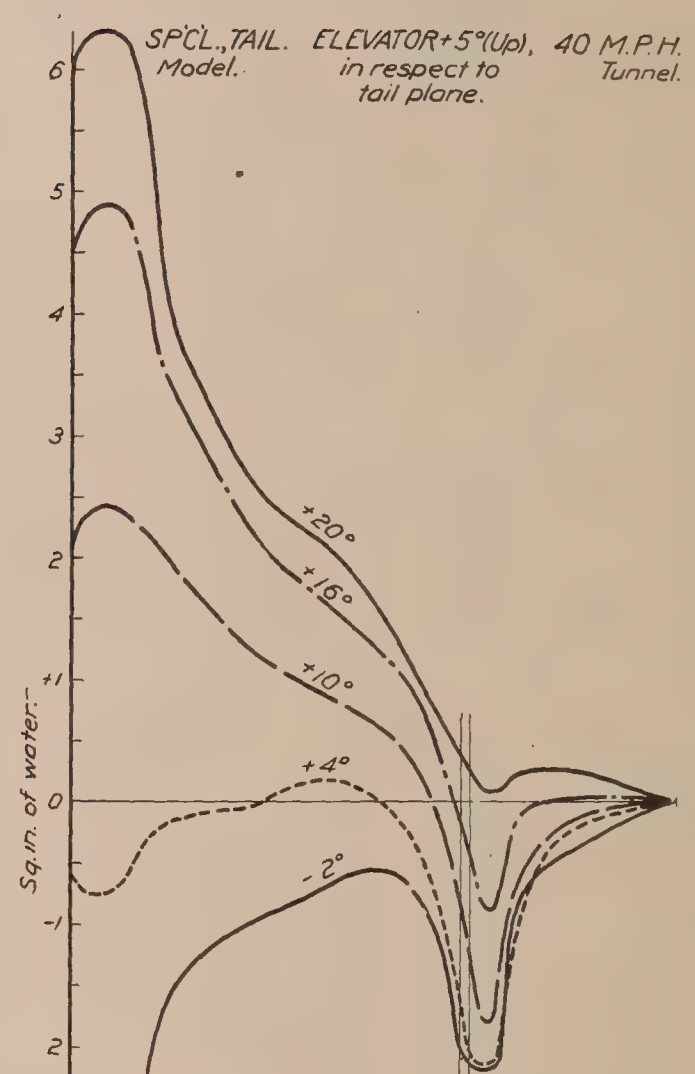
Angle of attack	Tailplane. Cu. in. of water.	Direction of action.	Elevator − 5° Cu. in of water.	Direction of action.
− 2°	1.47	Down.	0.71	Up.
+ 4°	0.46	Up.	0.93	"
+ 10°	3.22	"	1.08	"
+ 16°	4.95	"	1.23	"
+ 20°	5.94	"	1.49	"

Fig. 73



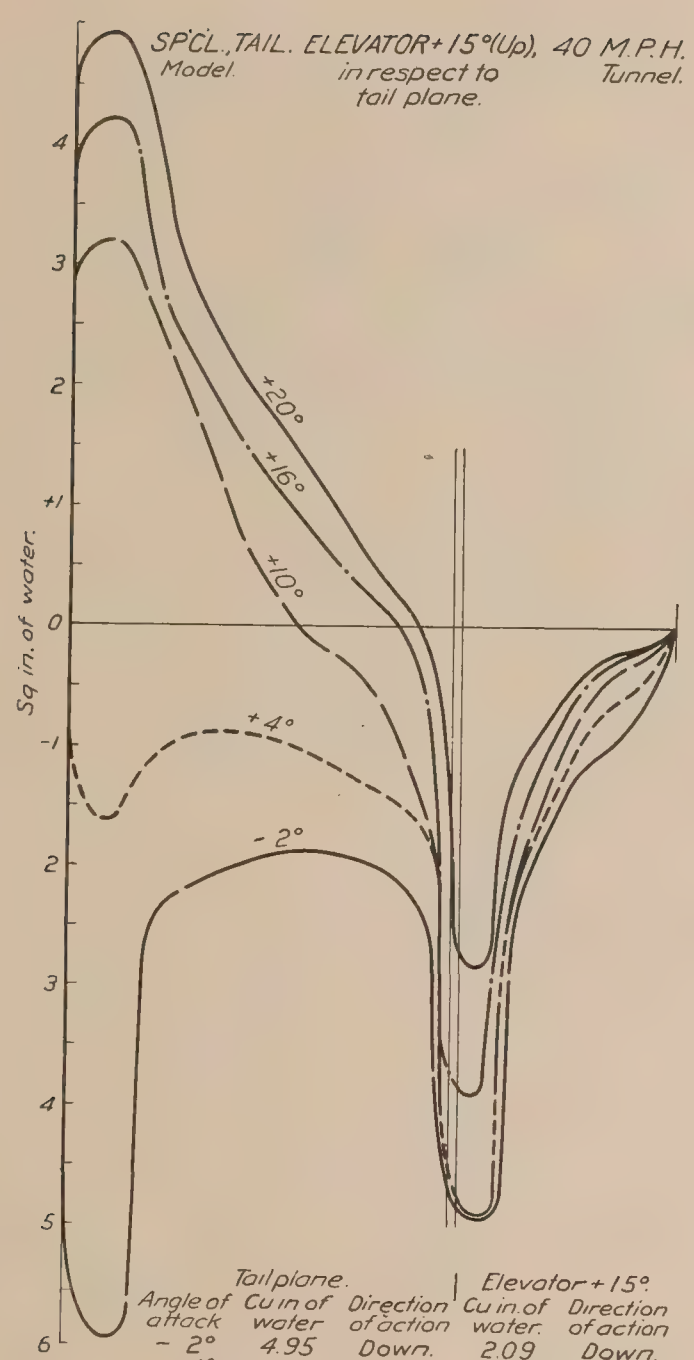
Tailplane			Elevator 0°	
Angle of attack.	Cu. in. of water.	Direction of action	Cu. in. of water.	Direction of action
− 2°	1.82	Down.	2.15	Down
+ 4°	0.04	Up	0.05	"
+ 10°	2.56	"	0.16	Up.
+ 16°	4.48	"	0.46	"
+ 20°	5.53	"	0.75	"

Fig. 74



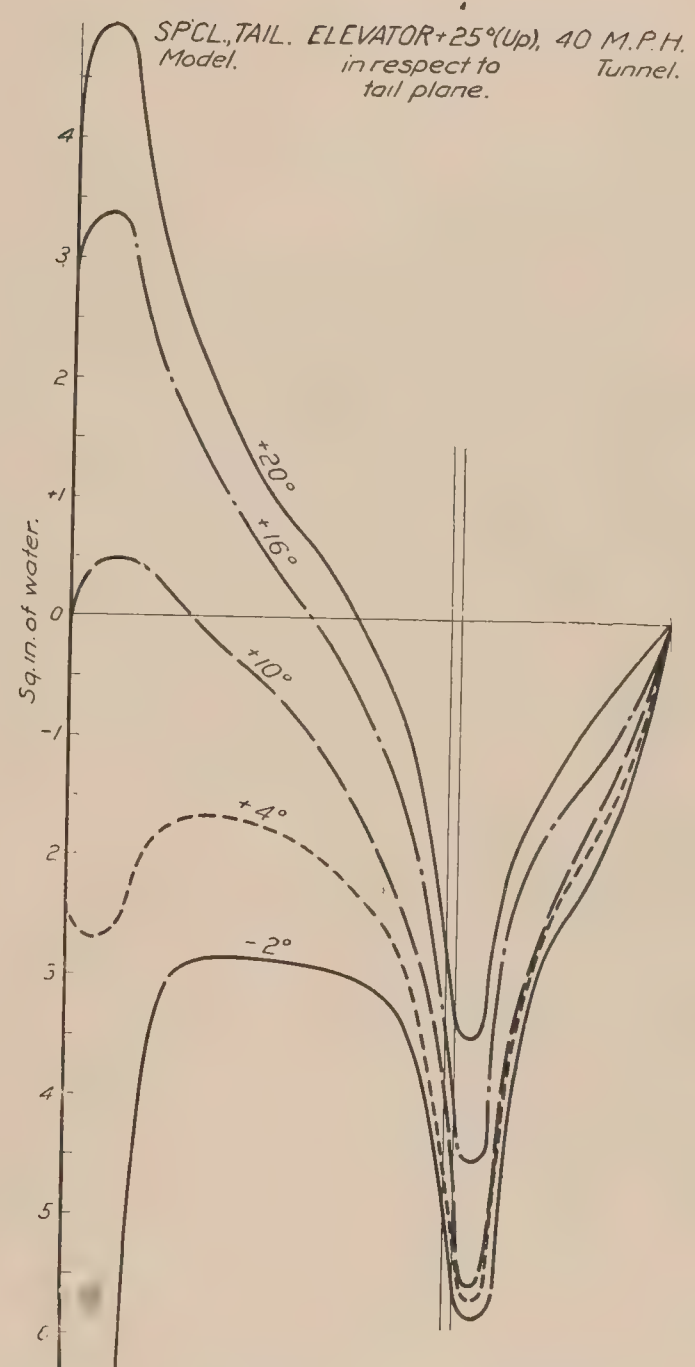
Tailplane.			Elevator + 5°.	
Angle of attack.	Cu. in. of water.	Direction of action.	Cu. in. of water.	Direction of action
− 2°	2.98	Down.	.76	Down.
+ 4°	.50	"	.74	"
+ 10°	2.43	Up.	.49	"
+ 16°	4.80	"	.16	"
+ 20°	6.09	"	.19	Up.

Fig. 75.



Tailplane.			Elevator + 15°.	
Angle of attack	Cu in of water	Direction of action	Cu in. of water.	Direction of action
− 2°	4.95	Down.	2.09	Down.
+ 4°	2.24	"	1.78	"
+ 10°	1.16	Up.	1.67	"
+ 16°	2.52	"	1.27	"
+ 20°	3.71	"	.93	"

Fig. 76.



Tailplane.			Elevator + 25°.	
Angle of attack.	Cu. in of water	Direction of action.	Cu. in. of water.	Direction of action.
− 2°	6.63	Down.	2.86	Down.
+ 4°	3.92	"	2.62	"
+ 10°	1.44	"	2.51	"
+ 16°	1.02	Up.	1.89	"
+ 20°	2.45	"	1.38	"

Fig. 77.

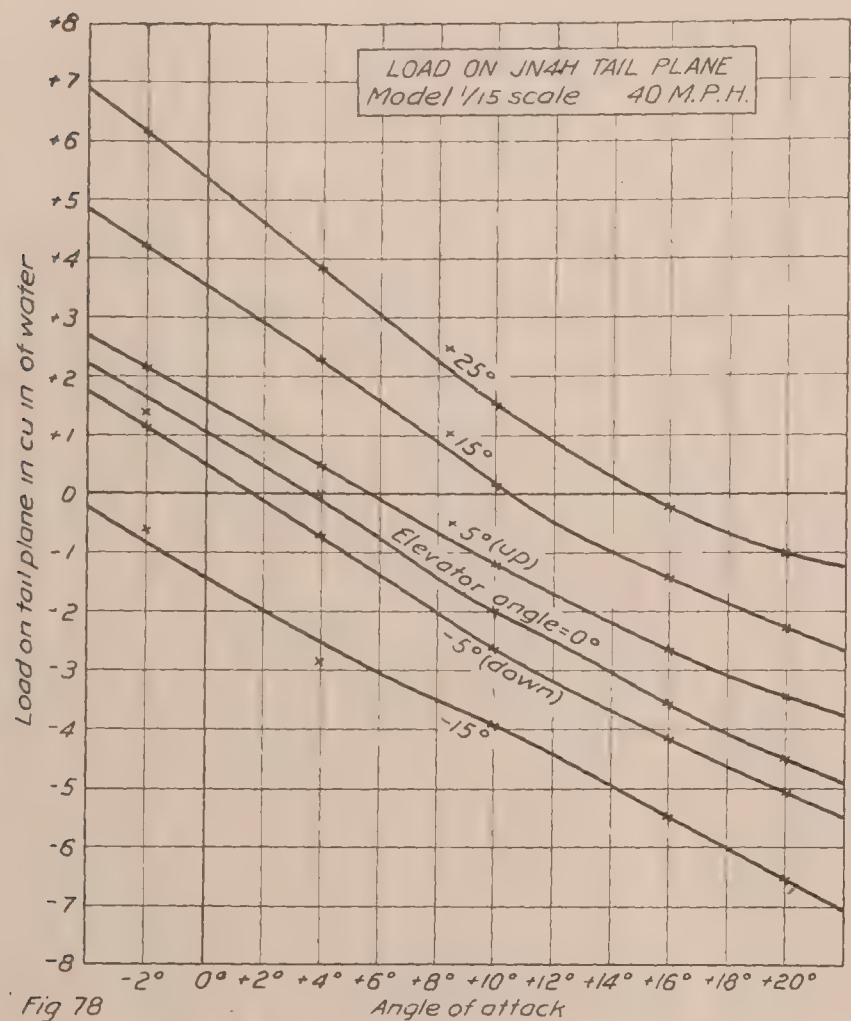


Fig 78

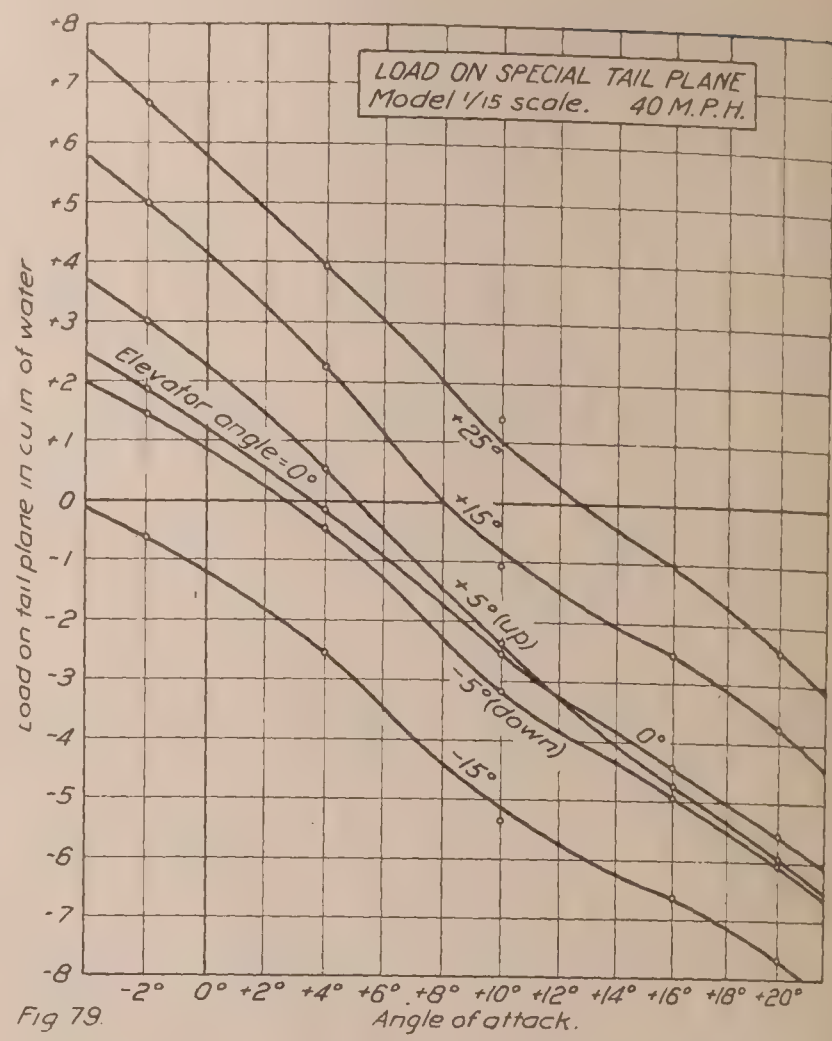


Fig 79.

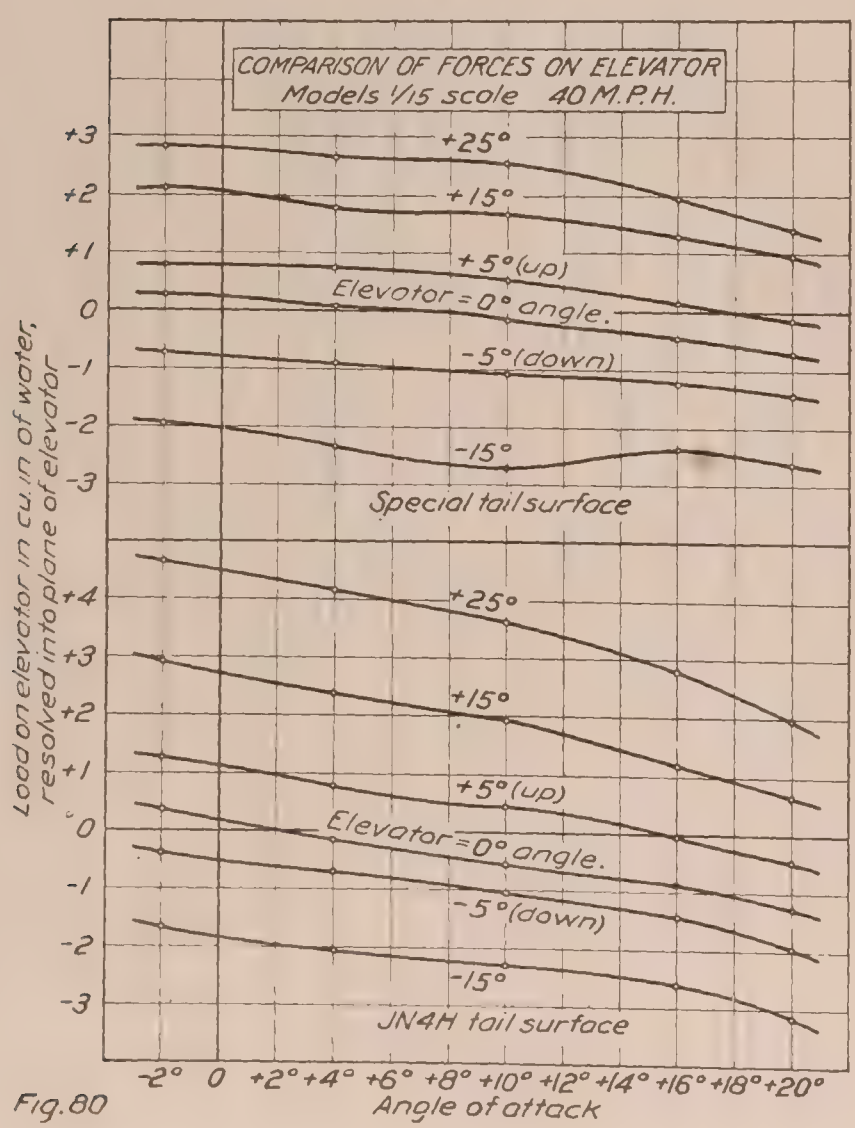


Fig.80

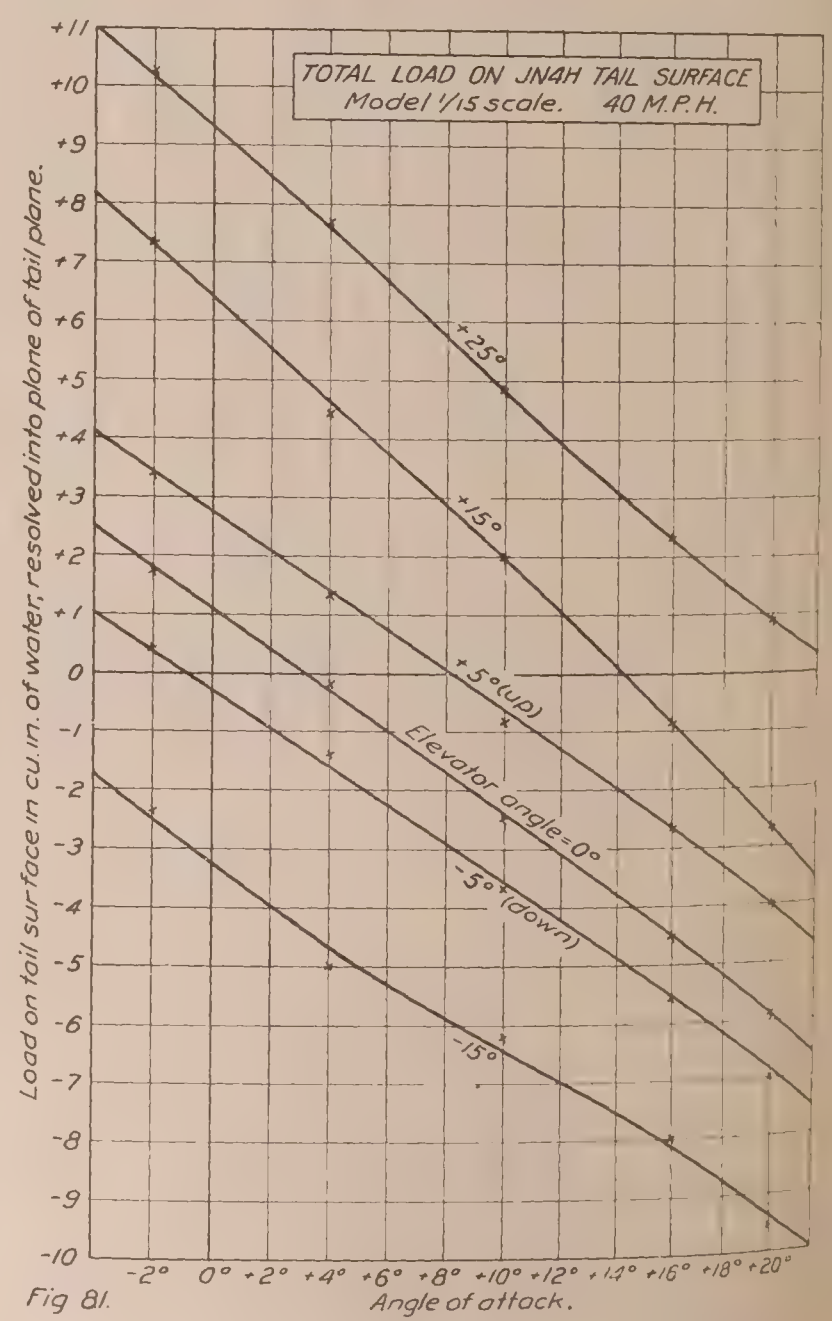


Fig 81.

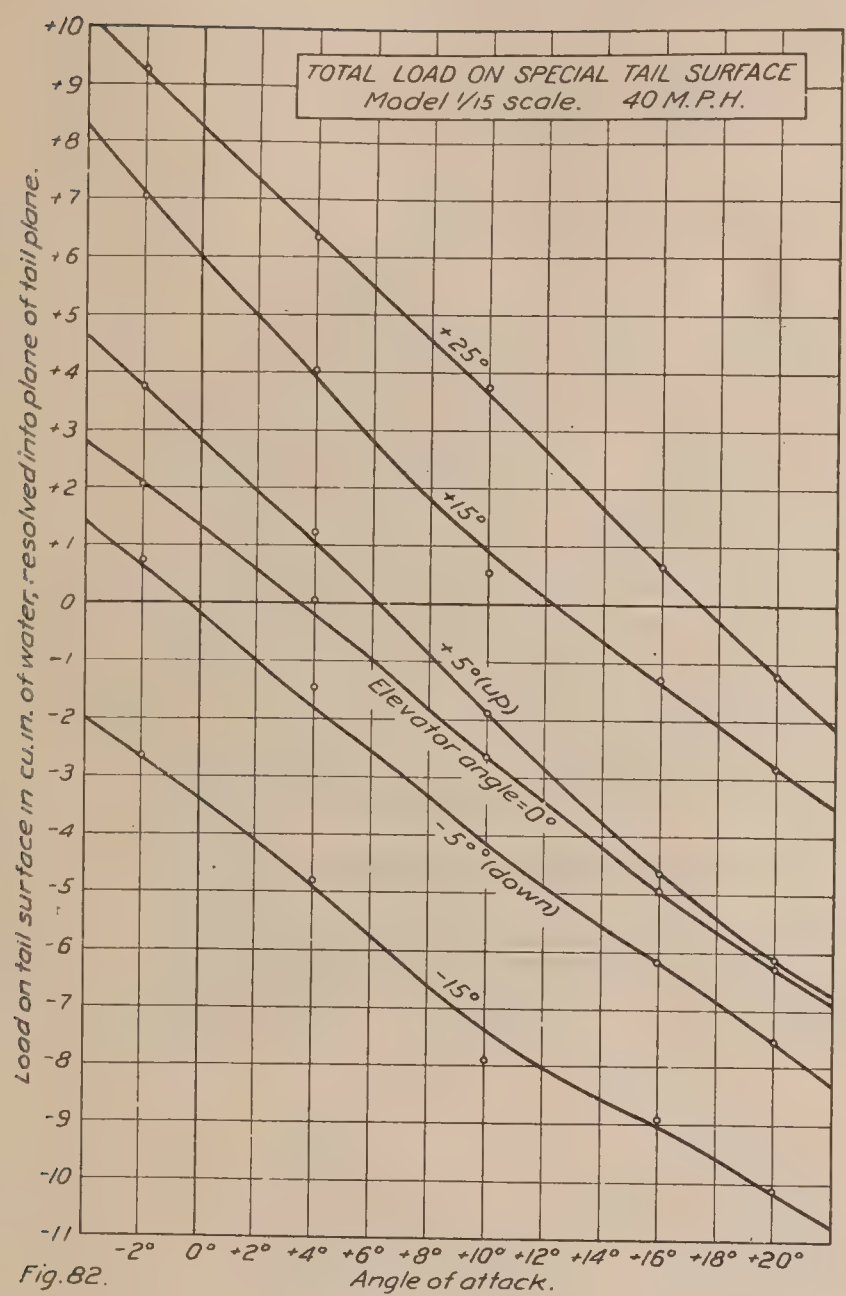


Fig. 82.

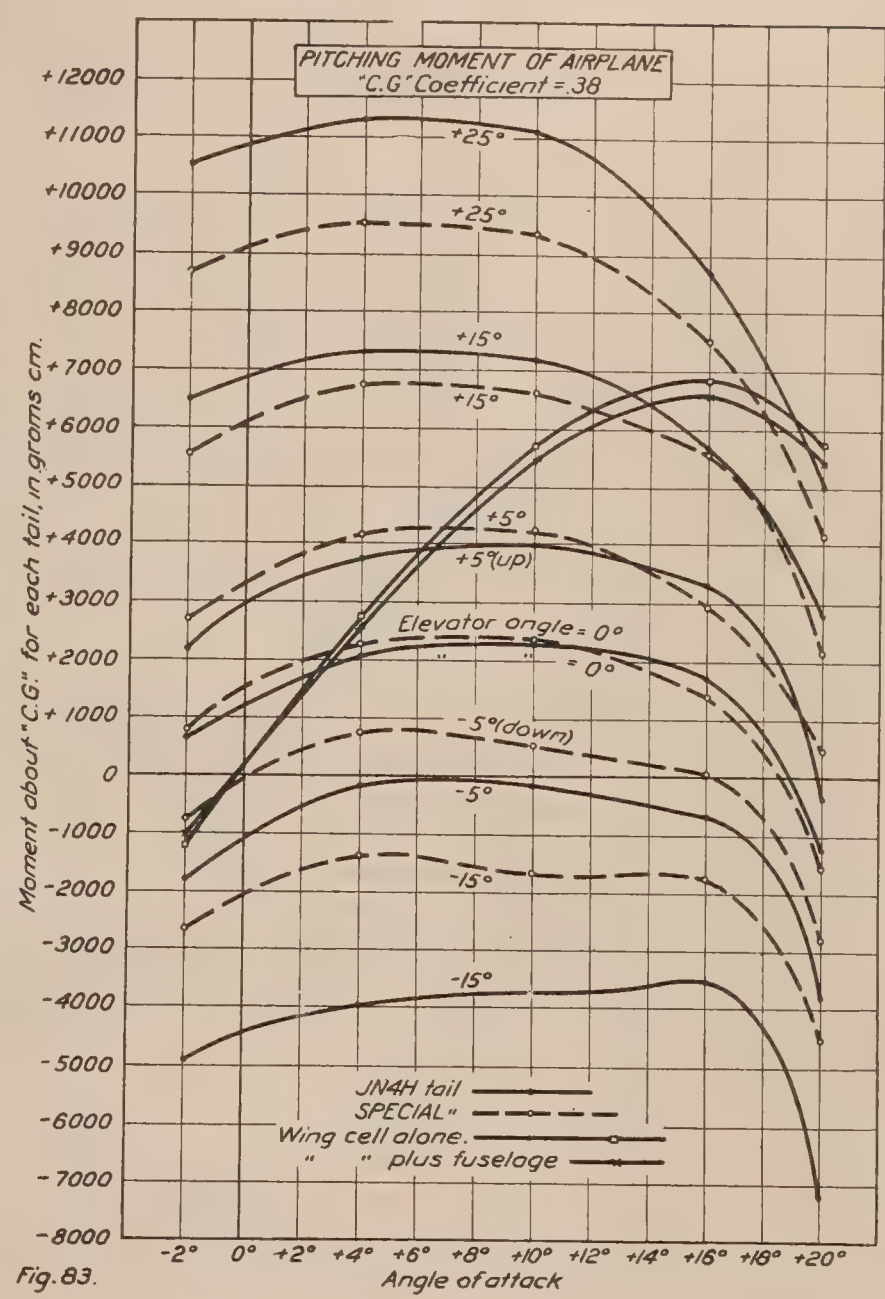


Fig. 83.

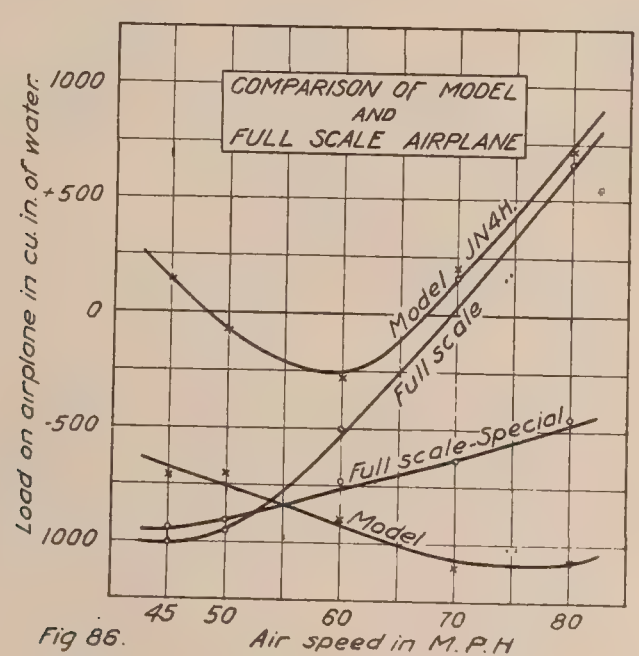


Fig 86.

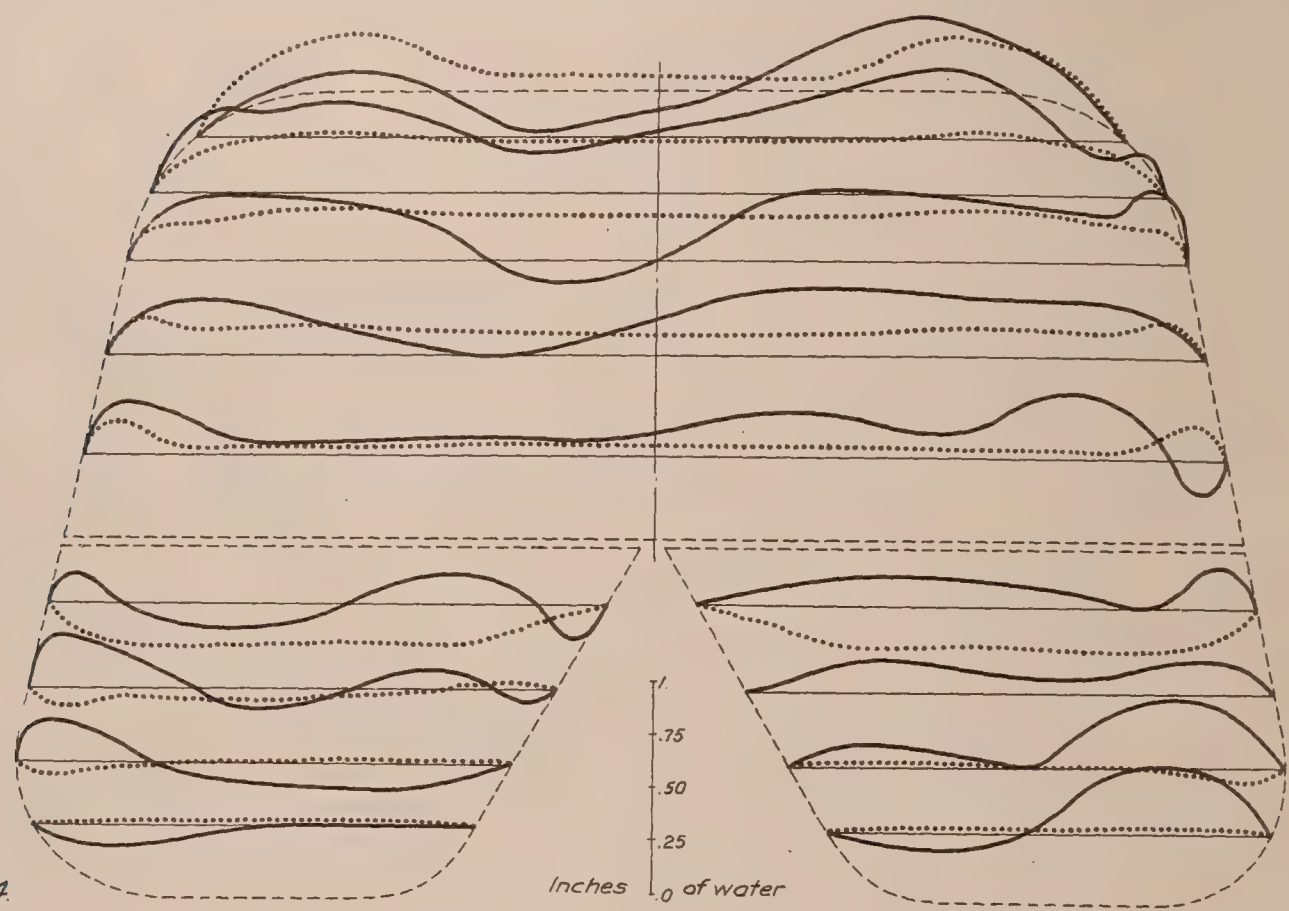

JN4H TAIL SURFACE
CASE I 600 R.P.M.
45 M.P.H. Free flight test
TAIL PLANE at +10° angle of attack. ELEVATOR +5° (up) in respect to tail plane.
40 M.P.H. Wind tunnel test
1.
.75
.50
.25
0
Inches of water

Fig. 84.

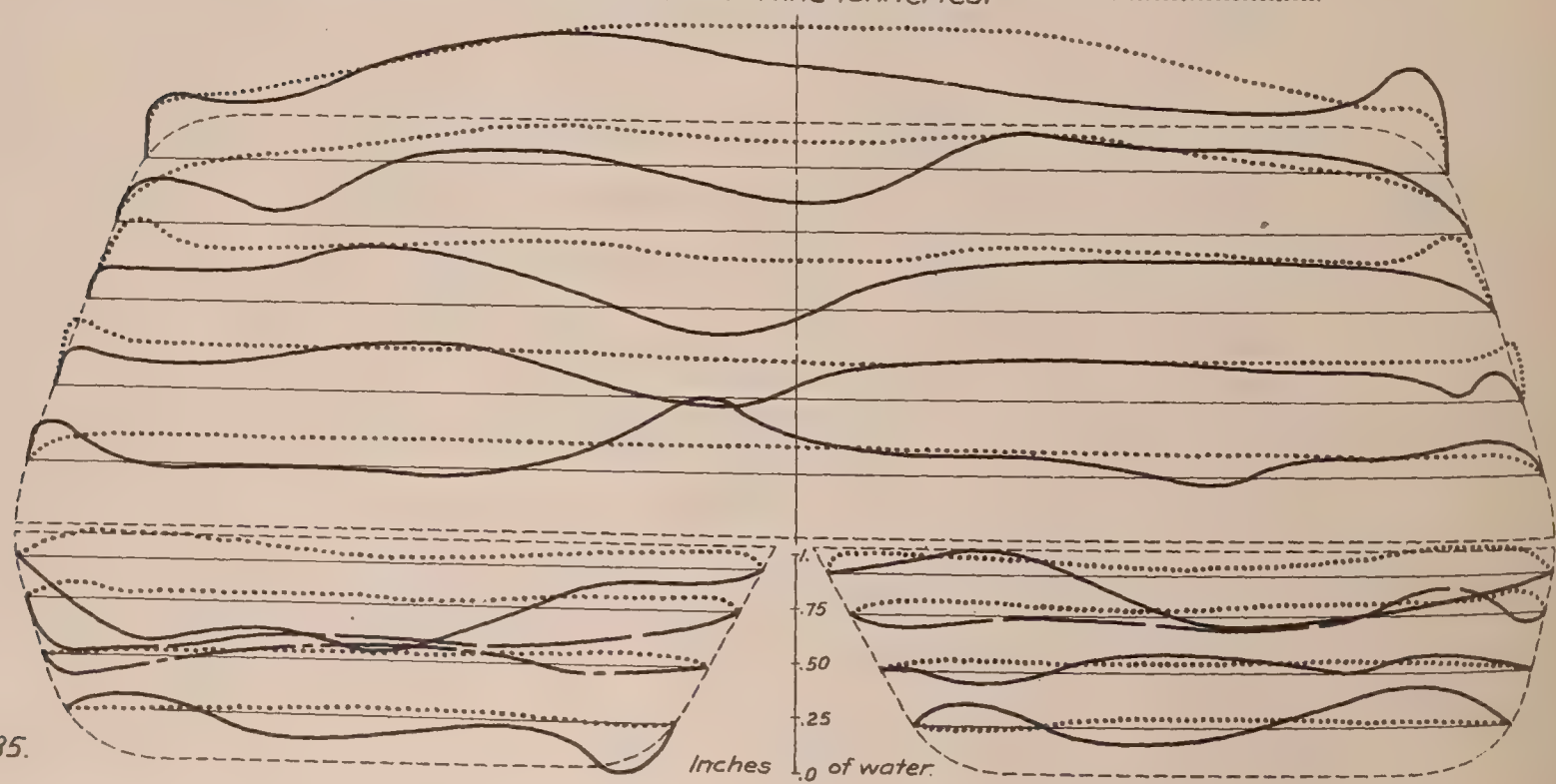

SPECIAL TAIL SURFACE
CASE VI 600 R.P.M.
50 M.P.H. Free flight test.
TAIL PLANE at +10° angle of attack ELEVATOR 0° in respect to tail plane
40 M.P.H. Wind tunnel test
1.
.75
.50
.25
0
Inches of water.

Fig. 85.

REPORT No. 120

PRACTICAL STABILITY AND CONTROLLABILITY OF AIRPLANES

By F. H. NORTON

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

REPORT No. 120.

PRACTICAL STABILITY AND CONTROLLABILITY OF AIRPLANES.

By F. H. Norton.

SUMMARY.

The effect of the characteristics of an airplane on balance, stability, and controllability, based on free flight tests, is discussed particularly in respect to the longitudinal motion. It is shown that the amount of longitudinal stability can be varied by changing the position of the center of gravity or by varying the aspect ratio of the tail plane, and that the stability for any particular air speed can be varied by changing the camber of the tail plane. It is found that complete longitudinal stability may be obtained even when the tail plane is at all times a lifting surface. Empirical values are given for the characteristics of a new airplane for producing any desired amount of stability and control, or to correct the faults of an airplane already constructed.

INTRODUCTION.

There has been a great deal of work done on stability, but the larger part of it has been either with models or with mathematical theory; so there is at present very little technical data to aid the designer in predicting the exact amount of stability an airplane will have when it is first taken into the air, or what changes are necessary to correct a particular fault. The feel of the controls on a new airplane determines probably more than any other one thing the test pilot's attitude toward it, and after all the pilot is the court of last appeal on the fitness of the airplane. There have been great numbers of airplanes built which in nearly every particular were excellent, but because of an undue force on the stick or too great an amount of stability or instability, were condemned to be "washouts." On the other hand, the few airplanes that do possess excellent longitudinal control in many cases are arrived at by copying some previously successful machine or are simply accidents. It is the object of this paper to bring together what practical data there are available from free flight tests on stability for the aid of the designer, and to indicate in what direction future research should be guided in order to solve some of the many existing problems.

There are always three ways to obtain the data for any design: First, from mathematical theory; second, from previous practice; and, third, from experimental research. In this subject of stability the first will not be of much aid to us, for there are so many unknown conditions that the theory must always follow the experimentation. In fact, the huge mass of mathematical theory on stability, a mass laboriously constructed on the insecure foundation of insufficient assumptions, has actually been a hindrance to a sound understanding of this problem. The second way will enable us to design a safe and satisfactory airplane, but if everyone followed this method there would be no general improvement in airplane design. The third way is the more effective, as it will not only tell us how to construct our airplanes so as to give the best results but it will also give us data on which we may base a satisfactory theory. Above everything else the pilot and the designer should get together, as only in this way can a satisfactory airplane be evolved.

In order to show how unnecessary are automatic stabilizing devices on properly designed airplanes, it is interesting to see what has actually been done in the way of inherent stability. A large number of machines show that without any radical departure from standard design it

is possible to make an airplane which will fly even in bumpy weather with absolutely no attention from the pilot. These airplanes will not only fly themselves, but they will acutally fly more steadily when left alone than when controlled by the pilot.

It should be noted that the results and conclusions in this report are based entirely on the data from tractor biplanes, and in some cases from only a single airplane, so that care should be taken not to assume a general applicability of the results. It is strongly recommended that all new airplanes be tested for stability and controllability and that the characteristics of the controls be recorded, in order to extend our empirical data on the subject. Such tests are fully as important as the usual performance tests.

BALANCE.

An airplane is said to be well balanced when there is no force on the stick or rudder with the airplane flying at its normal speed. It should be noted that the balance of the airplane is entirely

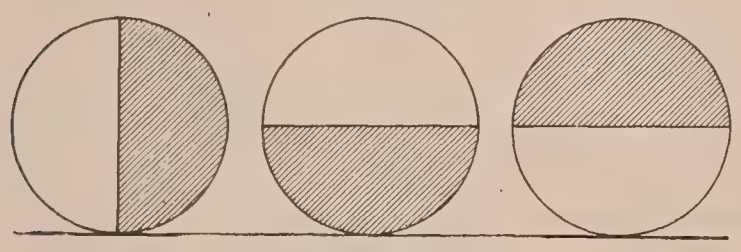


FIG. 1.—Illustration of balance.

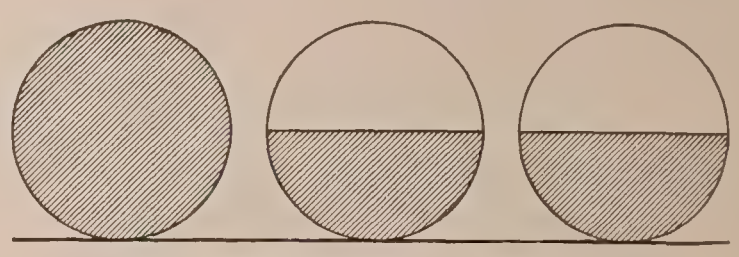


FIG. 2.—Illustration of static balance.

separate from its stability—that is, the airplane may be balanced and yet be unstable—but stability, of course, will have no significance until the airplane is balanced. In order to illustrate this more clearly reference may be made to figure 1, which represents the cross section of a cylinder which is divided along its diameter, one-half being of brass and the other half of aluminum. In the first position the cylinder is evidently unbalanced, for if it were left to itself it would roll over into a new position. In the second position the cylinder is evidently balanced, as no force is required to hold it in its position, but further than this it is also stable, because if it is moved from this position forces will be set up to bring it back. In the third position it is still balanced, but in this case unstable, because if moved even slightly from its equilibrium position it will turn completely over.

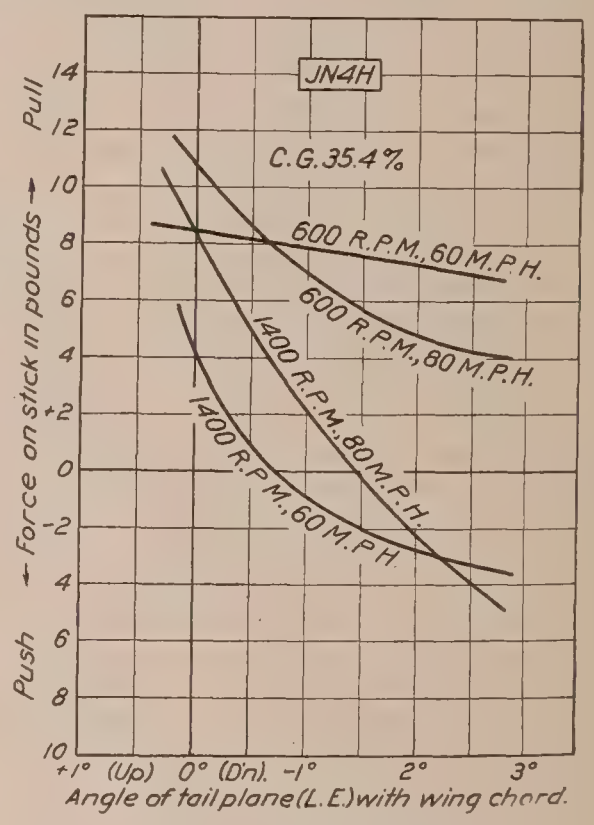


FIG. 3.—Effect of tail-plane angle on balance.

LONGITUDINAL BALANCE.

The following factors are the ones which mainly determine the longitudinal balance of an airplane:

1. The longitudinal position of the center of gravity.
2. The angular setting of the tail plane.
3. The weight of the elevators—that is, the moment produced about the elevator hinge by their own static weight.
4. The vertical position of the thrust line in respect to the center of gravity.

The only available data which shows the change in stick force with a given change in center of gravity position is that obtained on a JN4H; these results, however, should apply approximately to any tractor biplane. It was found on the JN4H that a movement of the center of gravity forward along 2 per cent of the chord required an additional pull on the stick of 1 pound to hold the machine at the same angle of flight.

The effect of the tail plane setting on the control force has been obtained both on the JN4H and on the DH4, the results being shown in figures 3 and 4. Although the change in stick force was not the same for all engine speeds, it may be safely assumed that when the tail plane is changed

1° the stick force changes by 4 pounds on the JN4H and by 6 pounds on the DH4 when the tail plane is turned through an angle of 1°. An adjustable tail plane is a most excellent method of balancing an airplane, and such a device should be applied to all but the smallest airplanes.

It should be noted that either moving the center of gravity forward or decreasing the angle of the tail plane increases the longitudinal stability of the airplane, so that by proper adjustment of these two factors it is possible to change the balance to any degree without appreciably affecting the stability characteristics.

The effect of the elevator weight on balance is quite obvious—that is, if the weight of the elevator is increased it will add a proportional pull on the stick. The effective weight of the elevator can be changed by a balance weight on some part of the control system—that is, if the stick gave a pull of 2 pounds at a speed where it is desired to have it balance, an effective weight of 2 pounds on the control system would give the required balance. Balance can also be obtained by springs, but it should be noted that this is not a true balance except for uniform flight, because in turns where the motion is accelerated the spring force would remain constant while the elevator weight would be proportional to the acceleration.

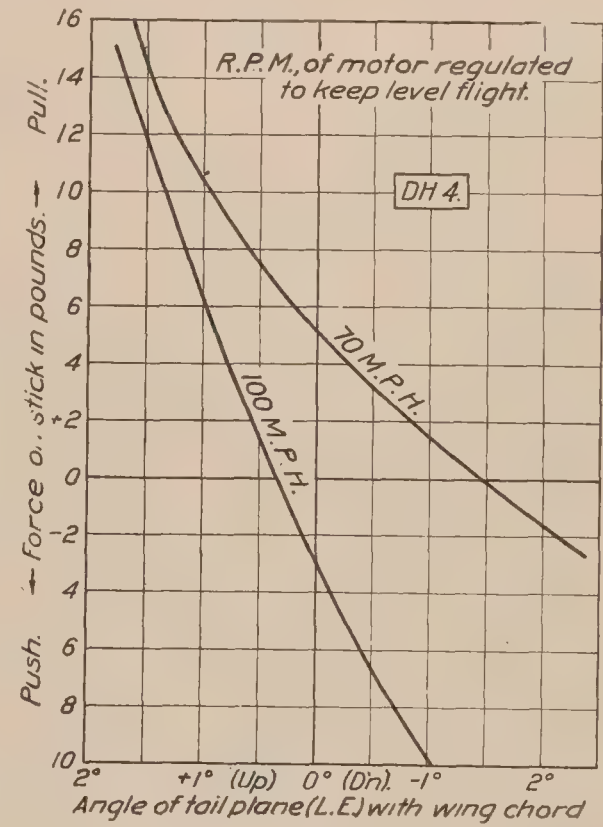


FIG. 4.—Effect of tail-plane angle on balance.

LATERAL BALANCE.

There have apparently been no tests made to determine the angle through which the tips of the wings must be changed in order to balance a given lateral force on the stick. Some tests have been made, however, in Germany[1] on a conventional biplane of about 400 square feet in area to determine at what angle the ailerons must be placed to balance a given angular warp in the wing itself. The results of these tests are shown in figure 5, and it is evident that a warping of the wing at the outer struts of 1° corresponds to 2° angle on the aileron.

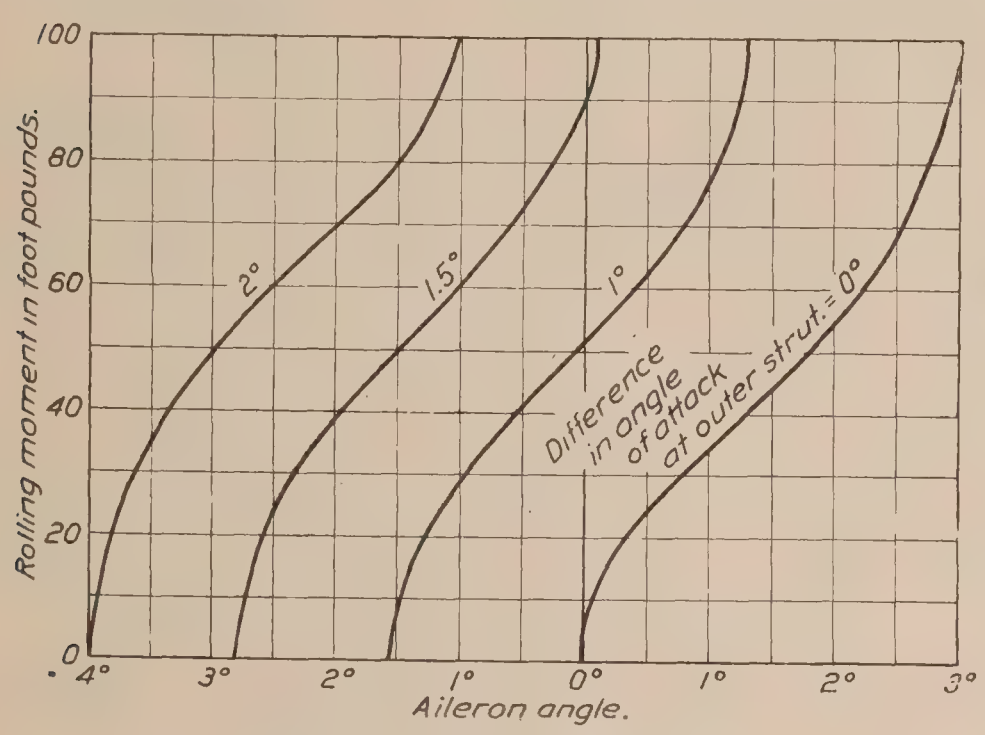


FIG. 5.—Effect of wing warp on lateral balance.

DIRECTIONAL BALANCE.

Directional balance, which depends upon the forces acting upon the rudder, is unfortunately quite different with the engine on and off. It is very simple to effect a balance of the rudder either by moving the leading edge of the fin or by applying an adjustable tension on one end of the rudder bar with an elastic cord. The airplane should always be balanced directionally after the wings have been properly aligned for balancing the airplane laterally, otherwise a small change in the angle of attack of the wing will considerably change the forces on the rudder. To design a rudder which is balanced at all engine speeds is a rather difficult problem and is perhaps best solved in some of the British machines, such as the SE5 where the rudder extends for a considerable distance below the fuselage so that it will be evenly acted upon by the rotation of the slip stream.

[1] N. A. C. A. Technical Note No. 38

STATIC STABILITY.

In order to define clearly the meaning of static stability, reference will be made to figure 2, which shows a cross section of the same cylinders as in figure 1. In the first position the cylinder is considered to be made of a homogeneous material, so that it will rest in any position in which it is placed; and in this case the stability is said to be neutral. In the second position the cylinder is evidently stable, for if it is displaced from its equilibrium position forces will be set up tending to return it to this position, but in the third position the cylinder is unstable, for if it is moved even slightly from its equilibrium position, forces will be set up tending to make it depart even further from its equilibrium position.

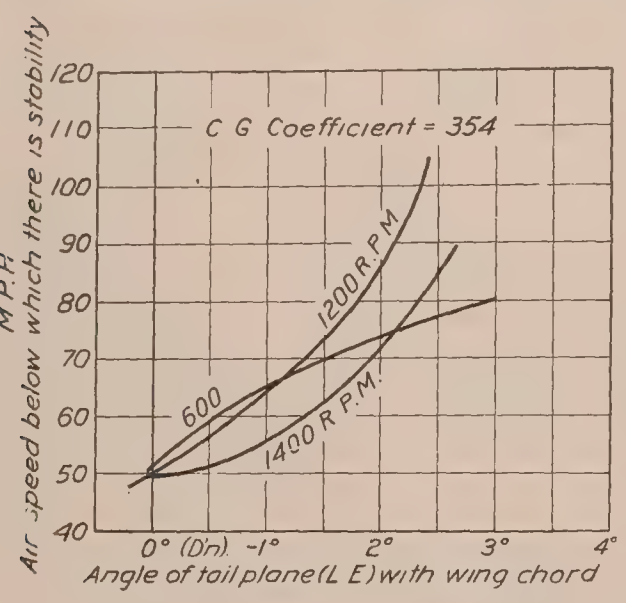


FIG. 6.—Stability of JN4H with free controls.

It is usually desirable to have only a small variation in control force for various throttle positions; but if there is a variation it is desirable to make the airplane more nose heavy with the throttle closed. The lower the thrust line is, compared with the center of gravity, the more will this tendency be, but in the usual case when the majority of the tail is in the slipstream the pitching effect will not be zero when the thrust line passes through the center of gravity, because of the large influence of the slipstream on the tail surface. By raising the thrust line high enough a position would of course be found where torque about the center of gravity would neutralize this downward force on the tail plane so that the tail plane would have the same balance for any throttle setting, but this is usually impossible to accomplish in the usual tractor airplane. As shown later, increasing the aspect ratio of the tail plane decreases the amount of area in the slipstream, thus giving more uniform balance at different engine speeds. By the use of a geared engine the thrust line is raised without changing the vertical position of the center of gravity, so that this type is of considerable advantage in giving a constant balance with changes in engine speeds.

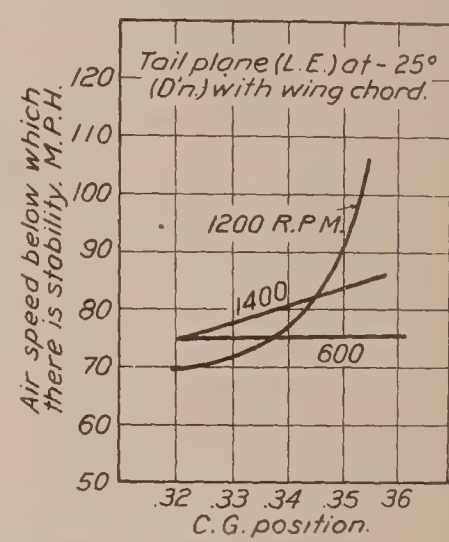


FIG. 7.—Stability of JN4H with free controls.

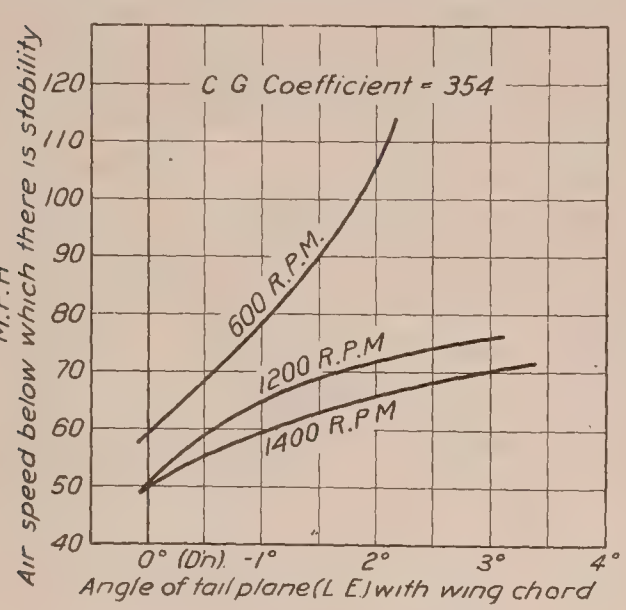


FIG. 8.—Stability of JN4H with locked controls.

In an airplane the question is slightly more complicated, as we may have stability in one case with the control surfaces locked in position and in the other with the control surfaces left free to rotate as they will. An airplane is said to be inherently stable with free controls when it is able to fly at some speed steadily without the attention of the pilot and is stable with locked controls when it will fly in the same way with the controls fastened in one position. When the control positions and forces are plotted, as in this report, stability is indicated by a negative slope of the curve. In the majority of cases, although not always, an airplane will be more stable with locked controls than with free controls, so that in making wind tunnel tests the control surfaces should be either entirely removed or else freely hinged. Due to the inevitable friction in the controls it is practically impossible to have free controls, so that what we call free controls is a combination of free and locked controls.

It is desirable in every airplane to have a small degree of static stability, sufficient to allow the machine to be flown even in very bumpy weather without using the controls, and yet not so stable that the airplane is not at all times subservient to the pilot's will.

LONGITUDINAL STABILITY.

The longitudinal stability is generally affected by the following factors:

1. Horizontal position of the center of gravity.
2. Angle of the tail plane.
3. The aspect ratio of the tail plane.
4. The section of the tail plane.
5. The area of the tail plane, and the length of the body.
6. The chord and center of pressure travel of the wing.

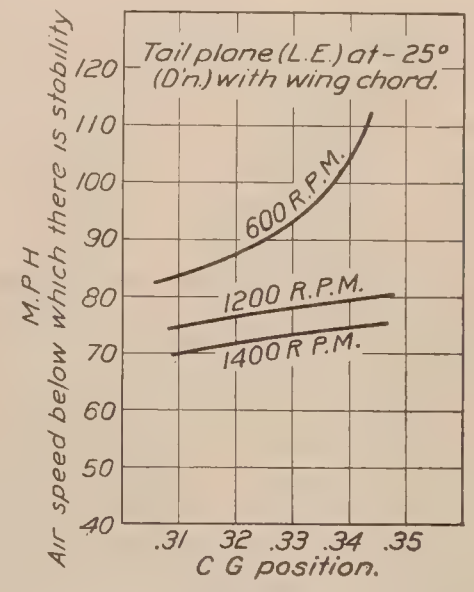


FIG. 9.—Stability of JN4H with locked controls.

It is not within the scope of this report to give a detailed analysis of the effect on longitudinal stability of changing the horizontal position of the center of gravity, but in figure 6 are shown elevator force curves which have been obtained in free flight on a JN4H airplane. The stability is somewhat influenced of course by the slip stream, but in general it may be said that the further forward the center of gravity the more stable will be the machine, not only in this particular case but on any machine. The stability with locked controls with a given change in center of gravity is shown in figures 8 and 9, and the same conclusions hold true for this case.

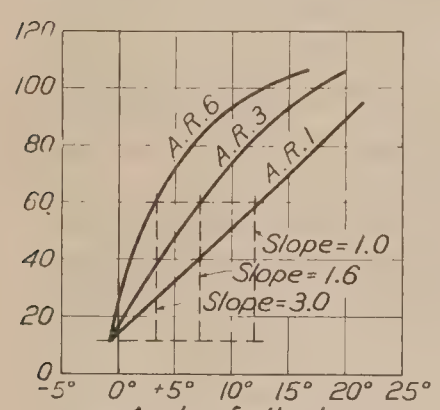


FIG. 10.—Effect of tail surface aspect ratio on lift.

The angle of the tail plane has some effect on stability, that is, a more positive angle decreases the stability. At the same time a more positive angle tends to reduce the difference in forces between various engine speeds. By using a tail of high aspect ratio it is possible to obtain stability even when the center of gravity is 42 per cent back on the mean chord, in which case the tail will be lifting at all times. It thus seems evident that for the sake of aerodynamic efficiency and equal balance at all engine speeds, it is desirable to place as much positive load on the tail as consistent with a moderate degree of stability. In figure 7 are shown curves obained in free flight, showing the effect on stability of a variation of the tail plane angle, but the data are not complete enough to give more than approximate results.

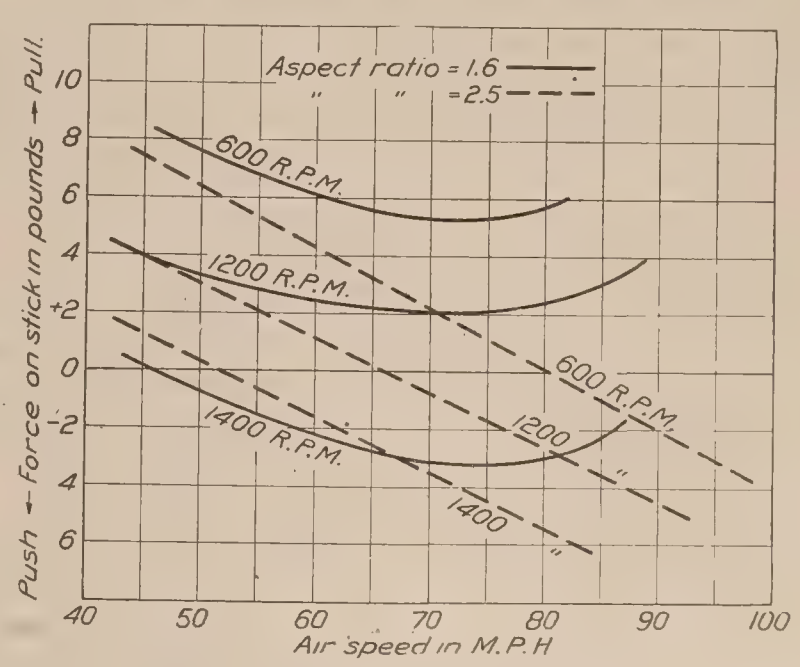


FIG. 11.—Effect of tail surface aspect ratio on stability with free controls.

The aspect ratio of the tail surface has more effect upon the stability in the usual tractor airplane than any other cause, and this is due not only to the increased slope of the lift curve with increased aspect ratio but also to the fact that a larger amount of area is outside of the slip stream, which of course increases the efficiency. The variation in slope of the lift curve with the change in aspect ratio is shown in figure 10, the results being taken from some of Eiffel's tests. The stabilizing action of the tail is evidently dependent upon the rate of change of lift with a unit change of inclination, so that the slope of the lift curve is a direct criterion of the stabilizing action. In other words, in order to have equal stabilizing

properties in two tails, one with an aspect ratio of 1, the other with an aspect ratio of 6, the latter would need to have only one-third of the area of the former. It should be noted, however, that this only applies to stability and that the small area would in all probability not give a sufficient amount of control.

In order to show the effect of changing the aspect ratio of the tail on the actual flight properties of an airplane, a JN4H was flown with a tail having an aspect ratio of 1.6 and another tail of the same area but with an aspect ratio of 2.5. In figure 11 are plotted the force curves obtained from these tests. The stability is shown by the slope of these curves, the negative slope indicating stability, and it will be noted how greatly the stability is increased by the tail of high aspect ratio even though the center of gravity position has not been changed between the two cases. Another interesting fact observed is that the force curves are closer together with the higher aspect ratio tail for various throttle settings, and this is undoubtedly due to the fact that a larger portion of the area of the high aspect ratio tail is outside of the slip stream. These are the only results available where the aspect ratio has been changed on the same airplane, but the results obtained for the stability on other airplanes seem to show that those airplanes having a tail of high aspect ratio such as the DH4 and the VE7 are stable, while the numerous machines with low aspect ratio tails are in every case unstable.

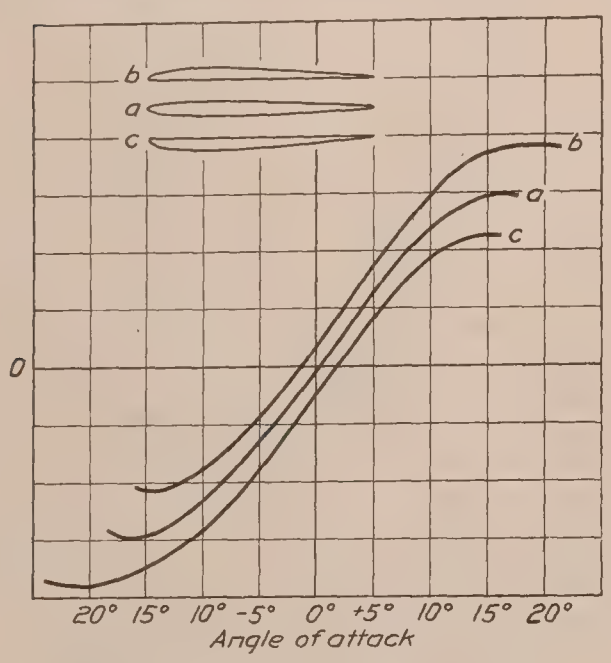


FIG. 12.—Effect of tail surface section on lift.

The characteristics of the airplane previously discussed affected only the degree of stability. The section of the tail plane,[2] however, has apparently the valuable property of changing the speed at which stability occurs. That is, if the camber of the tail plane is on the upper surface the stability will be greatest at low speeds. On the other hand, if the camber is highest on the lower surface the stability will be greatest at high speeds. The reason for this is made clear by referring to figure 12, where there are plotted the lift curves for three sections, the first a flat bottom section with the camber on the upper side, the second a symmetrical section, and the third a section flat on top and with the camber on the lower surface. It will be noted that the lift curve falls off as the burble point is approached and that the burble point is always earlier when the air strikes the cambered surface. As the stability is proportional to the slope of the lift curve it will be evident that varying the section will produce stability at various angles of the tail, which is equivalent to saying at various air speeds, so that by varying the section it is possible to produce stability at any given air speed.

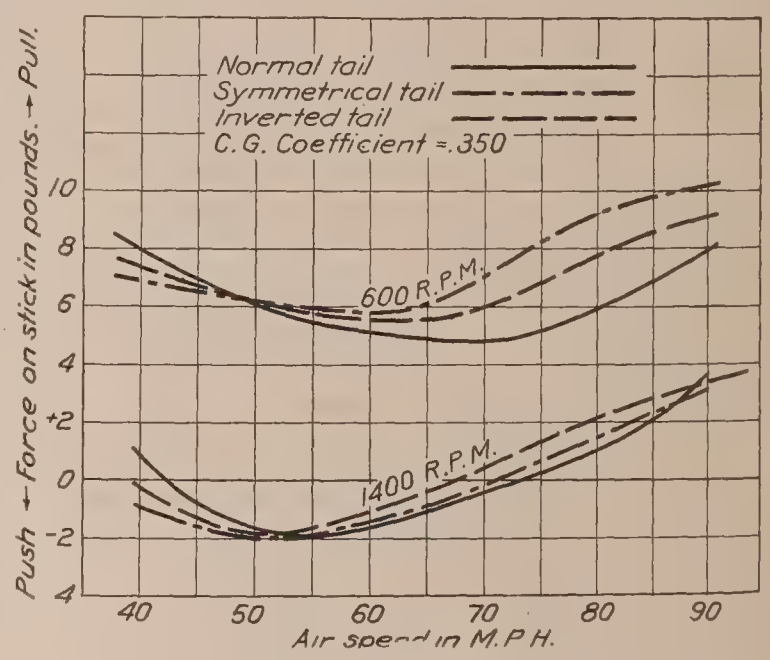


FIG. 13.—Effect of tail surface section on stability.

An actual test was carried out by placing on a JN4H three tails of the symmetrical, flat bottom and inverted section, and control forces and position were taken in the three cases. It was a little difficult to separate the effect of the tail section from the effect of tail setting and center of gravity position, which were necessarily changed somewhat between these different cases, but the tail setting was so placed in each case that the control forces at 60 miles an hour

[2] N. A. C. A. Report No. 96.

were the same for all of the sections. The results from this test, so far as the stability with free controls goes, are shown in figure 13.

There has been very little scientific investigation carried out in full flight on the effect of changing the length of the body or changing the area of the horizontal tail surfaces. This is mainly due to the fact that such alterations in the full-sized airplane are quite expensive. It is necessary, therefore, to fall back on the sizes that have been used in successful airplanes. Although this will undoubtedly give us figures enabling us to design a machine which will be satisfactory, it is simply following someone else and does not allow us to make any marked improvement in the design.

In order to obtain the characteristics of airplanes now in use there has been chosen from each type of airplane 10 successful airplanes and their average areas and lengths have been assembled in the following table. The length is given from the center of gravity to the elevator hinge in terms of the wing chord, and the areas of the control surfaces are all given in percentage of the total wing area, including the ailerons.

Type.	Tailplane area.	Elevator area.	Length.
	Per cent.	*Per cent.*	*Per cent.*
Training biplanes	6.8	5.9	3.2
2-seat service biplanes	6.4	5.8	3.2
1-seat service biplanes	7.8	6.2	2.9
Bombers, biplanes	7.0	3.6	3.7
Flying boats, biplanes	8.4	6.3	3.1
Average biplanes	7.5	5.5	3.2
Monoplanes	.....	.....	2.5
Triplanes	.....	.....	4.3
Airplanes having stability [1]	6.9	5.6	3.1

[1] Average of 5 machines considered to possess good stability: VE7, DH4, SE5, Avro 504K, Bristol fighter.

It will be observed from these figures that the area of the elevator is smaller on the large airplanes than it is on the others, and at the same time the fuselage is longer on these airplanes. This is probably due to the desire to reduce the forces on the controls. The average length of fuselage for the ordinary type of airplane is 3 chord lengths, while for the monoplane it is 2½ chord lengths, and for the triplane about 4.3 chord lengths. The average area of the tailplane is 7½ per cent and the average of the elevator 5½ per cent of the total wing area.

As it is quite impossible to give any definite values to stability without special tests, it was thought that the nearest approach to showing a relation between the longitudinal stability and the control areas would be to average up a group of airplanes which were known to have longitudinal stability. This has been done in the table and it will be observed that the areas of the tail surface and the length of the fuselage are very close to the average for the other airplanes, which shows quite definitely that it is not necessary to increase abnormally the area of the tail surfaces nor increase the length of the body in order to obtain satisfactory longitudinal stability. In fact, I believe that it is quite possible to obtain satisfactory longitudinal stability even when the areas of the horizontal tail surfaces are considerably reduced below the average and the length of the body markedly shortened. The advantages of reducing the length of the body are very great, as it allows a lighter fuselage and a much faster airplane in longitudinal maneuvers.

It has long been believed that the use of monoplanes with large chords would necessitate a longer body and a larger tail, but from actual practice it seems to have been proved that the stability and control are as great with the monoplane as they would be with a corresponding biplane of the same area with the same body and tail surfaces. For example, the Morane monoplane with a tail about 2½ chord lengths back of the center of gravity has most excellent longitudinal stability and can easily be flown without touching the controls. Again, the Stout monoplane seems to have sufficient longitudinal control, although I have no information on its stability, when the elevator hinge is only 1 chord length back of the center of gravity. It is also believed that a construction such as the JL6 (Junkers monoplane), where the wing is considerably below the level of the tail surface, increases the stabilizing efficiency.

The center of pressure travel on the wings may be reduced in a monoplane by varying the wing section, either by using a more nearly symmetrical surface, that is, by using a convex lower surface, or by turning up the trailing edge of the wing. The former method is much the more satisfactory, especially in thick wings, as it does not injure the aerodynamic properties nearly as much as the latter method. In a biplane, however, the center of pressure travel may be improved much more easily by using a large positive stagger or by using a small positive stagger and decalage. The internally braced wing used as a biplane with a large stagger obviously has a great advantage over the usual construction, where the obliquity of the struts materially reduces the structural efficiency.

LATERAL STABILITY.

There are much less available data on lateral stability than there are on longitudinal stability, mainly because the lateral stability is of considerably less importance in respect to the comfort of the pilot. Some work, however, has been done on the JN4H to determine the control forces and positions in circling flight [3] and in side slipping, but few alterations were made on the airplanes in order to determine their effect on the stability. It is evident, however, that any airplane will be moderately stable in straight level flight by the use of small dihedral on the wings, that is, a dihedral of between 2° and 6°. Lateral stability seems to be a much simpler problem than longitudinal stability. There are some airplanes, like the Morane monoplane, for instance, which are stable even in circles, that is, if the airplane is banked into a turn and the controls released it will continue to fly steadily in a circle until brought back to a level course. Just what properties make this stability possible are not known, but there are certainly no radical changes necessary to bring it about.

It may be stated here for those who would predict the lateral stability from mathematical analysis or model tests, that because several factors have been neglected which greatly influence the results, namely, the effect of the slipstream and the varying positions of the controls in full flight (where they have been assumed fixed in the model), the computed stability will have little resemblance to the actual performance of the airplane. The lateral derivatives Y_v, L_v, and N_v have been determined in full flight at Langley Field, and because of the erroneous assumptions mentioned above do not agree with the values obtained from the model.

DIRECTIONAL STABILITY.

Directional stability is exceedingly important, and fortunately is quite easy to obtain. Every airplane is probably directionally stable with locked rudder, but there are a great many airplanes which are unstable with a free rudder, which emphasizes the necessity for testing small models with the hinged part of the control surfaces free or entirely removed. The fore-and-aft position of the center of gravity has a very great effect on the directional stability. It was found in testing the JN4H that only a small change in the center of gravity position would change the airplane from unstable to stable with free rudder, which is due to the increased effectiveness of the fin when the moment arm to the center of gravity is increased, as the center of pressure of the fuselage and fin is very near the center of gravity.[4] Airplanes having large lateral area of the fuselage near the nose must have correspondingly greater fins to obtain directional stability. The Junkers JL6 airplane is directionally unstable because of this type of body. A high aspect ratio is aerodynamically desirable on the rudder and fin, but is structurally awkward.

The principal reason for having directional stability when the controls are free is that an airplane which is unstable in this particular when left to itself will immediately whip into a very rapid spin out of which it is quite impossible to get it without the use of the rudder; so should the rudder wires be broken on an airplane of this type it would be utterly impossible to keep the airplane from crashing. It should be noted that balancing the rudder is approximately equivalent to adding on to the stationary fin twice as much area as the balanced portion,

[3] N. A. C. A. Report No. 112, "Control in Circling Flight."

[4] Applied Aerodynamics, G. P. Thomson, p. 229.

so that by using a balanced rudder the effective fin area is greatly increased, thus increasing the directional stability with a free rudder.

Directional stability is almost as important when on the ground as in the air, for an unstable airplane in landing, losing most of its directional control due to the low air speed, is very apt to make a ground loop. This aspect of directional stability should be kept in mind by designers.

Below is given a table of rudder and fin areas of a number of airplanes in the same way as for longitudinal stability.

	Rudder.	Fin.
	Per cent.	*Per cent.*
Training	3.4	1.0
1-seat service	2.4	1.3
2-seat service	3.1	1.2
Bombers	2.1	1.6
Flying boats	3.2	3.0
Average	2.8	1.6

DYNAMIC STABILITY.

LONGITUDINAL STABILITY.

As dynamic stability can have no significance unless there is static stability, and as there are so few airplanes completely statically stable, there has been very little practical study made of the dynamic stability. Any statically stable airplane may be made to oscillate with its natural period, which ranges between 15 and 30 seconds for various airplanes. If the oscillation decreases in amplitude the airplane is said to be dynamically stable, but if the oscillation increases it is unstable. Of the airplanes that have been studied in this particular it is strange to notice that very few of them have shown any instability; that is, in most cases if the airplane is statically stable it automatically becomes dynamically stable also. A curve is shown in figure 14, the data for which were taken on a JN4H, showing the type of stable oscillation which usually occurs on an airplane. In general, it may be said that increasing the area of the tail or the length of the body will increase the damping and make the airplane more dynamically stable, which are exactly the things that increase the static stability. Extensive full-flight tests now being carried out by the committee show that the dynamic stability in this case does not agree with the theory or model test. At any rate, an airplane that is dynamically unstable in pitch is not at all dangerous; in fact, a pilot would have difficulty in determining whether there was instability or not.

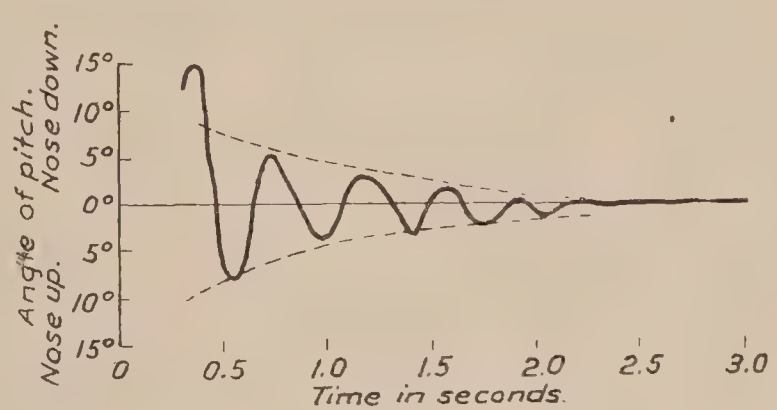


FIG. 14.—Free control oscillation of a JN4H with special tail.

LATERAL AND DIRECTIONAL STABILITY.

Very little is known in full flight of the directional or lateral stability in regard to oscillations, although a certain amount of oscillation has been studied in roll in order to determine the accuracy of bomb dropping.[5]

The spinning of an airplane is an important and much discussed subject, but up to the present there has been very little definitely learned about this matter.[6] It is certain, however, that it is a periodic phenomenon, as shown in the accelerometer record (fig. 15), and it appears as if the oscillation was decreasing in magnitude as the time increased. In some airplanes this dying out of the oscillation is much more marked, and completely disappeared in one or

[5] R. & M. No. 213, "Oscillation of an Airplane in Flight." British Advisory Committee.
[6] N. A. C. A. Report No. 105, "Angle of Attack and Air Speed in Manoeuvers."

two oscillations. The air speed is not at all high in a spin, seldom exceeding 60 to 80 miles per hour, and the angle of attack of the inner wing is often very high, sometimes reaching 90°.

As a large percentage of airplane accidents are due to spins it would be of the greatest value to construct an airplane which would not spin; but this problem is a rather complicated one, as it is connected with both longitudinal and lateral stability. It seems evident, however, from a considerable amount of investigation, that the ability to make an airplane come out of a spin by itself depends upon the area of the vertical fin. There are also some airplanes which can not be spun, but just why this should be is not at all clear.

CONTROLLABILITY.

Controllability is a rather difficult thing to define, as up to the present time there has been no quantitative method for measuring it. It may be logically divided, however, into three parts; the first is the power of the control, that is, the possibility of holding an airplane in any position, even against its natural stability or instability, if sufficient force is applied to the stick; the second, efficiency of control, demands the maximum possible controlling moment about the center of gravity of the airplane with a given force on the stick or rudder bar; and the third, which is quickness of control, demands a large moment about the center of gravity of the machine for a small movement of the control stick or rudder bar and is also of course dependent upon the moment of inertia of the airplane. To illustrate this distinction more completely, let us consider an airplane like the JN4H, which is an airplane with powerful controls, that is, even though the airplane is unstable at high speed and therefore a large force is required to hold the stick back, yet the controls are of sufficient power to pull the airplane out of the steepest dive provided the pilot has sufficient strength. An airplane such as the Salmson biplane is an example of an airplane with efficient control, that is, there are at no time more than 1 or 2 pounds force on the stick, so that it is a very comfortable airplane for the pilot to fly. An airplane that is quick on the controls may be illustrated by the VE7 or the Sopwith-Camel, where even the slightest movement of the controls produces a large angular velocity. It should be noted that the moment of inertia of the control system itself has a considerable influence on the quickness of control and for this reason should be made as light as possible. There seems to be no reason why all of these desirable characteristics may not be combined in one airplane.

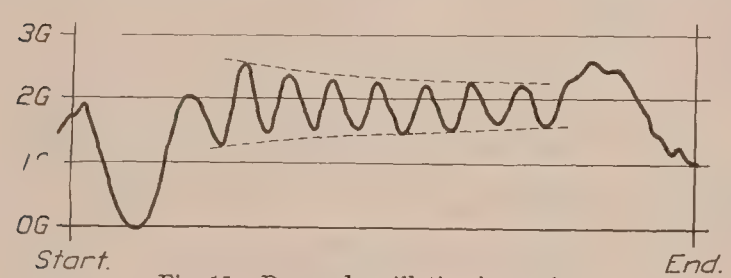


Fig. 15.—Damped oscillation in a spin.

The relation of controllability to stability should be recognized. For instance, it is possible to design an airplane which is so stable and which has so little control that it will be impossible to fly it at more than a small fraction of its normal flight range. In fact, it quite often happens that in stable airplanes it is impossible to reach the minimum speed, due to the fact that the elevator is pulled up as far as possible.[7] Again, an airplane which has powerful controls but is unstable is a rather uncomfortable airplane to fly, especially for commercial use, but is not dangerous. An airplane, however, which lacks both stability and control is extremely dangerous and should be avoided at all costs. In fighting airplanes it is desirable to have great controllability and small or neutral stability, but for commercial airplanes the stability should be considerable and the controllability only sufficient for ordinary flying.

LONGITUDINAL CONTROL.

The British[8] have done considerable work on the best size and shape of elevator, and the results are summarized in figure 16. It will be noticed that fairly close agreement is obtained between the model and the full-scale tests on the hinge moment, but that the results for the normal force on the whole tail surface in full flight show that there is a marked falling off as the size of the elevator is decreased, while the model shows nearly a constant force for any size

[7] The Factors that Determine the Minimum Speed of an Airplane. N. A. C. A. Technical Note No. 54.
[8] British Advisory Committee R. & M. No. 409 and No. 532, T 825.

of elevator, at least down to 30 per cent, so that the erroneous conclusion would be drawn from the latter results that a small elevator would be the best. It is necessary to compromise between efficiency and maximum control, the latter falling off with the size of the elevator, so that it may be concluded that an elevator containing an area of 33 per cent of the total tail surface will be satisfactory for commercial airplanes, which do not need to be stunted and where it is desirable to have an efficient control. On the other hand, for fighting airplanes, where quick maneuvers are essential, the area should be increased to about 42 per cent, which is about the average found in airplanes of this type.

These results are confirmed by some tests which have been made by the N. A. C. A. on a JN4H with two tail planes of the same area, one having an elevator of 33 per cent and one with an elevator of 43 per cent of the total horizontal tail surface. In the first case the airplane would control satisfactorily, but it appeared to the average pilot as sluggish, that is, the stick could be moved rapidly back and forth without producing any violent oscillations of the airplane, and it was very difficult to loop the airplane without having a negative loading at the top of the loop. The force on the stick, however, was quite small at all times, and for ordinary flying the control was considered to be quite satisfactory. The second tail plane, which had an elevator area of 43 per cent, was considerably more powerful, and although the airplane was not nearly as comfortable to fly, it could be stunted with ease.

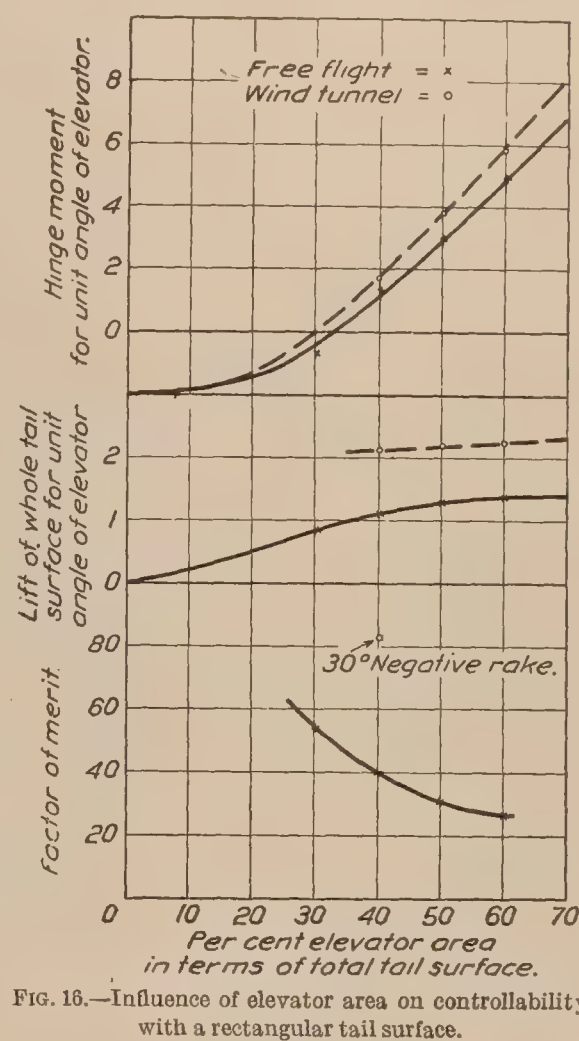


FIG. 16.—Influence of elevator area on controllability with a rectangular tail surface.

The range of elevator movement is quite important in determining the maximum control. The control efficiency can usually be increased by using smaller elevators and greater angular ranges than is now done. The angular ranges of elevator movement for several airplanes are given in the following table as actually measured on the field.

Type.	Elevator up.	Elevator down.
	°	°
JN4h	23	21
VE7	23	25
XB1A	27	19
DH4B	29	25
SE5	37	15
Martin bomber	30	14
Fokker D VII	30	18

Full flight tests were also made by the British to determine the effect of angle of rake at the tips of the tail surface. These results showed that for some inexplicable reason [9] the negative rake of 30°—that is, with the leading edge longest—gave much better results than with the usual positive rake.

LATERAL CONTROL.

There have been no satisfactory quantitative measurements made in free flight of aileron controllability, and it is necessary again to refer to the judgment of the pilot as to the lateral maneuverability of a particular airplane. In order to show what is the usual practice in area for ailerons, the following table gives the areas in percentage of the total wing area for several classes of airplanes.

[9] N. A. C. A. Report No. 119, "Distribution of Pressure Over the Tail Surfaces of an Airplane, II."

Training	11.6
2-seat service	13.8
1-seat service	11.0
Bombers	9.4
Flying boats	11.8
Average	11.5
Machines with good control	10.9

It will be noted that the average area for all of the airplanes is about 11½ per cent, while for the group of airplanes which are considered to be exceptionally controllable the area is only 11 per cent, showing that it is not area of aileron which gives great controllability. In fact, the Fokker, which is considered to have exceptional lateral control, has ailerons of only 5.2 per cent, so that the reason why this airplane should have such good controllability is unknown. As the ailerons on an airplane are out of the wash of the propeller, the results for model tests should give fairly accurate results when applied to the full-sized airplanes; therefore some British tests made on model ailerons may be considered.[10] It was found that the gap between the ailerons and the wings should be as small as possible, as a gap of one-half inch might reduce the effectiveness of the aileron by as much as 30 per cent. Contrary to the general belief, ailerons on the lower wing appear to be fully as effective as ailerons on the upper wing. In the same way as for the tail plane, the negative rake on the wing tip gives greater efficiency to the aileron, although the effect is not quite as marked in this case. As in the case of the elevator, reducing the chord of the aileron, keeping a constant breadth, increases the efficiency of the control and reduces the maximum control, especially at the lower speeds. Both full flight tests and model tests seem to show that the washed-out aileron, that is, ailerons where the tip is turned up, increases the efficiency at the lower speeds, and this method is employed on most of the German airplanes.

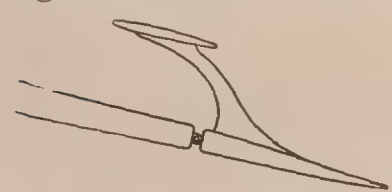


FIG. 17.—Aileron with biplane balance.

There has been a large amount of work done in the wind tunnel on balanced ailerons and some work in full flight, and the conclusion has been reached that satisfactory balances can not be obtained by an extension of the aileron at the wing tip, but that better results are obtained by an auxiliary surface above and ahead of the aileron, as shown in figure 17. From the satisfactory results which have been obtained, however, on the small and medium sized airplanes, which use no balancing, it seems that balancing would be unnecessary if the ailerons are properly designed, except for the larger airplanes.

Below is given a table of the angular range of ailerons on several airplanes.

Type.	Aileron up.	Aileron down.
	°	°
JN4H	19	24
VE7	28	25
XB1A	16	19
DH4B	17	17
SE5	14	12
Martin bomber	20	20
Fokker D VII	23	32

DIRECTIONAL CONTROL.

There can be little said on directional control, for if the rudder has enough power on the ground when taxing it usually has sufficient when in the air. The tendency of some airplanes to ground loop on landing can be attributed to insufficient directional stability and control. In stunting, a powerful rudder is not as essential as the other controls. It may be stated, however, that it is desirable to have the highest aspect ratio possible on the rudder and fin consistent with structural considerations.

[10] British Advisory Committee, T. 1247.

The angular range of rudder movement for several airplanes is given below:

Type.	Right rudder.	Left rudder.
	°	°
JN4H	30	31
VE7	38	36
XB1A	36	33
DH4B	19	20
SE 5	19	19
Martin bomber	30	27
Fokker D VII	43	40

CONCLUSIONS.

In the following paragraphs it is attempted to give as accurately as possible definite data so that the designer may produce an airplane which has the desired amount of stability and controllability. It should be realized, however, that the data on which these conclusions are based is rather meager and applies mainly to tractor airplanes with a single motor and that in some cases the results are obtained from one airplane, so that it can not be expected that this data will apply strictly to any airplane which is designed. Also, the conclusions will be modified as our information is increased. In fact, in the present state of the art it is quite impossible to design at the first trial an airplane which is perfect in stability and control, but it should be possible, however, to design an airplane which is fairly satisfactory and from tests on this airplane to deduce what changes it is necessary to make in order to correct any given faults.

LONGITUDINAL BALANCE.

1. To correct an unbalanced pull on the stick of 1 pound, move the center of gravity back 2 per cent of the chord or set the tail plane to ¼° more positive angle.

LONGITUDINAL STABILITY.

1. The center of gravity may be placed anywhere between 20 per cent and 40 per cent back on the mean chord of the wing, but 30 per cent is probably the most satisfactory.

2. The angle of the tail plane should be set at $-1°$ (zero lift line) to the wing chord for a center of gravity position of 30 per cent and up or down 0.125° for every per cent of the center of gravity from this position.

3. The area of the horizontal tail surfaces should be about 13 per cent of the wing area, and the length from the center of gravity to elevator hinge should be about 3 chord lengths.

4. With the above area and the center of gravity 30 per cent back on the wing chord, an aspect ratio of 2 on the horizontal tail surfaces will give about neutral stability. An aspect ratio of 3 will give great stability.

5. A tail section flat on the bottom will give the greatest stability at low speeds, while if it is flat on top it will give stability at high speeds. To obtain equal stability with free controls at all speeds use two-thirds of the camber above and one-third below.

LONGITUDINAL CONTROL.

1. The power of control is proportional to the area of the elevator. For great controllability use 45 per cent of the tail area in the elevator, for average airplanes 42 per cent, and for commercial airplanes, which require no stunting, and where small forces are desired on the control at all times, use 33 per cent. The efficiency of the elevator as a controlling member is found to be greatly increased by using about a 30 degree negative rake on the tips of the tail surface.

2. For the greatest quickness and lightness of control it is necessary to use small and light elevators with a large gearing between the elevator and the stick and to have a neutral or unstable airplane with locked controls. The quickness of control is also proportional to the longitudinal moment of inertia of the airplane.

LATERAL BALANCE.

1. In balancing a machine laterally, ½° of warp at the outer strut point will be equivalent to an angle of 1° on the aileron.

LATERAL STABILITY.

1. The lateral stability of the airplane does not depend primarily upon the shape or size of the aileron, but can be obtained most easily by using a dihedral angle of 3 to 6°.

LATERAL CONTROL.

1. The size of the aileron to obtain satisfactory lateral control should be about 11 per cent, but areas as low as 5 per cent have been found satisfactory on some airplanes. It is found that washing out the tips of the ailerons helps the controllability at low speeds.

DIRECTIONAL STABILITY.

1. With the usual length of fuselage a fin having an area of 2 per cent of the wings is sufficient to give directional stability with free controls.

DIRECTIONAL CONTROL.

1. It is found that a rudder having an area 2 per cent of the wing area is sufficient for ordinary controls. The rudder should have a high aspect ratio, and for a larger airplane be balanced.

REFERENCES.

Preliminary Report on Free Flight Tests. N. A. C. A. Report No. 70.
Angle of Attack and Air Speed in Manoeuvers. N. A. C. A. Report No. 105.
Statical Longitudinal Stability of Airplanes. N. A. C. A. Report No. 96.
Control in Circling Flight. N. A. C. A. Report No. 112.
The Factors that Determine the Minimum Speed of Airplanes. N. A. C. A. Technical Note No. 54.
Measurements of Control Moments on an Airplane During Flight. N. A. C. A. Technical Note No. 38. Translated from "Zeitschrift fur Flugtechnik und Motorluftschiffahrt," Vol. No. 21 and No. 22.
Systematische Versuche an Leitwertmodellen. Technische Berichte Nr. 5.

The following reports are from the British Advisory Committee:

Balancing of Wing Flaps. R. & M. No. 200.
Bodies with Fins and Rudders. R. & M. No. 201.
Controllability of Aeroplanes with Special Reference to Rapid Turning. R. & M. No. 212.
Full Scale Experiments on the Moment About the Hinge of the Air Forces of an Elevator. R. & M. No. 284.
Full Scale Stability Experiments on a B E 2 E with R. A. F. 15 Wing. R. & M. No. 326.
Full Scale Determination of Pitching Moment of a Biplane. R. & M. No. 400.
Full Scale Experiments with Elevators of Different Sizes. R. & M. No. 409.
Initial Control with Different Types of Wing Flaps. R. & M. No. 413.
Preliminary Tests on the Rolling and Pitching of a Handley-Page Machine. R. & M. 422.
Lateral Control of Various Aeroplanes. R. & M. 441.
The Longitudinal Control of an Aeroplane. R. & M. No. 470.
Full Scale Stability Experiments with R. A. F. 14 Wing Section. R. & M. No. 505
Full Scale Experiments with Different Shapes of Tailplanes. R. & M. No. 532.
An Investigation of the Aerodynamic Properties of Wing Ailerons. Pa 11I, R. & M. No. 550. Part II. R. & M. No. 615.
Oscillations of an Airplane in Flight. R. & M. No. 213.

REPORT No. 121

THE MINIMUM INDUCED DRAG OF AEROFOILS

By MAX M. MUNK
National Advisory Committee for Aeronautics

REPORT No. 121.

THE MINIMUM INDUCED DRAG OF AEROFOILS.

By Max M. Munk.

INTRODUCTION.

The following paper is a dissertation originally presented by the author to the University of Goettingen. It was intended principally for the use of mathematicians and physicists. The author is pleased to note that the paper has aroused interest in other circles, to the end that the National Advisory Committee for Aeronautics will make it available to a larger circle in America. The following introduction has been added in order to first acquaint the reader with the essence of the paper.

In the following development all results are obtained by integrating some simple expressions or relations. For our purposes it is sufficient, indeed, to prove the results for a pair of small elements. The qualities dealt with are integrable, since, under the assumptions we are allowed to make, they can not be affected by integrating. We have to consider only the relations between any two lifting elements and to add the effects. That is to say, in the process of integrating each element occurs twice—first, as an element producing an effect, and, second, as an element experiencing an effect. In consequence of this the symbols expressing the integration look somewhat confusing, and they require so much space in the mathematical expression that they are apt to divert the reader's attention from their real meaning. We have to proceed up to three dimensional problems. Each element has to be denoted twice (by a Latin letter and by a Greek letter), occurring twice in a different connection. The integral, therefore, is sixfold, six symbols of integration standing together and, accordingly, six differentials (always the same) standing at the end of the expression, requiring almost the fourth part of the line. The meaning of this voluminous group of symbols, however, is not more complicated and not less elementary than a single integral or even than a simple addition.

In section 1 we consider one aerofoil shaped like a straight line and ask how all lifting elements, which we assume to be of equal intensity, must be arranged on this line in order to offer the least drag.

If the distribution is the best one, the drag can not be decreased or increased by transferring one lifting element from its old position (a) to some new position (b). For then either the resulting distribution would be improved by this transfer, and therefore was not best before, or the transfer of an element from (b) to (a) would have this effect. Now, the share of one element in the drag is composed of two parts. It takes share in producing a downwash in the neighborhood of the other lifting elements and, in consequence, a change in their drag. It has itself a drag, being situated in the downwash produced by the other elements.

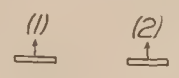


Fig 1

Considering only two elements, Fig. 1 shows that in the case of the lifting straight line the two downwashes, each produced by one element in the neighborhood of the other, are equal. For this reason the two drags of the two elements each produced by the other are equal, too, and hence the two parts of the entire drag of the wings due to one element. The entire drag

produced by one element has twice the value as the drag of that element resulting from the downwash in its environs. Hence, the entire drag due to one element is unchanged when the element is transferred from one situation to a new one of the same downwash, and the distribution is the best only if the downwash is constant over the whole wing.

In sections 2 to 6 it is shown that the two parts of the drag change by the same value in all other cases, too. If the elements are situated in the same transverse plane, the two parts are equal. A glance at Fig. 2 shows that the downwash produced by (1) at (2), (3), (4), and (5)

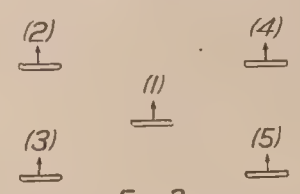


Fig. 2

is equal. But then it also equals the downwash due to (4), say, produced at (1). This holds true even for the component of the downwash in the direction of the lift if the elements are normal to each other (Fig. 3.); for this component is proportional $x.y/r^3$, according to the symbols

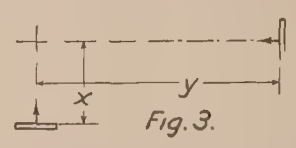


Fig. 3.

of the figure. Hence, it is proved for lift of any inclination, horizontal and vertical elements being able, by combination, to produce lift in any direction.

There remains only the question whether the two parts of the drag are also equal if the elements are situated one behind the other—that is to say, in different longitudinal positions. They are not; but their sum is independent of the longitudinal distance apart. To prove this, add in Fig. 4 to the lifting element (2) a second inverse lifting element (3) with inverse

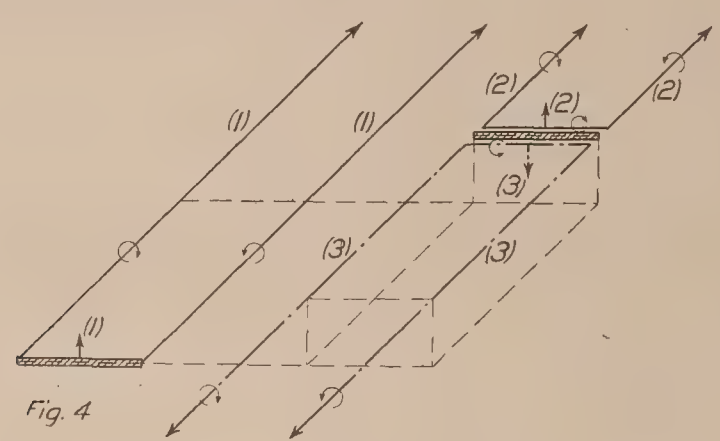


Fig. 4

linear longitudinal vortices in the inverse direction. The reader observes that the transverse vortices (2) and (3) neutralize each other; the longitudinal linear vortices, however, have the same sign, and all four vortices form a pair of vortices running from infinity to infinity. The drag, produced by the combination of (1) and this pair, is obviously independent of the longitudinal positions of (1) and (2). But the added element (3) has not changed the drag, for (1) and (3) are situated symmetrically and produce the same mutual downwash. The direction of the lift, however, is inverse, and therefore the two drags have the inverse sign, and their sum is zero.

If the two lifting elements are perpendicular to each other (chapter 5), a similar proof can be given.

Sections 6 and 7 contain the conclusions. The condition for a minimum drag does not depend upon the longitudinal coordinates, and in order to obtain it the downwash must be assumed to be constant at all points in a transverse plane of a corresponding system of aerofoils. This is not surprising; the wings act like two dimensional objects accelerating the air passing in an infinite transverse plane at a particular moment. Therefore the calculation leads to the consideration of the two dimensional flow about the projection of the wings on a transverse plane.

Section 8 gives the connection between the theory in perfect fluids and the phenomenon in true air. It is this connection that allows the application of the results to practical questions.

1. THE LIFTING STRAIGHT LINE.

A system of aerofoils moving in an incompressible and frictionless fluid has a drag (in the direction of its motion) if there is any lift (perpendicular to the direction of its motion). The magnitude of this drag depends upon the distribution of the lift over the surface of the aerofoils. Although the dimensions of the given system of aerofoils may remain unchanged, the distribution of the lift can be radically altered by changes in details, such as the aerofoil section or the angle of attack. The purpose of the investigation which is given in the following pages is to determine (*a*) the distribution of lift which produces the least drag, and (*b*) the magnitude of this minimum drag.

Let us first consider a single aerofoil of such dimensions that it may be referred to with sufficient exactness as a lifting straight line, which is at right angles to the direction of its flight. The length or span of this line may be denoted by l. Let the line coincide with the horizontal, or x axis of a rectangular system of coordinates having its origin at the center of the aerofoil. The density of the lift

$$A' = \frac{dA}{dx} \tag{1}$$

where A, the entire lift from the left end of the wing up to the point x, is generally a function of x and may be denoted by $f(x)$. Let the velocity of flight be v_0.

The modern theory of flight[1] allows the entire drag to be expressed as a definite double integral, if certain simplifying assumptions are made. In order to find this integral, it is necessary to determine the intensity of the longitudinal vortices which run from any lifting element to infinity in a direction opposite to the direction of flight. These vortices are generally distributed continuously along the whole aerofoil, and their intensity per unit length of the aerofoil is

$$\Gamma' = \frac{1}{v_0 \cdot \rho} \cdot \frac{dA'}{dx} \tag{2}$$

where ρ is the density of the fluid. Now, for each lifting element dx, we shall calculate the downwash w, which, in accordance with the law of Biot-Savart, is produced at it by all the longitudinal vortices. A single vortex, beginning at the point x, produces at the point $x=\xi$ the downwash

$$dw = \frac{1}{4\pi\rho v_0} \cdot dA' \cdot \frac{1}{\xi - x} \tag{3}$$

Therefore the entire downwash at the point ξ is

$$w = \frac{1}{4\pi\rho v_0} \int_{-\frac{l}{2}}^{+\frac{l}{2}} \frac{dA'}{dx} \cdot \frac{1}{\xi - x} dx \tag{4}$$

The integration is to be performed along the aerofoil; and the principal value of the integral is to be taken at the point $x=\xi$. This rule also applies to all of the following integrals. Hence it follows that the drag according to the equation

$$\frac{dW}{dx} = W' = \frac{w}{v_0} \cdot A' \tag{5}$$

is

$$W = \frac{1}{4\pi v_0^2 \rho} \int_{-\frac{l}{2}}^{+\frac{l}{2}} \left\{ \int_{-\frac{l}{2}}^{+\frac{l}{2}} \frac{dA'}{dx} \cdot \frac{1}{\xi - x} dx \right\} \cdot A' d\xi \tag{6'}$$

[1] See L. Prandtl, Tragflügeltheorie, I. Mitteilung. Nachrichten der Ges. d. Wiss. zu Göttingen, 1918.

or, otherwise expressed,

$$W=\frac{1}{4\pi v_0^2\rho}\int_{-\frac{l}{2}}^{+\frac{l}{2}}\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f'(x)\cdot f(\xi)}{\xi-x}dxd\xi \tag{6}$$

f' here signifies the derivative of f with respect to x or ξ. The entire lift is represented by

$$A=\int_{-\frac{l}{2}}^{+\frac{l}{2}}f(x)\,dx \tag{7}$$

Hence the solution of the problem to determine the best distribution of lift depends upon the determination of the function f so that the double integral

$$J_1=\int_{-\frac{l}{2}}^{+\frac{l}{2}}\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f'(x)\cdot f(\xi)}{\xi-x}dxd\xi \tag{8}$$

shall have a value as small as possible; while at the same time the value of the simple integral

$$J_2=\int_{-\frac{l}{2}}^{+\frac{l}{2}}f(x)dx=\text{const.} \tag{9}$$

is fixed.

The first step towards the solution of this problem is to form the first variation of J_1

$$\delta J_1=\int_{-\frac{l}{2}}^{+\frac{l}{2}}\left\{\delta f(x)\,dx\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f'(\xi)}{\xi-x}d\xi\right\}+\int_{-\frac{l}{2}}^{+\frac{l}{2}}\left\{\delta f'(\xi)d\xi\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f(x)}{\xi-x}d\xi\right\} \tag{10}$$

The second integral on the right side of (10) can be reduced to the first. By exchanging the symbols x and ξ and by partial integration with respect to x, considering $f'(\xi)$ as the integrable factor, there is obtained

$$\int_{-\frac{l}{2}}^{+\frac{l}{2}}\left\{\delta f'(\xi)d\xi\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f(x)}{\xi-x}dx\right\}=-\int_{-\frac{l}{2}}^{+\frac{l}{2}}\left\{\delta f(x)dx\cdot\frac{d}{dx}\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f(\xi)}{x-\xi}d\xi\right\} \tag{11}$$

The second member disappears since $f=O$ at the limits of integration.[2] Further, the right hand part of (11)

$$\frac{d}{dx}\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f(x)}{x-\xi}d\xi$$

upon substitution of the new variables x and $t=x-\xi$ for x and ξ, is transformed into

$$\frac{d}{dx}\int_{x+\frac{l}{2}}^{x-\frac{l}{2}}\frac{f(x-t)}{t}dt$$

[2] If this were not true, there would be infinite velocities at these points.

Now

$$\frac{d}{dx}\int_{x+\frac{l}{2}}^{x-\frac{l}{2}}\frac{f(x-t)}{t}\,dt=\frac{f\left(\frac{l}{2}\right)}{x-\frac{l}{2}}-\frac{f\left(-\frac{l}{2}\right)}{x+\frac{l}{2}}+\int_{x+\frac{l}{2}}^{x-\frac{l}{2}}\frac{f'(x-t)}{t}\,dt$$

or, since f disappears at the limits of integration,

$$\frac{d}{dx}\int_{x+\frac{l}{2}}^{x-\frac{l}{2}}\frac{f(x-t)}{t}\,dt=\int_{x+\frac{l}{2}}^{x-\frac{l}{2}}\frac{f'(x-t)}{t}\,dt$$

which, upon the replacement of the original variables, becomes

$$\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f'(\xi)}{x-\xi}\,d\xi$$

so that, finally,

$$\int_{-\frac{l}{2}}^{+\frac{l}{2}}\left\{\delta f'(\xi)\,d\xi\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f(x)}{\xi-x}\,dx\right\}=\int_{-\frac{l}{2}}^{+\frac{l}{2}}\left\{\delta f(x)\,dx\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f'(\xi)}{\xi-x}\,d\xi\right\} \tag{12}$$

Substituting this in (10) there finally results

$$\delta J_1=2\int_{-\frac{l}{2}}^{+\frac{l}{2}}\delta f(x)\,dx\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f'(\xi)}{\xi-x}\,d\xi \tag{13}$$

From which the condition for the minimum amount of drag, taking into consideration the second condition (9), is

$$\int_{-\frac{l}{2}}^{+\frac{l}{2}}\frac{f'(\xi)}{\xi-x}\,d\xi+\lambda=0 \tag{14}$$

or, when equation (4) is taken into consideration

$$w=\text{const.}=w_0 \tag{15}$$

The necessary condition for the minimum of drag for a lifting straight line is that the downwash produced by the longitudinal vortices be constant along the entire line.

That this necessary consideration is also sufficient results from the obvious meaning of the second variation, which represents the infinitesimal drag produced by the variation of the lift if it alone is acting, and therefore it is always greater than zero.

2. PARALLEL LIFTING ELEMENTS LYING IN A TRANSVERSE PLANE.

The method just developed may be applied at once to problems of a more general nature. If, instead of a single aerofoil, there are several aerofoils in the same straight line perpendicular to the direction of flight, only the limits of integration are changed in the development. The integration in such cases is to be performed along all of the aerofoils. However, this is nonessential for all of the equations and therefore the condition for the minimum drag (equation 15) applies to this entire system of aerofoils.

Let us now discard the condition that all of the lifting lines are lying in the same straight line, but retain, however, the condition that they are parallel to each other, perpendicular to the line of flight as before, and that they are all lying in a plane perpendicular to the line of flight. Let the height of any lifting line be designated by z or ζ. Equation (3) transforms into a similar one which gives the downwash produced at the point x, z by the longitudinal vortex beginning on the lifting element at the point ζ:

$$dw=\frac{1}{4\pi\rho v_0}\,dA'\,\frac{\xi-x}{(\xi-x)^2+(\zeta-z)^2} \tag{3a}$$

The expression, which must now be a minimum, is

$$J_1=\int\int\left[\frac{d}{dx}f(x,z)\right]\cdot f(\xi,\zeta)\,\frac{\xi-x}{(\xi-x)^2+(\zeta-z)^2}\,d\xi\,dx \tag{8a}$$

with the unchanged secondary condition

$$J_2=\int f(x,z)\,dx=\text{const.} \tag{9a}$$

These integrals are to be taken over all of the aerofoils.

This new problem may be treated in the same manner as the first.

$$\frac{\xi-x}{(\xi-x)^2+(\zeta-z)^2}$$

is always to be substituted for $\frac{1}{\xi-x}$. It may be shown that this substitution does not affect the correctness of equations (10) to (15). Therefore

$$w=\text{const.}=w_0 \tag{15a}$$

is again obtained as the necessary condition for the minimum of the entire drag.

Finally, this also holds true for the limiting case in which, over a limited portion of the transverse plane, the individual aerofoils, like venetian blinds, lie so closely together that they may be considered as a continuous lifting part of a plane. Including all cases which have been considered so far, the condition for a minimum of drag can be stated:

Let the dimensions of a system of aerofoils be given, those in the direction of flight being small in comparison with those in other directions. Let the lift be everywhere directed vertically. Under these conditions, the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift.

3. THREE DIMENSIONAL PARALLEL LIFTING ELEMENTS.

The three-dimensional problem may be based upon the two-dimensional one. Let now the dimensions in the direction of flight be considerable and let the lifting elements be distributed in space in any manner. Let y or η be the coordinates of any point in the direction of flight. For the time being, all lifting forces are assumed to be vertical.

The calculation of the density of drag for this case is somewhat more complicated than in the preceding cases. Consideration must be given not only to the longitudinal vortices, which are treated as before, but also to the transverse vortices which run perpendicular to the lift at any point and to the direction of flight. Their intensity at any point where there is a lifting element is

$$\Gamma=A'\cdot\frac{1}{v_0\cdot\rho}=f(x,y,z)\cdot\frac{1}{v_0\rho}.$$

The density of drag, W^1 now has two components, W_1 and W_2, the first being due to the transverse vortices and the second to the longitudinal vortices.

For the solution of the present problem only the total drag of all lifting elements

$$W=\int W'dx$$

is to be considered. In the first place it will be shown that the integral of those parts of the density resulting from the transverse vortices

$$W_1=\int W_1'dx$$

does not contribute to the total drag. A small element of one transverse vortex of the length dx at the point (x, y, z) produces at the point (ξ, η, ζ) the downwash

$$dw=\frac{1}{4\pi\rho v_0}\frac{\eta-y}{r^3}f(x, y, z)\cdot dx \tag{16}$$

where

$$r^2=(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2.$$

Therefore

$$W_1=\frac{1}{4\pi\rho v_0^2}\int\int f(x, y, z)\cdot f(\xi, \eta, \zeta)\frac{\eta-y}{r^3}dxd\xi. \tag{17}$$

This integration is to be extended over all the aerofoils. It is possible to write this expression in such a manner that it holds for a continuous distribution of lift over parts of surfaces or in space. This is true, moreover, for most of the expressions in this paper. Now, exchanging the variables x, y, z, for ξ, η, ζ, in equation (17) does not change the value of the integral, since the symbols for the variables have no influence on the value of a definite integral. On the other hand, the factor $(\eta-y)$, and therefore the integral also, changes its sign. Hence

$$W_1=-W_1=0 \tag{18}$$

and, as stated,

$$W=W_2. \tag{19}$$

Therefore the entire drag may be calculated without taking into consideration the transverse vortices.

The method of calculating the effect of the longitudinal vortices can be greatly simplified. At the point (ξ, η, ζ) that part of the density of drag resulting from a longitudinal vortex beginning at the point (x, y, z) is

$$W_2'=\frac{1}{v_0^2\rho}\int f(\xi, \eta, \zeta)f'(x, y, z)\cdot\psi dx \tag{20}$$

where

$$f'=\frac{d}{dx}f, \text{ resp. } \frac{d}{d\xi}f$$

and

$$\psi=\frac{1}{4\pi}\int_y^\infty\frac{\xi-x}{t^3}ds;\ t^2=(\xi-x)^2+(\eta-s)^2+(\zeta-z)^2. \tag{21}$$

The entire drag is

$$W=\int W_2'dx=\frac{1}{v_0^2\rho}\int\int f(\xi, \eta, \zeta)f'(x, y, z)\psi d\xi dx. \tag{22}$$

Now, in the double integral (22) the variables x, y, z may be exchanged with ξ, η, ζ, as before, without affecting the value of the definite integral. Partial integration may then be performed twice, first with respect to ξ and then with respect to x. The substitution results in

$$W=\frac{1}{v_0^2\rho}\int\int f(x, y, z)f'(\xi, \eta, \zeta)\bar{\psi}dxd\xi \tag{23}$$

$\bar{\psi}$ is obtained from ψ upon the exchange of variables. Its value is therefore

$$\bar{\psi}=\frac{1}{4\pi}\int_{y}^{\infty}\frac{x-\xi}{\bar{t}^3}\,ds;\ \bar{t}^2=(\xi-x)^2+(s-y)^2+(\zeta-z)^2. \tag{24}$$

When partially integrating with respect to $d\xi$, the integrable factor is $f'\ (\xi, \eta, \zeta)$

$$W=-\frac{1}{v_0^2\rho}\int\int f\ (\xi, \eta, \zeta)\ \frac{d}{d\xi}\bar{\psi}\cdot f\ (x, y, z)\ dxd\xi \tag{25}$$

In the subsequent partial integration with respect to dx, the integrable factor is $\frac{d}{d\xi}\bar{\psi}=-\frac{d}{dx}\bar{\psi}$.

$$W=-\frac{1}{v_0^2\rho}\int\int f\ (\xi, \eta, \zeta)\cdot f'\ (x, y, z)\ \bar{\psi}dxd\xi. \tag{26}$$

Finally, by addition of (22) and (26), there is obtained

$$2\,W=\frac{1}{v_0^2\rho}\int\int f\ (\xi, \eta, \zeta)\ f'\ (x, y, z)\ (\psi-\bar{\psi})\ dxd\xi. \tag{27}$$

$\bar{s}=y+\eta-s$ may now be substituted in (24) for the variable of integration s. Then $\bar{t}$ changes to t, and with the exception of the sign the integrand in (21) agrees with the resulting one in (22)

$$\bar{\psi}=-\frac{1}{4\pi}\int_{y}^{-\infty}\frac{x-\xi}{t^3}d\bar{s} \tag{28}$$

Subtracting (28) from (21) there results finally

$$\psi-\bar{\psi}=\frac{1}{4\pi}\int_{-\infty}^{+\infty}\frac{\xi-x}{t^3}ds. \tag{29}$$

Hence, $\psi-\bar{\psi}$ and therefore the entire right side of equation (22) is seen to be independent of the longitudinal coordinates y of the lifting elements.

Therefore the entire resistance of a three-dimensional system of aerofoils with parallel lifting elements does not depend upon the longitudinal positions of the lifting elements.

4. LIFTING ELEMENTS ARRANGED IN ANY DIRECTIONS IN A TRANSVERSE PLANE.

The problem considered in section 2 can also be generalized in another way. For the present the condition that all lifting elements be in one transverse plane may remain. However, they need no longer be parallel, and the lift may be due to not only a great number of infinitesimal lifts dA but also to similar transverse forces dB. In the first place let the direction of all lifting elements be arbitrary, but such that there is a minimum drag, and let this direction be an unknown quantity to be determined.

In the present problem it is desirable to consider a continuous distribution of lift over given areas instead of lines. The last case can be deduced from the first at any time by passing to the limit.

Let $A'=f(x, z)$ be the density of the vertical lift per unit area, and $B'=F(x, z)$ the density of the lateral force per unit area. The lateral force is considered positive when acting in the positive direction of the X-axis. Then the density of the transverse vortices has the components $\frac{1}{v_0\rho}\cdot A'$ and $-\frac{1}{v_0\rho}B'$. The density of the longitudinal vortex is the divergence of the density of the transversal vortex, or $\frac{1}{v_0\rho}\left(\frac{dA'}{dx}-\frac{dB'}{dz}\right)$. The longitudinal vortices beginning

at the point (x, z) therefore produce at the point (ξ,ζ) the downwash and the transverse velocity

$$dw=\frac{1}{4\pi v_o\rho}\left(\frac{dA'}{dx}-\frac{dB'}{dz}\right)dxdz\,\frac{\xi-x}{r^2} \tag{3b}$$

$$du=\frac{1}{4\pi v_o\rho}\left(\frac{dA'}{dx}-\frac{dB'}{dz}\right)dxdz\,\frac{z-\zeta}{r^2} \tag{3c}$$

According to the above, the density of the drag is

$$d\,W=A'\frac{w}{v_o}dxdz+B'\frac{u}{v_o}dxdz \tag{5b}$$

With these symbols there results for the total drag the expression

$$W=\frac{1}{4\pi v^2\rho}\Big\{\int\int\int\int f'\,(x,z)\,f\,(\xi,\zeta)\,\frac{\xi-x}{r^2}\,dxdzd\xi d\zeta-\int\int\int\int F'\,(x,z)\;F\,(\xi,\zeta)\,\frac{z-\zeta}{r^2}\,dxdzd\xi d\zeta+ \tag{30}$$
$$\int\int\int\int f'\,(x,z)\;F\,(\xi,\zeta)\,\frac{z-\zeta}{r^2}\,dxdzd\xi d\zeta-\int\int\int\int F'\,(x,z)\,f\,(\xi,\zeta)\,\frac{\xi-x}{r^2}\,dxdzd\xi d\zeta\Big\}$$

All of these integrals are to be taken over all of the lifting surfaces. Now the first two integrals have forms corresponding to the integral in (8), and therefore there is a possibility of substituting (12) for these. A similar relation also holds for the last two integrals. For example, the variation of the third integral is

$$\delta\int\int\int\int f'\,(x,z)\;F\,(\xi,\zeta)\,\frac{z-\zeta}{r^2}\,dxdzd\xi d\zeta=$$
$$\int\int\int\int\left[\delta f'\,(x,z)\cdot F\,(\xi,\zeta)\,\frac{z-\zeta}{r^2}+f'\,(x,z)\;\delta F\,(\xi,\zeta)\,\frac{z-\zeta}{r^2}\right]dxdzd\xi d\zeta \tag{31}$$

Now in the first term on the right-hand side the variables x and z may be exchanged with ξ and ζ. It may then be partially integrated with respect to $d\xi$, the integrable factor being $df^1(\xi,\zeta)$. This gives

$$\int\int\int\int\delta f'\,(x,z)\cdot F\,(\xi,\zeta)\,\frac{z-\zeta}{r^2}\,dxdzd\xi d\zeta=-\int\int\int\int\delta f\,(\xi,\zeta)\cdot\frac{d}{d\xi}\,F\,(x,z)\,\frac{\zeta-z}{r^2}\,dxdzd\xi d\zeta \tag{32}$$

This may be partially integrated with respect to dz, the integrable factor being

$$\frac{d}{d\zeta}\,\frac{\zeta-z}{r^2}=-\frac{d}{dz}\,\frac{\xi-x}{r^2}$$

$$\int\int\int\int\delta f'\,(x,z)\;F\,(\xi,\zeta)\,\frac{z-\zeta}{r^2}\,dxdzd\xi d\zeta=-\int\int\int\int\delta f\,(\xi,\zeta)\cdot F'\,(x,z)\,\frac{\xi-x}{r^2}\,dxdzd\xi d\zeta \tag{33}$$

Hence the first term of the variation of the third integral of (30) can be transformed into the second term of the variation of the fourth integral of this equation. In a similar manner the two other terms may be transformed into each other. It is therefore demonstrated that the variation of the entire drag may be written

$$\delta\,W=2\int\int\delta f\cdot w\cdot dxdz+2\int\int\delta F\cdot u\cdot dxdz \tag{13b}$$

Two problems of variation can now be stated. In the first place limited parts of the surfaces may be at our disposal, over which the vertical lift A and the horizontal transversal force B may have any distribution. Only the total lift

$$A=\int\int f\,(x,z)\,dxdz=\text{const.} \tag{9b}$$

will be given in this case.

Then

$$w=\text{const.}=w_0;\ u=o \tag{15b}$$

is the condition for the least drag.

If, however, the lifting parts are similar to lines, there is generally one other condition to fulfill. It is then required that the lift disappear everywhere along the direction of the aerofoils. That is to say,

$$f\sin\beta-F\cos\beta=o \tag{34}$$

where β is the angle of inclination of the aerofoil to the horizontal X-axis. In order to add the new requirement (34) a second Lagrange constant μ is introduced. The condition for the least drag is now

$$w+\lambda+\frac{\mu}{\cos\beta}=o,\ u-\frac{\mu}{\sin\beta}=o \tag{34a}$$

and after the elimination of μ

$$w\cos\beta+u\sin\beta=w_o\cos\beta \tag{15c}$$

the constant 2λ being replaced by $-w_0$, as before. In words:

If all lifting elements are in one transverse plane, the component of the velocity perpendicular to the wings, produced by the longitudinal vortices, must be proportional, at all lifting elements, to the cosine of the angle of lateral inclination.

5. LIFT DISTRIBUTED AND DIRECTED IN ANY MANNER.

The results obtained previously can be generalized not only for lifting elements distributed in a transverse plane but also for lifting elements distributed in any manner in space. That part of the total drag resulting from the transverse vortices is, in the general case

$$\begin{aligned}W_1=\frac{1}{4\pi\rho v^2}\Big[&\int\int\int\int\int\int f\,(x,y,z)\,f\,(\xi\,\eta,\zeta)\,\frac{n-y}{r^3}\,dxdydzd\xi d\eta d\zeta\\&+\int\int\int\int\int\int F\,(x,y,z)\,F\,(\xi,\eta,\zeta)\,\frac{n-y}{r^3}\,dxdydzd\zeta d\eta d\zeta\Big]\end{aligned} \tag{17a}$$

Both terms have the same form as the integral in (17). The demonstration for (17) therefore applies to both. In the general case also the total drag can be calculated from the longitudinal vortices without taking into consideration the transverse vortices:

$$\begin{aligned}W=\frac{1}{4\pi\rho v^2}\Big[&\int\int\int\int\int\int f(x,y,z)\,f\,(\xi,\eta,\zeta)\,\psi_1\,dxdydzd\xi d\eta d\zeta\\&+\int\int\int\int\int\int F\,(x,y,z)\,F\,(\xi,\eta,\zeta)\,\psi_2\,dxdydzd\xi d\eta d\zeta\\&-\int\int\int\int\int\int f(x,y,z)\,F'\,(\xi,\eta,\zeta)\,\psi_1\,dxdydzd\xi d\eta d\zeta\\&-\int\int\int\int\int\int F\,(x,y,z)\,f'\,(\xi,\eta,\zeta)\,\psi_2\,dxdydzd\xi d\eta d\zeta\Big]\end{aligned} \tag{22a}$$

In this as in (20),

$$\psi_1=\frac{1}{4\pi}\int_y^\infty\frac{\xi-x}{t^3}\,ds;\ t^2=(\xi-x)^2+(\eta-s)^2+(\zeta-z)^2$$

$$\psi_2=\frac{1}{4\pi}\int_y^\infty\frac{\zeta-z}{t^3}\,ds$$

The first two terms in (22a) have the same form as the right-hand side of (22), and the same conclusions are therefore valid for each. It can be proved directly for (22a) as for (22) that each of the two double integrals is independent of the longitudinal coordinates of the lifting elements. This proof can now be extended over the last two integrals of equation (22a).

The third integral, after changing the variables, becomes

$$\iiiint\!\!\iint f(x, y, z)\ F'(\xi, \eta, \zeta)\ \psi_1\, dxdydzd\xi d\eta d\zeta = \tag{35}$$
$$\iiiint\!\!\iint f(\xi, \eta, \zeta)\ F'(x, y, z)\ \psi_1\, d\xi d\eta d\zeta dxdydz$$

where

$$\bar{\psi}_1 = -\frac{1}{4\pi}\int_y^\infty \frac{\xi - x}{t^3}\, ds;\ t^2 = (\xi - x)^2 + (s - y)^2 + (\zeta - z)^2$$

Now, let F' be chosen as the integrable factor and be partially integrated with respect to z.

$$\iiiint\!\!\iint f(x, y, z)\ F'(\xi, \eta, \zeta)\ \psi_1 dxdydzd\xi d\eta d\zeta = \tag{36}$$
$$\iiiint\!\!\iint f(\xi, \eta, \zeta)\ F(x, y, z)\ \frac{d}{dz}\bar{\psi}_1 d\xi d\eta d\zeta dxdydz.$$

As in the previous cases, the second integral to be expected vanishes since f as well as F disappear at the limits of the integration. Next $\frac{d}{dz}\bar{\psi}_1, = -\frac{d}{d\xi}\bar{\psi}_2$ is chosen as the integrable factor and partially integrated with respect to x. By ψ_2, by analogy, is meant

$$\bar{\psi}_2 = -\frac{1}{4\pi}\int_y^\infty \frac{\zeta - z}{t^3}\, ds$$

$$\iiiint\!\!\iint f(x, y, z)\ F'(\xi, \eta, \zeta)\ \psi_1 dxdydzd\xi d\eta d\zeta = \tag{37}$$
$$\iiiint\!\!\iint f'(\xi, \eta, \zeta)\ F(x, y, z)\ \bar{\psi}_2 d\xi d\eta d\zeta dxdydz.$$

Now $\bar{\psi}_2$ may be transformed, the variable s in the defining equation being replaced by $\eta + y - s$. The result is that

$$\bar{\psi}_2 = \frac{1}{4\pi}\int_y^{-\infty} \frac{\xi - z}{t^3}\, ds;\ t^2 = (\xi - x)^2 + (\eta - s)^2 + (\zeta - z)^2.$$

It is seen that the integrand agrees with that of the defining integral ψ_2. Therefore, and since the right-hand side of (37) contains the same function under the double integral as the fourth turn in (22a), this fourth term can be combined with the transformed third member. This gives

$$\iiiint\!\!\iint f(x, y, z)\ F'(\xi, \eta, \zeta)\ \psi_1 dxdydzd\xi d\eta d\zeta + \tag{38}$$
$$\iiiint\!\!\iint F(x, y, z)\, f'(\xi, \eta, \zeta) \cdot \psi_2 dxdydzd\xi d\eta d\zeta =$$
$$\iiiint\!\!\iint F(x, y, z)\, f'(\xi, \eta, \zeta)\ (\psi_2 - \bar{\psi}_2)\ dxdydzd\xi d\eta d\zeta$$

where

$$\psi_2 - \bar{\psi}_2 = \frac{1}{4\pi}\int_{-\infty}^{+\infty} \frac{z - \xi}{t^3}\, ds;$$

$\psi_2 - \bar{\psi}_2$ and therefore the two sides of (38) are independent of y. This is therefore demonstrated for the whole right-hand side of (22a).

In general it can therefore be said:

The total resistance is always independent of the longitudinal coordinates of the lifting elements.

And further:

The most favorable distribution of the lift, with reference to the total drag, occurs when this is also the case for the projection of the lifting elements on a transverse plane.

That is to say, all of the lifting elements are projected on a plane perpendicular to the direction of flight, and any element so obtained has a lift equal to the sum of the lifts of all lifting elements projected onto it.

6. DETERMINATION OF THE SOLUTIONS.

The previous demonstrations show that the investigation for the distribution of lift which causes the least drag is reduced to the solution of the problem for systems of aerofoils which are situated in a plane perpendicular to the direction of flight. In addition, the condition for least drag (15c), which becomes the condition of uniform downwash (15) if the lift is vertical, leads to a problem which has often been investigated in the theory of two-dimensional flow with a logarithmic potential. The flow produced within the lifting transverse plane by the longitudinal vortices originating in it is, indeed, of this type. Each such vortex produces a distribution of velocity such as is produced by a two-dimensional vortex of half its intensity, and the whole distribution of velocity is obtained by adding the distributions produced by the longitudinal vortices. The potential flow sought is determined by the condition of (15c). Let it be combined with the flow of constant vertical upward motion $\omega = -\omega_0$. The resulting flow satisfies the condition at the boundaries

$$\omega \cos \beta + \mu \sin \beta = 0 \tag{39}$$

and there results, for the case of lifting lines:

The two dimensional potential flow is of the type that encircles the lifting lines, and at a great distance the velocity is directed upwards and has the value $w = -w_0$.

Within lifting surfaces the velocity is zero according to the condition (15b), and the fluid therefore flows around the contour.

The intensity of the longitudinal vortices at any point is twice the rotation of the two dimensional flow. In the case of the lifting lines, therefore, the density of the longitudinal vortices is double the discontinuity of velocity from one side to the other. The intensity of the transversal vortices is determined by integrating the longitudinal vortices along the aerofoils and therefore equals twice the difference of the velocity-integral produced on the two sides of the aerofoil. Now the integral of the velocity produced is identical with the potential and hence it appears:

The density of the lift perpendicular to the lifting line is proportional to the discontinuity of potential $\varphi_2 - \varphi_1$, and has the value

$$\sqrt{A'^2 + B'^2} = 2v_0\rho(\varphi_2 - \varphi_1) \tag{40}$$

Hence the total lift obtained by integrating over all aerofoils is

$$A = 2v_0\rho \int (\varphi_2 - \varphi_1) dx \tag{41}$$

Sometimes a transformation of this equation is useful. In order to obtain it, suppose that all of the lifting lines are divided into small parts. Then, on the two ends of each lifting element there begin two inverse longitudinal vortices, the effect of which on a distant point is that of a double vortex. Their velocity-potential φ and their stream function ψ may be combined in the complex function $\psi + i\varphi$, and, not considering the existence of a parallel flow, which is without any importance in the calculation, this complex function has the form for a lifting line,

$$d(\psi + i\varphi) = \frac{dA + idB}{z - z_0} \tag{42}$$

where z represents $x+iy$ and $z_0=x_0+iy_0$, x_0 and y_0 being the coordinates of the lifting elements of the line. For a lift distributed over areas a similar equation can be formed. The integration of (42) gives

$$\psi+i\varphi=\int\frac{dA+idB}{z-z_0} \tag{42a}$$

Now the residuum of the integrand at infinity is $dA+idB$ and therefore the residuum of the integral is $A+iB$. Therefore the expression can be written.

$$A=2v_0\rho R\,[\mathrm{Res}(\psi+i\varphi)] \tag{41a}$$

where the last part means the real part of the residuum of $\psi+i\varphi$ at infinity. In the most important case of horizontal aerofoils the residuum itself is real and can be used directly to calculate the lift. The density of drag at any point is proportional to the perpendical component of the density of lift and is $W'=\frac{w_0}{v_0}\cdot A'$, from which results $W=\frac{w_0}{v_0}\cdot A$. Making use of (41) one obtains

$$W=A^2\frac{1}{2v_0^2\rho}\frac{1}{\int\left(\frac{\varphi_2-\varphi_1}{w_0}\right)dx} \tag{43}$$

$$W=A^2\frac{1}{2v_0^2\rho}\frac{w_0}{R\,[\mathrm{Res}\,(\psi+i\varphi)]} \tag{43a}$$

The integral in the denominator of (43) represents an area characteristic of the system of aerofoils investigated. Frequently the easiest method of calculation is to assume from the beginning the velocity w_0 at infinity to be unity.

The case of the lift continuously distributed over single parts of areas is derived from the preceding one by passing to the limit. Since the vertical velocity w disappears at all points in the lifting surfaces, the velocity is zero at all points and the rotation vanishes.

Therefore, in the case of the most favorable distribution of lift, all of the longitudinal vortices from the continuously lifting areas begin at the boundaries of the areas.

Equations (43) and (43a) remain. The distribution of lift is indeterminate to a certain extent. On the other hand, it is possible to connect the points of the contour having the same potential φ by strips of any form, and it is only necessary that the lift be always perpendicular to the strip and its density have a constant value along the whole strip. According to equation (40) this equals the difference of the potential at the contour between the two borders of the strip. Worthy of note is the special case in which all of the strips run along the contour, thus coming again to the case of lifting lines. It appears that:

Closed lines have the same mimimum of drag as the enclosed areas when continuously loaded.

Especially important are those symmetrical contours which are cut by horizontal lines in only two points. With such the limitation to vertical lift does not involve an increase of the minimum drag. For this case it appears that:

The density of the vertical lift per unit area must be proportional to the vertical component of the velocity of the two-dimensional flow at the point of the contour of the same height z. It is

$$\frac{dA}{dF}=2v_0\rho\frac{d\varphi}{dz} \tag{44}$$

The corresponding density of drag is

$$\frac{dW}{dF}=2w_0\rho\frac{d\varphi}{dz} \tag{45}$$

7. EXAMPLES OF CALCULATIONS.

Examples of calculation of the previous demonstrations can be based on any calculated two-dimensional potential flow around parts of lines or areas. The simplest flow of the first kind is that around a single horizontal line. It leads to the problem investigated at the beginning of this paper.

In this case the potential is the real part of $\sqrt{p^2-1}$, where p denotes $x+iz$. The lifting line joins the two points $z=0$, $x=-1$ and $z=0$, $x=+1$, and has the length 2. The velocity at infinity is $w=1$. The discontinuity of potential along the lifting line is $\varphi_2-\varphi_1=2\sqrt{1-x^2}$. The density of lift is distributed according to the same law, therefore if plotted over the span the density of lift would be represented by the half of an ellipse.

The minimum drag is

$$W=A^2\frac{1}{v_0^2\rho/_2}\frac{1}{4\pi} \tag{46}$$

If, instead of the value 2, the span had the general value b, the minimum drag would be

$$W=A^2\cdot\frac{1}{v^2\rho/_2}\frac{1}{\pi b^2} \tag{47}$$

This same result has been obtained by Prof. Prandtl by another method.[3]

The simplest example for a lifting vertical area is the circle. Let its center coincide with the origin of the system of coordinates. Then the potential of the flow around this circle is

$$\varphi=\frac{z}{r^2}+z \tag{48}$$

where $r=\sqrt{x^2+z^2}$. At infinity $W_0=1$. Under the condition of and according to equation (40) the density of lift is

$$A'=2v_0\rho\frac{d}{dz}\left(\frac{z}{r^2}+z\right)_{r=1} \tag{49}$$

This results in a constant density of lift of $A'=2$. Therefore the drag is

$$W=A^2\cdot\frac{1}{2v_0^2\rho}\cdot\frac{1}{\iint 2dxdz}=A^2\frac{1}{v_0^2\rho/_2\cdot 8\pi} \tag{50}$$

The double integral is to be taken over the circle. If the general case for the diameter equal to D be considered, then the least drag is

$$W=A^2\cdot\frac{1}{v^2\rho/_2}\cdot\frac{1}{D^2 2\pi} \tag{51}$$

Hence in respect to the minimum drag the circle is equivalent to a lifting line having a length $\sqrt{2}$ times the diameter.

A lifting circular line would have the same minimum drag as the circular area.

This result was also obtained by Prof. Prandtl by another method.[4] A reduction of the original problem of variation to the two-dimensional flow sometimes enables a survey of the result to be made without calculation. For instance, let a third aerofoil be added between the two aerofoils of a biplane having a small gap. (The gap may be about one-sixth of the span.) Then, in order to find the most favorable distribution of lift, the double line about which the flow occurs is to be replaced by three lifting lines. Now, in the region of the middle lifting line the velocity is small, even before this line is introduced. Therefore the discontinuity of the potential along the middle line is very much smaller than that along the others. Hence it results that the middle aerofoil of a triplane should lift less than the other two.

[3] First communication concerning t in Zeitschrift für Flugtechnik und Motorl. 1914. S. 239, in a note by Betz.

[4] See Technische Berichte der Flugzeugmeisterei Bd. II Heft 3.

8. PROCEDURE FOR THE CASE OF FLUIDS WITH SMALL VISCOSITY.

The preceding results do not apply so much to the calculation of the most favorable distribution of lift as to the calculation of the least drag. For it appears, and the results are checked by calculation, that even considerable variations from the condition of most favorable distribution of lift do not increase the drag to any great extent. Usually the minimum drag can be considered as the real drag of the system of aerofoils and in order to allow for the effect of friction of the air it is sufficient to make an addition. This addition depends chiefly upon the aerofoil section; it also depends, omitting the Reynolds Number, only upon the area of the wings and on the dynamical pressure. It is independent of the dimensions of the system of wings themselves. It may be useful to have a name for that part of the density of drag, independent of the friction of the air, which results from the theory developed in this paper. It is called the "induced drag." Generally it is not the drag itself but an absolute coefficient which is considered. This coefficient is defined by

$$c_{wi}=\frac{W_i}{q\cdot F} \tag{52}$$

where W_i is the drag previously denoted by W, q is the dynamical pressure $v_0^2.\rho/2$, and F is the total area of the wings. Equation (43) can now be written

$$c_{wi}=\frac{c_a^2\cdot F}{\pi(k\cdot b)^2} \tag{53}$$

where c_a is the lift coefficient $\frac{A}{F\cdot q}$ corresponding to c_w. The greatest horizontal span b of the system of wings perpendicular to the direction of flight is arbitrarily chosen as a length characteristic of the proportions of the system, k is a factor characteristic of the system of aerofoils and has, according to the preceding, the value.

$$k=\sqrt{\frac{4}{\pi}\cdot\frac{1}{b^2}\int\frac{\varphi_2-\varphi_1}{w_0}dx} \tag{54}$$

It has a special physical significance.

Under the same conditions a single aerofoil with a span of k times the maximum span of a system of aerofoils has the same induced minimum resistance as the system.

9. REFINEMENT OF THE THEORY.

The demonstrations given rest on the assumption that the velocities produced by the vortices are small in comparison with the velocity of flight. The next assumption, more accurate, would be that only powers higher than the first power could be neglected.

In this case the solutions just found for lifting elements in a transverse plane can be considered as the first step towards the calculation of more exact solutions. The following steps must be taken: The exact density of drag is $W'=A'\frac{w}{v_0+v}$ where v is the horizontal velocity produced at the lifting elements by the transverse vortices. It can be calculated exactly enough from the first approximation. Now, the condition of least drag is

$$w\cdot\cos\beta+\mu\sin\beta=w_0\cos\beta\left(1+\frac{v}{v_0}\right) \tag{15d}$$

and the flow of potential, according to this condition at the boundary, is to be found. Compared with the first approximation the density below is in general somewhat increased and the density above is somewhat decreased. The minimum drag changes only by quantities of the second order.

If the lifting elements are distributed in three dimensions a similar refinement can easily be found. In this case there is to be taken into consideration a second factor which always comes in if the differences of the longitudinal coordinates of the lifting elements are considerable. The direction of the longitudinal vortices do not agree exactly with the direction of flight, but they coincide with the direction of the velocity of the fluid around the aerofoil. They are therefore somewhat inclined downwards. A better approximation is obtained by projecting the lifting elements not in the direction of flight but in a direction slightly inclined upwards from the rear to the front. This inclination is about $\frac{2w_0}{v_0}$. Except for this, the method of calculation remains unchanged.

REPORT No. 122

PRELIMINARY EXPERIMENTS TO DETERMINE SCALE AND SLIP-STREAM EFFECTS ON A 1/24TH SIZE MODEL OF A JN4H BIPLANE

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REPORT No. 122

PRELIMINARY EXPERIMENTS TO DETERMINE SCALE AND SLIP-STREAM EFFECTS ON A 1/24TH SIZE MODEL OF A JN4H BIPLANE.

By D. L. Bacon

SUMMARY.

The following report gives lift, drag, and longitudinal moment values obtained in tests of a particularly accurate model over a wide range of speeds. A measure of the slip stream corrections on lift and drag forces was obtained by the use of a power-driven model propeller.

The results are plotted, together with corresponding free flight data from Report No. 70, against the parameter *VL*.

Measurements were also made of forces and longitudinal moments for all angles from 0° to 360°.

INTRODUCTION.

This work was undertaken at the Langley Field Aerodynamic Laboratory of the National Advisory Committee for Aeronautics to obtain results on a small model of a complete airplane which might be used for comparison with corresponding tests made in full flight. Somewhat similar tests have been previously made at various other laboratories;[1] but, as certain discrepancies exist between corresponding tests in different tunnels, it has been deemed advisable to obtain a direct comparison for this particular installation.

The present work covers tests on a 1/24th scale model at speeds varying from 6.7 m./sec. (15 m. p. h.) to 40.2 m./sec. (90 m. p. h.). A slip stream correction has been obtained by the use of a small belt-driven propeller mounted in front of the model, and force coefficients thus obtained are compared with the measurements of the same forces made in full flight on a geometrically similar airplane.

EXPERIMENTAL METHODS.

These researches were carried out on an N. P. L. type balance[2] in the 5-foot wind tunnel at Langley Field. The model was built to the mean of actual measurements taken from the two full-size airplanes with which its characteristics are compared. The aluminum wings are everywhere within .002″ of the specified ordinates, the body and wheels within .010″, the engine compartment and engine are accurately represented and the radiator, made of a perforated brass plate, is so proportioned that its resistance per unit area is equal to that of a full-sized radiator. This fact was properly verified by a special test of the radiator alone. The air passing through the radiator circulates about the sump of the model engine and leaves the compartment in the same manner as with the full-sized airplane. The tail surfaces are cambered and the struts are to scale.

The truss and control wires have been omitted from the model, as have also the control horns, because the resistances of these parts can more readily be measured in full-sized tests of the individual members. This numerical addition of a separately determined drag has been fully justified by other experiments which show the interference effects to be negligible. The resistance of these omitted parts, amounting to .036 kg./m./sec. at full scale, or about 20 per cent of the total drag at low angles, has therefore been calculated by means of the usual coefficients,[3] and added to the values of drag as measured.

[1] Notably at the R. A. E. See A. C. A. R. & M. 656.
[2] For description see N. A. C. A. Report No. 72.
[3] U. S. Navy Aircraft Design Data.

As serious difficulty was encountered in previous experiments with deflection of the model and balance spindle, while testing at high speeds, a system of wire bracing was adopted which held the model with great rigidity in the desired position. Although this bracing proved highly satisfactory, the internal stresses in the model became so high at 90 m. p. h. (40.2 m./sec.) that it was thought incompatible with the safety of the model to increase the speed beyond this point.

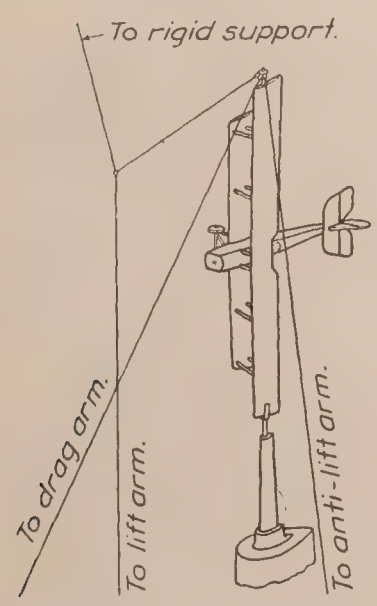

FIG 2.—Model set up for lift and drag runs.

The bracing used while measuring pitching moments is shown in Fig. 1. A mast is inserted in the upper extremity of the wing opposite to the holding spindle and in line with it. This mast is held in a bearing wired to the walls of the tunnel, thus permitting free rotation and ensuring that the Y axis of the airplane remains parallel to the axis of the balance.

While determining lift and drag forces a second method of wire bracing illustrated in Fig. 2 was employed. The mast was retained on the upper wing tip, and from it one wire was lead directly to the drag arm of the balance and another to the antilift arm. An equally simple method could not be used, however, on the lift arm because of interference with the lower wing. In place of this, wires leading from the model and from the lift arm were spliced to a ring which in turn was supported by a wire from the wall of the chamber. It is obvious that if the line of action of this latter wire passes through the pivot of the balance and if it lies in the plane of the lift arm the tension in the wire adds somewhat to the stability of the balance but does not otherwise affect its readings. In aligning these wires no painstaking measurements need be made of their various angles, the position of the external fastening need only be adjusted by trial until a change in the tension in the wire ttached to it is found to have no effect on the equilibrium of the balance.

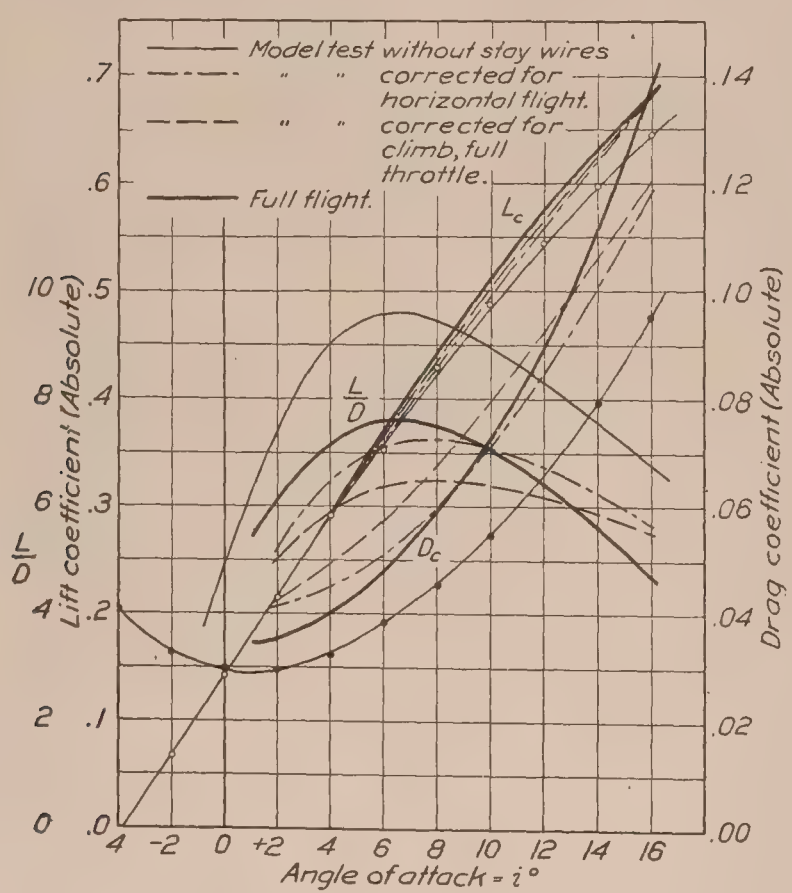

FIG. 4.—Results of model tests at VL=22.5 $\frac{m^2}{sec.}$ VL=16.5 M. P. H. × ft.

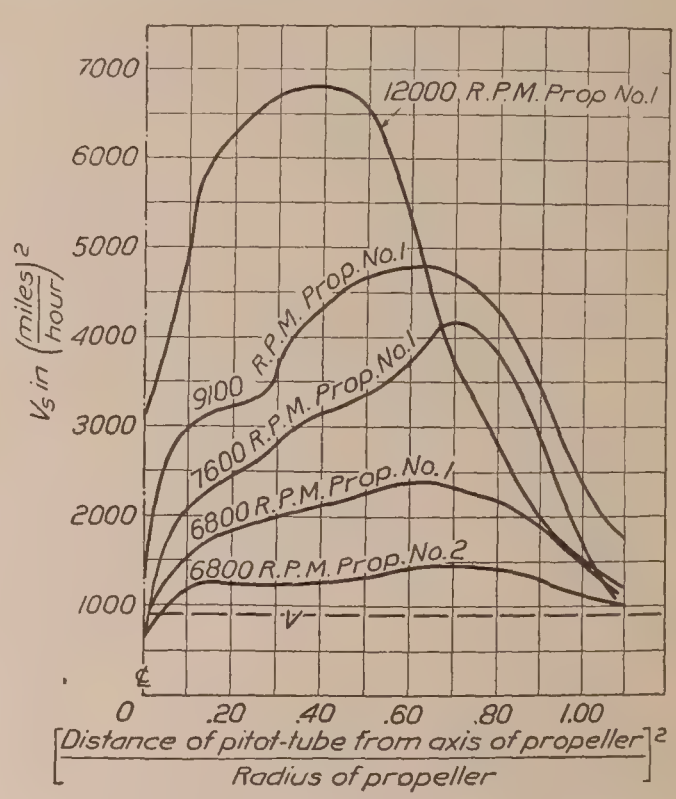

FIG. 5.—Exploration of slip stream of model propellers, section one diameter behind propeller.

The crude but satisfactory device used to reproduce slip stream effect upon the model consisted of a belt-driven ball-bearing spindle carrying a 4¼″ wooden propeller in its proper position relative to the model, as shown in Fig. 3. Because of the relatively low speeds at which the propeller was driven it was necessary to use a comparatively steep pitch and wide blade in

FIG. 1.—MODEL SET UP FOR MOMENT RUNS.

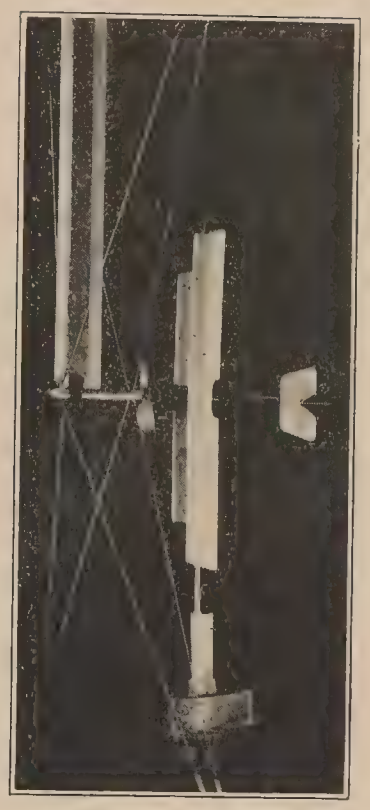

FIG. 3.—MODEL SET UP FOR SLIP STREAM CORRECTION.

order to obtain the desired slip stream velocities. A propeller geometrically similar to that used in full flight would require a rotational speed varying from 15,000 to 40,000 r. p. m., which was above the range available when these tests were made. The high pitch of the propeller used undoubtedly increased the rotation of the slip stream but this is of relatively small importance. Apparatus is now being developed for turning a model propeller at strictly proportionate speeds.

Two different propellers were used having different pitches. In order to determine the actual slip stream velocities (V_s) a complete traverse of the slip stream area was made, one diameter behind the blade, and the square root of the mean value of V_s^2 when plotted against the square of the distance from the propeller axis in terms of the radius, taken as the nominal slip stream velocity (see Fig. 5).

In order to determine the appropriate values of $\frac{V_s}{V}$ for all angles of flight, a similar exploration was made of the slip stream in actual flight, taking simultaneous readings of V, V_s, and r. p. m. of the propeller. This was done under two conditions of flight, namely, horizontal flight in which the throttle is wide open at maximum speed, gradually closed until the speed of minimum power is reached near $i=6°$, and then opened wide at minimum speed. The second series was made at wide throttle, beginning with high speed horizontal flight, then pulling the airplane up through its entire range of climbing angles and ending with minimum speed in horizontal flight. The two curves thus intersect at maximum and minimum flying speeds. The values of $\frac{V_s}{V}$ thus determined are plotted against both angle of attack and flying speed in Fig. 6. A few points computed on the basis of the momemtum theory are plotted for comparison.

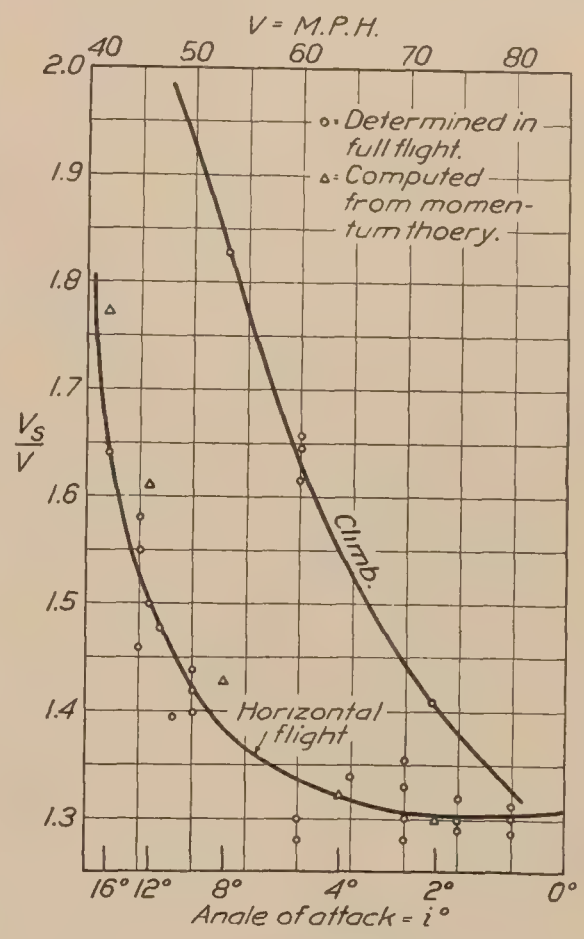

FIG. 6.—Slip stream velocity from free flight tests.

In measuring the lift and drag increments caused by the slip stream, a constant tunnel speed of 13.4 m./sec. (30 m. p. h.) was maintained and a number of different slip stream speeds produced by changing the rate of revolution or the pitch of the propeller. Data were thus obtained for a large range of $\frac{V_s}{V}$ ratios at each angle of attack. For the present report, however, only those force measurements for values of $\frac{V_s}{V}$ corresponding to horizontal flight and to climb with wide open throttle have been used. These are shown in Fig. 7, where the force increment due to the slip stream in per cent of the force on the model without slip stream is plotted against the angle of attack.

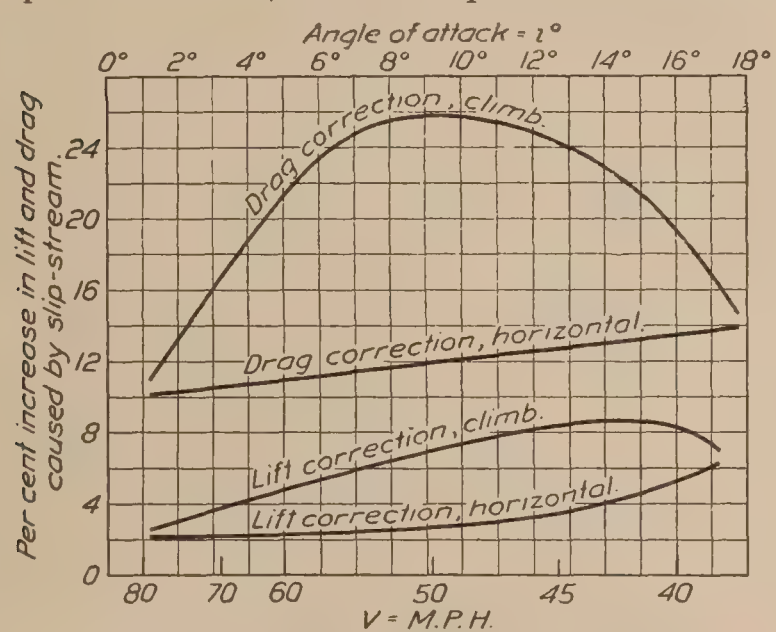

FIG. 7.—Increments caused by slip stream.

ACCURACY OF DATA.

Lift and drag measurements have a precision of approximately 1% except for drag values near the angle of maximum $\frac{L}{D}$, where the error in drag may be as great as 3%, due to inherent difficulties of measurement.

Air speed measurements during these tests were made by means of a side plate orifice previously calibrated against the N. P. L. standard pitot tube belonging to the Massachusetts Institute of Technology, assuming a constant of 1.00. The steadiness of flow was of the order of $\pm 1\%$ at all speeds.

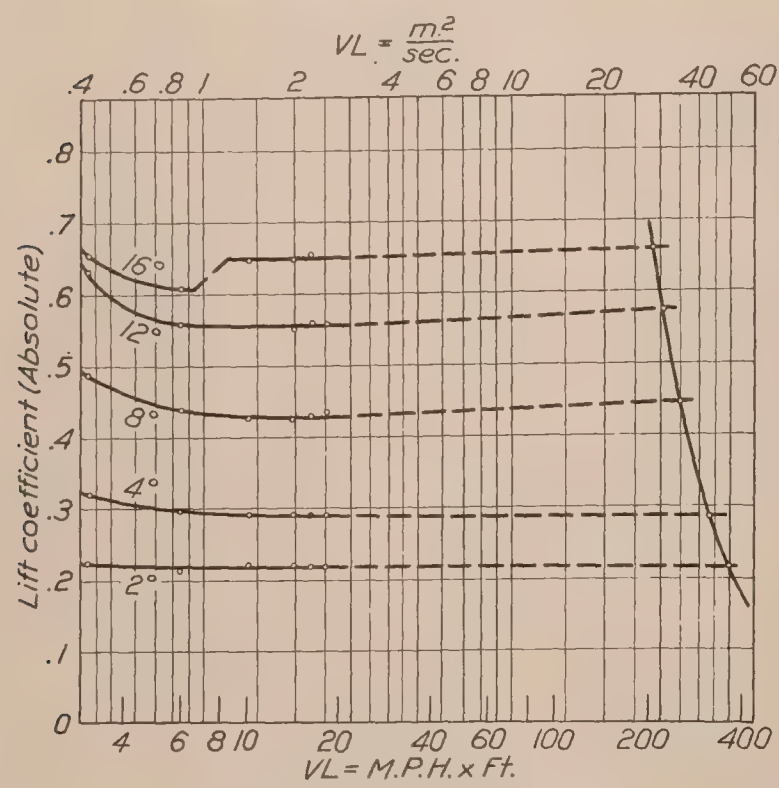

FIG. 8.—Change in L_c with change in VL.

Spindle drag and interference corrections were obtained by the method outlined in N. A. C. A. Technical Note No. 37. The additional drag of the bracing wires was determined by removing the model and supporting these wires from an extension spindle. No interference effects due to the wires could be detected.

In order to get an idea of the change in angle of attack through warping of the model at high air speeds, the upper end of the top wing was observed through a telescope. The warp was estimated at one-fourth of one degree.

The difficulty of accurately measuring either drag force or angle of attack during full flight tests, especially in the neighborhood of maximum and minimum flying speeds, is such that serious errors may be introduced. It is, therefore, possible that the values given in report No. 70 of lift on the full size airplane are no closer than 5% and that the corresponding drag values may be 20% in error.[4]

DISCUSSION OF RESULTS.

A representative set of curves for a single speed is shown in Fig. 4. The heavy lines shown on this chart are those of standard wind tunnel tests and do not include either slip stream or stay wire corrections. The corresponding dotted lines indicate the effect of adding calculated drag for parts not included on the model and the increase in both lift and drag forces due to the slip stream. Emphasis should be laid on the very considerable increase in drag when climbing, caused by the slip stream. The customary method of calculating the rate of climb by the formula:

$$\text{Rate of climb in ft./min.} = \frac{33{,}000 \times (\text{Reserve horsepower})}{\text{Total weight of airplane}}$$

where reserve horsepower is taken as the difference between the horsepower required for horizontal flight and the thrust horsepower available, usually gives a rate of climb considerably higher than that realized in actual flying tests. This is evidently due to the failure to consider the higher slip stream velocity obtained under climbing conditions.

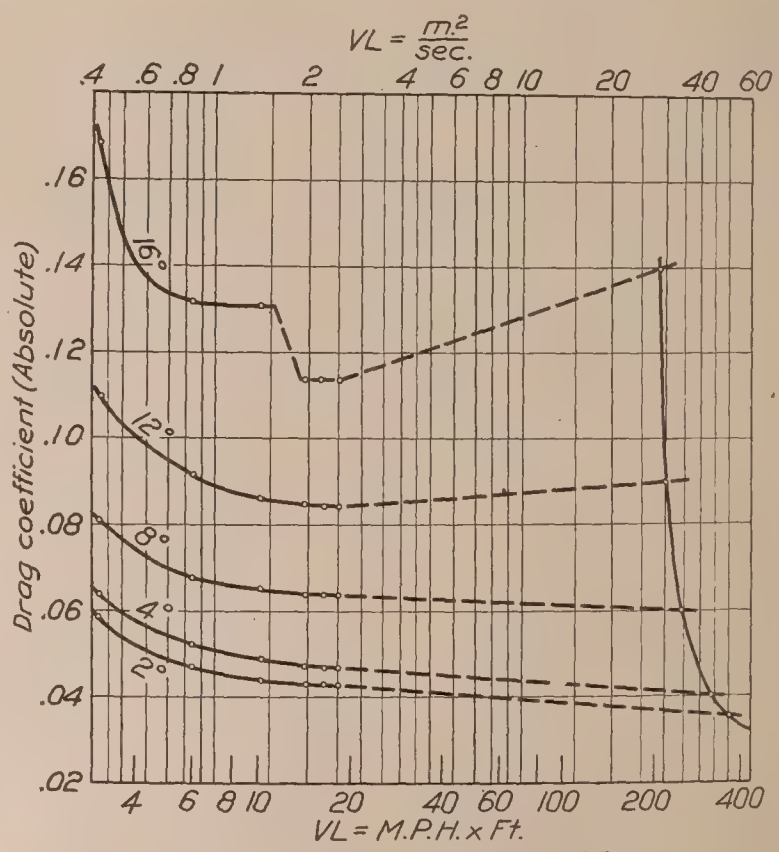

FIG. 9.—Change in D_c with change in VL.

The absolute lift and drag coefficients and the $\frac{L}{D}$ ratio are plotted against the scale of test in Figs. 8, 9, and 10, respectively. The scale VL is expressed in both metric and English units, V being the relative speed in meters per second or miles per hour and L the length of the wing

[4] Since these tests were made, much more accurate instruments have been devised which will soon provide comparatively accurate full scale data.

chord in meters or feet. The series of points on the right-hand side of the chart represent full flight conditions as obtained by averaging the measurements given for airplanes 1 and 2 of Report No. 70.

A critical region of flow will immediately be noticed at 16° for $VL = 1.4$ m.²/sec. At lower scale values for this angle of attack turbulent flow occurs with consequently reduced lift and increased drag. That either type of flow may exist at $VL = 1.4$ m.²/sec. is indicated by the fact that while measuring drag the break came at a somewhat higher speed and while measuring lift it occurred slightly below.

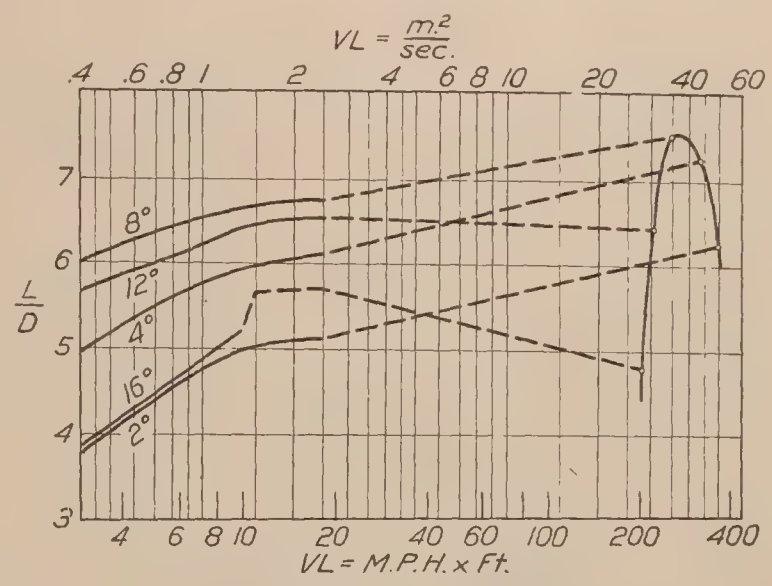

FIG. 10.—Change in L/D with change in VL.

It appears from these curves that both force coefficients decrease with increase of VL at low scale values, while at values of VL greater than 1.0 m.²/sec. the lift coefficient remains constant within the limits of experimental error. If the full scale drag measurements can be trusted the drag coefficients for small angles continue to decrease slowly with increase of scale.[5] All of the force measurements were made with the elevators set neutral. This procedure is justified by the curves given in Fig. 12, which show that in steady flight the elevators are never far enough from neutral to appreciably affect the total force on the machine except at the very highest angles of attack. In the neighborhood of 16° the elevator must be pulled up to its highest position, thus greatly adding to the resistance of the airplane. Unfortunately, no direct measurements of this increased drag were made under slip stream conditions, but those made without slip stream indicate this elevator drag increment to be in the neighborhood of 25% of the total drag at 16° angle of attack.

In order to determine whether a simple exponential equation could be derived for the drag coefficient in terms of VL the drag curves were replotted on full logarithmic coordinates. The resulting lines, however, were not straight and no further mathematical analysis was attempted.

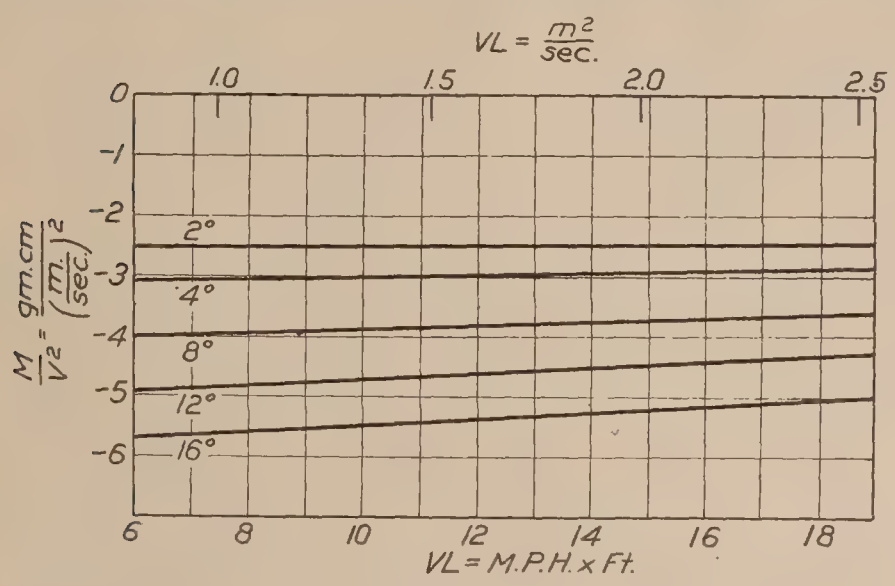

FIG. 11.—Pitching moment about holding spindle. (Stalling moment is positive.)

The lift being constant with respect to VL, $\frac{L}{D}$ becomes inversely proportional to the drag coefficient and for normal flying angles continues to improve at the highest speeds of the model tests. The effect of elevator drag corrections would be to make the 12-degree and 16-degree model tests at high VL agree closely with the full scale.

In Fig. 11, $\frac{M}{V^2}$ is plotted against VL for the range of model tests only, as no strictly comparable full flight data are at present available. The readings at $VL = 1.40$m.²/sec. do not lie on the curves and indicate either that the model was incorrectly aligned during this run or that some irregularity of air flow occurs at this speed, either as a peculiarity of the model or as an inherent characteristic of some part of the wind tunnel installation. With this exception the moment coefficients about the balance spindle decrease with great regularity as the speed of test increases.

The elevator angle required at 13.4 m./sec. (30 m. p. h.) to produce zero moment about the C. G. or to trim the model is plotted in Fig. 12 against the same angle determined during full

[5] W. Margoulis, Critical Review of Aeronautical Works. 6510 NAC/1.

flight. There may be some doubt regarding these latter values, as they were interpolated from a series of trimming angles at different engine speeds in order to get conditions of zero thrust which would simulate wind tunnel tests without slip-stream. The curves given show that it may be necessary to lower the elevator somewhat more than the model test would indicate while gliding at high speed.

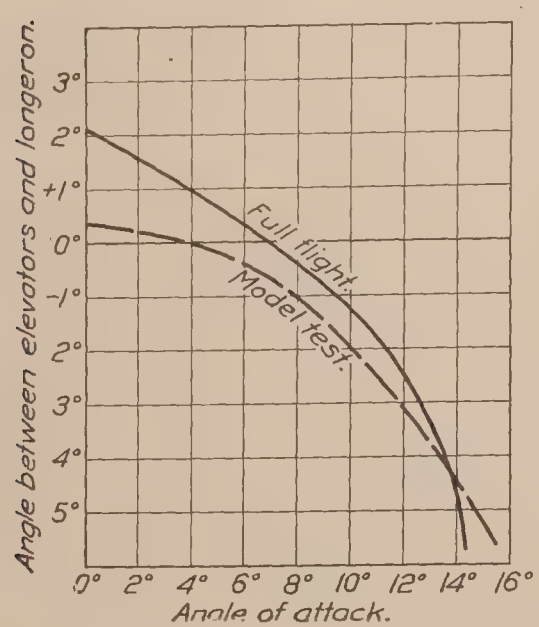

FIG. 12.—Tests without slip stream.

No attempts have yet been made to determine the effect of slip stream while measuring moments on models.

While doing the above work one run was made at 13.4 m./sec. (30 m. p. h.) in which the angle of attack was changed throughout a complete revolution from zero to 360°, in order to get some idea of the nature of the conditions encountered during stunting. Fig. 13 shows the corresponding changes in lift, drag, $\frac{L}{D}$, and moment about the C. G. for this test. Normal flight is confined between the limits of 0° and 18°, stalling and pancaking occurs roughly from 18° to 90°, tail sliding near 160° and sustained upside down flight near 320°. It will be noticed that the lift has a remarkably high peak at 170° and a corresponding $\frac{L}{D}$ of about 6, but the airplane is very unstable in this position, which anyone will acknowledge who has snapped out of a long tail slide.

The minimum gliding angle during a tail slide would be cot^{-1} 6.0 or 9½°, and that during an inverted glide cot^{-1} 2.7 or 20°. In this position the stability is almost neutral and it is evident that prolonged glides in this position are by no means difficult; in fact, they might be made unpleasantly easy by slightly increasing the angle of the tail plane.

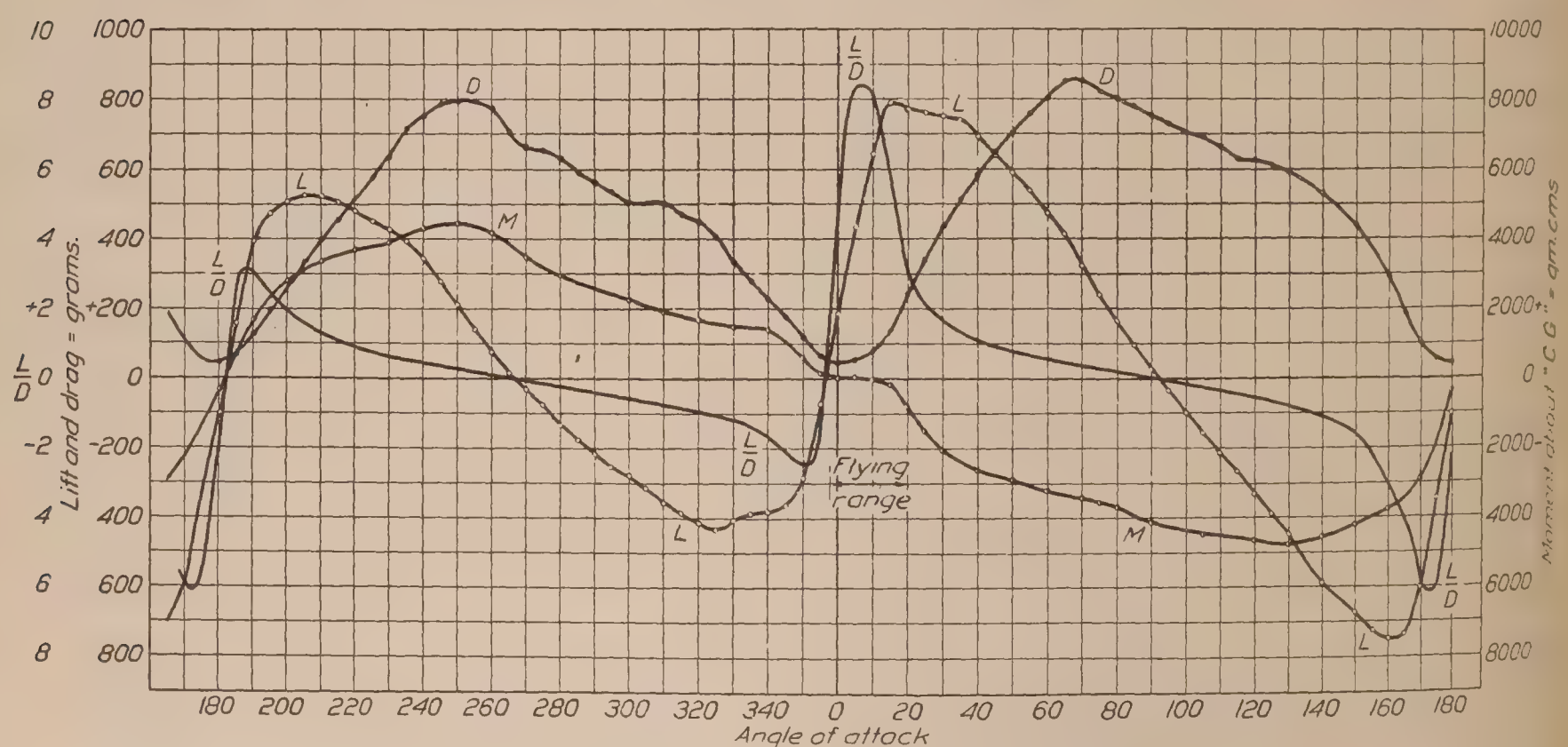

FIG. 13.—Forces and moments on complete JN4H model around 360°.

CONCLUSION.

It appears from the data here presented that by choosing model test conditions so that VL is greater than 1.200 m.²/sec. (8.8 m. p. h. x ft.) the lift of a full-size airplane of a type analogous to the JN 4H may be predicted within the usual accuracy of full-scale measurements.

The accuracy of the drag prediction can not be fully determined from these tests, as sufficiently precise free-flight data are lacking. It seems probable that minimum drag may be esti-

mated within 10 per cent, but that drag at high angles may be considerably in error unless proper corrections are made for elevator drag.

The additional labor of reproducing slip stream effects upon the model is undoubtedly justified when accurate results are required, as it eliminates the necessity of making several doubtful assumptions preparatory to the usual slip stream correction computations.

Preparations are now being made to increase the accuracy of full-flight drag determinations to a degree which will make them comparable with laboratory tests. Not until then will it be possible to say whether extreme exactitude in model making may be rewarded by proportionately accurate absolute values applicable to full scale airplanes, or even whether the relative order of merit of a series of small models will be maintained in actual practice.

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BIBLIOGRAPHY.

SIMILITUDE.

Thomson, G. P., "Applied Aerodynamics," p. 50–54.

Bairstow, L., "Applied Aerodynamics," p. 372–394.

Flugtechnik & Motorluftschiffahrt, 1919, Heft 9 & 10, by H. Kumbruch (translated into English by N. A. C. A. and issued as Technical Note No. 53).

W. Margoulis, Critical Review of Aeronautical Works, No. 1, November, 1919, p. 44 (N. A. C. A. 1115. 5–49A).

W. Margoulis, Practical Value of the Results of Model Tests in Present-Day Aerodynamical Laboratories (N. A. C. A. 5341–57).

British Advisory Committee for Aeronautics, R. & M. Nos. 72, 110, 148, 196, 362, 374, 415, 450, 656.

U. S. Navy Aircraft Design Data, vol. 1, note 9.

SLIP STREAM CORRECTIONS.

Slip Stream Corrections in Performance Computations, N. A. C. A. No. 71.

Slow-speed Airplane, by F. H. Norton, Aviation, September 1, 1920.

U. S. Navy Aircraft Design Data, vol. 1, note 10.

REPORT No. 123

SIMPLIFIED THEORY OF THE MAGNETO

By F. B. SILSBEE
Bureau of Standards

REPORT No. 123

SIMPLIFIED THEORY OF THE MAGNETO.

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RÉSUMÉ.

This paper contains part of the results of ignition investigations being made for the National Advisory Committee for Aeronautics at the Bureau of Standards, and describes a type of circuit which has been found useful for representing the action of the high-tension magneto. While this equivalent circuit is relatively simple, and consequently can be used as a basis for deriving definite mathematical formulas for induced voltages and similar quantities, it has been found experimentally to correspond quite closely in its performance with the highly complicated electrical circuits of an actual magneto. In the paper formulas are given for the voltage induced in the secondary under various conditions of operation, and a number of numerical examples are worked out showing the application of the equations to a variety of practical problems.

INTRODUCTION.

The high-tension magneto is an exceedingly complex piece of apparatus and even the most detailed mathematical treatments as yet attempted can take account of only the more important features of its operation and are necessarily based upon numerous limiting assumptions.[1] This fact, together with the lack of familiarity of many automotive engineers with electrical concepts, has contributed to produce a general air of mystery regarding the operation of ignition apparatus and has permitted the growth of numerous fallacies and the exploitation by misleading claims of devices for remedying ignition troubles. There is consequently need for a fuller understanding of the principal characteristics of magnetos by engineers and even by manufacturers of ignition apparatus.

A very rough approximation is, however, for most purposes all that is required, since the device is always used in connection with spark gaps which are themselves subject to very great variations in breakdown voltage from a number of conditions, only a few of which are at present thoroughly understood. It is therefore possible by suitable simplification and the elimination of unnecessary elements to develop a model electric circuit which will represent all the important features of operation of a magneto in a qualitative manner and do this with an accuracy which is sufficiently good for much quantitative work. Such an abstract theoretical circuit, provided it exhibits the same principal characteristics as the actual machine, will be referred to as a "model" of the magneto.

It is the purpose of the present paper to develop the theory of such a model which, though appearing at first glance very remote from the actual device, has been found to give a fairly close approximation to the actual working of the magneto; to show the application of this model through the entire cycle of operations; and to give a few examples showing how it may be used to answer the various questions which may arise in practice as to the performance of ignition apparatus under various conditions. This theory is the result of the investigation of ignition apparatus, sponsored by the National Advisory Committee for Aeronautics, which has been carried on at the Bureau of Standards for the past several years and has proved very useful in correlating the experimental results obtained in the course of the work.

[1] Young, A. P., Automobile Engineer 7, pp. 191, 237, 262, 298, 1917; Taylor-Jones, E., Phil. Mag. 36, p. 145, August, 1918; Silsbee, F. B., Scientific Paper of the Bureau of Standards, "The Mathematical Theory of the Induced Voltage in the High-Tension Magneto."

DEVELOPMENT OF MODEL.

The high-tension magneto serves two distinct functions. First, as an electric generator it transforms mechanical energy supplied to it by the driving shaft into stored electromagnetic energy of the primary current flowing in the inductive primary circuit. This action is performed by the primary coil alone and is similar to the behavior of any type of generator and hence need not be discussed in much detail. The second, and less understood function, is that of an induction coil which by a sudden effort delivers the stored energy at a voltage sufficiently high to break down the spark gap in the engine cylinder and produce a spark.

This latter action is analogous to that of a motor-truck driver endeavoring to take his vehicle up a curbing or to jerk it out of a crust of frozen mud in which it has stood over night, and involves two distinct actions. In the first place, by speeding up his engine and letting in the clutch suddenly, the driver makes use of the inertia of the flywheel to exert on the clutch shaft a much greater torque than the engine could deliver continuously. Secondly, the multiplying action of the transmission gears delivers at the driving axle a torque which has been still further increased in proportion to the gear ratio. In a similar manner, by establishing a considerable primary current (speed) and suddenly interrupting it at the breaker, there is induced in the primary circuit (clutch shaft) a voltage (torque) much larger than that originally generated by the rotation of the armature. This voltage is then in effect multiplied in the secondary winding (axle) by the ratio of turns between the two windings.

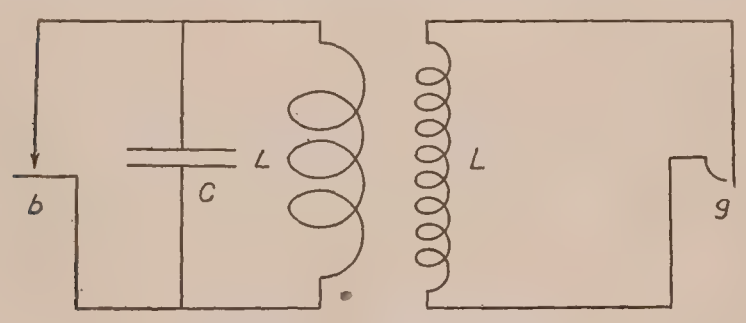

FIG. 1.—Circuits of actual magneto.

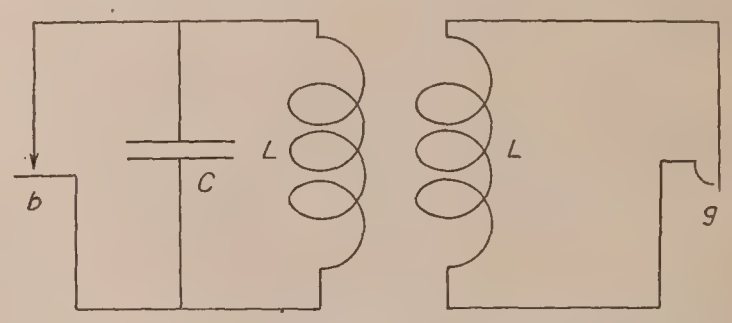

FIG. 2.—First intermediate type of circuit with secondary constants reduced to primary side.

Now the increase in voltage due to the great number of secondary turns is primarily a multiplication by the ratio (n) of secondary to primary turns, and we may consequently make the first simplifying step by replacing the circuits of figure 1 by those of figure 2 in which the secondary has been replaced with a fictitious winding having the same number of turns as the primary but occupying the same space as the actual secondary winding. Any values of voltage computed for this type of circuit may be accurately converted to those corresponding to the actual machine by multiplying by (n). It may be shown that the effect of the turn ratio upon current is the reverse of that upon voltage, so that the secondary current in the actual machine is $1/n$th of that in the model shown in figure 2. This relationship is of course parallel to the inverse relation between torque and speed in cases where energy is transmitted through gearing. Furthermore, in the electrical case, it may be shown that any resistance or inductance in the secondary circuit of figure 1 should be replaced in figure 2 by one having $1/n^2$ times this value, while any electrostatic capacity should be replaced by one of n^2 times the value. This transformation from the actual circuits of figure 1 to the fictitious circuit of figure 2 is widely used in electrical engineering problems involving transformers or telephone repeating coils. This transformation may be extended to any apparatus connected to the secondary circuit, as well as to the parts of the secondary winding itself, since a group of circuits whose constants have been modified as outlined above will obey the same electromagnetic laws (Ohm's law, for example) as did the unmodified circuit.

The next step in the simplification in figure 2 is to note that the two coils which now have the same number of turns are wound upon a common iron core and that by far the greater part of any magnetic flux which may be produced by the action of one coil must necessarily link the turns of the other coil. Experiments have shown that the coefficient of coupling k, which is

defined as the ratio of the mutual inductance between the two coils to the geometric mean of their respective self-inductances, is usually as great as 0.98 in the case of magnetos and 0.95 in the case of ignition coils in which the iron circuit is not as completely closed. In either case the amount of energy associated with the leakage flux which does not link both coils is very small, and we may therefore make the approximation that the coils are strictly closely coupled as is indicated by figure 3.

The next step is to replace the two closely coupled coils of figure 3, which, by our hypothesis, are identical in all their magnetic effects, by the single coil of the same number of turns and the

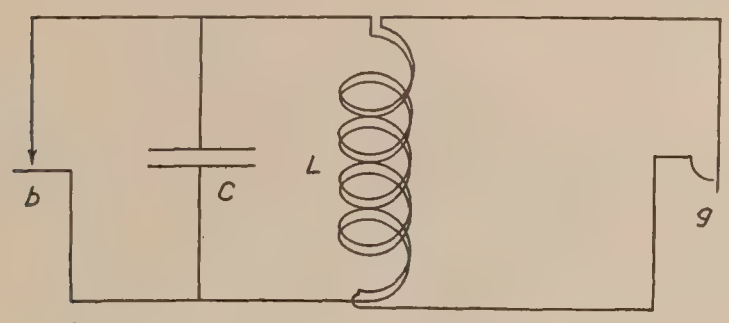

FIG. 3.—Second intermediate type of circuit with close magnetic coupling between primary and secondary.

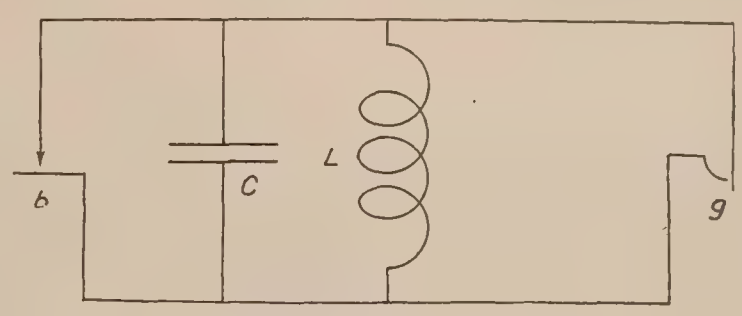

FIG. 4.—Third intermediate type of circuit with direct coupling between primary and secondary.

same self-inductance shown in figure 4. The only physical change which has been made in this step is that we no longer have the two coils separated by an insulating barrier, but since this insulation does not directly affect the calculation of voltages, currents, etc., it need not concern us here.

In any device for transforming energy there is, of course, always a certain amount of loss which, in the case of a magneto, is partly due to the resistance of the copper winding and in part to the hysteresis and eddy current loss in the iron core. We must, therefore, so arrange our model as to account for such losses. This can be done very easily by inserting the resistance R in series with the coil. (See fig. 5.) It should be noted, however, that when we are concerned with relatively slow changes in current and voltage, as for example during the building up of current by the rotation of the armature, only the copper resistances are of importance, while when very rapid changes are in progress, as during the building up of voltage immediately after the primary contact points have opened, the eddy current loss in the core produced by the rapid change of flux becomes much larger than the energy dissipation in the copper winding.

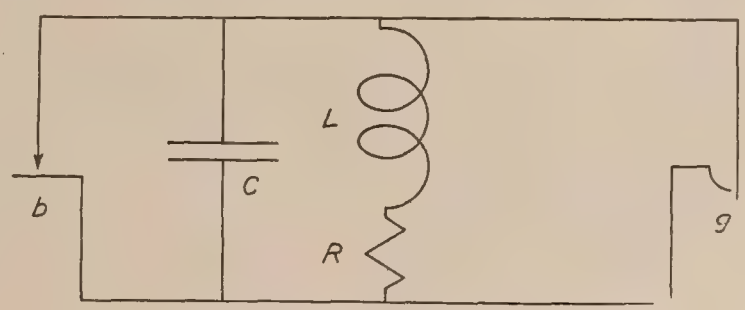

FIG. 5.—Complete single-coil model with series resistance.

Consequently, the numerical value to be assigned to the resistance R should be taken at different values according to the part of the cycle of operation under consideration.

A detailed discussion of methods for the determination of numerical values of the resistance in these cases and also of the inductance and capacity, if the model is to represent any given magneto, is beyond the scope of the present paper, as such methods have been treated elsewhere.[2]

In the paper referred to several lines of attack are suggested, the most direct of which is the measurement, by means of the Wheatstone bridge supplied with alternating current, of the inductance and resistance of the magneto winding at a frequency approximately the same as

[2] See the latter part of Scientific Paper of Bureau of Standards, "The Mathematical Theory of the Induced Voltage in the High-Tension Magneto." This paper also gives a more detailed discussion of the single-coil model and compares results obtained by it with those computed from other types of model and with the results of direct experiment.

that of the oscillations which would occur in the model. The proper value of the capacity to use is obtained by combining the easily measured value of the primary condenser with a value of the secondary capacity (multiplied of course by n^2) obtained by measurements made at radio frequencies. The ratio of turns of the magneto can readily be obtained by a null method, such as described in National Advisory Committee for Aeronautics Report No. 58, Part II. For most qualitative applications of the model, a precise knowledge of the values of the circuit constants is not necessary and, as will be indicated below, the equations which will be developed enable one to apply the model to numerous quantitative problems without requiring a complete knowledge of the constants.

FORMULAS AND APPLICATIONS OF MODELS.

During the first part of the cycle of operations of the magneto, when the rotation of the armature is building up current in the primary winding through the closed breaker contacts, the simplified model is of relatively little assistance. The wave form of the E. M. F. induced by the rotation is quite complex and depends entirely upon the shape of armature, pole pieces, and other parts of the magnetic circuit. Certain general considerations may, however, be drawn from the model which during this entire period (called "Period 1") may be considered as consisting merely of the portion shown in figure 6, since the breaker contacts form an effective short circuit upon the condenser and prevent it from affecting the operation.

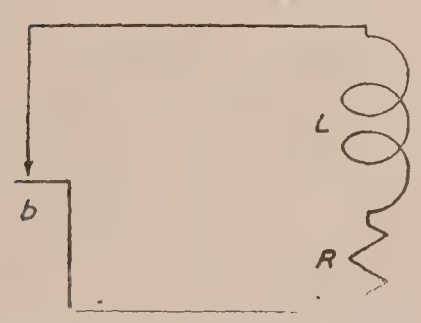

FIG. 6.—Effective circuit of model during Period 1 while current is being generated.

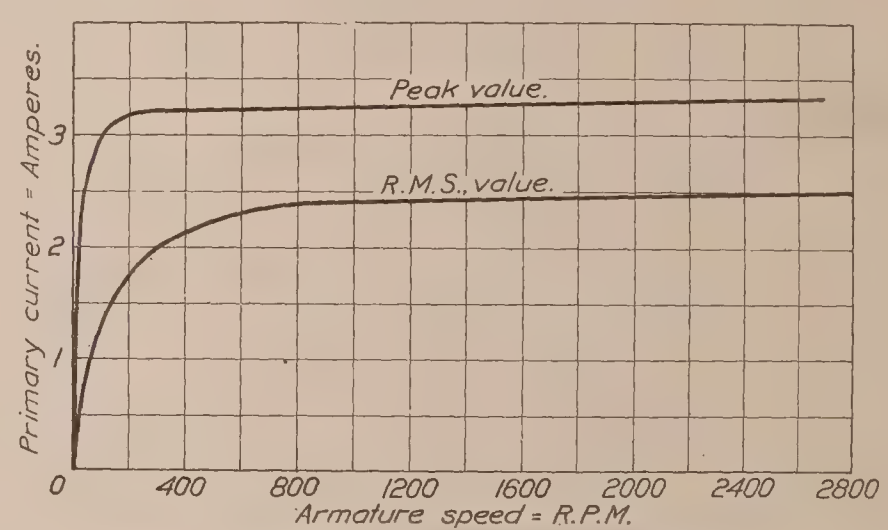

FIG. 7.—Data showing constancy of primary current of magneto after critical speed is attained.

The circuit as thus reduced consists of a resistance in series with an inductance supplied by an alternating E. M. F. This E. M. F. is proportional to the speed of the armature, the number of turns in the primary winding, and the total magnetic flux. If the voltage were sinusoidal, the current would consequently be given by the equation

$$I=\frac{E}{\sqrt{R^2+(2\pi fL)^2}}=\frac{K\,n_p f}{\sqrt{R^2+(2\pi fL)^2}} \tag{1}$$

where R is the resistance, n_p the number of turns, and L the inductance of the primary coil, and f is the frequency (or in a 2-pole machine the revolutions per second). If the speed of rotation is small, the term in the denominator involving the inductance is small relative to the resistance, and we note that the current will be proportional to the voltage and hence to the speed. At higher speeds, however, the inductance term becomes preponderant, and since it increases in proportion to the frequency, the current becomes independent of the frequency. As an example of this, figure 7 shows the effective (root-mean-square) current on short circuit and also the peak values of the same current observed in a certain magneto at several armature speeds. The effect of the inductance is greater upon any overtones which may be present in the current wave form, and consequently the wave shape changes considerably with frequency and the peak value of the current becomes independent of speed at a somewhat lower value than does the effective value. As a critical speed, above which the current at break and hence

the amount of energy available for the rest of the operation may be deemed substantially constant in value, we may take that defined by the equation

$$S \text{ (in r. p. m.)} = \frac{60}{2\pi} \frac{R \text{ (in ohms)}}{L \text{ (in henrys)}} \text{ approx.} \tag{2}$$

The energy which is stored in the magnetic field as a result of the current thus established is given by equation

$$W = \tfrac{1}{2} LI^2 \tag{3}$$

In some cases when the cam is set to open the contacts early in the rotation an additional amount of energy may be supplied to the spark by the rotation of the armature after the spark gap has broken down and provided a conducting path between its electrodes. This additional energy, however, does not contribute to the production of voltage to break down the gap except to the extent that the E. M. F. of rotation of the secondary winding in the magnetic field is added to the E. M. F. induced by the interruption of the primary current.

After the breaker contacts have been separated by the cam, we have an entirely distinct set of conditions which exists for a very short interval which may be called "Period 2," which lasts until the spark gap breaks down. Although this interval is very short, and there is usually but little transformation of energy during it, the phenomena which occur are extremely important

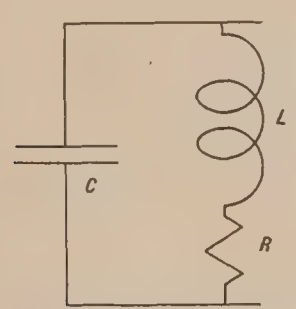

FIG. 8.—Effective circuit of model during Period 2 after breaker has opened and while voltage is building up.

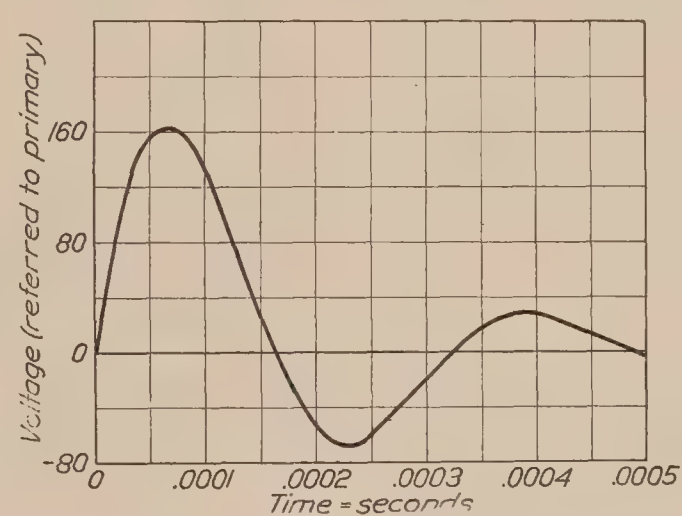

FIG. 9.—Voltage oscillation as computed for single coil model having L=.012 h; C=.215 μ f; R=127 ohms: assuming that the spark gap does not break down.

since they determine the amount of voltage which is produced in the secondary and, consequently, whether or not any spark at all is produced. The simplified model is developed especially to represent the magneto during this period and the portions of the model which are effective are shown in figure 8, in which it will be noted that the breaker circuit does not enter into the process. If such a circuit were left to itself undisturbed, the voltage across the condenser would rise to a maximum, at which time the current in the coil would have fallen to zero and would then decrease again oscillating thus until all the energy had been dissipated by the resistance. Figure 9 shows the variation of voltage with time in such a hypothetical case and is computed in a model where $L = 0.012\ h$, $C = 0.215 \mu f$, $R = 127\Omega$. The complete equation for the voltage in such a case is

$$v = \frac{I_0}{\beta C} e^{\alpha t} \sin \beta t \text{ ..... (volts)} \tag{4}$$

where

I_0 = current at break (amperes).

$$\alpha = \frac{-R}{2L} \text{ (seconds}^{-1}\text{).} \tag{5}$$

$$\beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ (seconds}^{-1}\text{).} \tag{6}$$

t = time (seconds).

e = Napierian base = 2.718

In most problems, however, we are interested merely in the value of the first maximum, since no spark will pass unless this maximum value is greater than the breakdown voltage of the gap at the spark plug. The value of this maximum is given by the equation

$$V_m = I_0\sqrt{\frac{L}{C}} \times F \text{ ---------- (volts)} \quad (7)$$

TABLE I.—*Damping factor F (for use in equation 7).*

$$F = e^{\frac{\alpha}{\beta}\tan^{-1}\frac{-\beta}{\alpha}}$$

$\frac{\alpha}{\beta}$	F
−0.0	1.000
.1	.863
.2	.760
.3	.681
.4	.621
.5	.575
.6	.539
.7	.511
.8	.488
.9	.470
1.0	.456

TABLE II.—*Damping factor F′ (for use in equation 12).*

$$F' = \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\frac{\alpha_1}{\alpha_2}}{1-\frac{\alpha_1}{\alpha_2}}}$$

$\frac{\alpha_1}{\alpha_2}$	F
0.0	1.000
.1	.774
.2	.669
.3	.597
.4	.543
.5	.500
.6	.465
.7	.435
.8	.410
.9	.387
1.0	.368

The factor F in equation (7) is an abbreviation for the expression

$$F = e^{\frac{\alpha}{\beta}\tan^{-1}\frac{-\beta}{\alpha}} \quad (8)$$

Table (I) gives values of this factor for various values of the ratio $\frac{\alpha}{\beta}$

In certain cases where the rate of dissipation of energy is very great, the voltage wave no longer oscillates, but having risen to a maximum subsides gradually to zero. Under these conditions, the equations (4) and (7) no longer apply, and the equation expressing the voltage as a function of time becomes

$$v = \frac{I_0}{C(\alpha_1 - \alpha_2)}\left[e^{\alpha_1 t} - e^{\alpha_2 t}\right] \text{ ---------- (volts)} \quad (9)$$

where

$$\alpha_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \text{ (seconds}^{-1}\text{)} \quad (10)$$

$$\alpha_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \text{ (seconds}^{-1}\text{)} \quad (11)$$

While the maximum voltage of the impulse is given by equation

$$V_m = \frac{-I_0}{C\alpha_2} \times F' \text{ ..... (volts)} \quad (12)$$

The factor F' in this equation is analogous to the factor F in the preceding case and is an abbreviation for the expression

$$F' = \left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\alpha_1}{\alpha_2}\,1-\frac{\alpha_1}{\alpha_2}} \quad (13)$$

Table (II) gives values of this factor for various values of the ratio $\frac{\alpha_1}{\alpha_2}$

In normal operation the circuits are usually such as to give the oscillatory case first considered, and the factor F has approximately the value 0.75, although this depends to a considerable extent upon the construction, in particular upon the amount of lamination of the magnetic circuit.

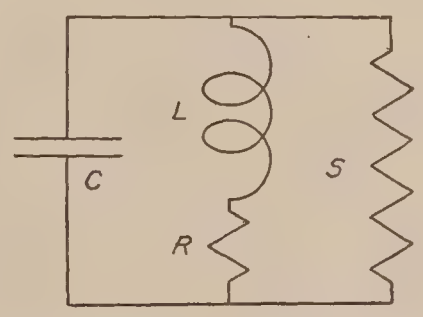

FIG. 10.—Effective circuit of model during Period 2 if spark plug is fouled with carbon.

In case the magneto secondary is shunted by a resistance, as is the case when a spark plug is fouled with carbon deposits, we have the somewhat more complicated circuit shown in figure 10. This can, however, be simplified to the circuit shown in figure 8 by making the following substitutions:

$$L' = L, \text{ (henrys) } C' = C \times \frac{S}{R+S} \text{ (farads)}$$
$$R' = R + \frac{L}{SC} \text{ (ohms) } I_0' = I_0 \text{ (amperes).} \quad (14)$$

In most cases this substitution has relatively little effect upon the value of capacity, but the term $\frac{L}{SC}$ in equation (14) which results from a moderately low value of the shunting resistance S in the circuit of figure 10 is so great that the original resistance R in the unshunted combination contributes practically a negligible amount to the damping. Under these circumstances, the maximum voltage is given by the equation

$$V_m' = \frac{I_0\sqrt{LC'}}{C} F \text{ ..... (volts)} \quad (15)$$

or in case the oscillation is overdamped, by

$$V_m' = \frac{I_0'}{C\alpha_2'} F' \text{ ..... (volts)} \quad (16)$$

in which should be noted that the value of C and not C' is to be used in the denominator. The factor F or F', however, is the same as occurs in equation (8 or 13) above, provided the primed values obtained by the substitutions (14) are used.

It may be noted that the presence of a shunt, as shown in figure 10, gives rise to effects very similar to those due to eddy currents in the iron core, and the substitutions defined by equation (14) indicate the reason for the effect of these currents in increasing the effective series resistance of the circuit of figure 8.

When the shunting is very heavy, as is usually the case when the magneto is loaded nearly to the point of misfiring, equation (16) reduces to the much more simple form

$$V_m = I_o S \tag{17}$$

The physical reason for this may be seen by the fact that if the full current I_o should flow through the shunting resistance S, it would maintain the initial magnetism in the coil unchanged. With such a constant flux no voltage would be induced to maintain the current flow. Consequently this initial value constitutes an upper limit to the current which will flow through such a shunting resistance and will, of course, produce at the terminals of the resistance the voltage given by equation (17).

While the oscillations and maximum voltages indicated by the preceding equations would be produced if the circuits in figure 8 (or 10) were left undisturbed, in the normal operation of an ignition system the breakdown voltage of the spark plugs is normally adjusted to a value much smaller than the maximum attainable by the system so as to insure firing by a factor of safety of 4 or 5. As soon as the spark gap breaks down, it provides a conducting path in parallel

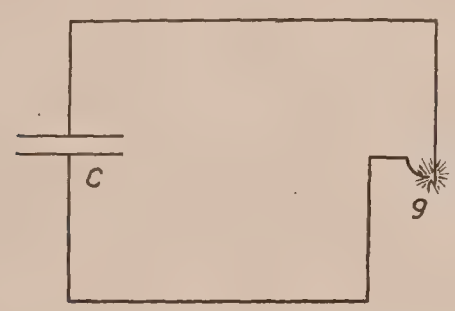

FIG. 11.—Effective circuit of model during Period 3 when spark gap has just broken down.

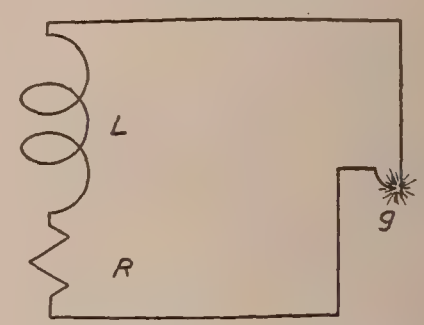

FIG. 12.—Effective circuit of model during Period 4 after the capacities have discharged but while the coil discharge maintains the spark.

with the model and completely changes the electrical system so that from this instant on we have to deal with the circuit shown in figure 11. The principal action which first occurs (and which may be called "Period 3") is the discharge of the condenser through the spark gap, and because of the very small resistance and inductance of the leads and of the spark itself, the resulting current may attain very considerable values for a short interval of time; consequently the slight contribution to the total current from the inductance itself is negligible and the coil has been omitted from figure 11.

In the actual magneto, the slight amount of leakage flux which is present because the two windings are not closely coupled becomes of some importance in the very rapid discharge which takes place during this Period 3, and consequently the first rush of current comes only from that part of the condenser due to the secondary leads and winding. The contribution from the primary condenser follows, however, very shortly afterward, and the entire phenomenon may be considered as a unit. The energy delivered to the spark gap during this period is given by

$$W = \tfrac{1}{2} CV^2 \tag{18}$$

where V is the voltage at which the gap broke down. This energy is usually only a small fraction of the original magnetic energy stored in the coil, but is probably sufficient to produce ignition of the gas mixture in the engine cylinder.

After the condensers have discharged we are left with the spark gap still in a conducting condition, and consequently have the circuit shown in figure 12 during what may be called "Period 4." Such a coil left to itself would tend to send a current nearly equal to that originally flowing through the breaker circuit, since such a current would be sufficient to maintain the

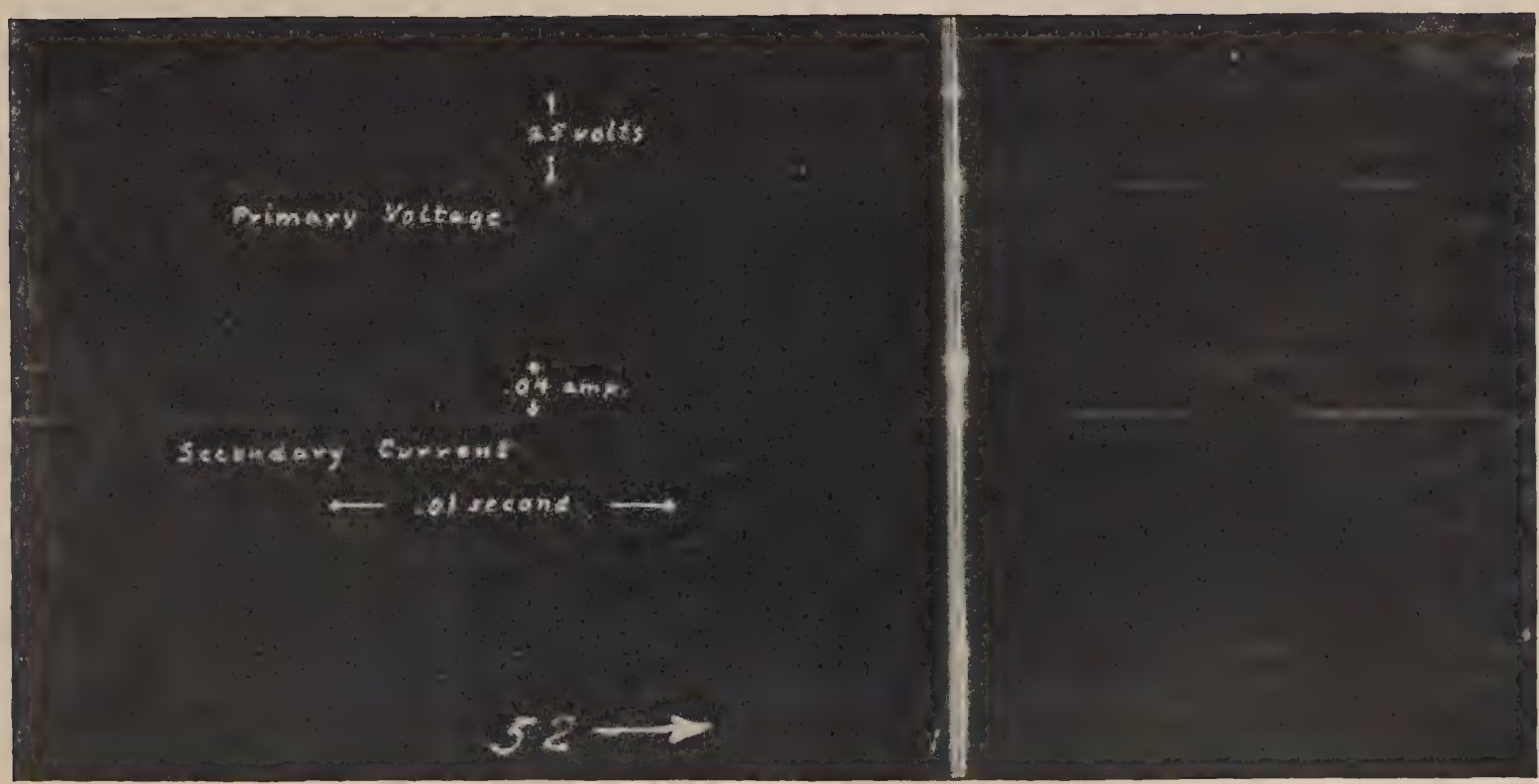

FIG. 13.—OSCILLOGRAM OF SLEEVE TYPE MAGNETO SHOWING IN UPPER RECORD THE NEARLY CONSTANT VOLTAGE SUSTAINING THE SPARK, AND THE LOWER RECORD THE NEARLY LINEAR DECREASE OF CURRENT WITH TIME DURING PERIOD 4.

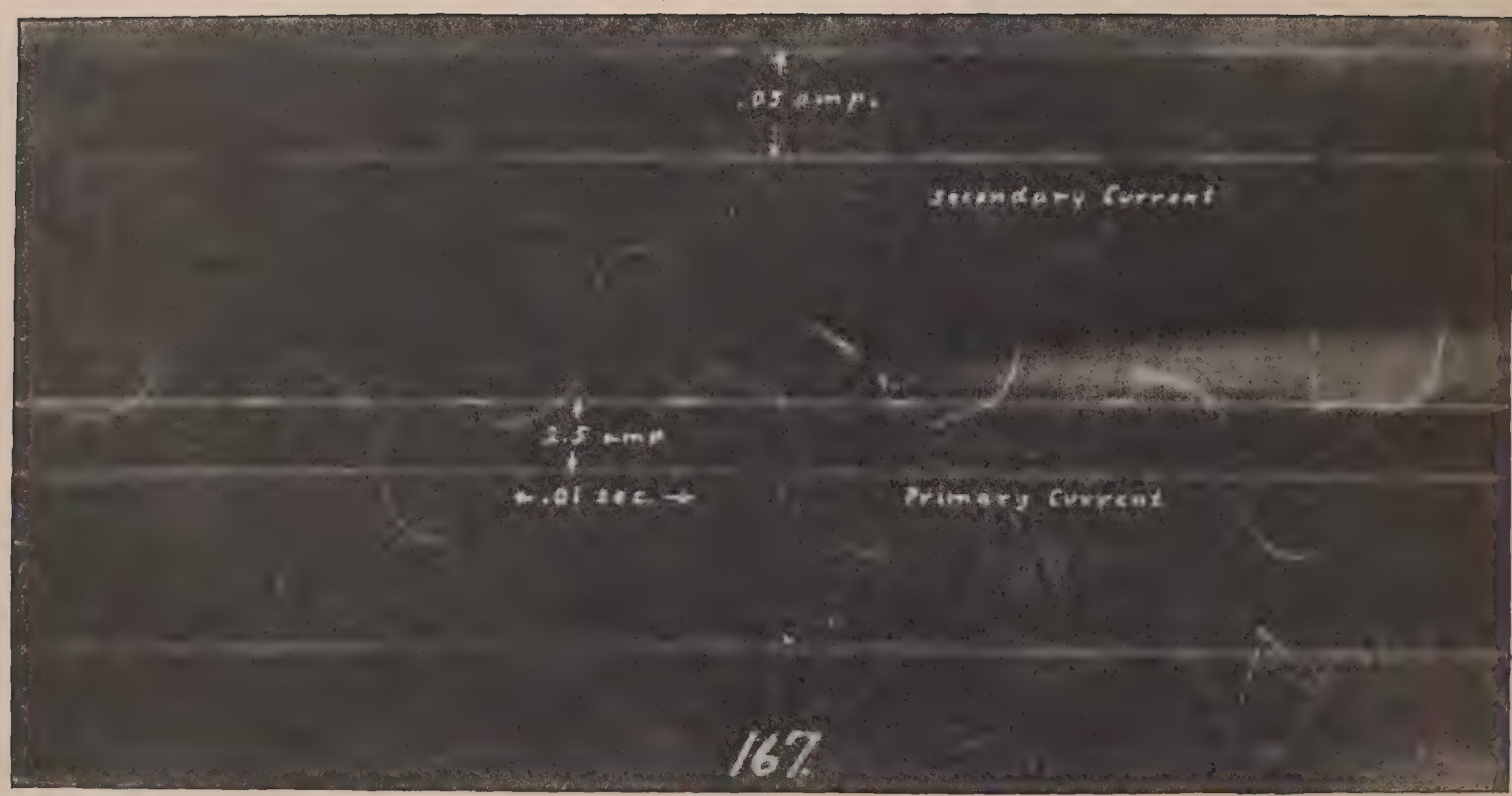

FIG. 15.—OSCILLOGRAMS OF SECONDARY (UPPER) AND PRIMARY (LOWER) CURRENT OF SHUTTLE TYPE MAGNETO. NOTE THE CORRESPONDENCE BETWEEN THE DISTURBANCES IN THE PRIMARY CURRENT CAUSED BY THE MECHANICAL "CHATTERING" OF THE BREAKER AND THE CORRESPONDING FLUCTUATIONS IN THE SPARK CURRENT. PLOTTED TO THE SAME SCALE OF AMPERE TURNS THE PEAKS OF ONE CURVE FIT INTO THE HOLLOWS OF THE OTHER.

magnetic flux undiminished. The actual current, therefore, closely approximates that which was present at the end of Period 2 and is roughly the same as that existing at the end of Period 1. It is a curious property of the type of spark gap used in engine cylinders that they require to maintain the current across them a voltage which is substantially constant during the duration of the spark. The value of this voltage depends upon the length of the gap and other conditions, and is of the order of magnitude of 1,000 volts. Since the flux is changing but slowly during the period now being considered, the effective resistance of the model is small and the IR drop in the winding is relatively slight. We consequently have roughly a constant rate of fall of current which thus maintains a constant voltage $\left(L\frac{di}{dt}\right)$ across the gap. This is illustrated by figure 13, which shows oscillograms of the secondary current through the spark gap and of the voltage across the primary coil terminals during the passage of the spark. Unless interrupted by the early closing of the breaker, this discharge of the coil continues until the current has dropped off uniformly to zero and the total energy delivered to the spark gap is given by

$$W=(EI)_{\text{ave}}t \tag{19}$$

This contribution is usually the bulk of the original magnetic energy stored in the coil, but in case the magneto has been working near its limit, so that the breakdown voltage of the spark gap was nearly equal to V_m, the energy will be definitely less than the original supply by the amount of electrostatic energy contributed from the condensers during Period 3.

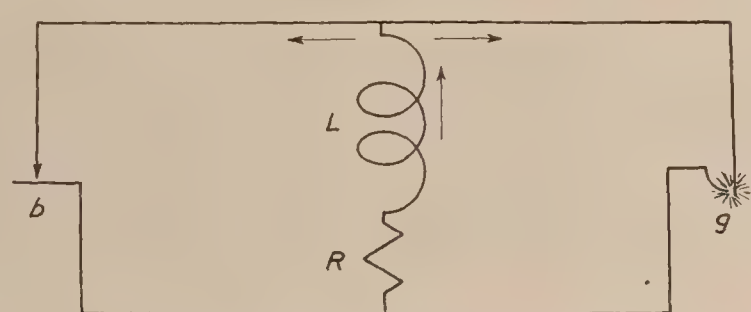

FIG. 14.—Effective circuit of model while spark is being extinguished by the closing of the breaker. (Period 5.)

In case the cam allows the primary breaker to close before the current through the spark gap has entirely died out, we get the condition shown in figure 14. On account of the changes in ionization it happens that, in the case of conduction across spark gaps, the resistance increases as the current flow through the gap decreases. Consequently, when the current begins to divide between the two parallel paths (b and g) shown in the figure, the resistance of path g rises as the current through it diminishes, with the result that the entire current is rapidly shifted to the circuit through the breaker, and the spark is extinguished. If for any reason the contacts bounce open again, as may occur at high speeds of operation, the current flow is again shifted to the spark gap, which is usually sufficiently ionized to remain conducting. Figure 15 shows this transfer of the current from primary to secondary, and vice versa, the upper curve in the record being that of the secondary current to a scale of 0.004 ampere per millimeter, while the lower curve gives the primary current to a scale of 0.3 ampere per millimeter. Since the ratio of turns in this particular magneto was 57:1, it will be seen that the ampere turns, and hence the current in the coil of the equivalent model, remains substantially constant in spite of the shift in the external path of the current.

EXAMPLES.

The following numerical examples may help to render the preceding discussion more definite and concrete and to illustrate the extent to which the model may be applied even in cases where a complete knowledge of the circuit constants is not available. In several cases the examples have been chosen to fit particular magnetos for which the answers have been verified experimentally.

EXAMPLE 1.

A certain magneto will barely fire a 6,100-volt spark gap when a current of 1 ampere passing through its primary is interrupted. If a condenser of 300 micro-microfarads capacity is connected to its secondary, it will then, with the same primary current, barely fire a 4,100-volt gap. What is its original effective capacity?

Let C_o be the total original capacity (referred for convenience to the secondary side). Then the total energy available under the first condition is

$$W=\frac{(6,100)^2}{2}\cdot C_o$$

The energy in the second condition is

$$W=\frac{(4,100)^2}{2}\cdot(C_o+300\times10^{-12})$$

Since the same primary current was used in both cases, the initial energy is the same in both, and we may equate these two expressions, getting the relation

$$\frac{(6,100)^2}{2}C_o=\frac{(4,100)^2}{2}(C_o+300\times10^{-12})$$

which may be solved to give

$$C_o=246\times10^{-12}\text{ farads.}$$

EXAMPLE 2.

In the magneto of example 1, assuming that the damping factor F is 0.75 and that the ratio of turns is 60, what is the effective inductance and resistance?

The current at break referred to the secondary is $\frac{1}{60}$ amperes and by equation (7) we have

$$6,100=\frac{1}{60}\sqrt{\frac{L}{246.10^{-12}}}\cdot 0.75$$

whence

$$L=59\text{ henrys.}$$

Now a factor F of 0.75 corresponds to $\frac{\alpha}{\beta}=-0.2$ approximately and

$$\beta=\frac{1}{\sqrt{LC}}\text{approx.}=\frac{1}{\sqrt{59\times246\ .10^{-12}}}=8,330.$$

The corresponding frequency is

$$f=\frac{8,330}{2\pi}=1,330$$

and

$$\alpha=-0.2\times8,330=-1,670.$$

hence

$$R=-2\ L\ \alpha\text{ by equation (5)}$$

$$=2\times59\times1,670=197,000\text{ ohms.}$$

Referred to the primary side, the various quantities would be

Current at break			$=1.0$ amperes.
Crest voltage	$=$	$\frac{6,100}{60}$	$=102$ volts.
Resistance (for period 2)	$=$	$\frac{197,000}{(60)^2}$	$=54.7$ ohms.
Inductance	$=$	$\frac{59}{(60)^2}$	$=.0164$ henrys.
Capacity	$=246\times10^{-12}\times(60)^2=.886\times10^{-6}$ farad.		

If the primary condenser has 0.25 microfarad capacity what is the secondary capacity?

Secondary capacity (considering the equations of the model to be referring to primary quantities) is equal to

$$0.886\times10^{-6}-0.25\times10^{-6}=0.636\times10^{-6} \text{ farads.}$$

Referring this to the secondary again gives

$$\frac{0.636\times10^{-6}}{(60)^2}=177\times10^{-12} \text{ farads.}$$

EXAMPLE 3.

If the magneto referred to above is used with leads 2 meters long of a type of high voltage cable which has an electrostatic capacity of 300 micro-microfarads per meter, by how much is the crest voltage reduced?

As in example 1 we may equate the electrostatic energies in the two cases and write

$$\frac{1}{2}\cdot(6{,}100)^2\times246\times10^{-12}=\frac{1}{2}(V_m)^2\times(246\times10^{-12}+2\times300\times10^{-12})$$

which gives $V_m=3{,}300$, or a decrease of

$$\frac{6{,}100-3{,}300}{6{,}100}=46 \text{ per cent.}$$

(NOTE.—The decrease observed experimentally on the magneto which gave the values taken in example 1 was 46 per cent when 600 $\mu\mu$ were added.)

EXAMPLE 4.

If an accumulation of dirt in the distributor amounts to a shunting resistance of 500,000 ohms from the secondary terminal to ground, by how much is the crest voltage reduced?

We may refer all quantities to the secondary and then make the substitutions indicated by equation (14) obtaining

$$L'=L=59 \text{ henrys}$$

$$S=500{,}000 \text{ ohms}$$

$$R'=R+\frac{L}{SC}=197{,}000+\frac{59}{500{,}000\times246\times10^{-12}}=677{,}000 \text{ ohms}$$

$$C'=\frac{CS}{R+S}=\frac{246\times10^{-12}\times500{,}000}{197{,}000+500{,}000}=176\times10^{-12}$$

Inserting these in equations (5 and 6) gives

$$\alpha'=\frac{-R'}{2L'}=-5{,}740$$

$$\beta'=\sqrt{96.\times10^6-33.\times10^6}=7{,}940$$

since α'^2 is less than $\frac{1}{LC'}$ the complete wave form is oscillatory and equation (7) applies.

$$\frac{\alpha'}{\beta'}=-0.723, \text{ hence } F=.506 \text{ by Table (I)}$$

and by equation (15)

$$V_m=\frac{I_o\sqrt{59\times176\times10^{-12}}}{246\times10^{-12}}\times0.506=210{,}000\ I_o$$

For 1 ampere in the primary, I_o referred to the secondary is $\frac{1}{60}$ and hence

$$V_m = \frac{210,000}{60} = 3,500 \text{ volts}$$

Decrease resulting from shunting is

$$\frac{6,100 - 3,500}{6,100} = 43 \text{ per cent.}$$

(NOTE.—The decrease observed experimentally on this magneto was 48 per cent.)

EXAMPLE 5.

If this magneto is connected to a spark plug which has the gap set to breakdown at 2,000 volts, what is the lowest value (S) of the resistance of a carbon deposit on the spark-plug insulator at which the magneto will fire the plug, if its primary current at break is 4 amperes?

For this case the approximate equation (17) is applicable. The current at break referred to the secondary is $\frac{4}{60} = 0.067$ ampere, and we therefore can write

$$2,000 = S \times 0.067$$

whence

$$S = 30,000 \text{ ohms.}$$

EXAMPLE 6.

If the primary condenser could be reduced to 0.15 microfarad without causing excessive arcing, what would be the gain in crest voltage?

The present capacity referred to the primary is (see example 2) 0.886 microfarad.

The present crest voltage referred to primary is 102 volts.

New capacity will be 0.786 microfarad.

Since total energy is the same in the two cases, we have

$$\frac{1}{2}(102)^2 \times 0.886 \times 10^{-6} = \frac{1}{2}(V_m)^2 \times .786 \times 10^{-6}$$

whence

$$V_m = 108$$

$$\text{the gain} = \frac{108 - 102}{102.} = 5.9 \text{ per cent.}$$

(NOTE.—The gain observed experimentally on the secondary was 5.9 per cent.)

EXAMPLE 7.

If the magneto operating with a primary current at break of 4 amperes is connected to a clean spark plug which has its gap set to break down at 2,000 volts: (*a*) What is the energy in the condenser part of the spark? (*b*) Neglecting losses, what is the total energy in the spark?

The condenser energy by equation (18)

$$W = \frac{1}{2} .(2000)^2 \times 246 \times 10^{-12} = .0005 \text{ joule.}$$

The total energy is by equation (3)

$$W = \frac{1}{2} .0.0164 \times (4)^2 = 0.131 \text{ joule.}$$

EXAMPLE 8.

Assuming that a constant voltage of 800 volts is required to maintain the spark at the spark plug, how long would the spark last if it were not interfered with by the closing of the primary breaker?

The initial current in the spark (during Period 4) is approximately

$$I=\frac{4}{60}=0.067 \text{ ampere.}$$

The final current is zero and the current decreases uniformly.

The average current is therefore

$$\frac{0.067+0}{2}=0.033 \text{ ampere.}$$

Now the energy (assuming no losses) is equal to

$$W=E\,I\,t=800\times 0.033\times t=0.131-0.0005$$

whence

$$t=0.0049 \text{ second.}$$

EXAMPLE 9.

If the cam holds the breaker open during a rotation of 60°, at what speed will the closing of the breaker begin to interfere with the spark?

The time of opening is $\frac{60°}{360°}$ or one-sixth of a revolution, and if this is to be 0.0049 second, one revolution must occupy $6\times .0049=0.03$ second, corresponding to 2,000 revolutions per minute.

NOTATION.

C = electrostatic capacity.
E = effective (root-mean-square) voltage.
e = base of Napierian logarithms = 2.718 . . . . . .
f = frequency (cycles per second).
F, F' = correction factors for damping.
I = effective (root-mean-square) current.
I_0 = current at instant of break.
K = any constant.
L = self-inductance.
n_p = number of turns of primary winding.
R = resistance.
s = speed of rotation of magneto armature.
t = time.
v = voltage at any instant.
V_m = maximum voltage.
W = energy.

$$\alpha=\frac{-R}{2\,L}$$

$$\alpha_1=-\frac{R}{2\,L}+\sqrt{\frac{R^2}{4\,L^2}-\frac{1}{L\,C}}$$

$$\alpha_2=-\frac{R}{2\,L}-\sqrt{\frac{R^2}{4\,L^2}-\frac{1}{L\,C}}$$

$$\beta=\sqrt{\frac{1}{L\,C}-\frac{R^2}{4\,L^2}}$$

REPORT No. 124

AERODYNAMIC CHARACTERISTICS OF AEROFOILS—II.

CONTINUATION OF REPORT No. 93

BY

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT No. 124.

AERODYNAMIC CHARACTERISTICS OF AEROFOILS—II.

CONTINUATION OF REPORT NO. 93.

By National Advisory Committee for Aeronautics.

INTRODUCTION.

This collection of data on aerofoils has been made from the published reports of a number of the leading aerodynamic laboratories of this country and Europe.[1] The information which was originally expressed according to the different customs of the several laboratories is here presented in a uniform series of charts and tables suitable for the use of designing engineers and for purposes of general reference.

It is a well-known fact that the results obtained in different laboratories, because of their individual methods of testing, are not strictly comparable even if proper scale corrections for size of model and speed of test are supplied. It is, therefore, unwise to compare too closely the coefficients of two wing sections tested in different laboratories. Tests of different wing sections from the same source, however, may be relied on to give true relative values.

The absolute system of coefficients has been used, since it is thought by the National Advisory Committee for Aeronautics that this system is the one most suited for international use and yet is one for which a desired transformation can be easily made. For this purpose a set of transformation constants is included in this report.

Each aerofoil section is given a reference number, and the test data are presented in the form of curves from which the coefficients can be read with sufficient accuracy for design purposes. The dimensions of the profile of each section are given at various stations along the chord in per cent of the chord, using as datum the line shown on the curves. The shape of the section is also shown in reasonable accuracy to enable one to more clearly visualize the section under consideration, together with its characteristics.

The authority for the results here presented is given as the name of the laboratory at which the experiments were conducted, with the size of model, wind velocity, and date of test.

TRANSFORMATION COEFFICIENTS.

For the convenience of those who prefer to use a system of units other than the absolute system, there is given below a table of transformation constants based on the standard condition adopted by the National Advisory Committee for Aeronautics of—

Temperature = 15.6° C.
Pressure = 760 mm. Hg.
Humidity = 0.
Gravity = 9.80 m./sec.^{2} = 32.2 ft./sec.^{2}

thus giving values of specific weight of air

$$W = 0.1223 \text{ kg./m.}^3 = 0.07635 \text{ lbs./ft.}^3$$

and of density

$\rho = 0.01250$ in the French engineering or kilogram, meter, second system.

Or

$= 0.00238$ in the English or foot, pound, second system.

In absolute units	$P = C\rho V^2$.
In kg./m.^{2} (m./sec.)	$P = .1250\ CV^2$.
In kg./m.^{2} (km./hr.)	$P = .009645\ CV^2$.
In lbs./sq. ft. (ft./sec.)	$P = .002378\ CV^2$.
In lbs./sq. ft. (mi./hr.)	$P = .005116\ CV^2$.

[1] A previous collection on aerofoil sections 1 to 214 and charts 1 to 4 may be found in N. A. C. A. Report No. 93.

INDEX.

Three separate types of index are given—chart indexes which make it possible for a designer to select the wing section most suitable for the particular design in which he is interested; a group index which is arranged by countries and laboratories at which tests were conducted, each section also being designated by a reference number; and an alphabetical index.

CHART INDEX.

In order that the designer may easily pick out a wing section which is suited to the type of airplane on which he is working, four index charts are given which classify the wings according to their aerodynamic and structural properties. In the charts of this report a lower case letter is placed adjacent to the reference number giving VL values, so that a comparison can be made without referring to the individual drawings.

In chart No. 5 the minimum drag is plotted against the L/D at one-fourth the maximum lift. This chart should be used in choosing a wing section for a high-speed airplane, the wing sections being more suited for this use the farther they are from the lower left-hand corner.

In chart No. 6 the mean spar depth is plotted against the maximum lift in order to show the possible strength and lightness of the wing structure. The higher the maximum lift coefficient is the smaller will be the wing area and the lighter the structural weight, and in the same way the greater the depth of the spars the lighter will be their weight, so that the sections the greatest distance from the lower left-hand corner will give the lightest and strongest wings.

The maximum L/D is plotted against the maximum lift in chart No. 7, which is of use in choosing the wing section for a slow and efficient airplane. In the same way as before, the sections farthest from the lower left-hand corner are the best for this purpose.

In chart No. 8 the L/D at two-thirds the maximum lift is plotted against the maximum lift, so that this chart can be used for choosing a section that will give an efficient climb or a long range at cruising speed. The best sections for this purpose will be the farthest from the lower lefthand corner of the chart.

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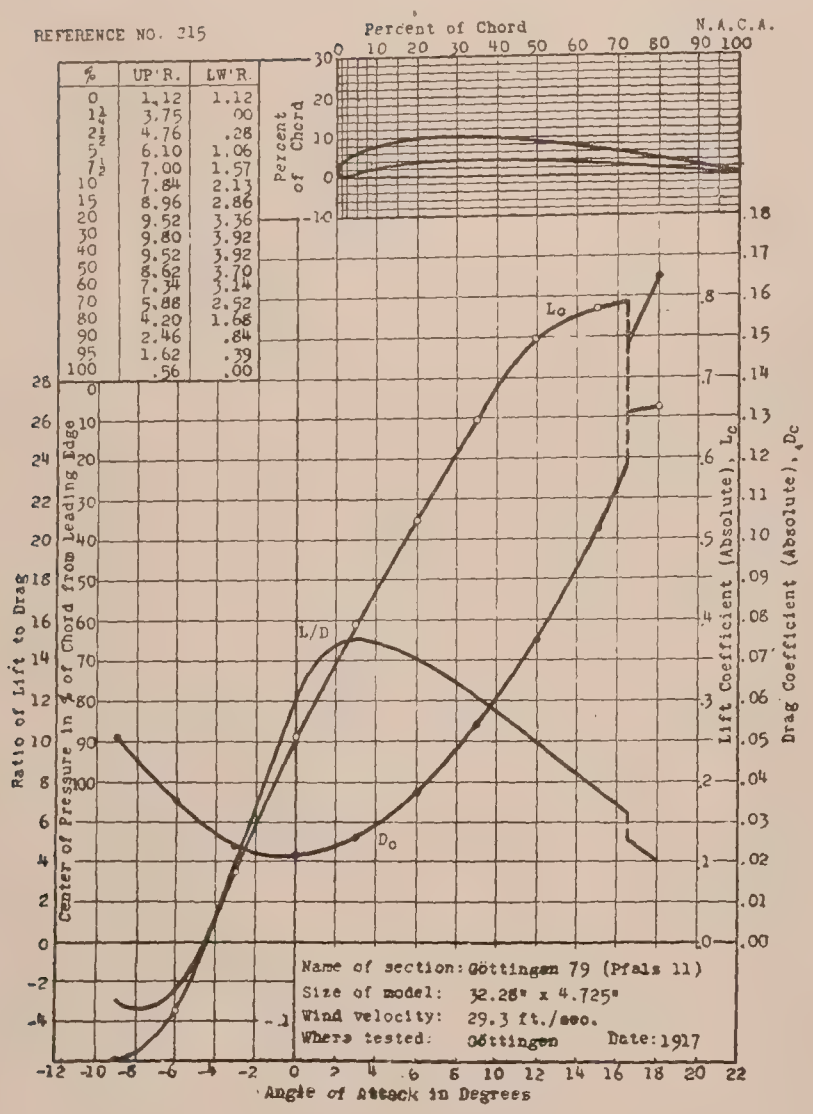
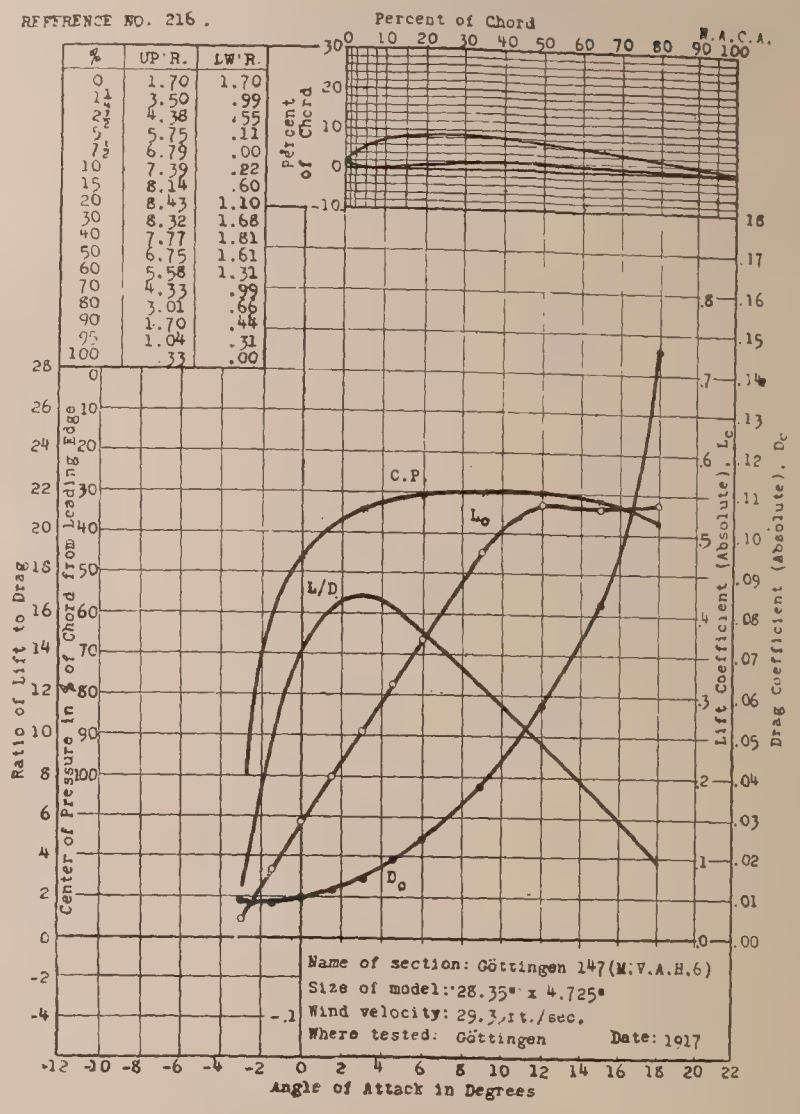
REFERENCE NO. 215

%	UP'R.	LW'R.
0	1.12	1.12
1¼	3.75	.00
2½	4.76	.28
5	6.10	1.06
7½	7.00	1.57
10	7.84	2.13
15	8.96	2.86
20	9.52	3.36
30	9.80	3.92
40	9.52	3.92
50	8.62	3.70
60	7.34	3.14
70	5.88	2.52
80	4.20	1.68
90	2.46	.84
95	1.62	.39
100	.56	.00



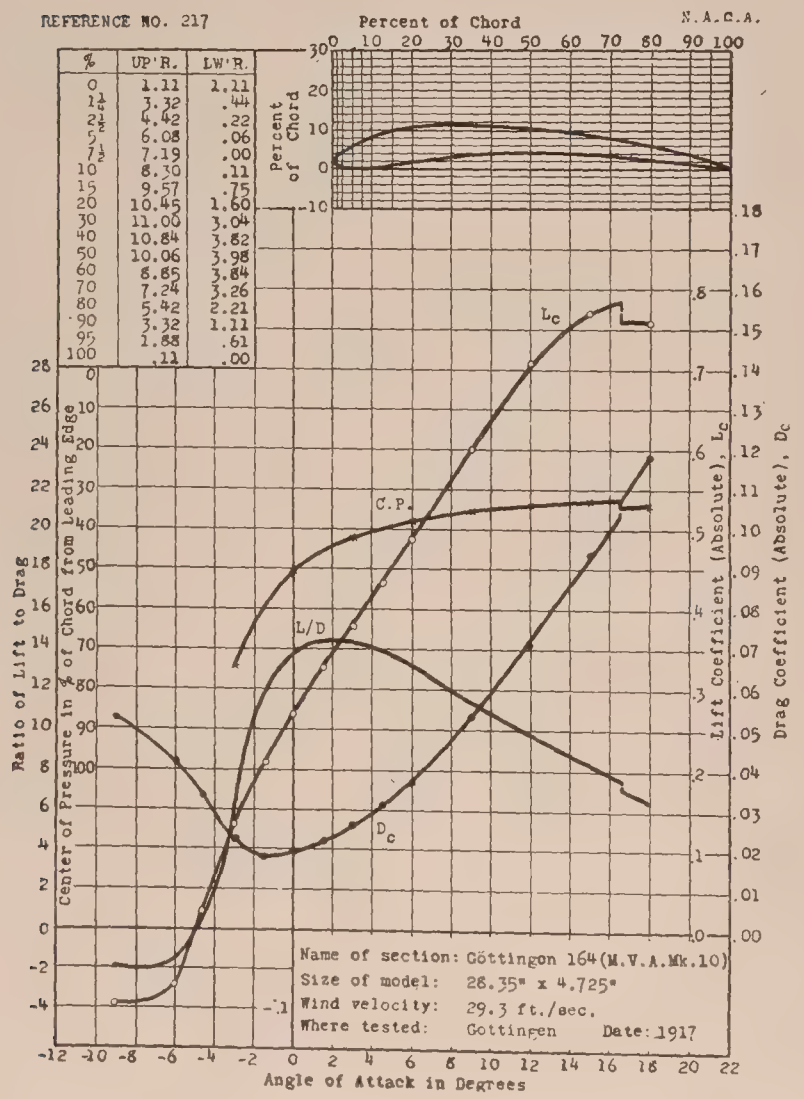
REFERENCE NO. 216.

%	UP'R.	LW'R.
0	1.70	1.70
1¼	3.50	.99
2½	4.38	.55
5	5.75	.11
7½	6.79	.00
10	7.39	.22
15	8.14	.60
20	8.43	1.10
30	8.32	1.68
40	7.77	1.81
50	6.75	1.61
60	5.58	1.31
70	4.33	.99
80	3.01	.66
90	1.70	.44
95	1.04	.31
100	.33	.00



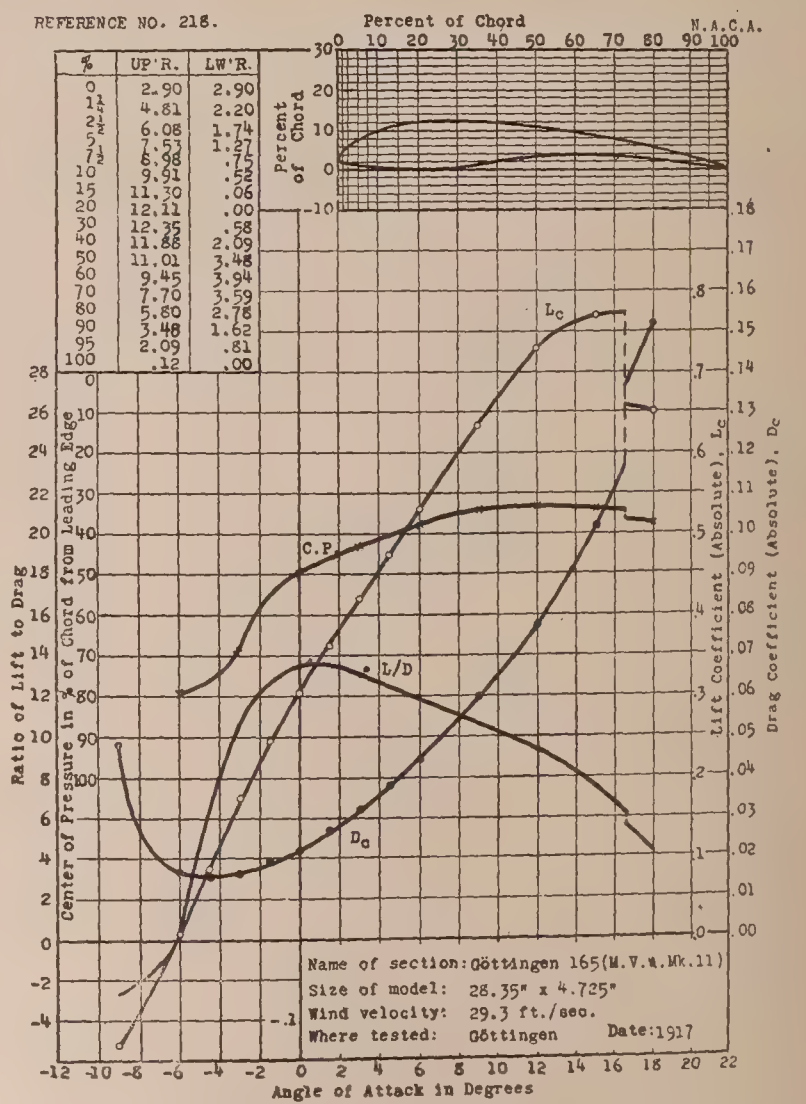
REFERENCE NO. 217

%	UP'R.	LW'R.
0	1.11	1.11
1¼	3.32	.44
2½	4.42	.22
5	6.08	.06
7½	7.19	.00
10	8.30	.11
15	9.57	.75
20	10.45	1.60
30	11.00	3.04
40	10.84	3.82
50	10.06	3.98
60	8.85	3.84
70	7.24	3.26
80	5.42	2.21
90	3.32	1.11
95	1.88	.61
100	.11	.00



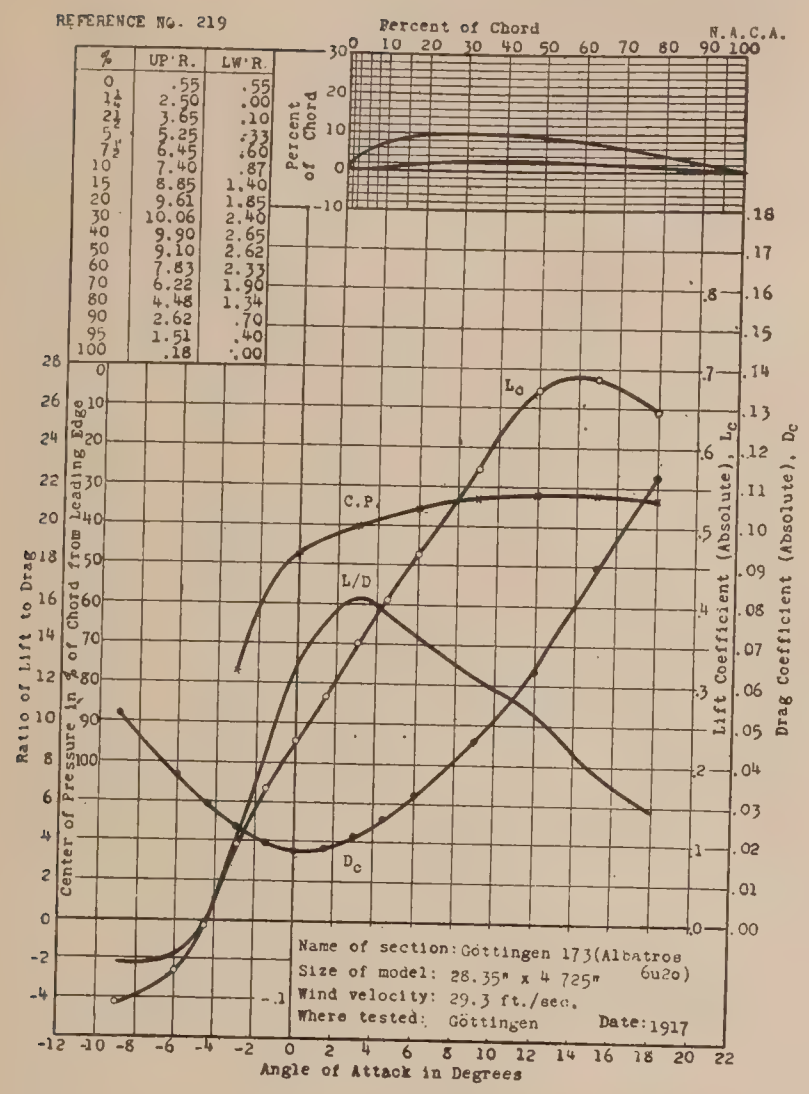
REFERENCE NO. 218.

%	UP'R.	LW'R.
0	2.90	2.90
1¼	4.81	2.20
2½	6.08	1.74
5	7.53	1.27
7½	8.98	.75
10	9.91	.52
15	11.30	.06
20	12.11	.00
30	12.35	.58
40	11.88	2.09
50	11.01	3.48
60	9.45	3.94
70	7.70	3.59
80	5.80	2.78
90	3.48	1.62
95	2.09	.81
100	.12	.00



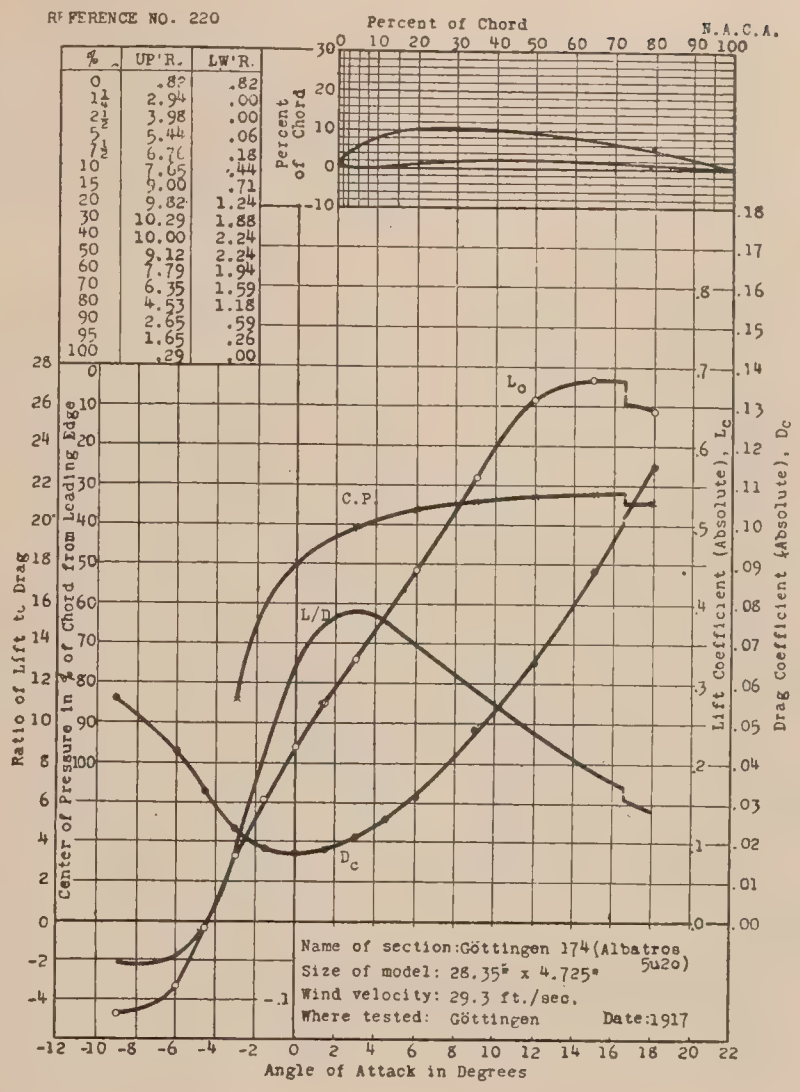
REFERENCE NO. 219

%	UP'R.	LW'R.
0	.55	.55
1¼	2.50	.00
2½	3.65	.10
5	5.25	.33
7½	6.45	.60
10	7.40	.87
15	8.85	1.40
20	9.61	1.85
30	10.06	2.40
40	9.90	2.65
50	9.10	2.62
60	7.83	2.33
70	6.22	1.90
80	4.48	1.34
90	2.62	.70
95	1.51	.40
100	.18	.00



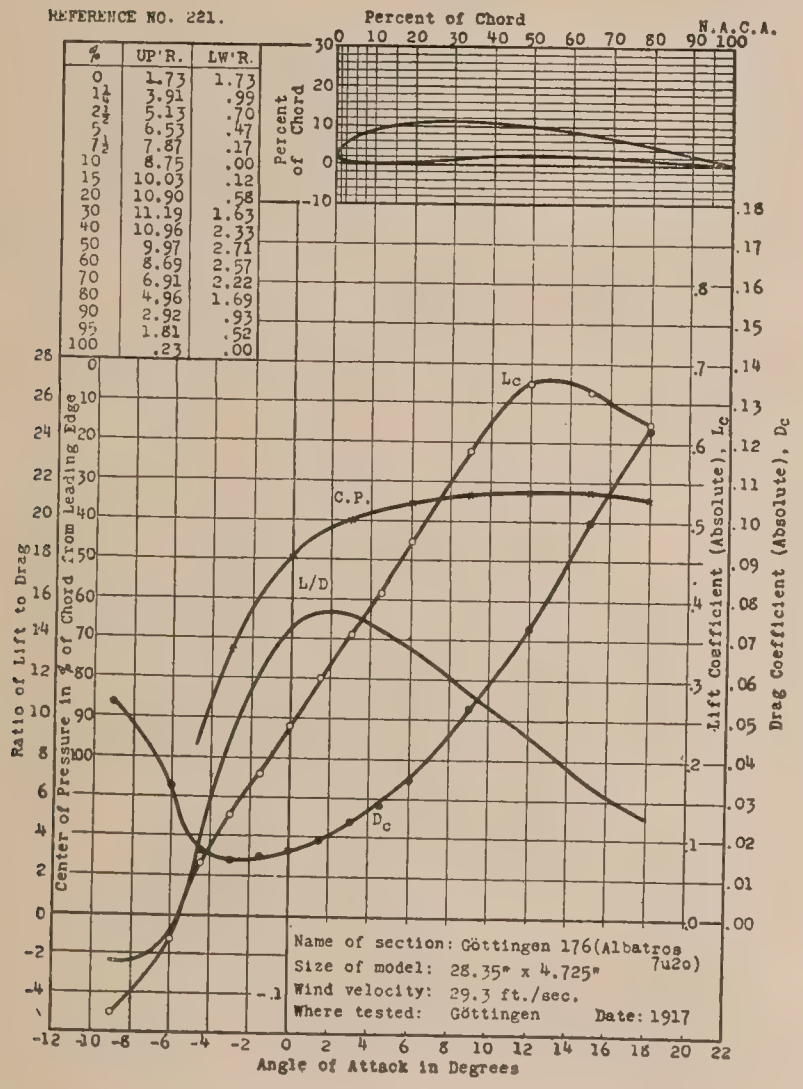
REFERENCE NO. 220

%	UP'R.	LW'R.
0	.82	.82
1¼	2.94	.00
2½	3.98	.00
5	5.44	.06
7½	6.70	.18
10	7.65	.44
15	9.00	.71
20	9.82	1.24
30	10.29	1.88
40	10.00	2.24
50	9.12	2.24
60	7.79	1.94
70	6.35	1.59
80	4.53	1.18
90	2.65	.59
95	1.65	.26
100	.29	.00



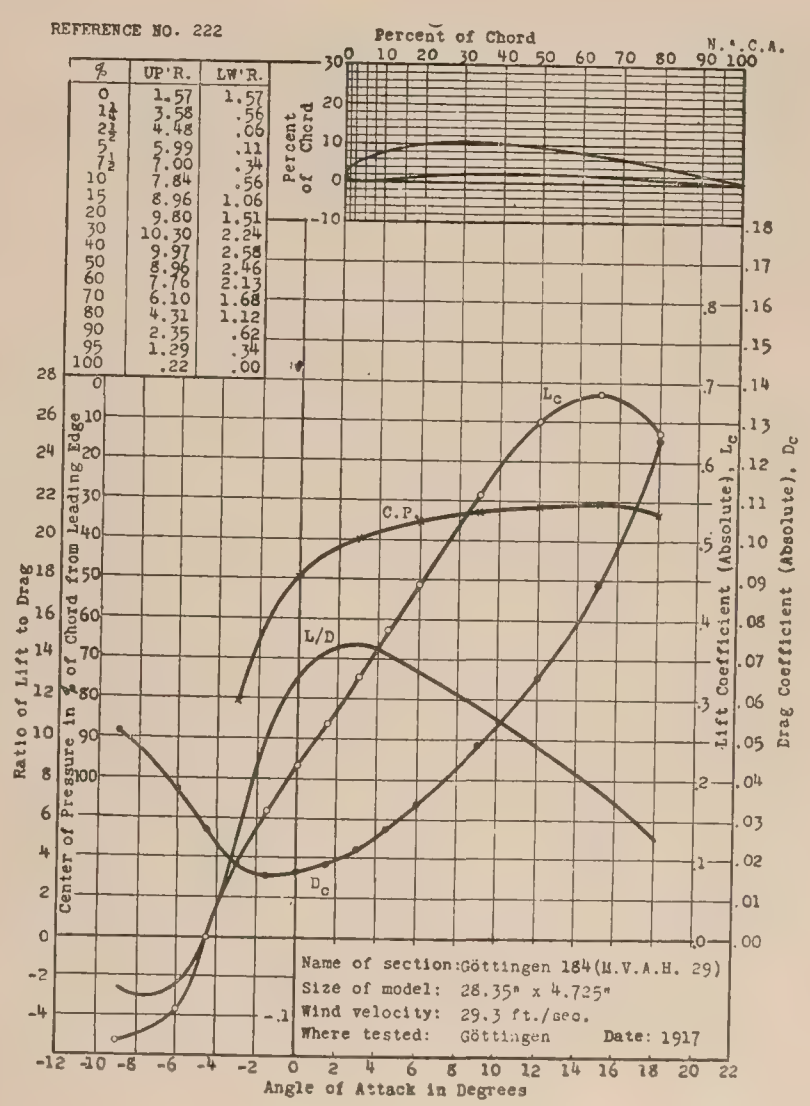
REFERENCE NO. 221.

%	UP'R.	LW'R.
0	1.73	1.73
1¼	3.91	.99
2½	5.13	.70
5	6.53	.47
7½	7.87	.17
10	8.75	.00
15	10.03	.12
20	10.90	.58
30	11.19	1.63
40	10.96	2.33
50	9.97	2.71
60	8.69	2.57
70	6.91	2.22
80	4.96	1.69
90	2.92	.93
95	1.81	.52
100	.23	.00

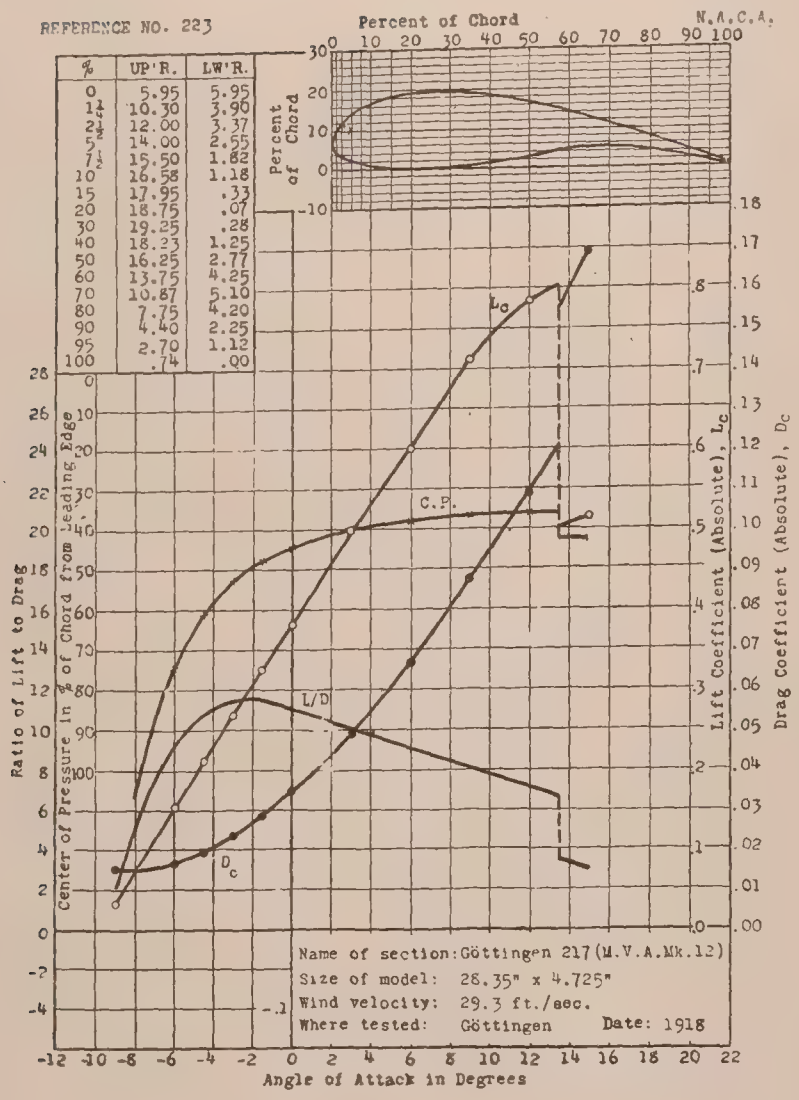


REFERENCE NO. 222

%	UP'R.	LW'R.
0	1.57	1.57
1¼	3.58	.56
2½	4.48	.06
5	5.99	.11
7½	7.00	.34
10	7.84	.56
15	8.96	1.06
20	9.80	1.51
30	10.30	2.24
40	9.97	2.58
50	8.96	2.46
60	7.76	2.13
70	6.10	1.68
80	4.31	1.12
90	2.35	.62
95	1.29	.34
100	.22	.00

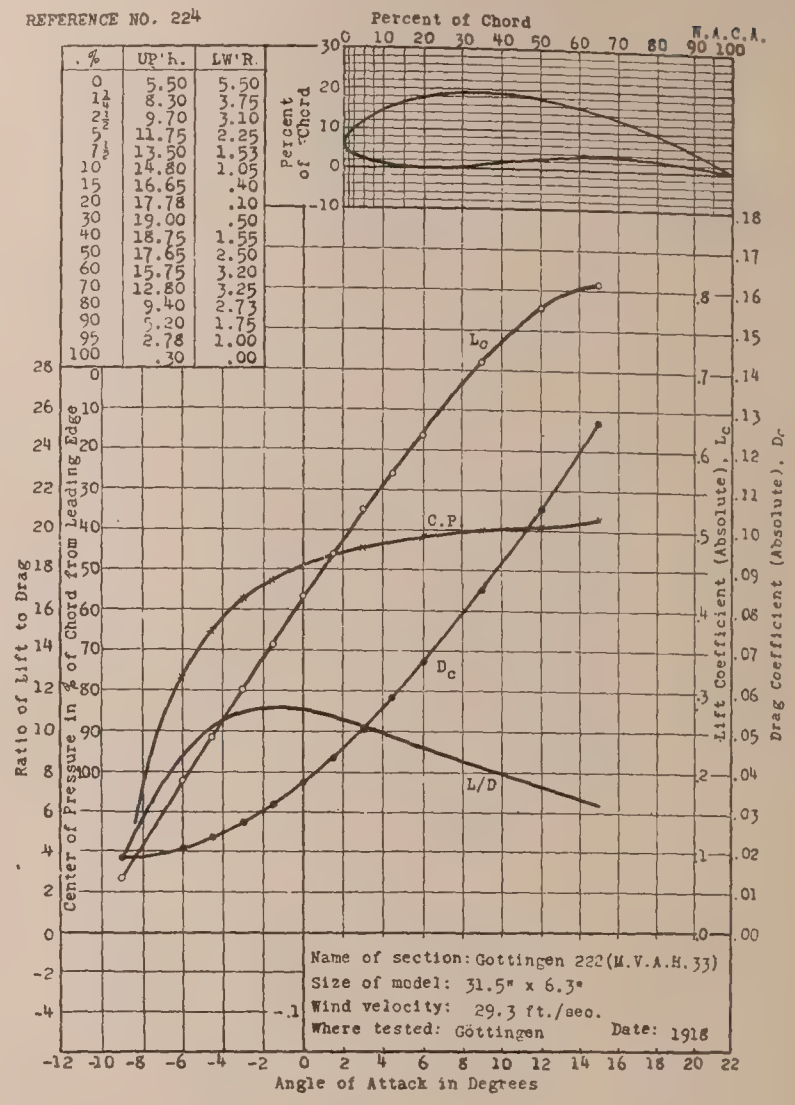


REFERENCE NO. 223



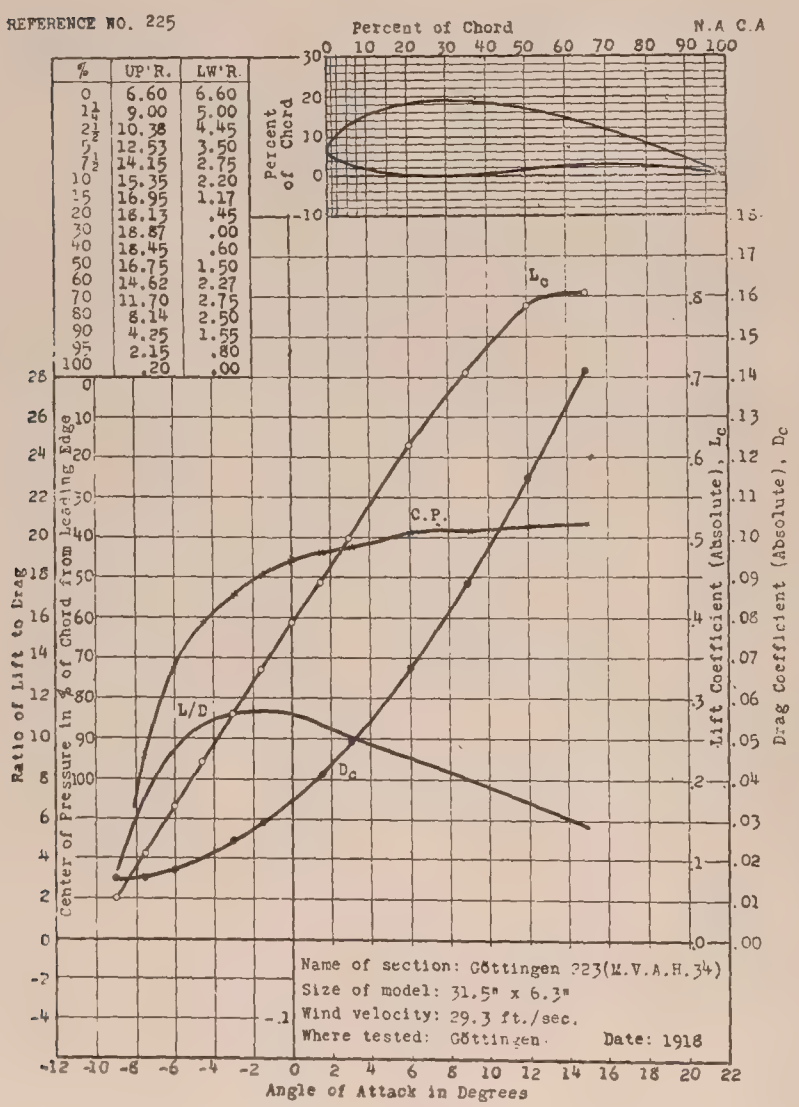
%	UP'R.	LW'R.
0	5.95	5.95
1¼	10.30	3.90
2½	12.00	3.37
5	14.00	2.55
7½	15.50	1.82
10	16.58	1.18
15	17.95	.33
20	18.75	.07
30	19.25	.28
40	18.23	1.25
50	16.25	2.77
60	13.75	4.25
70	10.87	5.10
80	7.75	4.20
90	4.40	2.25
95	2.70	1.12
100	.74	.00

REFERENCE NO. 224



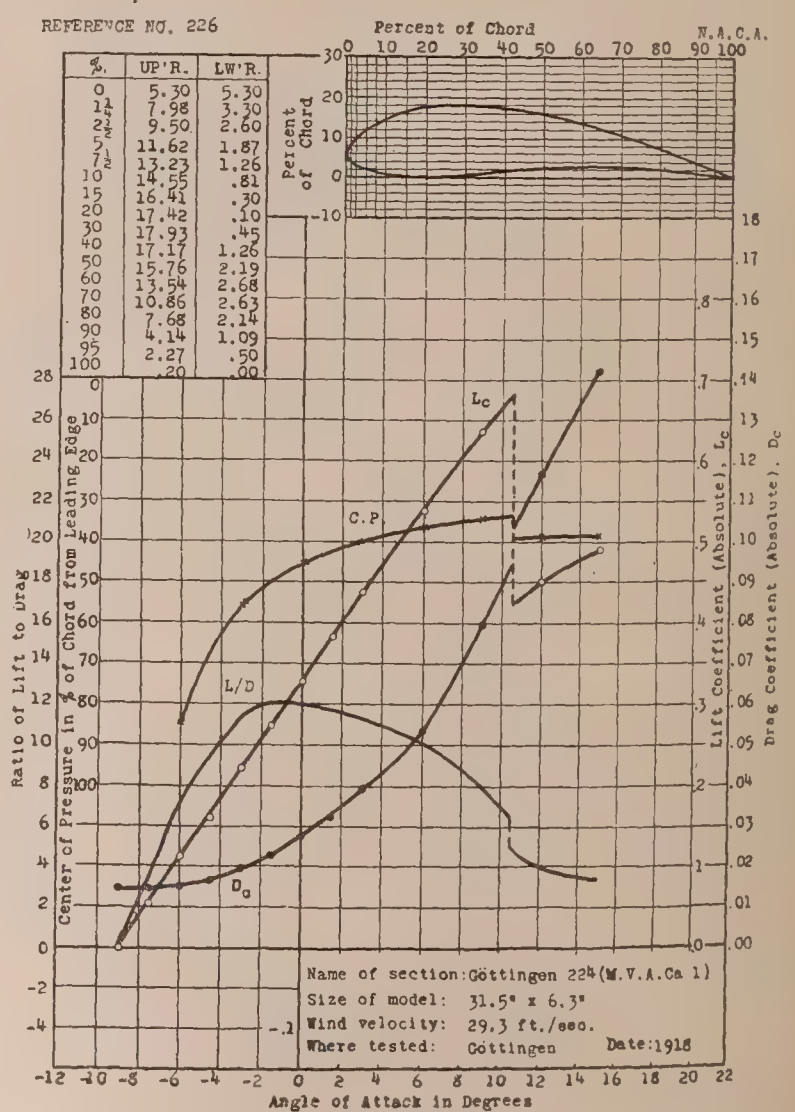
%	UP'R.	LW'R.
0	5.50	5.50
1¼	8.30	3.75
2½	9.70	3.10
5	11.75	2.25
7½	13.50	1.53
10	14.80	1.05
15	16.65	.40
20	17.78	.10
30	19.00	.50
40	18.75	1.55
50	17.65	2.50
60	15.75	3.20
70	12.80	3.25
80	9.40	2.73
90	5.20	1.75
95	2.78	1.00
100	.30	.00

REFERENCE NO. 225



%	UP'R.	LW'R.
0	6.60	6.60
1¼	9.00	5.00
2½	10.38	4.45
5	12.53	3.50
7½	14.15	2.75
10	15.35	2.20
15	16.95	1.17
20	18.13	.45
30	18.87	.00
40	18.45	.60
50	16.75	1.50
60	14.62	2.27
70	11.70	2.75
80	8.14	2.50
90	4.25	1.55
95	2.15	.80
100	.20	.00

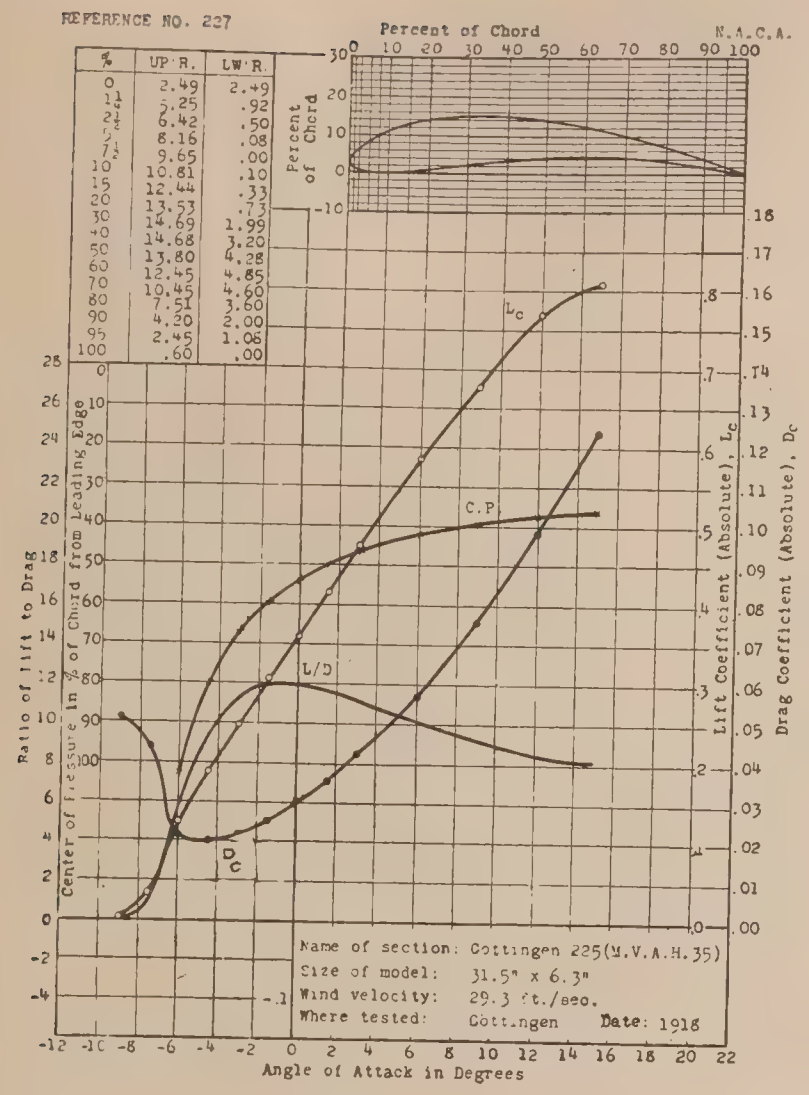
REFERENCE NO. 226



%.	UP'R.	LW'R.
0	5.30	5.30
1¼	7.98	3.30
2½	9.50	2.60
5	11.62	1.87
7½	13.23	1.26
10	14.55	.81
15	16.41	.30
20	17.42	.10
30	17.93	.45
40	17.17	1.26
50	15.76	2.19
60	13.54	2.68
70	10.86	2.63
80	7.68	2.14
90	4.14	1.09
95	2.27	.50
100	.20	.00

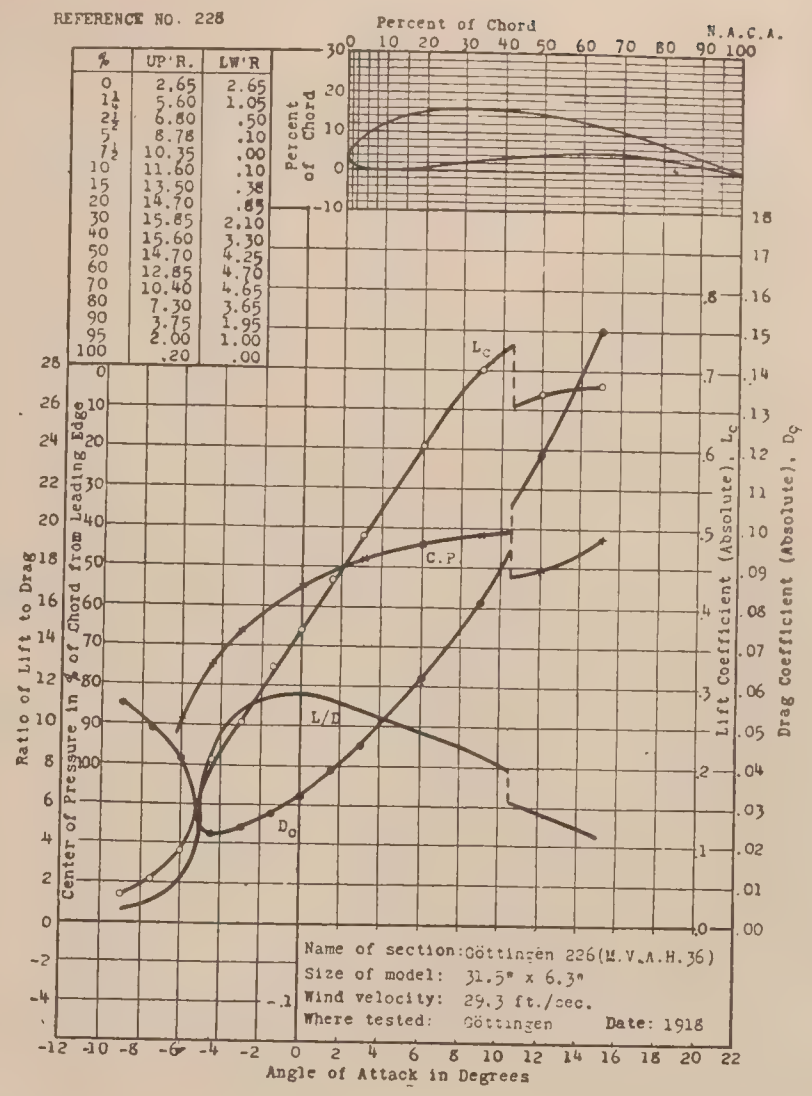
REFERENCE NO. 227

%	UP'R.	LW'R.
0	2.49	2.49
1¼	5.25	.92
2½	6.42	.50
5	8.16	.08
7½	9.65	.00
10	10.81	.10
15	12.44	.33
20	13.53	.73
30	14.69	1.99
40	14.68	3.20
50	13.80	4.28
60	12.45	4.85
70	10.45	4.60
80	7.51	3.60
90	4.20	2.00
95	2.45	1.05
100	.60	.00



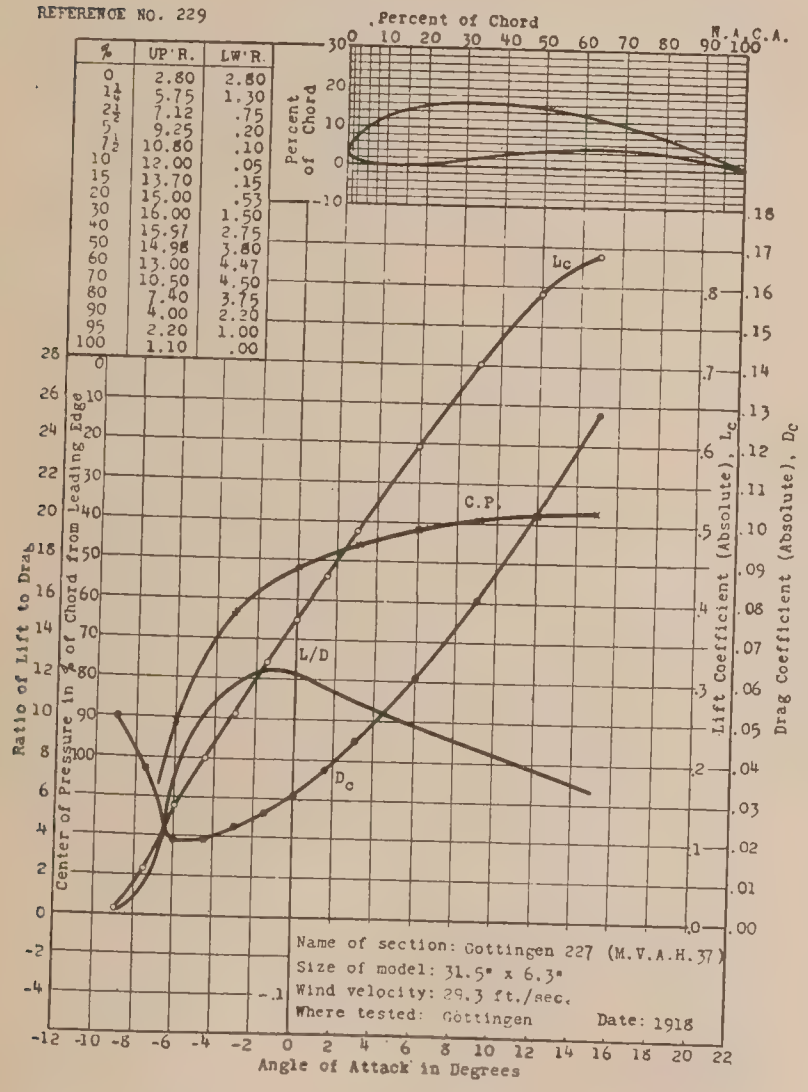
REFERENCE NO. 228

%	UP'R.	LW'R
0	2.65	2.65
1¼	5.60	1.05
2½	6.80	.50
5	8.78	.10
7½	10.35	.00
10	11.60	.10
15	13.50	.38
20	14.70	.85
30	15.85	2.10
40	15.60	3.30
50	14.70	4.25
60	12.85	4.70
70	10.40	4.65
80	7.30	3.65
90	3.75	1.95
95	2.00	1.00
100	.20	.00



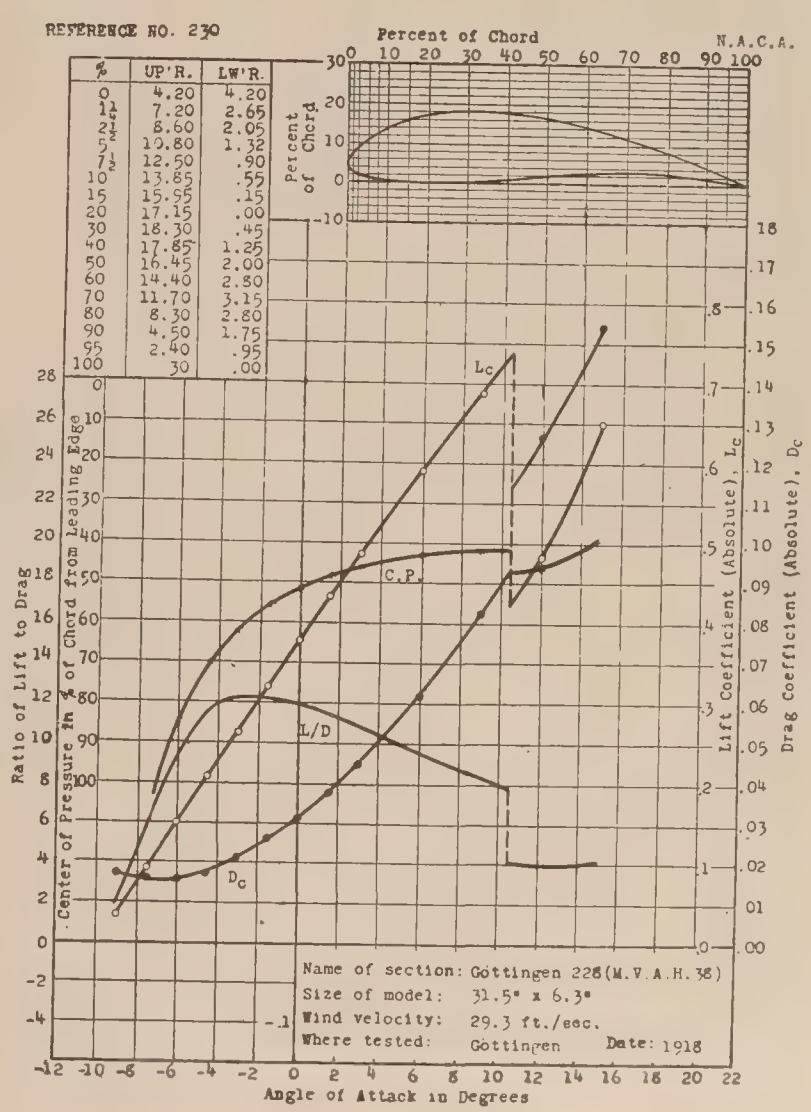
REFERENCE NO. 229

%	UP'R.	LW'R.
0	2.80	2.80
1¼	5.75	1.30
2½	7.12	.75
5	9.25	.20
7½	10.80	.10
10	12.00	.05
15	13.70	.15
20	15.00	.53
30	16.00	1.50
40	15.97	2.75
50	14.98	3.80
60	13.00	4.47
70	10.50	4.50
80	7.40	3.75
90	4.00	2.20
95	2.20	1.00
100	1.10	.00



REFERENCE NO. 230

%	UP'R.	LW'R.
0	4.20	4.20
1¼	7.20	2.65
2½	8.60	2.05
5	10.80	1.32
7½	12.50	.90
10	13.85	.55
15	15.55	.15
20	17.15	.00
30	18.30	.45
40	17.85	1.25
50	16.45	2.00
60	14.40	2.80
70	11.70	3.15
80	8.30	2.80
90	4.50	1.75
95	2.40	.95
100	30	.00



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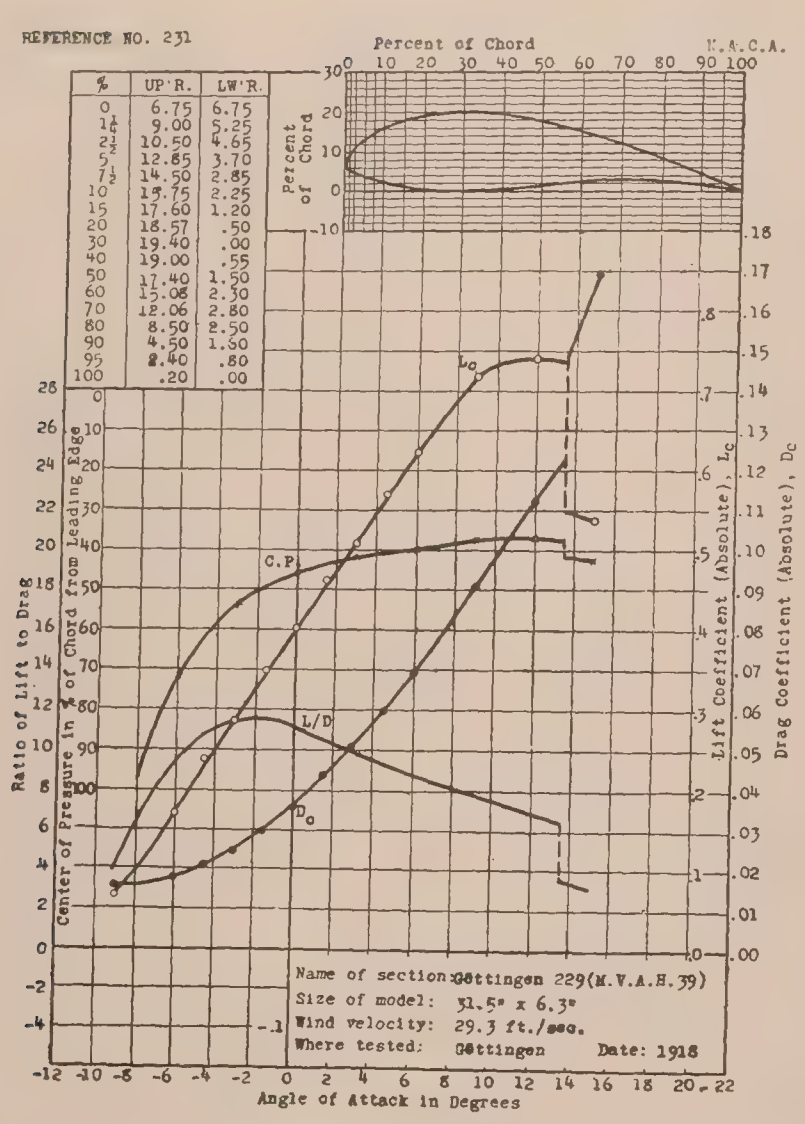
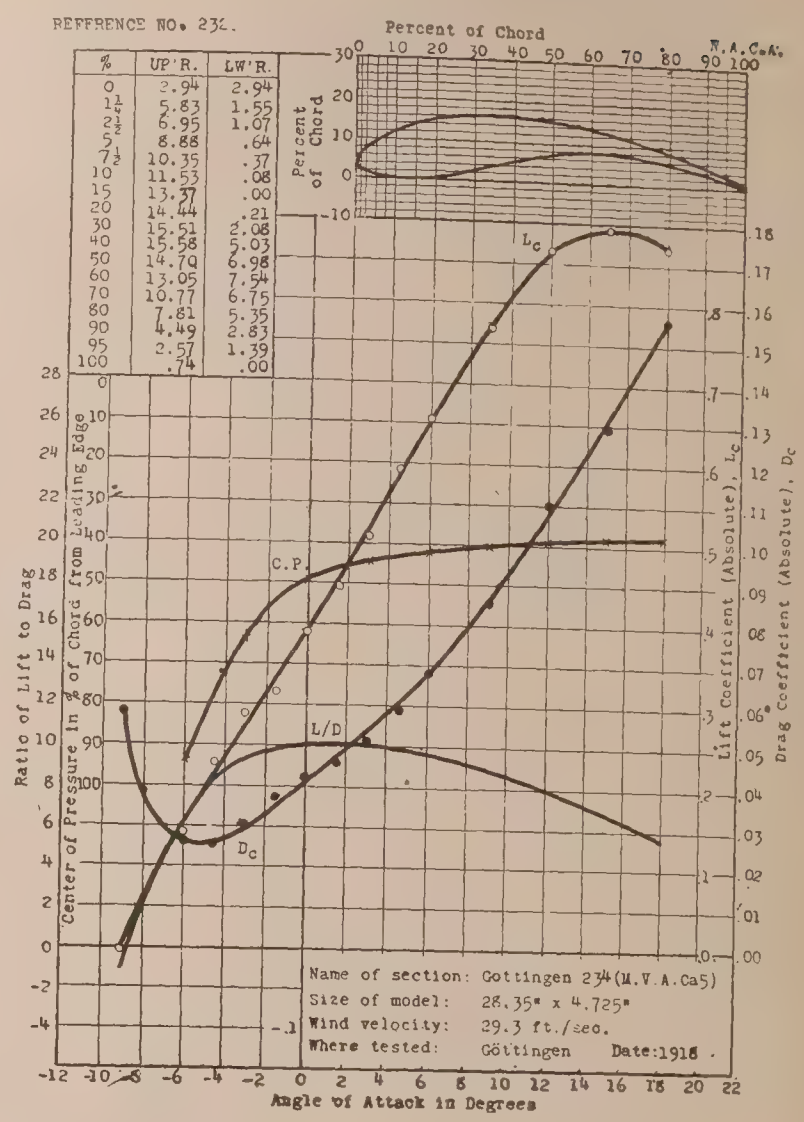


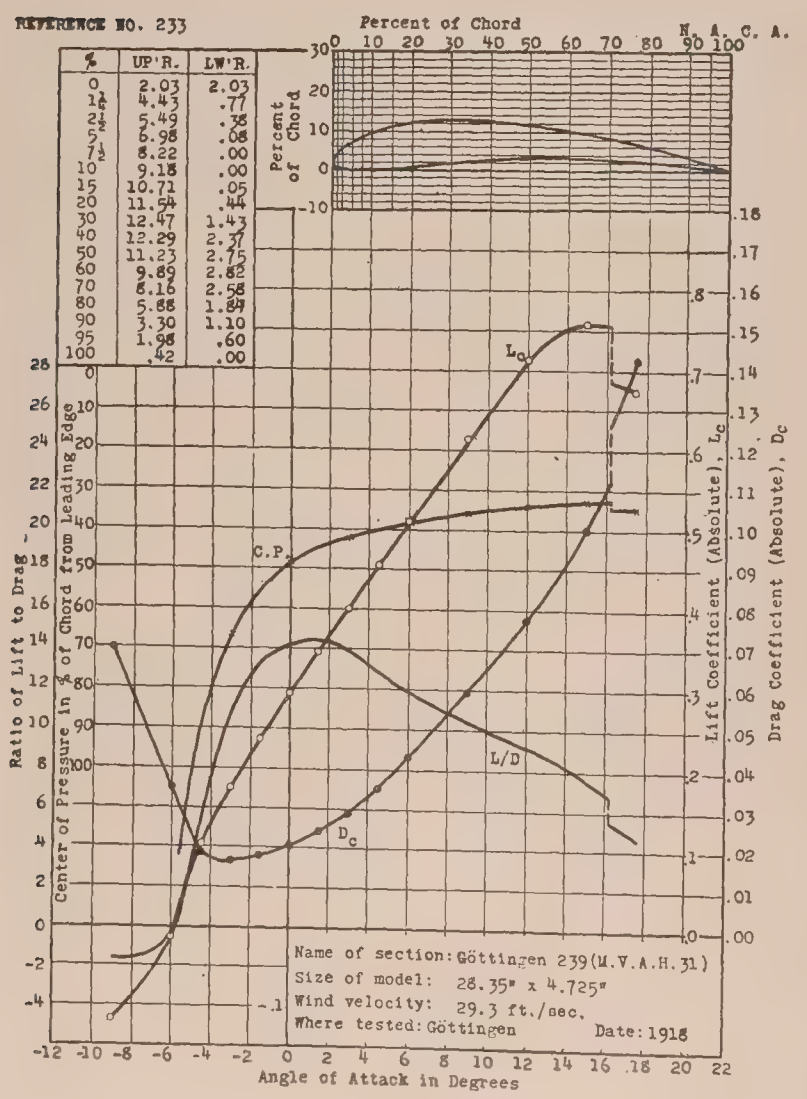
%	UP'R.	LW'R.
0	6.75	6.75
1¼	9.00	5.25
2½	10.50	4.65
5	12.85	3.70
7½	14.50	2.85
10	15.75	2.25
15	17.60	1.20
20	18.57	.50
30	19.40	.00
40	19.00	.55
50	17.40	1.50
60	15.08	2.30
70	12.06	2.80
80	8.50	2.50
90	4.50	1.60
95	2.40	.80
100	.20	.00

REFERENCE NO. 232.



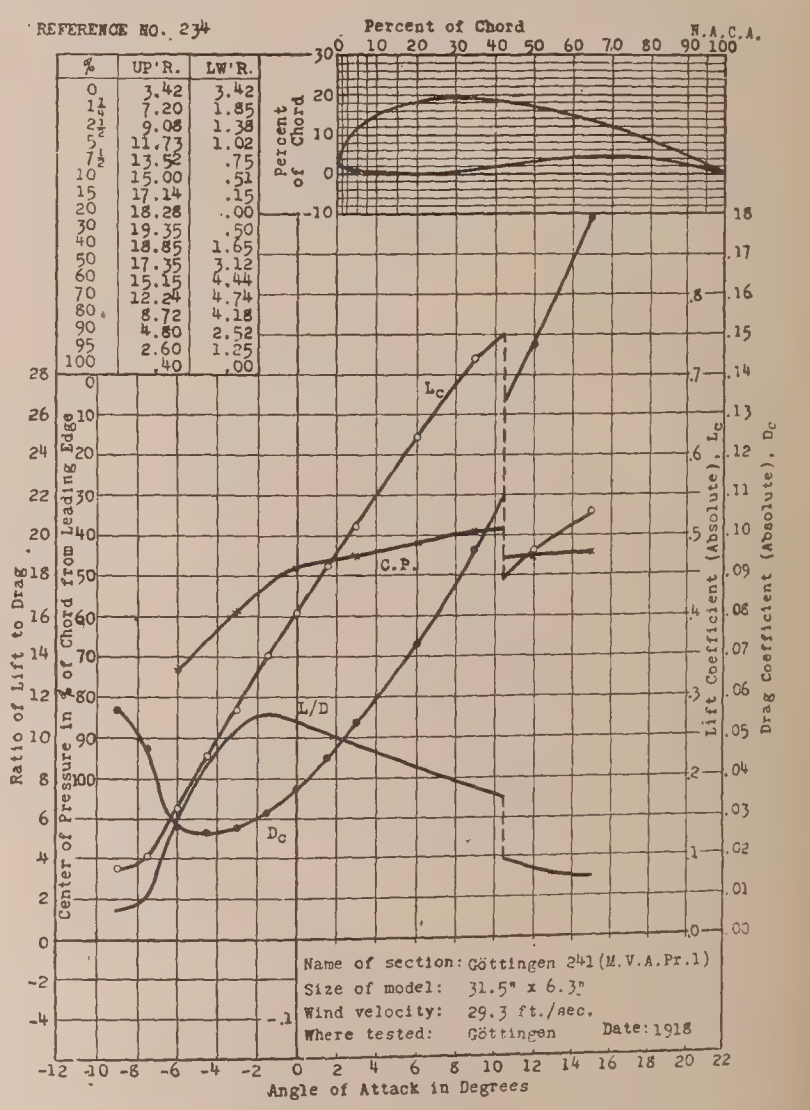
%	UP'R.	LW'R.
0	2.94	2.94
1¼	5.83	1.55
2½	6.95	1.07
5	8.88	.64
7½	10.35	.37
10	11.53	.08
15	13.37	.00
20	14.44	.21
30	15.51	2.08
40	15.58	5.03
50	14.70	6.98
60	13.05	7.54
70	10.77	6.75
80	7.81	5.35
90	4.49	2.83
95	2.57	1.39
100	.74	.00

REFERENCE NO. 233



%	UP'R.	LW'R.
0	2.03	2.03
1¼	4.43	.77
2½	5.49	.38
5	6.98	.08
7½	8.22	.00
10	9.18	.00
15	10.71	.05
20	11.54	.44
30	12.47	1.43
40	12.29	2.37
50	11.23	2.75
60	9.89	2.82
70	8.16	2.58
80	5.88	1.84
90	3.30	1.10
95	1.98	.60
100	.42	.00

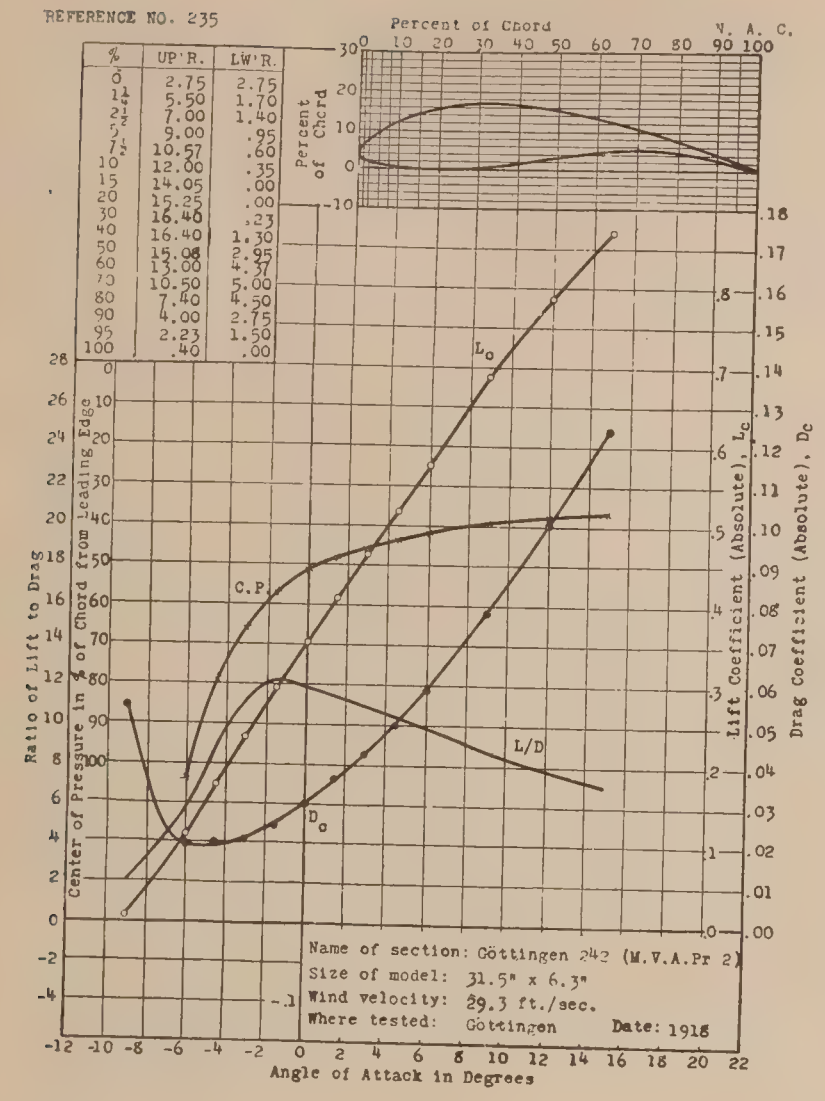
REFERENCE NO. 234



%	UP'R.	LW'R.
0	3.42	3.42
1¼	7.20	1.85
2½	9.08	1.38
5	11.73	1.02
7½	13.52	.75
10	15.00	.51
15	17.14	.15
20	18.28	.00
30	19.35	.50
40	18.85	1.65
50	17.35	3.12
60	15.15	4.44
70	12.24	4.74
80	8.72	4.18
90	4.80	2.52
95	2.60	1.25
100	.40	.00

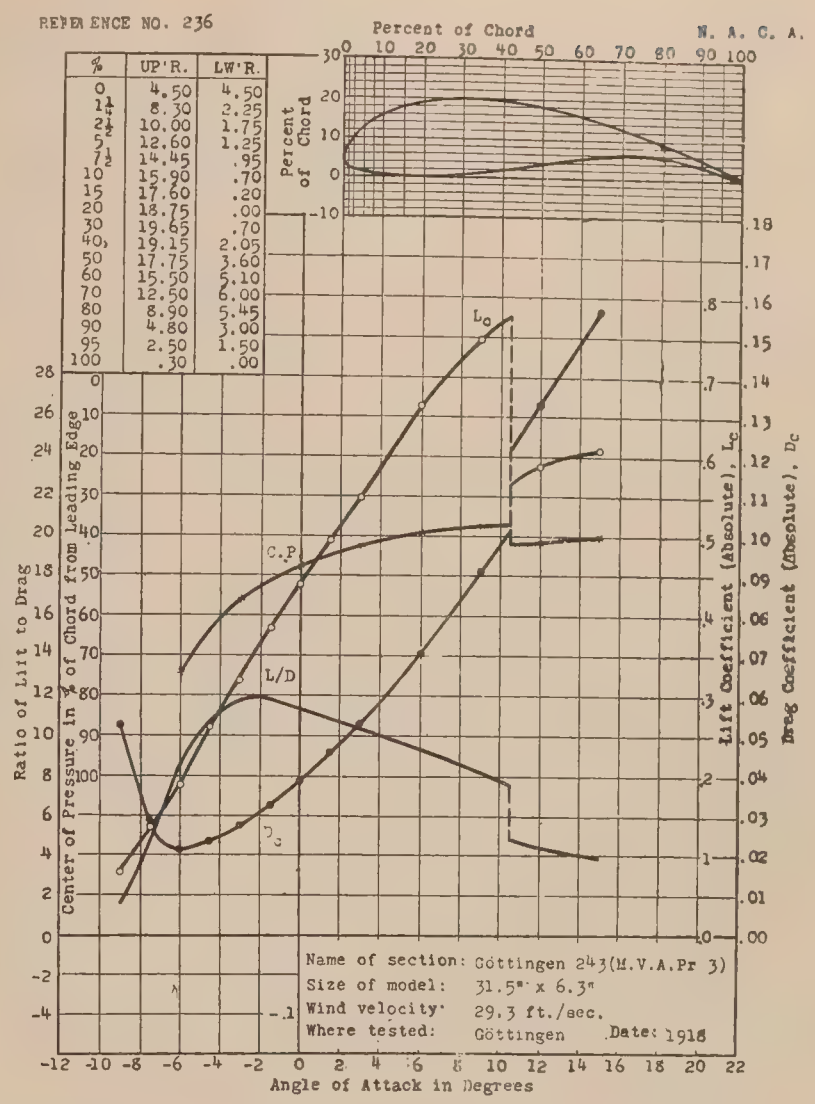
REFERENCE NO. 235

%	UP'R.	LW'R.
0	2.75	2.75
1¼	5.50	1.70
2½	7.00	1.40
5	9.00	.95
7½	10.57	.60
10	12.00	.35
15	14.05	.00
20	15.25	.00
30	16.40	.23
40	16.40	1.30
50	15.05	2.95
60	13.00	4.37
70	10.50	5.00
80	7.40	4.50
90	4.00	2.75
95	2.23	1.50
100	.40	.00



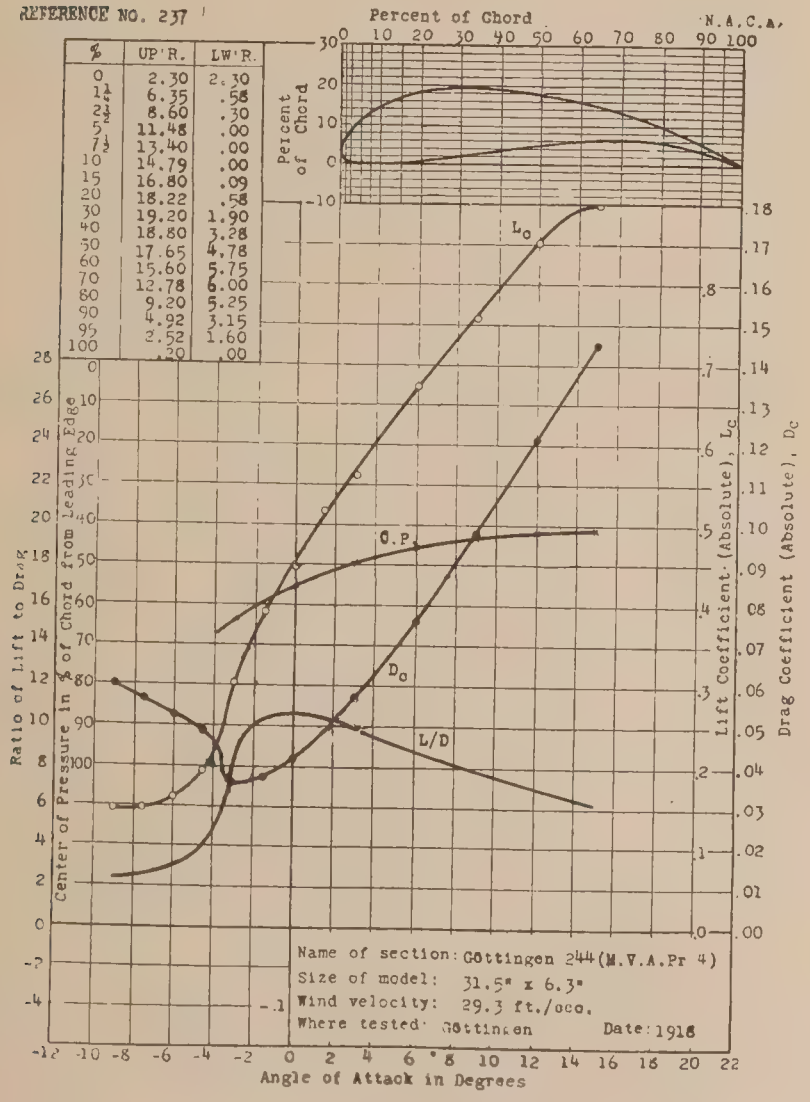
REFERENCE NO. 236

%	UP'R.	LW'R.
0	4.50	4.50
1¼	8.30	2.25
2½	10.00	1.75
5	12.60	1.25
7½	14.45	.95
10	15.90	.70
15	17.60	.20
20	18.75	.00
30	19.65	.70
40	19.15	2.05
50	17.75	3.60
60	15.50	5.10
70	12.50	6.00
80	8.90	5.45
90	4.80	3.00
95	2.50	1.50
100	.30	.00



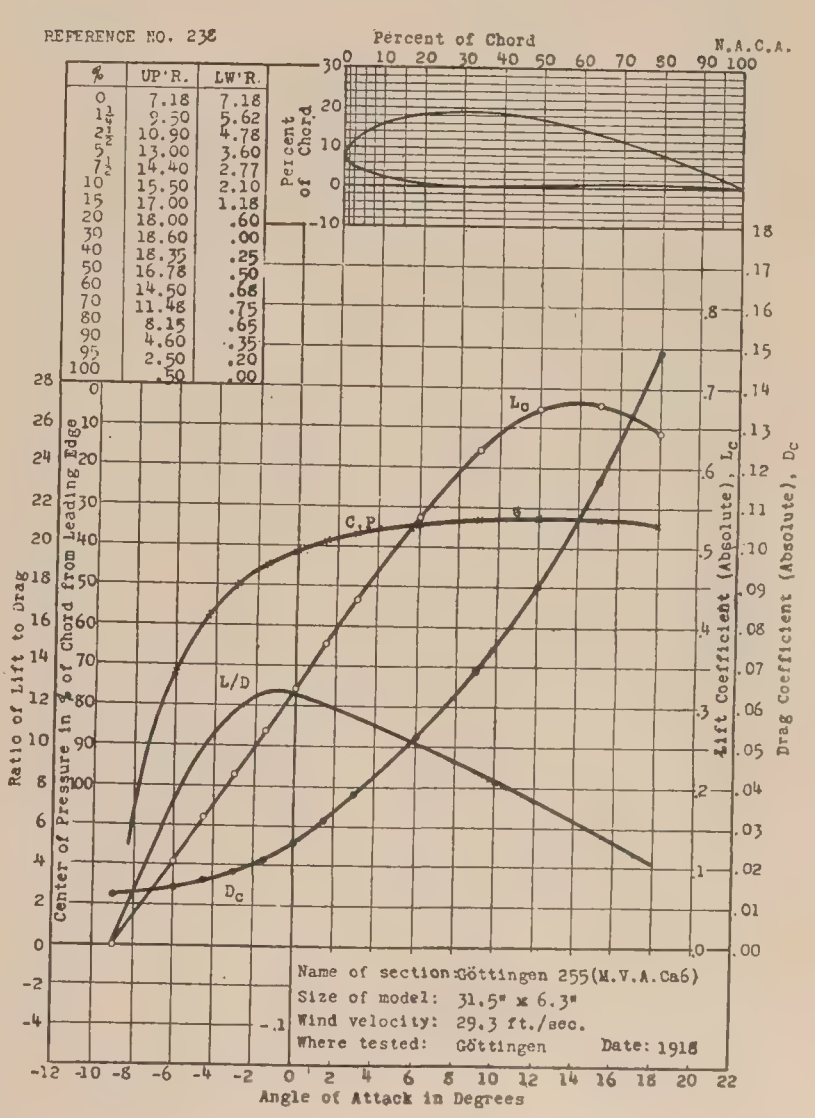
REFERENCE NO. 237

%	UP'R.	LW'R.
0	2.30	2.30
1¼	6.35	.58
2½	8.60	.30
5	11.48	.00
7½	13.40	.00
10	14.79	.00
15	16.80	.09
20	18.22	.58
30	19.20	1.90
40	18.80	3.28
50	17.65	4.78
60	15.60	5.75
70	12.78	6.00
80	9.20	5.25
90	4.92	3.15
95	2.52	1.60
100	.20	.00



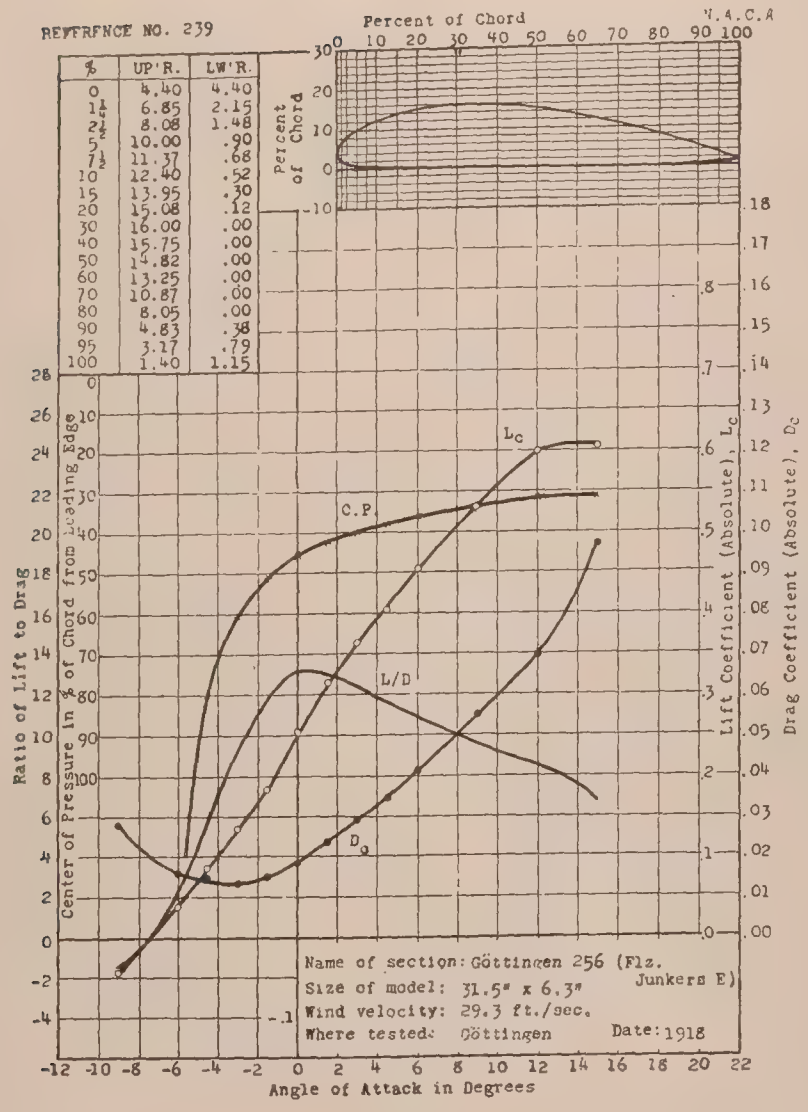
REFERENCE NO. 238

%	UP'R.	LW'R.
0	7.18	7.18
1¼	9.50	5.62
2½	10.90	4.78
5	13.00	3.60
7½	14.40	2.77
10	15.50	2.10
15	17.00	1.18
20	18.00	.60
30	18.60	.00
40	18.35	.25
50	16.78	.50
60	14.50	.68
70	11.48	.75
80	8.15	.65
90	4.60	.35
95	2.50	.20
100	.50	.00



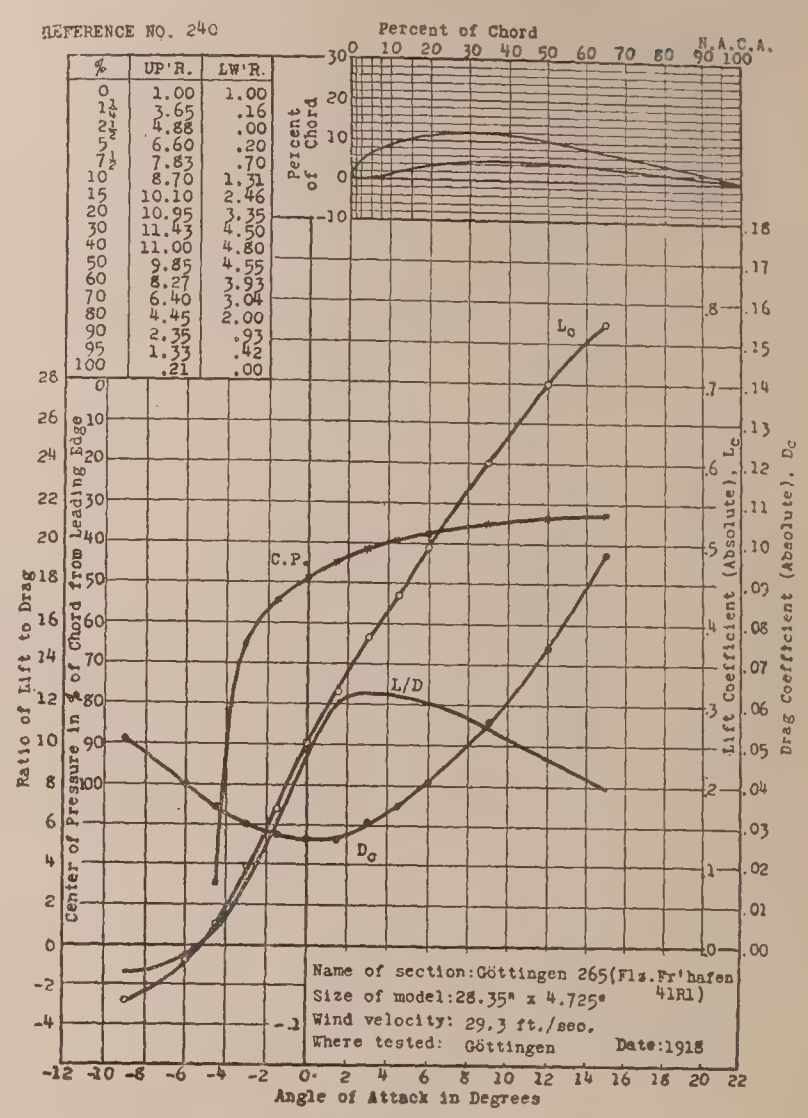
REFERENCE NO. 239

%	UP'R.	LW'R.
0	4.40	4.40
1¼	6.85	2.15
2½	8.08	1.48
5	10.00	.90
7½	11.37	.68
10	12.40	.52
15	13.95	.30
20	15.08	.12
30	16.00	.00
40	15.75	.00
50	14.82	.00
60	13.25	.00
70	10.87	.00
80	8.05	.00
90	4.83	.38
95	3.17	.79
100	1.40	1.15



REFERENCE NO. 240

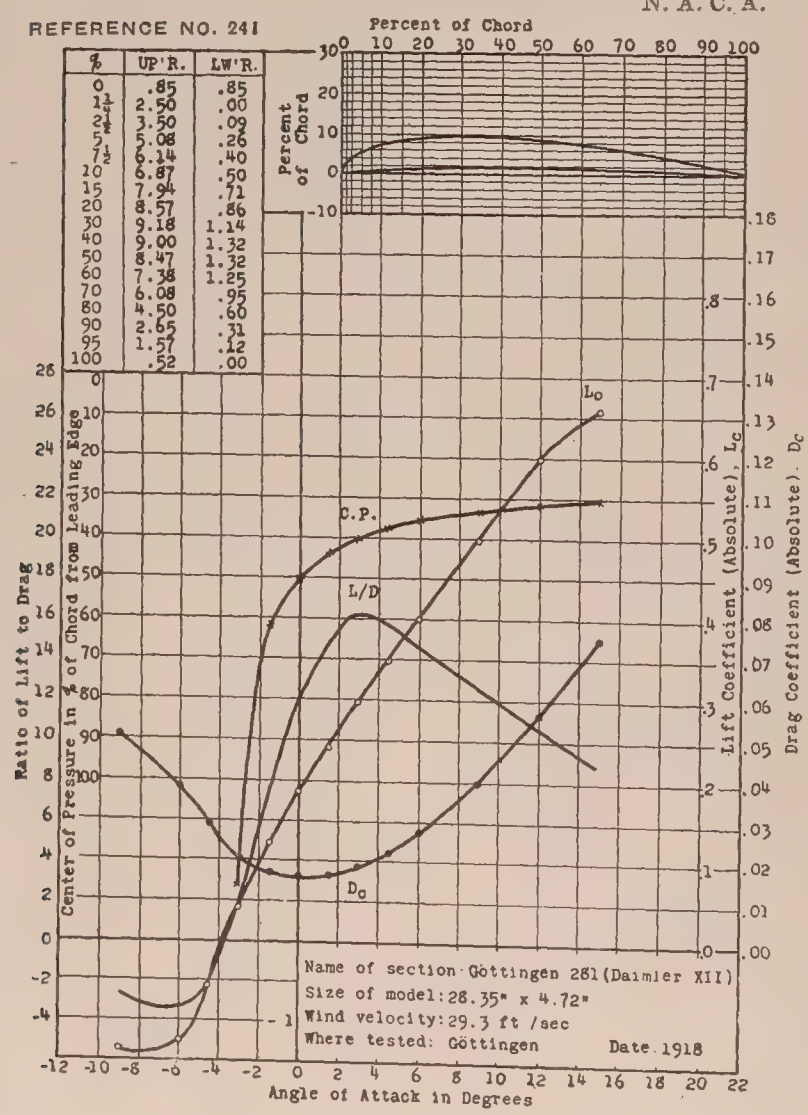
%	UP'R.	LW'R.
0	1.00	1.00
1¼	3.65	.16
2½	4.88	.00
5	6.60	.20
7½	7.83	.70
10	8.70	1.31
15	10.10	2.46
20	10.95	3.35
30	11.43	4.50
40	11.00	4.80
50	9.85	4.55
60	8.27	3.93
70	6.40	3.04
80	4.45	2.00
90	2.35	.93
95	1.33	.42
100	.21	.00



N. A. C. A.

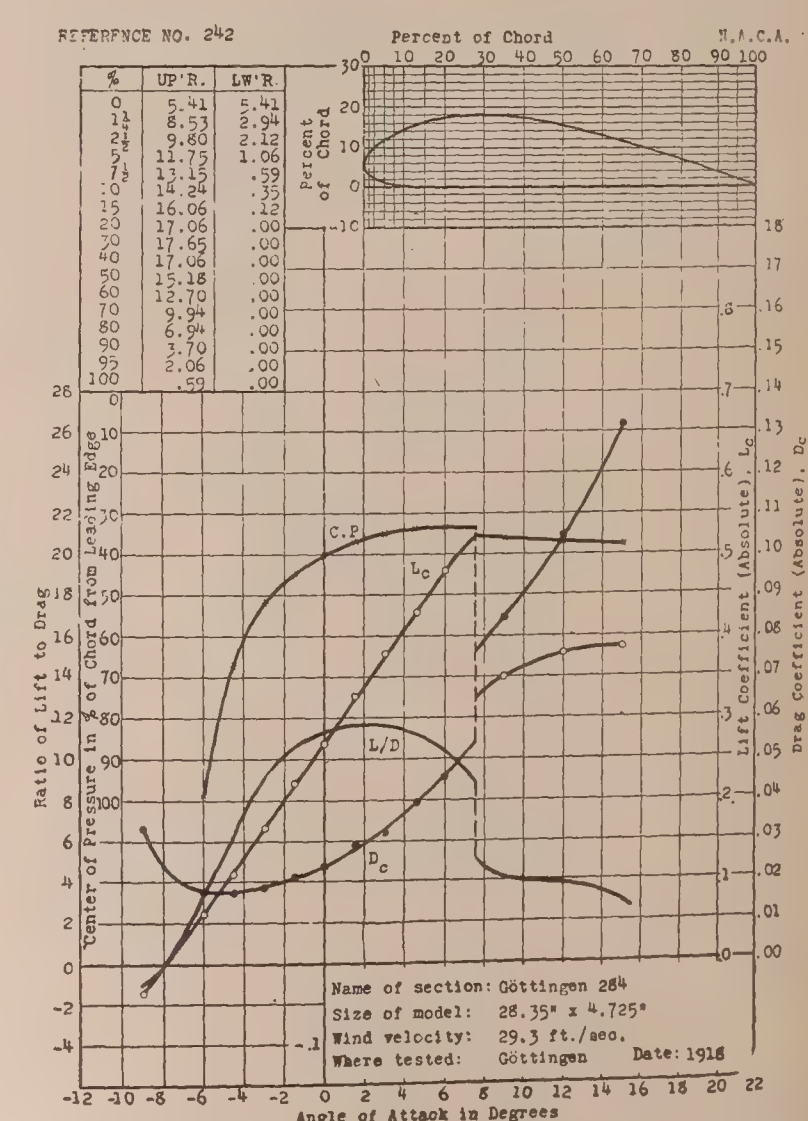
REFERENCE NO. 241

%	UP'R.	LW'R.
0	.85	.85
1¼	2.50	.00
2½	3.50	.09
5	5.08	.26
7½	6.14	.40
10	6.87	.50
15	7.94	.71
20	8.57	.86
30	9.18	1.14
40	9.00	1.32
50	8.47	1.32
60	7.38	1.25
70	6.08	.95
80	4.50	.60
90	2.65	.31
95	1.57	.12
100	.52	.00



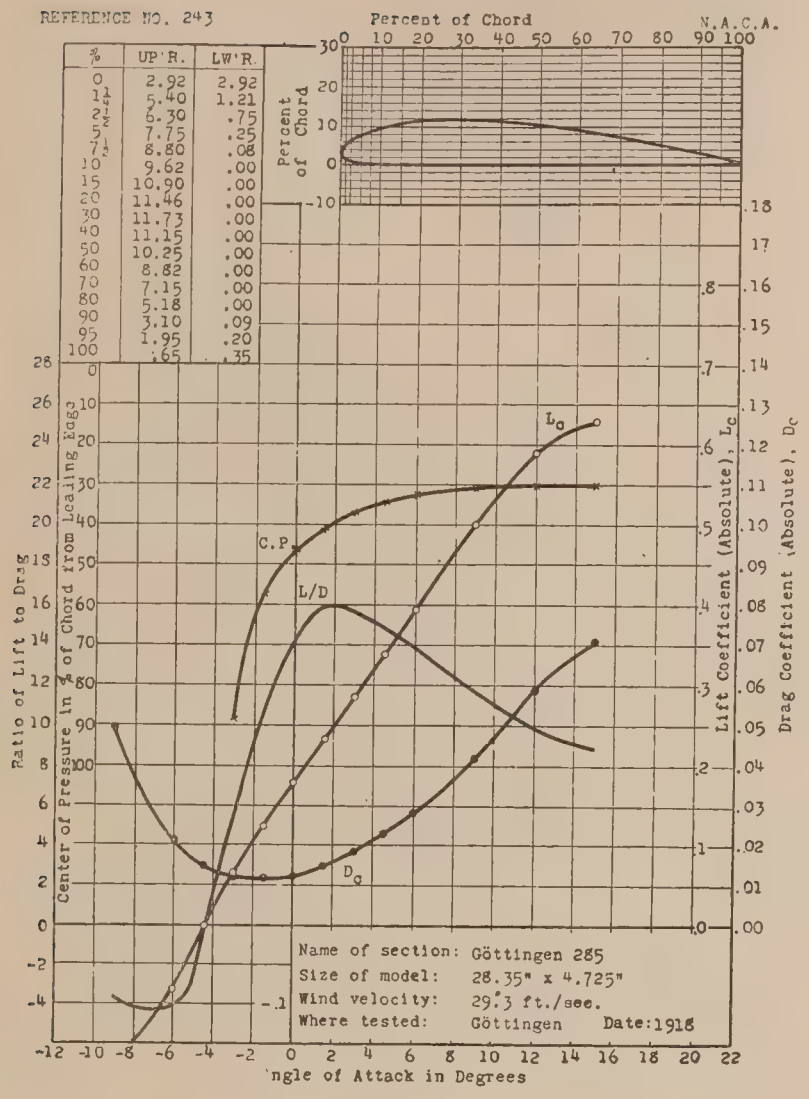
REFERENCE NO. 242

%	UP'R.	LW'R.
0	5.41	5.41
1¼	8.53	2.94
2½	9.80	2.12
5	11.75	1.06
7½	13.15	.59
10	14.24	.35
15	16.06	.12
20	17.06	.00
30	17.65	.00
40	17.06	.00
50	15.18	.00
60	12.70	.00
70	9.94	.00
80	6.94	.00
90	3.70	.00
95	2.06	.00
100	.59	.00



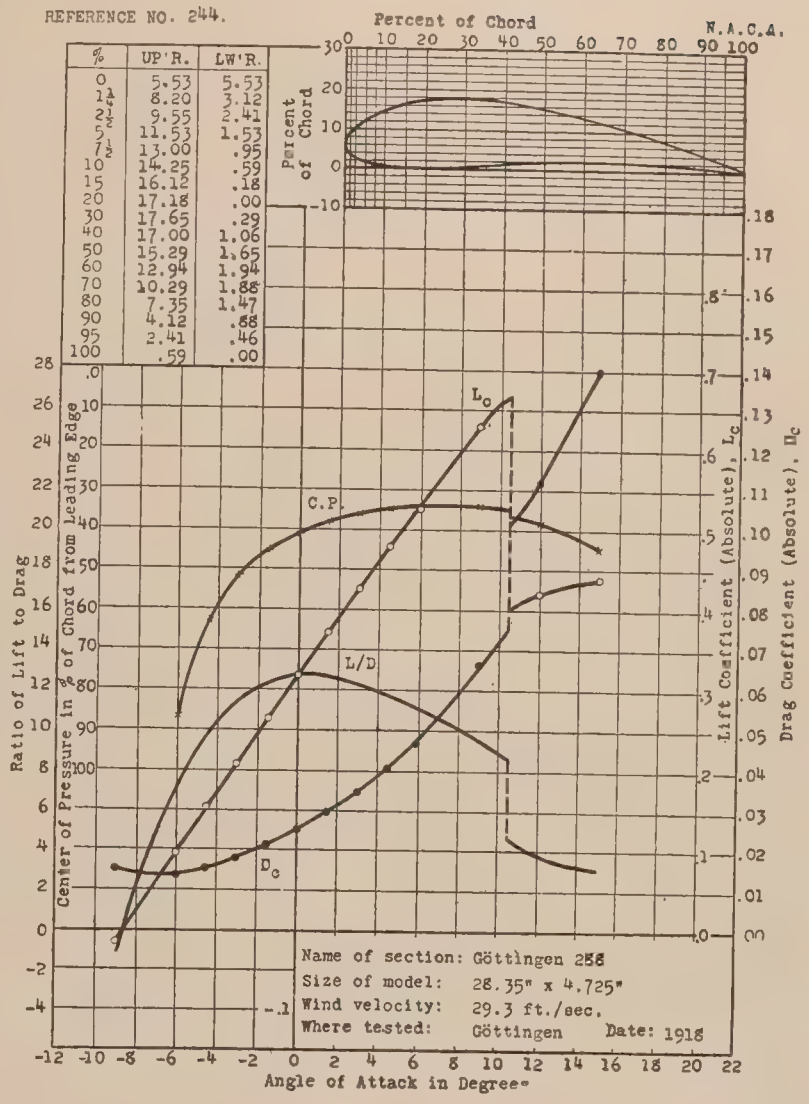
REFERENCE NO. 243

%	UP'R.	LW'R.
0	2.92	2.92
1¼	5.40	1.21
2½	6.30	.75
5	7.75	.25
7½	8.80	.08
10	9.62	.00
15	10.90	.00
20	11.46	.00
30	11.73	.00
40	11.15	.00
50	10.25	.00
60	8.82	.00
70	7.15	.00
80	5.18	.00
90	3.10	.09
95	1.95	.20
100	.65	.35



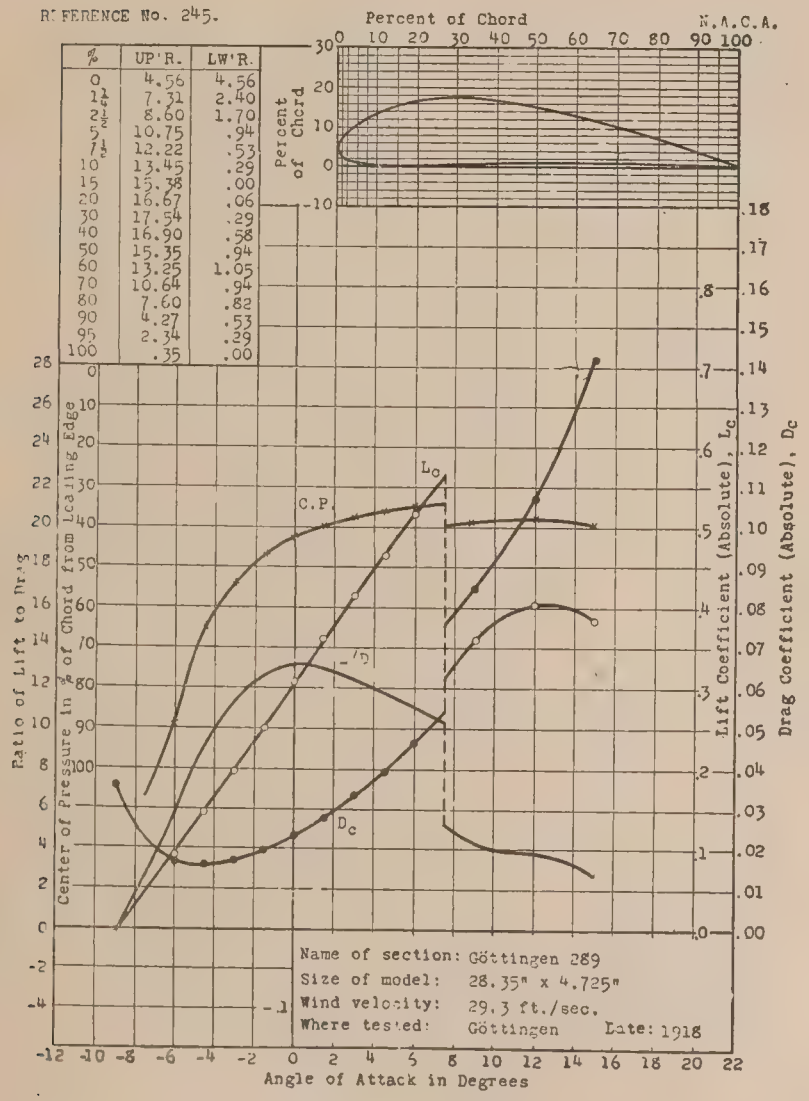
REFERENCE NO. 244.

%	UP'R.	LW'R.
0	5.53	5.53
1¼	8.20	3.12
2½	9.55	2.41
5	11.53	1.53
7½	13.00	.95
10	14.25	.59
15	16.12	.18
20	17.18	.00
30	17.65	.29
40	17.00	1.06
50	15.29	1.65
60	12.94	1.94
70	10.29	1.88
80	7.35	1.47
90	4.12	.88
95	2.41	.46
100	.59	.00



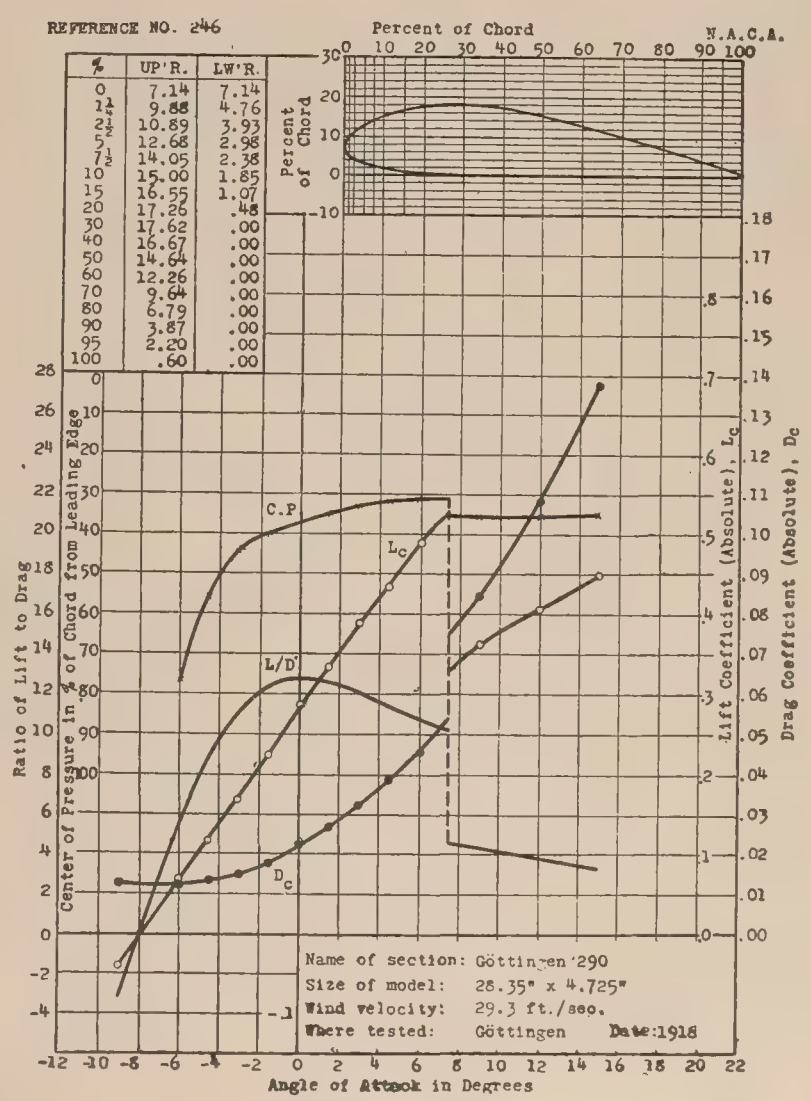
REFERENCE No. 245.

%	UP'R.	LW'R.
0	4.56	4.56
1¼	7.31	2.40
2½	8.60	1.70
5	10.75	.94
7½	12.22	.53
10	13.45	.29
15	15.38	.00
20	16.67	.06
30	17.54	.29
40	16.90	.58
50	15.35	.94
60	13.25	1.05
70	10.64	.94
80	7.60	.82
90	4.27	.53
95	2.34	.29
100	.35	.00



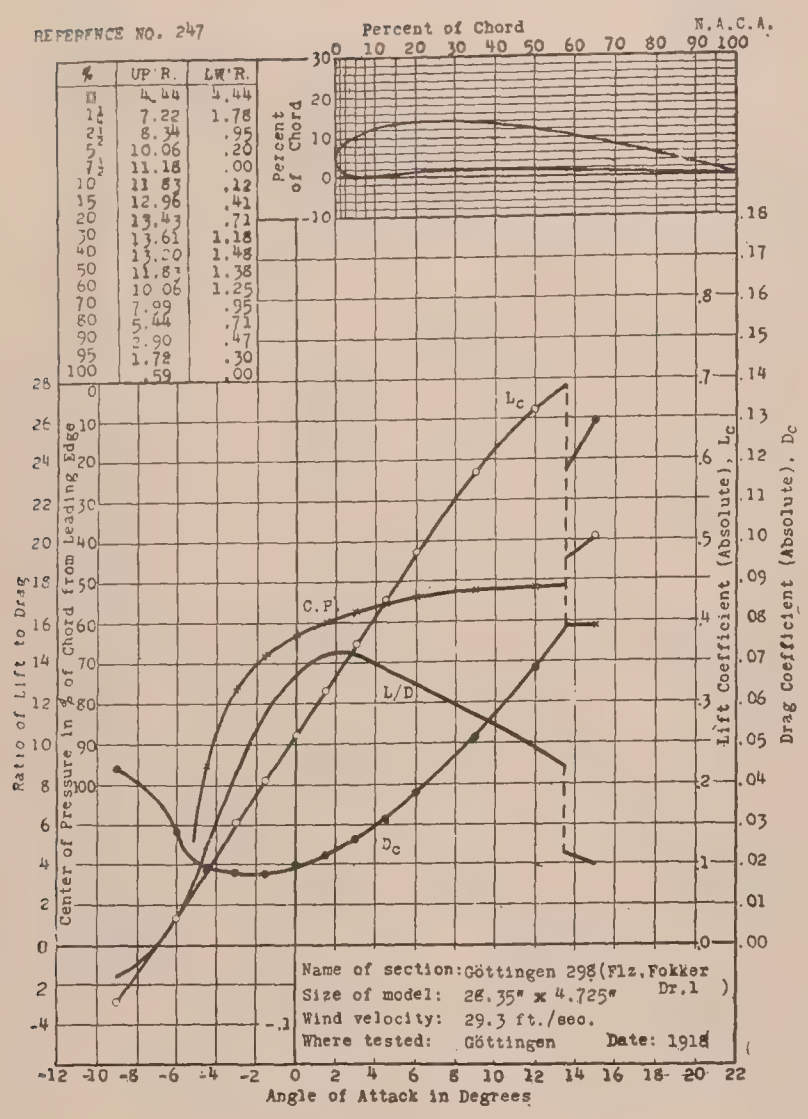
REFERENCE NO. 246

%	UP'R.	LW'R.
0	7.14	7.14
1¼	9.88	4.76
2½	10.89	3.93
5	12.68	2.98
7½	14.05	2.38
10	15.00	1.85
15	16.55	1.07
20	17.26	.48
30	17.62	.00
40	16.67	.00
50	14.64	.00
60	12.26	.00
70	9.64	.00
80	6.79	.00
90	3.87	.00
95	2.20	.00
100	.60	.00



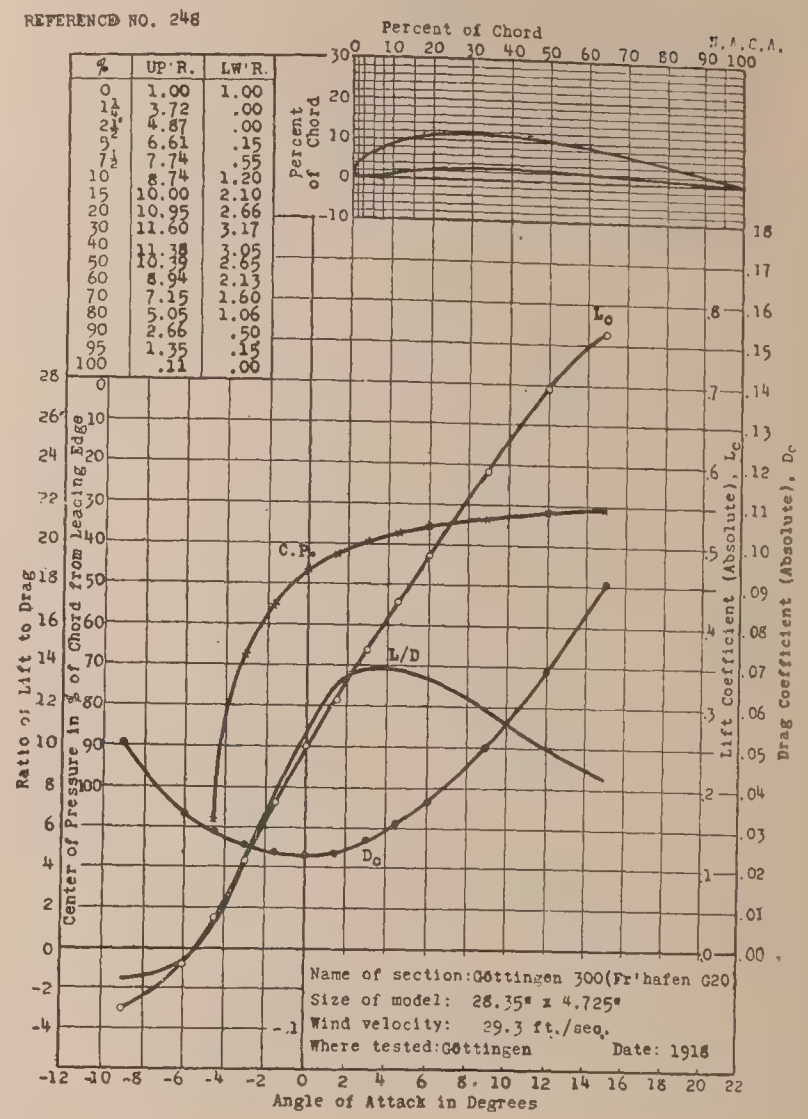
REFERENCE NO. 247

%	UP'R.	LW'R.
0	4.44	4.44
1¼	7.22	1.78
2½	8.34	.95
5	10.06	.20
7½	11.18	.00
10	11.83	.12
15	12.96	.41
20	13.43	.71
30	13.61	1.18
40	13.20	1.48
50	11.83	1.38
60	10.06	1.25
70	7.99	.95
80	5.44	.71
90	2.90	.47
95	1.72	.30
100	.59	.00



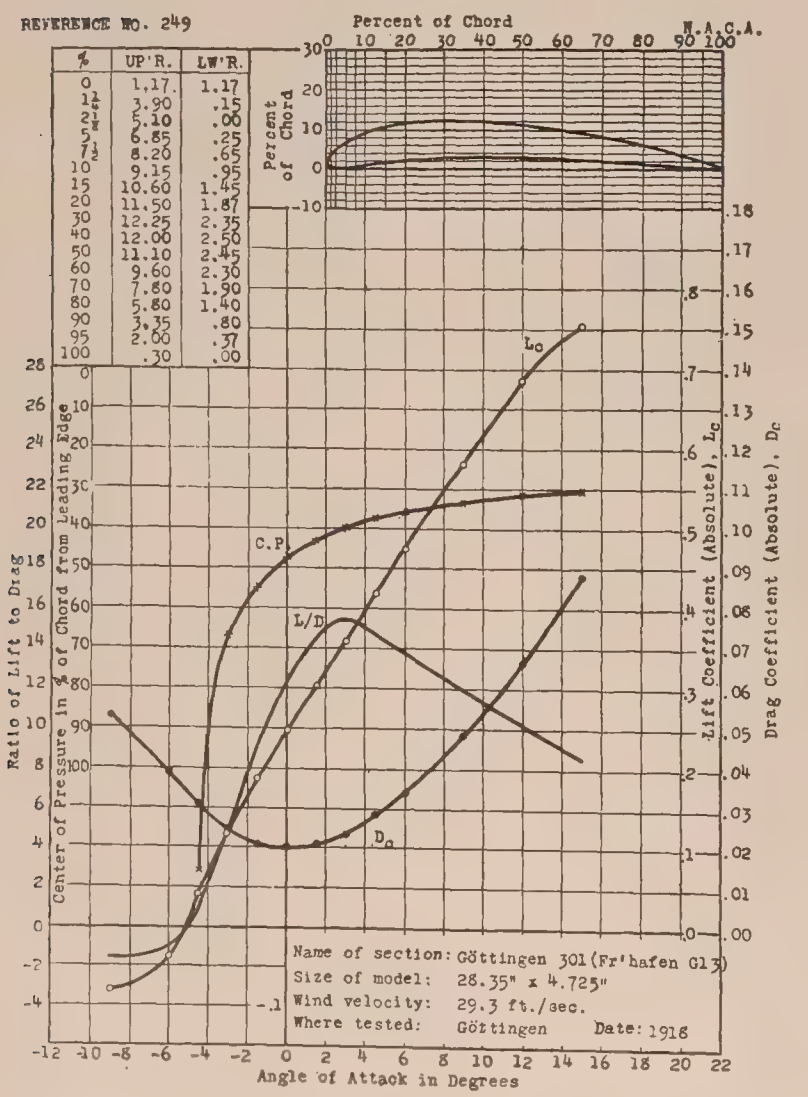
REFERENCE NO. 248

%	UP'R.	LW'R.
0	1.00	1.00
1¼	3.72	.00
2½	4.87	.00
5	6.61	.15
7½	7.74	.55
10	8.74	1.20
15	10.00	2.10
20	10.95	2.66
30	11.60	3.17
40	11.38	3.05
50	10.39	2.65
60	8.94	2.13
70	7.15	1.60
80	5.05	1.06
90	2.66	.50
95	1.35	.15
100	.11	.00



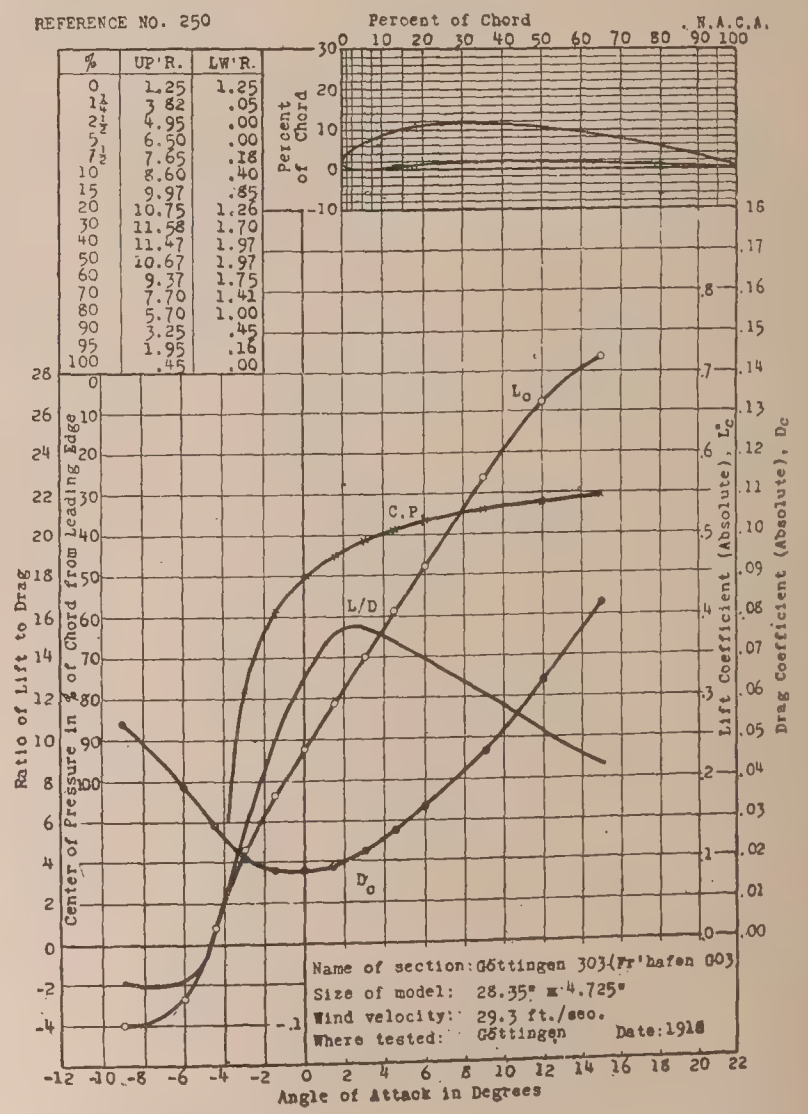
REFERENCE NO. 249

%	UP'R.	LW'R.
0	1.17	1.17
1¼	3.90	.15
2½	5.10	.00
5	6.85	.25
7½	8.20	.65
10	9.15	.95
15	10.60	1.45
20	11.50	1.87
30	12.25	2.35
40	12.00	2.50
50	11.10	2.45
60	9.60	2.30
70	7.80	1.90
80	5.80	1.40
90	3.35	.80
95	2.00	.37
100	.30	.00



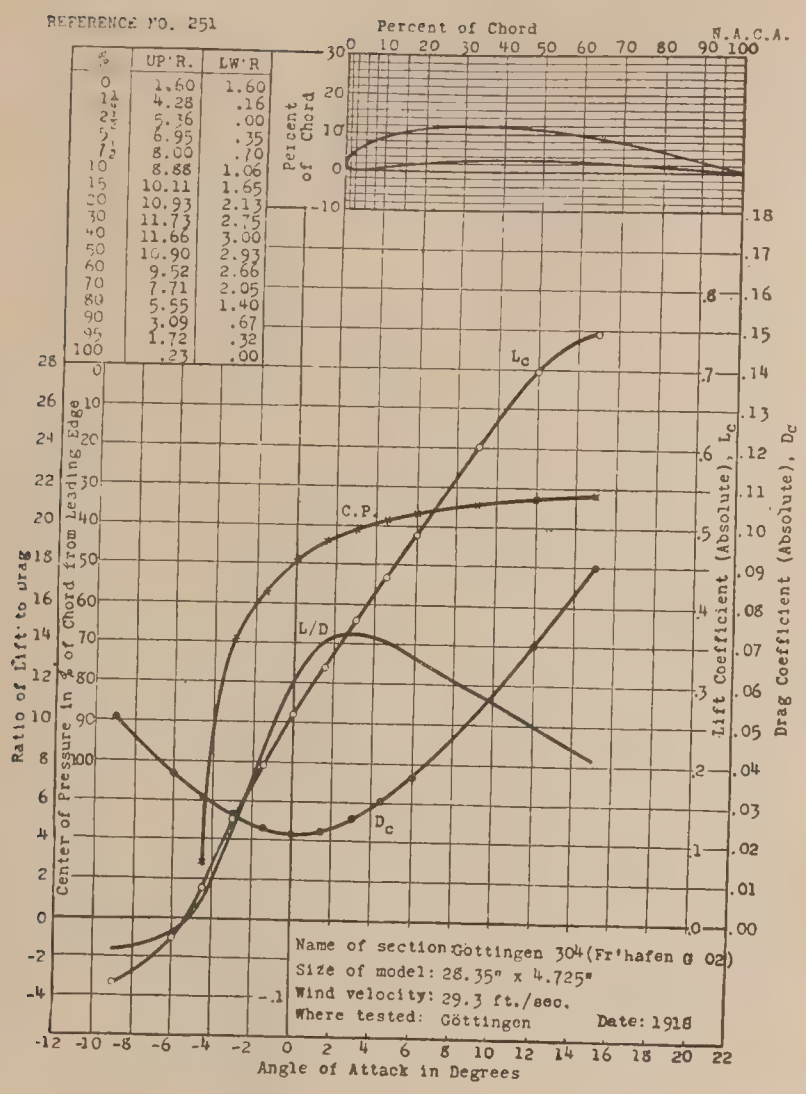
REFERENCE NO. 250

%	UP'R.	LW'R.
0	1.25	1.25
1¼	3.82	.05
2½	4.95	.00
5	6.50	.00
7½	7.65	.18
10	8.60	.40
15	9.97	.85
20	10.75	1.26
30	11.58	1.70
40	11.47	1.97
50	10.67	1.97
60	9.37	1.75
70	7.70	1.41
80	5.70	1.00
90	3.25	.45
95	1.95	.16
100	.45	.00



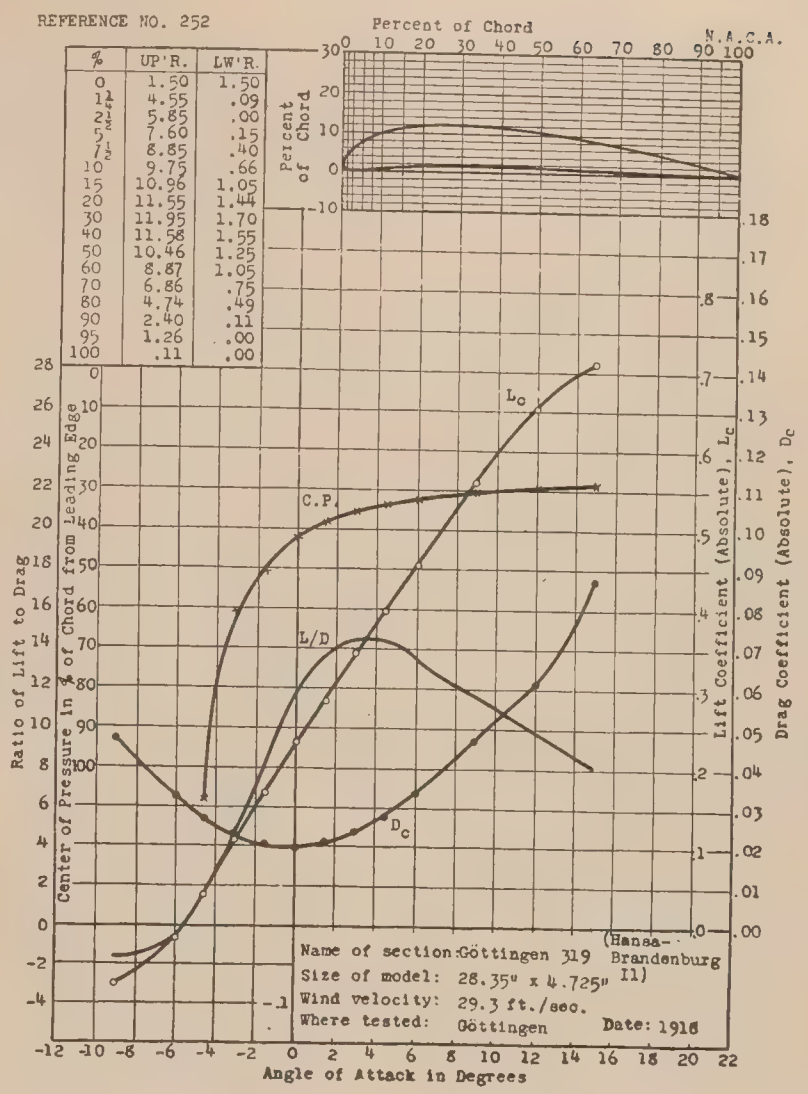
REFERENCE NO. 251

%	UP'R.	LW'R.
0	1.60	1.60
1¼	4.28	.16
2½	5.36	.00
5	6.95	.35
7½	8.00	.70
10	8.88	1.06
15	10.11	1.65
20	10.93	2.13
30	11.73	2.75
40	11.66	3.00
50	10.90	2.93
60	9.52	2.66
70	7.71	2.05
80	5.55	1.40
90	3.09	.67
95	1.72	.32
100	.23	.00



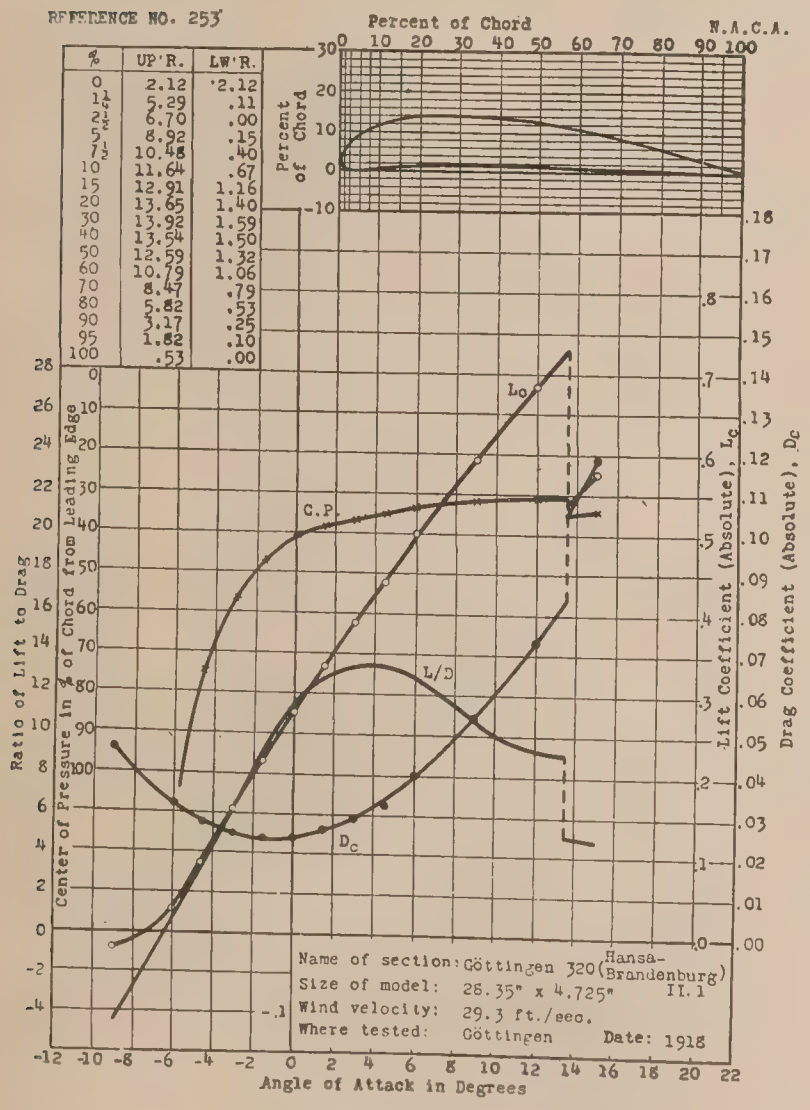
REFERENCE NO. 252

%	UP'R.	LW'R.
0	1.50	1.50
1¼	4.55	.09
2½	5.85	.00
5	7.60	.15
7½	8.85	.40
10	9.75	.66
15	10.96	1.05
20	11.55	1.44
30	11.95	1.70
40	11.58	1.55
50	10.46	1.25
60	8.87	1.05
70	6.86	.75
80	4.74	.49
90	2.40	.11
95	1.26	.00
100	.11	.00



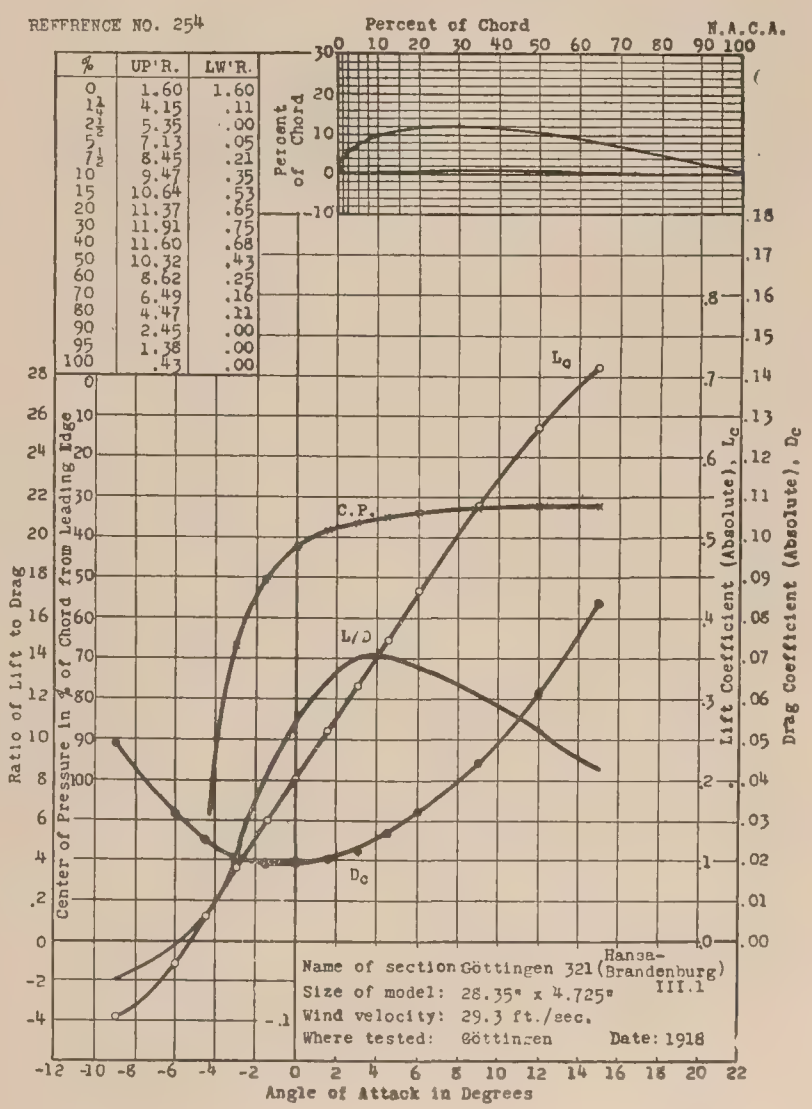
REFERENCE NO. 253

%	UP'R.	LW'R.
0	2.12	2.12
1¼	5.29	.11
2½	6.70	.00
5	8.92	.15
7½	10.48	.40
10	11.64	.67
15	12.91	1.16
20	13.65	1.40
30	13.92	1.59
40	13.54	1.50
50	12.59	1.32
60	10.79	1.06
70	8.47	.79
80	5.82	.53
90	3.17	.25
95	1.82	.10
100	.53	.00



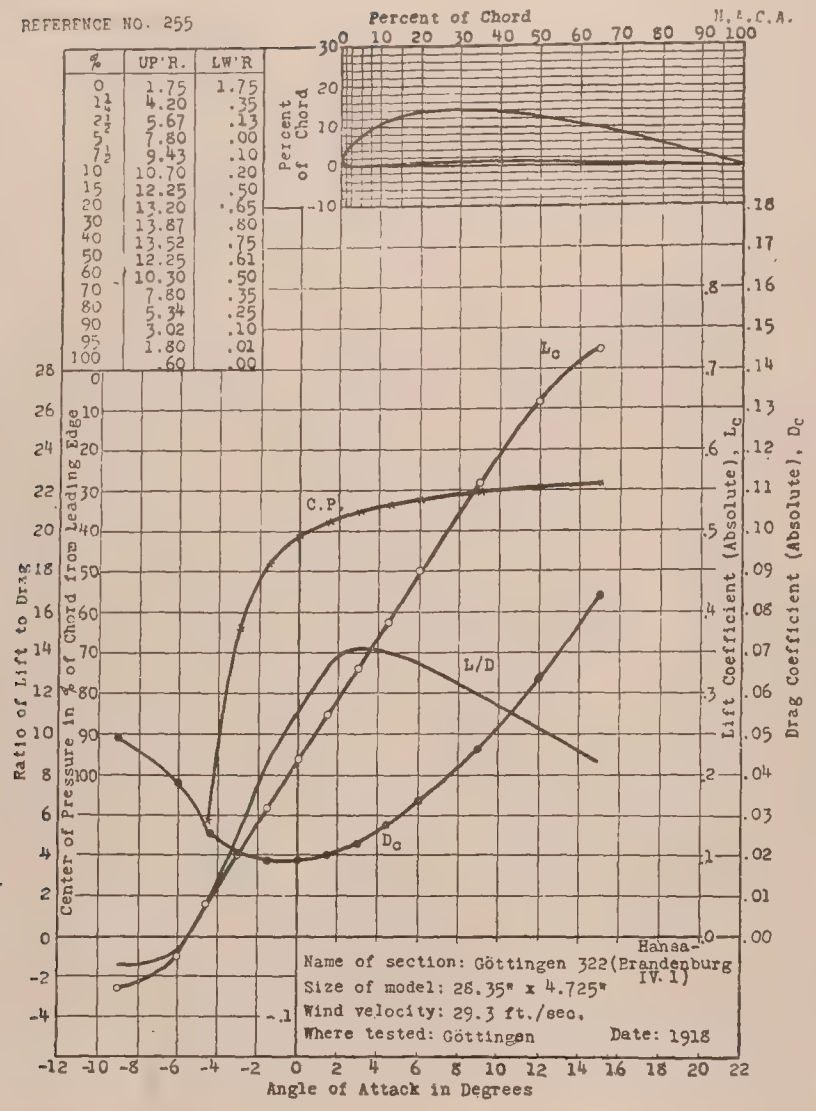
REFERENCE NO. 254

%	UP'R.	LW'R.
0	1.60	1.60
1¼	4.15	.11
2½	5.35	.00
5	7.13	.05
7½	8.45	.21
10	9.47	.35
15	10.64	.53
20	11.37	.65
30	11.91	.75
40	11.60	.68
50	10.32	.43
60	8.62	.25
70	6.49	.16
80	4.47	.11
90	2.45	.00
95	1.38	.00
100	.43	.00



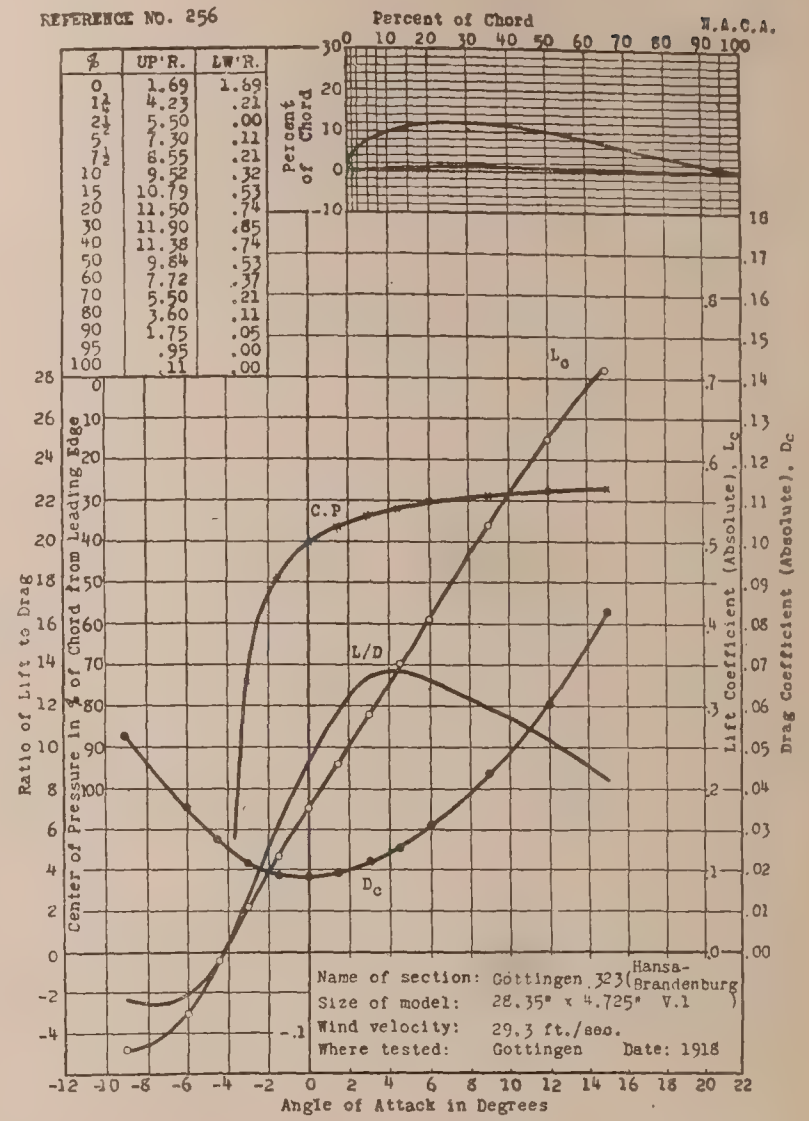
REFERENCE NO. 255

%	UP'R.	LW'R.
0	1.75	1.75
1¼	4.20	.35
2½	5.67	.13
5	7.80	.00
7½	9.43	.10
10	10.70	.20
15	12.25	.50
20	13.20	.65
30	13.87	.80
40	13.52	.75
50	12.25	.61
60	10.30	.50
70	7.80	.35
80	5.34	.25
90	3.02	.10
95	1.80	.01
100	.60	.00



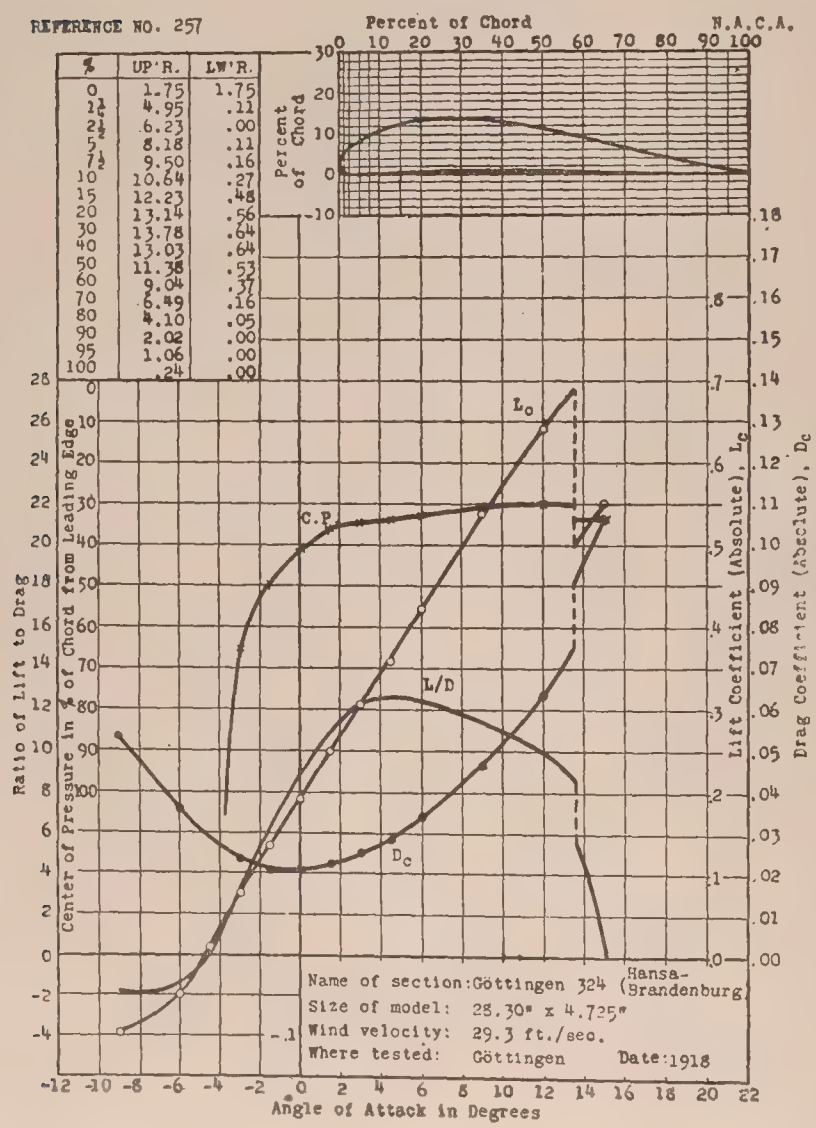
REFERENCE NO. 256

%	UP'R.	LW'R.
0	1.69	1.69
1¼	4.23	.21
2½	5.50	.00
5	7.30	.11
7½	8.55	.21
10	9.52	.32
15	10.79	.53
20	11.50	.74
30	11.90	.85
40	11.38	.74
50	9.84	.53
60	7.72	.37
70	5.50	.21
80	3.60	.11
90	1.75	.05
95	.95	.00
100	.11	.00



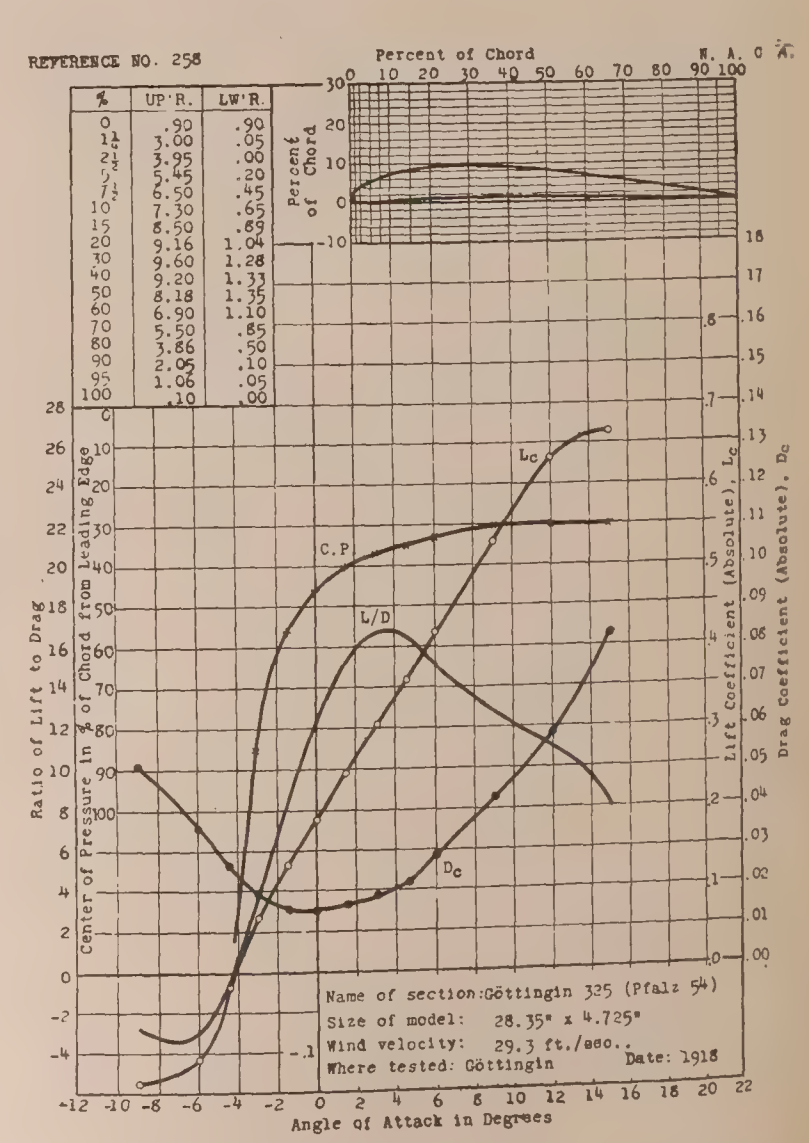
REFERENCE NO. 257

%	UP'R.	LW'R.
0	1.75	1.75
1¼	4.95	.11
2½	6.23	.00
5	8.18	.11
7½	9.50	.16
10	10.64	.27
15	12.23	.48
20	13.14	.56
30	13.78	.64
40	13.03	.64
50	11.38	.53
60	9.04	.37
70	6.49	.16
80	4.10	.05
90	2.02	.00
95	1.06	.00
100	.24	.00



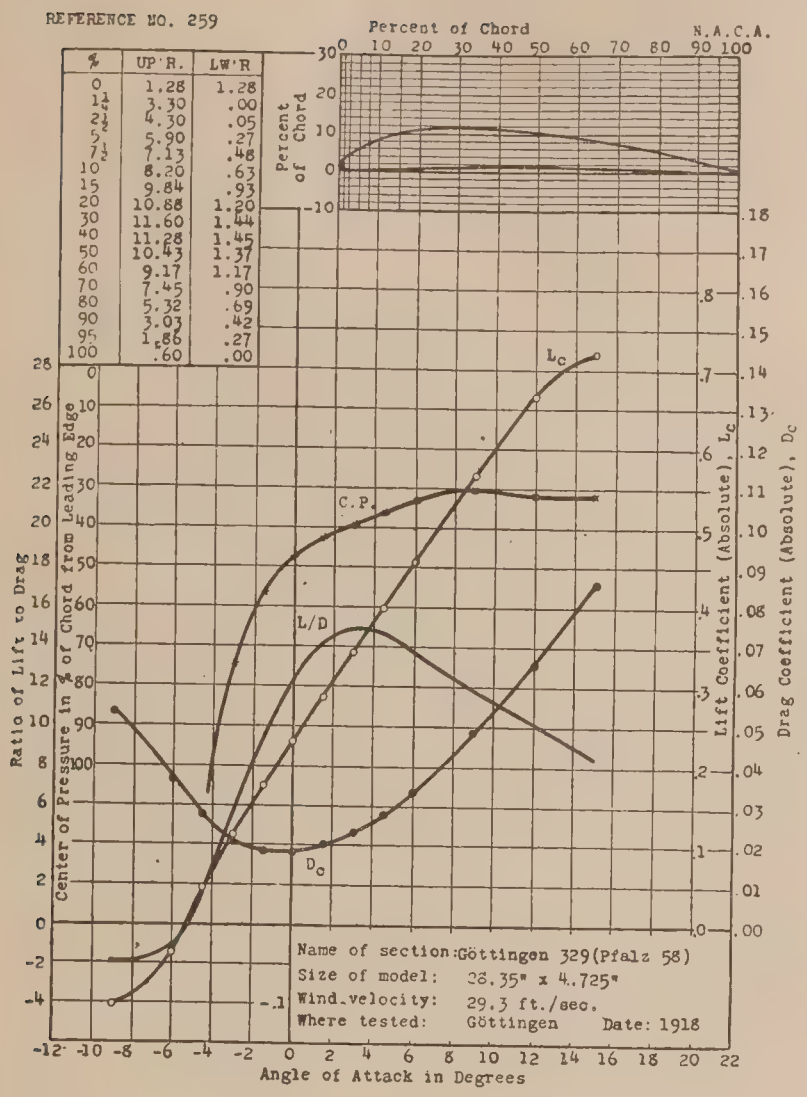
REFERENCE NO. 258

%	UP'R.	LW'R.
0	.90	.90
1¼	3.00	.05
2½	3.95	.00
5	5.45	.20
7½	6.50	.45
10	7.30	.65
15	8.50	.89
20	9.16	1.04
30	9.60	1.28
40	9.20	1.33
50	8.18	1.35
60	6.90	1.10
70	5.50	.85
80	3.86	.50
90	2.05	.10
95	1.06	.05
100	.10	.00



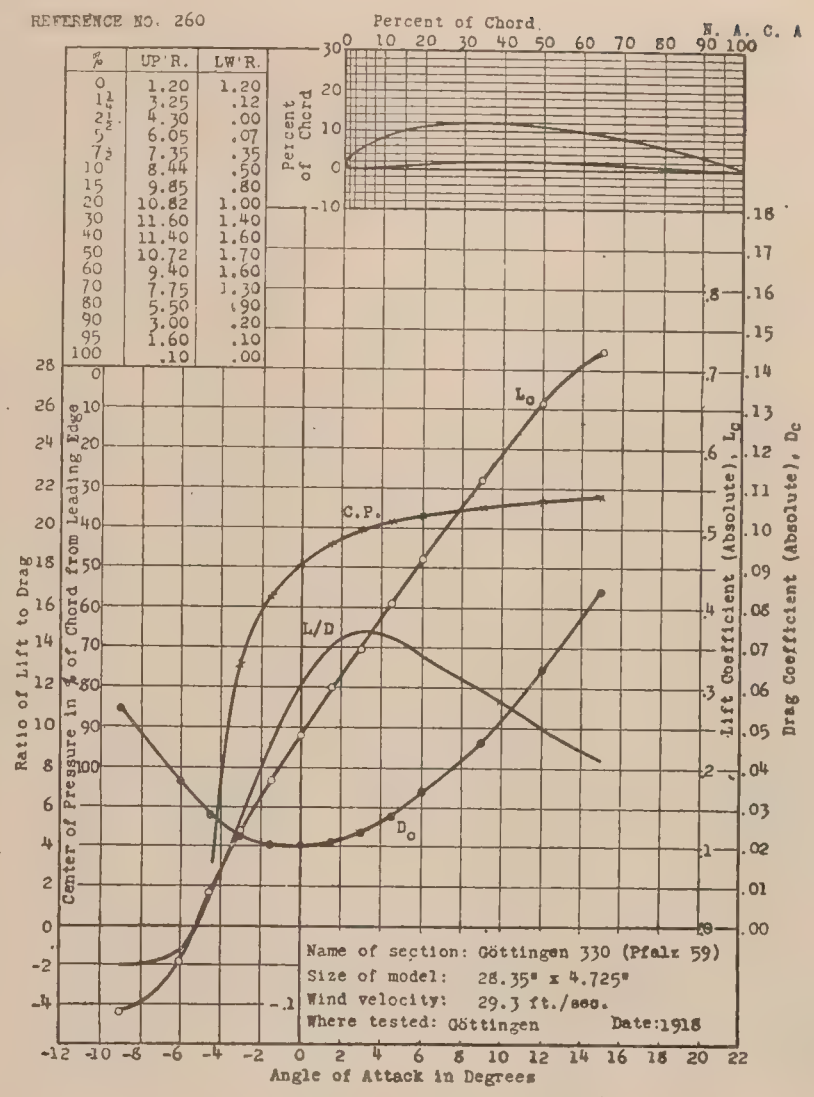
REFERENCE NO. 259

%	UP'R.	LW'R
0	1.28	1.28
1¼	3.30	.00
2½	4.30	.05
5	5.90	.27
7½	7.13	.48
10	8.20	.63
15	9.84	.93
20	10.88	1.20
30	11.60	1.44
40	11.28	1.45
50	10.43	1.37
60	9.17	1.17
70	7.45	.90
80	5.32	.69
90	3.03	.42
95	1.86	.27
100	.60	.00



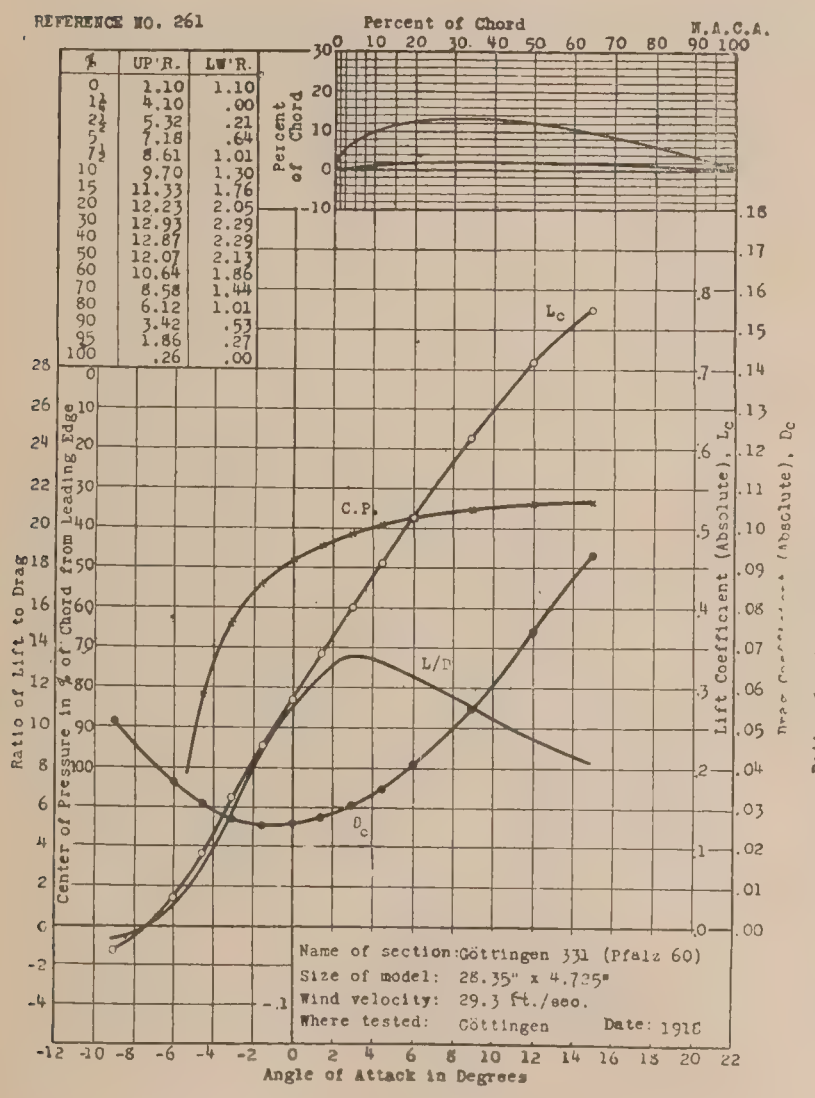
REFERENCE NO. 260

%	UP'R.	LW'R.
0	1.20	1.20
1¼	3.25	.12
2½	4.30	.00
5	6.05	.07
7½	7.35	.35
10	8.44	.50
15	9.85	.80
20	10.82	1.00
30	11.60	1.40
40	11.40	1.60
50	10.72	1.70
60	9.40	1.60
70	7.75	1.30
80	5.50	.90
90	3.00	.20
95	1.60	.10
100	.10	.00



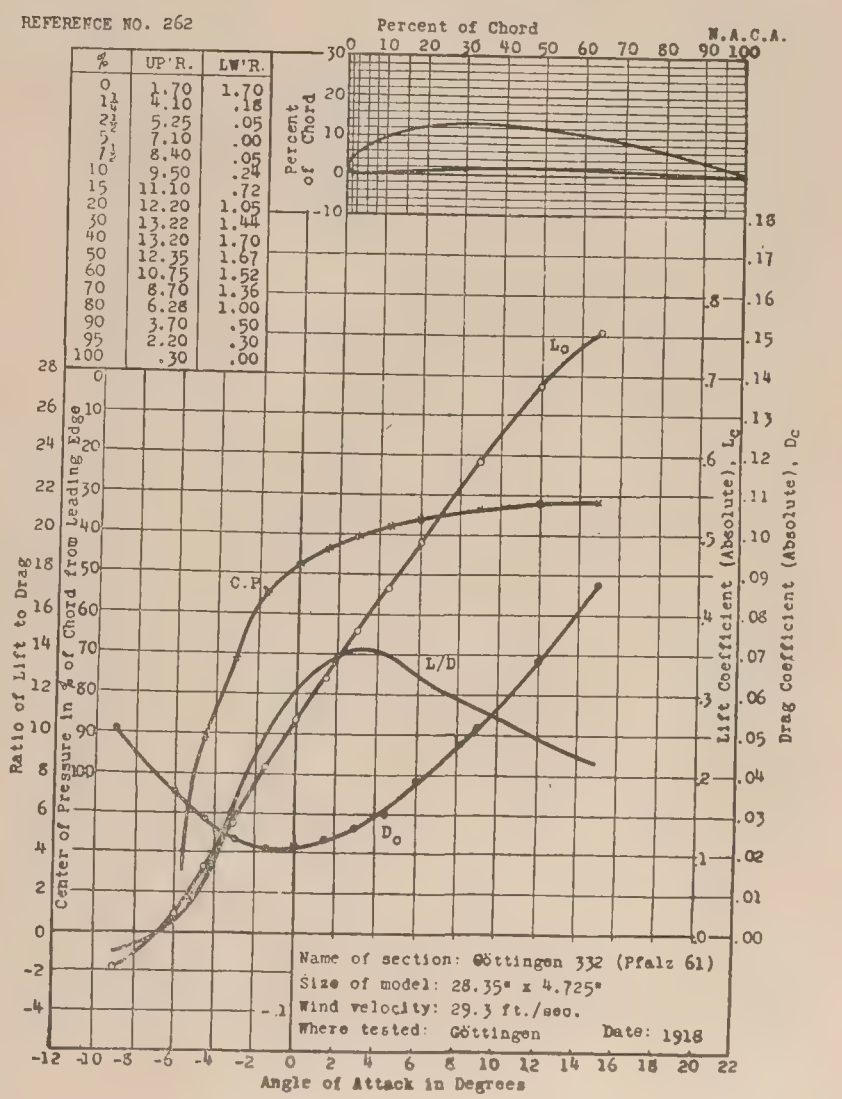
REFERENCE NO. 261

%	UP'R.	LW'R.
0	1.10	1.10
1¼	4.10	.00
2½	5.32	.21
5	7.18	.64
7½	8.61	1.01
10	9.70	1.30
15	11.33	1.76
20	12.23	2.05
30	12.93	2.29
40	12.87	2.29
50	12.07	2.13
60	10.64	1.86
70	8.58	1.44
80	6.12	1.01
90	3.42	.53
95	1.86	.27
100	.26	.00



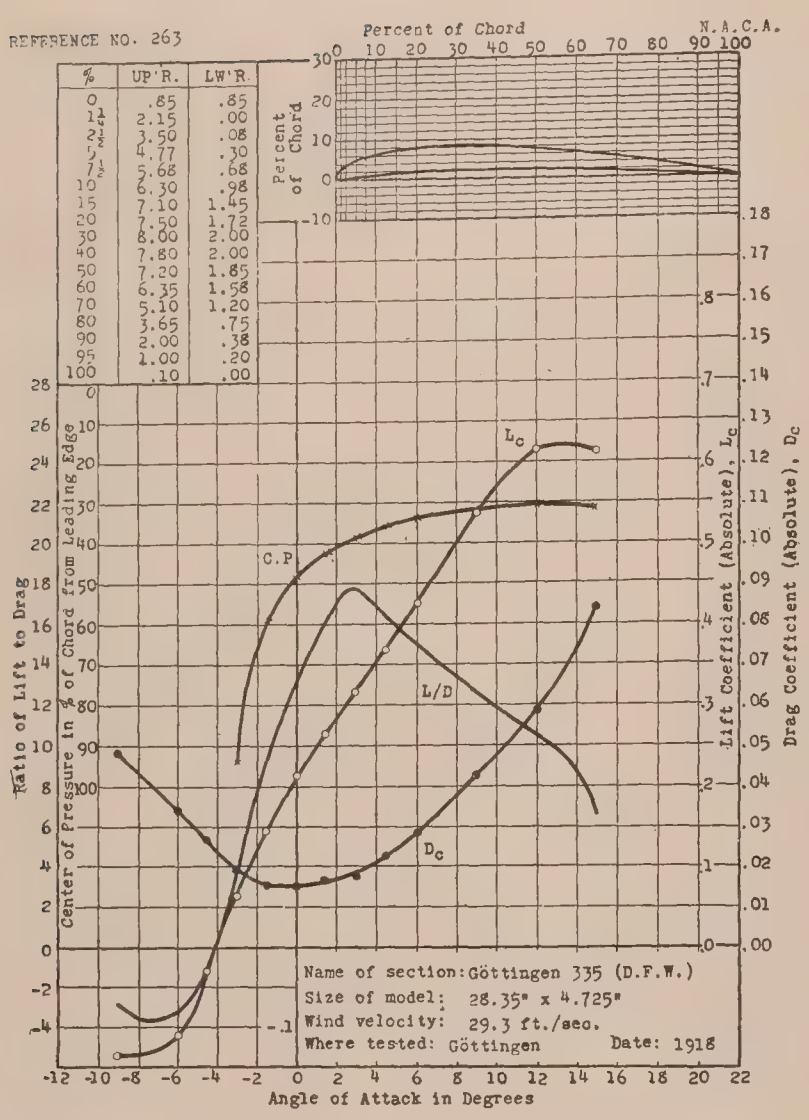
REFERENCE NO. 262

%	UP'R.	LW'R.
0	1.70	1.70
1¼	4.10	.18
2½	5.25	.05
5	7.10	.00
7½	8.40	.05
10	9.50	.24
15	11.10	.72
20	12.20	1.05
30	13.22	1.44
40	13.20	1.70
50	12.35	1.67
60	10.75	1.52
70	8.70	1.36
80	6.28	1.00
90	3.70	.50
95	2.20	.30
100	.30	.00



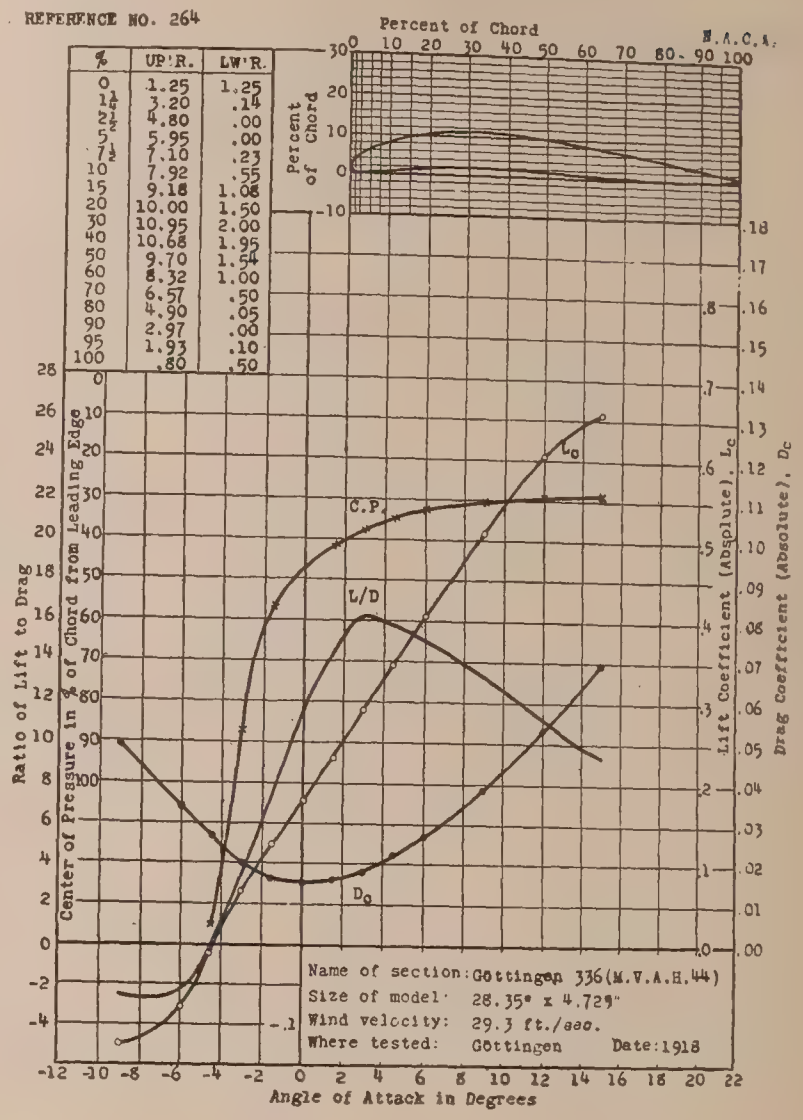
REFERENCE NO. 263

%	UP'R.	LW'R.
0	.85	.85
1¼	2.15	.00
2½	3.50	.08
5	4.77	.30
7½	5.68	.68
10	6.30	.98
15	7.10	1.45
20	7.50	1.72
30	8.00	2.00
40	7.80	2.00
50	7.20	1.85
60	6.35	1.58
70	5.10	1.20
80	3.65	.75
90	2.00	.38
95	1.00	.20
100	.10	.00



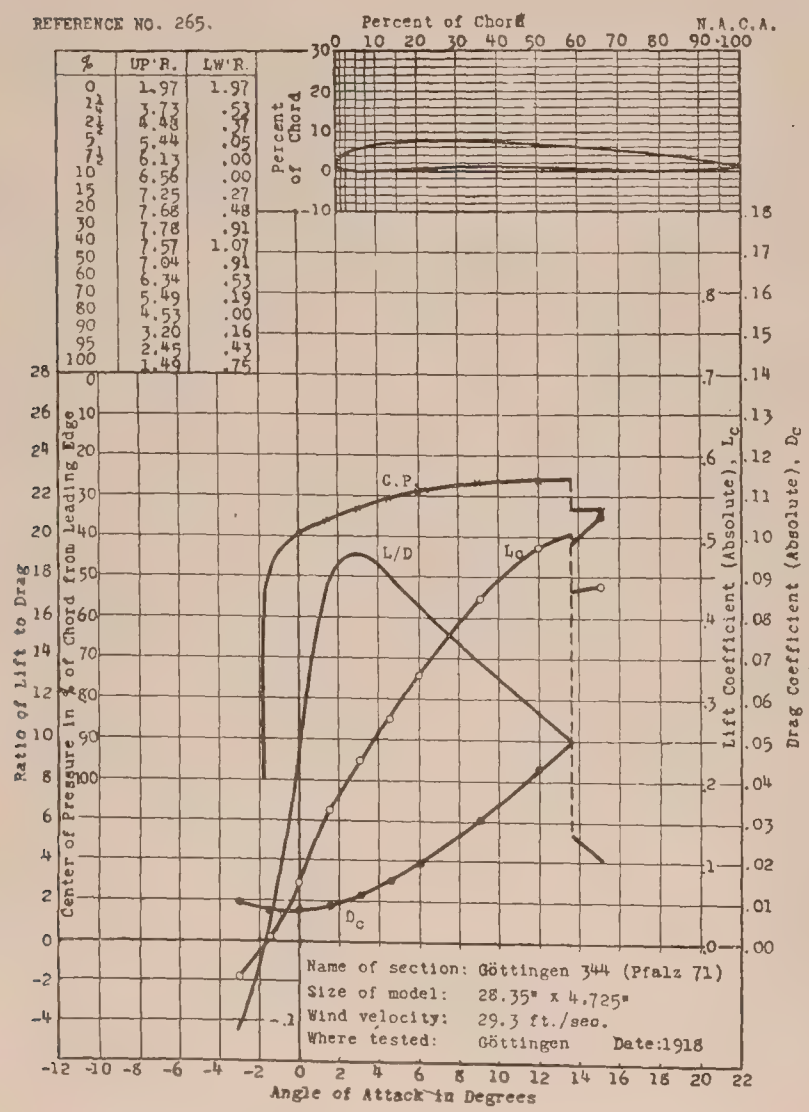
REFERENCE NO. 264

%	UP'R.	LW'R.
0	1.25	1.25
1¼	3.20	.14
2½	4.80	.00
5	5.95	.00
7½	7.10	.23
10	7.92	.55
15	9.18	1.08
20	10.00	1.50
30	10.95	2.00
40	10.68	1.95
50	9.70	1.54
60	8.32	1.00
70	6.57	.50
80	4.90	.05
90	2.97	.00
95	1.93	.10
100	.80	.50



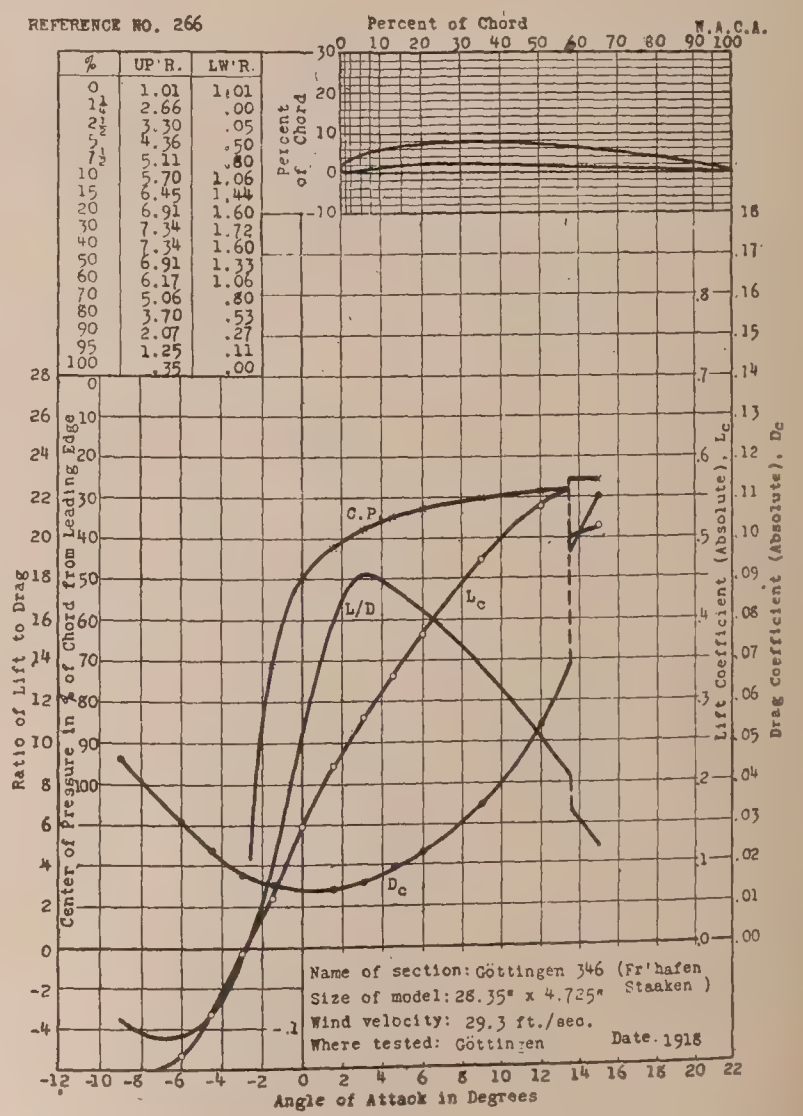
REFERENCE NO. 265.

%	UP'R.	LW'R.
0	1.97	1.97
1¼	3.73	.53
2½	4.48	.37
5	5.44	.05
7½	6.13	.00
10	6.56	.00
15	7.25	.27
20	7.68	.48
30	7.78	.91
40	7.57	1.07
50	7.04	.91
60	6.34	.53
70	5.49	.19
80	4.53	.00
90	3.20	.16
95	2.45	.43
100	1.48	.75



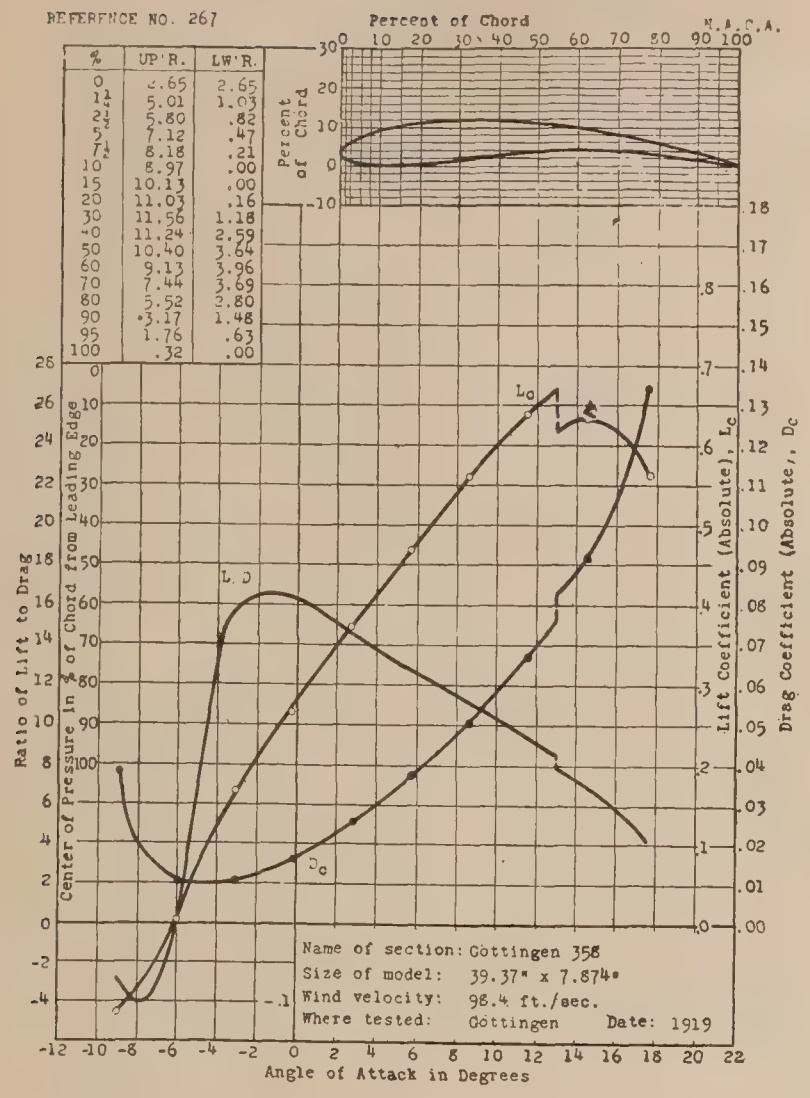
REFERENCE NO. 266

%	UP'R.	LW'R.
0	1.01	1.01
1¼	2.66	.00
2½	3.30	.05
5	4.36	.50
7½	5.11	.80
10	5.70	1.06
15	6.45	1.44
20	6.91	1.60
30	7.34	1.72
40	7.34	1.60
50	6.91	1.33
60	6.17	1.06
70	5.06	.80
80	3.70	.53
90	2.07	.27
95	1.25	.11
100	.35	.00



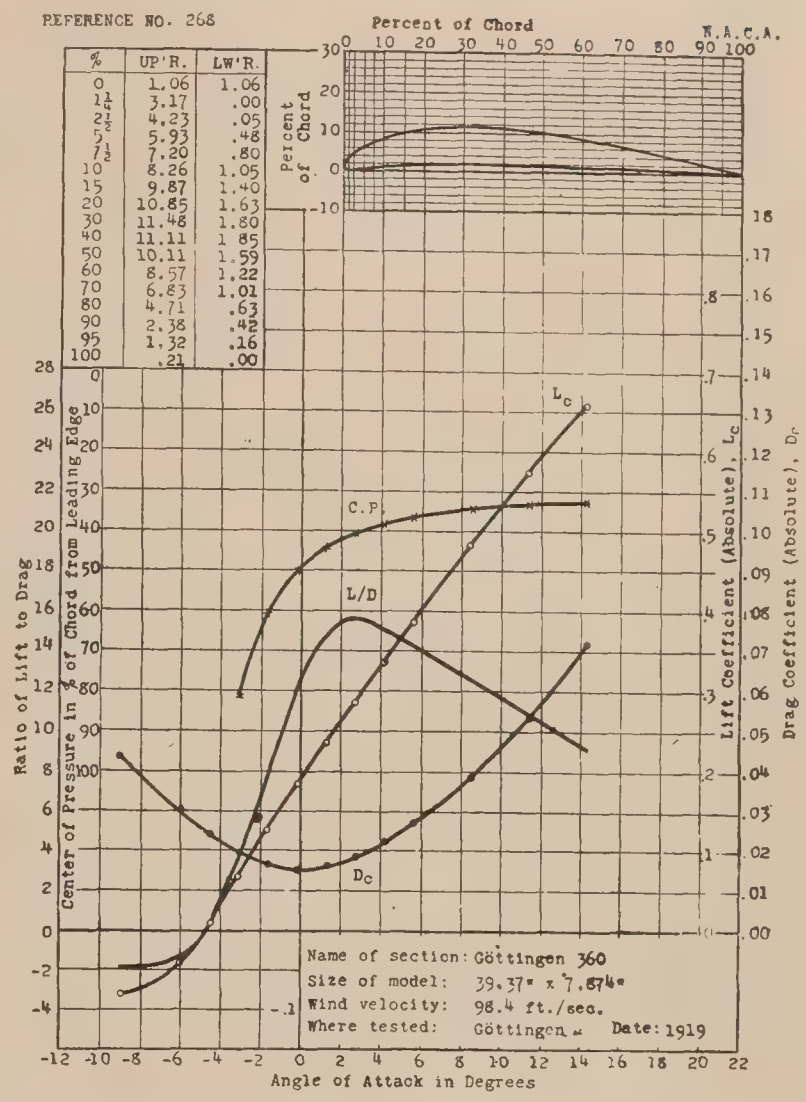
REFERENCE NO. 267

%	UP'R.	LW'R.
0	2.65	2.65
1¼	5.01	1.03
2½	5.80	.82
5	7.12	.47
7½	8.18	.21
10	8.97	.00
15	10.13	.00
20	11.03	.16
30	11.56	1.18
40	11.24	2.59
50	10.40	3.64
60	9.13	3.96
70	7.44	3.69
80	5.52	2.80
90	3.17	1.48
95	1.76	.63
100	.32	.00



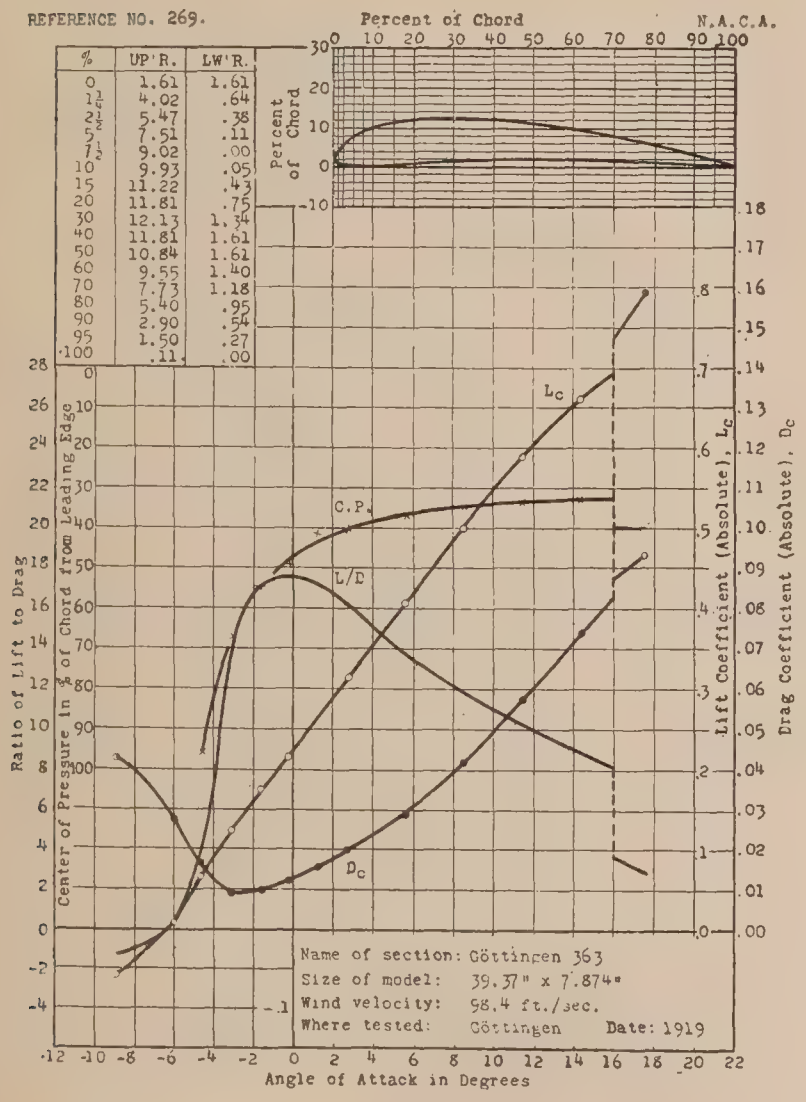
REFERENCE NO. 268

%	UP'R.	LW'R.
0	1.06	1.06
1¼	3.17	.00
2½	4.23	.05
5	5.93	.48
7½	7.20	.80
10	8.26	1.05
15	9.87	1.40
20	10.85	1.63
30	11.48	1.80
40	11.11	1.85
50	10.11	1.59
60	8.57	1.22
70	6.83	1.01
80	4.71	.63
90	2.38	.42
95	1.32	.16
100	.21	.00



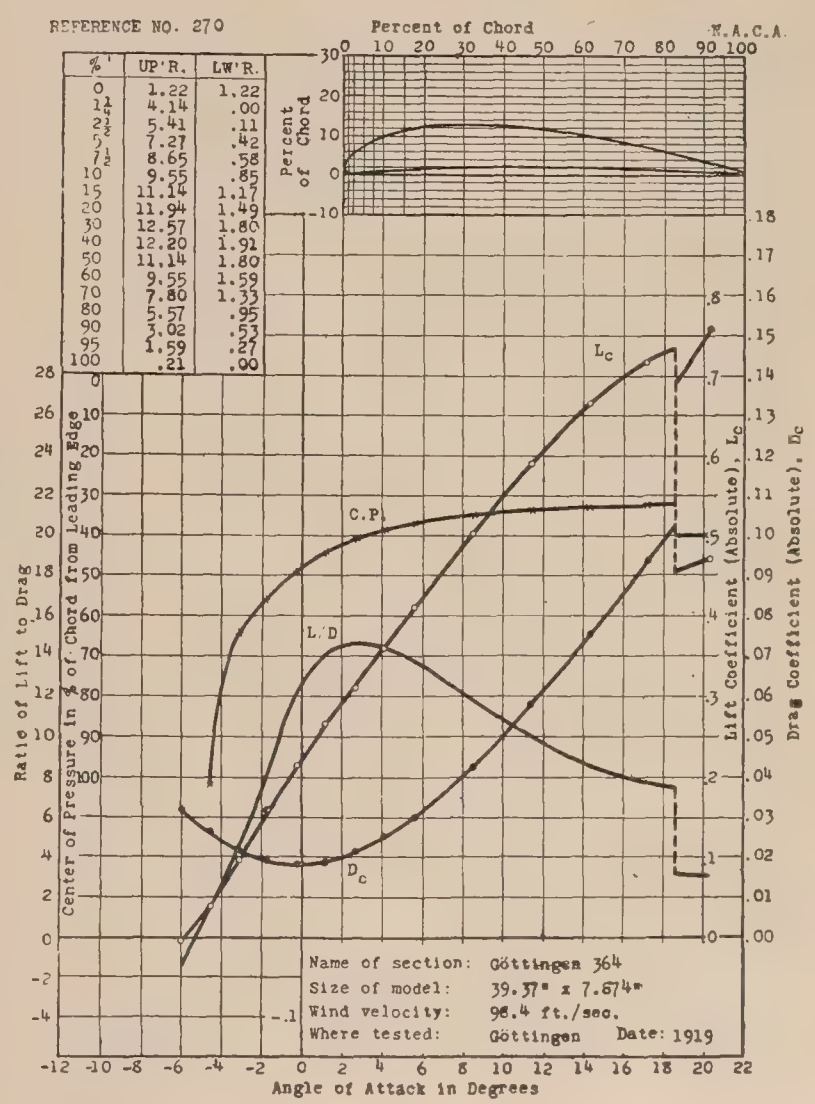
REFERENCE NO. 269.

%	UP'R.	LW'R.
0	1.61	1.61
1¼	4.02	.64
2½	5.47	.38
5	7.51	.11
7½	9.02	.00
10	9.93	.05
15	11.22	.43
20	11.81	.75
30	12.13	1.34
40	11.81	1.61
50	10.84	1.61
60	9.55	1.40
70	7.73	1.18
80	5.40	.95
90	2.90	.54
95	1.50	.27
100	.11	.00



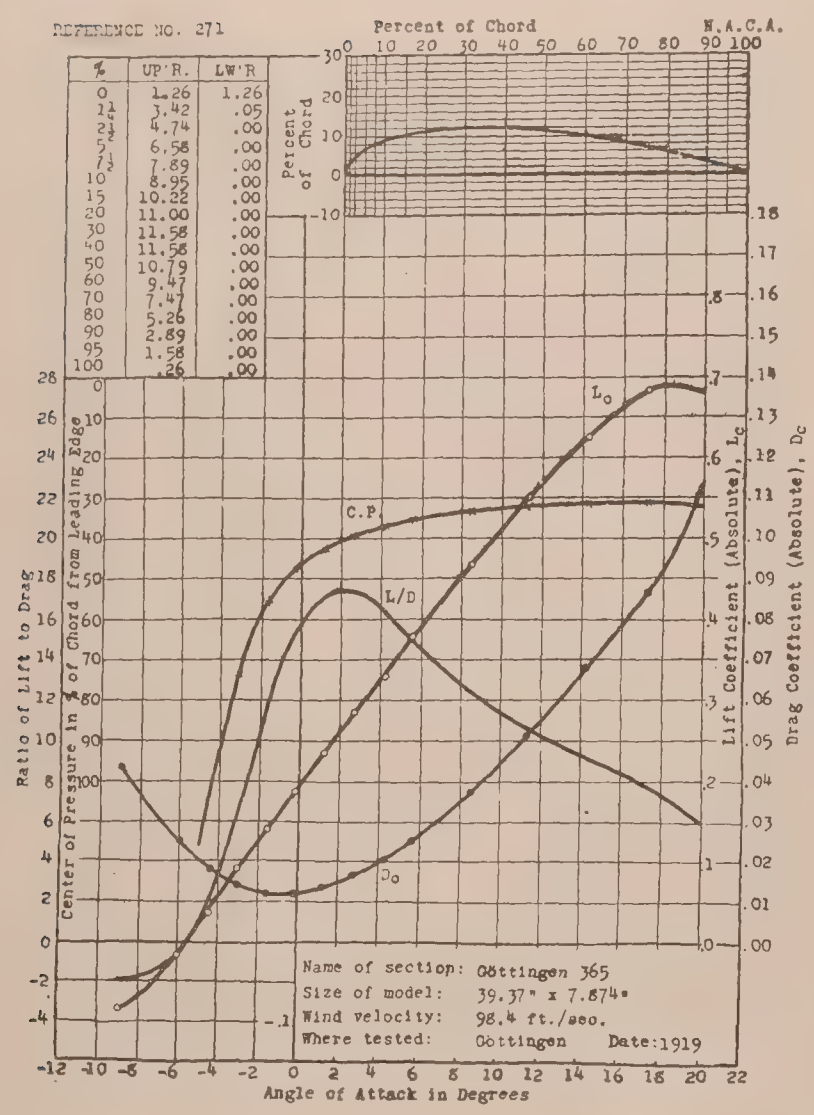
REFERENCE NO. 270

%	UP'R.	LW'R.
0	1.22	1.22
1¼	4.14	.00
2½	5.41	.11
5	7.27	.42
7½	8.65	.58
10	9.55	.85
15	11.14	1.17
20	11.94	1.49
30	12.57	1.80
40	12.20	1.91
50	11.14	1.80
60	9.55	1.59
70	7.80	1.33
80	5.57	.95
90	3.02	.53
95	1.59	.27
100	.21	.00



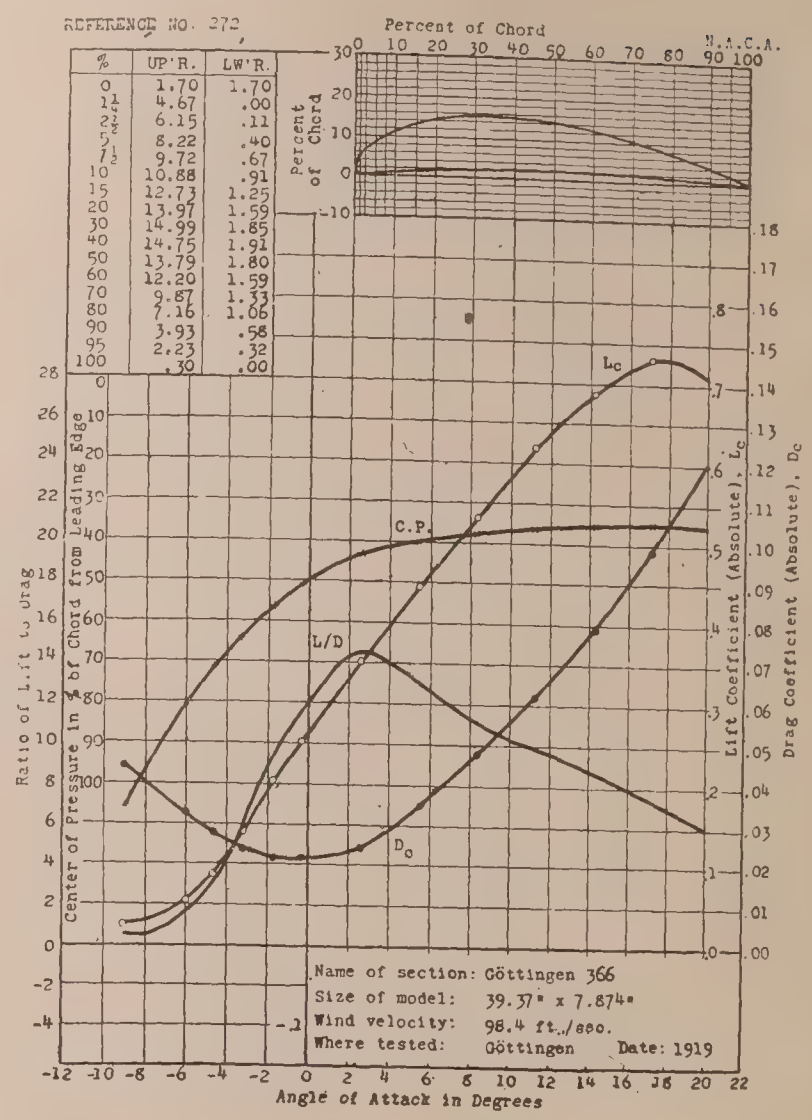
REFERENCE NO. 271

%	UP'R.	LW'R
0	1.26	1.26
1¼	3.42	.05
2½	4.74	.00
5	6.58	.00
7½	7.89	.00
10	8.95	.00
15	10.22	.00
20	11.00	.00
30	11.58	.00
40	11.58	.00
50	10.79	.00
60	9.47	.00
70	7.47	.00
80	5.26	.00
90	2.89	.00
95	1.58	.00
100	.26	.00



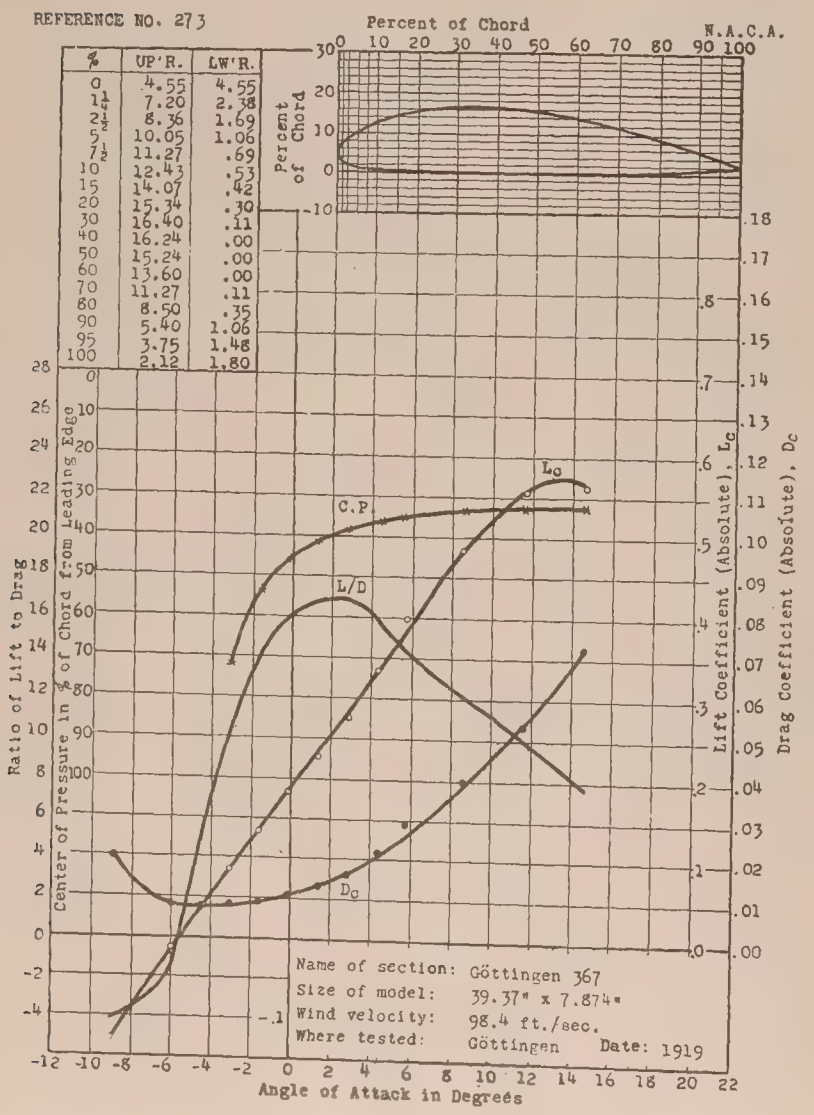
REFERENCE NO. 272

%	UP'R.	LW'R.
0	1.70	1.70
1¼	4.67	.00
2½	6.15	.11
5	8.22	.40
7½	9.72	.67
10	10.88	.91
15	12.73	1.25
20	13.97	1.59
30	14.99	1.85
40	14.75	1.91
50	13.79	1.80
60	12.20	1.59
70	9.87	1.33
80	7.16	1.06
90	3.93	.58
95	2.23	.32
100	.30	.00



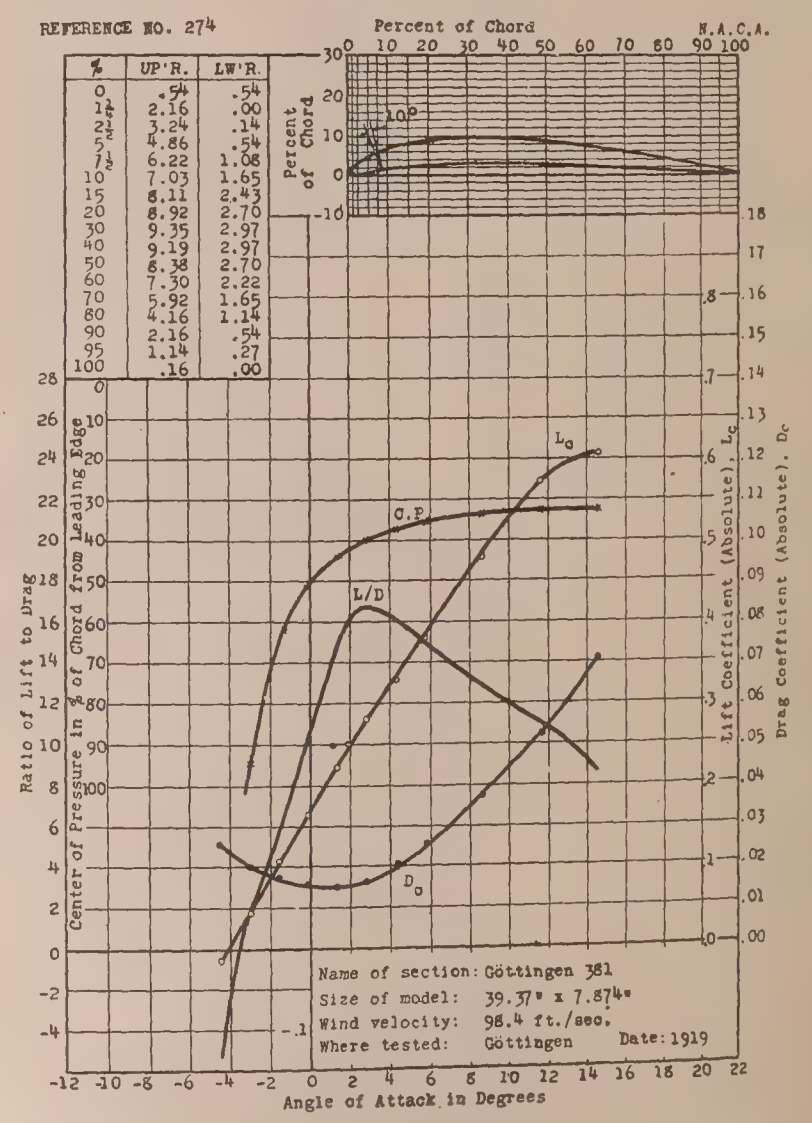
REFERENCE NO. 273

%	UP'R.	LW'R.
0	4.55	4.55
1¼	7.20	2.38
2½	8.36	1.69
5	10.05	1.06
7½	11.27	.69
10	12.43	.53
15	14.07	.42
20	15.34	.30
30	16.40	.11
40	16.24	.00
50	15.24	.00
60	13.60	.00
70	11.27	.11
80	8.50	.35
90	5.40	1.06
95	3.75	1.48
100	2.12	1.80



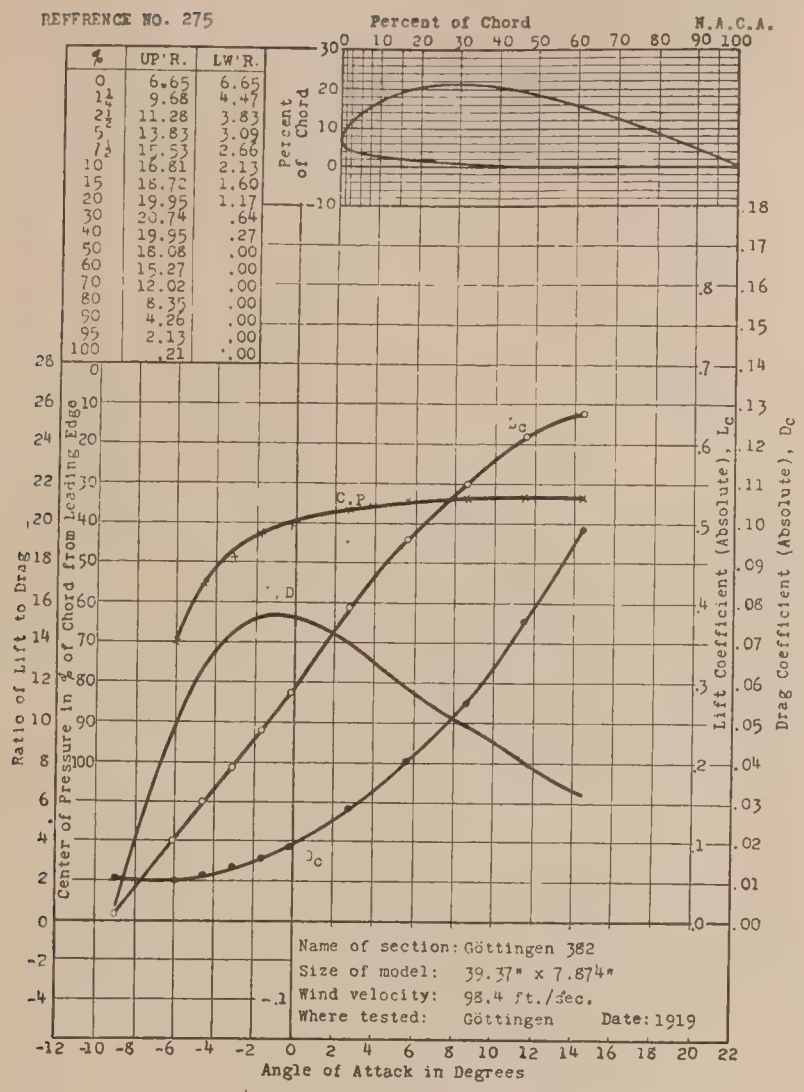
REFERENCE NO. 274

%	UP'R.	LW'R.
0	.54	.54
1¼	2.16	.00
2½	3.24	.14
5	4.86	.54
7½	6.22	1.08
10	7.03	1.65
15	8.11	2.43
20	8.92	2.70
30	9.35	2.97
40	9.19	2.97
50	8.38	2.70
60	7.30	2.22
70	5.92	1.65
80	4.16	1.14
90	2.16	.54
95	1.14	.27
100	.16	.00



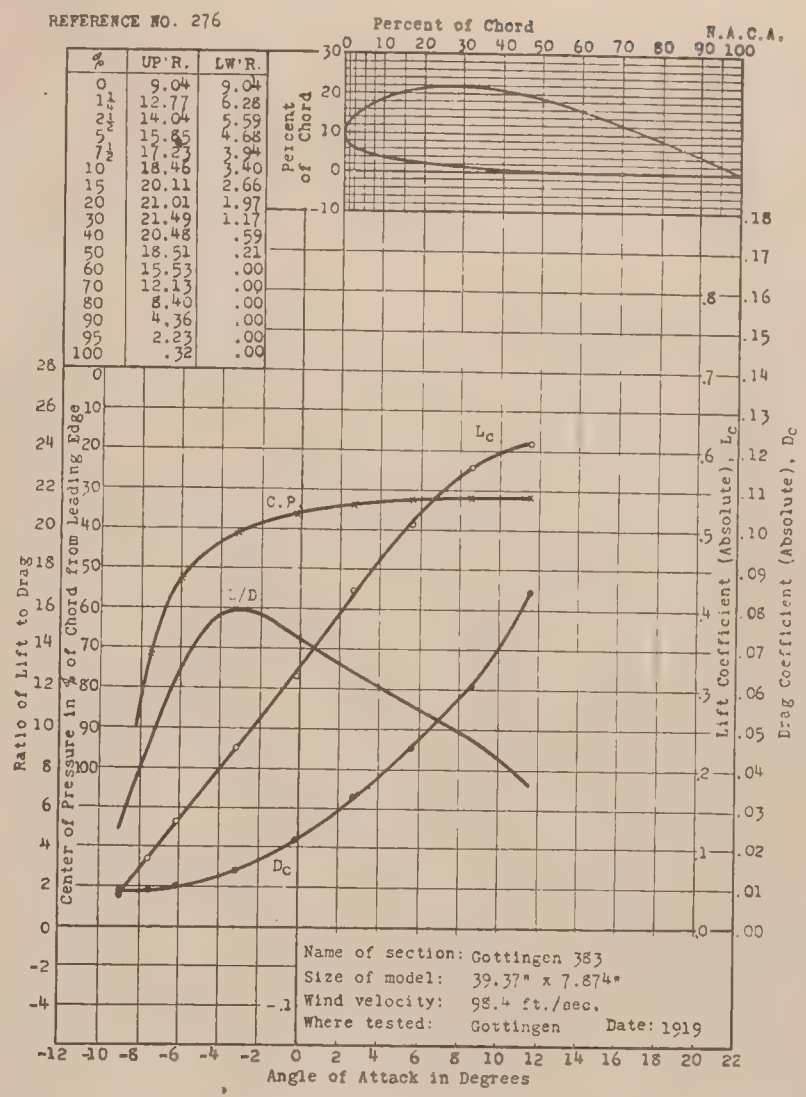
REFFRENCE NO. 275

%	UP'R.	LW'R.
0	6.65	6.65
1¼	9.68	4.47
2½	11.28	3.83
5	13.83	3.09
7½	15.53	2.66
10	16.81	2.13
15	18.72	1.60
20	19.95	1.17
30	20.74	.64
40	19.95	.27
50	18.08	.00
60	15.27	.00
70	12.02	.00
80	8.35	.00
90	4.26	.00
95	2.13	.00
100	.21	.00



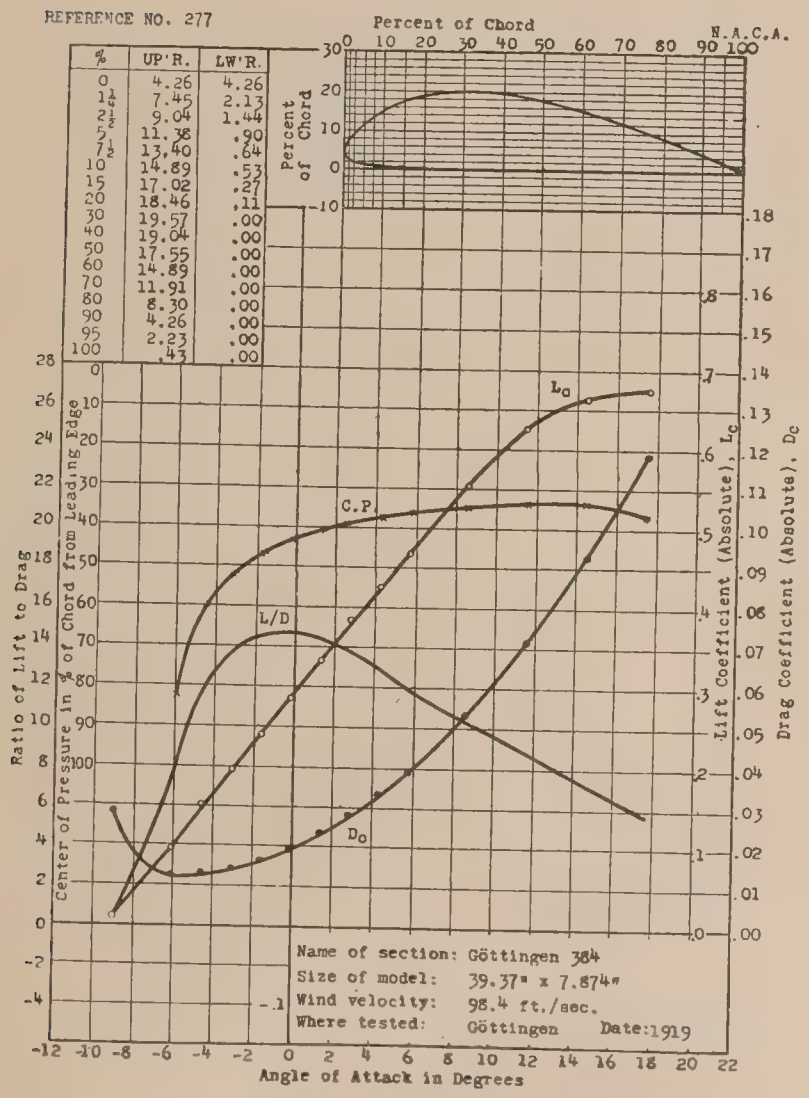
REFERENCE NO. 276

%	UP'R.	LW'R.
0	9.04	9.04
1¼	12.77	6.28
2½	14.04	5.59
5	15.85	4.68
7½	17.23	3.94
10	18.46	3.40
15	20.11	2.66
20	21.01	1.97
30	21.49	1.17
40	20.48	.59
50	18.51	.21
60	15.53	.00
70	12.13	.00
80	8.40	.00
90	4.36	.00
95	2.23	.00
100	.32	.00



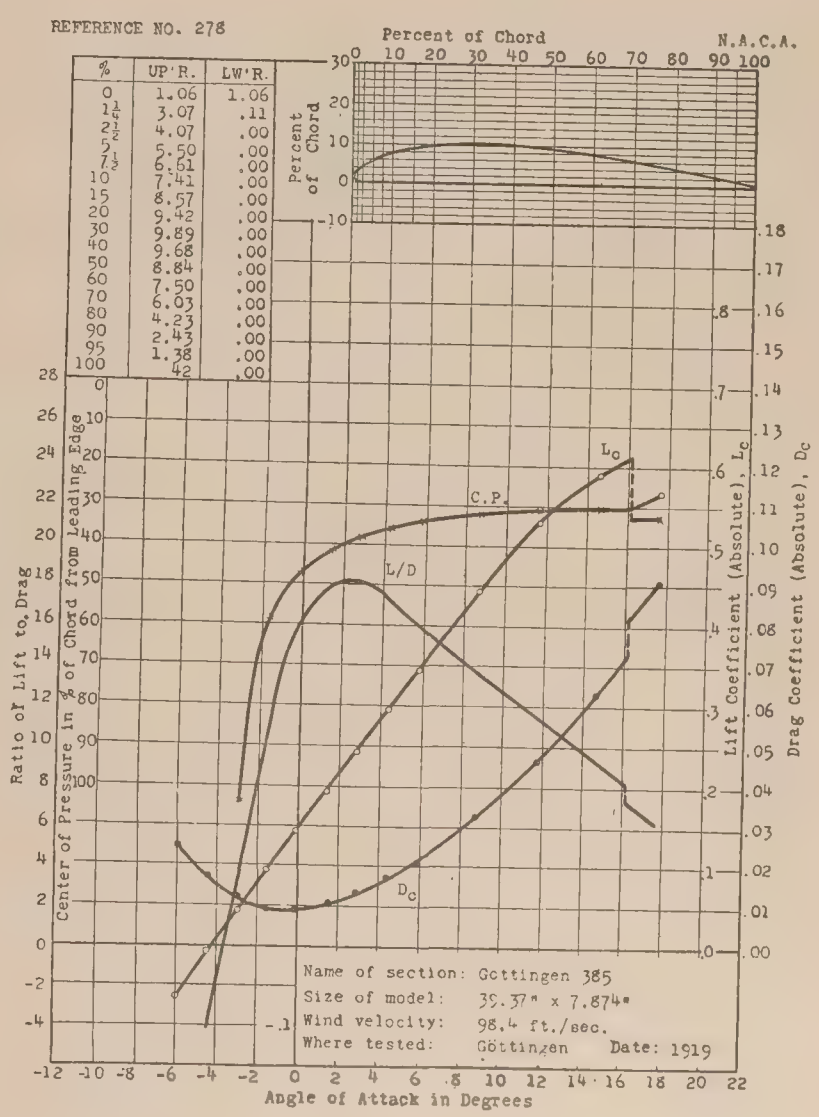
REFERENCE NO. 277

%	UP'R.	LW'R.
0	4.26	4.26
1¼	7.45	2.13
2½	9.04	1.44
5	11.38	.90
7½	13.40	.64
10	14.89	.53
15	17.02	.27
20	18.46	.11
30	19.57	.00
40	19.04	.00
50	17.55	.00
60	14.89	.00
70	11.91	.00
80	8.30	.00
90	4.26	.00
95	2.23	.00
100	.43	.00



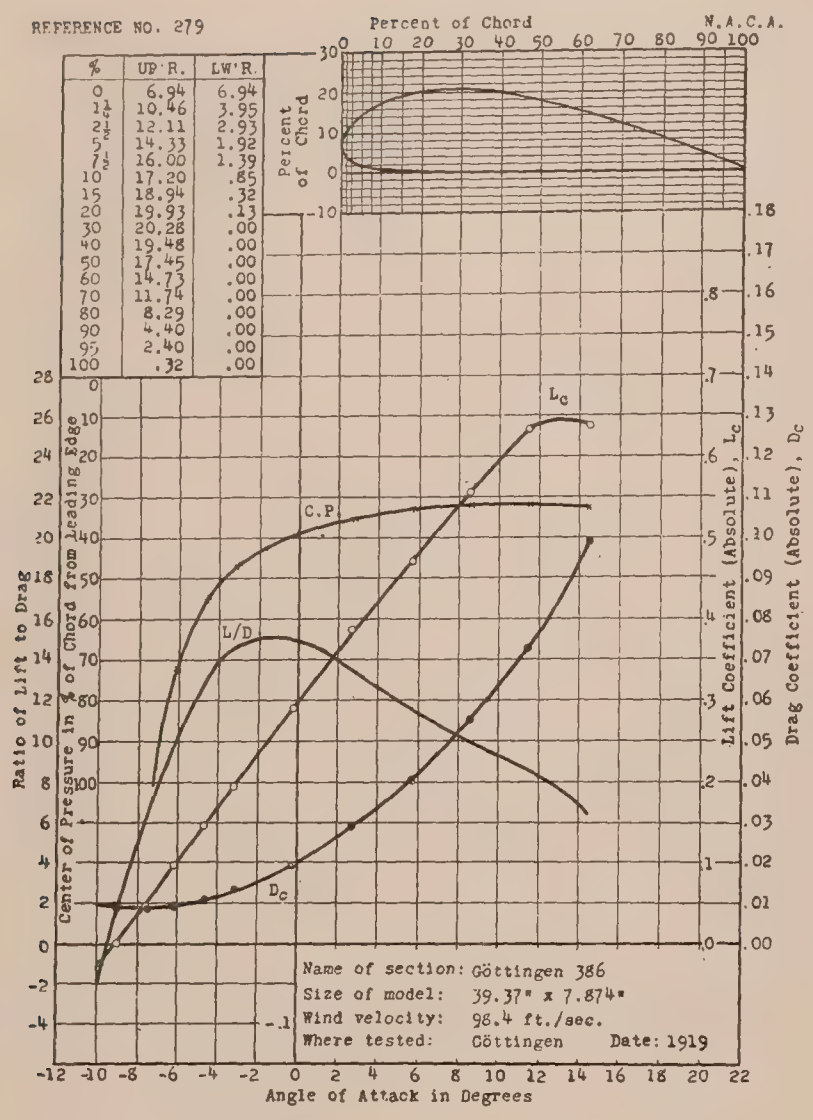
REFERENCE NO. 278

%	UP'R.	LW'R.
0	1.06	1.06
1¼	3.07	.11
2½	4.07	.00
5	5.50	.00
7½	6.61	.00
10	7.41	.00
15	8.57	.00
20	9.42	.00
30	9.89	.00
40	9.68	.00
50	8.84	.00
60	7.50	.00
70	6.03	.00
80	4.23	.00
90	2.43	.00
95	1.38	.00
100	42	.00



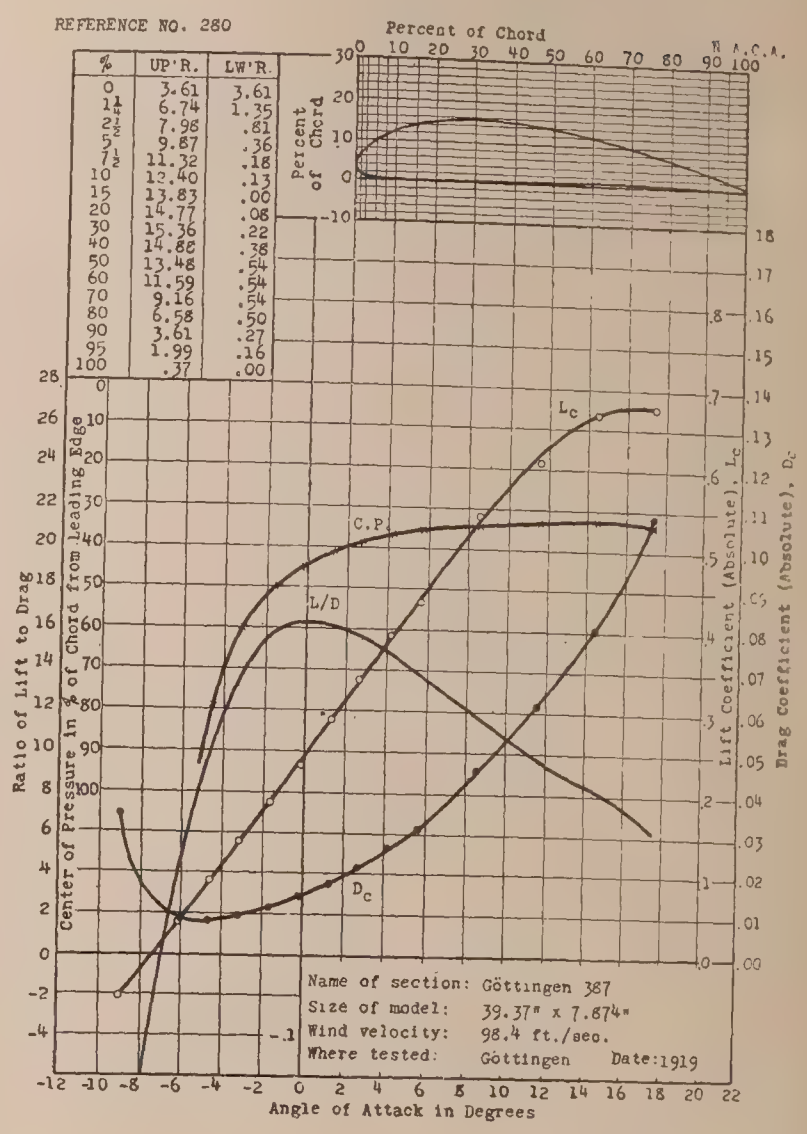
REFERENCE NO. 279

%	UP'R.	LW'R.
0	6.94	6.94
1¼	10.46	3.95
2½	12.11	2.93
5	14.33	1.92
7½	16.00	1.39
10	17.20	.85
15	18.94	.32
20	19.93	.13
30	20.28	.00
40	19.48	.00
50	17.45	.00
60	14.73	.00
70	11.74	.00
80	8.29	.00
90	4.40	.00
95	2.40	.00
100	.32	.00



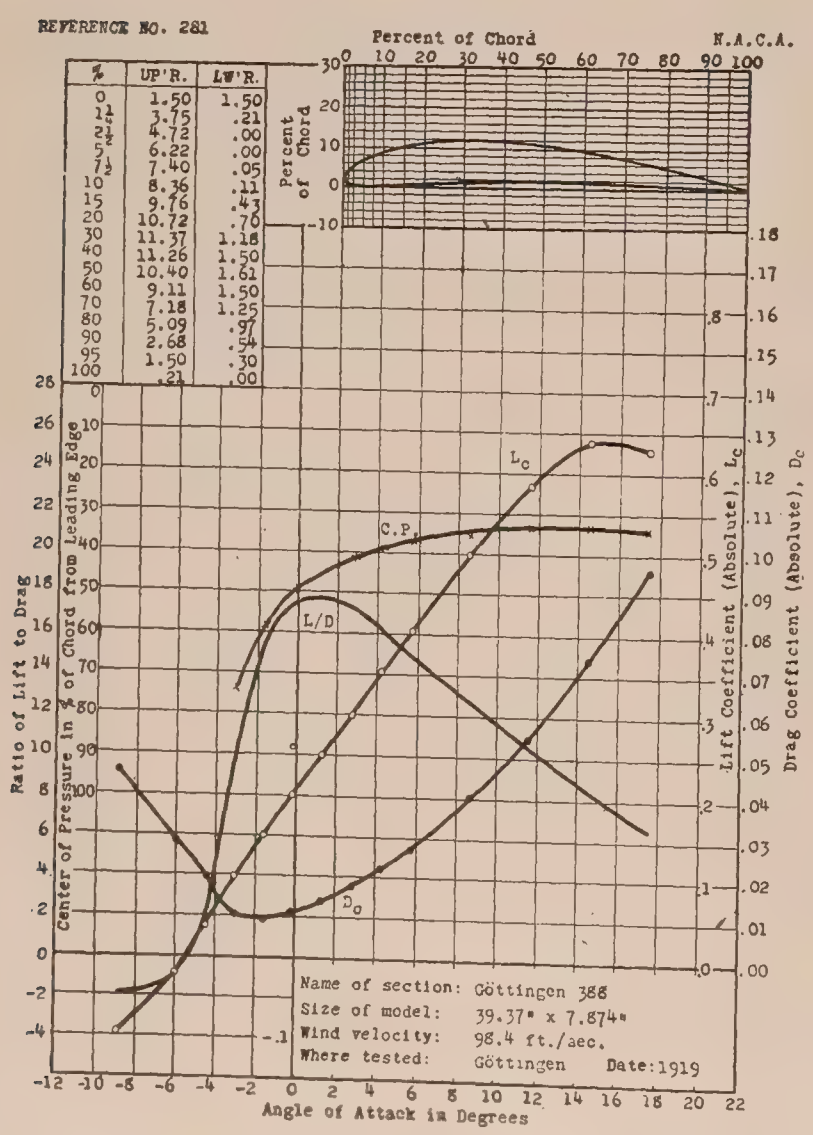
REFERENCE NO. 280

%	UP'R.	LW'R.
0	3.61	3.61
1¼	6.74	1.35
2½	7.98	.81
5	9.87	.36
7½	11.32	.18
10	12.40	.13
15	13.83	.00
20	14.77	.08
30	15.36	.22
40	14.88	.38
50	13.48	.54
60	11.59	.54
70	9.16	.54
80	6.58	.50
90	3.61	.27
95	1.99	.16
100	.37	.00



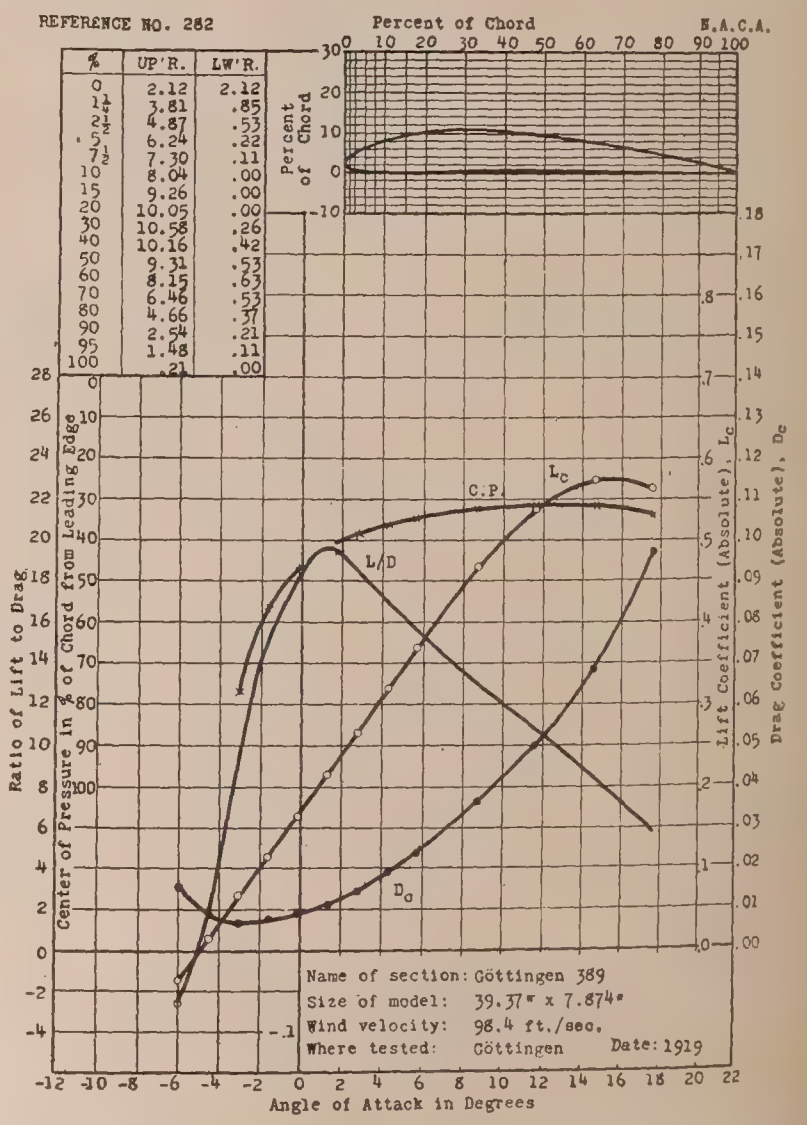
REFERENCE NO. 281

%	UP'R.	LW'R.
0	1.50	1.50
1¼	3.75	.21
2½	4.72	.00
5	6.22	.00
7½	7.40	.05
10	8.36	.11
15	9.76	.43
20	10.72	.70
30	11.37	1.18
40	11.26	1.50
50	10.40	1.61
60	9.11	1.50
70	7.18	1.25
80	5.09	.97
90	2.68	.54
95	1.50	.30
100	.21	.00



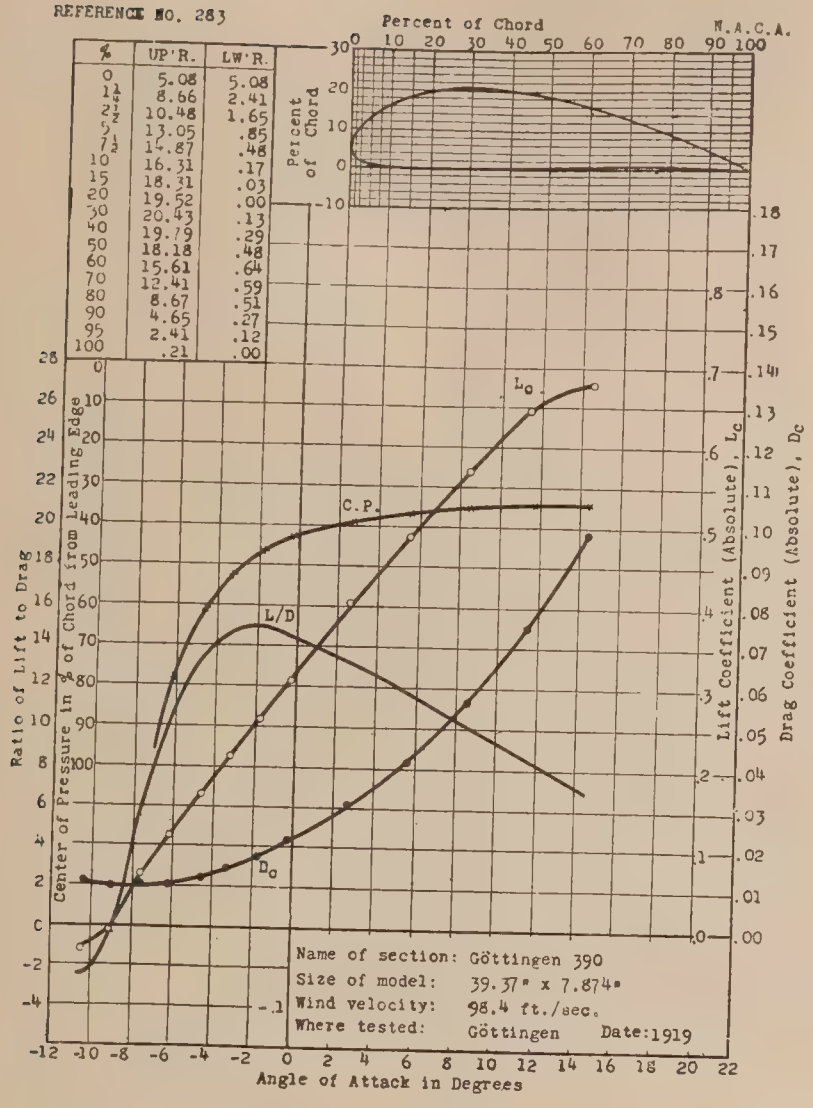
REFERENCE NO. 282

%	UP'R.	LW'R.
0	2.12	2.12
1¼	3.81	.85
2½	4.87	.53
5	6.24	.22
7½	7.30	.11
10	8.04	.00
15	9.26	.00
20	10.05	.00
30	10.58	.26
40	10.16	.42
50	9.31	.53
60	8.15	.63
70	6.46	.53
80	4.66	.37
90	2.54	.21
95	1.48	.11
100	.21	.00



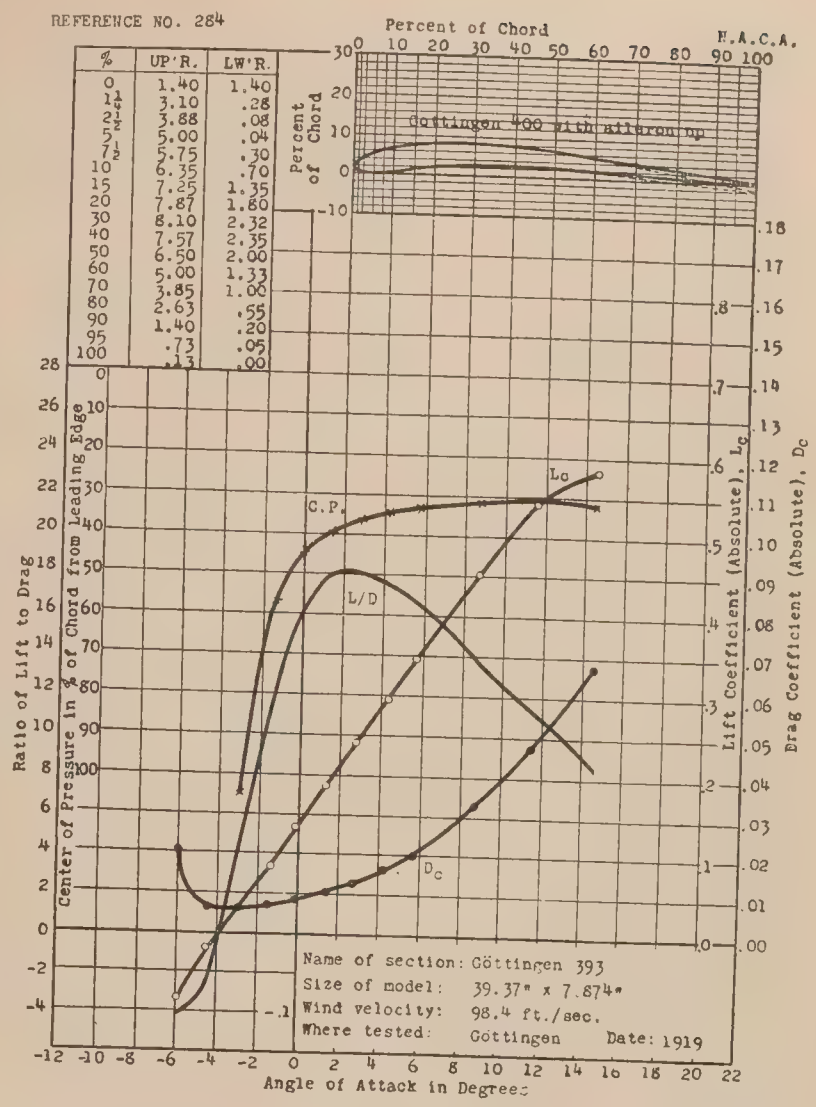
REFERENCE NO. 283

%	UP'R.	LW'R.
0	5.08	5.08
1¼	8.66	2.41
2½	10.48	1.65
5	13.05	.85
7½	14.87	.48
10	16.31	.17
15	18.31	.03
20	19.52	.00
30	20.43	.13
40	19.79	.29
50	18.18	.48
60	15.61	.64
70	12.41	.59
80	8.67	.51
90	4.65	.27
95	2.41	.12
100	.21	.00



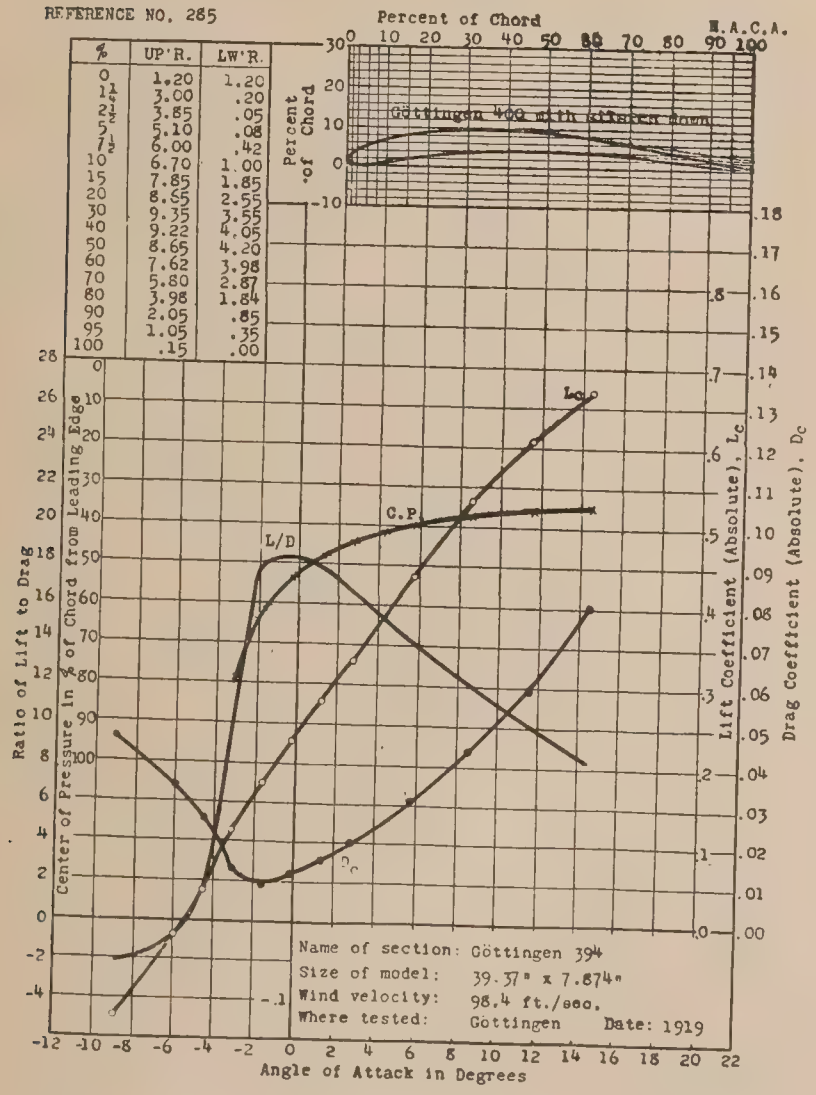
REFERENCE NO. 284

%	UP'R.	LW'R.
0	1.40	1.40
1¼	3.10	.28
2½	3.88	.08
5	5.00	.04
7½	5.75	.30
10	6.35	.70
15	7.25	1.35
20	7.87	1.80
30	8.10	2.32
40	7.57	2.35
50	6.50	2.00
60	5.00	1.33
70	3.85	1.00
80	2.63	.55
90	1.40	.20
95	.73	.05
100	.13	.00



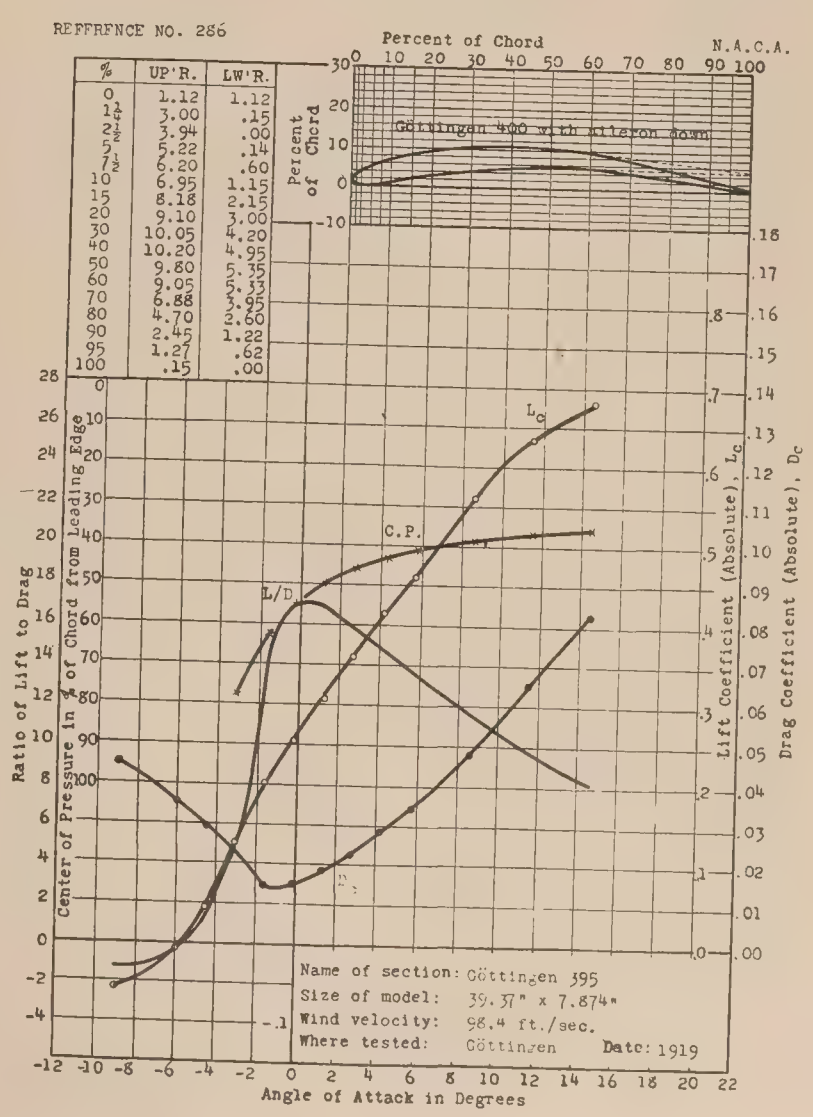
REFERENCE NO. 285

%	UP'R.	LW'R.
0	1.20	1.20
1¼	3.00	.20
2½	3.85	.05
5	5.10	.08
7½	6.00	.42
10	6.70	1.00
15	7.85	1.85
20	8.55	2.55
30	9.35	3.55
40	9.22	4.05
50	8.65	4.20
60	7.62	3.98
70	5.80	2.87
80	3.98	1.84
90	2.05	.85
95	1.05	.35
100	.15	.00

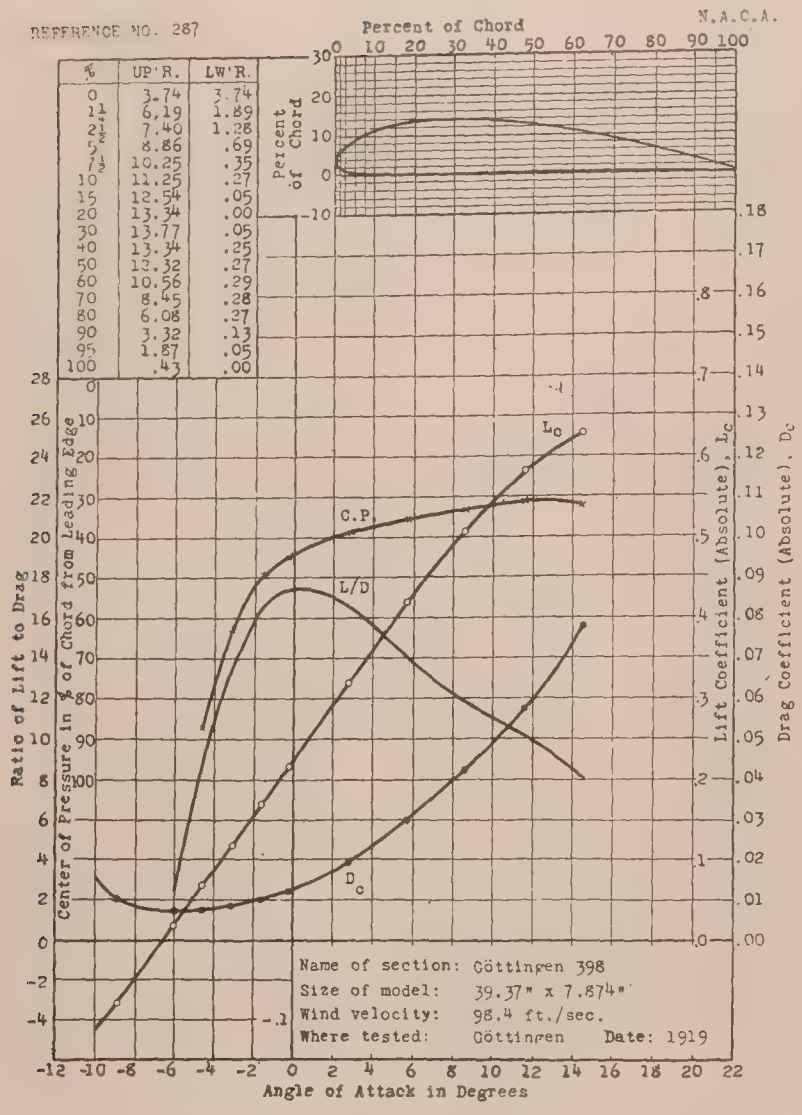


REFERENCE NO. 286

%	UP'R.	LW'R.
0	1.12	1.12
1¼	3.00	.15
2½	3.94	.00
5	5.22	.14
7½	6.20	.60
10	6.95	1.15
15	8.18	2.15
20	9.10	3.00
30	10.05	4.20
40	10.20	4.95
50	9.80	5.35
60	9.05	5.33
70	6.88	3.95
80	4.70	2.60
90	2.45	1.22
95	1.27	.62
100	.15	.00

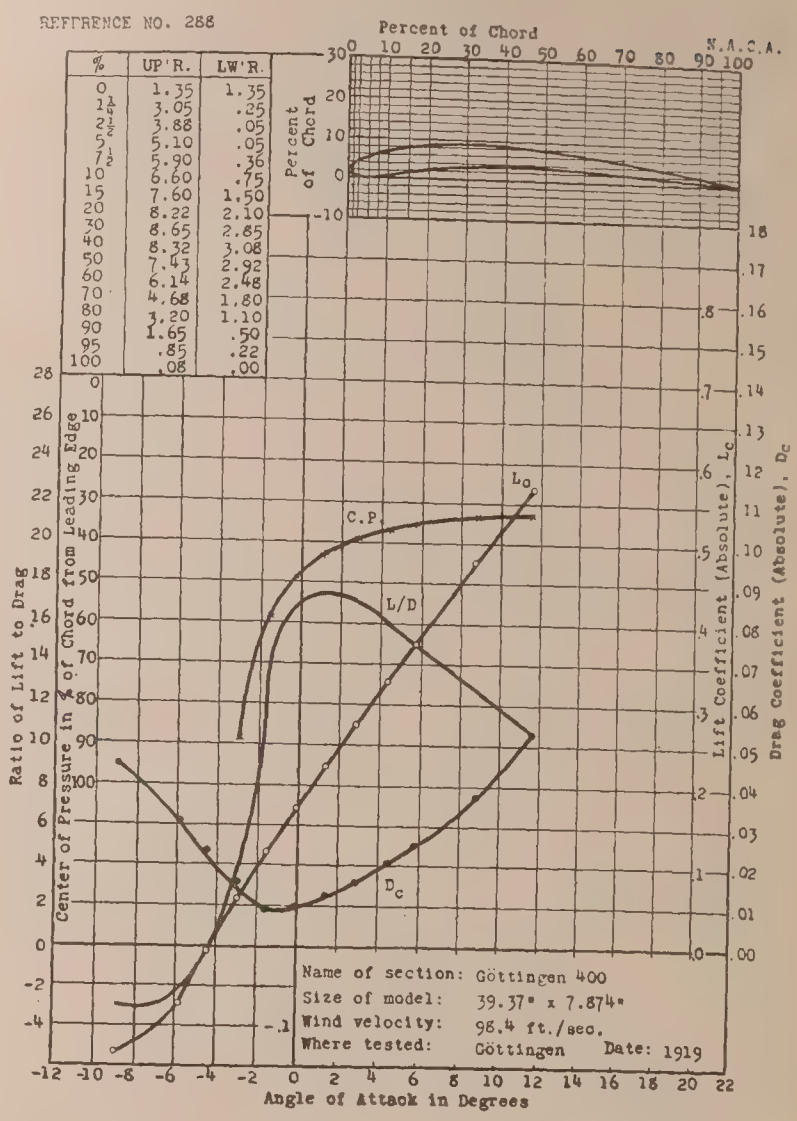


REFERENCE NO. 287



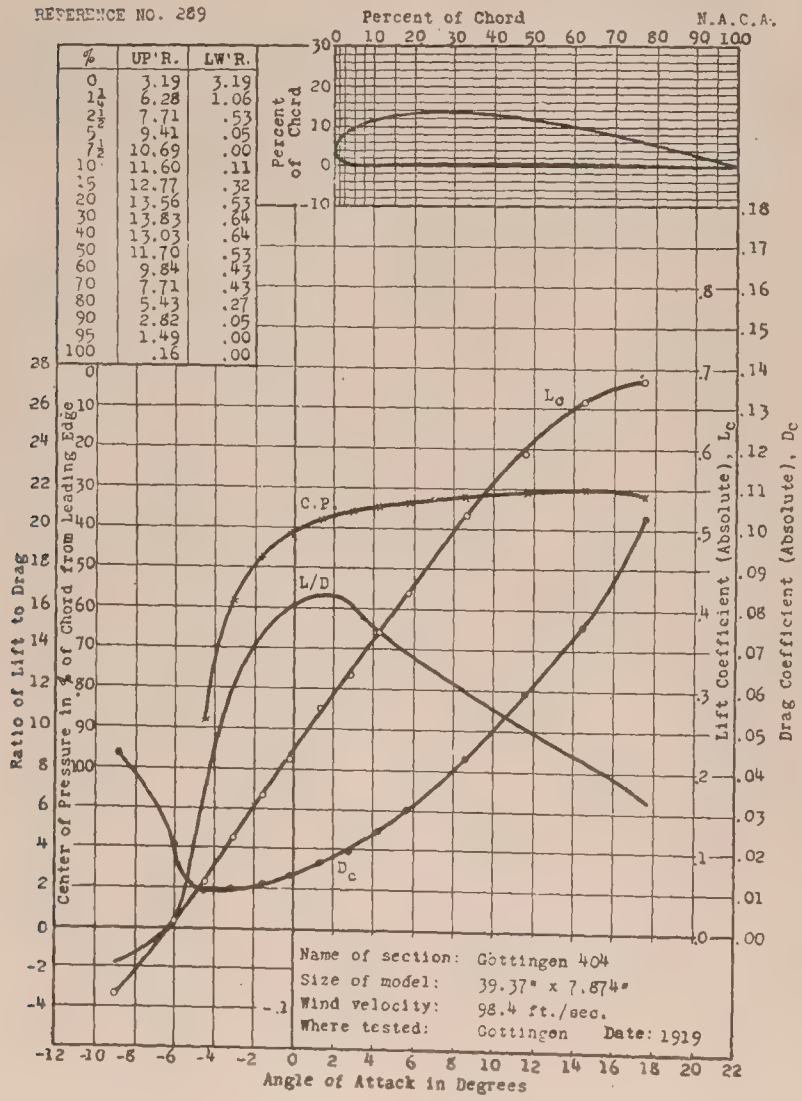
%	UP'R.	LW'R.
0	3.74	3.74
1¼	6.19	1.89
2½	7.40	1.28
5	8.86	.69
7½	10.25	.35
10	11.25	.27
15	12.54	.05
20	13.34	.00
30	13.77	.05
40	13.34	.25
50	12.32	.27
60	10.56	.29
70	8.45	.28
80	6.08	.27
90	3.32	.13
95	1.87	.05
100	.43	.00

REFERENCE NO. 288



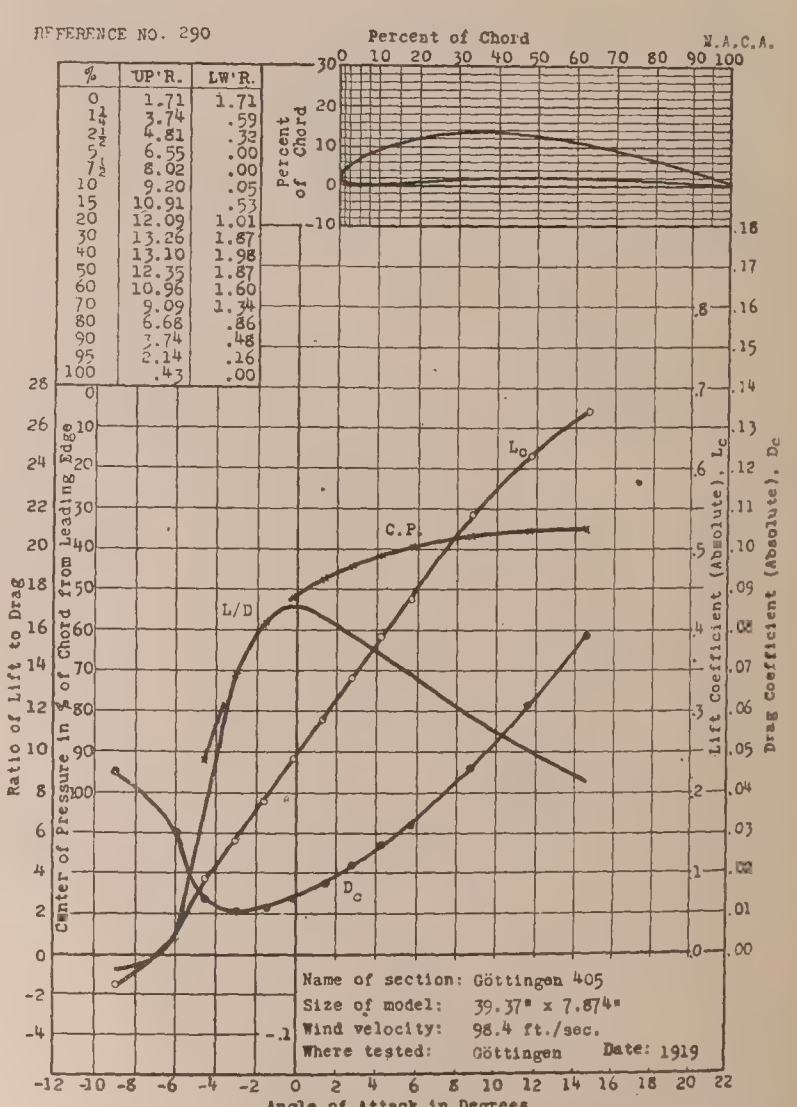
%	UP'R.	LW'R.
0	1.35	1.35
1¼	3.05	.25
2½	3.88	.05
5	5.10	.05
7½	5.90	.36
10	6.60	.75
15	7.60	1.50
20	8.22	2.10
30	8.65	2.85
40	8.32	3.08
50	7.43	2.92
60	6.14	2.48
70	4.68	1.80
80	3.20	1.10
90	1.65	.50
95	.85	.22
100	.08	.00

REFERENCE NO. 289



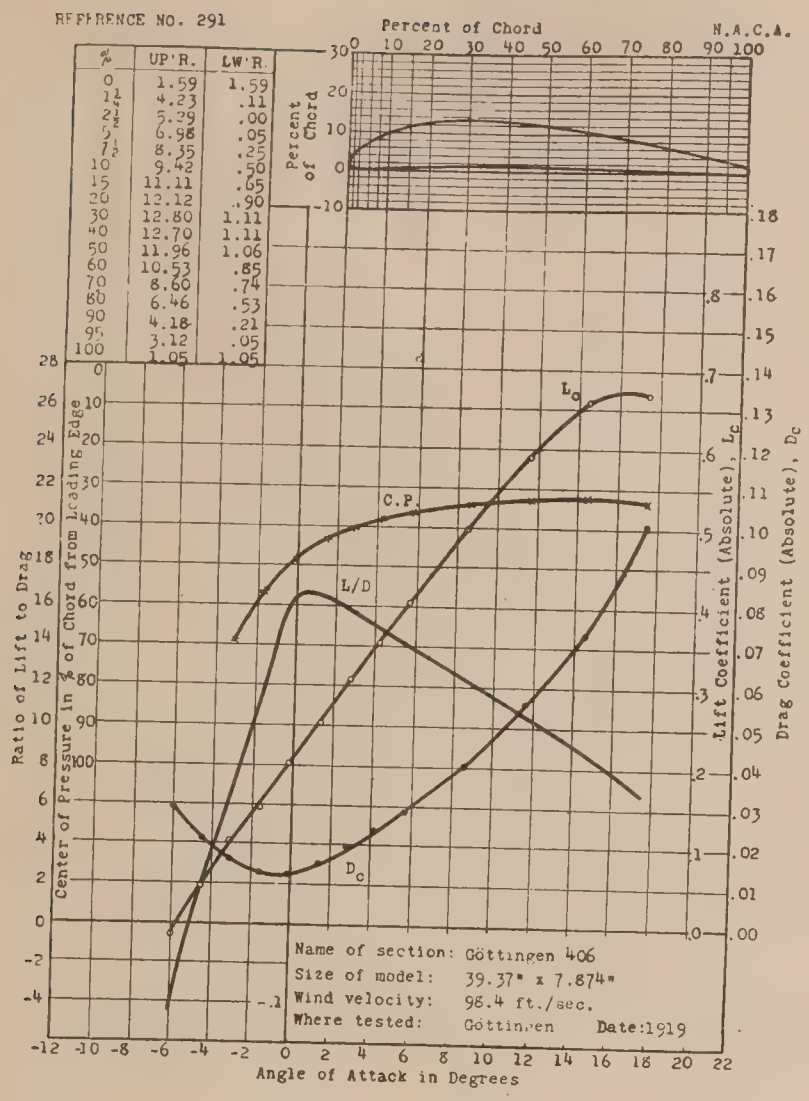
%	UP'R.	LW'R.
0	3.19	3.19
1¼	6.28	1.06
2½	7.71	.53
5	9.41	.05
7½	10.69	.00
10	11.60	.11
15	12.77	.32
20	13.56	.53
30	13.83	.64
40	13.03	.64
50	11.70	.53
60	9.84	.43
70	7.71	.43
80	5.43	.27
90	2.82	.05
95	1.49	.00
100	.16	.00

REFERENCE NO. 290



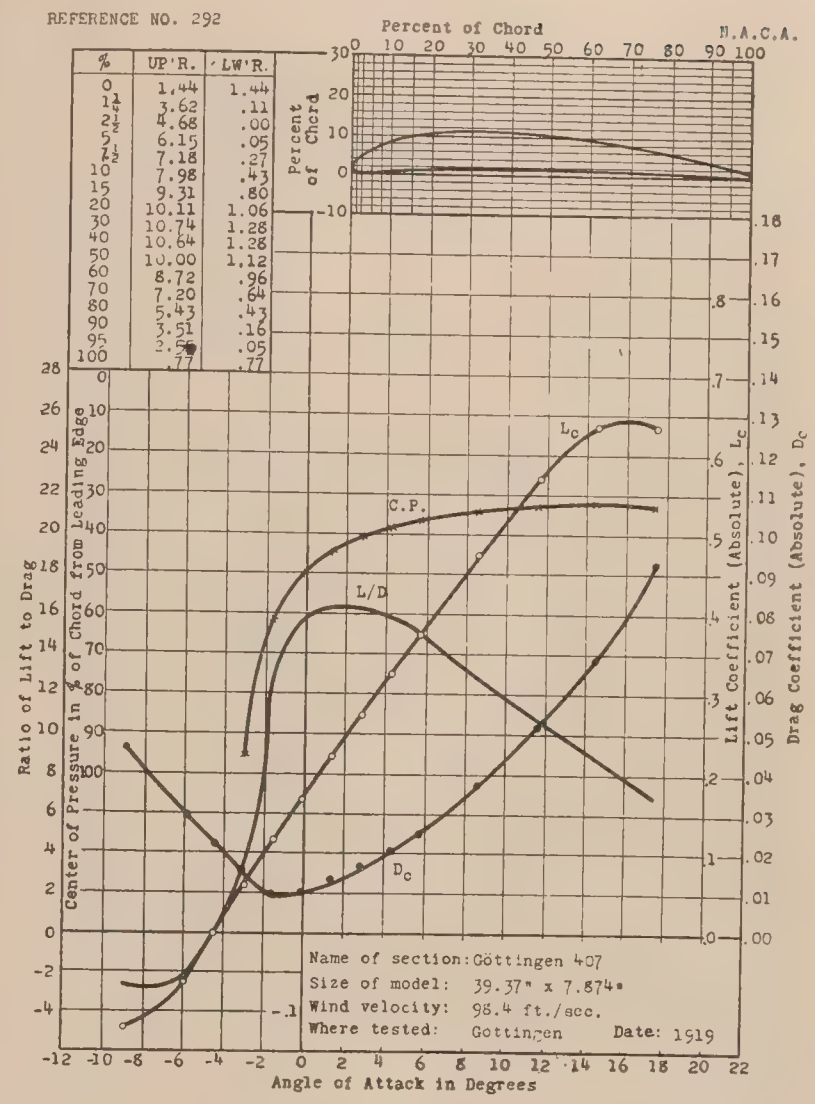
%	UP'R.	LW'R.
0	1.71	1.71
1¼	3.74	.59
2½	4.81	.32
5	6.55	.00
7½	8.02	.00
10	9.20	.05
15	10.91	.53
20	12.09	1.01
30	13.26	1.87
40	13.10	1.98
50	12.35	1.87
60	10.96	1.60
70	9.09	1.34
80	6.68	.86
90	3.74	.48
95	2.14	.16
100	.43	.00

REFERENCE NO. 291



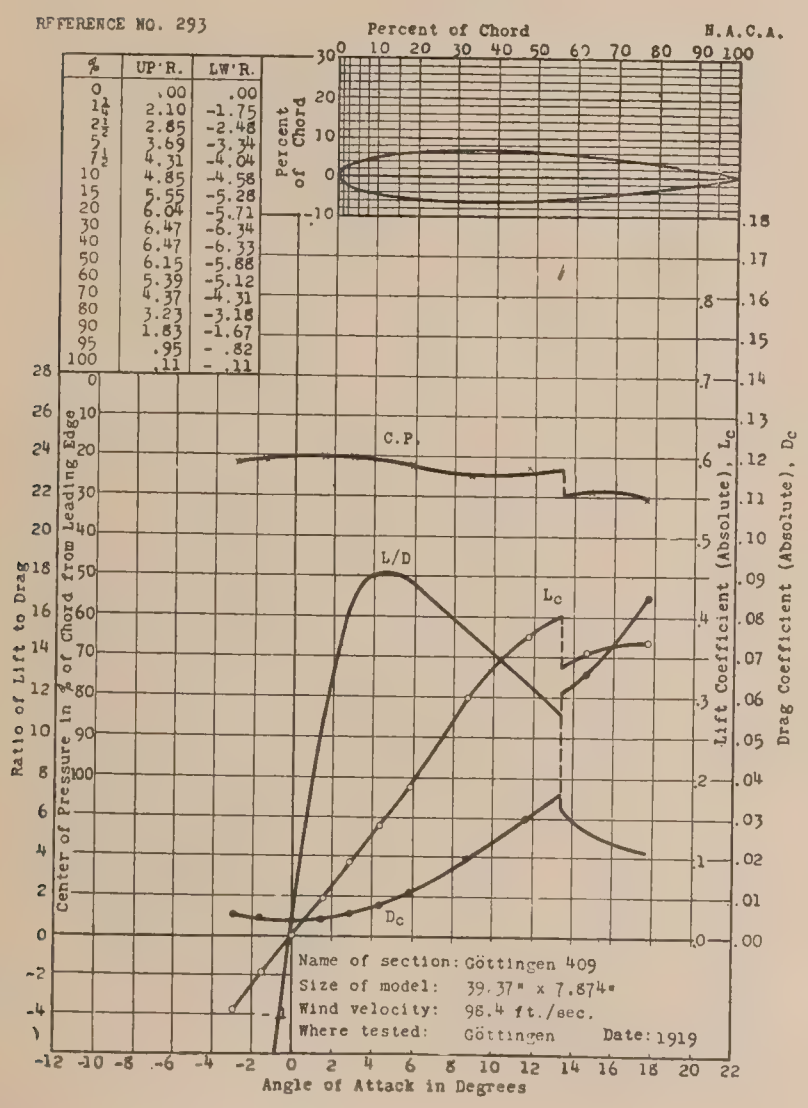
%	UP'R.	LW'R.
0	1.59	1.59
1¼	4.23	.11
2½	5.29	.00
5	6.98	.05
7½	8.35	.25
10	9.42	.50
15	11.11	.65
20	12.12	.90
30	12.80	1.11
40	12.70	1.11
50	11.96	1.06
60	10.53	.85
70	8.60	.74
80	6.46	.53
90	4.18	.21
95	3.12	.05
100	1.05	1.05

REFERENCE NO. 292



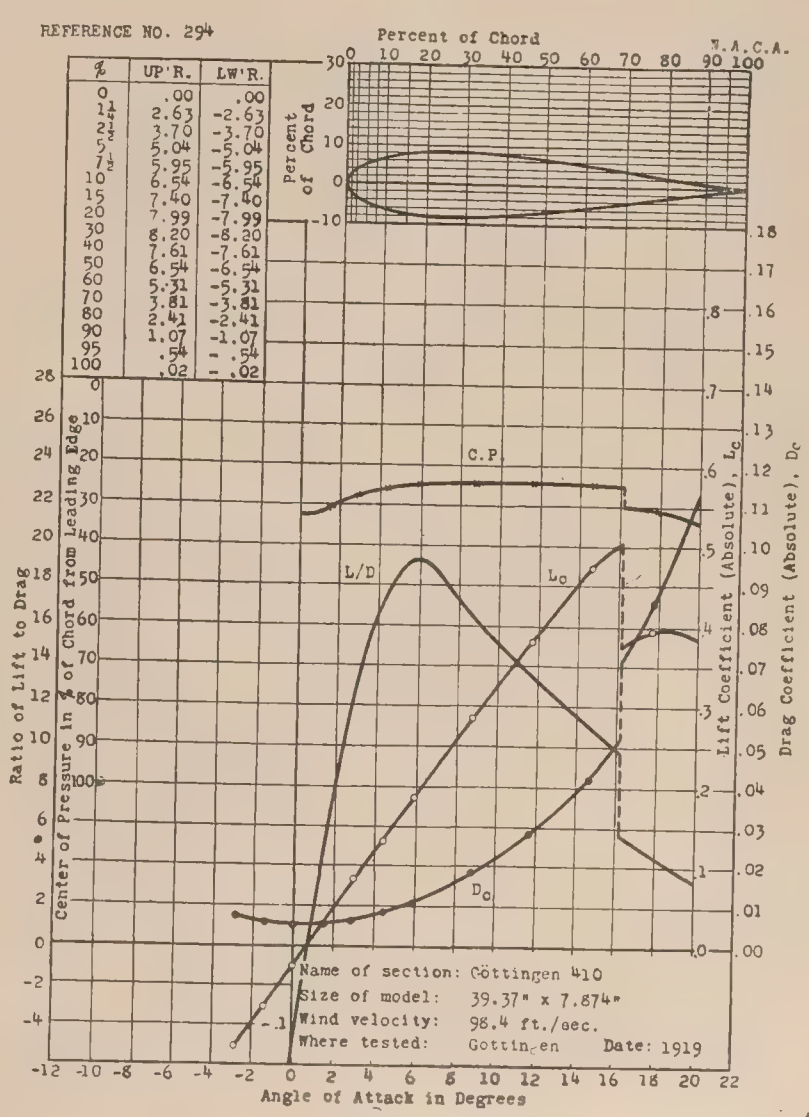
%	UP'R.	LW'R.
0	1.44	1.44
1¼	3.62	.11
2½	4.68	.00
5	6.15	.05
7½	7.18	.27
10	7.98	.43
15	9.31	.80
20	10.11	1.06
30	10.74	1.28
40	10.64	1.28
50	10.00	1.12
60	8.72	.96
70	7.20	.64
80	5.43	.43
90	3.51	.16
95	2.5[illegible]	.05
100	.77	.77

REFERENCE NO. 293



%	UP'R.	LW'R.
0	.00	.00
1¼	2.10	-1.75
2½	2.85	-2.48
5	3.69	-3.34
7½	4.31	-4.04
10	4.85	-4.58
15	5.55	-5.28
20	6.04	-5.71
30	6.47	-6.34
40	6.47	-6.33
50	6.15	-5.88
60	5.39	-5.12
70	4.37	-4.31
80	3.23	-3.18
90	1.83	-1.67
95	.95	- .82
100	.11	- .11

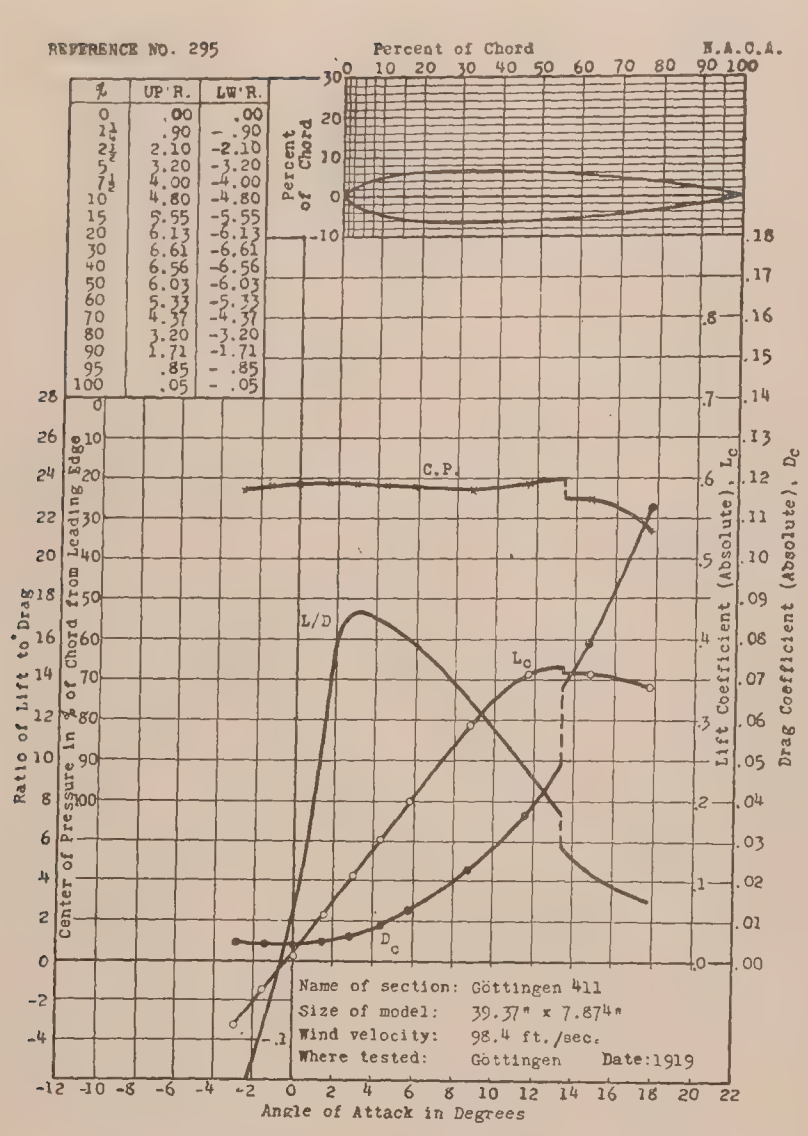
REFERENCE NO. 294



%	UP'R.	LW'R.
0	.00	.00
1¼	2.63	-2.63
2½	3.70	-3.70
5	5.04	-5.04
7½	5.95	-5.95
10	6.54	-6.54
15	7.40	-7.40
20	7.99	-7.99
30	8.20	-8.20
40	7.61	-7.61
50	6.54	-6.54
60	5.31	-5.31
70	3.81	-3.81
80	2.41	-2.41
90	1.07	-1.07
95	.54	- .54
100	.02	- .02

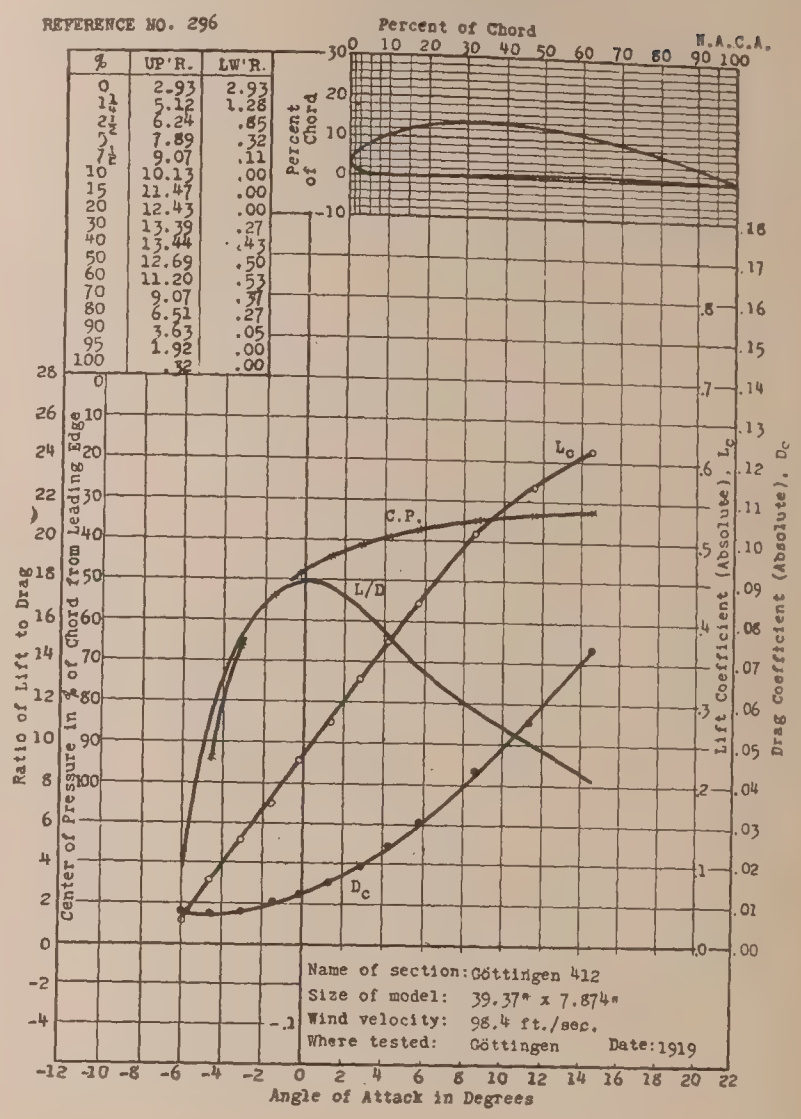
REFERENCE NO. 295

%	UP'R.	LW'R.
0	.00	.00
1¼	.90	- .90
2½	2.10	-2.10
5	3.20	-3.20
7½	4.00	-4.00
10	4.80	-4.80
15	5.55	-5.55
20	6.13	-6.13
30	6.61	-6.61
40	6.56	-6.56
50	6.03	-6.03
60	5.33	-5.33
70	4.37	-4.37
80	3.20	-3.20
90	1.71	-1.71
95	.85	- .85
100	.05	- .05



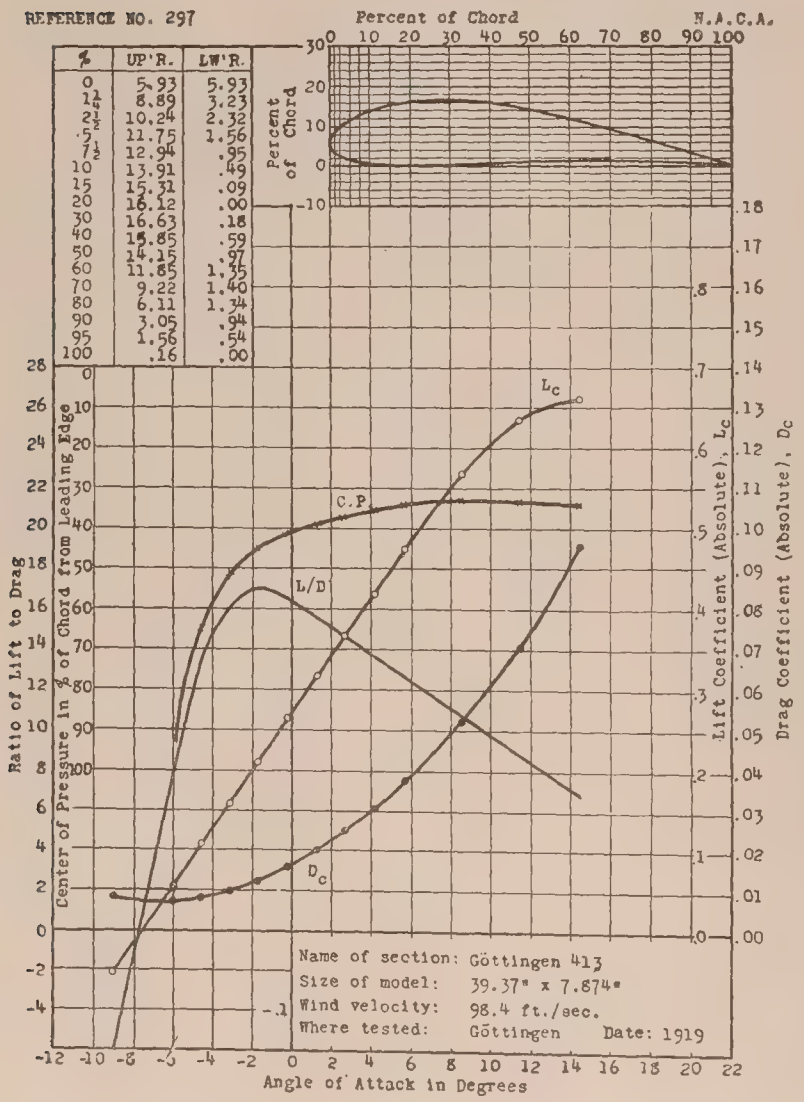
REFERENCE NO. 296

%	UP'R.	LW'R.
0	2.93	2.93
1¼	5.12	1.28
2½	6.24	.85
5	7.89	.32
7½	9.07	.11
10	10.13	.00
15	11.47	.00
20	12.43	.00
30	13.39	.27
40	13.44	.43
50	12.69	.50
60	11.20	.53
70	9.07	.37
80	6.51	.27
90	3.63	.05
95	1.92	.00
100	.32	.00



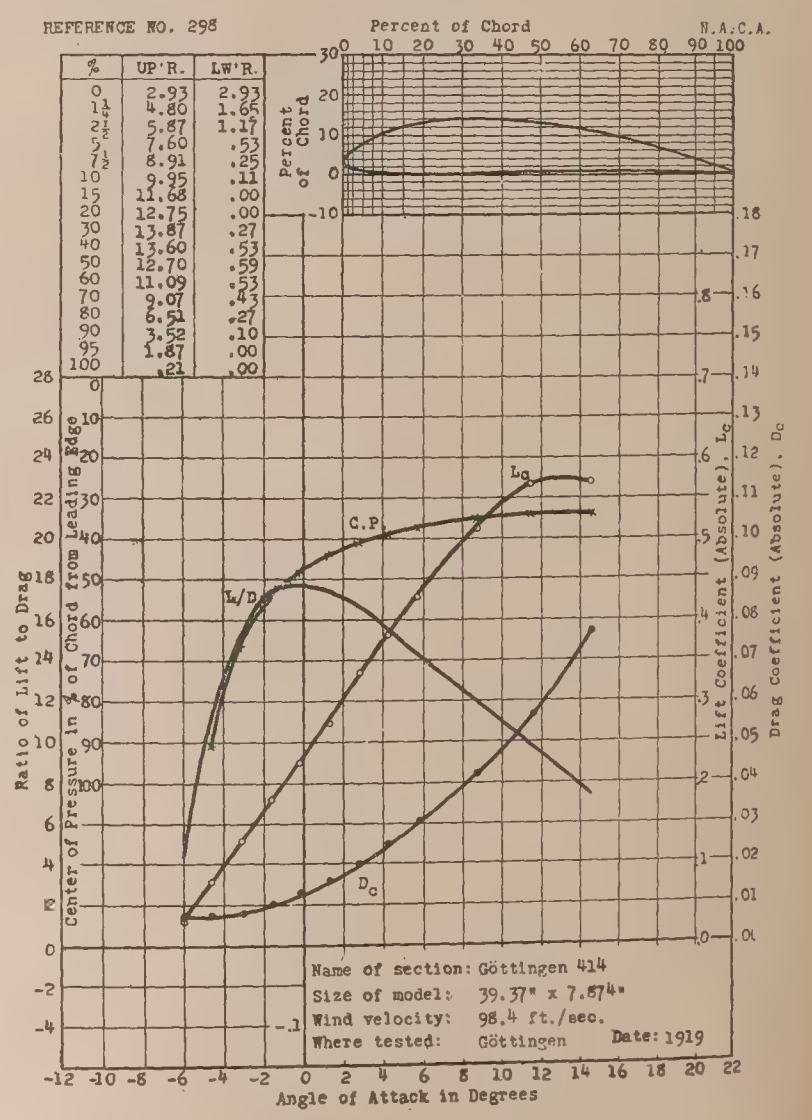
REFERENCE NO. 297

%	UP'R.	LW'R.
0	5.93	5.93
1¼	8.89	3.23
2½	10.24	2.32
5	11.75	1.56
7½	12.94	.95
10	13.91	.49
15	15.31	.09
20	16.12	.00
30	16.63	.18
40	15.85	.59
50	14.15	.97
60	11.85	1.35
70	9.22	1.40
80	6.11	1.34
90	3.05	.94
95	1.56	.54
100	.16	.00



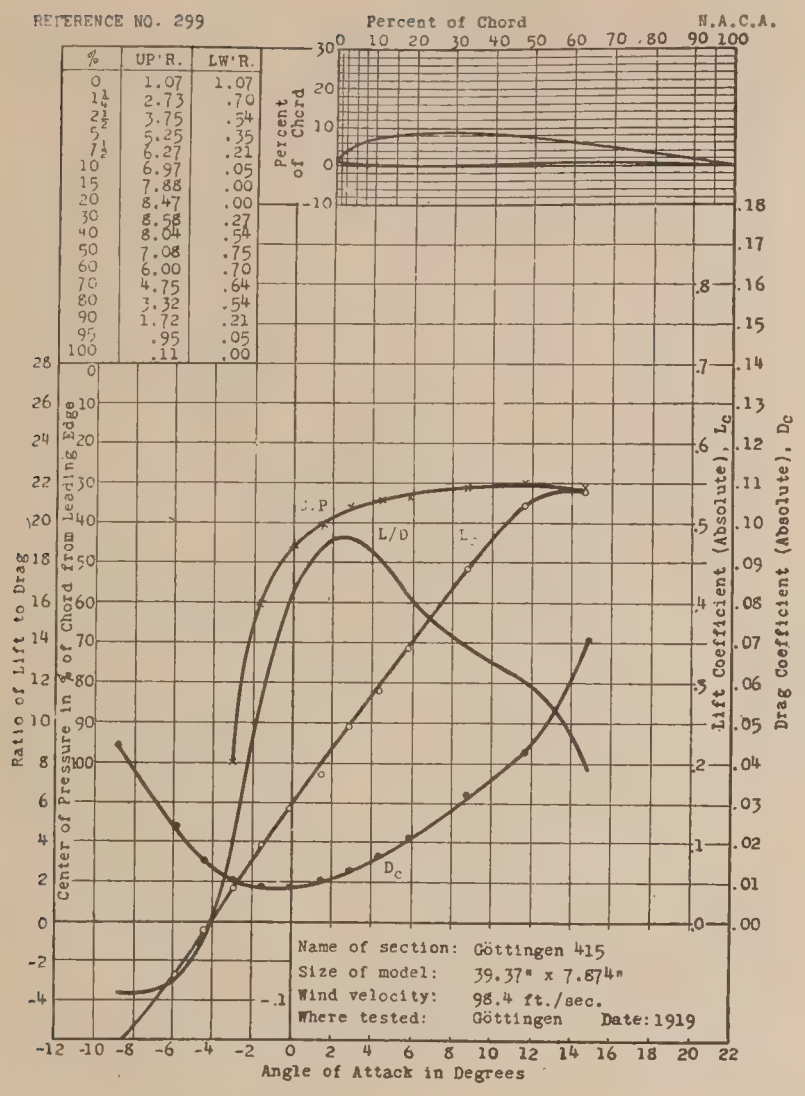
REFERENCE NO. 298

%	UP'R.	LW'R.
0	2.93	2.93
1¼	4.80	1.65
2½	5.87	1.17
5	7.60	.53
7½	8.91	.25
10	9.95	.11
15	11.68	.00
20	12.75	.00
30	13.87	.27
40	13.60	.53
50	12.70	.59
60	11.09	.53
70	9.07	.43
80	6.51	.27
90	3.52	.10
95	1.87	.00
100	.21	.00



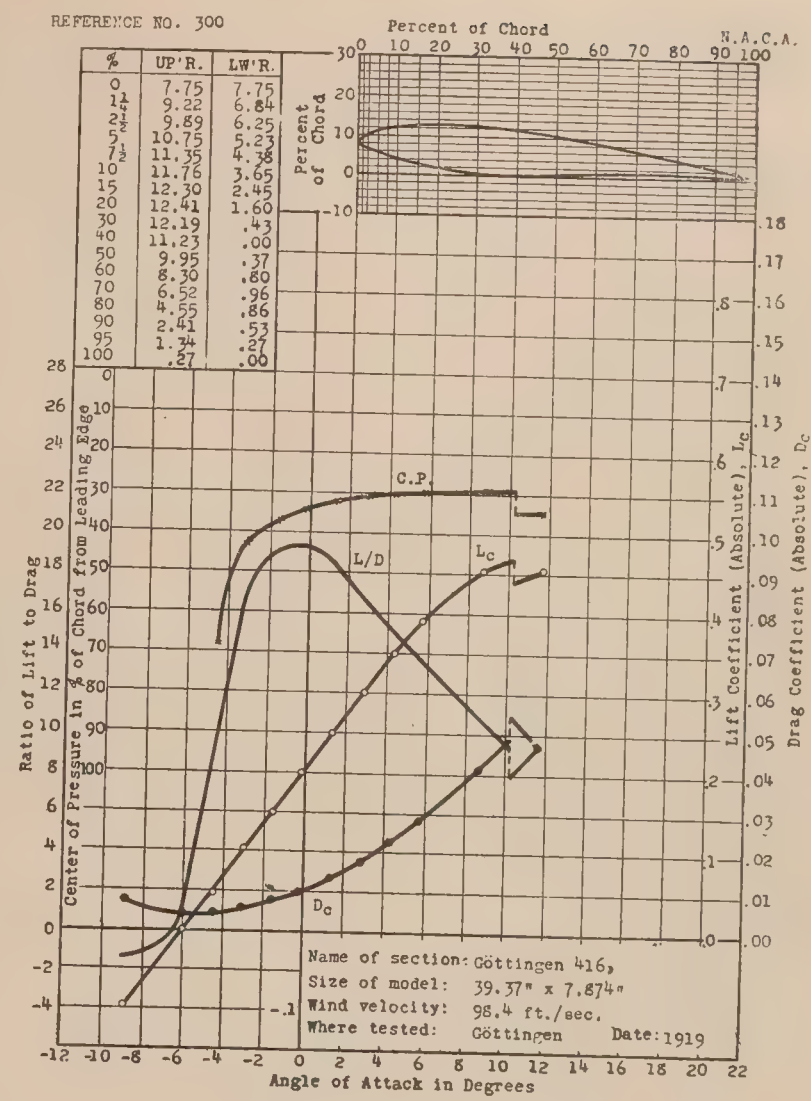
REFERENCE NO. 299

%	UP'R.	LW'R.
0	1.07	1.07
1¼	2.73	.70
2½	3.75	.54
5	5.25	.35
7½	6.27	.21
10	6.97	.05
15	7.88	.00
20	8.47	.00
30	8.58	.27
40	8.04	.54
50	7.08	.75
60	6.00	.70
70	4.75	.64
80	3.32	.54
90	1.72	.21
95	.95	.05
100	.11	.00



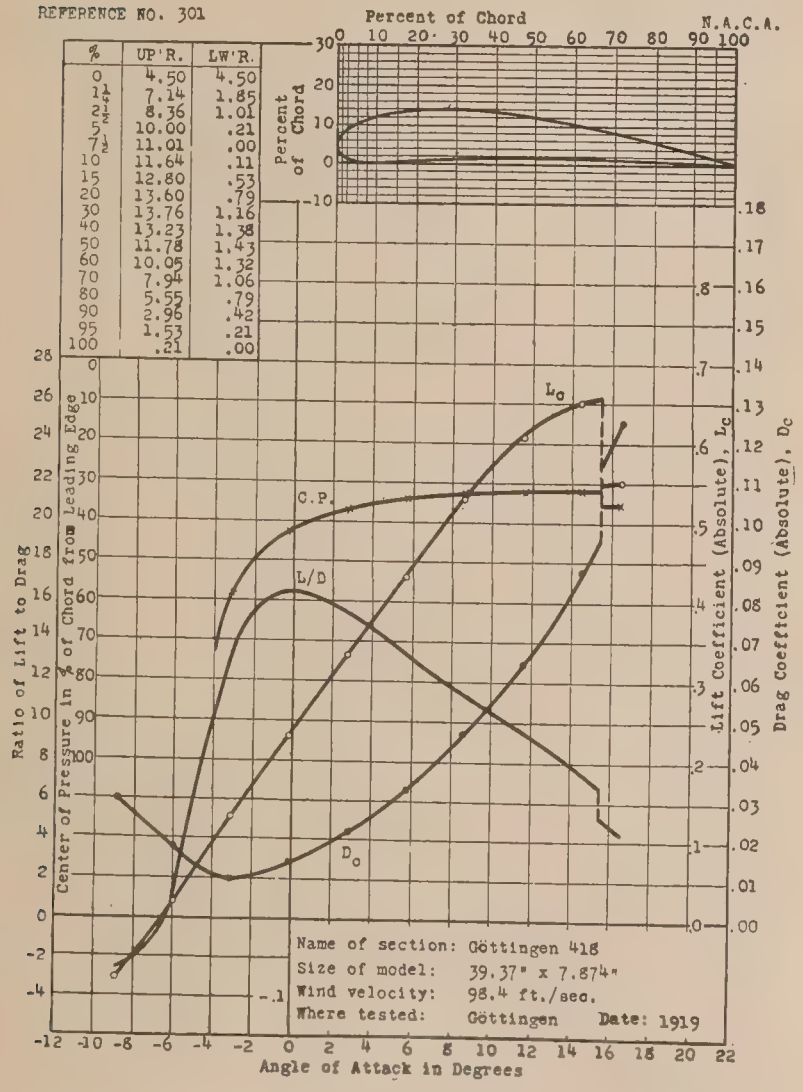
REFERENCE NO. 300

%	UP'R.	LW'R.
0	7.75	7.75
1¼	9.22	6.84
2½	9.89	6.25
5	10.75	5.23
7½	11.35	4.38
10	11.76	3.65
15	12.30	2.45
20	12.41	1.60
30	12.19	.43
40	11.23	.00
50	9.95	.37
60	8.30	.80
70	6.52	.96
80	4.55	.86
90	2.41	.53
95	1.34	.27
100	.27	.00



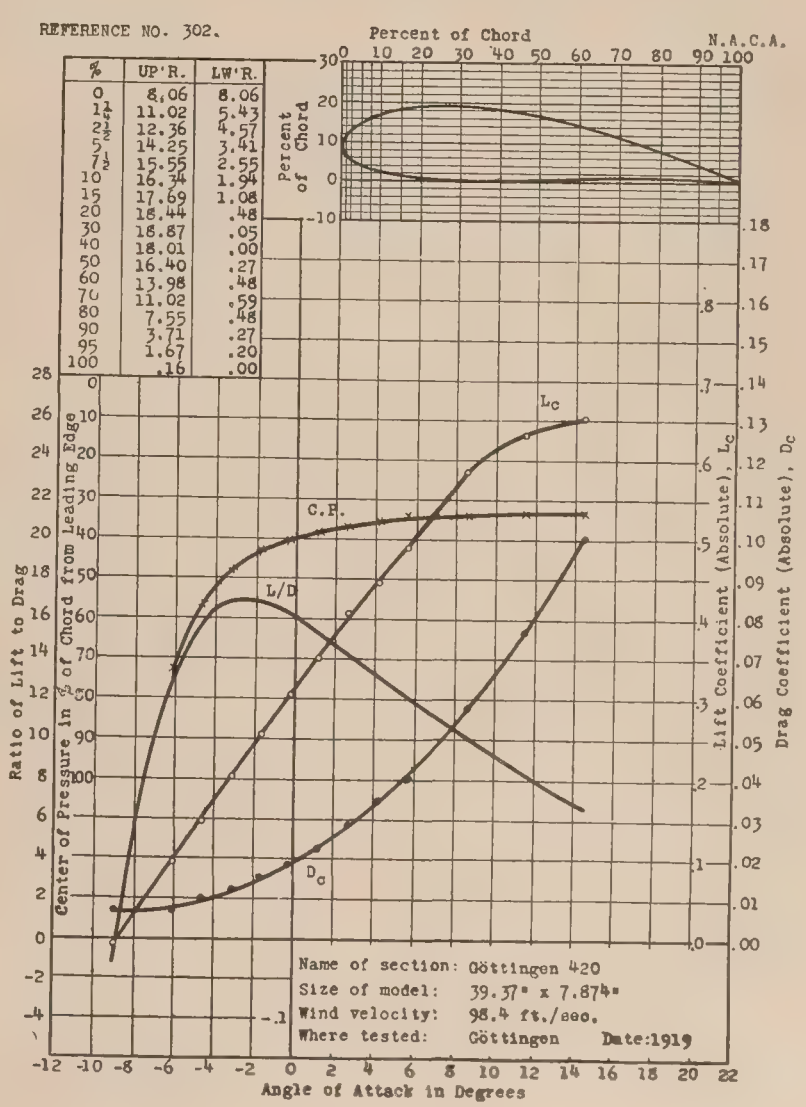
REFERENCE NO. 301

%	UP'R.	LW'R.
0	4.50	4.50
1¼	7.14	1.85
2½	8.36	1.01
5	10.00	.21
7½	11.01	.00
10	11.64	.11
15	12.80	.53
20	13.60	.79
30	13.76	1.16
40	13.23	1.38
50	11.78	1.43
60	10.05	1.32
70	7.94	1.06
80	5.55	.79
90	2.96	.42
95	1.53	.21
100	.21	.00



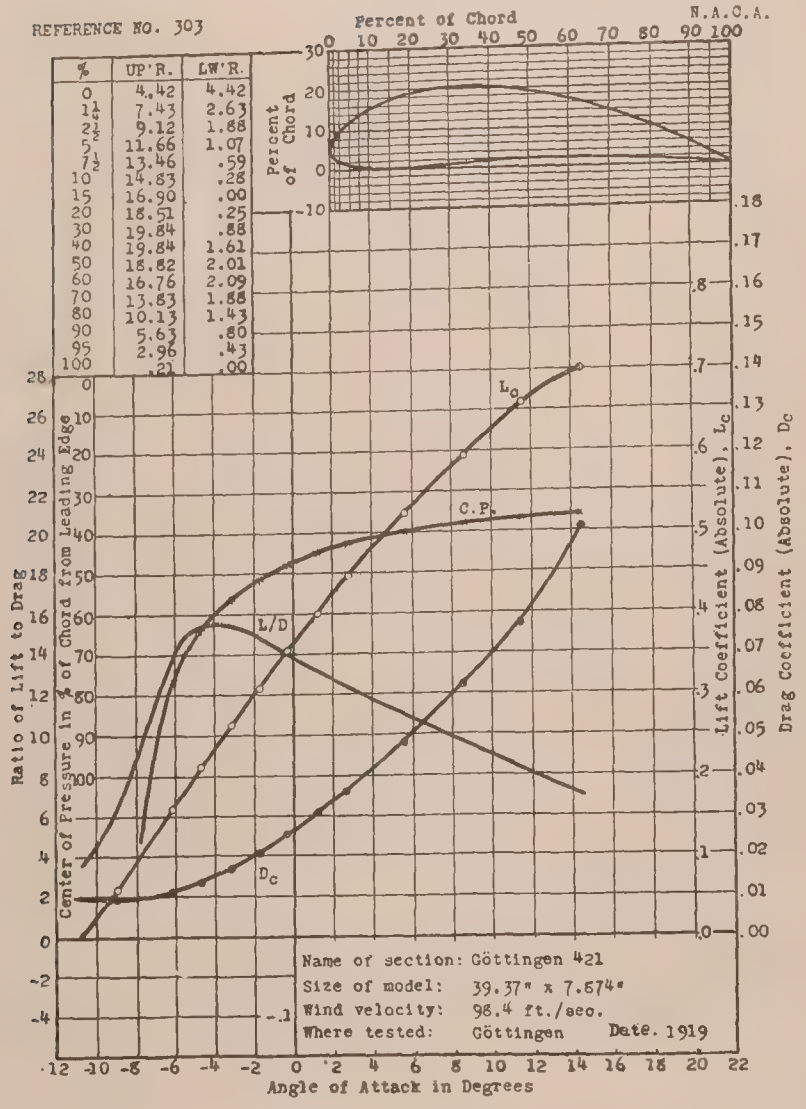
REFERENCE NO. 302.

%	UP'R.	LW'R.
0	8.06	8.06
1¼	11.02	5.43
2½	12.36	4.57
5	14.25	3.41
7½	15.55	2.55
10	16.34	1.94
15	17.69	1.08
20	18.44	.48
30	18.87	.05
40	18.01	.00
50	16.40	.27
60	13.98	.48
70	11.02	.59
80	7.55	.48
90	3.71	.27
95	1.67	.20
100	.16	.00



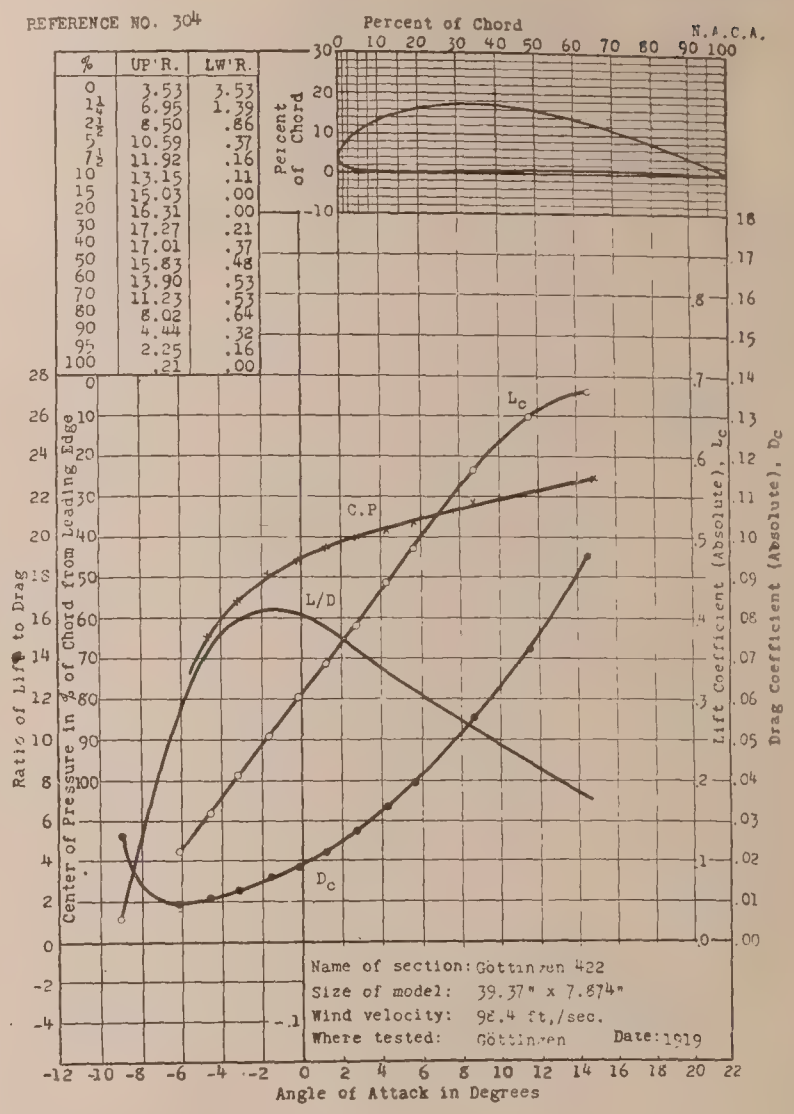
REFERENCE NO. 303

%	UP'R.	LW'R.
0	4.42	4.42
1¼	7.43	2.63
2½	9.12	1.88
5	11.66	1.07
7½	13.46	.59
10	14.83	.28
15	16.90	.00
20	18.51	.25
30	19.84	.88
40	19.84	1.61
50	18.82	2.01
60	16.76	2.09
70	13.83	1.88
80	10.13	1.43
90	5.63	.80
95	2.96	.43
100	.21	.00



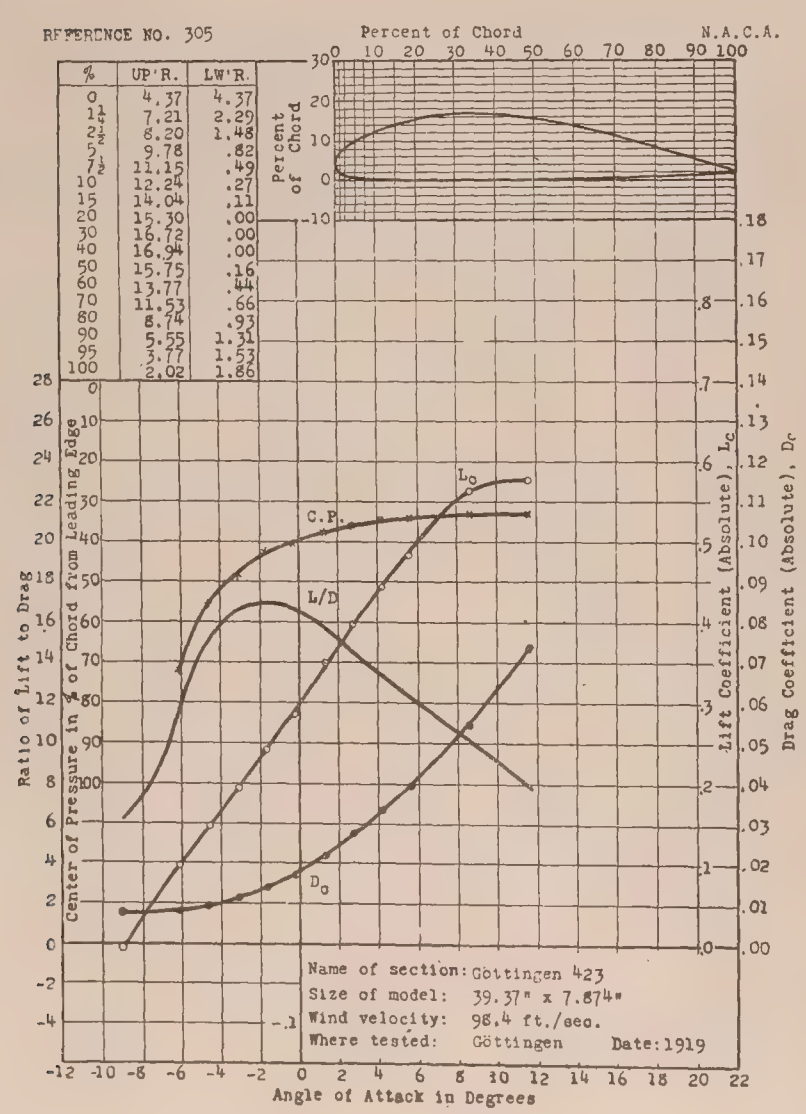
REFERENCE NO. 304

%	UP'R.	LW'R.
0	3.53	3.53
1¼	6.95	1.39
2½	8.50	.86
5	10.59	.37
7½	11.92	.16
10	13.15	.11
15	15.03	.00
20	16.31	.00
30	17.27	.21
40	17.01	.37
50	15.83	.48
60	13.90	.53
70	11.23	.53
80	8.02	.64
90	4.44	.32
95	2.25	.16
100	.21	.00



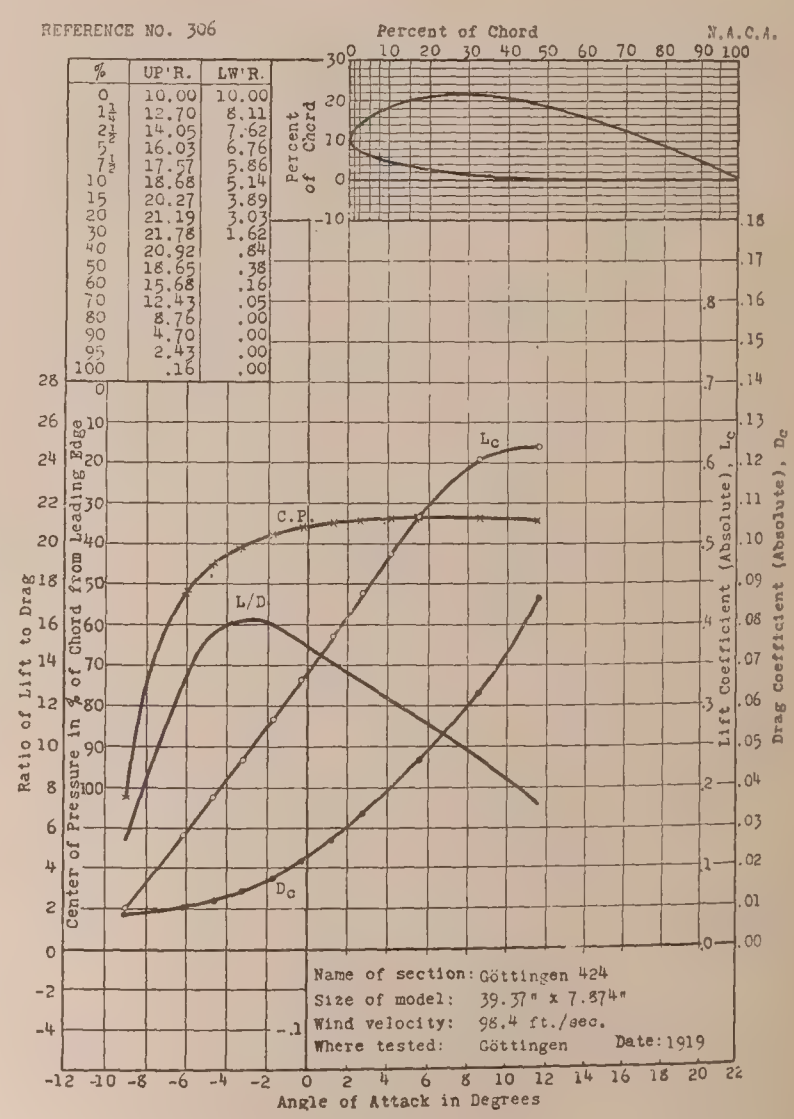
REFERENCE NO. 305

%	UP'R.	LW'R.
0	4.37	4.37
1¼	7.21	2.29
2½	8.20	1.48
5	9.78	.82
7½	11.15	.49
10	12.24	.27
15	14.04	.11
20	15.30	.00
30	16.72	.00
40	16.94	.00
50	15.75	.16
60	13.77	.44
70	11.53	.66
80	8.74	.93
90	5.55	1.31
95	3.77	1.53
100	2.02	1.86



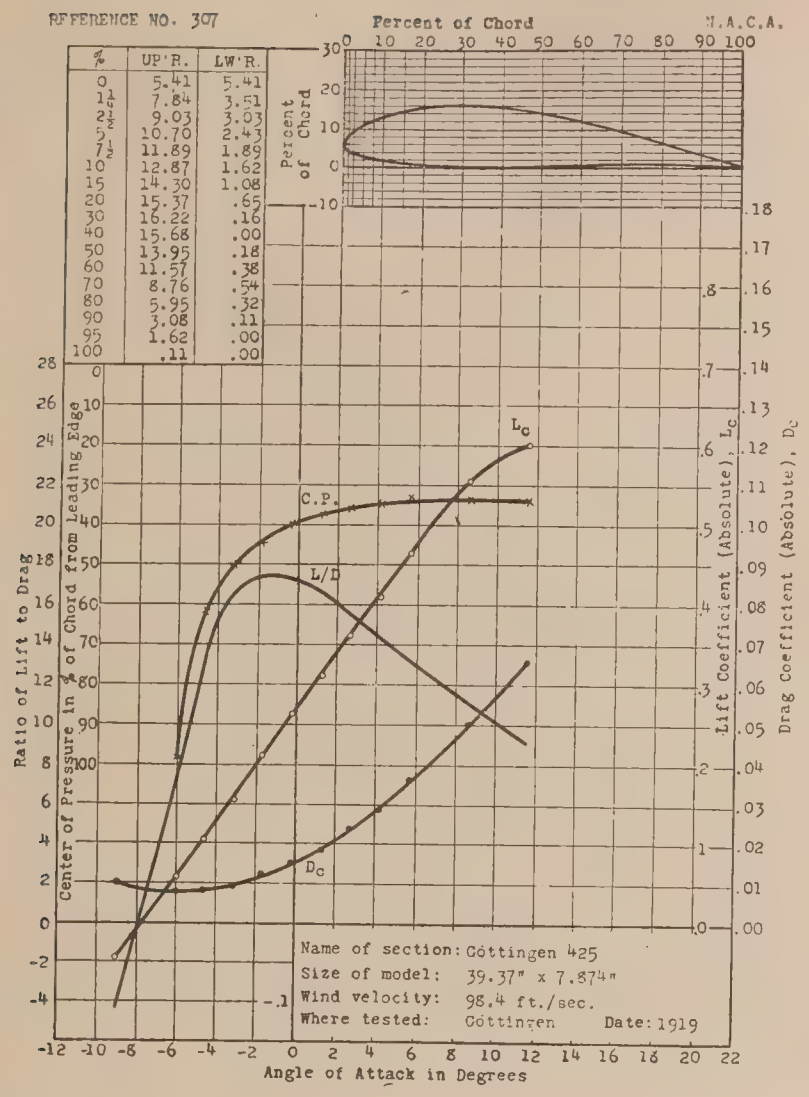
REFERENCE NO. 306

%	UP'R.	LW'R.
0	10.00	10.00
1¼	12.70	8.11
2½	14.05	7.62
5	16.03	6.76
7½	17.57	5.86
10	18.68	5.14
15	20.27	3.89
20	21.19	3.03
30	21.78	1.62
40	20.92	.84
50	18.65	.38
60	15.68	.16
70	12.43	.05
80	8.76	.00
90	4.70	.00
95	2.43	.00
100	.16	.00



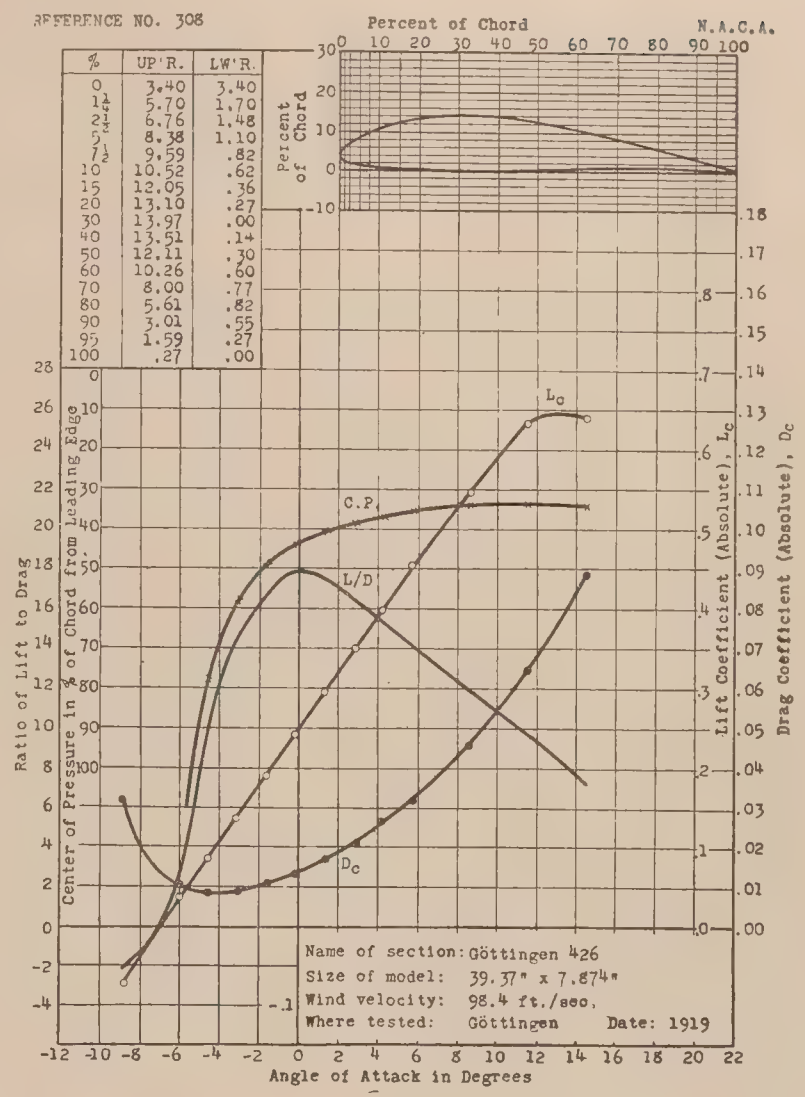
REFERENCE NO. 307

%	UP'R.	LW'R.
0	5.41	5.41
1¼	7.84	3.51
2½	9.03	3.03
5	10.70	2.43
7½	11.89	1.89
10	12.87	1.62
15	14.30	1.08
20	15.37	.65
30	16.22	.16
40	15.68	.00
50	13.95	.18
60	11.57	.38
70	8.76	.54
80	5.95	.32
90	3.08	.11
95	1.62	.00
100	.11	.00



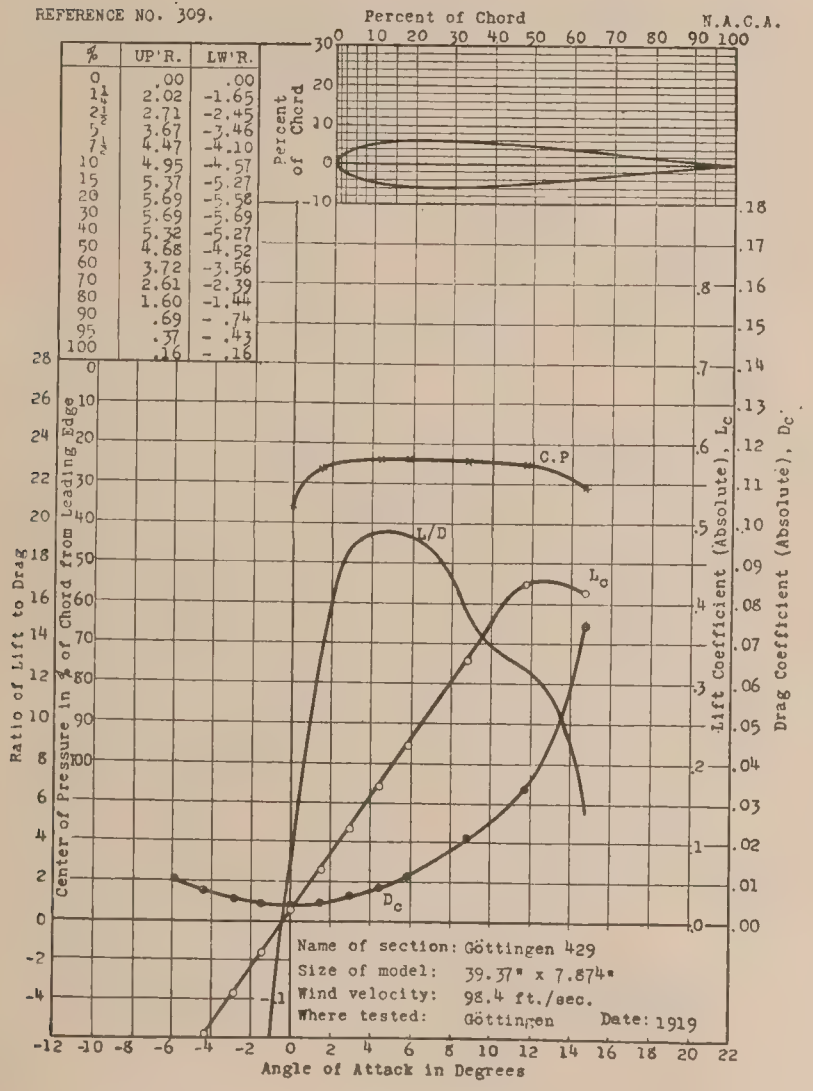
REFERENCE NO. 308

%	UP'R.	LW'R.
0	3.40	3.40
1¼	5.70	1.70
2½	6.76	1.48
5	8.38	1.10
7½	9.59	.82
10	10.52	.62
15	12.05	.36
20	13.10	.27
30	13.97	.00
40	13.51	.14
50	12.11	.30
60	10.26	.60
70	8.00	.77
80	5.61	.82
90	3.01	.55
95	1.59	.27
100	.27	.00



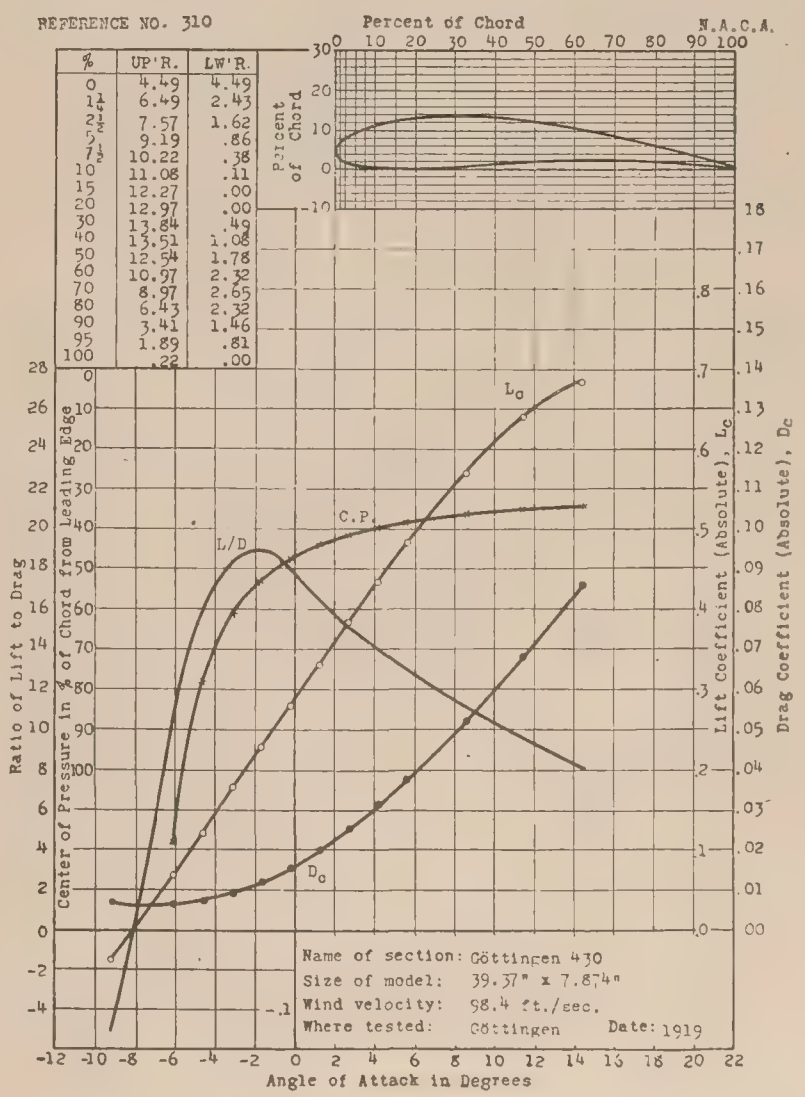
REFERENCE NO. 309.

%	UP'R.	LW'R.
0	.00	.00
1¼	2.02	-1.65
2½	2.71	-2.45
5	3.67	-3.46
7½	4.47	-4.10
10	4.95	-4.57
15	5.37	-5.27
20	5.69	-5.58
30	5.69	-5.69
40	5.32	-5.27
50	4.68	-4.52
60	3.72	-3.56
70	2.61	-2.39
80	1.60	-1.44
90	.69	- .74
95	.37	- .43
100	.16	- .16

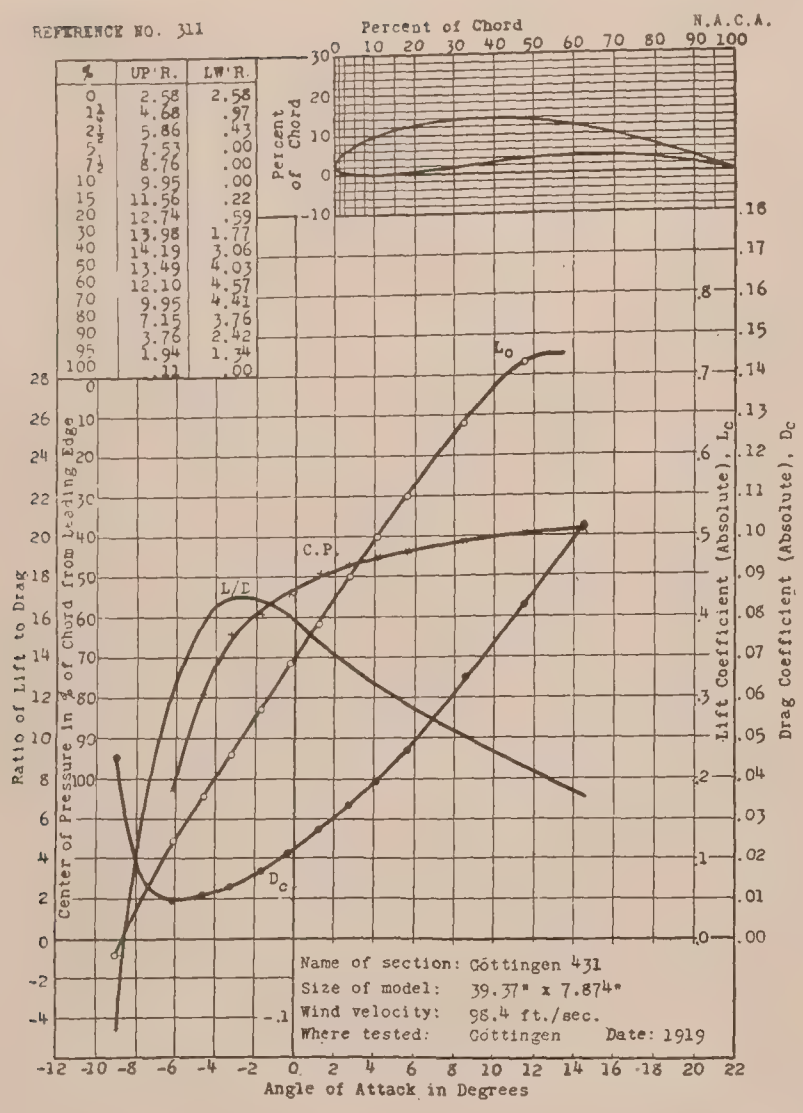


REFERENCE NO. 310

%	UP'R.	LW'R.
0	4.49	4.49
1¼	6.49	2.43
2½	7.57	1.62
5	9.19	.86
7½	10.22	.38
10	11.08	.11
15	12.27	.00
20	12.97	.00
30	13.84	.49
40	13.51	1.08
50	12.54	1.78
60	10.97	2.32
70	8.97	2.65
80	6.43	2.32
90	3.41	1.46
95	1.89	.81
100	.22	.00

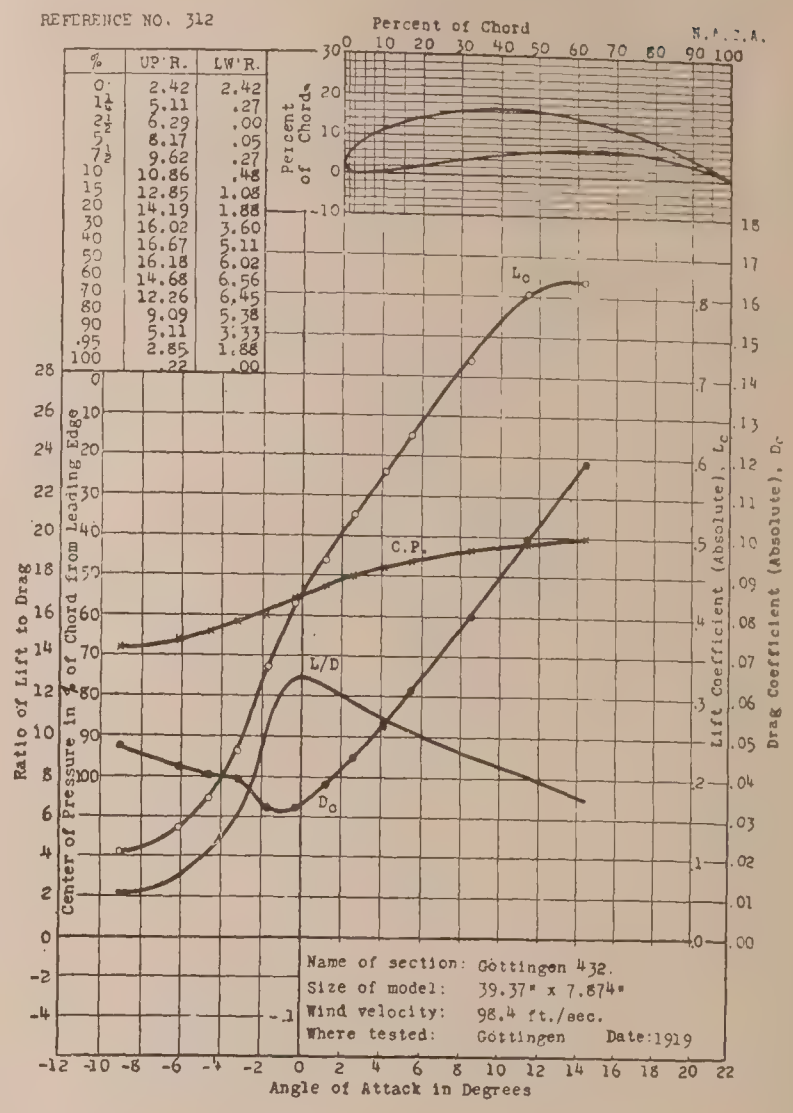


REFERENCE NO. 311



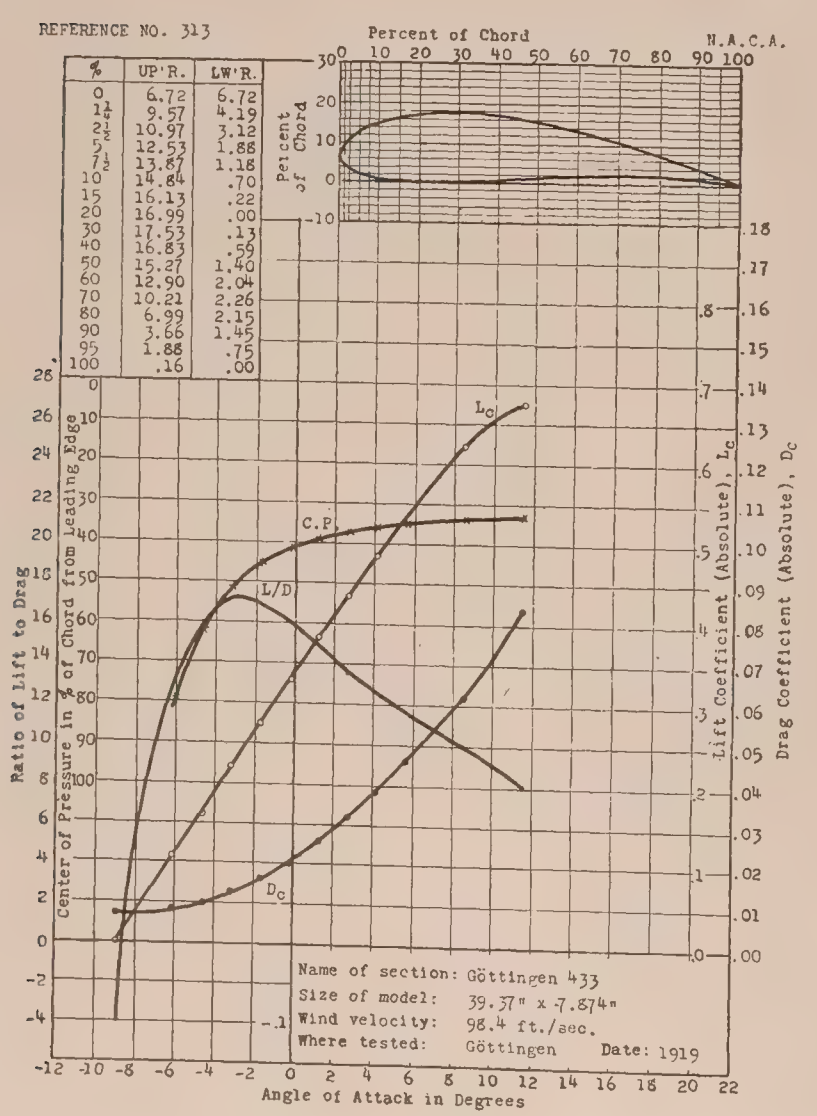
%	UP'R.	LW'R.
0	2.58	2.58
1¼	4.68	.97
2½	5.86	.43
5	7.53	.00
7½	8.76	.00
10	9.95	.00
15	11.56	.22
20	12.74	.59
30	13.98	1.77
40	14.19	3.06
50	13.49	4.03
60	12.10	4.57
70	9.95	4.41
80	7.15	3.76
90	3.76	2.42
95	1.94	1.34
100	.11	.00

REFERENCE NO. 312



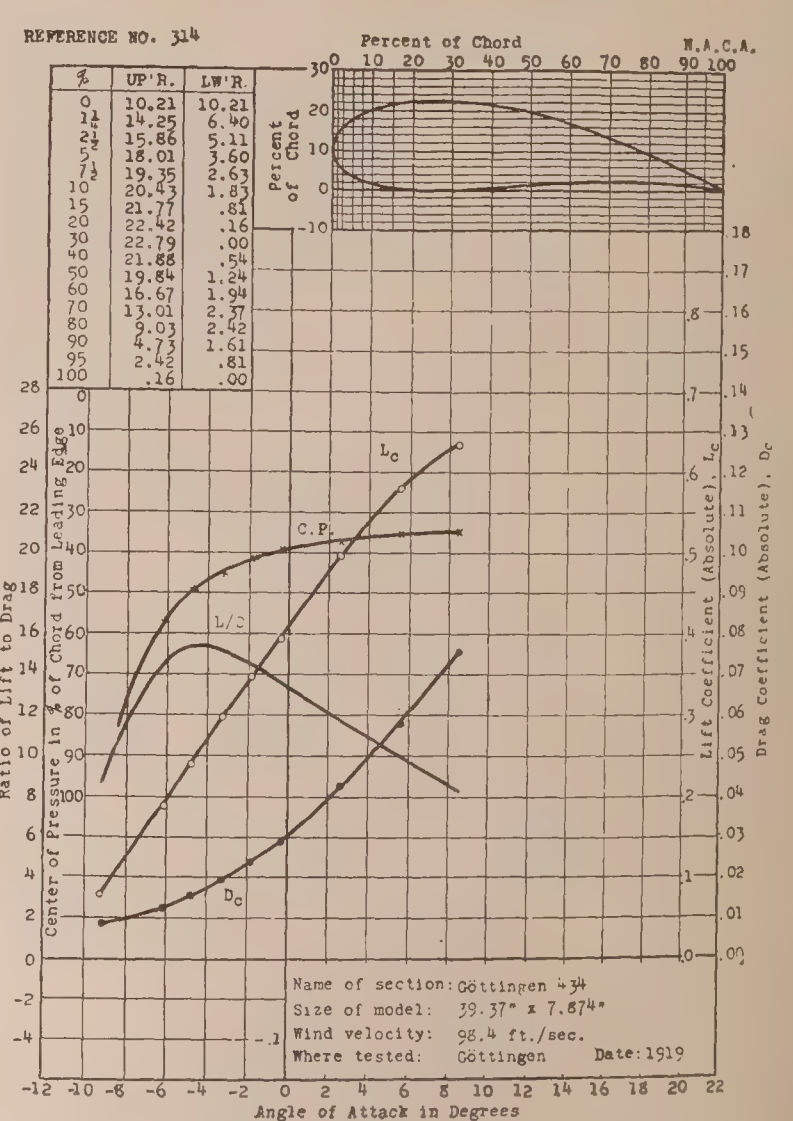
%	UP'R.	LW'R.
0	2.42	2.42
1¼	5.11	.27
2½	6.29	.00
5	8.17	.05
7½	9.62	.27
10	10.86	.48
15	12.85	1.08
20	14.19	1.88
30	16.02	3.60
40	16.67	5.11
50	16.18	6.02
60	14.68	6.56
70	12.26	6.45
80	9.09	5.38
90	5.11	3.33
95	2.85	1.88
100	.22	.00

REFERENCE NO. 313



%	UP'R.	LW'R.
0	6.72	6.72
1¼	9.57	4.19
2½	10.97	3.12
5	12.53	1.88
7½	13.87	1.18
10	14.84	.70
15	16.13	.22
20	16.99	.00
30	17.53	.13
40	16.83	.59
50	15.27	1.40
60	12.90	2.04
70	10.21	2.26
80	6.99	2.15
90	3.66	1.45
95	1.88	.75
100	.16	.00

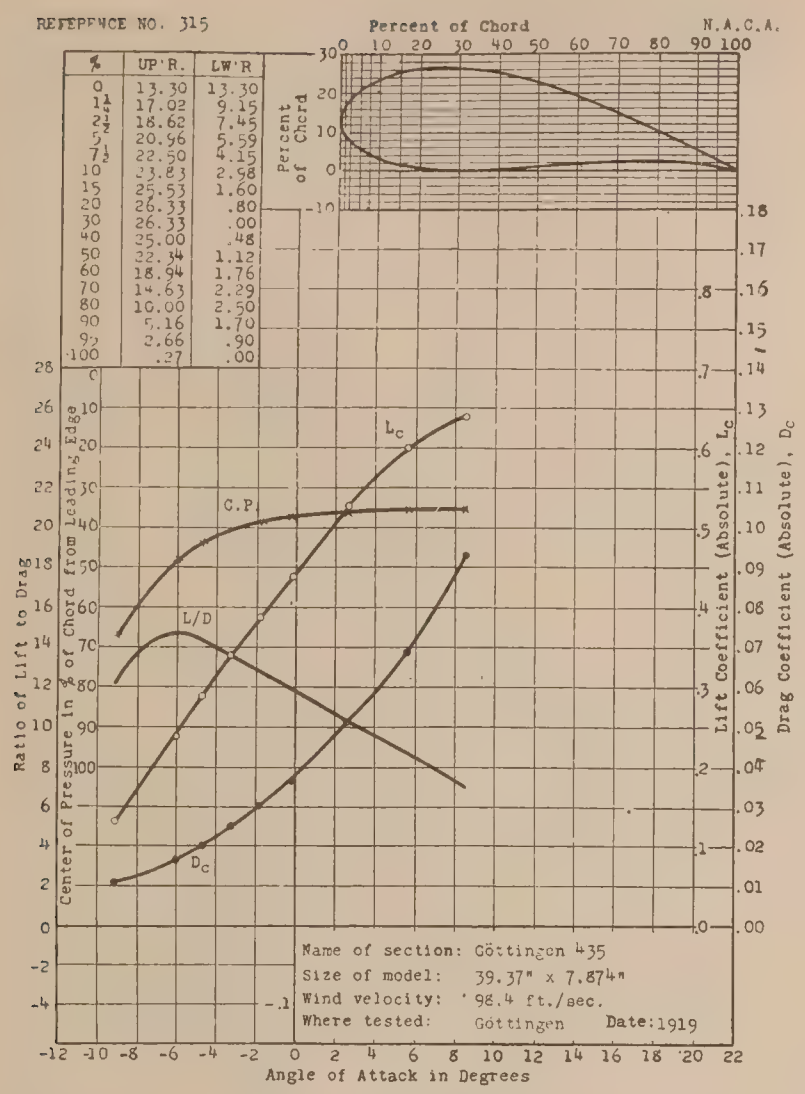
REFERENCE NO. 314



%	UP'R.	LW'R.
0	10.21	10.21
1¼	14.25	6.40
2½	15.86	5.11
5	18.01	3.60
7½	19.35	2.63
10	20.43	1.83
15	21.77	.81
20	22.42	.16
30	22.79	.00
40	21.88	.54
50	19.84	1.24
60	16.67	1.94
70	13.01	2.37
80	9.03	2.42
90	4.73	1.61
95	2.42	.81
100	.16	.00

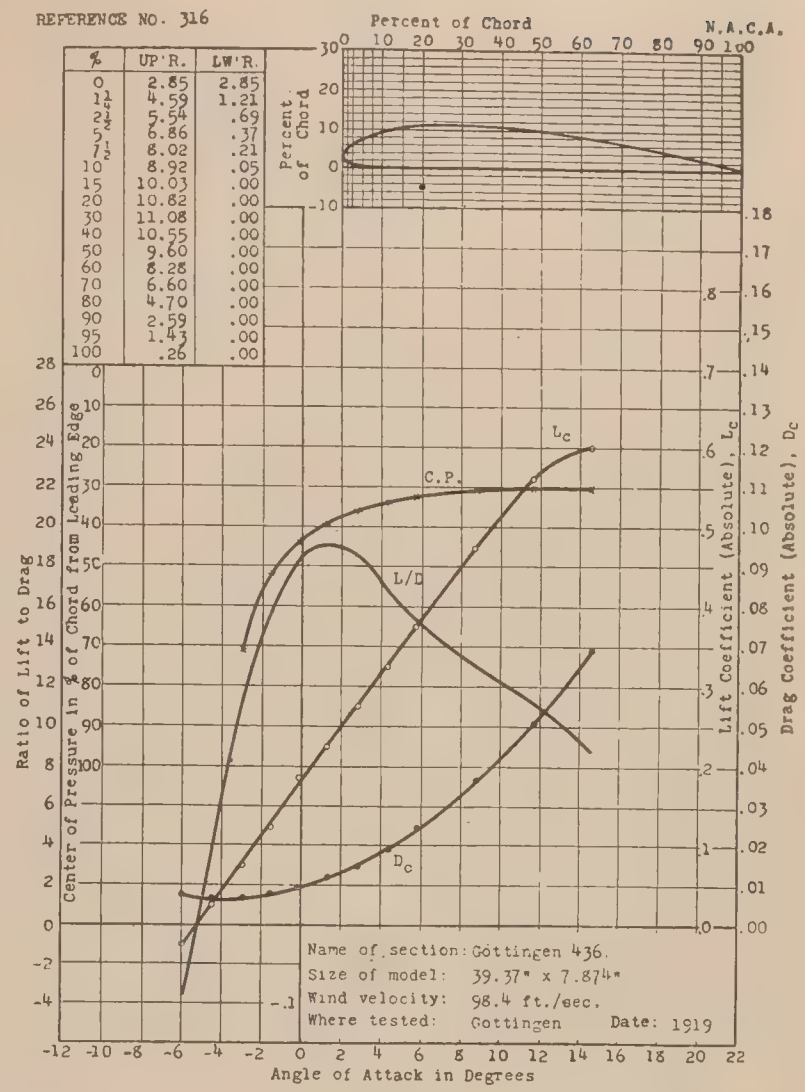
REFERENCE NO. 315

%	UP'R.	LW'R.
0	13.30	13.30
1¼	17.02	9.15
2½	18.62	7.45
5	20.96	5.59
7½	22.50	4.15
10	23.83	2.98
15	25.53	1.60
20	26.33	.80
30	26.33	.00
40	25.00	.48
50	22.34	1.12
60	18.94	1.76
70	14.63	2.29
80	10.00	2.50
90	5.16	1.70
95	2.66	.90
100	.27	.00



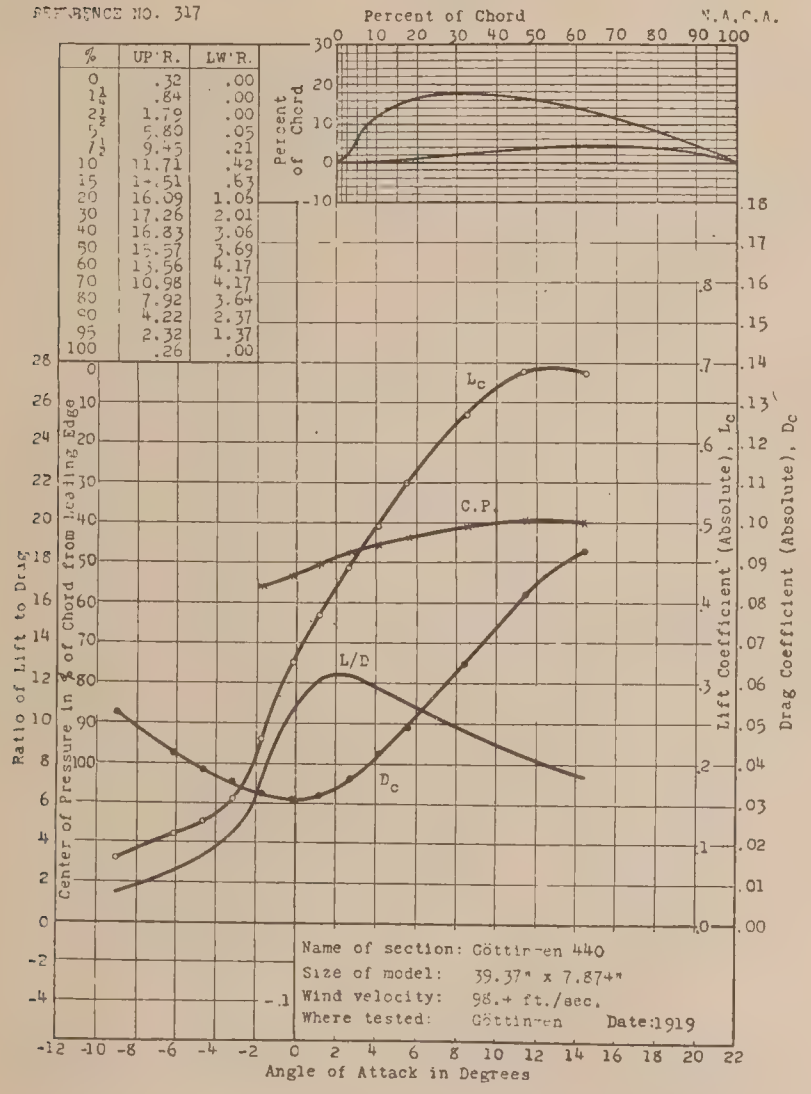
REFERENCE NO. 316

%	UP'R.	LW'R.
0	2.85	2.85
1¼	4.59	1.21
2½	5.54	.69
5	6.86	.37
7½	8.02	.21
10	8.92	.05
15	10.03	.00
20	10.82	.00
30	11.08	.00
40	10.55	.00
50	9.60	.00
60	8.28	.00
70	6.60	.00
80	4.70	.00
90	2.59	.00
95	1.43	.00
100	.26	.00



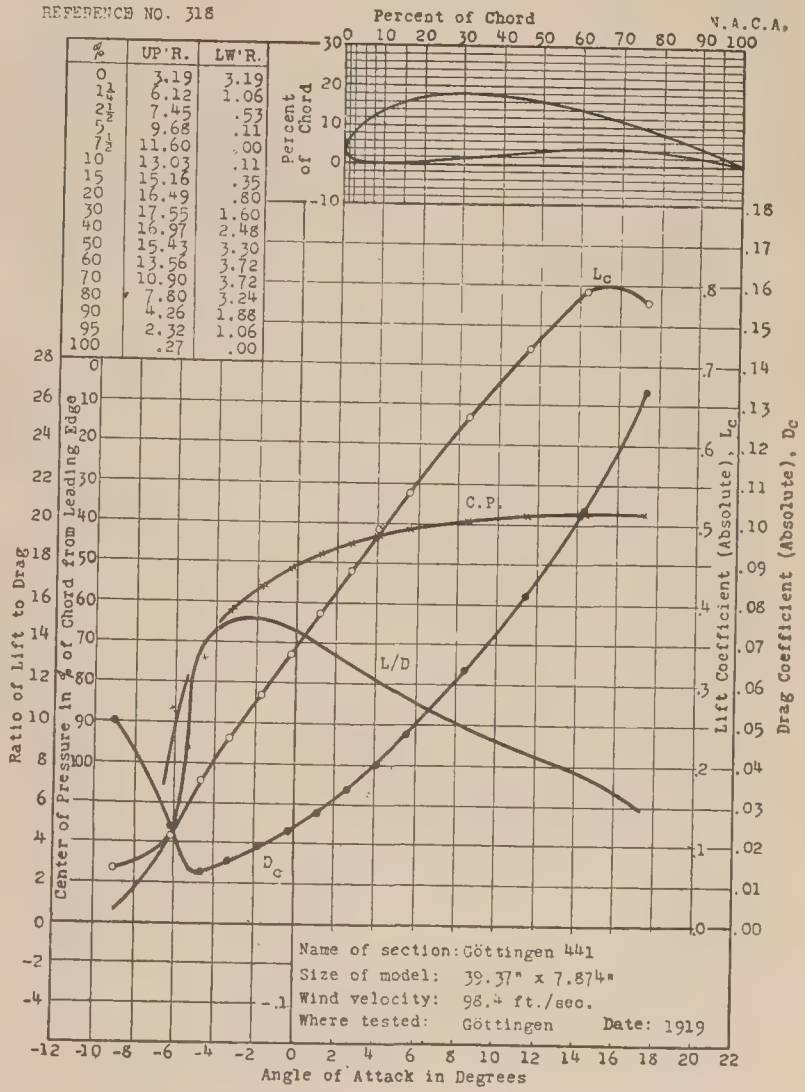
REFERENCE NO. 317

%	UP'R.	LW'R.
0	.32	.00
1¼	.84	.00
2½	1.79	.00
5	5.80	.05
7½	9.45	.21
10	11.71	.42
15	14.51	.63
20	16.09	1.06
30	17.26	2.01
40	16.83	3.06
50	15.57	3.69
60	13.56	4.17
70	10.98	4.17
80	7.92	3.64
90	4.22	2.37
95	2.32	1.37
100	.26	.00



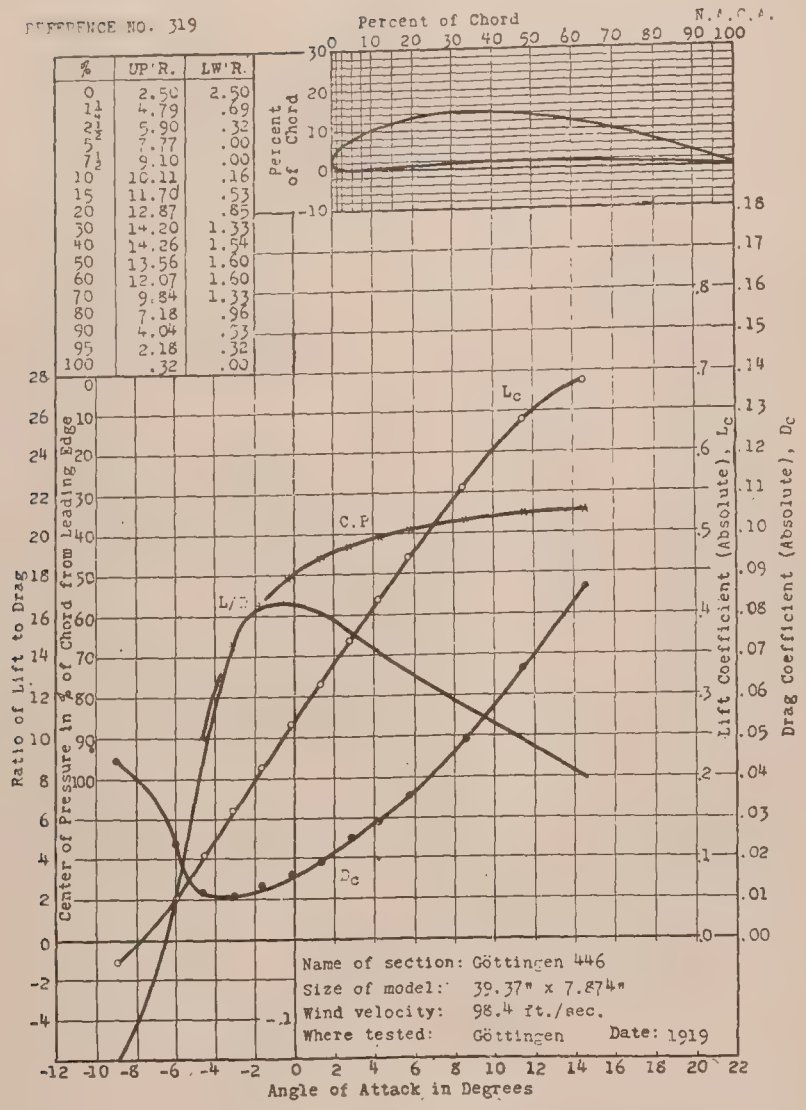
REFERENCE NO. 318

%	UP'R.	LW'R.
0	3.19	3.19
1¼	6.12	1.06
2½	7.45	.53
5	9.68	.11
7½	11.60	.00
10	13.03	.11
15	15.16	.35
20	16.49	.80
30	17.55	1.60
40	16.97	2.48
50	15.43	3.30
60	13.56	3.72
70	10.90	3.72
80	7.80	3.24
90	4.26	1.88
95	2.32	1.06
100	.27	.00



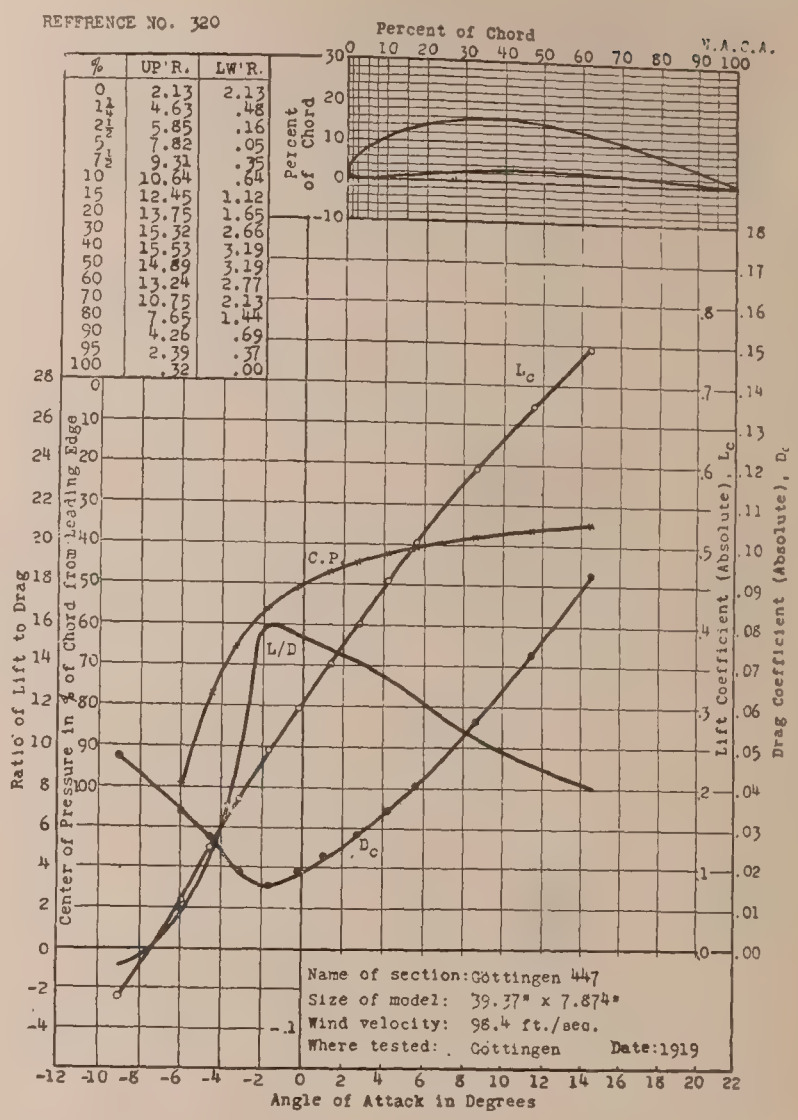
REFERENCE NO. 319

%	UP'R.	LW'R.
0	2.50	2.50
1¼	4.79	.69
2½	5.90	.32
5	7.77	.00
7½	9.10	.00
10	10.11	.16
15	11.70	.53
20	12.87	.85
30	14.20	1.33
40	14.26	1.54
50	13.56	1.60
60	12.07	1.60
70	9.84	1.33
80	7.18	.96
90	4.04	.53
95	2.18	.32
100	.32	.00



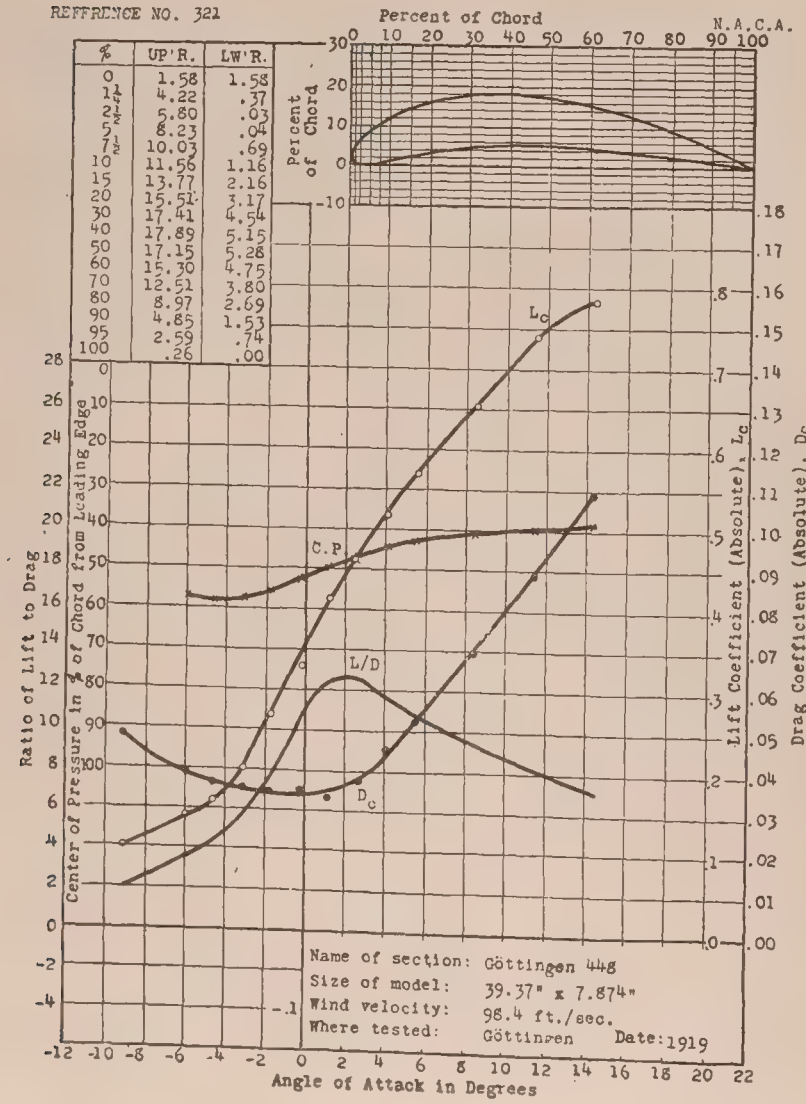
REFERENCE NO. 320

%	UP'R.	LW'R.
0	2.13	2.13
1¼	4.63	.48
2½	5.85	.16
5	7.82	.05
7½	9.31	.35
10	10.64	.64
15	12.45	1.12
20	13.75	1.65
30	15.32	2.66
40	15.53	3.19
50	14.89	3.19
60	13.24	2.77
70	10.75	2.13
80	7.65	1.44
90	4.26	.69
95	2.39	.37
100	.32	.00



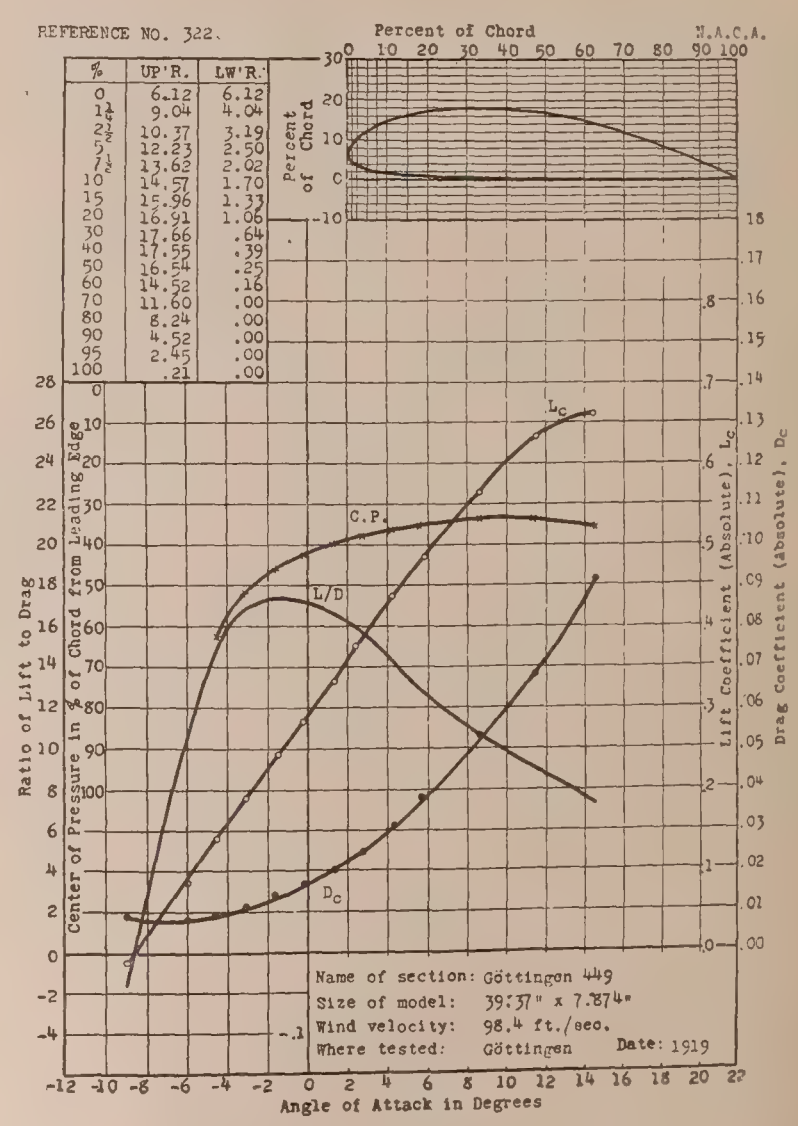
REFERENCE NO. 321

%	UP'R.	LW'R.
0	1.58	1.58
1¼	4.22	.37
2½	5.80	.03
5	8.23	.04
7½	10.03	.69
10	11.56	1.16
15	13.77	2.16
20	15.51	3.17
30	17.41	4.54
40	17.89	5.15
50	17.15	5.28
60	15.30	4.75
70	12.51	3.80
80	8.97	2.69
90	4.85	1.53
95	2.59	.74
100	.26	.00



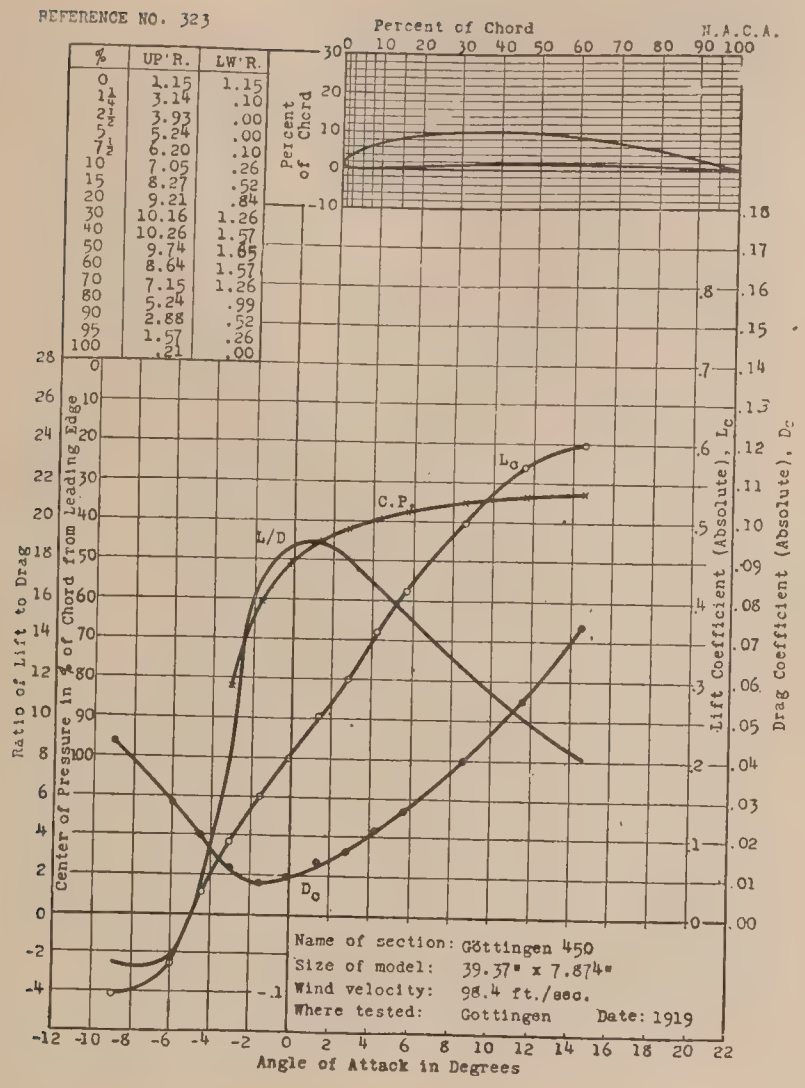
REFERENCE NO. 322.

%	UP'R.	LW'R.
0	6.12	6.12
1¼	9.04	4.04
2½	10.37	3.19
5	12.23	2.50
7½	13.62	2.02
10	14.57	1.70
15	15.96	1.33
20	16.91	1.06
30	17.66	.64
40	17.55	.39
50	16.54	.25
60	14.52	.16
70	11.60	.00
80	8.24	.00
90	4.52	.00
95	2.45	.00
100	.21	.00



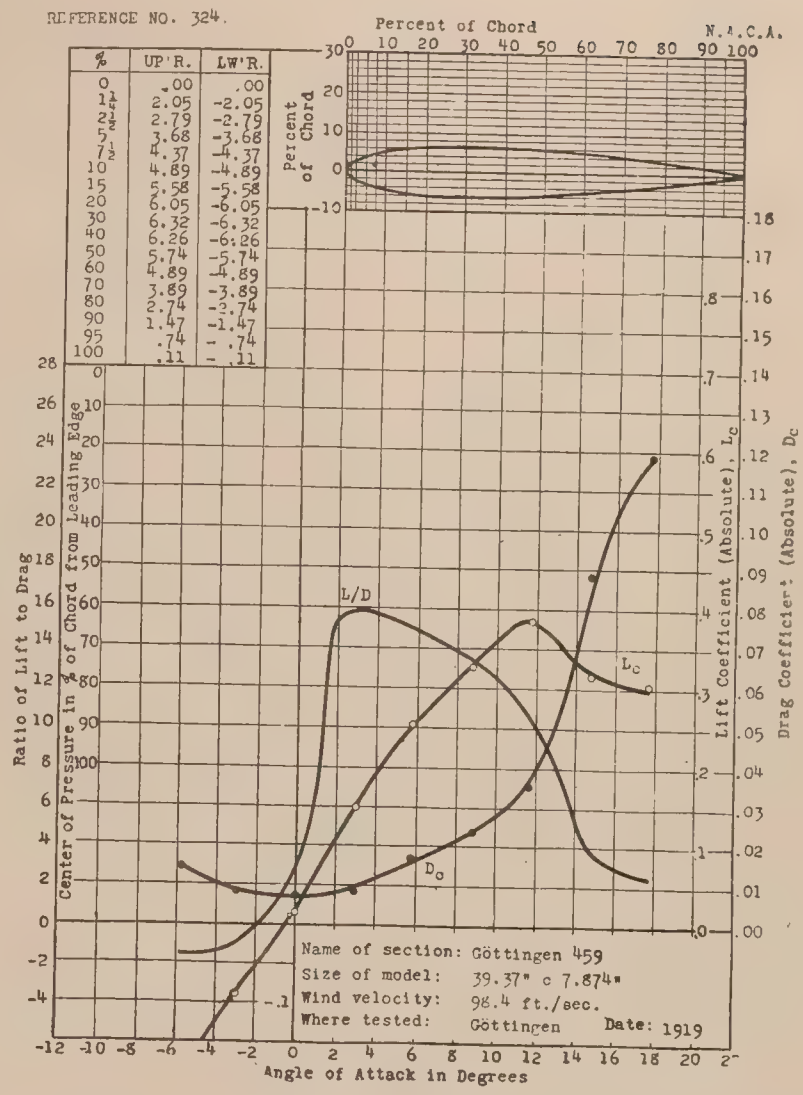
REFERENCE NO. 323

%	UP'R.	LW'R.
0	1.15	1.15
1¼	3.14	.10
2½	3.93	.00
5	5.24	.00
7½	6.20	.10
10	7.05	.26
15	8.27	.52
20	9.21	.84
30	10.16	1.26
40	10.26	1.57
50	9.74	1.85
60	8.64	1.57
70	7.15	1.26
80	5.24	.99
90	2.88	.52
95	1.57	.26
100	.21	.00



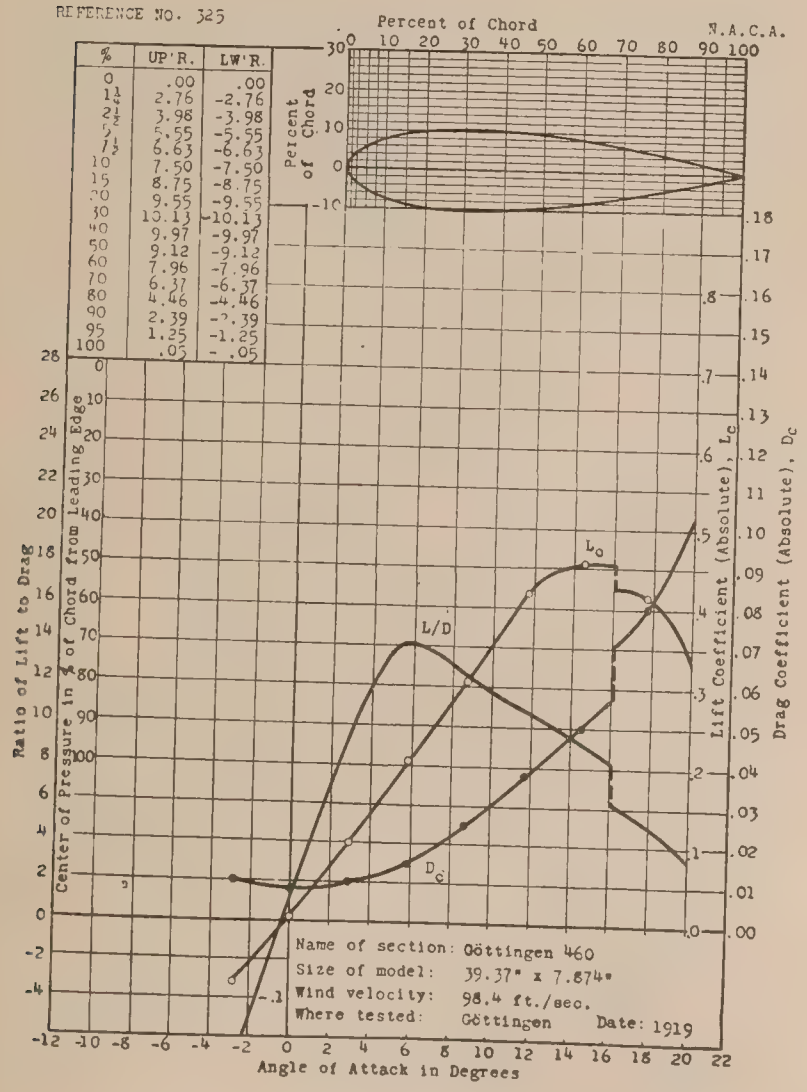
REFERENCE NO. 324.

%	UP'R.	LW'R.
0	.00	.00
1¼	2.05	-2.05
2½	2.79	-2.79
5	3.68	-3.68
7½	4.37	-4.37
10	4.89	-4.89
15	5.58	-5.58
20	6.05	-6.05
30	6.32	-6.32
40	6.26	-6.26
50	5.74	-5.74
60	4.89	-4.89
70	3.89	-3.89
80	2.74	-2.74
90	1.47	-1.47
95	.74	- .74
100	.11	- .11



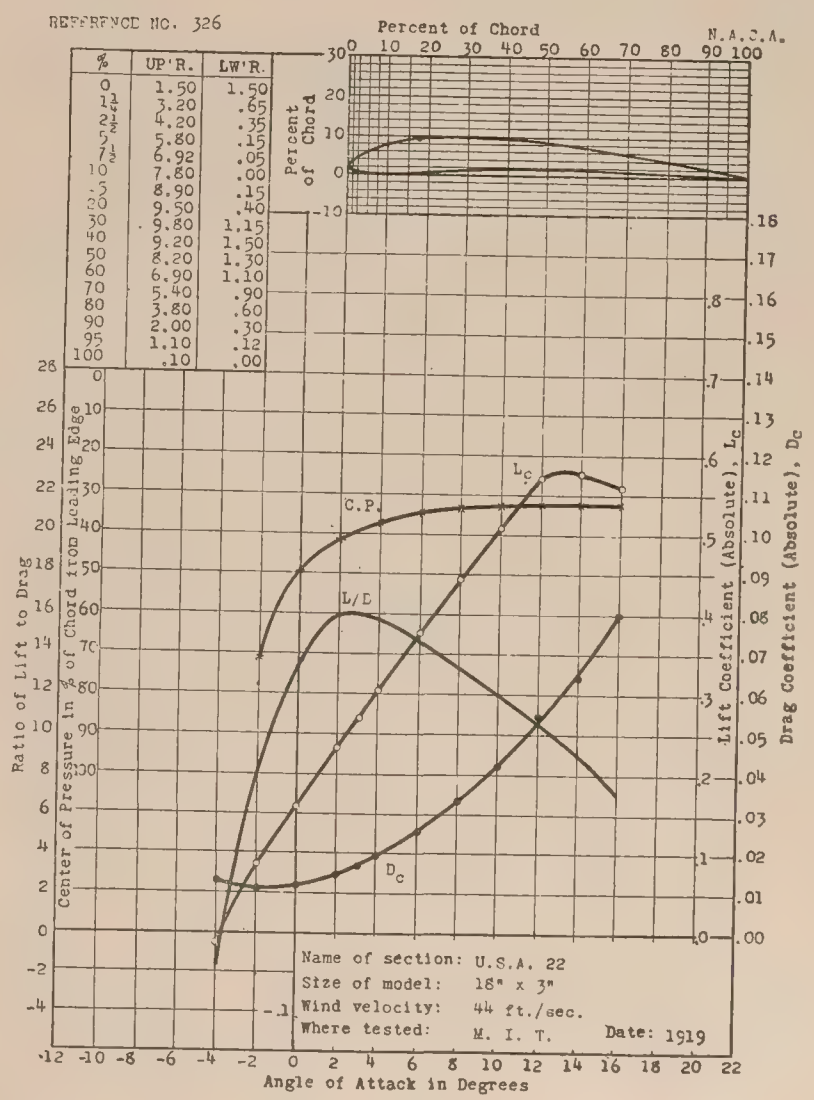
REFERENCE NO. 325

%	UP'R.	LW'R.
0	.00	.00
1¼	2.76	-2.76
2½	3.98	-3.98
5	5.55	-5.55
7½	6.63	-6.63
10	7.50	-7.50
15	8.75	-8.75
20	9.55	-9.55
30	10.13	-10.13
40	9.97	-9.97
50	9.12	-9.12
60	7.96	-7.96
70	6.37	-6.37
80	4.46	-4.46
90	2.39	-2.39
95	1.25	-1.25
100	.05	- .05

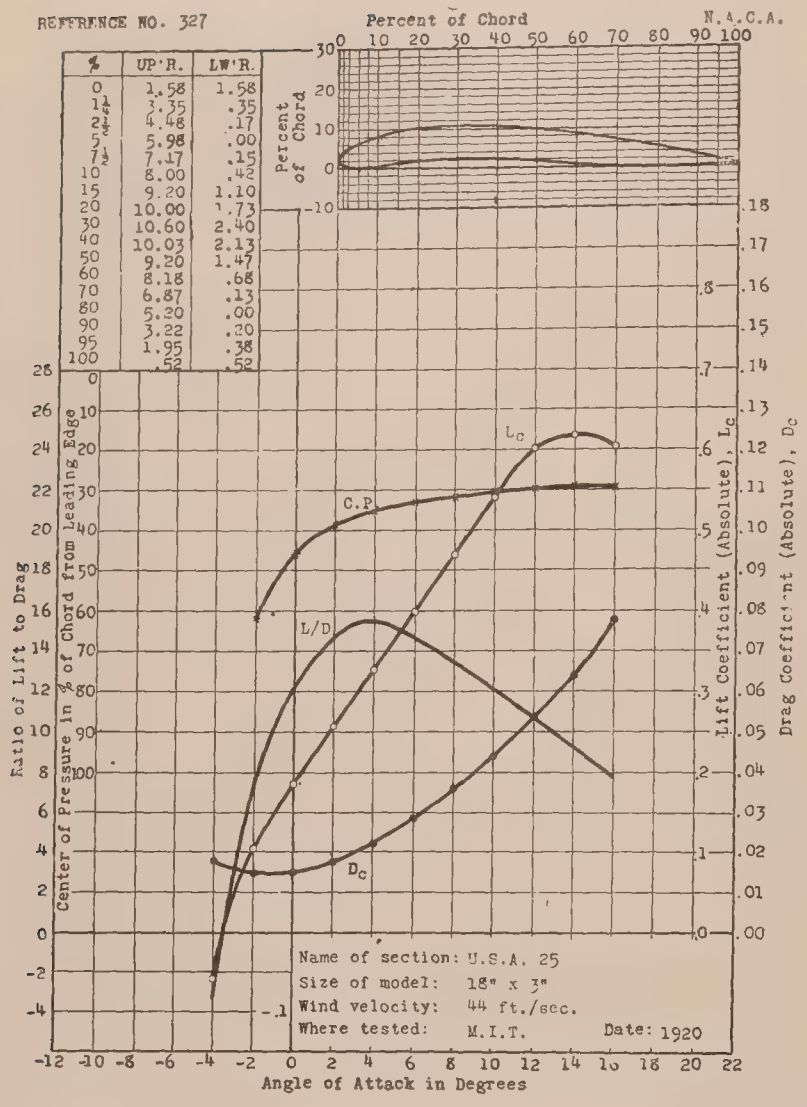


REFERENCE NO. 326

%	UP'R.	LW'R.
0	1.50	1.50
1¼	3.20	.65
2½	4.20	.35
5	5.80	.15
7½	6.92	.05
10	7.80	.00
15	8.90	.15
20	9.50	.40
30	9.80	1.15
40	9.20	1.50
50	8.20	1.30
60	6.90	1.10
70	5.40	.90
80	3.80	.60
90	2.00	.30
95	1.10	.12
100	.10	.00

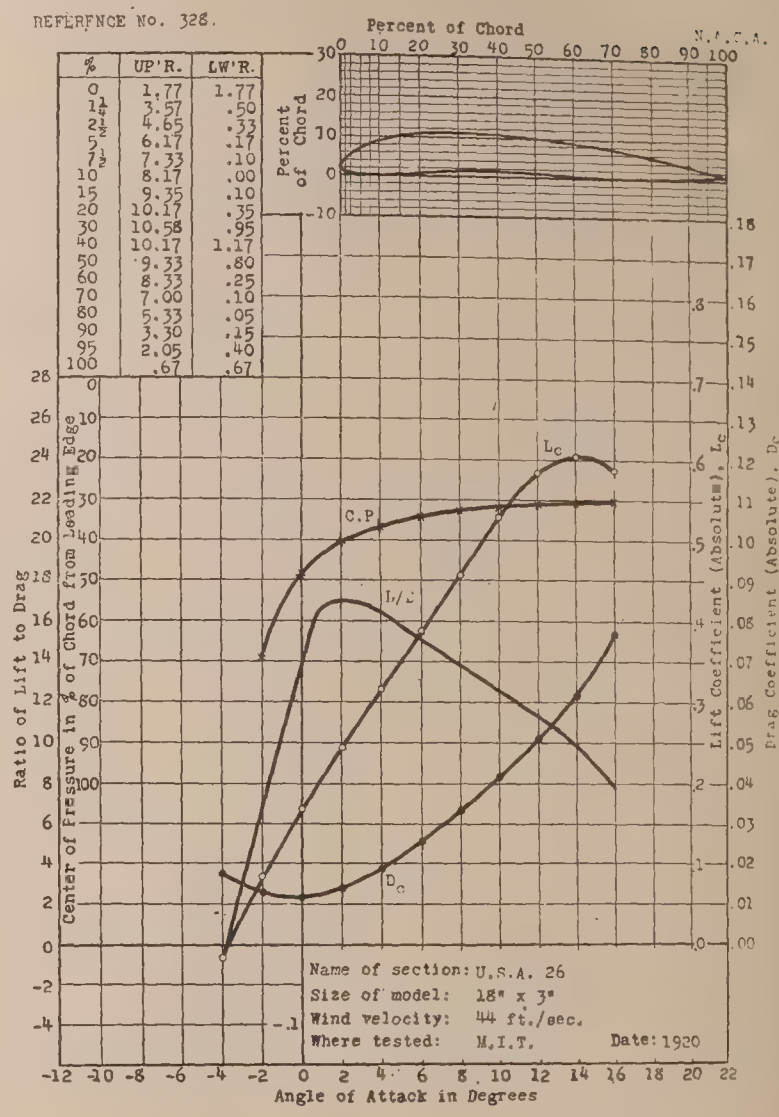


REFERENCE NO. 327



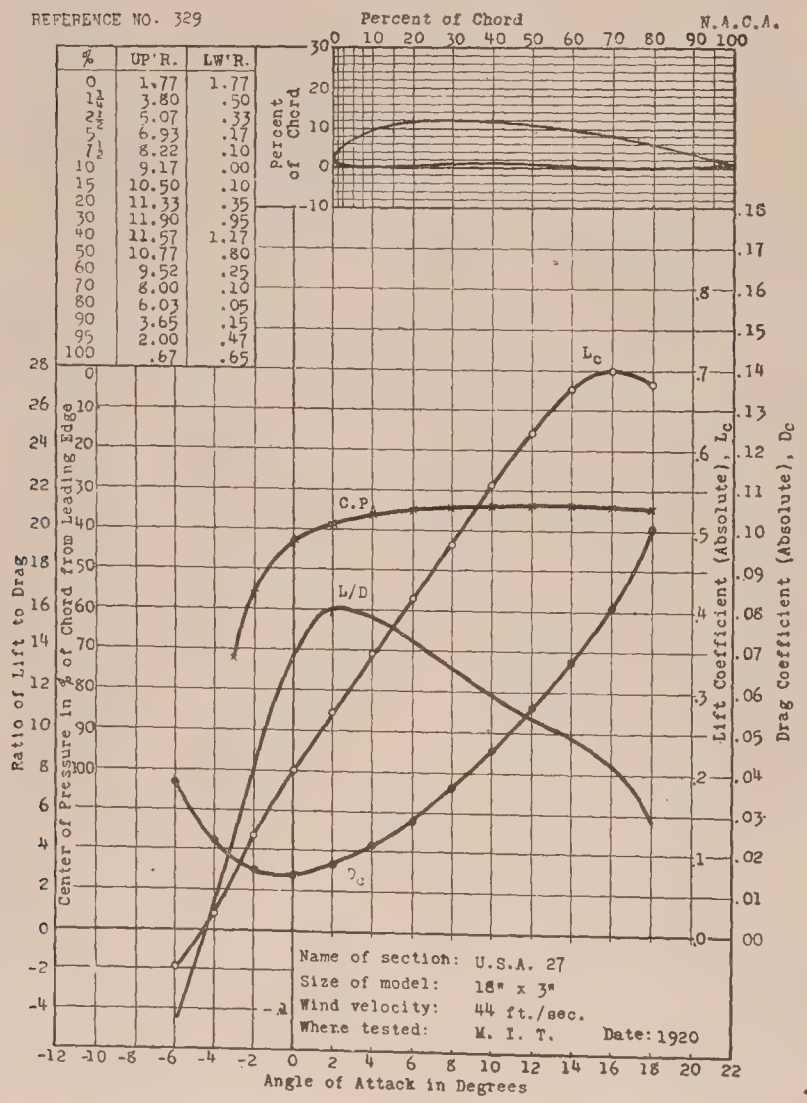
%	UP'R.	LW'R.
0	1.58	1.58
1¼	3.35	.35
2½	4.48	.17
5	5.98	.00
7½	7.17	.15
10	8.00	.42
15	9.20	1.10
20	10.00	1.73
30	10.60	2.40
40	10.03	2.13
50	9.20	1.47
60	8.18	.68
70	6.87	.13
80	5.20	.00
90	3.22	.20
95	1.95	.38
100	.52	.52

REFERENCE No. 328.



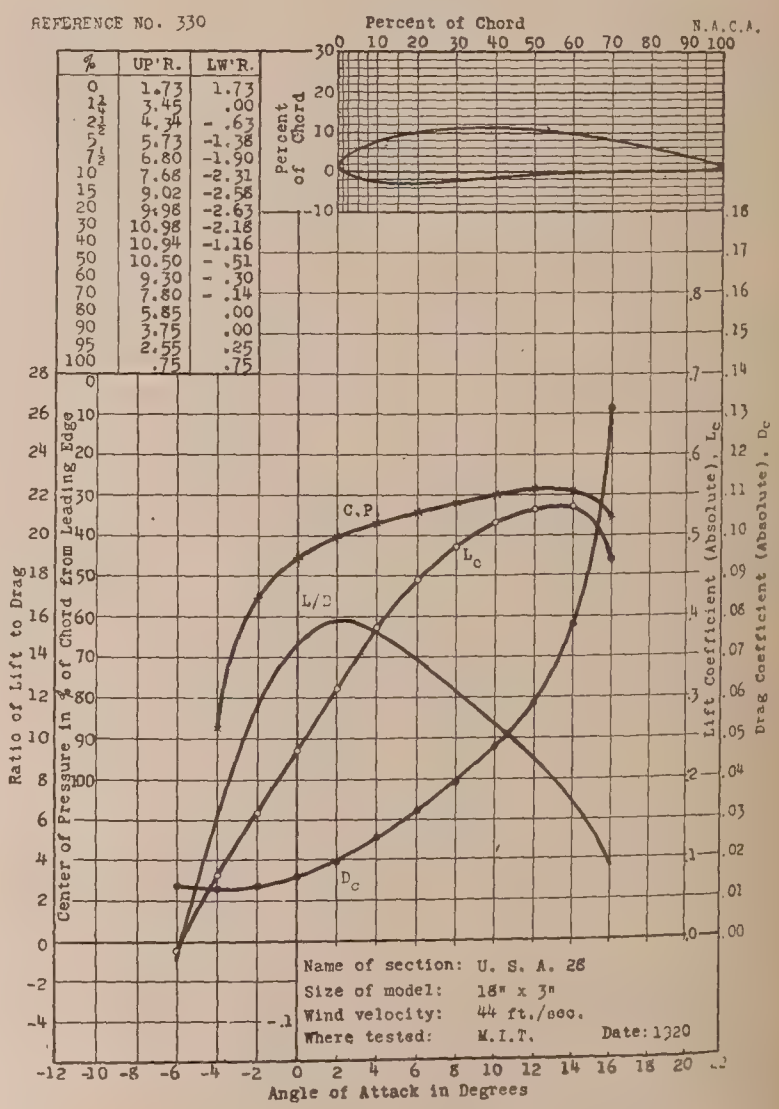
%	UP'R.	LW'R.
0	1.77	1.77
1¼	3.57	.50
2½	4.65	.33
5	6.17	.17
7½	7.33	.10
10	8.17	.00
15	9.35	.10
20	10.17	.35
30	10.58	.95
40	10.17	1.17
50	9.33	.80
60	8.33	.25
70	7.00	.10
80	5.33	.05
90	3.30	.15
95	2.05	.40
100	.67	.67

REFERENCE NO. 329



%	UP'R.	LW'R.
0	1.77	1.77
1¼	3.80	.50
2½	5.07	.33
5	6.93	.17
7½	8.22	.10
10	9.17	.00
15	10.50	.10
20	11.33	.35
30	11.90	.95
40	11.57	1.17
50	10.77	.80
60	9.52	.25
70	8.00	.10
80	6.03	.05
90	3.65	.15
95	2.00	.47
100	.67	.65

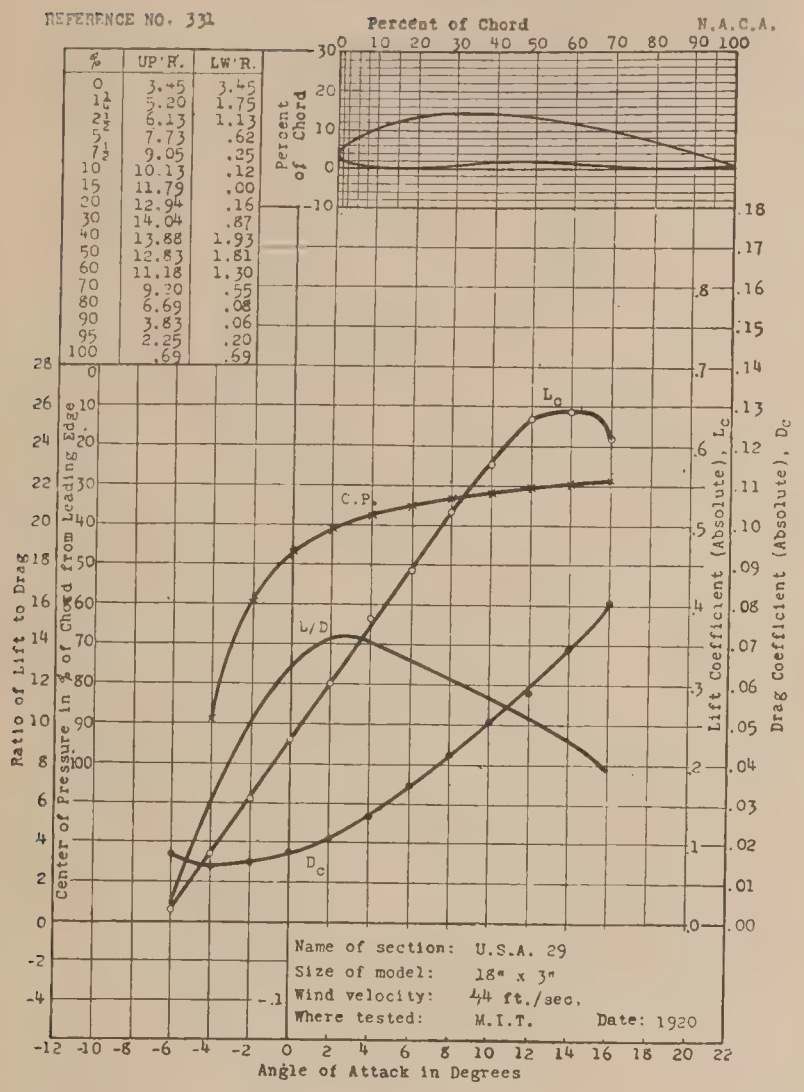
REFERENCE NO. 330



%	UP'R.	LW'R.
0	1.73	1.73
1¼	3.45	.00
2½	4.34	- .63
5	5.73	-1.38
7½	6.80	-1.90
10	7.68	-2.31
15	9.02	-2.58
20	9.98	-2.63
30	10.98	-2.18
40	10.94	-1.16
50	10.50	- .51
60	9.30	- .30
70	7.80	- .14
80	5.85	.00
90	3.75	.00
95	2.55	.25
100	.75	.75

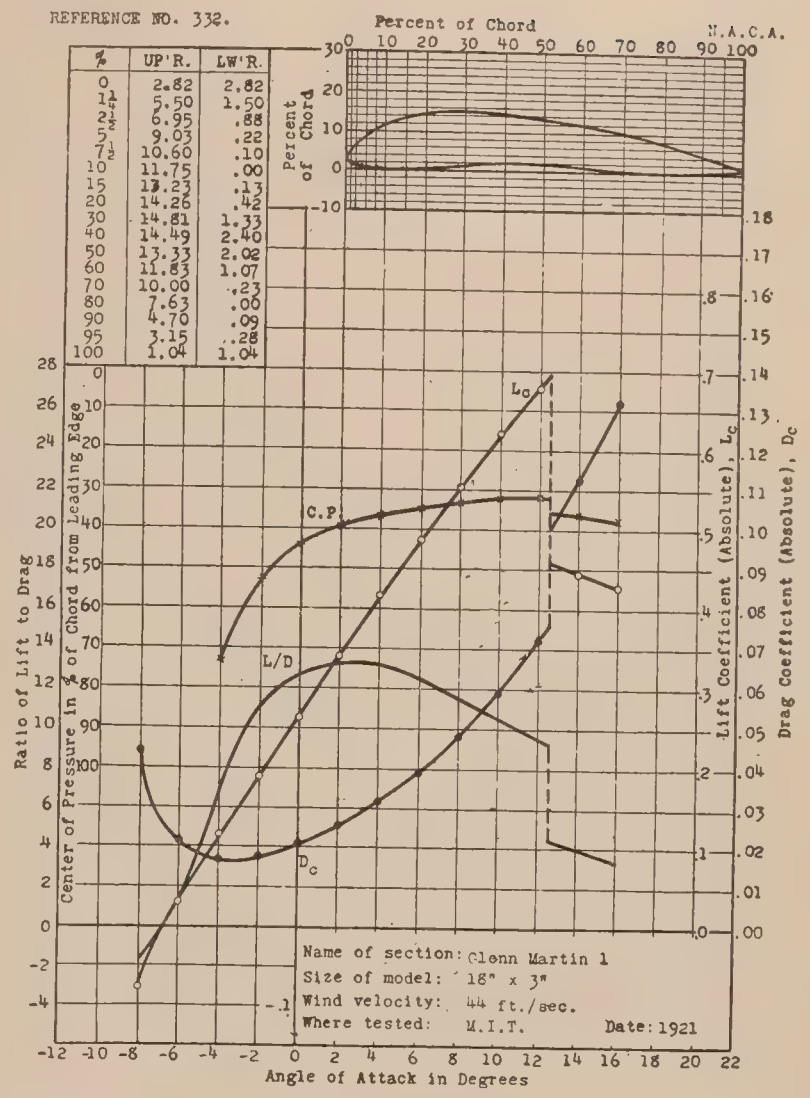
REFERENCE NO. 331

%	UP'R.	LW'R.
0	3.45	3.45
1¼	5.20	1.75
2½	6.13	1.13
5	7.73	.62
7½	9.05	.25
10	10.13	.12
15	11.79	.00
20	12.94	.16
30	14.04	.87
40	13.88	1.93
50	12.83	1.81
60	11.18	1.30
70	9.20	.55
80	6.69	.08
90	3.83	.06
95	2.25	.20
100	.69	.69



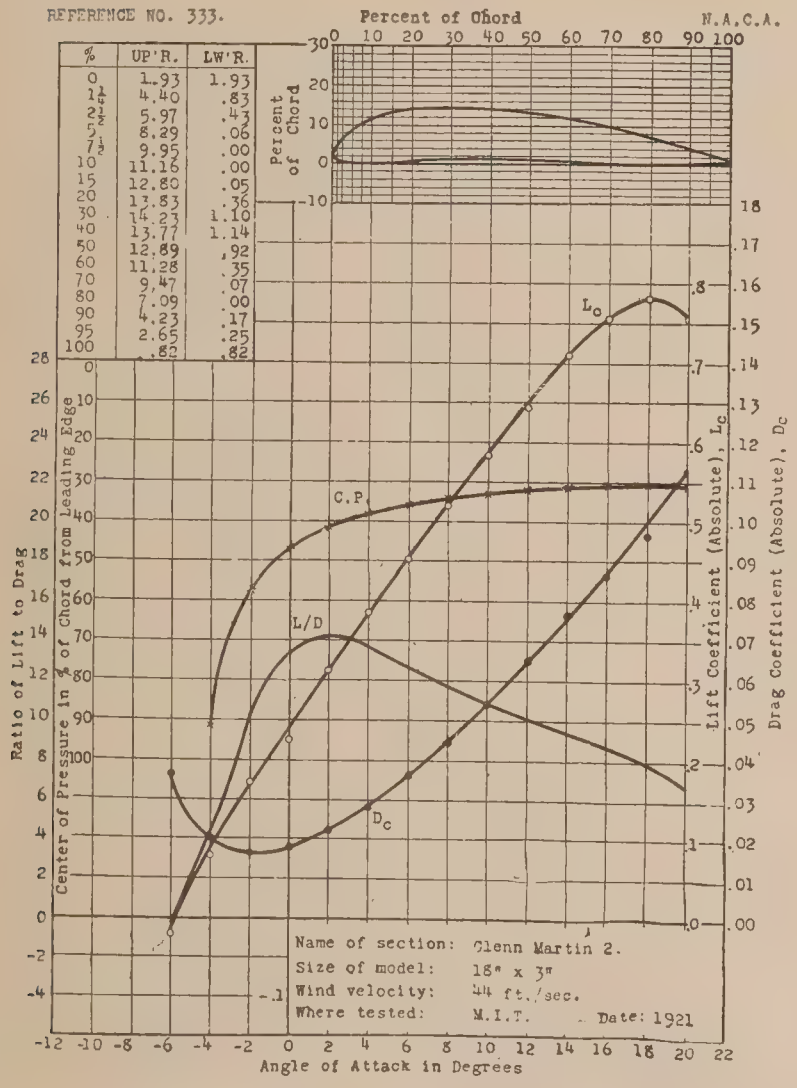
REFERENCE NO. 332.

%	UP'R.	LW'R.
0	2.82	2.82
1¼	5.50	1.50
2½	6.95	.88
5	9.03	.22
7½	10.60	.10
10	11.75	.00
15	13.23	.13
20	14.26	.42
30	14.81	1.33
40	14.49	2.40
50	13.33	2.02
60	11.83	1.07
70	10.00	.23
80	7.63	.06
90	4.70	.09
95	3.15	.28
100	1.04	1.04



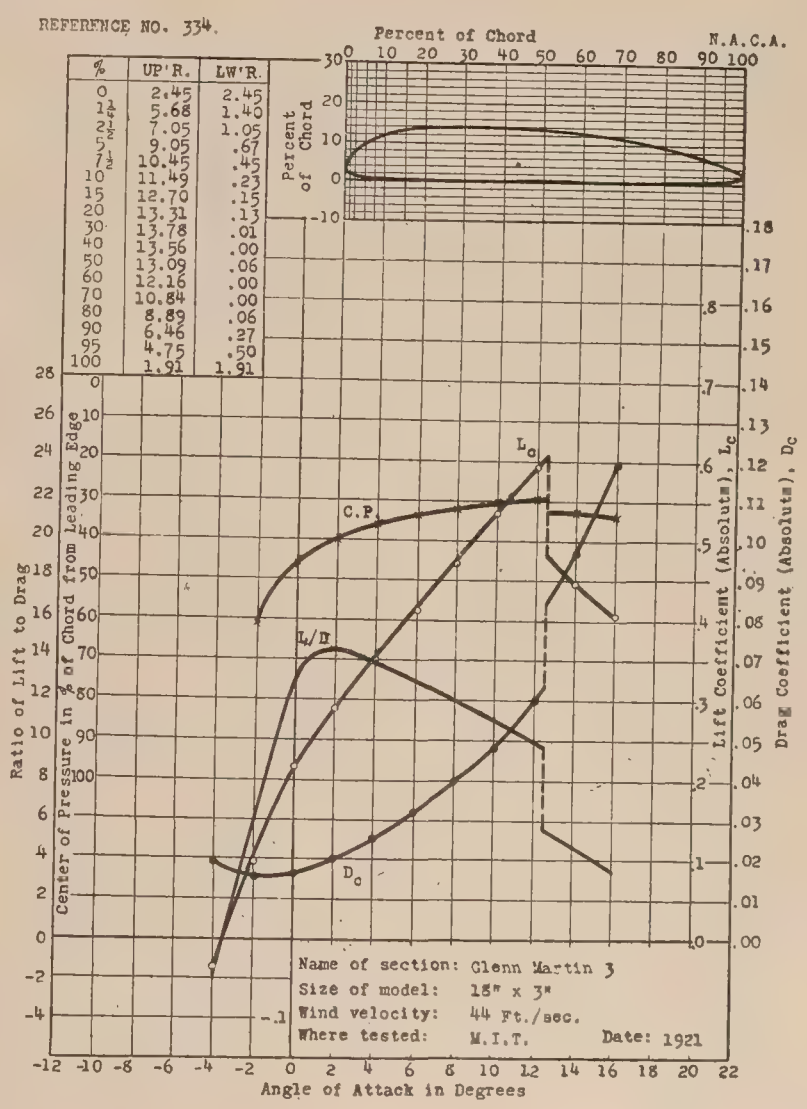
REFERENCE NO. 333.

%	UP'R.	LW'R.
0	1.93	1.93
1¼	4.40	.83
2½	5.97	.43
5	8.29	.06
7½	9.95	.00
10	11.16	.00
15	12.80	.05
20	13.83	.36
30	14.23	1.10
40	13.77	1.14
50	12.89	.92
60	11.28	.35
70	9.47	.07
80	7.09	.00
90	4.23	.17
95	2.65	.25
100	.82	.82



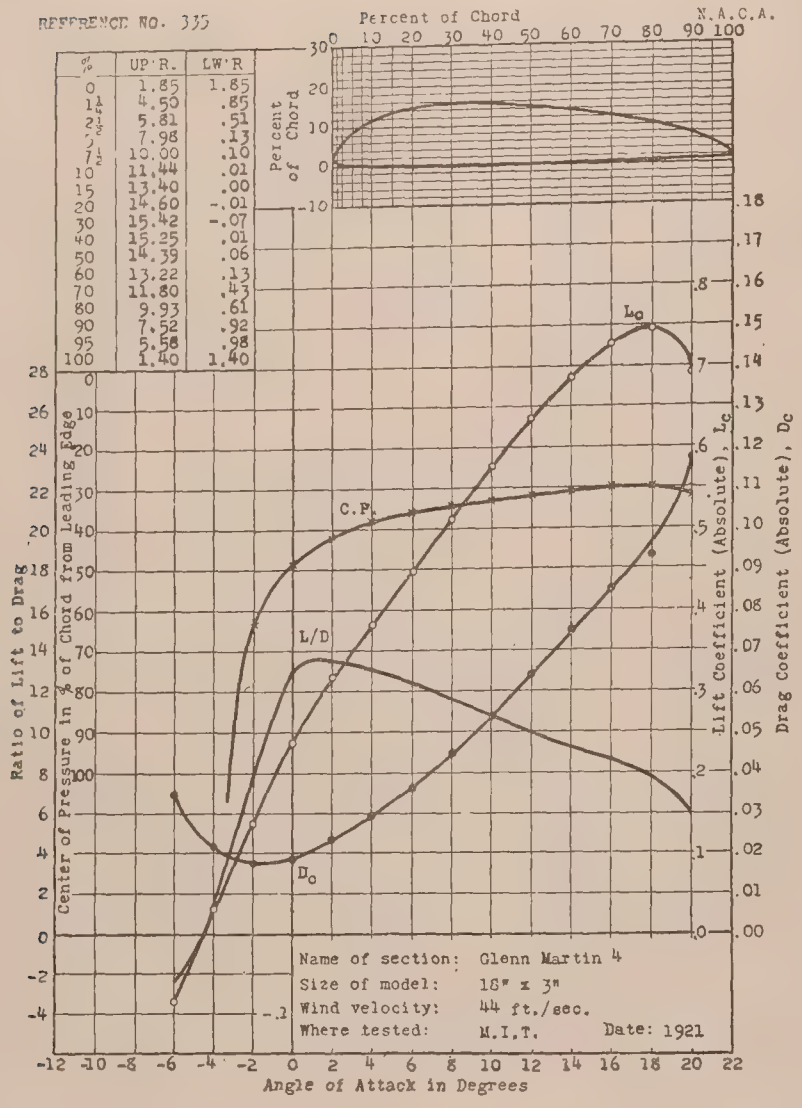
REFERENCE NO. 334.

%	UP'R.	LW'R.
0	2.45	2.45
1¼	5.68	1.40
2½	7.05	1.05
5	9.05	.67
7½	10.45	.45
10	11.49	.23
15	12.70	.15
20	13.31	.13
30	13.78	.01
40	13.56	.00
50	13.09	.06
60	12.16	.00
70	10.84	.00
80	8.89	.06
90	6.46	.27
95	4.75	.50
100	1.91	1.91



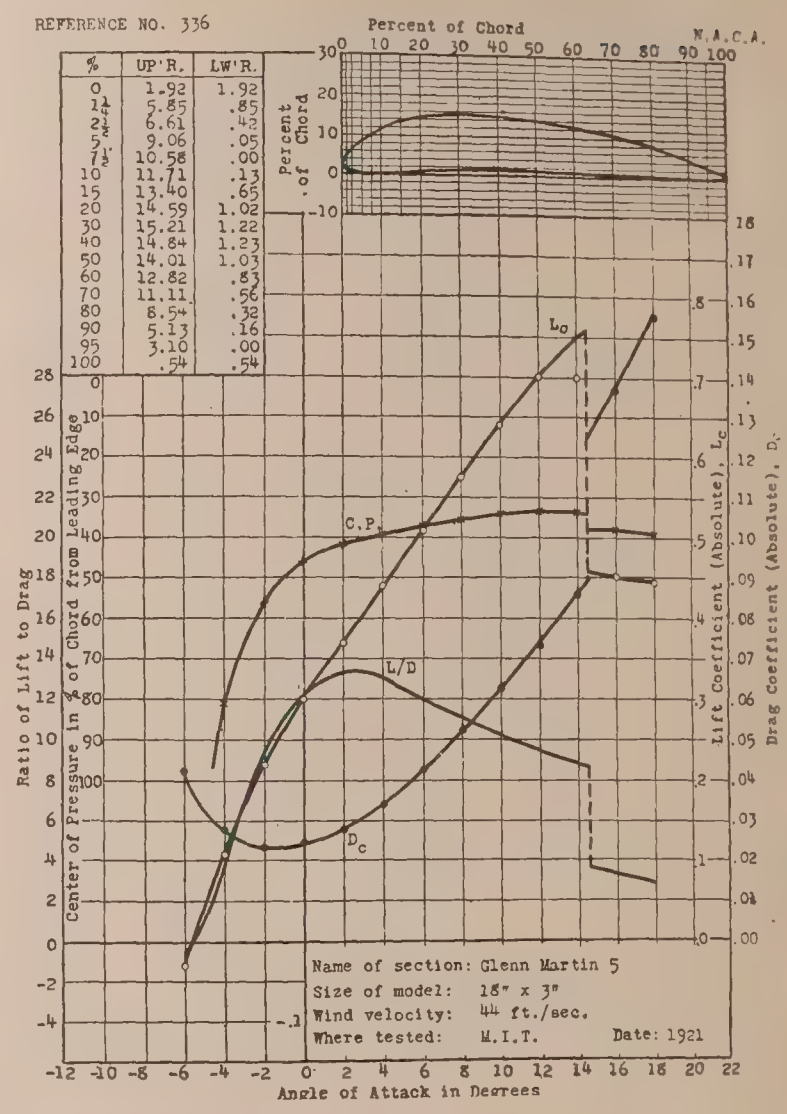
REFERENCE NO. 335

%	UP'R.	LW'R
0	1.85	1.85
1¼	4.50	.85
2½	5.81	.51
5	7.98	.13
7½	10.00	.10
10	11.44	.01
15	13.40	.00
20	14.60	-.01
30	15.42	-.07
40	15.25	.01
50	14.39	.06
60	13.22	.13
70	11.80	.43
80	9.93	.61
90	7.52	.92
95	5.58	.98
100	1.40	1.40



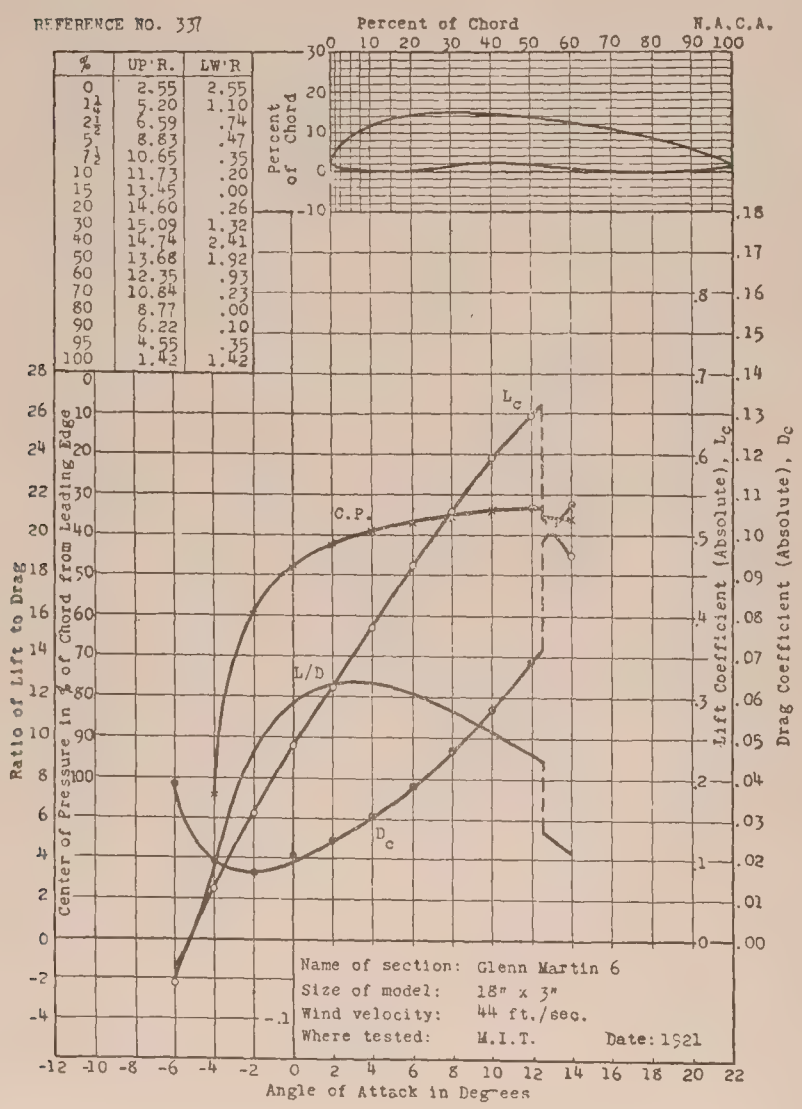
REFERENCE NO. 336

%	UP'R.	LW'R.
0	1.92	1.92
1¼	5.85	.85
2½	6.61	.42
5	9.06	.05
7½	10.58	.00
10	11.71	.13
15	13.40	.65
20	14.59	1.02
30	15.21	1.22
40	14.84	1.23
50	14.01	1.03
60	12.82	.83
70	11.11	.56
80	8.54	.32
90	5.13	.16
95	3.10	.00
100	.54	.54



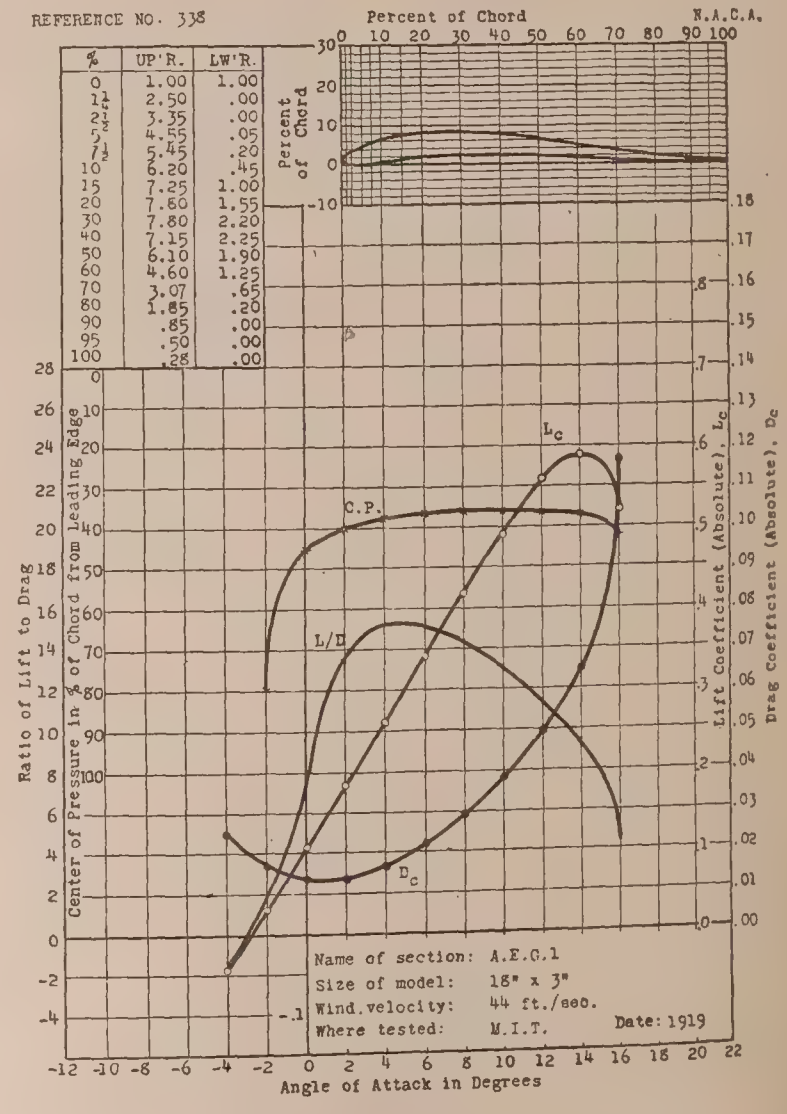
REFERENCE NO. 337

%	UP'R.	LW'R
0	2.55	2.55
1¼	5.20	1.10
2½	6.59	.74
5	8.83	.47
7½	10.65	.35
10	11.73	.20
15	13.45	.00
20	14.60	.26
30	15.09	1.32
40	14.74	2.41
50	13.68	1.92
60	12.35	.93
70	10.84	.23
80	8.77	.00
90	6.22	.10
95	4.55	.35
100	1.42	1.42



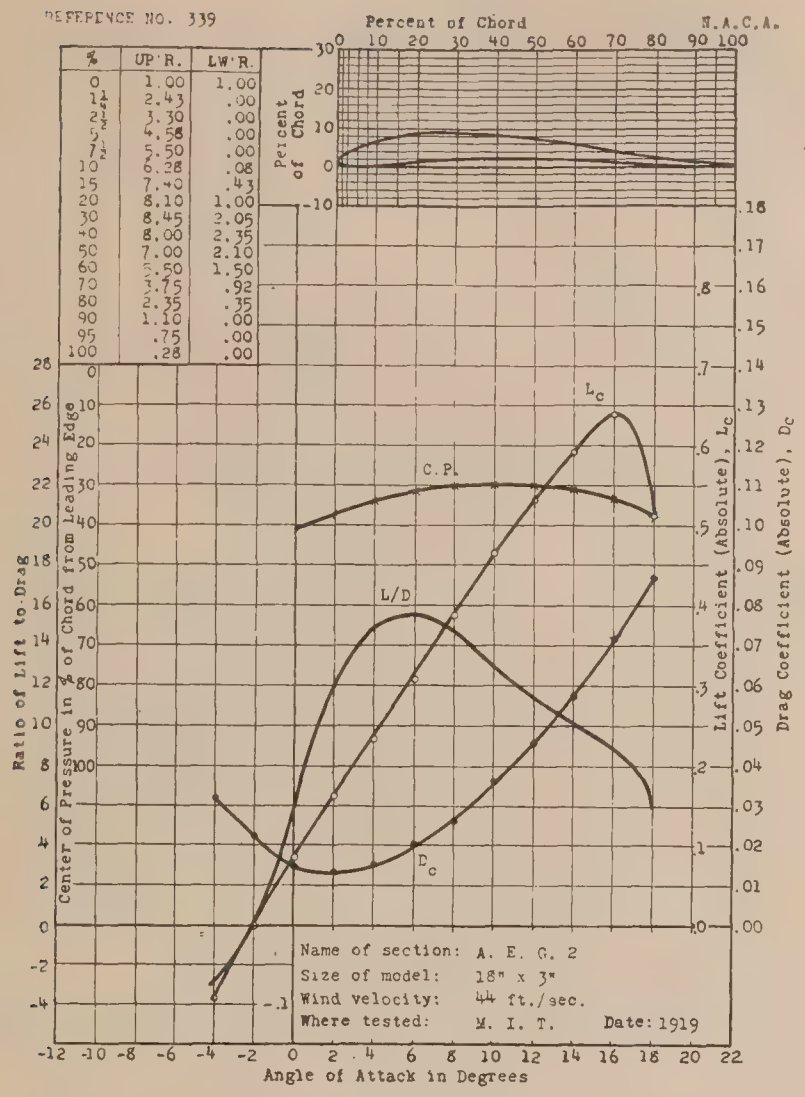
REFERENCE NO. 338

%	UP'R.	LW'R.
0	1.00	1.00
1¼	2.50	.00
2½	3.35	.00
5	4.55	.05
7½	5.45	.20
10	6.20	.45
15	7.25	1.00
20	7.80	1.55
30	7.80	2.20
40	7.15	2.25
50	6.10	1.90
60	4.60	1.25
70	3.07	.65
80	1.85	.20
90	.85	.00
95	.50	.00
100	.28	.00



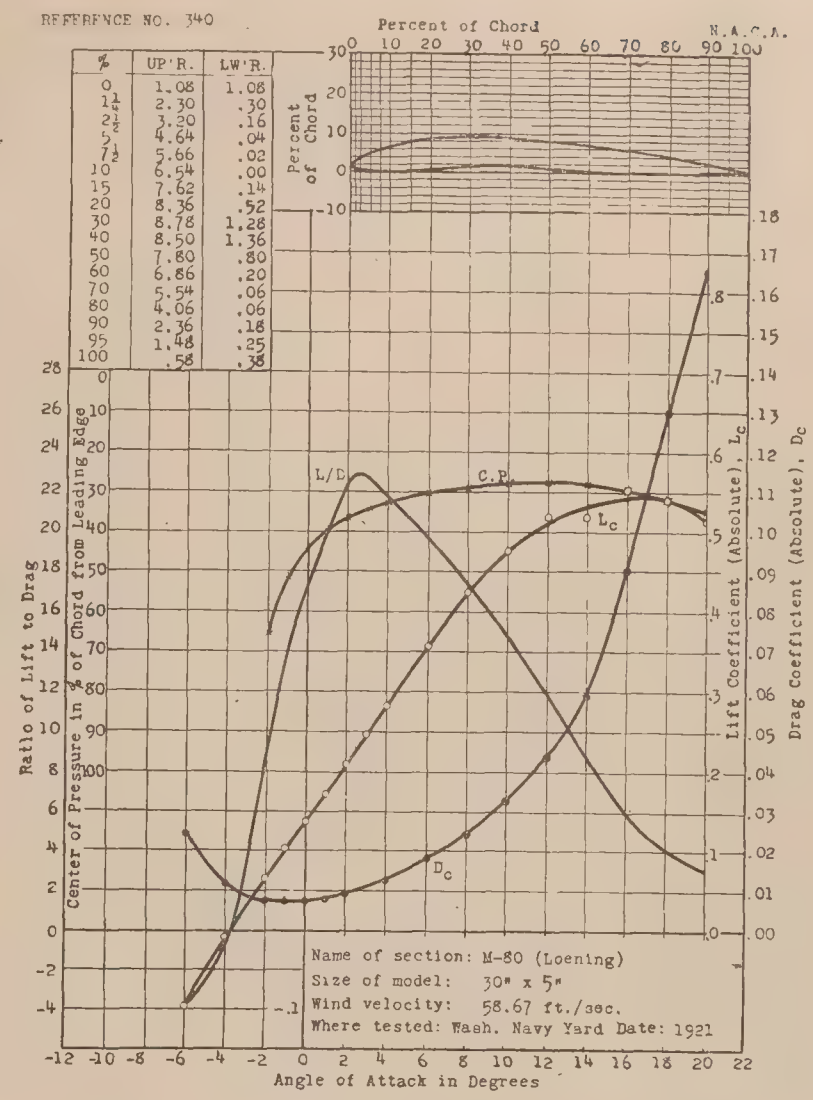
REFERENCE NO. 339

%	UP'R.	LW'R.
0	1.00	1.00
1¼	2.43	.00
2½	3.30	.00
5	4.58	.00
7½	5.50	.00
10	6.28	.08
15	7.40	.43
20	8.10	1.00
30	8.45	2.05
40	8.00	2.35
50	7.00	2.10
60	5.50	1.50
70	3.75	.92
80	2.35	.35
90	1.10	.00
95	.75	.00
100	.28	.00



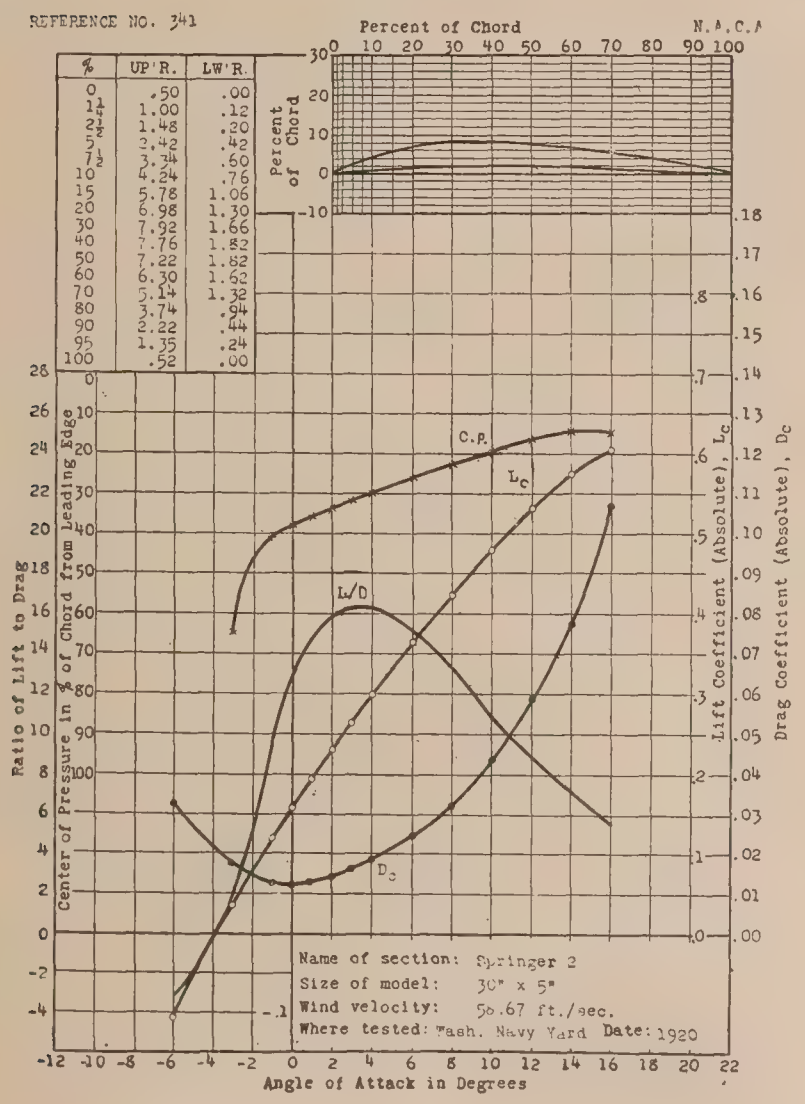
REFERENCE NO. 340

%	UP'R.	LW'R.
0	1.08	1.08
1¼	2.30	.30
2½	3.20	.16
5	4.64	.04
7½	5.66	.02
10	6.54	.00
15	7.62	.14
20	8.36	.52
30	8.78	1.28
40	8.50	1.36
50	7.80	.80
60	6.86	.20
70	5.54	.06
80	4.06	.06
90	2.36	.18
95	1.48	.25
100	.58	.38



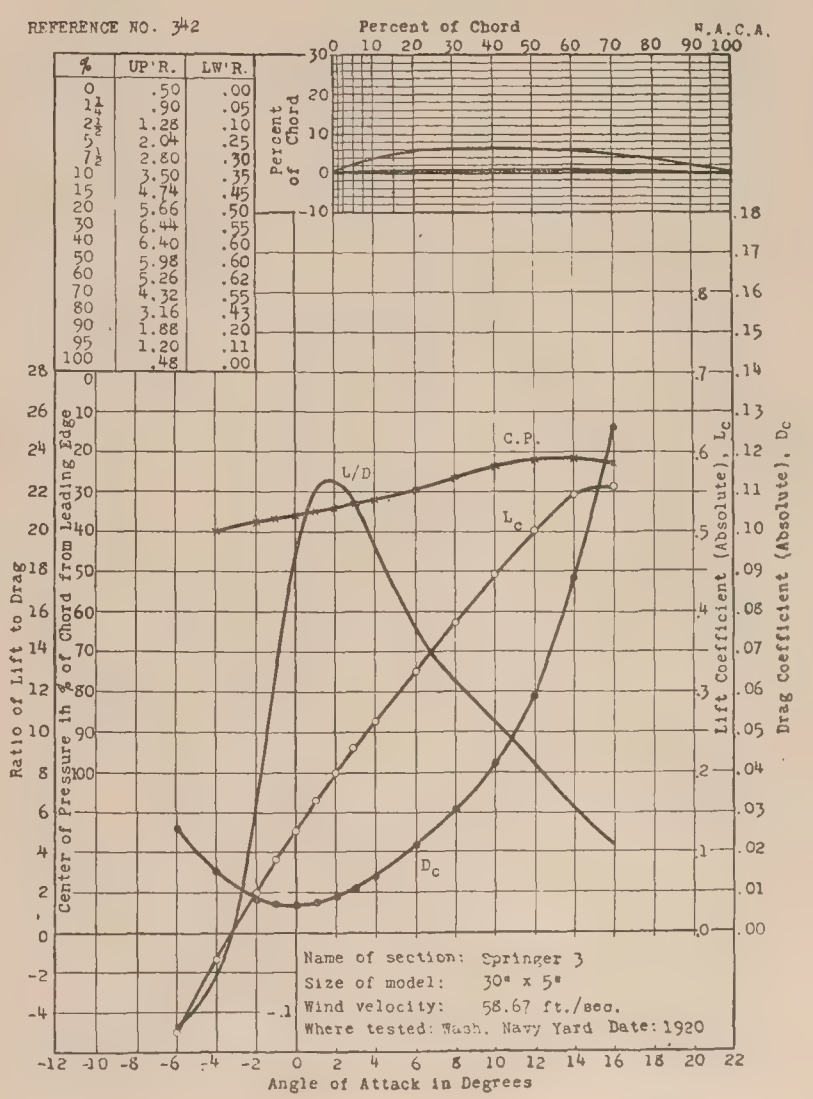
REFERENCE NO. 341

%	UP'R.	LW'R.
0	.50	.00
1¼	1.00	.12
2½	1.48	.20
5	2.42	.42
7½	3.34	.60
10	4.24	.76
15	5.78	1.06
20	6.98	1.30
30	7.92	1.66
40	7.76	1.82
50	7.22	1.82
60	6.30	1.62
70	5.14	1.32
80	3.74	.94
90	2.22	.44
95	1.35	.24
100	.52	.00

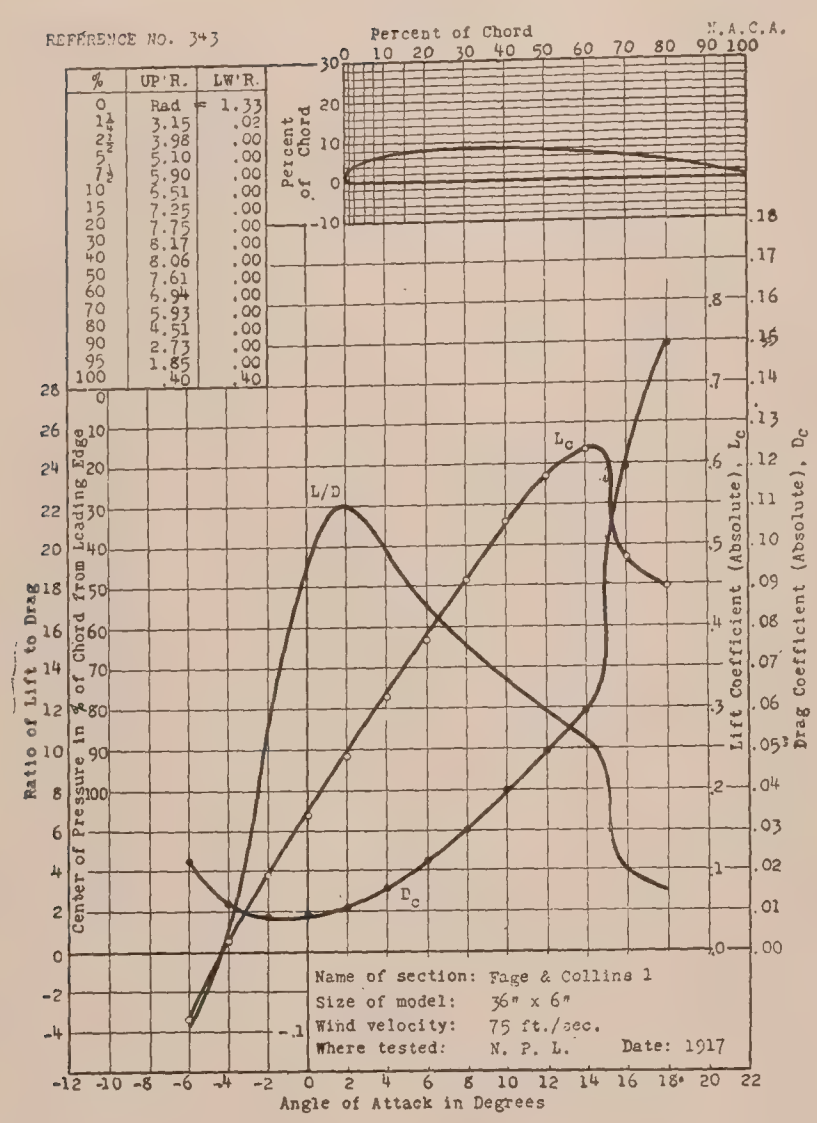


REFERENCE NO. 342

%	UP'R.	LW'R.
0	.50	.00
1¼	.90	.05
2½	1.28	.10
5	2.04	.25
7½	2.80	.30
10	3.50	.35
15	4.74	.45
20	5.66	.50
30	6.44	.55
40	6.40	.60
50	5.98	.60
60	5.26	.62
70	4.32	.55
80	3.16	.43
90	1.88	.20
95	1.20	.11
100	.48	.00

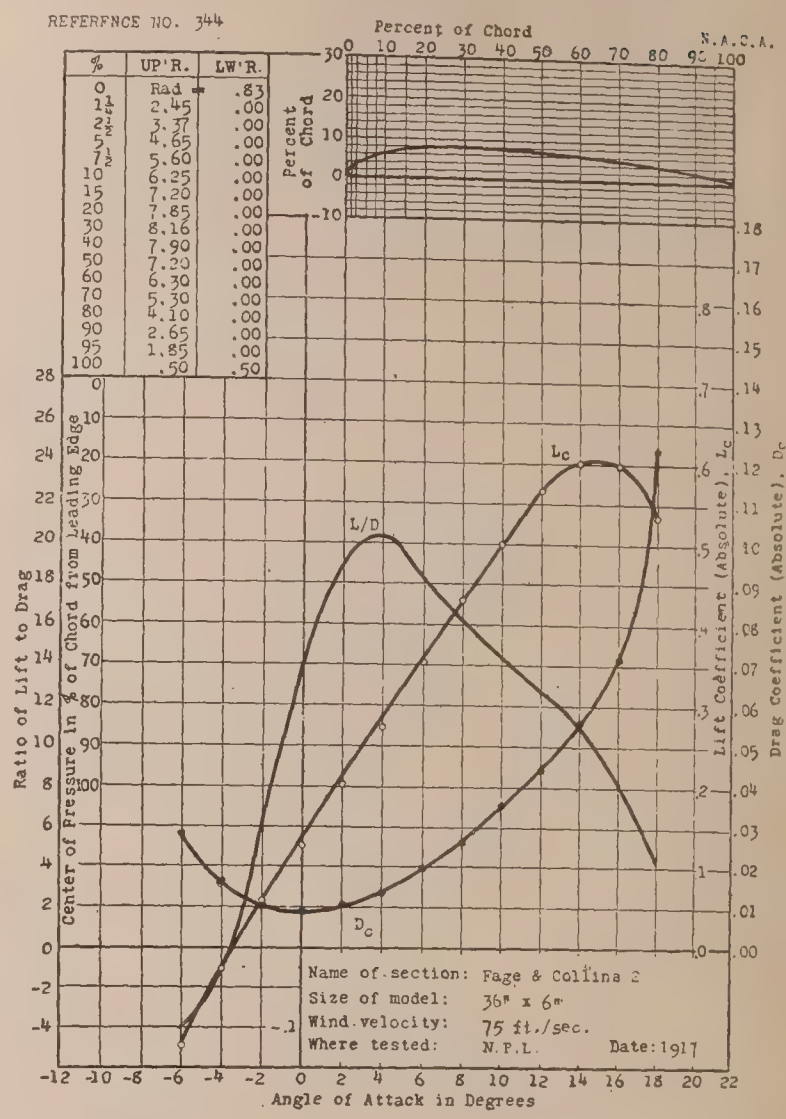


REFERENCE NO. 343



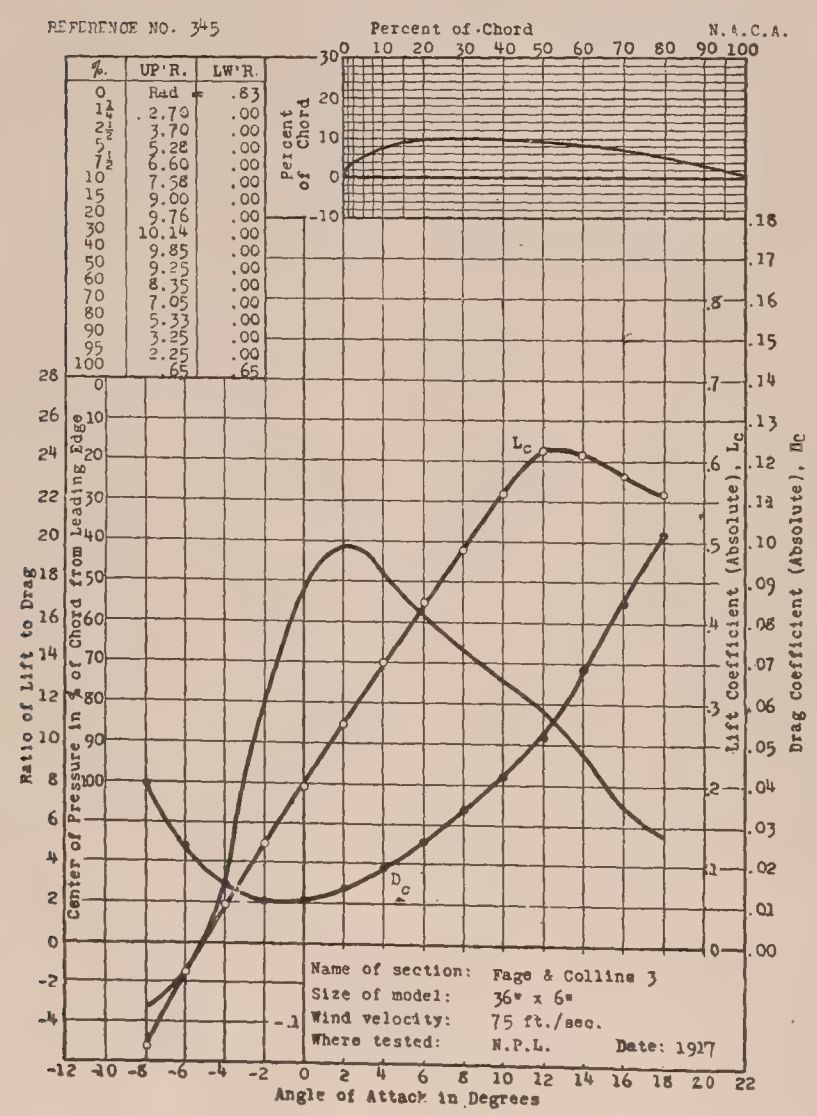
%	UP'R.	LW'R.
0	Rad =	1.33
1¼	3.15	.02
2½	3.98	.00
5	5.10	.00
7½	5.90	.00
10	6.51	.00
15	7.25	.00
20	7.75	.00
30	8.17	.00
40	8.06	.00
50	7.61	.00
60	6.94	.00
70	5.93	.00
80	4.51	.00
90	2.73	.00
95	1.85	.00
100	.40	.40

REFERENCE NO. 344



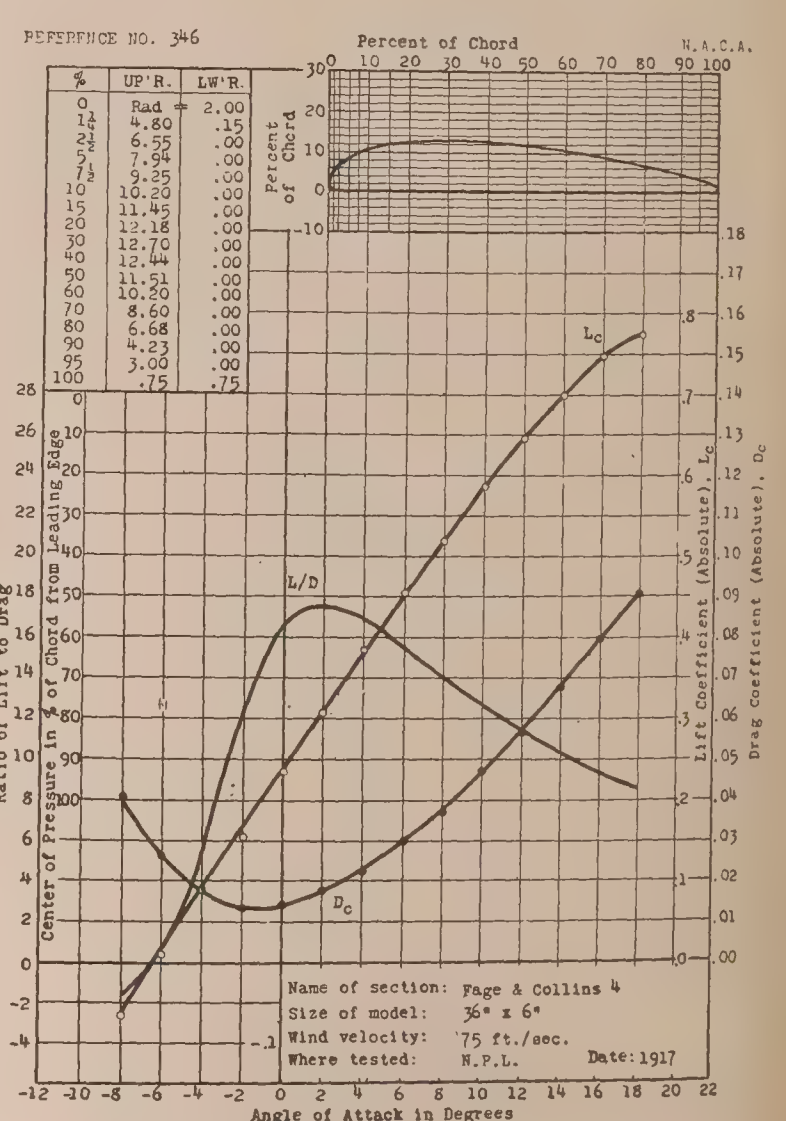
%	UP'R.	LW'R.
0	Rad =	.83
1¼	2.45	.00
2½	3.37	.00
5	4.65	.00
7½	5.60	.00
10	6.25	.00
15	7.20	.00
20	7.85	.00
30	8.16	.00
40	7.90	.00
50	7.20	.00
60	6.30	.00
70	5.30	.00
80	4.10	.00
90	2.65	.00
95	1.85	.00
100	.50	.50

REFERENCE NO. 345



%.	UP'R.	LW'R.
0	Rad =	.83
1¼	2.70	.00
2½	3.70	.00
5	5.28	.00
7½	6.60	.00
10	7.58	.00
15	9.00	.00
20	9.76	.00
30	10.14	.00
40	9.85	.00
50	9.25	.00
60	8.35	.00
70	7.05	.00
80	5.33	.00
90	3.25	.00
95	2.25	.00
100	.65	.65

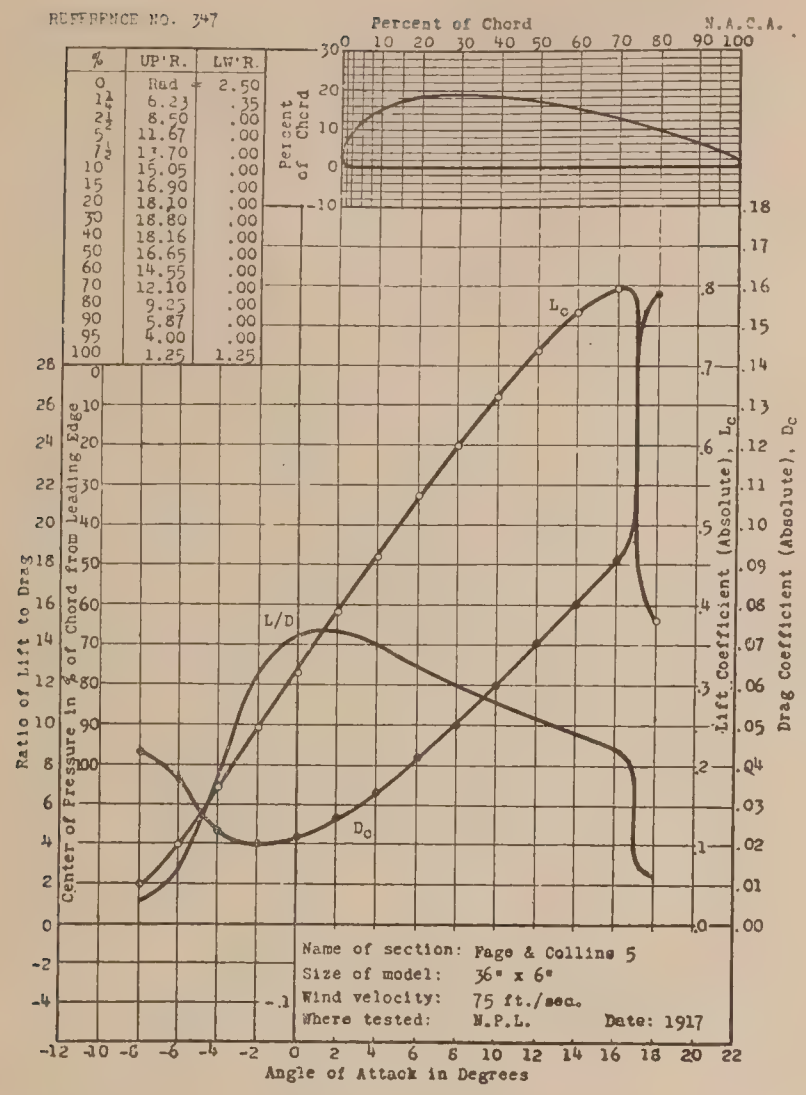
REFERENCE NO. 346



%	UP'R.	LW'R.
0	Rad =	2.00
1¼	4.80	.15
2½	6.55	.00
5	7.94	.00
7½	9.25	.00
10	10.20	.00
15	11.45	.00
20	12.18	.00
30	12.70	.00
40	12.44	.00
50	11.51	.00
60	10.20	.00
70	8.60	.00
80	6.68	.00
90	4.23	.00
95	3.00	.00
100	.75	.75

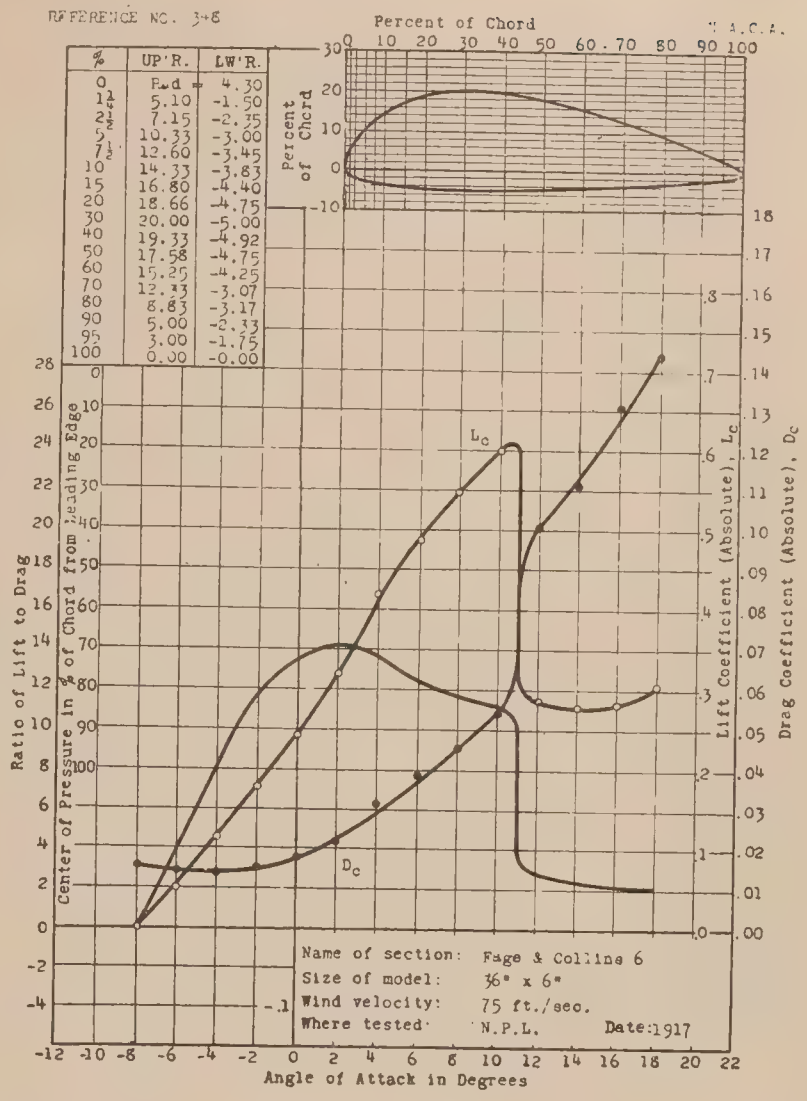
REFERENCE NO. 347

%	UP'R.	LW'R.
0	Rad =	2.50
1¼	6.23	.35
2½	8.50	.00
5	11.67	.00
7½	13.70	.00
10	15.05	.00
15	16.90	.00
20	18.10	.00
30	18.80	.00
40	18.16	.00
50	16.65	.00
60	14.55	.00
70	12.10	.00
80	9.25	.00
90	5.87	.00
95	4.00	.00
100	1.25	1.25



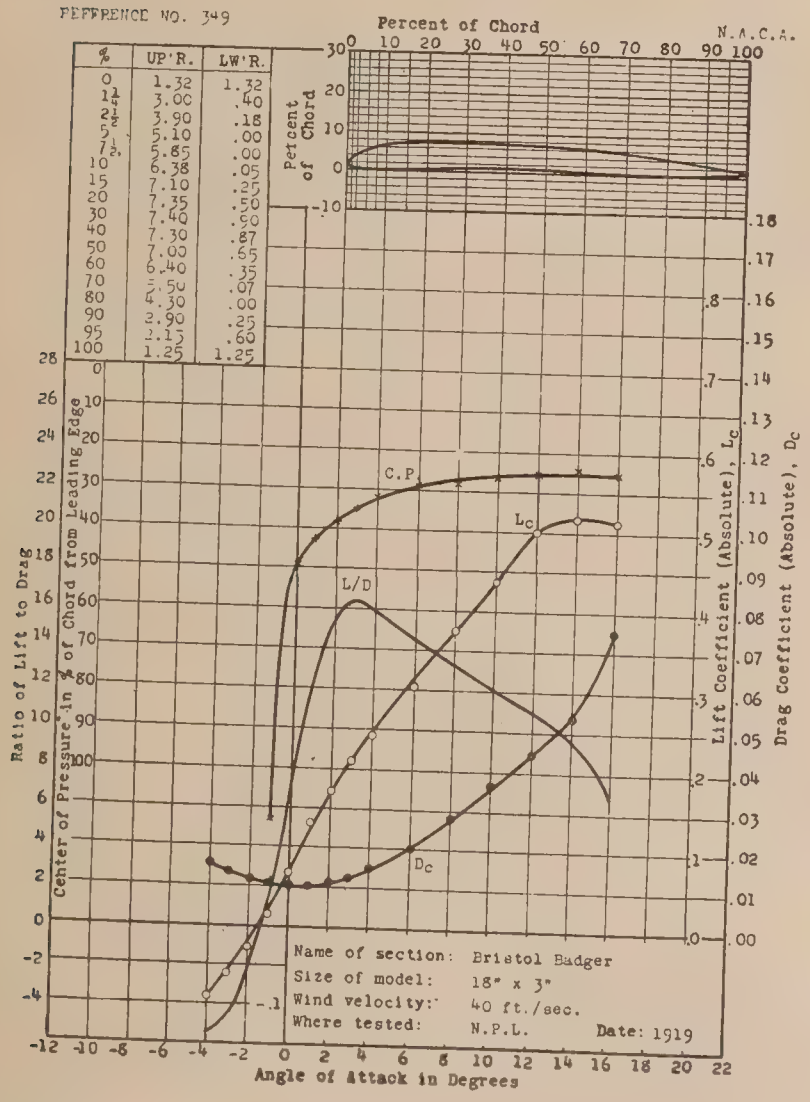
REFERENCE NO. 348

%	UP'R.	LW'R.
0	Rad =	4.30
1¼	5.10	-1.50
2½	7.15	-2.35
5	10.33	-3.00
7½	12.60	-3.45
10	14.33	-3.83
15	16.80	-4.40
20	18.66	-4.75
30	20.00	-5.00
40	19.33	-4.92
50	17.58	-4.75
60	15.25	-4.25
70	12.33	-3.07
80	8.83	-3.17
90	5.00	-2.33
95	3.00	-1.75
100	0.00	-0.00



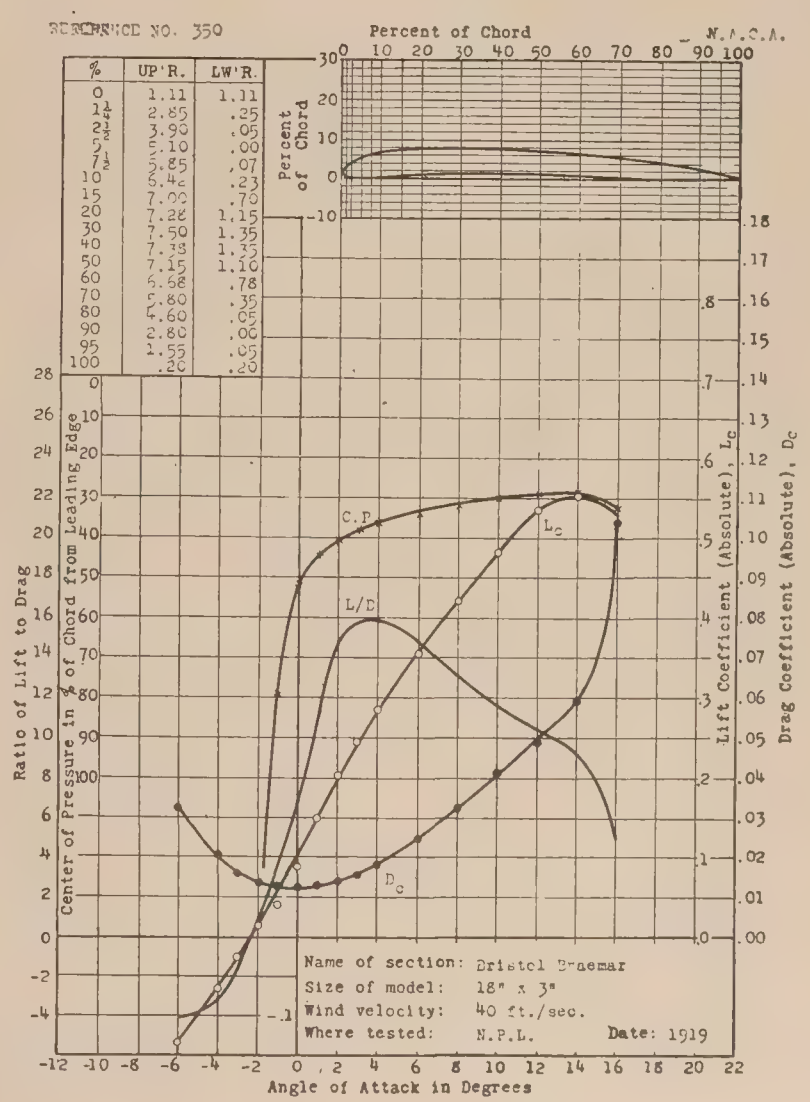
REFERENCE NO. 349

%	UP'R.	LW'R.
0	1.32	1.32
1¼	3.00	.40
2½	3.90	.18
5	5.10	.00
7½	5.85	.00
10	6.38	.05
15	7.10	.25
20	7.35	.50
30	7.40	.90
40	7.30	.87
50	7.00	.65
60	6.40	.35
70	5.50	.07
80	4.30	.00
90	2.90	.25
95	2.15	.60
100	1.25	1.25



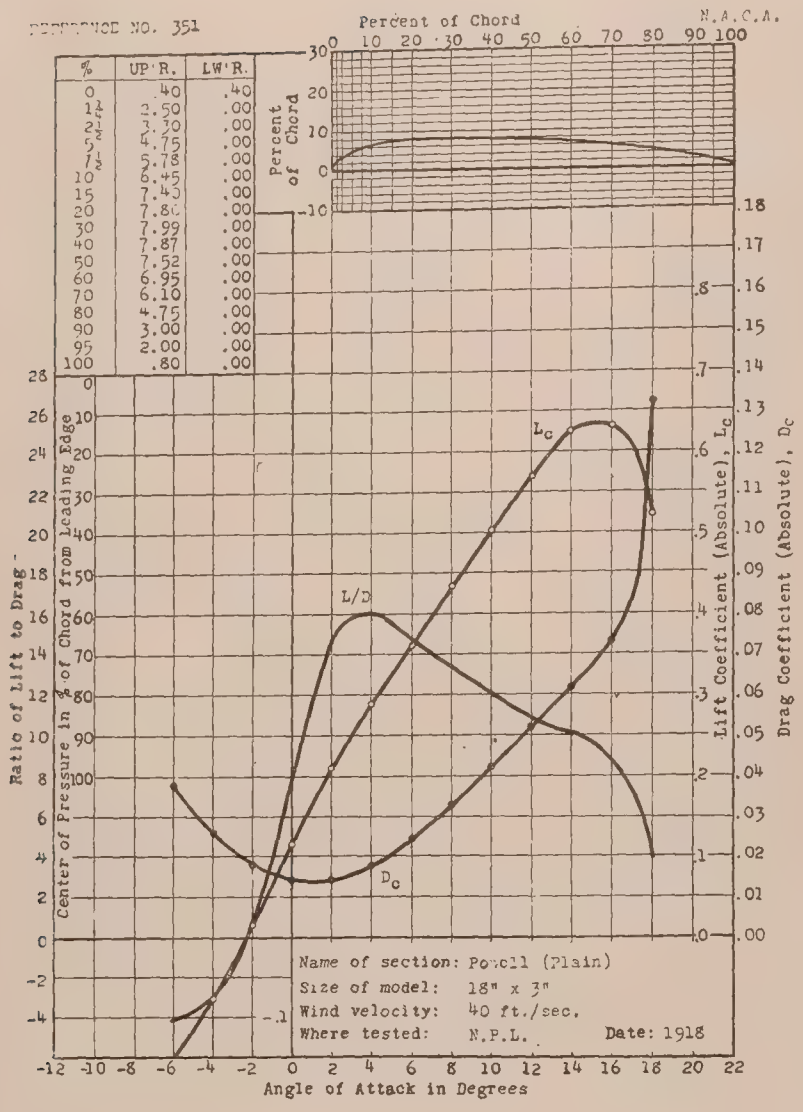
REFERENCE NO. 350

%	UP'R.	LW'R.
0	1.11	1.11
1¼	2.85	.25
2½	3.90	.05
5	5.10	.00
7½	5.85	.07
10	6.42	.23
15	7.00	.70
20	7.28	1.15
30	7.50	1.35
40	7.33	1.35
50	7.15	1.10
60	6.58	.78
70	5.80	.35
80	4.60	.05
90	2.80	.00
95	1.55	.05
100	.20	.20



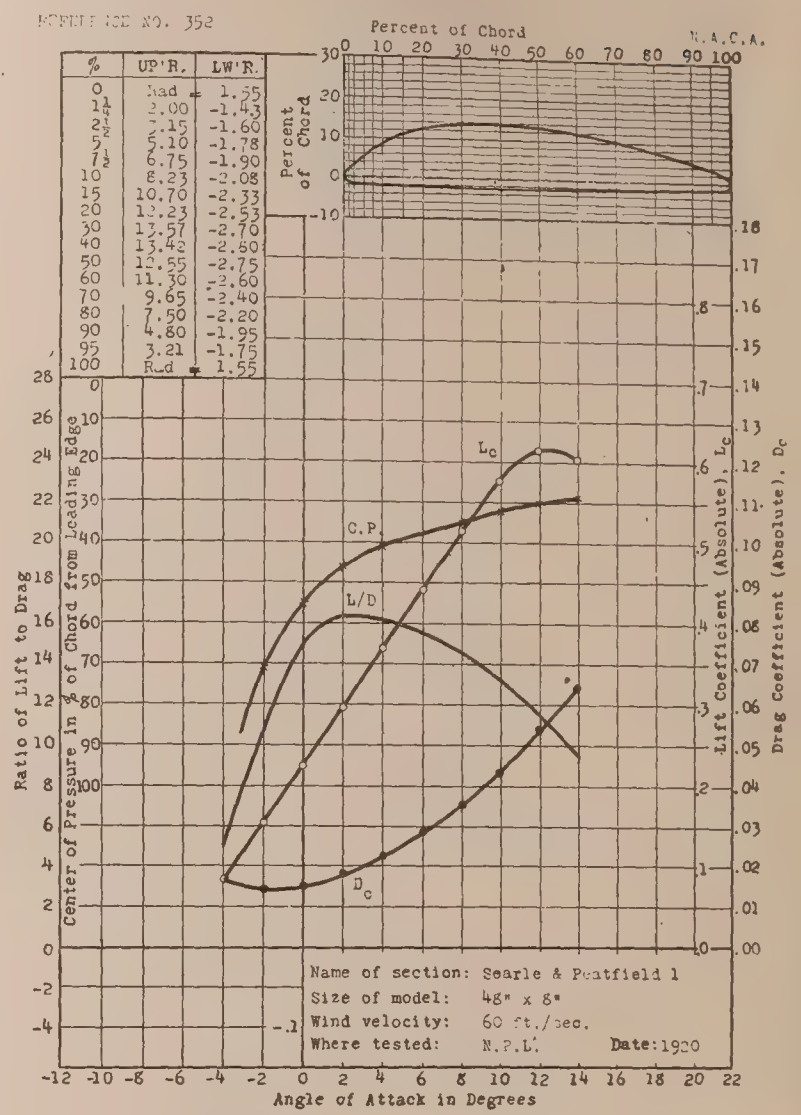
REFERENCE NO. 351

%	UP'R.	LW'R.
0	.40	.40
1¼	2.50	.00
2½	3.30	.00
5	4.75	.00
7½	5.78	.00
10	6.45	.00
15	7.40	.00
20	7.80	.00
30	7.99	.00
40	7.87	.00
50	7.52	.00
60	6.95	.00
70	6.10	.00
80	4.75	.00
90	3.00	.00
95	2.00	.00
100	.80	.00



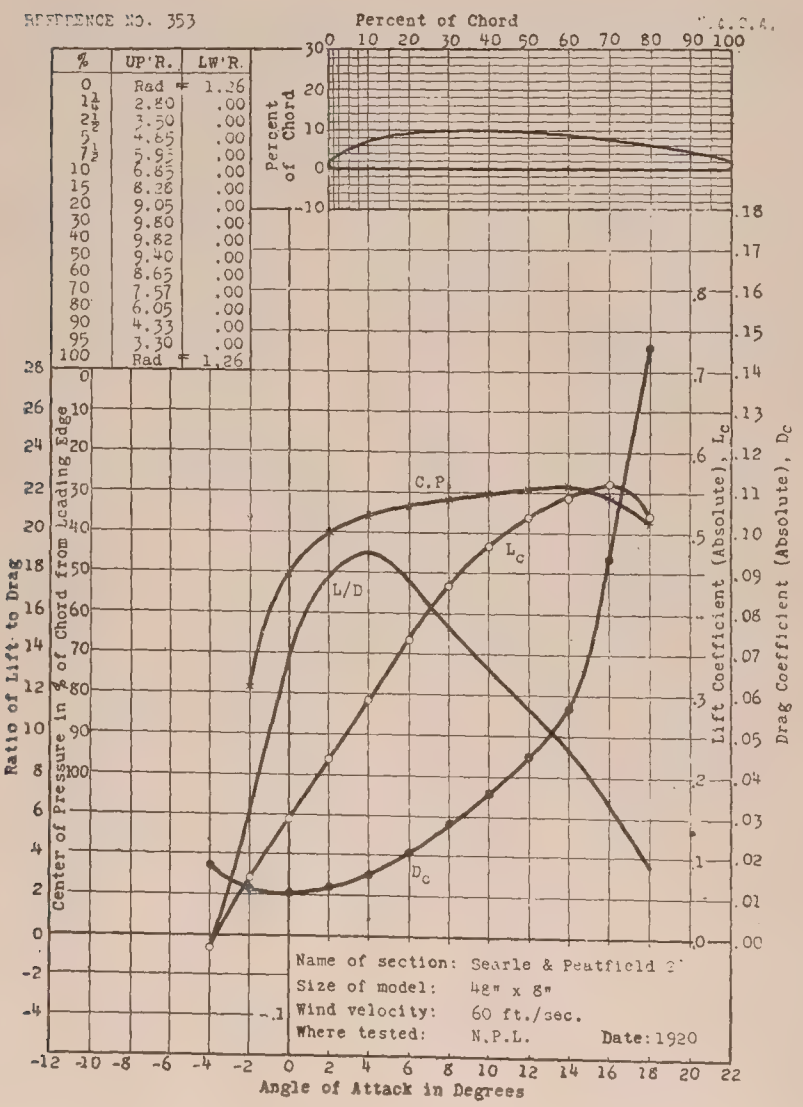
REFERENCE NO. 352

%	UP'R.	LW'R.
0	Rad =	1.55
1¼	2.00	-1.43
2½	2.15	-1.60
5	5.10	-1.78
7½	6.75	-1.90
10	8.23	-2.08
15	10.70	-2.33
20	12.23	-2.53
30	13.57	-2.70
40	13.42	-2.60
50	12.55	-2.75
60	11.30	-2.60
70	9.65	-2.40
80	7.50	-2.20
90	4.80	-1.95
95	3.21	-1.75
100	Rad =	1.55



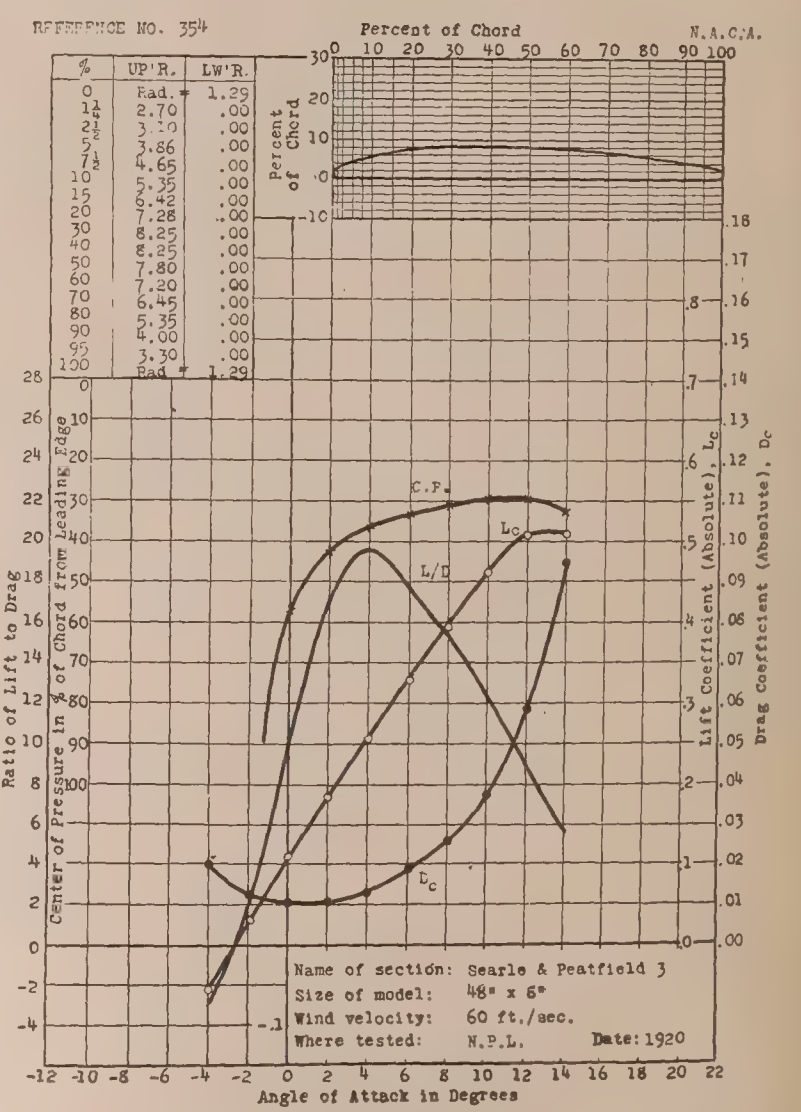
REFERENCE NO. 353

%	UP'R.	LW'R.
0	Rad =	1.26
1¼	2.80	.00
2½	3.50	.00
5	4.65	.00
7½	5.95	.00
10	6.85	.00
15	8.38	.00
20	9.05	.00
30	9.80	.00
40	9.82	.00
50	9.40	.00
60	8.65	.00
70	7.57	.00
80	6.05	.00
90	4.33	.00
95	3.30	.00
100	Rad =	1.26

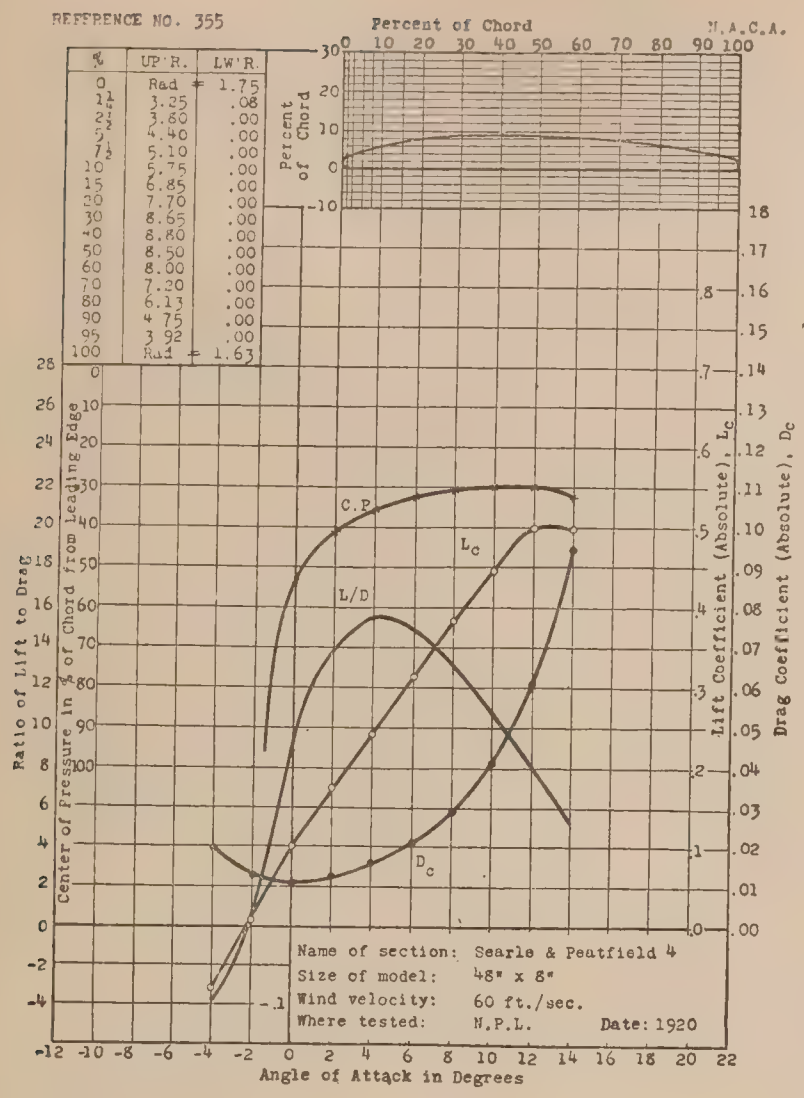


REFERENCE NO. 354

%	UP'R.	LW'R.
0	Rad. =	1.29
1¼	2.70	.00
2½	3.10	.00
5	3.86	.00
7½	4.65	.00
10	5.35	.00
15	6.42	.00
20	7.28	.00
30	8.25	.00
40	8.25	.00
50	7.80	.00
60	7.20	.00
70	6.45	.00
80	5.35	.00
90	4.00	.00
95	3.30	.00
100	Rad =	1.29

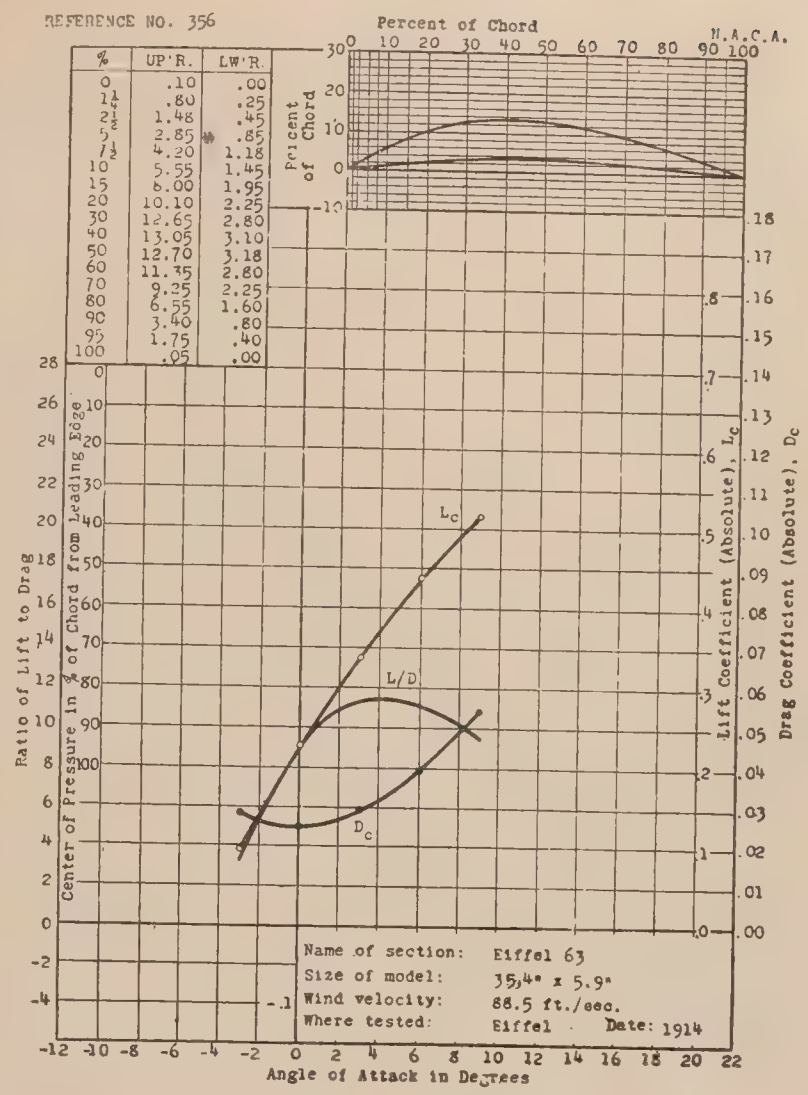


REFERENCE NO. 355



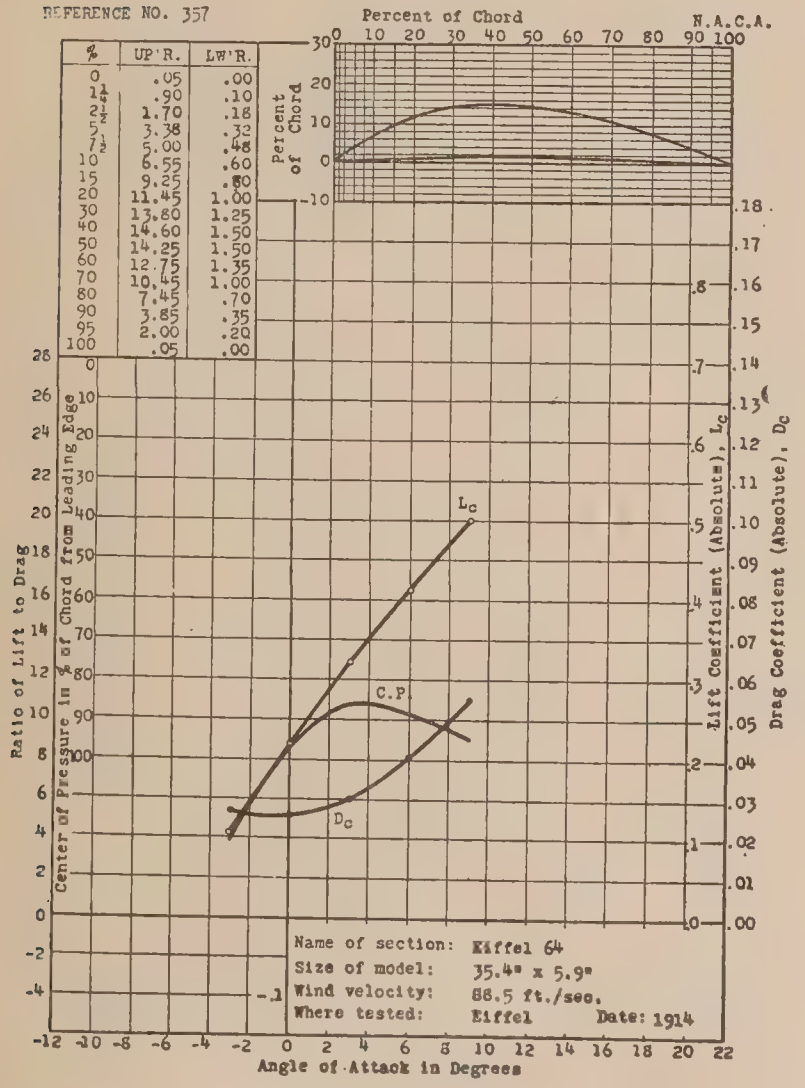
%	UP'R.	LW'R.
0	Rad =	1.75
1¼	3.25	.08
2½	3.60	.00
5	4.40	.00
7½	5.10	.00
10	5.75	.00
15	6.85	.00
20	7.70	.00
30	8.65	.00
40	8.80	.00
50	8.50	.00
60	8.00	.00
70	7.20	.00
80	6.13	.00
90	4.75	.00
95	3.92	.00
100	Rad =	1.63

REFERENCE NO. 356



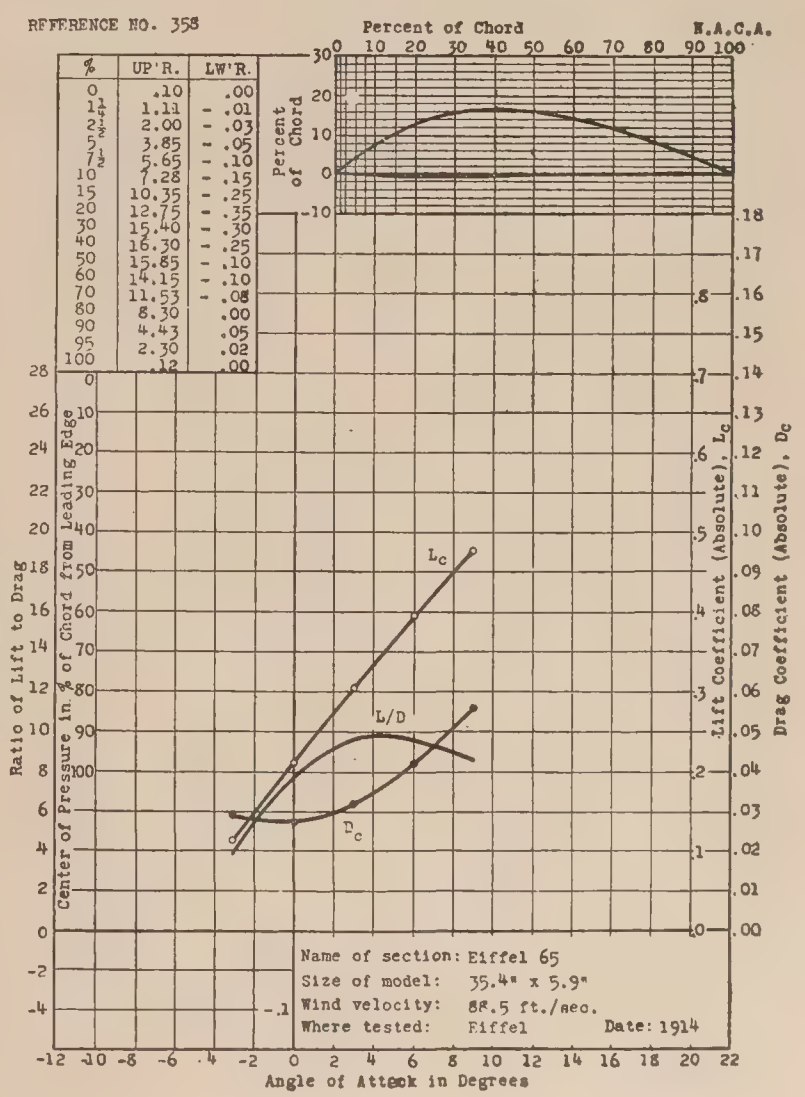
%	UP'R.	LW'R.
0	.10	.00
1¼	.80	.25
2½	1.48	.45
5	2.85	.85
7½	4.20	1.18
10	5.55	1.45
15	6.00	1.95
20	10.10	2.25
30	12.65	2.80
40	13.05	3.10
50	12.70	3.18
60	11.35	2.80
70	9.25	2.25
80	6.55	1.60
90	3.40	.80
95	1.75	.40
100	.05	.00

REFERENCE NO. 357



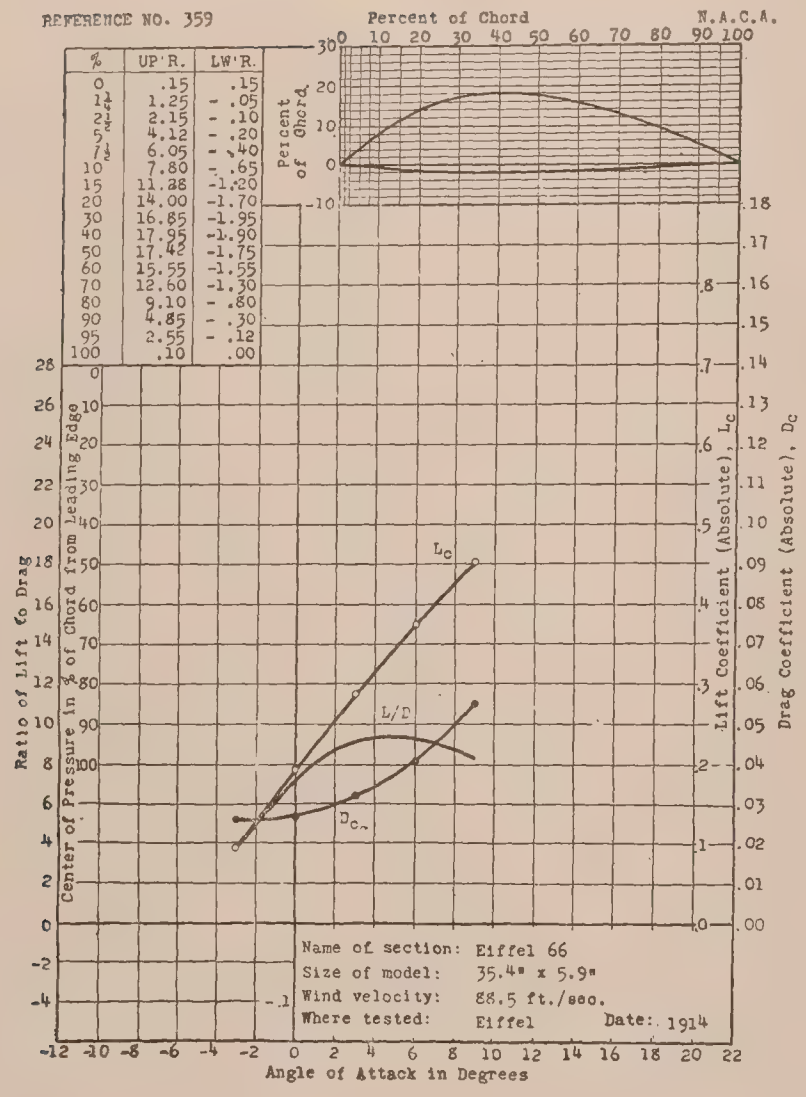
%	UP'R.	LW'R.
0	.05	.00
1¼	.90	.10
2½	1.70	.18
5	3.38	.32
7½	5.00	.48
10	6.55	.60
15	9.25	.80
20	11.45	1.00
30	13.80	1.25
40	14.60	1.50
50	14.25	1.50
60	12.75	1.35
70	10.45	1.00
80	7.45	.70
90	3.85	.35
95	2.00	.20
100	.05	.00

REFERENCE NO. 358



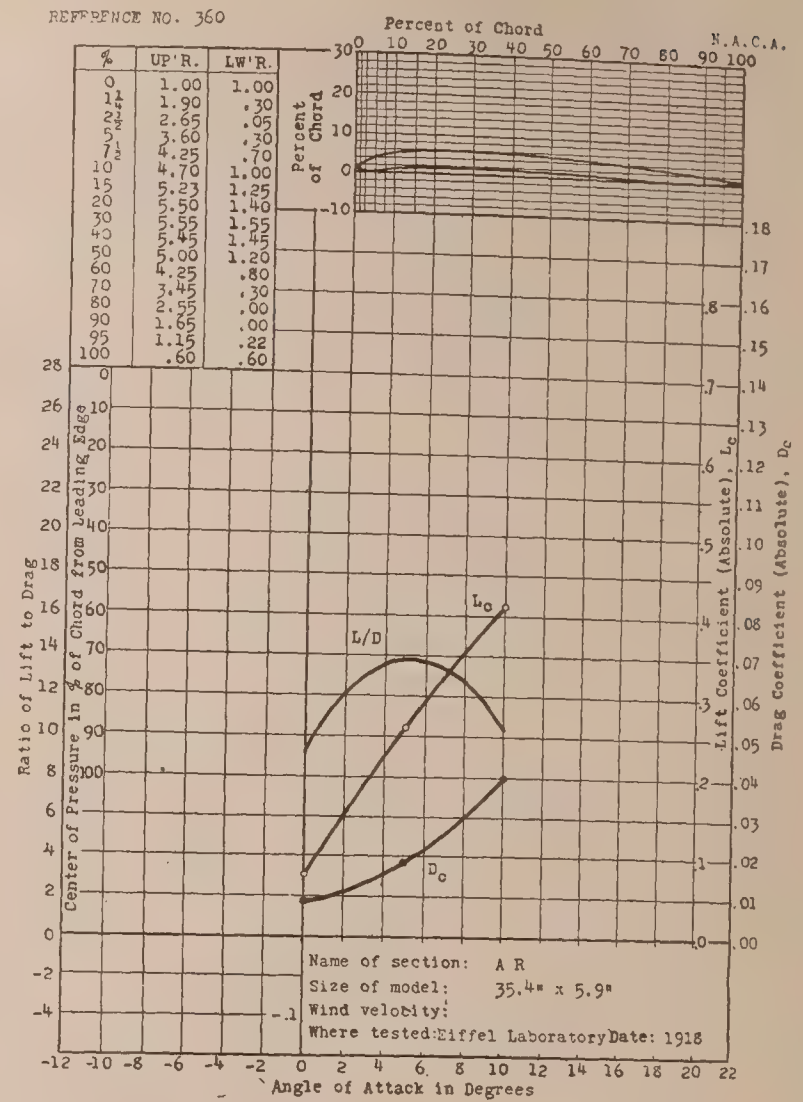
%	UP'R.	LW'R.
0	.10	.00
1¼	1.11	- .01
2½	2.00	- .03
5	3.85	- .05
7½	5.65	- .10
10	7.28	- .15
15	10.35	- .25
20	12.75	- .35
30	15.40	- .30
40	16.30	- .25
50	15.85	- .10
60	14.15	- .10
70	11.53	- .08
80	8.30	.00
90	4.43	.05
95	2.30	.02
100	.12	.00

REFERENCE NO. 359



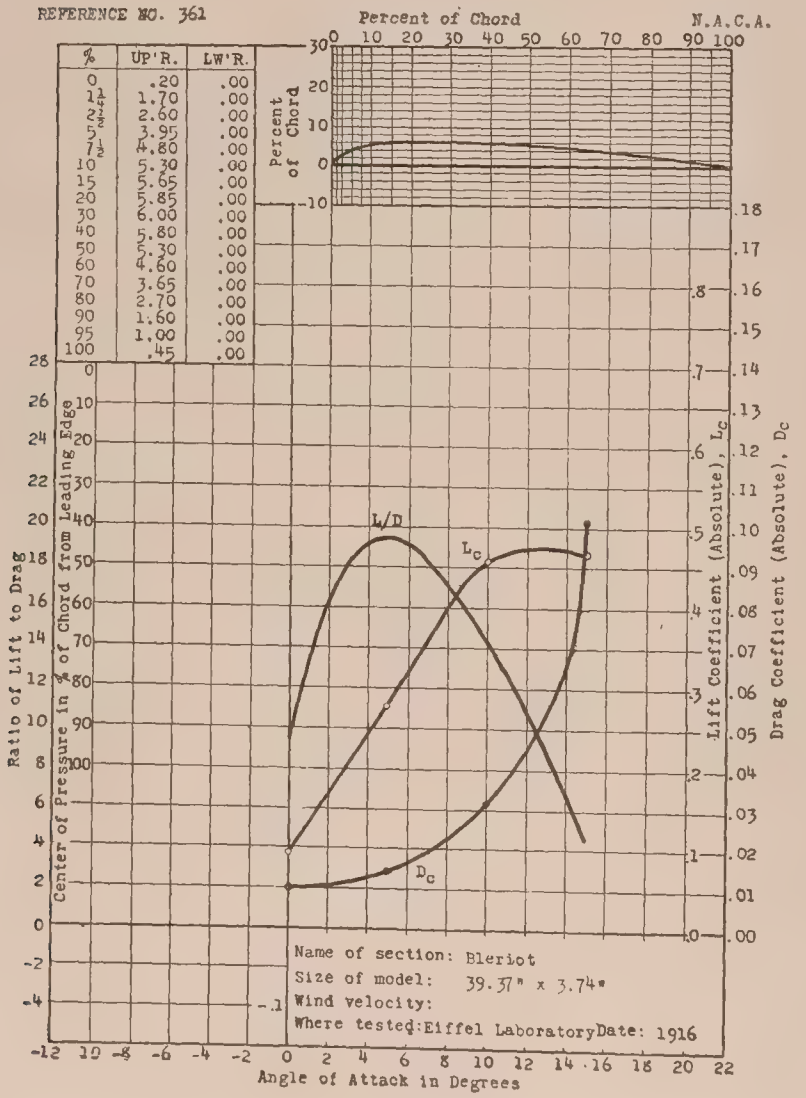
%	UP'R.	LW'R.
0	.15	.15
1¼	1.25	- .05
2½	2.15	- .10
5	4.12	- .20
7½	6.05	- .40
10	7.80	- .65
15	11.28	-1.20
20	14.00	-1.70
30	16.85	-1.95
40	17.95	-1.90
50	17.42	-1.75
60	15.55	-1.55
70	12.60	-1.30
80	9.10	- .80
90	4.85	- .30
95	2.55	- .12
100	.10	.00

REFERENCE NO. 360



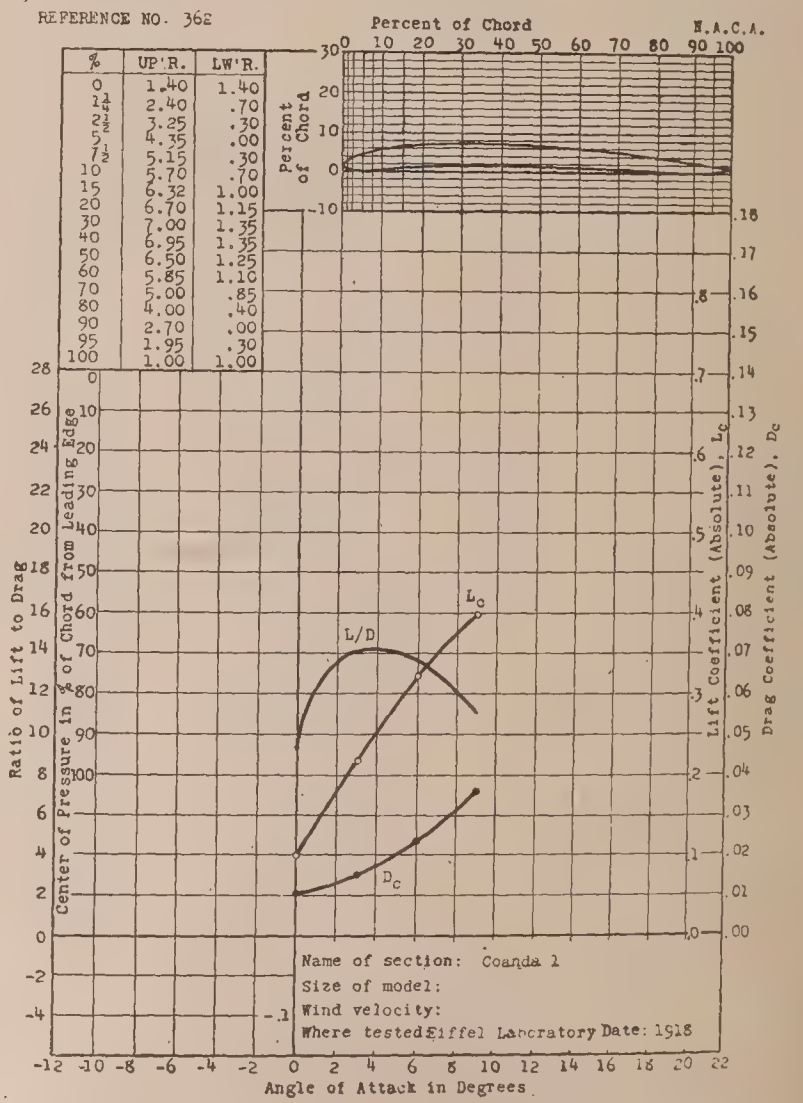
%	UP'R.	LW'R.
0	1.00	1.00
1¼	1.90	.30
2½	2.65	.05
5	3.60	.30
7½	4.25	.70
10	4.70	1.00
15	5.23	1.25
20	5.50	1.40
30	5.55	1.55
40	5.45	1.45
50	5.00	1.20
60	4.25	.80
70	3.45	.30
80	2.55	.00
90	1.65	.00
95	1.15	.22
100	.60	.60

REFERENCE NO. 361



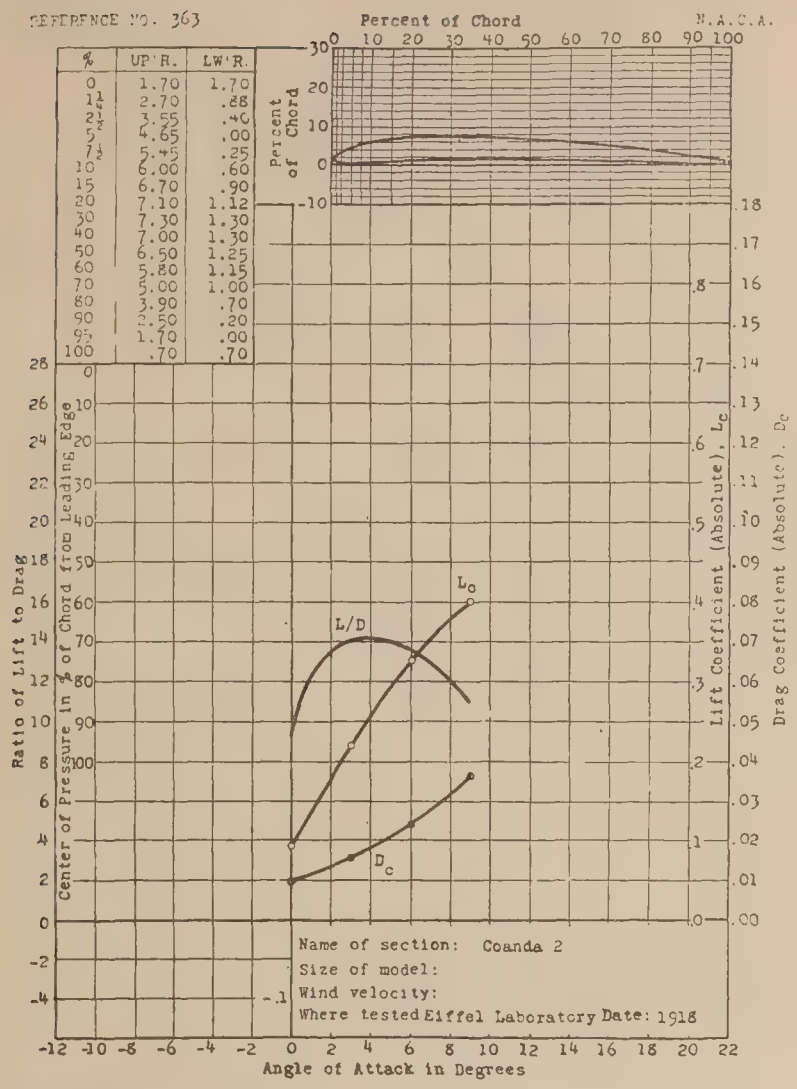
%	UP'R.	LW'R.
0	.20	.00
1¼	1.70	.00
2½	2.60	.00
5	3.95	.00
7½	4.80	.00
10	5.30	.00
15	5.65	.00
20	5.85	.00
30	6.00	.00
40	5.80	.00
50	5.30	.00
60	4.60	.00
70	3.65	.00
80	2.70	.00
90	1.60	.00
95	1.00	.00
100	.45	.00

REFERENCE NO. 362



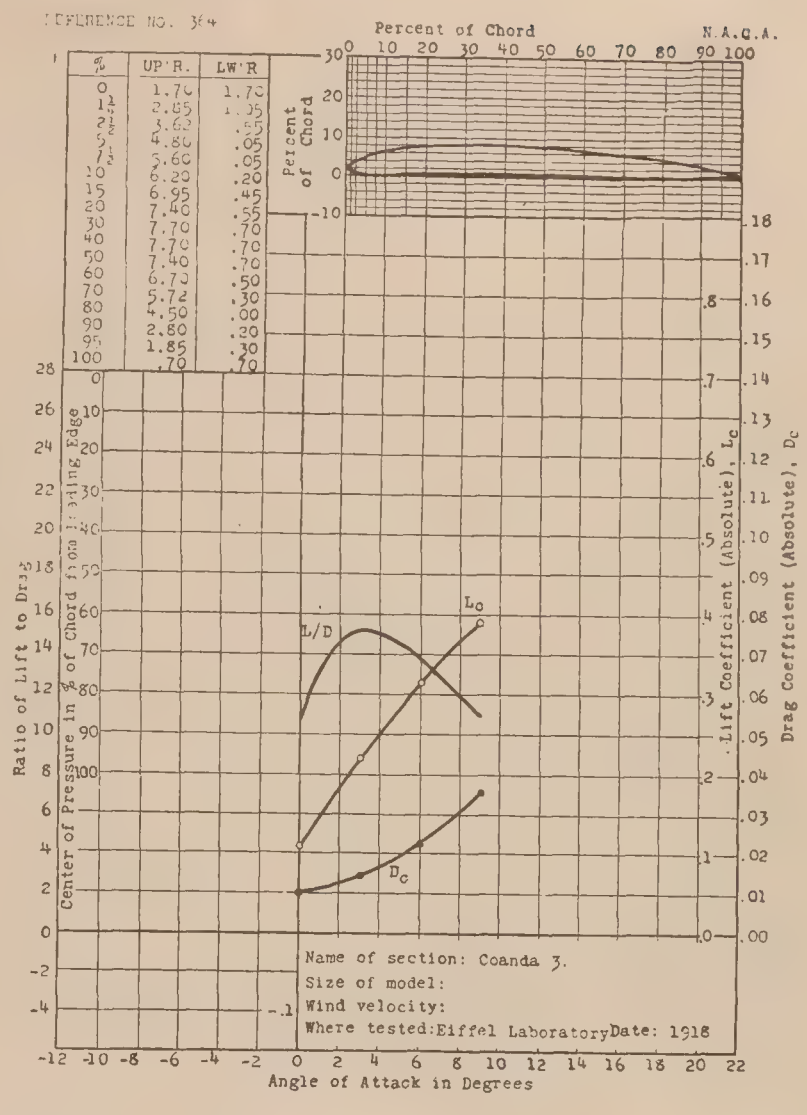
%	UP'R.	LW'R.
0	1.40	1.40
1¼	2.40	.70
2½	3.25	.30
5	4.35	.00
7½	5.15	.30
10	5.70	.70
15	6.32	1.00
20	6.70	1.15
30	7.00	1.35
40	6.95	1.35
50	6.50	1.25
60	5.85	1.10
70	5.00	.85
80	4.00	.40
90	2.70	.00
95	1.95	.30
100	1.00	1.00

REFERENCE NO. 363



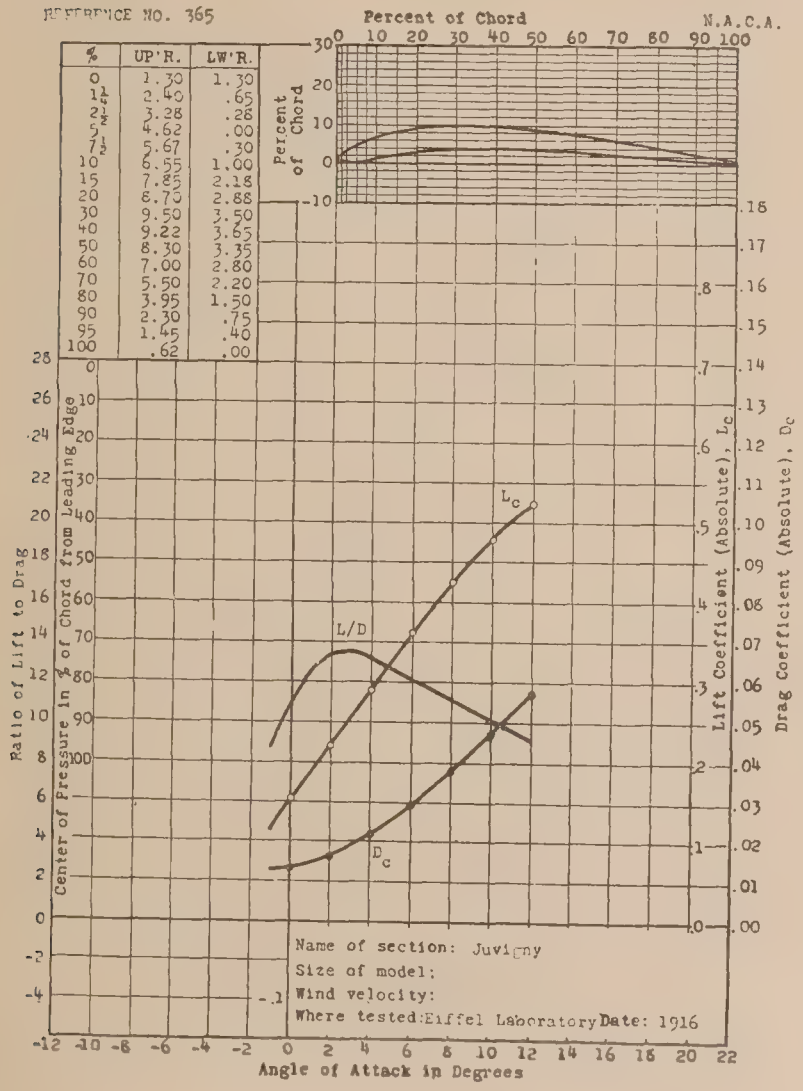
%	UP'R.	LW'R.
0	1.70	1.70
1¼	2.70	.88
2½	3.55	.40
5	4.65	.00
7½	5.45	.25
10	6.00	.60
15	6.70	.90
20	7.10	1.12
30	7.30	1.30
40	7.00	1.30
50	6.50	1.25
60	5.80	1.15
70	5.00	1.00
80	3.90	.70
90	2.50	.20
95	1.70	.00
100	.70	.70

REFERENCE NO. 364



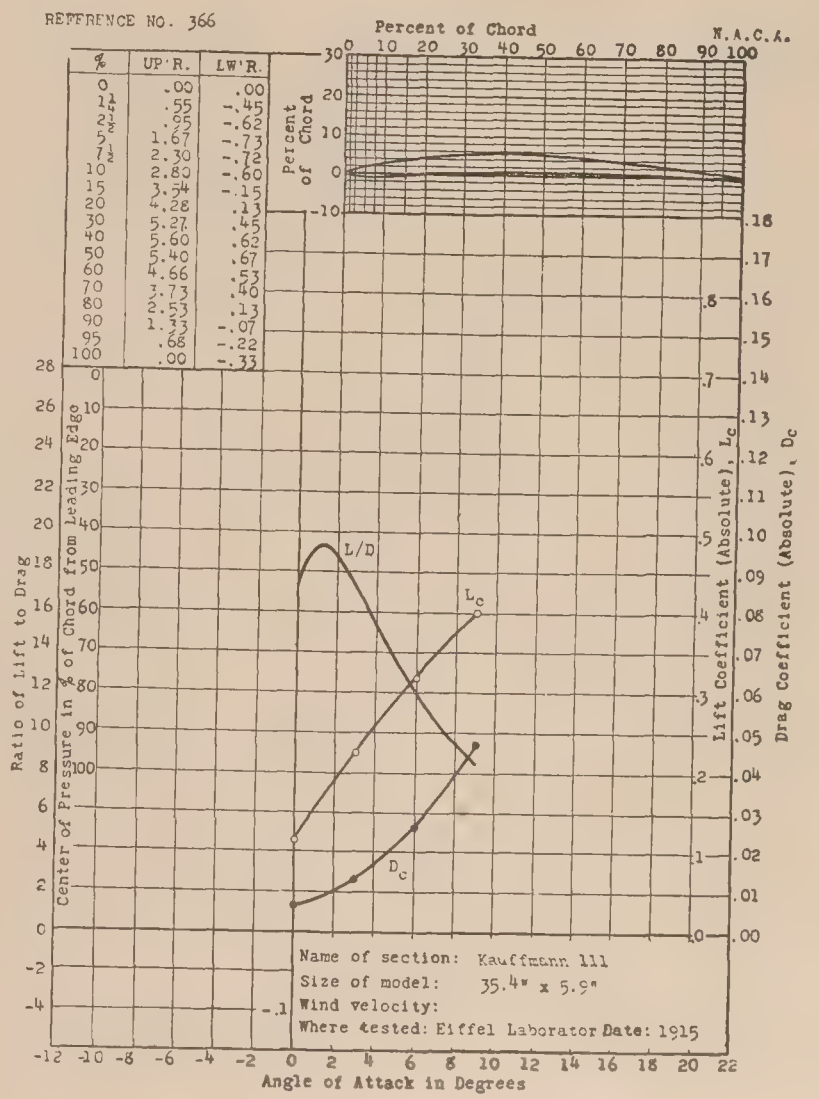
%	UP'R.	LW'R.
0	1.70	1.70
1¼	2.65	1.05
2½	3.63	.65
5	4.80	.65
7½	5.60	.65
10	6.20	.20
15	6.95	.45
20	7.40	.55
30	7.70	.70
40	7.70	.70
50	7.40	.70
60	6.70	.50
70	5.72	.30
80	4.50	.00
90	2.80	.20
95	1.85	.30
100	.70	.70

REFERENCE NO. 365



%	UP'R.	LW'R.
0	1.30	1.30
1¼	2.40	.65
2½	3.28	.28
5	4.62	.00
7½	5.67	.30
10	6.55	1.00
15	7.85	2.18
20	8.70	2.88
30	9.50	3.50
40	9.22	3.65
50	8.30	3.35
60	7.00	2.80
70	5.50	2.20
80	3.95	1.50
90	2.30	.75
95	1.45	.40
100	.62	.00

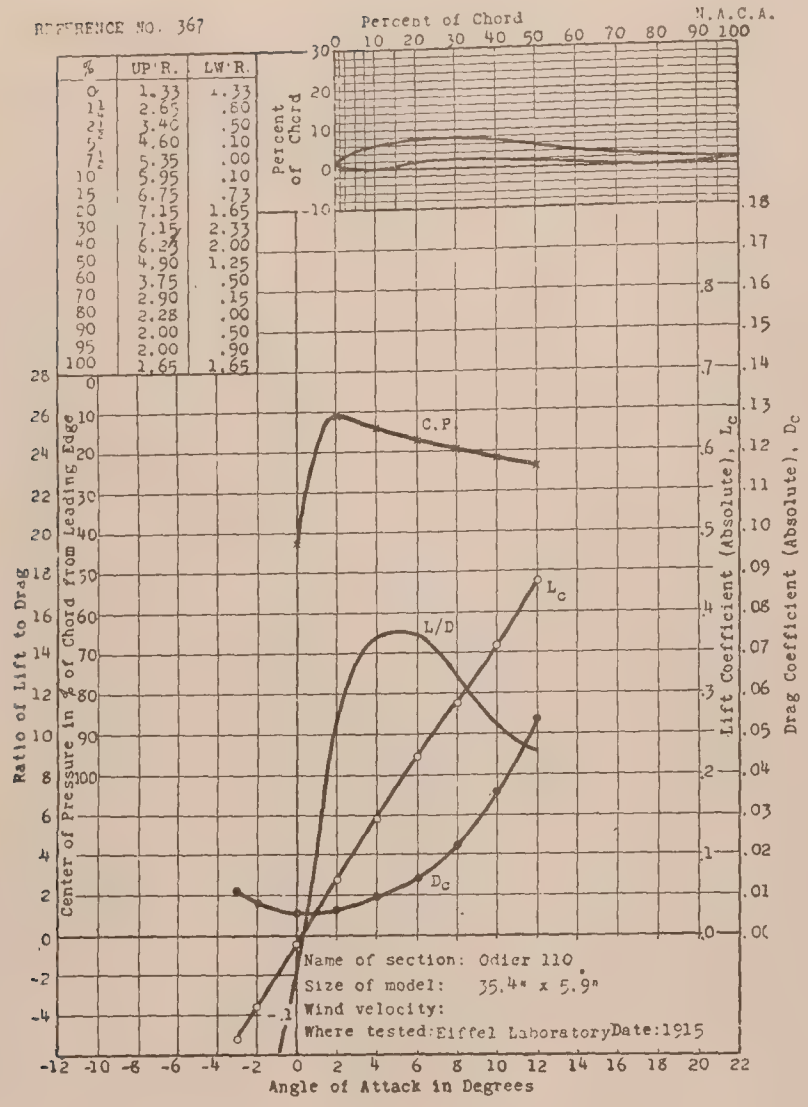
REFERENCE NO. 366



%	UP'R.	LW'R.
0	.00	.00
1¼	.55	-.45
2½	.95	-.62
5	1.67	-.73
7½	2.30	-.72
10	2.80	-.60
15	3.34	-.15
20	4.28	.13
30	5.27	.45
40	5.60	.62
50	5.40	.67
60	4.66	.53
70	3.73	.46
80	2.53	.13
90	1.33	-.07
95	.68	-.22
100	.00	-.33

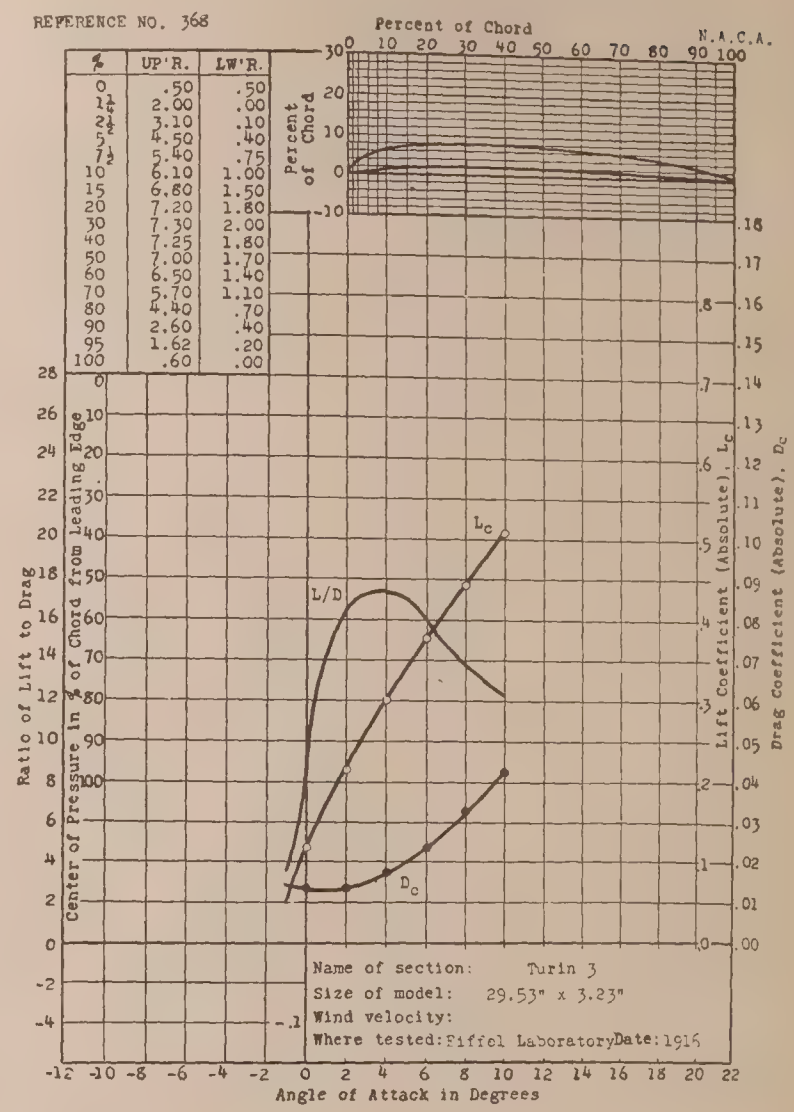
REFERENCE NO. 367

%	UP'R.	LW'R.
0	1.33	1.33
1¼	2.65	.80
2½	3.40	.50
5	4.60	.10
7½	5.35	.00
10	5.95	.10
15	6.75	.73
20	7.15	1.65
30	7.15	2.33
40	6.25	2.00
50	4.90	1.25
60	3.75	.50
70	2.90	.15
80	2.28	.00
90	2.00	.50
95	2.00	.90
100	1.65	1.65



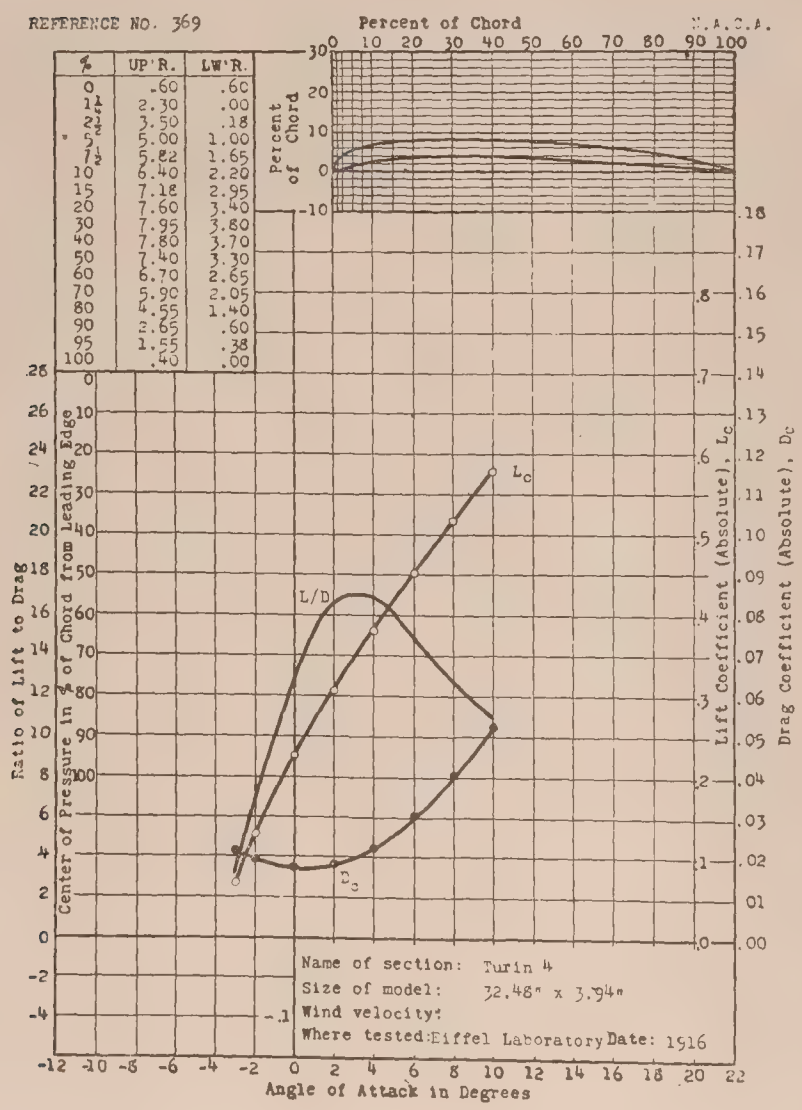
REFERENCE NO. 368

%	UP'R.	LW'R.
0	.50	.50
1¼	2.00	.00
2½	3.10	.10
5	4.50	.40
7½	5.40	.75
10	6.10	1.00
15	6.80	1.50
20	7.20	1.80
30	7.30	2.00
40	7.25	1.80
50	7.00	1.70
60	6.50	1.40
70	5.70	1.10
80	4.40	.70
90	2.60	.40
95	1.62	.20
100	.60	.00



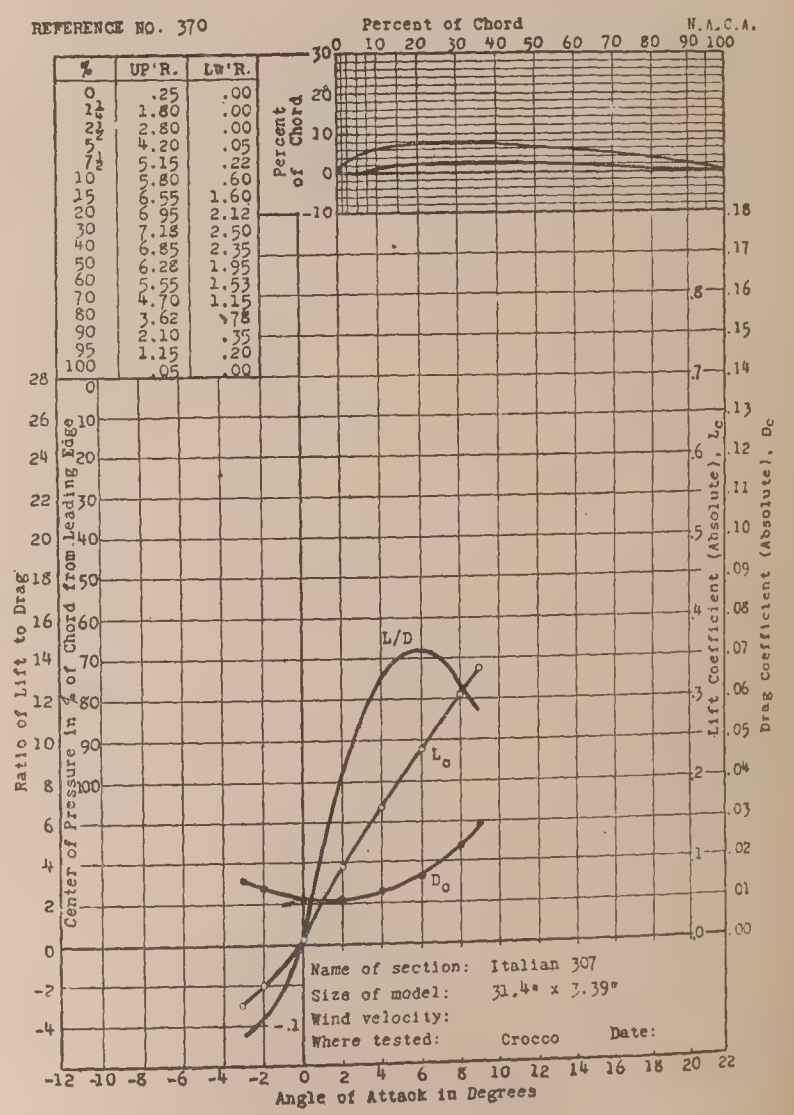
REFERENCE NO. 369

%	UP'R.	LW'R.
0	.60	.60
1¼	2.30	.00
2½	3.50	.18
5	5.00	1.00
7½	5.82	1.65
10	6.40	2.20
15	7.18	2.95
20	7.60	3.40
30	7.95	3.80
40	7.80	3.70
50	7.40	3.30
60	6.70	2.65
70	5.90	2.05
80	4.55	1.40
90	2.65	.60
95	1.55	.38
100	.40	.00



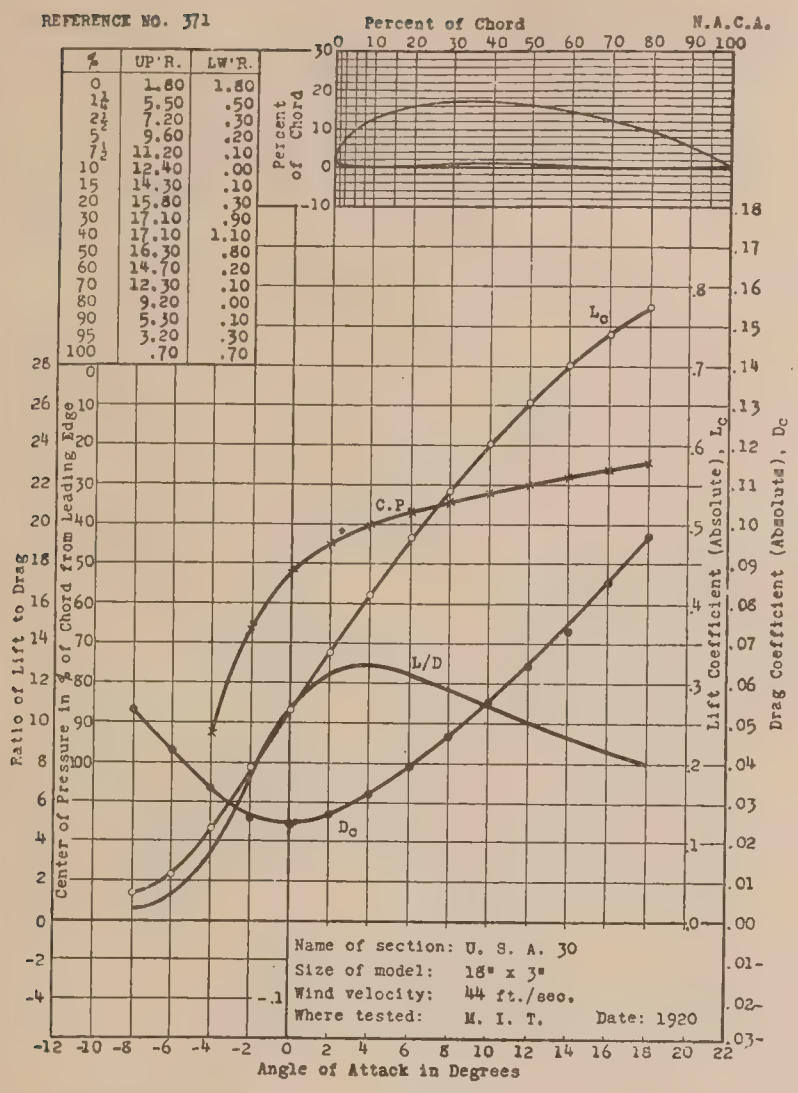
REFERENCE NO. 370

%	UP'R.	LW'R.
0	.25	.00
1¼	1.80	.00
2½	2.80	.00
5	4.20	.05
7½	5.15	.22
10	5.80	.60
15	6.55	1.60
20	6.95	2.12
30	7.18	2.50
40	6.85	2.35
50	6.28	1.95
60	5.55	1.53
70	4.70	1.15
80	3.62	.78
90	2.10	.35
95	1.15	.20
100	.05	.00



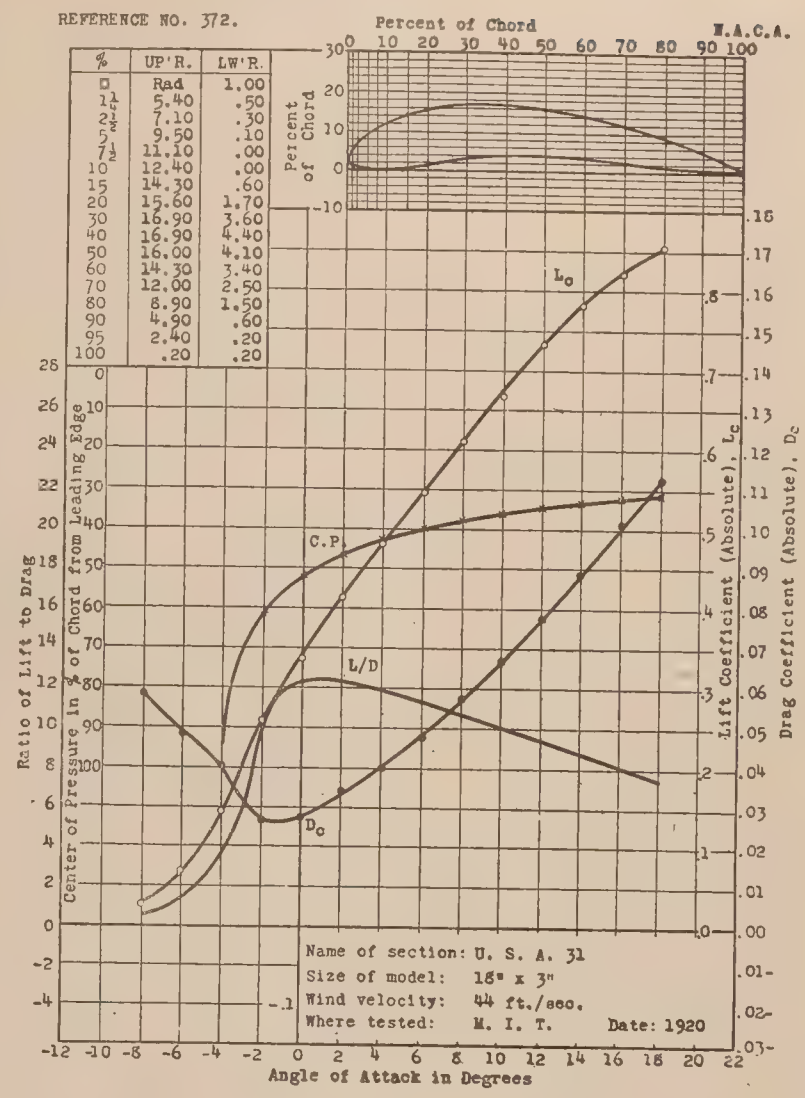
REFERENCE NO. 371

%	UP'R.	LW'R.
0	1.80	1.80
1¼	5.50	.50
2½	7.20	.30
5	9.60	.20
7½	11.20	.10
10	12.40	.00
15	14.30	.10
20	15.80	.30
30	17.10	.90
40	17.10	1.10
50	16.30	.80
60	14.70	.20
70	12.30	.10
80	9.20	.00
90	5.30	.10
95	3.20	.30
100	.70	.70



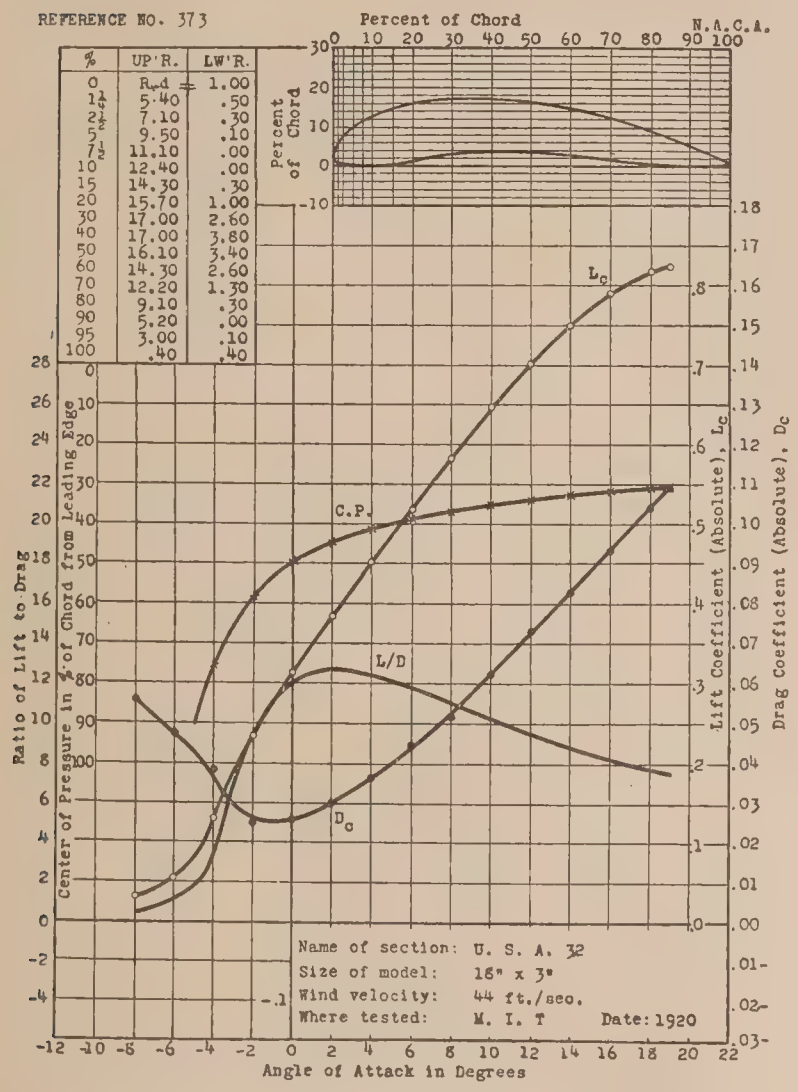
REFERENCE NO. 372.

%	UP'R.	LW'R.
0	Rad	1.00
1¼	5.40	.50
2½	7.10	.30
5	9.50	.10
7½	11.10	.00
10	12.40	.00
15	14.30	.60
20	15.60	1.70
30	16.90	3.60
40	16.90	4.40
50	16.00	4.10
60	14.30	3.40
70	12.00	2.50
80	8.90	1.50
90	4.90	.60
95	2.40	.20
100	.20	.20



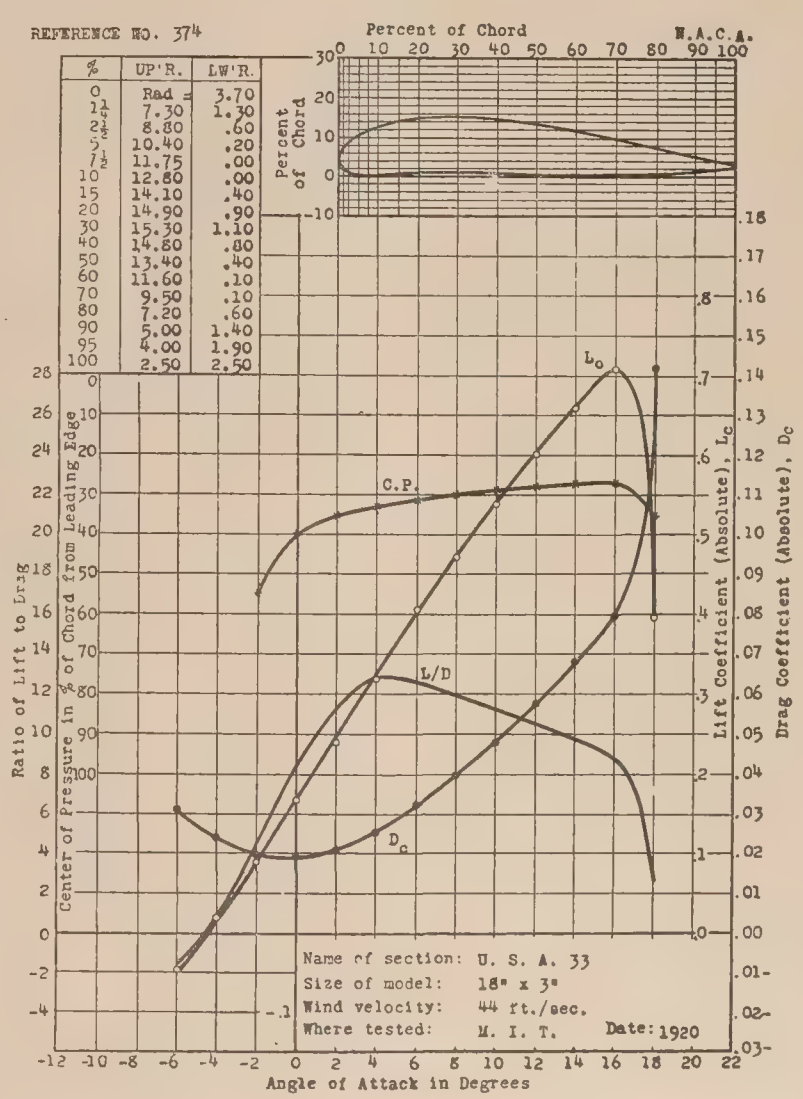
REFERENCE NO. 373

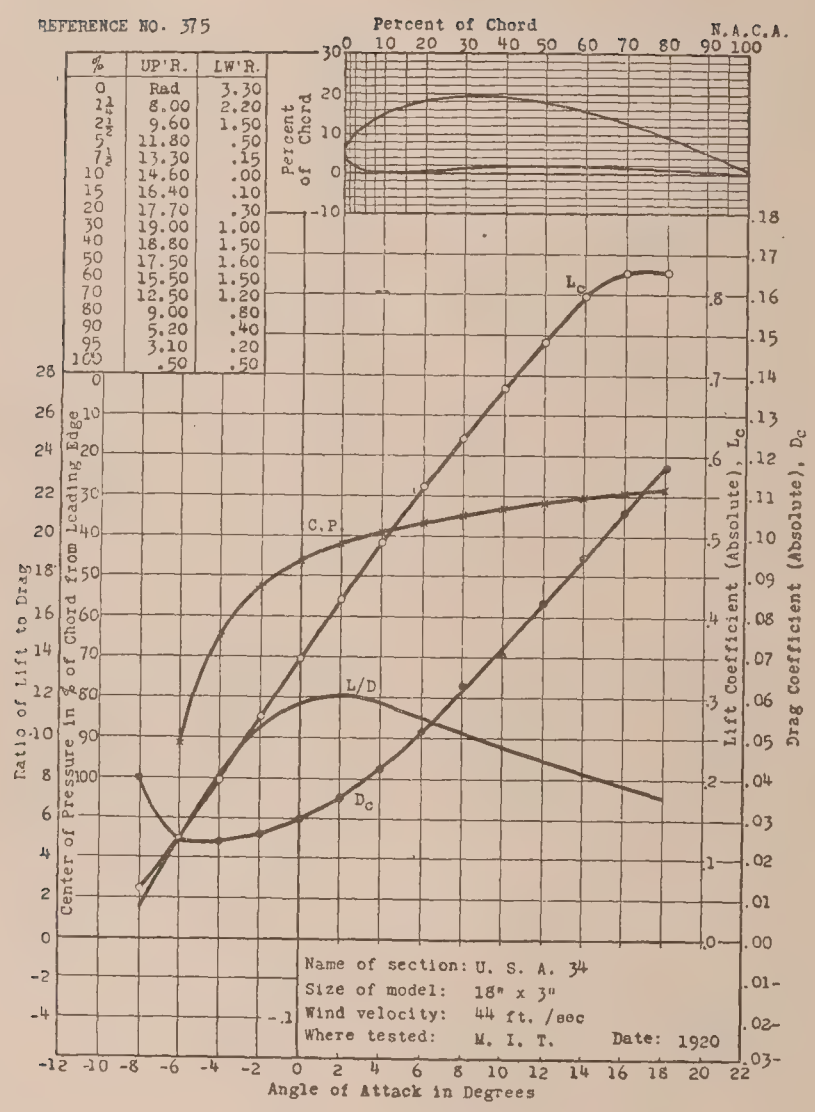
%	UP'R.	LW'R.
0	Rad	1.00
1¼	5.40	.50
2½	7.10	.30
5	9.50	.10
7½	11.10	.00
10	12.40	.00
15	14.30	.30
20	15.70	1.00
30	17.00	2.60
40	17.00	3.80
50	16.10	3.40
60	14.30	2.60
70	12.20	1.30
80	9.10	.30
90	5.20	.00
95	3.00	.10
100	.40	.40



REFERENCE NO. 374

%	UP'R.	LW'R.
0	Rad	3.70
1¼	7.30	1.30
2½	8.80	.60
5	10.40	.20
7½	11.75	.00
10	12.80	.00
15	14.10	.40
20	14.90	.90
30	15.30	1.10
40	14.80	.80
50	13.40	.40
60	11.60	.10
70	9.50	.10
80	7.20	.60
90	5.00	1.40
95	4.00	1.90
100	2.50	2.50





%	UP'R.	LW'R.
0	Rad	3.30
1¼	8.00	2.20
2½	9.60	1.50
5	11.80	.50
7½	13.30	.15
10	14.60	.00
15	16.40	.10
20	17.70	.30
30	19.00	1.00
40	18.80	1.50
50	17.50	1.60
60	15.50	1.50
70	12.50	1.20
80	9.00	.80
90	5.20	.40
95	3.10	.20
100	.50	.50

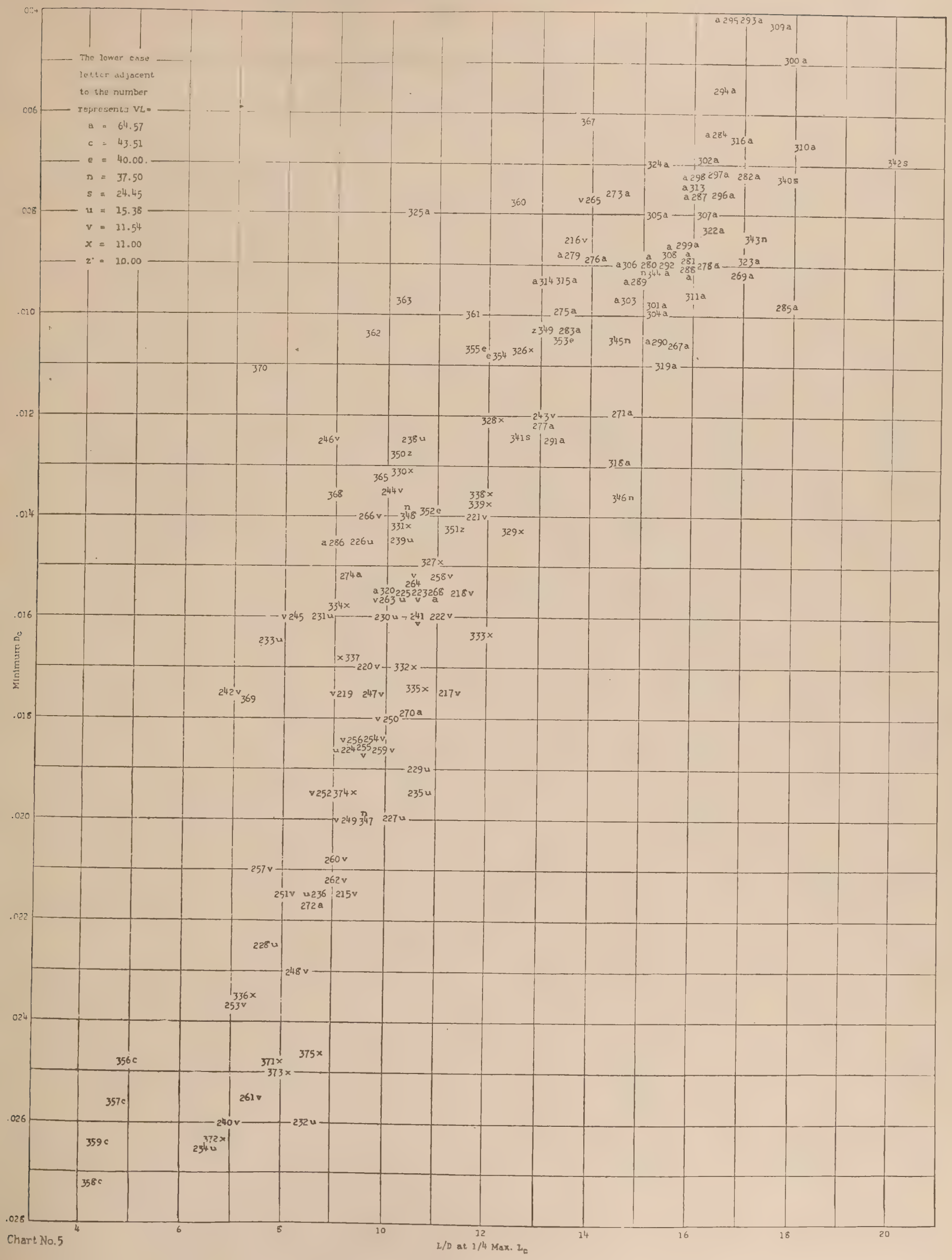
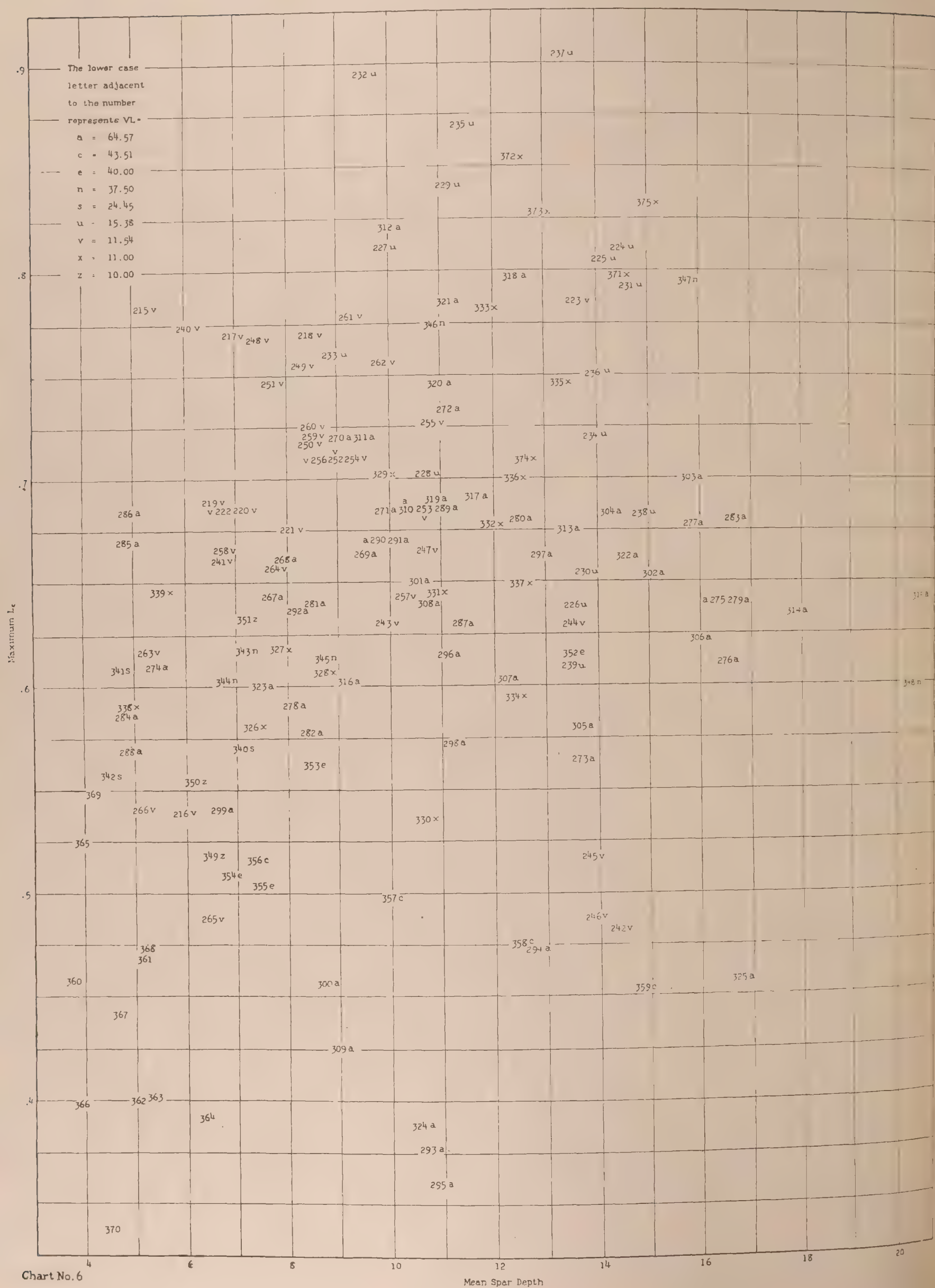

The lower case
letter adjacent
to the number
represents VL=
a = 64.57
c = 43.51
e = 40.00.
n = 37.50
s = 24.45
u = 15.38
v = 11.54
x = 11.00
z = 10.00
Minimum D_c
L/D at 1/4 Max. L_c
.004
.006
.008
.010
.012
.014
.016
.018
.020
.022
.024
.026
.028
4
6
8
10
12
14
16
18
20

Chart No. 5


The lower case letter adjacent to the number represents VL =
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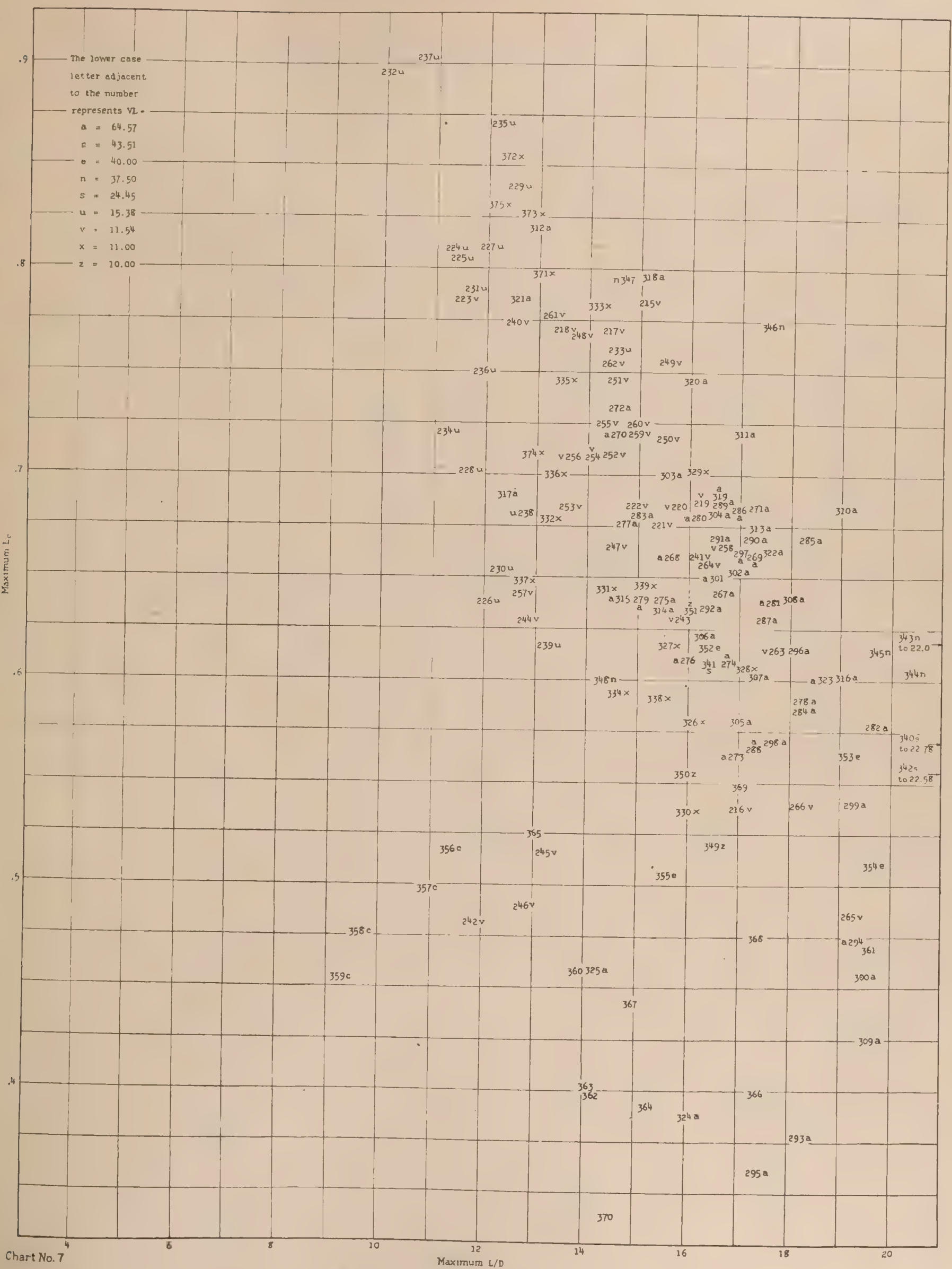
c = 43.51
e = 40.00
n = 37.50
s = 24.45
u = 15.38
v = 11.54
x = 11.00
z = 10.00
Maximum Lc
Mean Spar Depth
.9
.8
.7
.6
.5
.4
4
6
8
10
12
14
16
18
20

Chart No. 6


The lower case letter adjacent to the number represents VL.
a = 64.57
c = 43.51
e = 40.00
n = 37.50
s = 24.45
u = 15.38
v = 11.54
x = 11.00
z = 10.00
Maximum Lc
Maximum L/D
Chart No. 7

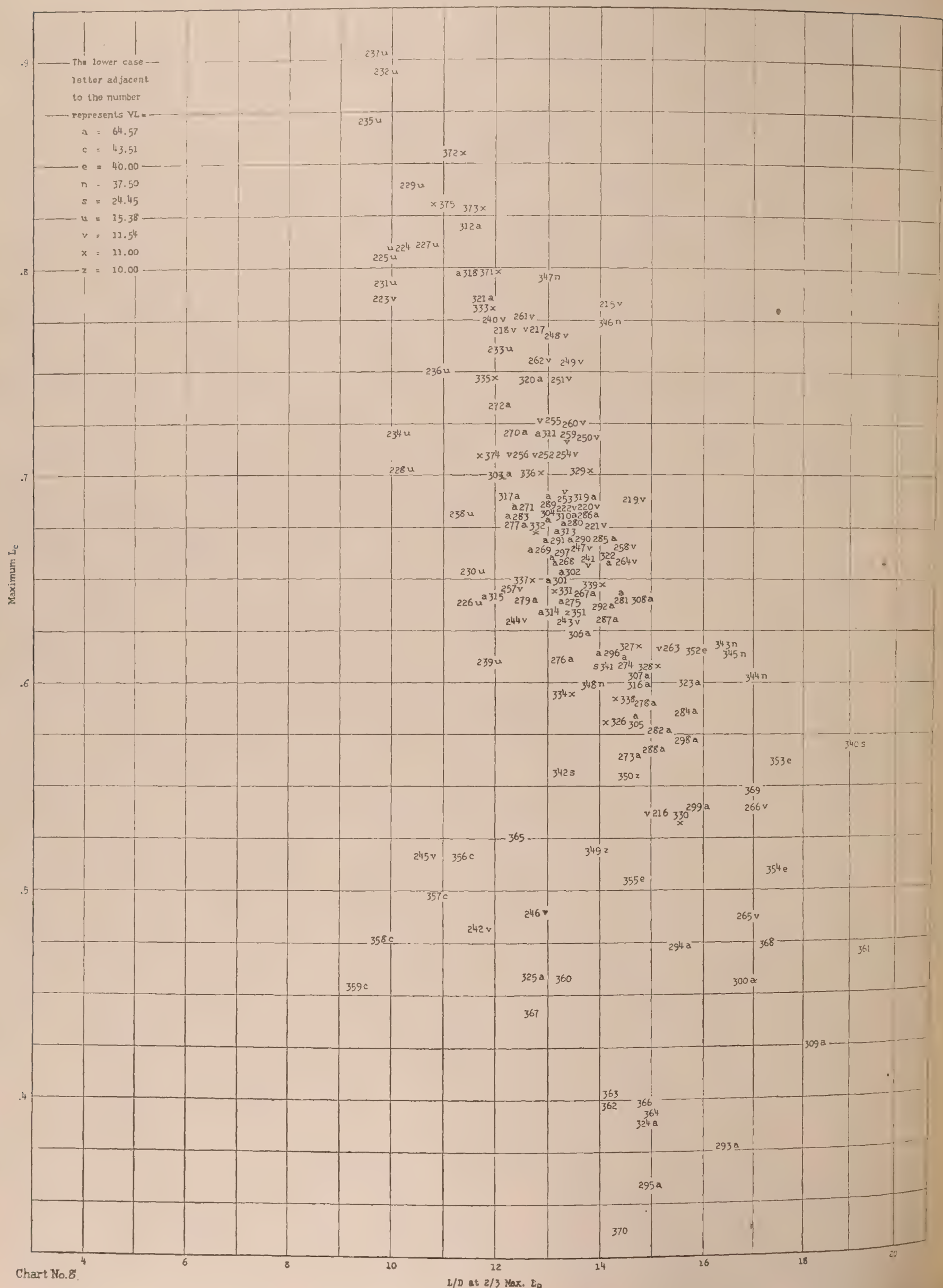

The lower case letter adjacent to the number represents VL =
a = 64.57
c = 43.51
e = 40.00
n = 37.50
s = 24.45
u = 15.38
v = 11.54
x = 11.00
z = 10.00
Maximum L_c
L/D at 2/3 Max. L_D
.9
.8
.7
.6
.5
.4
4
6
8
10
12
14
16
18
20

Chart No. 8.

REPORT No. 125

AERONAUTIC INSTRUMENTS
SECTION I

GENERAL CLASSIFICATION OF INSTRUMENTS AND PROBLEMS INCLUDING BIBLIOGRAPHY

By MAYO D. HERSEY
Bureau of Standards

REPORT No. 125.

GENERAL CLASSIFICATION OF INSTRUMENTS AND PROBLEMS, INCLUDING BIBLIOGRAPHY.

By Mayo D. Hersey.

INTRODUCTION.

This report is Section I of a series of papers, comprising a general report on aeronautic instruments, published as Technical Reports Nos. 125 to 132, inclusive, which contain the results of investigations on aeronautic instruments by the Bureau of Standards and which have been prepared under research authorizations formulated and recommended by the subcommittee on aerodynamics and approved by the National Advisory Committee for Aeronautics. Much of the material contained in this report was made available through the cooperation of the War and Navy Departments.

As authorized by the committee on aerodynamics, these reports include a complete account of the status of aeronautic instruments at the end of the war and cover the subject in detail up to the beginning of the year 1920. Since that date nearly a year and a half has been required for the actual preparation of the manuscripts, which represent the cooperative effort of eighteen individual authors, many of whom have left the Government service. Report No. 132, by Dr. F. L. Hunt, now Chief of the Aeronautic Instruments Section of the Bureau of Standards, serves to bridge this gap by giving a brief statement of recent developments. The bibliography also has been kept complete up to the moment of going to press.

Technical Reports Nos. 126 to 131, inclusive, contain a systematic, illustrated description of American, British, French, Italian, Swiss, Dutch, Danish, Austrian, and German aircraft instruments, together with methods of testing developed by the Bureau of Standards, and brief statements of investigation results. In compiling the material for these reports, separate papers have been written by experts on the respective types of instruments, as, for example, altimeters, tachometers, or oxygen apparatus.

Subjects which are common to instruments in general are treated in this report. Throughout the series of reports emphasis has been placed on the description of successful types of instruments and the exposition of fundamental scientific principles, while the space devoted to investigations and developments of transitory interest has been reduced to a minimum. In this way it is expected that the reports will be of permanent value for reference.

SUMMARY.

This report is intended as a technical introduction to the series of reports on aeronautic instruments. It presents a discussion of those subjects which are common to all instruments. In the first place, a general classification is given, embracing all types of instruments used in aeronautics. The arrangement of information dealing with these various instruments throughout the reports is then briefly indicated as a guide to the reader. Finally, a classification is given of the various problems confronted by the instrument expert and investigator. In this way the following groups of problems are brought up for consideration: First, problems of mechanical design; second, human factor; third, manufacturing problems; fourth, supply and selection of instruments; fifth, problems concerning the technique of testing; sixth, problems of installation; seventh, problems concerning the use of instruments; eighth, problems of maintenance; ninth, physical research problems. This enumeration of problems which are

common to instruments in general serves to indicate the different points of view which should be kept freshly in mind in approaching the study of any particular instrument.

CLASSIFICATION OF INSTRUMENTS.

Instruments used in aeronautics may be classified first of all into three general groups according as they are used on the ground; or sent up into the air on kites and balloons, solely for aerological purposes; or installed on board navigable aircraft. Briefly, these three groups may be designated as ground instruments, aerological instruments, and aircraft instruments.

Ground instruments comprise aircraft location instruments, meteorological, and laboratory instruments. Aircraft location instruments include, first, those used for detecting the approach of hostile aircraft, usually acoustic in principle; second, instruments for measuring the altitude or speed of aircraft from the ground, based upon transits or other sighting devices; third, course-tracing instruments, such as the camera-obscura and recording theodolites. Laboratory instruments for aeronautic work include, besides wind-tunnel and power-plant testing equipment, the necessary standard or master instruments for use in calibrating aircraft instruments.

Aerological instruments may be put into four groups according as they are sent up on kites and pilot balloons, or in captive balloons, in free balloons, or in navigable aircraft. In the first case only recording instruments are wanted, and the altitude range may need to be exceedingly high. Maximum indicators have an application here as well as those yielding continuous records. For captive balloons the need of instruments would seem to be a minimum, but might include the aneroid barometer and anemometer. For free balloons the anemometer, or air-speed indicator, would not serve any purpose, but the altimeter and compass and a ground-speed indicator would be desirable, and a statoscope indispensable for delicate indications of ascent or descent. For preserving a record of the trip and investigating air conditions encountered, the various aerographic instruments would be used, including barograph, thermograph, and hygrograph. Aerological instruments may, finally, be installed in airplanes and dirigibles for investigation of atmospheric conditions, or, as in the case of the strut thermometer, to secure data needed for reducing performance tests to standard atmospheric conditions. The term aerological is employed in these reports to distinguish instruments used in studying the structure of the atmosphere from those needed as an aid to flying. The latter will be referred to as aircraft instruments.

Aircraft instruments are fundamentally divided into two groups, those designed for experimental purposes and those furnished as a part of the regular service equipment of aircraft. Experimental instruments must be capable of more accurate results than service instruments, and the self-recording feature is desirable, but the designer has considerable latitude in attaining these objects. In contrast with this, service instruments, though less sensitive and precise, must be sufficiently reliable under a more severe range of conditions; and they must be direct-reading without elaborate corrections, not easily deranged, and possessing minimum bulk and weight.

Experimental instruments differ somewhat in operation and in requirements according as they are designed for routine performance testing, for steady flight investigations, for stunting and accelerated flight, or for competitive altitude records.

Service instruments, on the contrary, are designed more with reference to the type of aircraft than the character of the flight, since the latter is indeterminate. While the majority of service instruments are the same for all aircraft, some are specially adapted for seaplanes, others for multiple engine craft, and, finally, additional instruments such as gas-bag manometers and thermometers, water-ballast gauges, ballonet volume indicators, and hydrogen detectors may be needed for lighter-than-air craft.

These various classifications are shown concisely in Table I.

TABLE I.

INSTRUMENTS USED IN AERONAUTICS.

I. GROUND INSTRUMENTS.
- A. AIRCRAFT LOCATION INSTRUMENTS.
  - For detection.
  - For speed measurement.
  - For course tracing.
- B. METEOROLOGICAL INSTRUMENTS.
- C. LABORATORY INSTRUMENTS.
  - For wind tunnels, power-plant testing, etc.
  - For calibration of aircraft instruments.

II. AEROLOGICAL INSTRUMENTS.
- A. FOR KITES AND PILOT BALLOONS.
- B. FOR CAPTIVE BALLOONS.
- C. FOR FREE BALLOONS.

III. AIRCRAFT INSTRUMENTS.
- A. EXPERIMENTAL INSTRUMENTS.
  - For routine performance testing.
  - For steady flight investigations.
  - For stunting and accelerated flight.
  - For competitive altitude records.
- B. SERVICE INSTRUMENTS.
  - Instruments common to all aircraft.
  - Instruments adapted for seaplanes.
  - Instruments for multiple engine craft.
  - Additional instruments for airships.

Service instruments may further be classified with reference to the respective physical measurements made; this has been done more or less completely in Table II.

TABLE II.

SERVICE INSTRUMENTS.

1. ALTITUDE INSTRUMENTS.
   - (*a*) Altimeters and barographs.
   - (*b*) Statoscopes and rate-of-climb indicators
   - (*c*) Aerographic instruments.
2. SPEED INSTRUMENTS.
   - (*a*) Air speed indicators.
   - (*b*) Ground speed indicators.
3. DIRECTION INSTRUMENTS.
   - (*a*) Inclinometers and banking indicators.
   - (*b*) Stabilizers.
   - (*c*) Compasses and turn indicators.
4. POWER-PLANT INSTRUMENTS.
   - (*a*) Tachometers.
   - (*b*) Thermometers.
   - (*c*) Pressure gauges.
   - (*d*) Gasoline depth gauges and flow meters.
5. OXYGEN AND OTHER ACCESSORY INSTRUMENTS.
6. NAVIGATING INSTRUMENTS.

The first five groups meet the requirements of local aviation, while those of the sixth group (navigation instruments) are needed for long-distance flights. Local aviation instruments may be divided into general flying instruments, power-plant instruments, and accessory instruments. The general flying instruments are classified as altitude, speed, or direction instruments, and shown here by groups 1, 2, and 3. Power-plant instruments are represented by group 4. Oxygen apparatus for the aviator constitutes the most important of the accessory instru-

ments, group 5, although bombsights and other military or photographic equipment might also be included here, together with hydrogen detectors, buoyancy computers, and other airship accessories.

COMPOSITION OF THE SERIES OF REPORTS.

Substantially the foregoing classification (Table II) has been adopted for the framework of these reports, which deal primarily with service instruments. To a less extent experimental, aerological, and laboratory instruments are likewise discussed and grouped under the same general headings. The six classes in Table II correspond respectively to Reports Nos. 126 to 131, inclusive.

Thus, Report No. 126 covers the general field of altitude instruments and closely-related subjects in the following papers:

Part I, by A. H. Mears, H. B. Henrickson, and W. G. Brombacher, on altimeters and barographs;

Part II, by J. B. Peterson and J. R. Freeman, jr., on precision altimeter design;

Part III, by A. H. Mears, on statoscopes and rate-of-climb indicators;

Part IV, by J. A. C. Warner, on aerographic instruments.

Report No. 127, dealing with aircraft speed instruments, by F. L. Hunt and by H. O. Stearns, is composed of one part descriptive of air-speed indicators, a second on testing methods, and a third on principles of ground speed measurement.

Direction instruments, a complex group, are brought together in Report No. 128, in the following papers:

Part I, by W. S. Franklin and M. H. Stillman, covering inclinometers and banking indicators, including all necessary gyroscopic principles;

Part II, by R. L. Sanford, dealing specifically with methods adopted for testing magnetic compasses;

Part III, by J. A. C. Warner, comprehensively describing compasses;

Part IV, by R. C. Sylvander and Lieut. E. W. Rounds, U. S. N., describing turn indicators.

Report No. 129 includes the numerous power-plant instruments in five parts, as follows:

Part I, by G. E. Washburn, dealing with aircraft tachometers;

Part II, by R. C. Sylvander, on testing methods for tachometers;

Part III, by E. F. Mueller and R. W. Wilhelm, on thermometers for aircraft engines.

Part IV, by H. N. Eaton, on pressure gauges;

Part V, by J. A. C. Warner, on gasoline depth gauges and flow meters.

Report No. 130, by F. L. Hunt, presents a complete account of oxygen equipment for the aviator.

Report No. 131, by H. N. Eaton, is a comprehensive report on modern navigating instruments for long-distance flight, including radio equipment. This subject is approached from the standpoint of aerial navigation methods rather than that of the physical operation of the instruments. Passing from one problem of navigation to another, the instruments needed are described in turn. This paper naturally overlaps with several of the others; for example, in the case of air-speed indicators and compasses, because the navigator has to depend on all of the instruments to some extent in addition to his own special equipment. In such instances reference can be made to other reports of the series for the details of mechanical construction, so that it remains only to discuss those characteristics of the instrument pertaining to the navigation problem itself, especially questions concerning the accuracy attainable.

This understanding of the general arrangement of subject matter should make it possible for the reader to locate a given topic without any alphabetical index by reference to the table of contents at the beginning of each report. In one or another of these tables every instrument discussed in the body of the reports will be found listed, taking usually the maker's name or trade name as its designation. Altogether in Reports Nos. 126 to 131, inclusive, some 250 different instruments are specifically described, aside from testing equipment. (Practically

all of the more important of these instruments have been available for examination in the Government instrument collection maintained at the Bureau of Standards in Washington.)

While the general outline or plan of these reports was entirely the work of the Aeronautic Instruments Section, organized by the author under the personal supervision of Dr. Stratton, all departments of the bureau have been freely called upon for cooperation. Thus wind tunnel tests were made in cooperation with the staff of the aerodynamic laboratory, other tests were carried on with the facilities of the low temperature laboratory, and the contributions of Messrs. Freeman, Stillman, Sanford, and Mueller and Wilhelm were made possible through courtesy of the metallurgy, weights and measures, electrical, and heat divisions, respectively.

Any adequate attempt to acknowledge cooperative assistance received from outside the circle of those who participated directly in this work would develop almost into a complete catalogue of the aviation authorities and instrument experts in the United States and allied countries. However, the authors and compilers of these reports cordially recognize these innumerable courtesies, in the absence of which the reports could not have been made at all complete.

CLASSIFICATION OF PROBLEMS.

While the concrete details of the respective instruments will be thoroughly taken up in the subsequent papers, there are some problems common to all instruments which may well be considered first. Such problems, originating in the study of one particular instrument, oftentimes lead to a general solution which can later be applied with advantage to other instruments presenting analogous questions. Moreover, it ought to stimulate the imagination to take this bird's-eye view of instrument problems in the abstract, enabling one to study the remaining reports more critically and to be better prepared to anticipate the difficulties involved in any contemplated project of instrument development.

PROBLEMS OF MECHANICAL DESIGN.

The art of designing aeronautical instruments is at present almost wholly on a cut-and-try basis. It is to be expected that rational design will eventually supersede empirical design, so that the designer, utilizing available results of physical research, can go almost directly to the drawing board and lay out the necessary proportions for securing the desired performance.

In either case the design will be carried forward in two general stages, first accomplishing what might be called the functional design, and then modifying this design in conformity with practical restrictions that may have been imposed.

This preliminary or functional design includes in typical cases the design of the force element, the transmission mechanism, the indicating or recording element, and the compensation, in all their interrelations.

Practically every type of instrument is actuated by some force element. The force in question is produced by some effect corresponding in magnitude with the physical quantity to be measured. In the various aeronautic instruments this initial force action may be set up by hydrostatic pressure, impact pressure, centrifugal force, gyrostatic torque, viscous drag, thermal expansion, magnetic field, or otherwise. However produced, it is usually registered by the deformation of some elastic system, such as a steel spring, a German silver diaphragm, or some combination of elastic parts. In the consideration of spring design, regardless of the particular instrument in question, certain problems will come up. Among these are the determination of the stiffness needed; the calculation of the necessary dimensions or shape to produce this amount of stiffness; the amount of deflection possible without departing from a uniform scale relation; and means for securing sufficient rigidity in the attachment of the spring. Similar problems arise in diaphragm design, including the consideration of the effect of corrugations; thickness of metal; variability of thickness from center to circumference as a means of controlling the scale relation; relative advantage of combining diaphragms in series as contrasted with increasing the area of a single diaphragm; best method for joining the top and bottom diaphragms together at the rim. Many other such questions are involved in successful

diaphragm design, and it is notable that flexible diaphragms are used in a large number of the most diverse instruments. The design of the complete force element may also require the consideration of the properties of coupled systems.

The problems of transmission design are largely geometrical. Uniformity of scale in the finished instrument is usually desirable, and expedients for securing this by suitable arrangement of levers, cams, or other movements are of interest in connection with all instruments. The mechanism must also be provided with a sufficient degree of adjustability as regards both the zero point and the total multiplying power. Other considerations involved are the relative merits of jewelled or metallic bearings; of knife-edge contacts or flexible plate connections; of large multiplying power or large force action. In all mechanisms, finally, a suitable compromise has to be determined between friction and lost motion. Neither one can be completely eliminated without suffering too large an influence from the other.

Even the dial and pointer design involves scientific problems which have not been completely solved. The graduations may either be laid out empirically, after assembling and testing the instrument with a substitute dial, or they may be laid out in a uniform manner beforehand, with the expectation of subsequently adjusting the movement to agree with the dial. Maximum visibility and precision must be secured for both the dial and pointer. The pointer must be sufficiently close to the dial surface to reduce the parallax without danger of contact, and the attachment of the pointer to the spindle must be such as to permit readjustment of position without danger of slipping around while the instrument is in use.

The design of compensation devices is likewise difficult. It seems comparatively simple to compensate for gravitational effects (inclination of instrument) by suitable counterweights, but this has rarely been done with complete success. Compensation for angular accelerations such as are always present due to the vibration of the instrument board has rarely been undertaken. The effect of temperature on the reading of the instrument, other conditions remaining constant, can be compensated by thermal expansion devices, such as the bimetallic bar familiar in aneroid construction, but the compensation of instruments for change of sensitivity with temperature may practically be classed as an unsolved problem.

The final stage of design has mainly two objects in view: First, conformity with limitations of weight and size; second, interchangeability.

In regard to the former requirement, it is evident that, while a large force action is desirable to overcome friction in the mechanism, this on the one hand may, in the case of diaphragm instruments, require large areas and tend to make the total instrument too large; on the other hand large force action requires greater strength of supporting parts, thus tending to make the weight of the final instrument excessive. Here, as in so many other problems of mechanical design, a suitable compromise has to be determined upon.

Standardization of design for the purpose of interchangeability applies to the external size of case, arrangement of holes and flanges, connections to shafting or tubing, cover glasses, screw threads, etc. A beginning has been made toward international standardization on many of these details by the work of the International Aircraft Standards Board as temporarily organized in Washington during the war. This general subject of standardization would seem to merit further consideration in the future.

THE HUMAN FACTOR.

The reaction of the aviator to his instruments has to be considered, as well as the operation of the instruments themselves. This is evident enough in the case of appliances such as oxygen apparatus, intended solely for the comfort and efficiency of the aviator, or in the case of complicated instruments such as bomb sights. But it is equally true with the more simple, direct-reading instruments. It is not enough for such instruments to be mechanically correct; they must be, in the case of service instruments, readily intelligible to the pilot. The manipulation of the instrument must not make an appreciable demand on his time or attention. The visibility must be satisfactory both day and night. Finally, service instruments must be "fool proof." While much can be accomplished by technical instruction courses for aviation

personnel, still the personal prejudice of the average pilot has to be reckoned with. If the instrument for any reason fails to appeal to the individual pilot, he will take great chances rather than trouble to look at it. On the other hand, if the instrument pleases his fancy, he may grow so attached to it that he will claim he could not fly safely without it, even though the instrument be scientifically known to be incorrect. Curious examples of this circumstance were found in the popularity of the earlier liquid type Pitot tube among the British pilots and the spinning-top inclinometer among the French.

MANUFACTURING PROBLEMS.

For best results the instrument designer must look beyond the intrinsic problems of his instrument to consider the difficulties of actual construction, and especially of quantity production. The problems of casting, stamping, machining, tempering, assembling, and other processes may, of course, be such as to make one design more economical or capable of quicker production than another of the same quality.

Having agreed upon a given design, there still remain problems of production efficiency which the manufacturer must settle by himself. Take, for example, the question of uniform or empirical scale graduations already referred to. Machines are on the market for graduating mercury-in-glass thermometers automatically in such a manner that any three fixed points may be scratched on the stem and the remainder of the scale smoothly interpolated. The same process should be applicable to aeronautic instruments of the circular dial type. Now for quantity production, this method, whether executed automatically or by hand, is without doubt the most efficient for turning out properly adjusted instruments. It eliminates the time needed on uniform scale instruments for tinkering with each individual movement; and the empirical scale instrument passes inspection tests with practically no rejections for calibration error. Yet after being out in service for some time, if the respective instruments undergo exactly the same internal change, the uniform scale will now show the less error, as careful reasoning might predict. Thus, there is something to be said in favor of each method, and the problem can not be settled *a priori*.

Besides many such special problems, the manufacturer is always confronted by the general question whether it will pay better to force production rapidly, permitting a high percentage of test rejections, or to more carefully examine the material entering each instrument, thus slowing down the process but insuring a nearly perfect product in each case. While this problem exists to some degree in all manufacturing work, it is a specially prominent and interesting one for measuring instruments.

SUPPLY AND SELECTION OF INSTRUMENTS.

Proper coordination of instrument orders, not merely speed of production, was found to be of critical importance during the war. Such coordination requires both a correct relative amount of different items and correct choice of corresponding parts. Questions of this kind can not be satisfactorily settled by office personnel. Technical knowledge and personal contact with conditions at the airdromes are necessary.

The control of quality of output by systematic inspection is generally necessary. Such inspection in this country and England is based on written specifications. In France such specifications were not in vogue during the war, greater reliance being placed on the artistic pride and professional skill of the numerous instrument manufacturers of established reputation.

An important difference between the inspection system followed in England and in this country was that in England practically all of the inspection and testing was done at one large central station, the Royal Aircraft Establishment at Farnborough, while in this country inspectors were sent out by the Government and stationed at the different factories. As it was, the inspectors were trained for their work at the Bureau of Standards, and a certain proportion of the instrument production from all parts of the country was shipped to the Bureau of Standards for more complete tests.

Factory inspection tests may legitimately be accelerated in various ways if the instruments will be individually tested, as they always should be, immediately before installation on the aircraft. Unless this is done, the instruments are liable to errors incurred during transportation. If it is done, the factory tests, at least those conducted by the Government inspection staff, may be directed to the rejection of defective instruments rather than to the observation of calibration errors. The distinction between defects and errors is a fundamental one. Defects—that is, mechanical imperfections—are avoidable and need not be permitted. Errors, on the other hand, must be present in every measuring instrument; they can not be avoided in kind, but only diminished in magnitude.

This leads up to the general matter of instrument selection, which is best accomplished by written specifications. The routine of preparing, revising, and enforcing instrument specifications can be facilitated by separating the technical specifications from other items and dividing them into construction and performance specifications.

Opinions differ regarding the need of construction specifications. One view is that the purchaser, except in special instances, should not bind the manufacturer to any particular set of constructive details. It is held that since ultimately the purchaser can only be interested in the performance of his instrument, the expedients available for securing this performance may better be left to the discretion of the manufacturer. In fact, at times rigid insistence upon construction specifications has resulted in eliminating valuable improvements. On the other hand, performance specifications formulated in ignorance of the mechanical details of the instrument are liable to overlook some item which might give rise to serious sources of error not provided for. In such a case the instrument, though worthless, could not legitimately be rejected. The best practice seems to be a compromise, leaving the construction specifications as liberal as possible but protecting the purchaser by making the performance specifications quite complete. This situation is also helped by not placing unduly large orders each time, thus leaving the manufacturer free to propose improvements which can be adopted from time to time with a revision of specifications.

The art of writing specifications, particularly performance specifications, involves two main problems: First, that of simplifying and standardizing the form of the specifications; second, that of securing sufficient flexibility in the requirements laid down.

Test reports are necessary in order to show whether an instrument conforms to the performance specifications, and unless the specifications have been prepared with just this difficulty in mind, the corresponding test report will drag out to an inconvenient length, requiring a voluminous set of curves, perhaps, to represent the observations on a single instrument. By long study of this problem in the case of altimeter testing it was found possible to select and define mathematically a minimum number of performance characteristics which could be represented by single numerical magnitudes. As a result, it became possible to give the essential test results of a large group of instruments on one single sheet of paper, a significant economy when instruments are tested in quantity, and also an advantage when investigating statistically the progressive improvement in the quality of instruments from time to time.

The other problem, flexibility of requirements, may be illustrated by an example. Suppose that a given instrument is required by the specifications to show satisfactory performance in five particulars, A, B, C, D, and E, each determined by some laboratory test. The simplest way of writing the specifications is to say that the instrument will be accepted if each of the errors, A, B, C, D, and E, is less than a stated numerical amount; otherwise, rejected. This system is satisfactory for instruments showing uniformly large or small errors throughout the schedule. But suppose that two sample instruments are submitted by rival concerns, X and Y, seeking contracts. Suppose, further, that the X instrument shows practically perfect performance on the last four items, but just barely falls below the limit on item A, while instrument Y just barely slips by on all five items. Which instrument is the better for practical use? Obviously the X instrument, because of its exceptionally good performance in the majority of the tests. Therefore, the simple system of rejection limits formulated above is unfair to the manufacturer and disadvantageous to the purchaser.

Two suggestions at least have been made for solving this problem, but nothing has yet been settled upon. One proposal is to formulate two sets of limits, a maximum and a minimum for each of the errors A, B, C, D, and E. If the instrument exceeds the maximum error for any one item, it is rejected; if it exceeds the minimum error for all items, it is likewise rejected; but if it exceeds the minimum limit in a sufficiently small number of items only, say, A and B, while showing figures below the minimum limit for the remaining items C, D, and E, then it is not rejected. The other plan is to assign numerical weights to the different errors reported in order to derive a figure of merit or average result, such as is furnished in the civil service report of a candidate's examination. Calling numerical errors A, B, C, D, and E, and the arbitrary weighting factors a, b, c, d, and e, one computes the quotient:

$$\frac{Aa+Bb+Cc+Dd+Ee}{a+b+c+d+e}$$

The resulting figure shall not exceed a prescribed numerical magnitude. This sytem has been followed for some time in the testing of timepieces at the Bureau of Standards and elsewhere, but it can not be adopted hastily on account of the difficulty of establishing suitable values for the relative weights, a, b, c, d, and e.

The determination of numerical limits for performance specifications, after agreement upon the form of the specifications, depends on two sources of information—first, production possibilities; second, actual needs in the air on the part of the pilot. Very much further study is needed on the subject of instrument specifications.

PROBLEMS CONCERNING THE TECHNIQUE OF TESTING.

The fundamental problems of laboratory testing are: First, the development of suitable standards of measurement; second, the reproduction or simulation in the laboratory of the essential physical conditions experienced in flight. The question of standards will be found discussed in each of the subsequent papers.

The most significant conditions experienced in flight are these five:

1. Extreme change of temperature;
2. Change of pressure or density;
3. Inclination or acceleration;
4. Vibration;
5. Time elapsed during flight.

Practically all instruments are liable to be influenced by change of temperature. For the testing of completed instruments it is convenient to control the temperature of a confined body of air, inside of which the instrument will then be operated and tested in the usual manner. But for preliminary testing and adjustment in the factory time can be saved by immersing the mechanism alone in a liquid bath before assembling. For eliminating very imperfect instruments it is sufficient to secure the desired temperature change by heating. But for accurate results cooling is likewise necessary, for some instruments give sharply parabolic temperature curves, showing fair compensation in the warm region but sloping off steeply toward the cold.

Change of pressure and density can be regulated in the laboratory by placing the instrument in some air-tight container, connected to an insulated air chamber of large volume in which the requisite vacuum is maintained. Expedients necessary for operating most of the instruments under reduced pressure are described in the subsequent papers, while such observations on air speed nozzles have already been presented in Report No. 110, The Altitude Effect on Air Speed Indicators, published by the National Advisory Committee for Aeronautics (Sixth Annual Report).

The effects of inclination and of linear acceleration are equivalent, because either is equivalent to a change in the effective component of gravity. Linear acceleration errors equivalent to a decrease of gravity can be reproduced in the laboratory then by tilting or inverting the instrument. Accelerations corresponding to an increase of gravity may be realized by the use

of a whirling table. This has been done in connection with the laboratory testing of accelerometers and gyro turn indicators.

The vibrations of an airplane instrument board have been observed to consist mainly of torsional vibrations in the plane of the board. They can be reproduced in the laboratory by the elastic suspension of a structure carrying an unbalanced motor. Such devices, capable of regulation, are described in Report No. 126, Part I; Report No. 127, Part II; and Report No. 129, Part II. Vibrations may obscure the reading, shift the reading, or cause progressive changes in the mechanism of an instrument. Most instruments also show less hysteresis (lag) under vibration than when tested on a stationary support. Such tests are particularly important with new types of instruments and should be run for an extended period.

The absolute amount of time elapsed during any given part of a flight is an essential factor when the instrument is subject to irreversible effects, such as temperature lag or elastic lag. For the exact determination of the errors of an instrument in such cases, the same variation of temperature and instrument reading from moment to moment which occurred during the flight must be reproduced in the laboratory. This is known as a flight-history test, and although difficult, has been found necessary in establishing competitive altitude records. Short-cut methods for discovering the magnitude of these irreversible effects in instruments based on elastic action are the observation of drift, hysteresis, or after-effect. These tests are explained in the paper on altimeters and barographs, Part I of Report No. 126, but are equally applicable in principle to other instruments.

Besides the foregoing physical problems, an interesting field is offered for efficiency engineering in the development of appliances for testing a great quantity of instruments simultaneously, and in perfecting the procedure for recording and computing observations with a minimum pay-roll.

PROBLEMS OF INSTALLATION.

The problems of installation are hardly less important than those relating to the design of the instruments themselves, because a good instrument can be made ineffective by improper location or faulty connections.

The force element of typical instruments, referred to above under the head of mechanical design, consists of two parts, the collecting or receiving element and some elastic system. In general these two parts are connected by long-distance transmissions (wires, tubing, or shafting), but in particular cases the two parts are consolidated. Table III shows the usual arrangement of the receiving and indicating elements for the more familiar instruments. The modern tendency is toward the separated type, particularly with large aircraft.

TABLE III.—*Installation of receiving and indicating elements.*

Consolidated.		Separated.
In cockpit.	Remote.	
Altimeters and barographs. Statoscopes and rate-of-climb indicators. Aerographic instruments. Inclinometers and banking indicators. Most compasses and turn indicators. Timepieces.	Proposed barograph with static collector. Strut thermometer. Pensuti and similar air speed indicators. Morell anemometer. Certain compasses. Gasoline float gauges. Wind driven turn indicator.	Proposed altimeter with static collector. Vertimeter. Pressure-head air speed indicators. Angle of incidence, side slip and yaw meters. Bamberg remote indicating compass. Flexible shaft and electric tachometers. Radiator thermometers. Hydrostatic and electric gasoline gauges. Static head turn indicator.

Problems of installation can be investigated from the standpoint of the receiving element, the long-distance transmission, and the indicating element. Compasses, for example, may be in serious error unless the magnetic element can be separated and placed sufficiently far away from the engine; likewise the pressure head of an air-speed indicator might as well be left off entirely as to be installed too close to the propeller slip stream. These difficulties have been analyzed by Prof. S. Herbert Anderson, in a recent discussion of aeronautic instruments.[1]

[1] Trans. Am. Soc. Mech. Eng., *42*: 107–109, 1920.

Again the difficulties of long-distance transmissions are the tendencies toward time lag, excessive weight, and breakage. The breakage difficulty was particularly noticed in France with radiator thermometer capillaries. A great proportion of the planes sent back from the front for salvage at Romorantin were found to have these broken transmissions. It was concluded that in many cases the breakage occurred through delicacy of the fine metal tubing, but that in numerous other cases the mechanics had cut the tubes as one would cut wire with the intention of soldering them together again. Breakage of tachometer shafts occurred through stresses caused by sudden twisting. Air-speed indicator tubing proved difficult to keep intact, while it is well known that the hydrostatic long-distance gasoline gauges were abandoned on combat planes because of fire risk and loss of gasoline caused by exposure of the tubing to gunfire.

The study of best location for indicating elements leads to the general question of instrument board design. The instruments must be so placed, without mutual interference, as to offer the greatest convenience for the pilot. This is not easy with multiple engine planes where each of the power-plant instruments has to be repeated from two to four times. The problem is still more difficult in relation to visibility at night. Three plans have been tried for artificial illumination of instruments. One of the earliest was to provide small incandescent lamps underneath translucent dials. The plan most extensively followed during the early part of the war consisted in the use of luminous paint. The requirements for satisfactory illumination by this process are complex and have been fully investigated by Dr. N. E. Dorsey.[2] There was a tendency to provide too great luminosity, which was found objectionable by many pilots and night bombers. For observation of the landscape the natural sensitivity of the eye at night has to be relied upon, and this was destroyed by the glare from the luminous paint, even in the case of dials seemingly capable of giving out only the very faintest glow. The demand consequently arose for small shielded incandescent lamps adjacent to the respective instruments, which could be put on and off at will.

Many further problems have been considered regarding instrument board design, such as the development of antivibration boards; the construction of a curved instrument board with dials normal to the line of sight; while the possibility of thermostatically maintaining a constant temperature for the instruments by the use of a circulating fluid bath has been considered as a means of eliminating temperature errors. This last expedient might have had the advantage during the war of speeding up the supply of instruments by eliminating the test for temperature compensation.

In general the further investigation of installation improvements is very much to be desired.

PROBLEMS CONCERNING THE USE OF INSTRUMENTS.

For best results in the practical use of instruments, attention must be given to their proper adjustment and care, to the application of the necessary corrections, and to the proper interpretation of observations.

Instruments must be adjusted where possible to read zero when not in action. This is important especially with altimeters at the start of a flight; otherwise the altimeter will be no help in making a landing. Instruments which are not adjustable should be read at the start of a flight. For this reason instruments provided with stops, as is very common with pressure gauges, are objectionable. In such cases there is no way to tell, before operating an instrument over its scale, whether it is in working order or not. Instruments should be tapped before starting to see that the pointer swings freely, and all connections looked over, tubes being tested to see that they are not plugged. Such details will be brought out in the separate papers, but it may be stated in general that the most important precaution to secure accuracy is to make sure that the instrument in use has been recently given an authoritative laboratory test.

[2] Report No. 33, National Advisory Committee for Aeronautics, 4th Annual Report.

Corrections may be divided into two classes, theoretical and instrumental. The theoretical correction is the amount which would still have to be applied even if the instrument were mechanically perfect. For example, the usual altimeter is subject to correction for the temperature of the atmosphere, an item which is oftentimes forgotten though it may readily amount to a thousand feet or more. Similarly the air speed meter has to be corrected for the density of the atmosphere and the compass for the departure of magnetic north from true north. Instrumental corrections, on the other hand, have to be discovered by laboratory test in comparison with a suitable standard. The instrumental corrections might well be noted down in tabular form on a small card placed beside each instrument in front of the pilot. This practice is already quite common with compasses.

The interpretation of observations, after the necessary corrections have been applied, depends on the object sought; this subject is quite fully discussed in the paper on navigating instruments, Report No. 131.

PROBLEMS OF MAINTENANCE.

Instruments which are satisfactory when used the first time may not remain so indefinitely. Attention must be given to questions of inspection, calibration, readjustment, and salvage.

The instrument equipment of aircraft should be regularly inspected by experts who are able to tell at a glance whether anything is wrong. If any derangement is discovered, the instrument should be plainly marked, "Out of order," and, of course, replaced when possible. Some of the defects which may be discovered by ground inspection are the following: Failure of pointer to read zero; failure to vibrate readily when vigorously tapped; bent or loose pointer; dial in wrong position; cover glass cracked; illumination out of order; connections broken; leakage from liquid-filled instruments.

In addition to the periodical inspection, instruments should be specially inspected after a crash or transportation, and before starting on any important flight. And besides such records, obtained by regular inspectors, a record may profitably be kept of troubles reported by the flyers themselves. These reports will furnish raw material for important scientific study in the future, regardless of whether the troubles reported are genuine or only psychological.

At long intervals instruments should be detached and tested in the laboratory as a matter of precaution, even if no troubles have been reported either by the pilots or inspectors. The reason for this is that serious errors may accompany the full scale deflection of an instrument, even though nothing is visibly wrong on the ground. Such changes of calibration often arise from deterioration due to continuous vibration, the presence of dust or rust (the latter especially on seaplanes), and even from true progressive changes in the quality of the constituent material. These last, which are known as secular changes, may become apparent several years after the instrument has been manufactured; they have been demonstrated to occur not only in materials like fabric and rubber, but also in metallic diaphragms and steel springs.

Such calibration tests need not be as elaborate as those originally given. It is usually sufficient to determine the correction of the instrument at two points—the zero point and one point near the end of the working range. The temperature test, if it has really been made once, need not be repeated on this occasion. From the two readings taken an approximate percentage correction factor is available, although an average factor, based on a complete set of observations, is still better.

Unless the percentage correction is small, the instrument should be readjusted. This may at times be done sufficiently well by merely shifting the pointer; at other times it will be necessary to alter the movement internally, which should not be undertaken without expert knowledge. If only a small correction has been found, it is better to let it go or issue a correction card, for small mechanical readjustments are liable to slip back again or otherwise change during the next flight.

Instruments which are broken need not be discarded. If the original manufacturer is unable to handle repair business, the service should maintain some central salvaging station, equipped with expert personnel and facilities for such work. This was found to be a very

necessary function of the supply services in France. American instruments were salvaged at Colombey-les-belles and at Romorantin; British instruments were sent back to Farnborough, where at the Royal Aircraft Establishment instrument salvaging was done on a very large scale in the most scientific manner.

The salvaging problem consists of taking a certain number of damaged instruments and reassembling them to provide a smaller number of good instruments. The percentage of instruments redeemed will, of course, be increased if, in addition to the wrecked instruments, spare parts from the factory are also kept on hand. The readjustment of the resulting instruments, including temperature compensation, forms an important part of the salvage problem. A further study of scientific expedients for quickly making the necessary adjustment and preserving a uniform scale ought to be seriously taken up. Some instruments are better adapted than others for readjustment; hence since it is the destiny of every instrument to be eventually readjusted, this consideration should be given some weight in selecting instruments for purchase.

Instrument salvaging is not only one of the essential problems of maintenance of equipment, but the tabulation of the defects thus brought to light should be of great scientific value as a basis for the future improvement of instruments.

PHYSICAL RESEARCH PROBLEMS.

The foregoing general problems have been taken up more or less in chronological order. First, an instrument has to be designed, and then it has to be manufactured, and so on along the line. This sequence begins and ends with the subject of physical research. Rational design, spoken of at the beginning, is an application of whatever physical laws and data are available at the time. Instrument development can not progress indefinitely without further installments of such research results.

Among the applications of physical research discussed in the subsequent papers are the following: Design of elastic system of precision altimeter; theory of bimetallic temperature compensation; compensation for angular acceleration; determination of the minimum number of data needed for defining the behavior of instruments with respect to imperfect elasticity; development of rate-of-climb indicators with high sensitivity and small lag; theory of dynamical ground speed indicators without mechanical integration; theory of the motion of a spinning top in an airplane going around a bank; design of a new type of gyroscopic mechanism with experimental determination of the requisite data on friction; and the development of an absolute method for testing oxygen apparatus without the need of flow-metering devices. Some of the more general problems along which, in the interest of instrument development, further progress is to be desired are: The laws of solid friction; fluid dynamics; synthetic development of liquids with a minimum temperature coefficient of viscosity; development of alloys having a minimum change of elasticity with temperature; and a more complete determination of the laws of elastic hysteresis.

No field of practical work could therefore be more fascinating to the student of theoretical or experimental physics, or more completely dependent on this branch of science, than that of aeronautic instruments.

BIBLIOGRAPHY.[3]

1892–93.

Abaque pour la Détermination du Point à la Mer. MM. Favé et Bollet de L'Isle. Annales Hydrographique, 1892; and Revue Maritime, Jan., 1893.

1897.

Fleuriais Artificial Horizon. E. Caspani, Journal de Physique, 6: 229.

1900.

Ein Neues Barometer ("Luftdruckaräometer"). Karl T. Fischer, Ill. Aeron. Mitt., 4, Nr. 3, Juli 1900: 68–70; also, Physikalische Zeitschrift, 1, Nr. 37, 1900: 394.

[3] Prepared in collaboration with Mr. John A. C. Warner, foreign representative of the Bureau of Standards during 1921; the French references having been obtained by Mr. Warner through courtesy of M. A. de Grammont de Guiche and M. R. E. Badin, and the German contributions through Dr. E. Everling.

1903.

Ein Neuer Registrierapparat für Sondierballons. Nach Assmann, Ill. Aeron. Mitt., 7, Nr. 4, April 1903: 122.

1904.

Notice sur l'Horizon Gyroscopique de Fleuriais. Annales Hydrographique, 1904.

Taschenbuch zum Praktischen Gebrauch für Flugtechniker und Luftschiffer. H. W. L. Moedebeck. 2. Aufl. Berlin W., 1904, W. H. Kühl.

1905.

Vérification des Altitudes Barométriques par la visée Directe des Ballons-sondes. L. T. de Bort, Comptes-Rendus de l'Acad. d. Sci., 1905: 153–155.

1906.

Altitude, Azimuth, and Geographical Position. G. W. Littlehales. Lippincott, 1906.

Recherches sur les Instruments et les Méthodes Propres pour la Détermination du Point en Ballon. Mémoires du IIIème Congrès international d'Aéronautique, 1906.

Astronomische Ortsbestimmung im Ballon. Alfred Wegener, Ill. Aeron. Mitt., 10, Nr. 4, April 1906: 116–121; auch Astronomische Ortsbestimmungen des Nachts bei der Ballonfahrt vom 5. bis 7. April, 1906; Ebenda, Nr. 6, Juni 1906: 205–207.

1907.

Ueber eine neue Automatische Abstellvorrichtung der Schreibfedern von Registrierapparaten für Unbemannte Freiballons und eine neue Methode der Fixierung der Diagramme. R. Nimführ, Ill. Aeron. Mitt., 11, Nr. 3, März 1907: 78–81.

Die Astronomische Ortsbestimmung im Ballon und ihre Bedeutung für die Luftschiffahrt. Ad. Marcuse, Ill. Aeron. Mitt., 11, Nr. 1, Jan. 1907: 20–25.

1908.

Aneroid Barometer for Aeronauts. C. J. Asten, Airship. Lond., 37, 1908.

Ein Ballonvariometer zur Messung der Vertikalgeschwindigkeit. A. Bestelmeyer, Ill. Aeron. Mitt., 12, Nr. 21, 21. Okt. 1908: 656–657; auch Physikalische Zeitschrift, 9, 1908: 863.

Der Kreisel als Richtungsweiser auf der Erde mit Besonderer Berücksichtigung seiner Anwendbarkeit in Luftschiffen. A. F. nach Anschütz-Kämpfe, Ill. Aeron. Mitt., 12, Nr. 25, Dez. 1908: 792–793.

1909.

Thermograph zur Registrierung der Füllungstemperatur von Luftschiffen und Freiballons. K. V. Bassus, Ill. Aeron. Mitt., 13, Nr. 17, 25. Aug. 1909: 755–757.

Der Peilkompass im Freiballon. H. H. Kreitzinger, Ill. Aeron. Mitt., 13, Nr. 22, 3. Nov. 1909: 984–987.

Ueber die Magnetische Ortsbestimmung im Ballon. Bidlingmeier, Ill. Aeron. Mitt., 13, Nr. 22, 3. Nov. 1909: 975–983.

Sauerstoff- und Atmungsvorrichtungen für Luftschiffe. Werner Bleines, Ill. Aeron. Mitt., 13, Nr. 20 und 23, 6. Okt. und 17. Nov. 1909: 900–903 u. 1029–1040.

Ein Verfahren zur Auswertung Astronomischer Positions-bestimungen. A. Brill, Ill. Aeron. Mitt., 13, Nr. 22, 3. Nov. 1909: 988–994; auch Ilas-Wochenrundschau, Nr. 4.

Neuer Beitrag zur Frage der Ortsbestimmung im Ballon. Ad. Marcuse, Ill. Aeron. Mitt., 13, Nr. 22, 3. Nov. 1909: 984.

Anleitung zur Astronomischen Ortsbestimmung im Ballon. Ad. Marcuse. Berlin 1909, G. Reimer.

Ein Beitrag zur Frage der Balloninstrumente. P. A. Meckel, Ill. Aeron, Mitt., 13, Nr. 26, 29. Dez. 1909: 1151–1152

1910.

Zur Theorie des Ballonvariometers. A. Bestelmeyer, Physikalische Zeitschrift, 11, 1910: 763–768.

Ueber Statische und Dynamische Luftdruckmessungen. Böninger, Flugsport 2, Nr. 2, 22. Jan. 1910: 35–42.

Zur Benutzung des Kompasses im Ballon. A Bestelmeyer, Deutsche Zeitschrift für Luftschiffahrt, 14, Nr. 24, 30. Nov. 1910: 17–18.

Ein Peilkompass für Luftschiffe. Anon., Flug- und Motortechnik, 4, Nr. 23, 10. Dez. 1910: 929–931.

Le Point sans l'Horizon de la mer. Horizon Gyroscopique de l'Amiral Fleuriais. Revue Maritime, 1910.

Ueber eine Neue Einheitliche Methode zur Nautischen und Aeronautischen Ortsbestimmung aus Gestirnsbeobachtungen. A. Brill, Frankfurt a. M.; auch Annalen der Hydrographie und Maritimen Meteorologie, 1910. Nr. 11, und 1911. Sonderabdruck. Berlin, 1911.

Ueber Ortsbestimmung bei der Luftschiffahrt. Nach Marcuse, Flug- und Motortechnik, 4, Nr. 5, 10. März 1910: 169–170.

Ueber das Registrieren Horizontaler Luftschwankungen bei Luftschiffen und Flugapparaten. Emil Kraft v. Helmhacker, Flug- und Motortechnik, 4, Nr. 4 u. 5, 25. Febr. und 10. März 1910: 140–144 u. 170–172.

1911.

Determination of the Altitude of Aeroplanes. R. W. Willson. Proc. Am. Acad. Arts & Sci., 47: 22–43. 1911.

Barometry and Aeronautic Instruments. Annual Report of Director Bureau of Standards, 1911: p. 12; 1912: pp. 13–14; 1913: p. 8; 1914: p. 19; 1915: pp. 20–21; 1916: p. 21; 1917: pp. 27–28; 1918: pp. 125–127; 1919: pp. 174–181; 1920: pp. 167–173.

Die Mechanisch-Graphische Lösung des Höhenproblems mit dem Voigtschen Instrument. H. Boykow, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 2, Nr. 9, 13. Mai 1911: 116–120.

Die Höhenmessungen bei Flugveranstaltungen. A Schmauss, Deutsche Zeitschrift für Luftschiffahrt, 15, Nr. 6, 22. März 1911: 11–16.

Ein Verbesserter Ballonapparat, das Vertikal Doppel-Anemometer. Anon., Deutsche Zeitschrift für Luftshiffahrt, 15, Nr. 5, 7. März 1911: 22–23.

Mitteilungen des Luftschiffbau Zeppelin. I. Die Ermittelungen der Momentanen Eigengeschwindigkeiten von Luftfahrzeugen mit Hilfe der Pitotschen Röhre. A. Frhr. V. Soden und Cl. Dornier, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 2, Nr. 19, 14. Okt. 1911: 241–245.

Die Anémo-Bussole. Anon., Oesterr. Flug-Zeitschrift, 5, Nr. 1, 10. Jan. 1911: 20.

Navigation Mittels Derivators. H. Boykow, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 2, Nr. 11, 16. Juni 1911: 145–147.

Die Orientierungsfrage. Anon., und "Orion," ein neuer Auswertungsapparat für astronomische Ortsbestimmung in der Luft und auf der See. E. Wippich, Der Motorwagen, 14, Nr. 10, 10. April 1911: 224–227 u. 227–228.

Instrument zur Graphischen Auswertung Astronomischer Positionsbestimmungen nach der Standlinienmethode. A. Brill, Luftschiffahrt, Flugtechnik und Sport, Nr. 24, 31. August 1911; auch Deutsche Zeitschrift für Luftschiffahrt, 15, Nr. 17, 23. Aug. 1911: 18–21.

Ein Beitrag zur Astronomischen Ortsbestimmung mit dem Voigtschen Instrument. W. Hormel, Deutsche Zeitschrift für Luftschiffahrt, 15, Nr. 5, 7, März 1911: 14–18.

Astronomische Ortsbestimmung im Luftfahrzeug. O. Voight, Vortrag vom. 14. Jan. 1911.

Astronomische Aeronavigation. Anon., Österr. Flug-Zeitschrift, 5, Nr. 15, u. 16, 10 u 25, August 1911: 403–406. u. 430–432.

Bemerkungen zur Astronomischen Ortsbestimmung im Luftfahrzeug. Marcuse, Deutsche Zeitschrift für Luftschiffahrt, 15, Nr. 24, 29. Nov. 1911: 24–25.

Praktische Flugzeug-Navigation. H. Boykow, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 2, Nr. 5 u. 6, 11. u. 24. März 1911: 66–68 u. 77–79.

Aeronavigation. Hans Boykow, Oesterr. Flug-Zeitschrift, 5, Nr. 14, 25. Juli 1911: 374–376.

The Aerial Age. Walter Wellman, A. R. Keller Co., New York.

1912.

Ballonvariometer. A. Bestelmeyer, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 25, 11. Dez. 1912: 615.

Aneroidvariometer zur Feststellung der Vertikalgeschwindigkeit im Freiballon. G. von dem Borne, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 22, 24. Okt. 1912: 538.

Ballon-Thermometer. O. Ebersbach, Der Motorwagen, 15, Nr. 33, 30. Nov. 1912: 767–768.

Geschwindigkeitsmessungen von Luftschiffen. E., Der Motorwagen, 15, Nr. 12, 30. April 1912: 314.

Theorie eines Windschreibers. G. von dem Borne, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 3, Nr. 20, 26. Okt. 1912: 261 u. 264.

Ein Einfacher Winddruckschreiber. G. von dem Borne, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 3, Nr. 14, 27. Juli 1912: 188–189.

Ueber Kompasse für Luftfahrzeuge. G. Gehlhoff, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 15, 24. Juli 1912, 369–370.

Ein Neuer Kompass für Flugzeuge. H. H. Kistzinger, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 11, 29. Mai 1912: 278–279.

Der Kompass im Flugzeug. R. Topp, Der Motorwagen, 15, Nr. 35, 20. Dez. 1912: 809–813.

Tachometer für Flugzeuge. Anon., Flugsport, 4, Nr. 22, 23. Okt. 1912: 832–833.

On Gyroscopic Oscillations. W. S. Franklin, Phys. Rev., 34: 48–52, 1912.

Ueber ein Neuer Instrument für Ermittlung der Steuerwinkels und der resultierenden Geschwindigkeit bei Flugzeugen und Ueber die Anwendung der Nomographie für Lösung Solcher Aufgeben. K. Hoetken, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 24, 27, Nov. 1912: 577–580.

Instrument zur Graphischen Auswertung Astronomischer Positions-Bestimmungen nach der Standlinien-Methode. A. Brill. Frankfurt a. M., 1912, Hartmann & Braun.

Instrument zur Auswertung von Gestirnshöhenbeobachtungen in der Karte. A. Wedemeyer, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 3, Nr. 1, 13. Jan. 1912, S. 3/5.

Astronomische Ortsbestimmmungen mit besonderer Berücksichtigung der Luftschiffahrt. W. Leick. 8°. S. 130. Leipzig, Quelle u. Meyer, 1912.

Astronomische Ortsbestimmungen mit Hilfe von Nomogrammen. W. Leick, Zeitschrift für Flugtechnick und Motorluftschiffahrt, 3, Nr. 17, 14. Sept. 1912: 230–231.

Zur Frage der Ortsbestimmung nach Fixsternen mit dem Libellenquadranten. S. v. Kolbe, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 5. 6. März 1912: 105–106.

Ein Libellenquadrant in Neuer Form für Astronomische Ortsbestimmungen (Zahnbogen-Höhenmesser). E. Hartmann, Annalen der Hydrographie und Maritimen Meteorologie, 40, Nr. 9, 1912: 486–488; auch Deutsche Luftfahrer-Zeitschrift, 16, Nr. 20, 2. Okt. 1912: 495–496.

Ein Libellenquadrant in Neuer Form für Astronomische Ortsbestimmungen. W. Lindt, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 11, 29, Mai 1912: 273–274; ferner Annalen der Hydrographie und Maritimen Meteorologie 40, Nr. 1, 1912.

Orientierung und Navigation im Luftfahrzeug. H. Boykow, Deutsche Luftfahrer-Zeitschrift, 16, Nr. 22, 24. Okt. 1912: 530–534.

Mitteilungen des Luftschiffbau Zeppelin-Friedrichshafen. V. Temperaturmessungen des Füllgases und der Luft im Z-Schiff und deren Bedeutung für die Höhennavigation. Gludd u. Fhr. V. Soden, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 3, Nr. 7, 13, April, 1912: 97–101.

1913.

The Measurement of True Static Pressure in a Moving Fluid-Application to an Aeroplane Barograph. A. F. Zahm, Journal of the Franklin Institute, v. 175, 503–509, Jan., 1913.

Aneroidvariometer zur Feststellung der Vertikalgeschwindigkeit im Balloon. G. von dem Borne, Österr. Flug-Zeitschrift, 7, Nr. 4, 25. Febr. 1913: 91.

Enregistrement de la Vitesse Relative des Aeroplanes. A. Toussaint et G. Lepére, Bull. de l'Institut Aérotcehde l'Univ. de Paris, fasc. 3, 1913.

Expériences sur l'Ajutage de venturi. A. Toussaint et G. Lepére, Bull. de l'Institut Aerotech. de l'Univ. de Paris, fasc. 3, 1913.

Der Luftgeschwindigkeitsmesser der Siemens und Halska A.-G. (Vortrag). H. Gerdien, Verhandlungen der Gesselschaft Deutscher Naturforscher und Ärzte, 85. Versammlung zu Wien, 2. Teil, 1. Hälfte, 1913: 234–238.

Preisausschreiben für einen Wettbewerb um einen Aufzeichnenden Beschleunigungsmesser für Flugzeuge. Anon., Zeitschrift für Flugtechnik und Motorluftschiffahrt, 4, Nr. 14, 26. Juli, 1913: 199–200.

The Theory of the Earth Inductor as an Inclinometer. N. E. Dorsey, Terrestrial Magnetism & Atmospheric Electricity, 18, 1913: 1–38.

Bemerkungen zu dem Bericht über Kompass-Beobachtungen im Luftschiff "Hansa." Karl Bamberg, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 4, Nr. 21, 15. Nov. 1913: 292.

Zur Dämpfungsfrage bei Luftfahrzeugkompassen. H. Boykow, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 4, Nr. 18, 27. Sept. 1913: 248–250.

Bericht über Kompassbeobachtungen im Luftschiff "Hansa." Meldau u. J. Möller, Mitteilungen des Luftschiffbau Zeppelin. VIII. Bericht über Astronomische und Magnetische Ortsbestimmungsversuche an Bord des Luftschiffes "Hansa," J. Möller, Mitteilungen des Luftschiffbau Zeppelin, IX. Zeitschrift für Flugtechnik und Motorluftschiffahrt, 4, Nr. 18, 27. Sept. 1913: 243–245 bezw. 245–246.

Ueber einen Neuer Kreiselkompass. Th. Bruger, Jahrbuch der Wissenschaftliche Gesellschaft für Flugtechnik (bis 1914) 1, 1912–13. Berlin, 2. Lief., 1913: 125–129.

Der Better Sauerstoff in Seinen Beziehungen zur Luftfahrt Dräger-Hefte. I. W. Haase-Spe, Nr. 11, Mai 1913: 81, 84–87.

Ermittelung der Navigationsdaten im Luftschiff. H. Boykow, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 4, Nr. 20, 25. Okt. 1913: 273–279.

Äusserung zu dem Aufsatz des Herrn Dr. H. H. Kritzinger zur Theorie des Libellenquadranten: V. Kobbe, Deutsche Luftfahrer-Zeitschrift, 17, Nr. 10, 14. Mai 1913: 236.

Zur Theorie des Libellenquadranten. H. H. Kritzinger, Deutsche Luftfahrer-Zeitschrift, 17, Nr. 4, u. 10, 19. Febr. u. 14. Mai 1913: 85–87 u. 236.

Libellenhorizont und Libellensextant. K. Schwarzschild, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 4, Nr. 13, 12. Juli 1913: 177–180.

Sternzeit-Transformator. A. Schütze, Annalen der Hydrographie und Maritimen Meteorologie, Febr. 1913.

Ort- und Zeitbestimmungen. Grosse, Deutsche Luftfahrer-Zeitschrift, 17, Nr. 10, 14. Mai 1913: 235–236.

Wissenschaftliche Instrumente. Anon., Deutsche Luftfahrer-Zeitschrift, 17, Nr. 5, 5. März 1913: 103–114.

Eine Ausstellung von Messapparaten. R. Wachsmuth, Jahrbuch der Wissenschaftliche Gesellschaft für Flugtechnik (bis 1914) 1, 1912–13. Berlin, 2. Lief., 1913: 133–151.

Physikalische Untersuchungen im Freiballon I. A. Wigand u. G. Lutze, unter Mitarbeit von E. Everling, und G. Jenrich, Abhandlungen der Naturforschenden Gesellschaft. Halle a. S., n. F., Nr. 2, 1913: 39.

Luftschiffnavigation im Nebel. H. Boykow, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 4, Nr. 15, 16. Aug. 1913: 206–208.

1914.

The Testing of Barometers. Bureau of Standards Circular No. 46, 1914: 12.

Ein Fluggeschwindigkeitsmesser mit Luftdruck und Temperaturkompensation. G. Fuhrmann, Jahrbuch der Deutsche Versuchsanstalt für Luftfahrt, 1, 1912–13. Berlin, 1914: 232–236.

Ein Apparat zur Untersuchung des Windstruktur (Anemoklinograph) der Siemens und Halska A.-G. H. Gerdien, Jahrbuch der Wissenschaftliche Gesellschaft für Flugtechnik (bis 1914) 2, 1913–14. Berlin, 1914, 1. Lief., 67–78; Diskussion: 79–80.

Preisausschreiben für einen Wettbewerb um einen Aufzeichnenden Beschleunigungsmesser für Flugzeuge. (Reissner), Jahrbuch der Wissenschaftliche Gesellschaft für Flugtechnik (bis 1914), 1913–14. Berlin, 1. Lief., 1914: 31–34.

Discussion of the Action of Different Types of Air Speed Indicator. L. Bairstow, British Adv. Com. for Aero., 1913–14: 461–466.

Mesures sur Aéroplanes en Vol au Moyen d'Appareils Enregistreurs. A. Toussaint, G. Lepére & Gouin, La Technique Aéronautique, Mars 1, 15, et Avril 1, 1914.

Recherches Expérimentals sur les Moulinets Anémométriques. M. C. E. Brazier, Annales du Bureau Central Météorologique de France (mémoires).

Neue Aufzeichende Kraftmessgeräte und einige Messungen im Flugzeug. W. Hoff u. F. Bendemann, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 5, Nr. 1. u. 2, 17, u. 31. Jan. 1914: 3–5 u. 17–22.

Flugrichtungskompass, System Wäller. Gustav E. Macholz, Österr. Flug-Zeitschrift, 8, Nr. 3–4, Febr. 1914: 42–43.

Gyrostats and Gyrostatic Action. A. Gray, Smithsonian Report for 1914: 193–208 (Publication 2327).

Report on Gyroscopic Theory. Greenhill, British Adv. Com. for Aero. Reports and Memos. No. 146, 1914: 277.

Der Ripograph. R. Katzmayr, Österr. Flug-Zeitschrift, 8, Nr. 22–23, 1. Dez. 1914: 360–361.

Physikalische Untersuchungen im Freiballon II. G. Lutze u. E. Everling, unter Mitarbeit von A. Wigand u. G. Jenrich, Abhandlungen der Naturforschenden Gesellschaft. Halle a. S., n. F., Nr. 3, 1914: 79.

Bericht des Ausschusses für Messwesen. Jahrbuch der Wissenschaftliche Gesellschaft für Flugtechnik (bis 1914) 2, 3. Lief.,1913–14: 223–236.

Kritischer Bericht des Prüfungsausschusses zur Beurteilung von Erfindungen. v. Parseval, Jahrbuch der Wissenschaftliche Gesellschaft für Flugtechnik (bis 1914) 2, 1913–14. Berlin, 3. Lief., 1914: 193–195.

1915–16.

Aneroid Barometer Investigations. M. D. Hersey, Phys. Rev., 6, 1915: 75–76.

Investigation of Pitot Tubes. W. H. Herschel (Pt. I) and E. Buckingham (Pt. II), Report No. 2, Nat. Adv. Com. for Aero., First Annual Report, 1915: 77–110.

Kompasstörungen im Nebel. Wilhelm Krebs, Der Motorwagen, 18, Nr. 1, 10. Jan. 1915: 12–13.

Report on the Accuracy with which the Temperature Errors in Determining Heights by the Barometer may be Corrected. G. I. Taylor, British Adv. Com. for Aero., 1915–16: 576–578.

Calibration of Tube Anemometers at High Velocities. J. R. Pannell, British Adv. Com. for Aero., 1915–16: 529–533.

On some Proposed Electrical Methods of Recording Gas Flow in Channels and Pipes Based on the Linear Hot Wire Anemometer. L. V. King, Journal Franklin Institute, Aug. 1916: 191.

Geschwindigkeitsmesser für Flugzeuge. Anon., Mitteilungen des K. K. Österr. Aero-Clubs 1916; abgedruckt im Bulletin des Schweiz. Aero-Clubs, Okt. 1916: 152–154.

Zur Geschwindigkeitsmessung im Luftfahrzeug. Anon., Bulletin des Schweiz. Aero-Clubs, Nov. 1916: 162.

The Magnetic Compass for Air Navigation. Creagh-Osborne. Glasgow, James Brown & Son.

Report on the Errors of Compasses on Aeroplanes. K. Lucas, British Adv. Com. for Aero., 1915–16: 554–566.

Compass Deviation in Aircraft, British Adv. Com. for Aero., 1915–16: 533–536.

Compass Deviation Due to Vibration and Friction. A. Mallco, British Adv. Com. for Aero., 1915–16: 536–538.

Compass Deviation Due to Vibration Without Friction. G. Greenhill, British Adv. Com. for Aero., 1915–16: 539–553.

On a New Type of Magnetic Compass for Use on Aeroplanes. K. Lucas, British Adv. Com. for Aero., 1915–16; 567–575.

Elektrisches Fernthermometer zur Messung der Kühlwasser-Temperatur in Explosionsmotoren, in Flug-Zeugen und Automobilen. Otto Hauser, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 7, Nr. 7–8, 29. April 1916: 49–50.

Eine Vereinfachte Atmungsmaske für Flugzeuge. E. Everling. Deutsche Luftfahrer-Zeitschrift 20, Nr. 3–4, Febr. 1916: 35–36.

The Oscillations of an Aeroplane in Flight. K. Lucas, British Adv. Com. for Aero., Report, 1915–16: 267–277.

General Specifications for Aeronautic Instruments. Report No. 8, Nat. Adv. Com. for Aeronautics, Second Annual Report, 1916: 25–28.

1917.

Aerial Navigation over Water. Elmer A. Sperry, Trans. Soc. Automotive Engs., Sec. 1, 1917: 153.

Creep Errors in Altimeters Due to Hysteresis. T. G. Hull, British Adv. Com. for Aero., Report, 1916–17: 668–670.

A Barothermograph for Use on Aeroplanes. Aeroplane Testing Squadron, British Adv. Com. for Aero., Report, 1916–17: 673–674.

A Rate-of-Climb Indicator. Commanding Officer, Testing Squadron, Royal Flying Corps, British Adv. Com. for Aero., Report, 1916–17: 699–701.

A Method of Measuring the Speed of an Aeroplane at a Height. Commandant, Central Flying Station, British Adv. Com. for Aero., 1916–17: 744–751.

Die Messung von Flugzeuggeschwindigkeiten. E. Jacobi. Technische Berichte der Flugzeugmeisterei bezwi II, Nr. 1, 20. Dez. 1917: 99–110.

Behaviour of Levels when Subject to Vibration. A. Mallock, British Adv. Com. for Aero., Report, 1916–17: 671–672.

The Northerly Turning Error of the Magnetic Compass. F. A. Lindemann, British Adv. Com. for Aero., Report, 1916–17: 678–698.

Turn Indicators for Aeroplanes. F. A. Lindemann, British Adv. Com. for Aero., Reports and Memoranda, No. 525, Dec. 1917: 1–8.

Tachometer mit Ablesevorrichtung. Wilke, Zeitschrift des Vereines deutscher Ingenieure, 61, 31. März 1917: 297.

A Petrol Flowmeter for use on Aeroplanes. Aeroplane Testing Squadron, British Adv. Com. for Aero., Report; 1916–17: 675–677.

Flossendruckmessungen. E. Everling, M. Iklé und E. Sieber, Technische Berichte der Flugzeugmeisterei bezw. der Inspektion der Fliegertruppen I, Nr. 6, 15. Okt. 1917: 302–312.

Vorläufiges Ergebnis der Versuchsflüge mit der Messnabe. E. Everling, Technische Berichte der Flugzeugmeisterei bezw. der Inspektion der Fliegertruppen I, Nr. 2, 1. April 1917: 54–56.

Altitude Azimuth and Hour Angle Diagram. G. W. Littlehales, Proceedings U. S. Naval Inst., Nov. 1917.

Die Physikalischen Grundlagen der Höhennavigation. K. Bassus. München, 1917.

Das Verhalten des Flugzeuges im Fluge. E. Everling und W. Zabel, Technische Berichte der Flugzeugmeisterei bezw. der Inspektion der Fliegertruppen I, Nr. 6, 15. Okt. 1917: 313–318.

Die Messung von Flugleistungen in England. E. Everling, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 8, Nr. 19–20, 21–22 und 23–24, 27. Okt., 29. Nov. u. 29. Dez. 1917: 154–155, 163–166, u. 182–185.

Das Wesen und die Eigenschaften der Radioaktiven Leuchtfarben. P. Ludewig, Österr. Flug-Zeitschrift, 11, Nr. 1–2, Jan. 1917: 23–24.

Die Skalenberechnung zum Deutschen Bombenvisier für Flugzeuge. O. Prochnow, Technische Berichte der Flugzeugmeisterei bezw. der Inspektion der Fliegertruppen II, Nr. 1, 20. Dez. 1917: 135–141.

1918.

Altimètres à Cadrans et Baromètres Enregistreurs. Gourdou, Bull. du S. T. Aéron. No. 4, Septembre.

Fehler in den Angaben von Aneroidbarometern. Wilh. Schmidt, Österr. Flug-Zeitschrift 12, Nr. 15–16, Aug. 1918: 170–171.

Tests of Swivelling Pressure Heads. Aerodynamics Staff of R. A. E., British Adv. Com. for Aero., Reports and Memoranda No. 567, Oct., 1918: 1–4.

A Hot-Wire Anemometer which is Sensitive over a Large Range of Wind Speed. A. Fage, British Adv. Com. for Aero., Reports and Memoranda No. 556, Nov., 1918: 1–8.

Development of Air Speed Nozzles. A. F. Zahm, Report No. 31, Nat. Adv. Com. for Aeronautics, Fourth Annual Report, 1918: 322–334.

Neuere Geschwindigkeitsmessungen. V. Heidelberg und A. Hölzel, Technische Berichte der Flugzeugmeisterei bezw. der Inspektion der Fliegertruppen III, Nr. 5, 1918: 174–179.

Ein Neues Instrument zur Geschwindigkeitsmessung auf Flugzeugen. W. Hort, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 9. Nr. 11–12, 28. Juni 1918: 67–71.

Flugzeugkompasswesen und Flugsteuerkunde. W. Immler, Berlin-Charlottenburg, 2, 1918.

Der Flugzeugkompass. Anon., aus "Ueberall," Ill. Zeitschrift für Heer und Marine, Juli 1918; Die Deutsche Luftwaffe, Nr. 10, 1918: 833–839.

Drehzahl- und Fahrtfernanzeiger für Flugzeuge und Luftschiffe. Wilke, Der Motorwagen 21, Nr. 34–36, 10. u. 31, Dez. 1918: 461–463 u. 492–494.

Gyrostatics and Rotational Motion. Andrew Gray, MacMillan & Co., Ltd., London, 1918: 530.

Untersuchungen über Fliehkraft-Tachometer nach dem Drehpendelprinzip. Wilke, Zeitschrift des Vereines deutscher Ingenieure, 62, 16. u. 23. Nov. 1918: 801 u. 829.

Vorrichtung zum Eichen von Aspiratoren auf ihre Fördermenge. Schmdt, Zeitschrift für Instrumentenkunde, 38.-Nov., 1918: 180–181.

L'Inhalation d 'Oxygène dans les Vols à Haute Altitude. Garsaux, Bull. de la S. T. de l'Aéro. Militaire, No. 2, Juillet.

Navigation und Seemanschaft im Seeflugzeug. Th. E. Sönnichsen. Berlin W., 62, R. C. Schmidt u. Co., 1918.

The Search for Instrumental Means to Enable Navigators to Observe the Altitude of a Celestial Body when the Horizon is not Visible. Proceedings U. S. Naval Institute, v. 44, No. 8, Aug., 1918.

Altitude Azimuth and Line of Position Tables. U. S. Hydrographic Office Publication No. 200.

The Chart as a Means of Finding Geographical Position by Observations of Celestial Bodies in Aerial and Marine Navigation. G. W. Littlehales, Proc. U. S. Naval Inst., Mar., 1918.

Simplified Navigation for Ships and Aircraft. C. L. Poor. New York: Century Co. (126 pp.+), 1918.

Die Messnabe für Flugzeuge. W. Steiber, Technische Berichte der Flugzeugmeisterei bezw. der Inspektion der Fliegertruppen III, Nr. 6, 1918: 221–229.

"La Nature" Bordinstrumente der Deutschen Flugzeuge. Lefranc, Flugsport 10, Nr. 23, 6. Nov. 1918: 553–560.

Ueber Flugtechnische Instrumente. K. Bennewitz, Technische Berichte der Flugzeugmeisterei bezw. der Inspektion der Fliegertruppen III, Nr. 5, 1918: 160–165.

Free Flight Investigations. Annual Report of Director, Bureau of Standards, 1918: 128.

Self-Luminous Materials. N. E. Dorsey, Natl. Adv. Com. for Aeronautics, Fourth Annual Report, 1918: 339–349.

1919.

Les Altimètres. L. Condroyer, Bulletin, S. T. Aé., 19 Février 1919.

Altimètre à pression Statique. Goffin, "Aeronautics," Sept. 19.

Atmosphère Standard. Toussaint, L'Aéronautique, October.

Ueber Aneroide. E. Warburg und W. Heuse, Zeitschrift für Instrumentenkunde, Febr. 1919: 41; abstracted in Technical Note No. 72, N. A. C. A.

Die Wahre Neigung von Flugzeugen. E. Everling, Der Motorwagen 22, Nr. 28, 10. Okt. 1919: 531–533.

Ueber ein Neues Instrument zum Bestimmen der Vertikalgeschwindigkeit. A. Gockel, Bulletin des Schweiz. Aero-Clubs, 13, 1919: 101–102

The Relation of Wind to Distribution of Barometric Pressure. Sir Napier Shaw, Manual of Meteorology, part IV.

Der Höhenregulator. Popper-Lynkeus u. Löwy, Physikalische Zeitschrift, 20, 1. Okt. 1919: 433–439.

Indicateurs de Vitesse Relative. L. Condroyer, Bull. de la S. T. Aé. Militaire, No. 28, Juin.

General Investigation of Air Speed Indicators. F. L. Hunt, Technology Press, June, 1919 (abstract of investigations submitted: 1–5).

Notes on the Design of a Recording Three-Dimensional Accelerometer for Use in Aeroplanes. G. F. C. Searle, British Adv. Com. for Aero. Reports and Memoranda, No. 627: 1–16, June, 1919.

Ueber die Bestimmung der Inklination mit dem Induktions Inklinator. W. Uljanin, Terrestrial Magnetism, 24: 113–117. 1919.

Zwei Staugeräte. Holm. Physikalische Zeitschrift, 20, 15. Jan. 1919: 27–30.

Untersuchung eines Staugerätes Englischer Herkünft. R. Katzmayr, Oesterr. Flug-Zeitschrift, 13, Nr. 2, Febr. 1919: 25–27.

Bemerkungen zum Artikel: Zur Störungstheorie des Kreiselpendels von R. Grammel. H. Boykow, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 10, Nr. 11–12, 28. Juni 1919: 124–125. Erwiderung dazu. R. Grammel, Ebenda: 125.

Zur Störungstheorie des Kreiselpendels. R. Grammel, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 10, Nr. 1–2, 25 Jan. 1919: 1–12.

Ueber Geodätische Anwendungen des Kreisels. Alfred Lechner, Oesterr. Flug-Zeitschrift, 13, Nr. 1, Jan. 1919: 5–10.

Technische Anwendungen der Kreiselbewegung. H. Lorenz, Erweiterter Sonderabdruck aus der Zeitschrift des Vereines deutscher Ingenieure, 63, 6. u. 13. Dez. 1919: 64, 17. Jan. 1920: 1224–1231, 1250–1257 u. 68–69.

Der Kreisel als Lotse im Luftmeer. A. Neuburger, Motor, März, April 1919: 135 u. 203; auch Deutsche Optische Wochenschrift, Nr. 43–44; 1919: 309–312.

Registrierender Flüssigkeitskompass als Fahrtrichtungs-Kontroll-Apparat. Anon., Flugsport, 11, Nr. 19, 17. Sept. 1919: 675–676.

General Investigation of Oxygen Instruments. F. L. Hunt, Technology Press, June, 1919 (abstract of investigations submitted: 6–8).

Der Dräger-Sauerstoff-Apparat für Höhenflüge. C. W. Vogelsang, Automobil- und Flugverkehr, 24 Sept. 1919: 239–240.

Richtungsbestimungen im Luftfahrzeug mittels Drahtloser Telegraphie. Thieme, Luftfahrt, 23, Febr. 1919: 21.

Calculations in Aerial Photography. L. P. Clerc, British Journal of Photography, 66, 1919: 295–299.

Report on the Navigation of Aircraft by Sextant Observations. H. N. Russell, Proc. Astron. Soc. Pacific, 31, 1919: 129–149.

Le Point Estimé en Navigation Aérienne. L. Dunoyer, l'Aéronautique, September: 133.

Variation in Direction of Propagation of Long Electro-Magnetic Waves. Lt. Comdr. A. Hoyt Taylor, U. S. N. R F., Bureau of Standards Scientific Paper No. 353.

Principles of Radio Transmission and Reception with Antenna and Coil Aerials. J. H. Dellinger, Bureau of Standards Scientific Paper No. 354.

Wireless Navigation for Aircraft. J. Robinson, Electrician, vol. 83, Oct. 1919: 420.

Die Messnabe für Schrauben- und Motorprüfungen im Fluge. O. Enoch, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 10, Nr. 19, 15. Okt. 1919: 208–215.

Die Messnabe und das Schwungrad des Motors. E. Everling, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 10, Nr. 17–18, 27. Sept. 1919: 181–182.

Spannseildehnungsmesser. R. Katzmayr, Oesterr. Flug-Zeitschrift, 13, Nr. 9, Sept. 1919: 142–143.

Instrumente. F. Schieferl, Oesterr. Flug-Zeitschrift, 13, Nr. 5, Mai 1919: 65–69.

Instrumentenkunde des Fliegers. R. Thebis u. G. Römmler, Berlin W., 1919.

Eine Methode zur Messung der Sicht. A. Wigand, Physikalische Zeitschrift, 20, 1. April 1919: 151–160; auch, Annalen der Hydrographie und maritimen Meteorologie, Mai–Juni 1919: 134–140.

Les Méthodes et les Calculs d'Essais des Avions Nouveaux. Bull. de la S. T. de l'Aéro. Militaire, No. 26, Mai 1919.

Aircraft Incidence Meter. A. F. Zahm, Aeronautics, Nov. 12, 1919.

1920.

The Altitude Effect on Air Speed Indicators. M. D. Hersey, F. L. Hunt, and H. N. Eaton, Report No. 110 (27 pp.), preprint from Sixth Annual Report, Nat. Adv. Com. for Aero., 1920.

Sur la Vitesse du Vent aux Différentes Altitudes. Toussaint, L'Aéronautique, December.

Mitteilungen über Fortschritte an Meteorologischen Beobachtungsinstrumenten. Georgi, Jahrbuch der Wissenschaftliche Gesellschaft für Luftfahrt (siet 1914) 5, 1920, Berlin 1920: 35-36. (Kurzer Auszug.)

Instrumente für Geschwindigkeits- bezw. Volumenmessung Gasförmiger und Tropfbarflussiger Körper. P. Biber, Zeitschrift für Feinmechanik, 28, 1919; 29, 1920: 71–74, 81–84, 89–91, 97–98, 105–107, 113–114, 120–122, 129–131, 136–137, 146–147, 152–154, 161–163, 169–171, 178–180, 184–185, 3–5, 13–14. (Nr. 1 und 2); auch als Sonderdruck, Berlin, Nikolassee, 1921.

Die Wildsche Windmesstafel. Georgi, Flugsport, 12, 29. Sept. 1920: 460–461; ferner, 27. Okt.: 512.

Ueber einen neuen Luftgeschwindigkeitsmesser (I). H. Gerdien u. R. Holm, Veröffentlichungen aus dem Siemens-Konzern 1, Nr. 1, 1920: 107–121.

Ueber die Nacheilung der Zwangläufigen Geschwindigkeitsmesser. W. Hort, Zeitschrift für technische Physik, Nr. 11, 1920: 243-246.

Acceleration in Flight. F. H. Norton and E. T. Allen, Report No. 99, preprint from Sixth Ann. Rep. Nat., Adv. Com. for Aero., 1920.

Special Inclinometers and Indicators. Prepared in the Office of the Director of Air Service, Aviation and Aircraft Journal, 9, 1920: 322.

Was Muss man Wissen, um einen Luftfahrerkompass zu Kompensieren. H. Wilke, Schweizer Luftfahrt 2, 10. April 1920.

Der Fernkompass. W. Friedensburg, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 11, Nr. 15, 15. Aug. 1920: 217–220.

Le Compas de Navigation. L'Aéronautique, Féyrier.

Indicateurs de Virage. Aeronautics, Flight (1919).

Der Kreisel im Flugzeug. H. Boykow, Der Luftweg, 4, Nr. 36/27, 23. Sept. 1920: 7–8.

Neuzeitliche Bordgeräte zur Flugorientierung. Drexler (Bericht über Vortrag), Nachrichten für Luftfahrer, 1, Nr. 4, 29. Okt. 1920: 70–71.

Der Kreisel, seine Theorie und seine Anwendungen. R. Grammel, Braunschweig, 1920.

Flugzeug-Steurzeiger. A. Neuburger, Verkehrstechnik, 1, 5. Febr. 1920: 51–53.

Zur Störungstheorie des Kreiselpendels. M. Schuler, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 11, 15, Mai 1920: 129.

Der Drexler-Steuranzeiger, ein Unentbehrliches Kreiselgerät für das Fliegen in Wolken, bei Nacht und Nebel. Wertheim, Flugsport, 12, 3. März 1920: 104–114.

Benzinstandmesser. Anon., Ill, Flug-Welt, 2, Nr. 4, 18. Febr. 1920: 116.

Flugzeug-Kontrolluhr (DRP 324–284). Anon., Industrie und Handelszeitung, 3. Okt. 1920.

Le cercle Calculateur de Routes et de Vitesses. Duval, L'Aéronautique, Sept.

Le Correcteur de Route. Lt. de vaisseau Le Prieur, L'Aéronautique, No. 18, Nov.–Dec. 1920: 227.

Le Derivomètre Brion. L'Aéronautique, Mars 1920.

Nouvelle Méthode de Navigation Aérienne á l'Estime. Y. Le Prieur, L'Aéronautique, Décembre.

Traité Pratique de Navigation Aérienne. L. Hébrard et A. B. Duval, L'Aéronautique, 1920–21.

Die Instrumentellen Hilfsmittel des Fliegens. H. Boykow (Bericht über Vortrag), Nachrichten für Luftfahrer, 1, Nr. 8, 26. Nov. 1920: 122.

Messverfahren zur Untersuchung von Lüftanlagen auf Schiffen. Freudenthal, Zeitschrift des Vereines deutscher Ingenieure, 64, 15. Mai 1920: 371–374.

Efficiency of Small Bearings in Instruments of the Type Used in Aircraft. F. H. Norton, Report No. 94 (10 pp.), preprint from Sixth Ann. Rep., Nat. Adv. Com. for Aero., 1920.

Aeronautic Instruments: General Principles of Construction, Testing, and Use. M. D. Hersey (with Discussion by Prof. S. H. Anderson, Com. G. C. Westervelt, Dr. F. A. Carpenter, Prof. A. McAdie, Mr. E. A. Sperry, Lieut. A. F. Hegenberger, Rear-Adml. J. A. Hoogewerff, Prof. T. D. Cope), Trans. Am. Soc. Mech. Engs., 42, 1920: 81–118.

Design of Instruments for the Navigation of Aircraft. G. M. B. Dobson, Geographical Journal, Vol. LVI, No. 5, Nov., 1920: 370. Royal Geographical Society of London.

Methods of Measurement. L. Bairstow, Chapter III, 73–115, Applied Aerodynamics, New York; Longmans Green & Co., 1920.

A Primer of Air Navigation. H. E. Wimperis. London, Constable, 1920: 128.

Aerial Navigation. J. E. Dumbleton. London. Crosby, Lockwood & Son, 1920: 168.

Flying the Atlantic in Sixteen Hours. Sir Arthur Whitten Brown. Frederick A. Stokes Co.

The Prospective Utilization of Vessel-to-Shore Radio-Compass Bearings in Aerial Navigation. G. W. Littlehales, Journal American Society of Naval Engineers, vol. 32, No. 1, Feb., 1920.

Direction and Position Finding. H. J. Round, Journal Institution of Electrical Engineers (British), 58, Mar., 1920: 224–257.

1921.

Altimètre. Gourdou et Lesseurre, tome I, 93. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre, Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

The Testing of Aneroid Barometers at the Laboratory of the Dominion Lands Survey. Bull. 42: 1–9 (Ottawa).

Das Messen der Wahren Neigung. E. Everling, Das Weltall, 21, Nr. 9–10 u. 11–12, Febr., März 1921: 67–73.

Neigungs-und Kurvenmessung bei Flugzeugen. E. Everling, Der Motorwagen, 24, Nr. 24, 31. August 1921: 481–493.

Methodik Aerophysikalischer Flüge. A. Wigand. I. Ein Flugzeug-Meteorograph, A. Wigand und A. Wieneke, Methodik aerophysikalischer Flüge II. Die Anbringung von Meteorographen im Flugzeuge, A. Wigand, und A. Wieneke, Methodik aerophysikalischer Flüge II. Temperaturmessungen im Flugzeuge, Beiträge für Physik der freien Atmosphäre, 9, Nr. 4, 1921: 137–147, 148–162, 163–175.

Étude d'une Girouette à Applications Multiples. Louis Constantin, tome II, 91. Rapports du Premier Congrès International de la Navigation Aérienne. Paris, 15–25 Novembre; Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

On the Theory of Air Speed Indicators. Mayo Dyer Hersey, tome II, 79. Rapports du Premier Congrès International de la Navigation Aérienne. Paris, 15–25 Novembre; Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

N. A. C. A. Recording Air Speed Meter. F. H. Norton, Aerial Age Weekly, Vol. 14, No. 7: 151–154. Technical Note No. 64, N. A. C. A.

Ein Manometer für Aufzeichnung der Fluggeschwindigkeiten. C. Wieselsberger, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 12, Nr. 1, 15 Jan. 1921: 4–6.

Der Flügel-Beanspruchungsmesser von Klemperer. E. M., Der Luftweg, 5, 7. April 1921: 105–106.

Druckmessungen am Fliegenden Flugzeug. A. Pröll, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 12, Nr. 12, 30. Juni 1921: 177–180.

Neuzeitliche Bordgeräte zur Flugorientierung. F. Drexler, Berichte und Abhandlungen der Wissen schaftliche Gesellschaft für Luftfahrt (seit 1914), Nr. 4, April 1921: 14. (Kurzer Bericht.)

Anschütz Artificial Horizon. A. Gradenwitz, Aeronautics, 20, June 2, 1921: 390–391.

Les Indicateurs Gyroscopiques de Virage. Raoul Badin, tome I, 96. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre. Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

Indicateur de Virage. A. de Grammont de Guiche, tome II, 87. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre. Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

Inhalateur d'Oxygen. Gourdou et Lesseurre, tome II, 170. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre. Ed. Blondel la Rougery, Editeur, 7, rue St. Lazare, Paris.

L'Échelle Rectiligne et les Appareils à Sensibilité Constante. Barbillon, L'Aéronautique, Mars.

Pratique de la Navigation Aérienne. Hébrard et Duval, L'Aéronautique, Février, Mars, Avril.

Old and New Problems of Aeronautic Instruments. M. D. Hersey, Jour. Wash. Acad. Sci., 11, 1921: 22–23.

Aeronautic Instruments. C. E. Mendenhall, Jour. Franklin Institute, 191, 1921: 57–86.

The Aveline Automatic Pilot. Aerial Age, 12, 1921: 656–658; 667.

Radio Communication with Postal Aeroplanes. J. L. Bernard and L. E. Whittemore, Aerial Age, 13, 1921: 105–106.

Position Plotting by Radio Bearings. Elmer B. Collins, Proc. U. S. Naval Inst., vol. 47, No. 2, Feb. 1921.

The Radio Direction Finder and Its Application to Navigation. F. A. Kolster and F. W. Dunmore. (Forthcoming Bureau of Standards publication.)

Appareils de Repèrage de la Position d'un Aéronef. Capitaine Volmerange, tome I, 87. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre. Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

La Navigation à l'Estime. Pierre Maffert, tome II, 73. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre. Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

Le "Toposcope" (Chercheur de Point). Détermination graphique du point en avion indépendant du cap et chemin parcouru. Capitaine Vucetic, tome II, 96. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre. Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

Graphiques pour la Détermination des Routes Orthodromiques sur la Projection de Mercator. Comptes-rendus de l'Academie des Sciences, Janvier 31, 1921.

Progrès Techniques en 1920 dans la Navigation Aérienne. Capitaine A. Volmerange, L'Aéronautique, No. 19–20, Janvier 1921: 319.

Correcteur de Routes Coutinho-Sacadura. Capitaine de Corvette Sacadura-Cabral, tome II, 112. Rapports du Premier Congrès International de la Navigation Aérienne, Paris, 15–25 Novembre. Ed. Blondel la Rougery, 7, rue St. Lazare, Paris.

Die Instrumentelle Unterstützung des Fliegens. H. Boykow, Zeitschrift für technische Physik, 2, Nr. 9, Sept. 1921: 238–244.

REPORT No. 126

AERONAUTIC INSTRUMENTS
SECTION II

ALTITUDE INSTRUMENTS

IN FOUR PARTS

AERONAUTIC INSTRUMENTS SECTION
BUREAU OF STANDARDS

REPORT No. 126.

ALTITUDE INSTRUMENTS.

PART I.

ALTIMETERS AND BAROGRAPHS.

By A. H. Mears, H. B. Henrickson, and W. G. Brombacher.

INTRODUCTION.

This report is Section II of a series of reports on aeronautic instruments (Technical Reports Nos. 125 to 132, inclusive) prepared by the aeronautic instruments section of the Bureau of Standards under research authorizations formulated and recommended by the subcommittee on aerodynamics and approved by the National Advisory Committee for Aeronautics. Much of the material contained in this report was made available through the cooperation of the War and Navy Departments.

This part discusses briefly barometric altitude determinations, and describes in detail the principal types of altimeters and barographs used in aeronautics during the recent war. This is followed by a discussion of performance requirements for such instruments and an account of the methods of testing developed by the Bureau of Standards. The paper concludes with a brief account of the results of recent investigations.

Altimeters and barographs are used for determining the altitude of aircraft above the ground, the former giving direct indications on a dial, and the latter furnishing a continuous record on a chart.

For accurate measurements of altitude, reference must also be made to thermometer readings of atmospheric temperature, since the altitude is not fixed by atmospheric pressure alone. This matter is discussed in the following section on barometric altitude determination. Obviously, any instrument which purports to determine altitude solely from observations of atmospheric condition is liable to some uncertainty unless very complete observations of the state of the atmosphere are taken throughout the entire period of time and over the entire region of space corresponding to the flight, including simultaneous observations on the ground.

Thus the determination of the most probable altitude from barometric observations requires an elaborate system of observations and intricate computations. Fortunately for many purposes extreme accuracy is not necessary. It is possible to make altitude observations by direct reading of the altimeter without any computations or supplementary observations, provided an accuracy of 10 or 15 per cent is deemed sufficient. This is done by having the altimeter dial graduated directly in altitude units, based on the assumption of some particular constant value for atmospheric temperature.

BAROMETRIC ALTITUDE DETERMINATION.

It is a comparatively simple matter to deduce mathematically the relation connecting pressure, temperature, and altitude for a perfectly stationary atmosphere treated as an ideal gas. If proper account is taken of the actual variation of temperature vertically throughout the air column such a relation will be sufficient for most purposes of altitude determination. This relation may be referred to as the general altitude equation, which may be written in either of the forms

$$h = -\frac{R}{g}\int_{p_0}^{p} \theta \frac{dp}{p} \tag{1}$$

or

$$\int_0^h \frac{dh}{\theta} = \frac{R}{g} \log_e \frac{p_0}{p} \tag{2}$$

Evidently the choice between the two forms in any practical problem depends whether the absolute temperature, θ, has been given as a function of pressure p or as a function of altitude h. In the foregoing equations R is the gas constant which occurs in the characteristic equation of an ideal gas—

$$pv = R\theta \tag{3}$$

where v is the specific volume (reciprocal of density); g denotes as usual the acceleration of gravity; and p_o represents the barometric pressure at the ground, where the altitude h is zero.

An example of a case where the temperature is given as a function of pressure is found in Radau's law. This is the empirical law assumed in graduating the dials of most of the altimeters manufactured in France. Evidently when θ is given explicitly as some algebraic function of p with numerical constants, the integration can be completely worked out. Upon substituting suitable numerical values for the actual gas constant of the atmosphere R and for gravity g in appropriate units, there results a definite working formula connecting altitude with pressure.

Radau's law makes the temperature a linear function of pressure. The advantages, however, of assuming the temperature to be a linear function of altitude rather than pressure have recently been urged by some authorities, and in this case the other form of the general equation would be used.

The chief value of the general relation given above is, however, for the determination of probable altitude in cases where the temperature variation has not been given mathematically but by actual observations taken throughout the flight. In such instances the integration may be worked out graphically or by some numerical step-by-step method. Unless the temperature has been actually observed all the way up the air column, it is of course impossible to arrive at any very accurate altitude determination.

So much for the use of the general altitude equation as it stands. This relation may also be simplified or extended. An example of the altitude pressure relation when reduced to its very simplest form is found in the graduation of British and American altimeters.

In both cases this simplification consists in assuming for the air column a strictly uniform temperature of 50° F. (10° C.). Thus the temperature comes outside of the integral sign, making the altitude proportional to the logarithm of the pressure ratio. The value of the gas constant and other constants adopted from the Smithsonian Meteorological Tables as a basis for American practice are such as to lead to the formula:

$$h = 62900 \log_{10} \frac{29.90}{p} \tag{4}$$

In this equation, h represents the altitude in feet corresponding to a pressure p in inches of mercury. All American altimeter dials are graduated in accordance with this formula. British altimeters are graduated by means of substantially the same formula, the difference being negligible for practical purposes.

While British and American altimeter scales are in substantial agreement, those adopted on the continent of Europe differ considerably and are not interchangeable.

When the available data and the importance of the determination warrant an extension of the general altitude relation, some of the factors which may well be taken account of are the following: Variation of barometric pressure on the ground during the time of flight; variation of the gas constant or even departure from the law of an ideal gas, due to humidity; effect of the wind in modifying the normal static distribution of pressure; small correction for variation of gravity with altitude, etc.

The foregoing discussion is intended to indicate some of the difficulties inherent in the determination of altitude from a knowledge of barometric pressure even if the pressure-measuring instrument itself were mechanically perfect.

PRINCIPLES OF ALTIMETER CONSTRUCTION.

The altimeter is not necessarily different from any other form of aneroid barometer except that the principal scale on the dial is graduated in altitude units, while the pressure scale is usually omitted. As a matter of fact the altitude scale is usually also a scale of equal parts. This is a great convenience because it would otherwise not be mathematically legitimate for the altitude scale to be rotated in order to make the zero setting at the start of the flight. This error in design occurs in the older forms of aneroid barometer, such as the familiar pocket size used in surveying and mountain climbing, in which the pressure scale is usually equally spaced. Consequently the 1,000-foot intervals on the altitude scale are about twice as close together in the neighborhood of the 20,000-foot point as they are at sea level, in accordance with the logarithmetic relation (equation 4). Suppose, now, that the pressure on the ground has shifted half an inch from its normal value. With the older style aneroid, having a movable altitude scale on the bezel ring, the zero setting would naturally be made by turning the scale half an inch of pressure, which means about 500 feet at sea level but about 1,000 feet at the other end if the total altitude is, say, 20,000 feet. Thus an error of 500 feet is inadvertently introduced. For accurate work an unequally spaced altitude scale should not be rotated. It should be clamped in position and the altitude found by subtracting the initial altitude reading (on the ground) from the final altitude reading (at the top of the climb).

This awkward procedure is eliminated in the altimeter, where the equally spaced divisions of the altitude scale do permit rotation. The action of an altimeter is otherwise similar to that of any aneroid barometer.

The essential parts of an aneroid are the pressure measuring element, the transmission or multiplying mechanism, and the indicating element. In addition, auxiliary devices are sometimes introduced to compensate for possible sources of error, notably temperature changes. Throughout the discussion of the different altimeter designs which follows later, the description will in every case be taken up successively with reference to these four features; namely, the pressure element, the mechanism, the indicating element, and the compensation.

While one of the earliest forms of aneroid employed a Bourdon tube for the pressure element, the forms adopted in aviation invariably have for the pressure element some combination of flexible diaphragm and stiff steel spring. The springs serve to weigh the total force due to air pressure acting on the diaphragms, after the fashion of any spring balance. The larger the diaphragm, the greater will be the total force available to actuate the spring. As a general principle, in instruments of this class a large force action is desirable in order that friction and similar sources of error may have a relatively small influence. The diaphragm boxes or capsules are commonly called vacuum boxes, because there is usually a fairly high vacuum inside the box. This would not be necessary so far as the mechanical operation of the instrument is concerned, provided the temperature could be held constant. The practice of exhausting the boxes to a high degree is merely to avoid change of pressure due to expansion or contraction of the confined air when heated or cooled. The diaphragms are corrugated in order to make them more flexible. They are commonly constructed of German silver (nickel brass) although other materials have been used to a less extent.

The mechanism for multiplying the motion of the spring, so that a large deflection of the pointer may be realized, has been developed in a great variety of forms. The numerical multiplying power ranges in different types from about 200 to 800. Various combinations of levers, sometimes together with cam motions or gearing, will be found in the descriptions which follow.

The indicating element in altimeters consists ordinarily of a pointer moving over a graduated dial. Various methods are used for graduating the dial and controlling the zero adjustment. The microscopes, verniers, and micrometer screws which have been used on surveying aneroids are not found in the aviation type. Optical indicating devices have been tried but are not in common use.

The compensation of altimeters almost invariably includes static balancing of the mechanism by counterweights so as to overcome as far as possible the error due to inclination

of the instrument, or to linear accelerations. If not otherwise specified it may be assumed that such balancing has been satisfactorily accomplished. Angular accelerations are also always present at the instrument board of an airplane. Compensation for such accelerations is more difficult, and seems not to have been seriously attempted except in a recent French instrument. Some of the altimeters developed at the beginning of the war were rejected on account of faulty balancing in this respect.

Temperature compensation is frequently provided for by the bimetallic method, or by regulating the amount of air in the vacuum box when the instrument is manufactured, or by both methods. The bimetallic method consists in providing a compound bar in some part of the lever system; for example, a brass lever with a thin steel strip welded onto it. Change of temperature produces curvature in such a bar if normally straight; and it should be so designed that the amount of curvature per unit change of temperature will compensate for the deflection of the pointer which would otherwise take place at constant pressure due to temperature change. It will be observed that this form of compensation does not necessarily compensate for the change in the sensitivity of the instrument with temperature. Even if an instrument is so compensated that the pointer will not respond to temperature changes while the pressure is constant, it is still possible that the amount of deflection for unit change of pressure will be different at different temperatures. Without bimetallic compensation there may be a partial degree of compensation, satisfactory over a very limited temperature interval at atmospheric pressure, provided for by the admission of air into the vacuum box before sealing it up.

FIG. 1.—Short & Mason altimeter.

ALTIMETERS OF THE D-SPRING TYPE.

SHORT & MASON.

One of the oldest and best known aneroid movements is the Short & Mason (fig. 1). The Tycos altimeter, the Neko produced during the war, and several other British instruments such as the A. T. Reynolds, and S. Smith & Sons altimeters are substantially identical in general design with the Short & Mason instrument so far as a brief description would show.

These instruments form the principal group coming under what is known as the D-spring type. The Short & Mason instrument will be taken as an example and described in detail. Other instruments need be described only so far as they deviate from this one.

Figure 2 shows a top and side view assembly drawing of the Short & Mason altimeter. The large steel mainspring (10) has somewhat the form of a letter D in cross section and is known to the trade as a D-spring; hence the designation of this type of construction. Historically the original form of this mechanism, in its main features, was devised by Naudet in France. The present Naudet altimeter will be described later.

The pressure element in the Short & Mason altimeter consists of the D-spring (10), together with one vacuum box which is coupled to it by means of a knife-edge. The vacuum box is made up of two corrugated diaphragms which are formed so as to overlap at the rim where they are joined together by soldering. The bottom diaphragm is attached to the base plate (6) by means of a very stout screw and nut (15). If the top and bottom were not forcibly held apart during exhaustion the vacuum box would collapse under atmospheric pressure. The tension in the mainspring is so adjusted by the carriage screws (8 and 9) as to hold the top and bottom diaphragms approximately parallel under normal atmospheric pressure—if anything, slightly concave. If the air pressure is now decreased, as it will be in flying to a higher altitude, the atmospheric pressure on the surface of the vacuum box diminishes, relieving the tension on the spring and allowing the free end to deflect upward. This movement is of the order of 1 millimeter in 20,000 feet of altitude.

The movement of the free end of the mainspring is transmitted to the pointer by means of the multiplying mechanism. This, in the Short & Mason instrument, begins with the main

lever (32) and a floating link (20) which connects the end of the main lever with the regulator spindle (17). The link is not attached to the rigid part of the regulator spindle but to the regulator spring (18), which is a flexible strip offset from the axis of rotation of the regulator spindle by a distance fixed by the position of a small adjusting screw shown in the drawing. Thus the upward motion of the D-spring due to a diminution of air pressure causes the floating link (20) to pull up on the regulator spring, thus rotating the regulator spindle counterclockwise in the drawing. From the middle of the regulator spindle a straight lever (19) projects in an upward direction. A fine brass chain (48) transmits the movement of the upper end of this lever (19) over to the chain arbor (49). The chain winds and unwinds on this arbor, which is connected with a hairspring (43) for taking up lost motion. The chain arbor forms a solid part of the pointer spindle (47). Thus the motion of the vacuum box and spring is transmitted successively through the main lever (32), the link (20), the regulator spindle (17), the lever (19), chain (48), and chain arbor (49) to the pointer or hand (52) which moves over the graduated dial (4).

From the side-view drawing in figure 2 it will be seen that the initial position of the lever (19) has a good deal to do with the uniformity of the altitude scale. The multiplying power may be constant in an ordinary aneroid, but must be made to vary logarithmatically in the altimeter. This is not always a simple matter and constitutes one of the reasons for the great variety of multiplying mechanisms in existence.

The indicating element of the Short & Mason instrument consists of the pointer (52); the dial (4), which is movable and which carries the altitude graduations; and the dial (5) which is stationary but not graduated. Teeth are cut in the outside circumference of the movable dial which mesh with pinion (31). This pinion is actuated by means of the knurled knob (28). In order, therefore, to adjust the zero point of the altitude scale to coincide with the position of the pointer at the start of a flight it is only necessary to turn the knob (28).

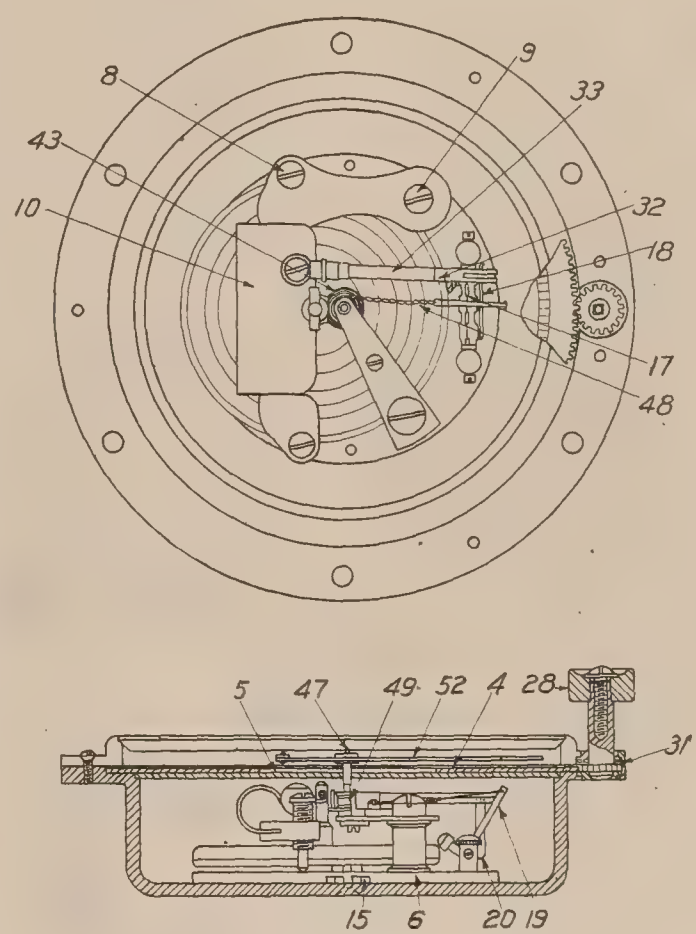


FIG. 2.—R. A. F. Mark V aneroid.

The temperature compensation of the Short & Mason instrument is accomplished at sea-level pressure by the usual bimetallic method. The main lever (32) has a thin steel strip (33) welded onto the top side. For this reason the main lever is commonly known as the compound bar.

The details referred to in figure 2 may also be recognized in the photographic view of the dismantled Short & Mason altimeter (fig. 3).

SCHNEIDER.

In the Schneider altimeter the pressure element is substantially the same as in the Short & Mason type. In place of the Maxwell point, groove, and plane mounting of the spring carriage, the carriage is held in position by three screws, two of which end in points, and the third in a polished plane. Lock nuts are provided on the carriage screws. This company has also made aneroids with steel vacuum boxes.

The multiplying mechanism, although similar in arrangement to the Short & Mason, is different in one respect: The chain arbor block is machined with a spiral groove of variable radius so as to produce a uniformly spaced altitude scale.

The indicating element is similar to the usual form but without the stationary dial.

The temperature compensation is bimetallic. Ordinarily the bimetallic bar has been incorporated in the main lever, but in some samples the lever from the regulator spindle has been used instead.

NAUDET.

The Naudet instrument (fig. 4) is simply a modification for aeronautic use of the original aneroid barometer of the D-spring type, which they were the first to develop. The descriptions of the Short & Mason instrument above will serve to give an idea of the essentials of the Naudet construction.

The pressure element differs from the Short & Mason in that the main spring carriage is swung on cylindrical bearings. The modified form of the regulator spindle in this instrument should be noted (fig. 4). The regulator spring (18) is free to move about an axis perpendicular to the axis of the regulator spindle (17) and is therefore capable of adjustment in two planes. This adjustment is obtained by means of screws (21) and (21a).

Only one dial is provided, which is movable and is graduated in meters. The usual bimetallic compensation is provided.

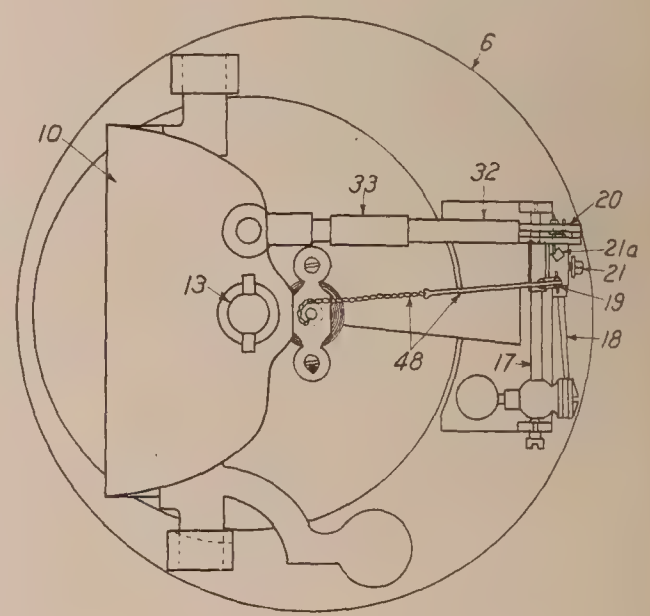


FIG. 3.—Short & Mason altimeter disassembled.

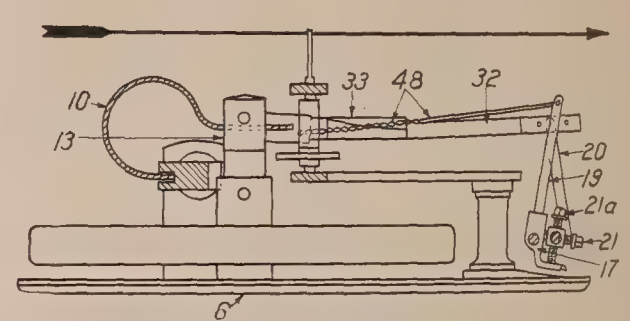


FIG. 4.—Diagram mechanism of Naudet altimeter.

COLOMBEL.

Another French altimeter is very similar to the Naudet. In addition to the movable altitude scale, there is also provided a stationary dial graduated in millimeters of mercury.

The pointer of this instrument may be set to the true pressure by means of the adjustment of the carriage screw. This is accomplished by adjusting the regulator screw through a hole in the bottom of the instrument case, as is the practice in surveying and weather aneroids.

KNUDSEN.

This instrument of Danish construction has a pressure element similar to the others except that the center portion of the spring is machined out.

The multiplying mechanism is also different. In place of the usual regulator spring there is a pin which slides in and out of a transverse hole in the regulator spindle and which is clamped in position by a set screw.

The movable altitude dial graduated in meters is rotated by means of the knurled bezel. It moves over a stationary dial, a small sector of which is graduated in millimeters. The temperature compensation is bimetallic.

GERMAN ALTIMETER.

This altimeter (monogram GL) has been found in two slightly different forms (figs. 5 and 6). The carriage screws point upward and fit in conical cups in the carriage. The carriage-adjusting screw is placed in the base of the instrument and acts against the arm of the carriage.

The chain and hairspring are arranged contrary to the usual manner so that the pointer moves clockwise for increasing altitude. The main lever connecting the D-spring with the mechanism is threaded, thus allowing adjustment of its length. The regulator spindle and spring are made in one piece of metal. The hairspring is of phosphor bronze.

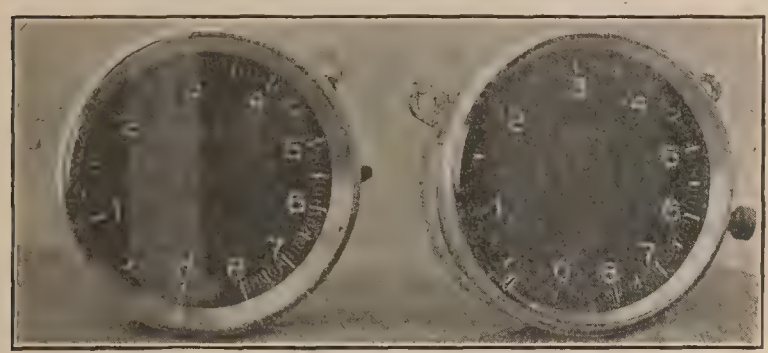


FIG. 5.—German altimeters.

FIG. 6.—German altimeters with dial removed.

In the two forms referred to, only the indicating element is essentially different. One type has both a stationary and movable dial. The movable dial is rotated by pushing on a projecting knob which comes out sideways through a slot in the case. In a later form there is only a single dial, which is fixed to the case. The entire aneroid mechanism is revolved by a simple form of gearing shown in the photograph (fig. 6) operated by a knob on the outside of the case. The usual bimetallic compensation is provided.

OTTO BOHNE.

The pressure element of this altimeter has carriage screws pointing upward and fitting in conical cup and slot. The carriage adjusting screw is in the base of the instrument as in the other German altimeter. There is a link between the vacuum box and the D-spring as in the De Giglio altimeter.

The regulator spindle and spring are made in one piece of metal. The indicating element is provided with both a pressure scale graduated in millimeters and a movable altitude scale graduated in meters rotated by gearing and knurled knob. The temperature compensation is bimetallic.

HELICAL SPRING TYPE.

RICHARD AND SIMILAR INSTRUMENTS.

The helical-spring type is well exemplified by the Richard instrument. It differs from the D-spring type in that the mainspring consists of a vertical helical spring under compression. The spring is not directly coupled to the vacuum box, but the downward pull of the vacuum box is transmitted to the helical spring through a flat plate having its fulcrum on two steel points. The movement is shown in figure 7. The disphragm (11) is coupled to the flat plate lever (7) by the knife-edge (14) working on a steel collar (14a). The tension of the helical spring (10) is varied by the screw fitting on which the spring rests. This can be adjusted by means of a screw reached through a hole in the base of the instrument.

Any movement of the diaphragm (11) is multiplied in the ratio of about five to one at the end of the long lever arm. This movement is transmitted through the multiplying system through the pin (20). This pin strikes the curved surface of a cam causing it to rotate about the axis (25). Attached to the cam is a flat brass pin (19) whose motion is transmitted by

contact with an arm projecting from the sector gear (48). The teeth of this sector gear mesh with a small pinion on the pointer spindle. Backlash is taken up by a spiral hairspring. Thus the movement of the top of the vacuum box is transmitted to the cam through contact with a pin; the rotation of the cam and upright pin (19) transmits the movement to the sector gear, thence to the pointer spindle.

The indicating element consists of a movable dial graduated in meters revolving over a stationary dial graduated in millimeters.

Temperature compensation is accomplished by the relative expansion of the brass and steel parts of the movement, but without any bimetallic bar.

HATOT.

In this instrument, also of French construction, special care is taken with the mounting of the flat plate lever or carriage (see fig. 8). The carriage screws are of steel with sharp points and are held in small cylindrical brass posts inserted into the base plate.

The bearing surface for the carriage screws are small cylindrical pieces of steel inserted into the carriage plate and are cupped to receive the points of the carriage screws. The connection of the diaphragm to the carriage plate should be noted. There is a square hardened steel pin inserted transversely in the diaphragm pillar similar to the ordinary knife-edge. The surface of this pin facing the diaphragm is cupped to receive two steel points which are inserted in the carriage. The diaphragm is held distended by a helical spring as in the case of the Richard. The tension of this spring can be adjusted by means of a screw which raises and lowers a brass fitting on which the spring rests.

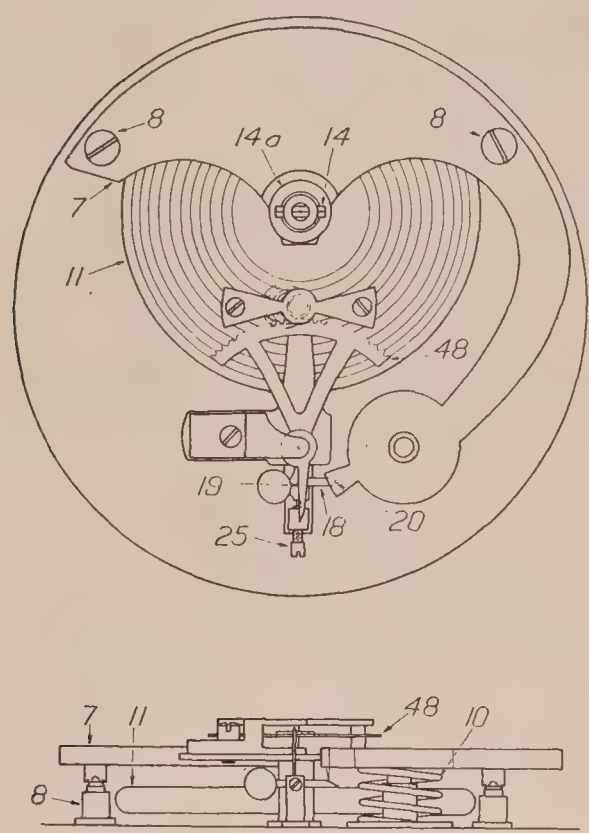


FIG. 7.—Diagram for Richard altimeter.

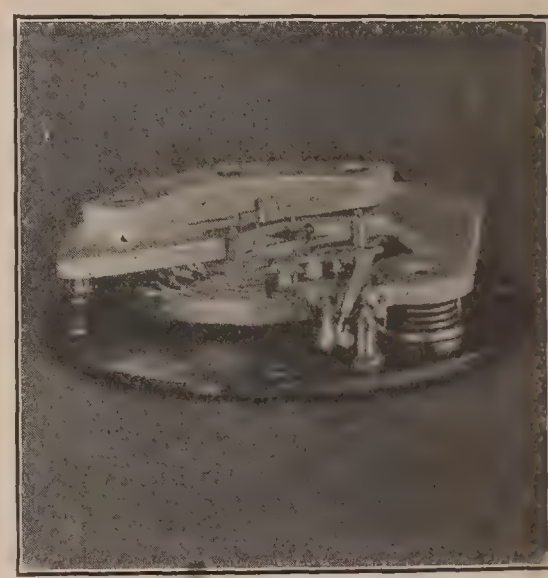


FIG. 8.—Mechanism of Hatot altimeter.

Referring to figure 8, there is shown a screw in the offset from the circular head of the carriage plate. In the end of this screw a steel pin is inserted eccentrically. This steel pin rests on the polished face of the multiplying lever. By rotating the screw that holds the steel pin the multiplying ratio can be adjusted.

The multiplication of the motion of the carriage is accomplished in the following manner: The steel pin in the carriage rests on the polished steel surface of a lever shaped somewhat like a bell crank. The other arm of the bell crank is in the form of a steel knife-edge, and takes the place of the upright pin of the Richard instrument. This lever is counterbalanced. The steel knife-edge is in contact with the tail of a toothed sector. The motion is transmitted to the indicating hand by the sector meshing with a pinion mounted on the same arbor shaft with the hand. A tension is maintained on this lever by means of a light steel spring. Backlash is taken up by a steel hairspring.

The indicating mechanism is a fixed dial graduated with a pressure scale in millimeters. Over this fits a movable illuminated dial provided with a crown gear. The dial is rotated by means of a small pinion mounted in the instrument case which is turned by a knurled thumb screw. No bimetallic compensation is used in this instrument.

DE GIGLIO.

The pressure element of this instrument differs from the Richard by providing an extra connecting link between the vacuum box and the fitting which carries the knife-edge. The mechanism is similar to that of the D-spring altimeters except that the rotation of the hand is clockwise for increasing altitudes.

The indicating element has both a stationary pressure scale graduated in millimeters and a movable altitude scale graduated in meters. There is no bimetallic temperature compensation, but the static balancing is done as in the Richard, by the insertion of a strip of lead into the back end of the flat plate lever.

FILOTECNICA.

The Filotecnica altimeter, also an Italian make, is very similar to the De Giglio except that the movable dial is rotated by turning the knurled bezel instead of a geared thumb screw.

OTHER TYPES.

MAXANT.

The pressure element of this French altimeter (fig. 9) consists of a vacuum box with an internal spring. This construction is familiar in barographs, but unusual with altimeters.

The multiplying mechanism starts with a long lever consisting of a brass rod into which is inserted transversely a long steel upright. Offset from this lever is an arm (19) which engages the top of the vacuum box. The bearings (25) of this long lever are mounted in a bridge over the vacuum box. The long steel upright actuates a geared brass sector (48). This sector meshes with a pinion on the index arbor. The zero of the instrument is adjusted by turning a steel screw which raises or lowers one end of the mechanism bridge. Backlash is taken care of by a small spiral spring.

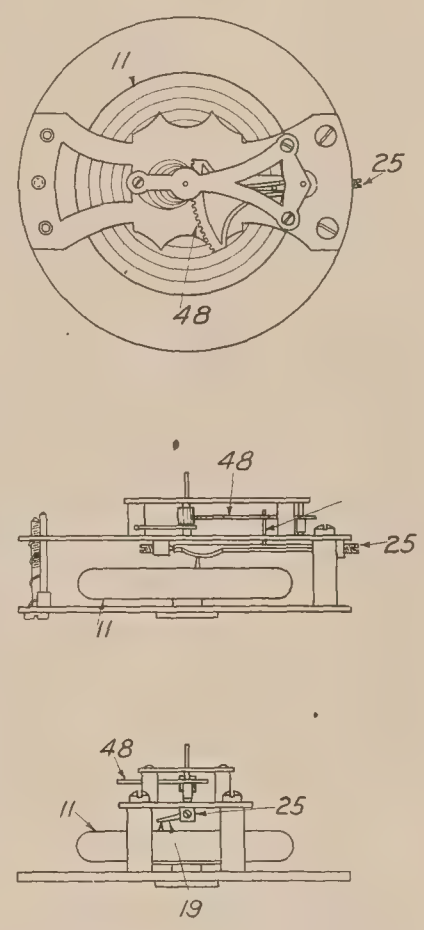


FIG. 9.—Diagram of Maxant altimeter.

The indicating element has been made in two forms, in each of which there is both a stationary and movable dial. In one form the movable dial giving the altitude scale in meters is rotated by gearing actuated by a knurled thumb screw. In the other form the movable dial is revolved in a knurled bezel.

SPERRY.

The pressure element of this movement is made up of a battery of two vacuum boxes with internal springs which are mounted on a U-shaped brass frame. This U-shaped casting holds the entire mechanism and is screwed to the case of the instrument.

A bimetallic bridge spans the two legs of the U-shaped piece and supports the multiplying mechanism in the center. This mechanism consists of a carefully machined helical groove in the pointer arbor. This arbor is free to turn in a polished hole in the bridge. It is caused to rotate in proper relation to the movement of the upper vacuum box by the vertical movement of a pin traveling in the helical groove. There is a spiral hairspring to take up backlash.

TAGLIABUE.

The pressure element of this altimeter consists of a single vacuum box with an internal spring. Any motion of the upper surface of the diaphragm is communicated to the multiplying mechanism by means of the jeweled bearing soldered to the top of the upper diaphragm.

The design of this instrument is unique and offers several advantages in the adjustment for calibration. The motion of the diaphragm actuates a short lever arm projecting from the regulator spindle, which rests on the jeweled bearing of the diaphragm. This lever is punched integral with the regulator spindle, and, since the material is of spring steel, considerable adjustment of its length can be accomplished by the lateral motion produced by a regulator screw. The vertical movement of the diaphragm serves to rotate the regulator spindle, which carries with it an upright. The movement of this upright in turn rotates a cam to which it is connected by a link. The cam rotates about a vertical axis. By means of a hairspring mounted on the cam shaft lost motion is taken up and a slight pressure maintained against the jeweled bearing. In contact with the cam surface there is a small roller bearing which is attached to the tail of a geared sector. Thus any rotation of the cam causes a corresponding movement of the geared sector. The cam is so designed that with increasing altitudes there is a gradually increasing movement of the sector for equal movements of the diaphragm. The geared sector meshes with the pinion on pointer spindle.

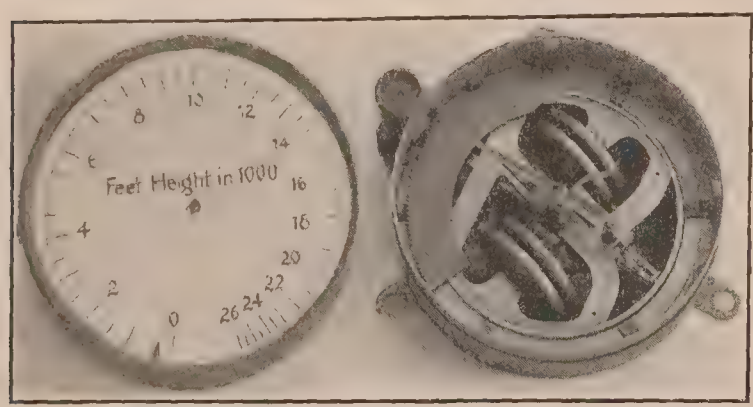


FIG. 10.—German altimeter with double action.

The stationary and movable dials are constructed in the usual way and the latter is turned by a knurled knob.

GERMAN ALTIMETERS WITH DOUBLE ACTION.

The pressure element of this altimeter consists of two batteries of two diaphragms each. The springs are internal. The vacuum boxes are mounted edgewise so that the direction of their motion on deflection is in a plane parallel to the dial of the instrument. The two batteries deflect in opposite directions as shown by figures 10 and 11.

The deflections of the diaphragms cause a rotation of the multiplying lever (17) about an axis, perpendicular to plane of motion of the diaphragms, the two arms of this lever being connected by links to the diaphragms. The motion of the multiplying lever (17) is transmitted by means of a short pin (21) to a second slotted lever (20) carrying the sector (48). The pin (21) slides in this slot when the lever (17) is displaced. The pin (21) is attached to lever (17) by a lock nut and washer and is adjustable in the slot at the end of lever (17), thereby making it possible to change the multiplying ratio of the lever system and hence the deflection of the pointer. The geared sector engages with a small pinion on the pointer spindle. A hairspring is mounted on the indicating arbor.

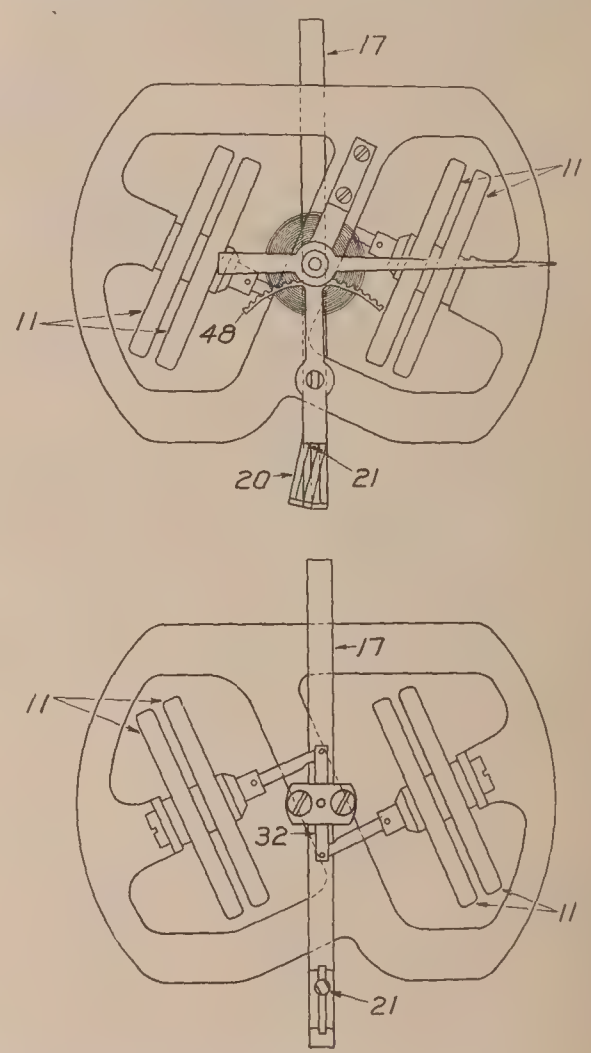


FIG. 11.—Diagram mechanism of German altimeter with double action.

The indicating element has an unequally divided altitude scale which is fixed in position. The entire inside mechanism is rotated by means of a geared sector engaging in a pinion mounted in an offset in the instrument case. The altitude scale is considerably cramped at the high altitude end. The rotation of the hand for a change of altitude of 1,000 feet near the 25,000-foot point is only one-third that at sea level.

Temperature compensation is effected by the general design of the lever system.

PRINCIPLES OF BAROGRAPH CONSTRUCTION.

Barographs may be described with reference to the pressure element, multiplying mechanism, recording element, and compensation.

The pressure element is invariably such as to give a larger amount of deflection than in the case of altimeters. This is necessary in order to reduce the demand on the multiplying mechanism, as will be seen. The increased sensitivity of the pressure element is usually attained by having a battery of vacuum boxes one on top of another. The steel springs may either be internal or external in arrangement, the former being more common.

In barographs the mechanism has considerably less multiplying power than in altimeters. This is necessary in order to insure a sufficiently firm and powerful movement of the tracing point; otherwise the unavoidable friction of this point on the chart might cause trouble.

Various optical and other devices have been tried for the recording element, some of them with complete success, but none of these has come into very common use. Ordinarily the recording element consists of a pen-and-ink record on a paper chart. The chart is placed on a revolving drum whose time of revolution ranges in different instruments from a half hour to 24 hours or longer, according to the requirements of the work. The scale provided by the chart is rarely as open in the barograph as in altimeters.

The term altigraph has frequently been used for consistency with the term altimeter to designate a barograph whose chart is equally spaced with respect to altitude, and some charts are provided carrying only the altitude scale.

Temperature compensation is usually accomplished in barographs only so far as is possible by means of admitting air to the vacuum boxes. Bimetallic compensation is rarely provided. Balancing is also seldom attempted, for it is expected that the barograph will be held in an upright position.

INTERNAL SPRING BAROGRAPHS.

THE RICHARD BAROGRAPH AND SIMILAR INSTRUMENTS.

The most widely used and probaly one of the oldest types of barographs is the Richard. The Green barograph is very similar to the Richard except for the use of a tubular-shaped pen instead of the usual form consisting of a V-shaped trough.

The pressure element is a battery of two internal spring diaphragms attached securely at the base to a flexible flat steel plate or spring. A square-headed screw operated by a key adjusts the height of the boxes above the base plate by causing an up-and-down motion of the spring and serves as a means of adjusting the zero setting of the pen. The Richard movement is shown in figure 12.

The motion of the upper diaphragms, pillar (A), is transmitted by means of a link (B) to a multiplying lever which oscillates about the axis (O). The long pen arm is actuated by means of a link connecting it to the multiplying lever and an upright (D) from the pen arm. A flat spring is provided which keeps a slight tension on the pen arm and takes up backlash.

The pen arm of the recording element is a long flexible flat spring steel shaft. This long thin arm is in turn fastened to the last rigid lever arm by two screws. It is sprung away from this lever arm and then brought back toward it by a thumb screw. This method of regulating the tension in the pen arm serves to adjust the amount of pressure at the contact of the pen with the chart. The pen can be removed from the surface of the chart by means of an arrestment actuated by a shaft projecting outside of the case.

HÜE.

The pressure element of this barograph is similar to that of the Richard except for the zero adjustment being made by a knurled thumbscrew.

The mechanism serves to transmit the deflection of the diaphragms to the pen arm in the following manner: The movement is first transmitted to a multiplying lever by means of an upright extending from the uppermost vacuum box. Both arms of the multiplying lever are

adjustable and are held in place by set screws. The pen is actuated by means of a link which connects this multiplying lever with an adjustable arm attached to the pen arm. Thus a further degree of adjustment is possible than in the case of the Richard barograph. The recording element is similar to the Richard except in certain details. The pen arrestment is operated by a lever extending outside the case of the instrument, to which is attached a brake serving to stop the clockwork whenever the pen is thrown off the chart. The mechanism and base plate slide into a wooden case. The pen arm is counterbalanced.

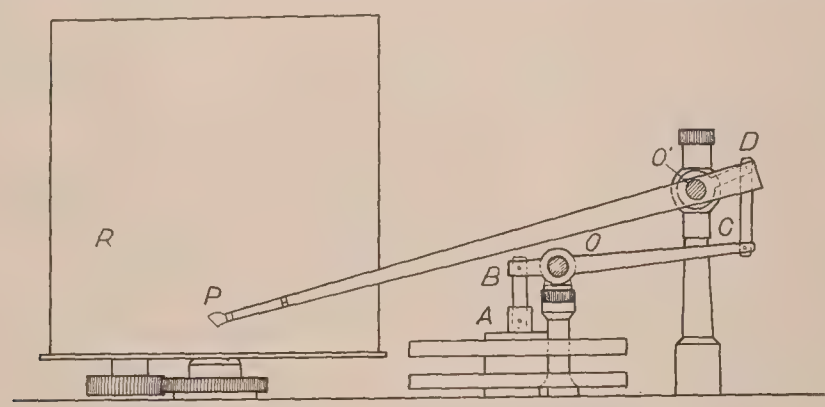


FIG. 12.—Diagram mechanism of Richard altigraph.

SHORT & MASON.

A barograph developed in England and made for a limited time in this country is shown in figure 13. The pressure element consists of a battery of three internal spring vacuum boxes. Unusual attention was given to the quality of the clockwork and to care in construction.

The multiplying lever (C) is mounted on an arbor having conical bearings suspended on a bridge between two uprights. The height of this mounting can be varied by turning a knurled thumbscrew (S), thus adjusting the zero of the instrument. The multiplying lever is connected by means of a link (D) to the pen arm. A helical spring keeps the tension on the pen arm and serves to take up backlash.

This instrument was the first to provide a true altigraph scale. It has also an unusually open scale. This was made possible by a very high drum. The period of rotation of the drum in some forms is as short as 30 minutes, adapting it especially for performance testing and experimental work.

In this barograph a bimetallic bar is provided in the multiplying lever (C). It is the only barograph known in which bimetallic compensation has been attempted.

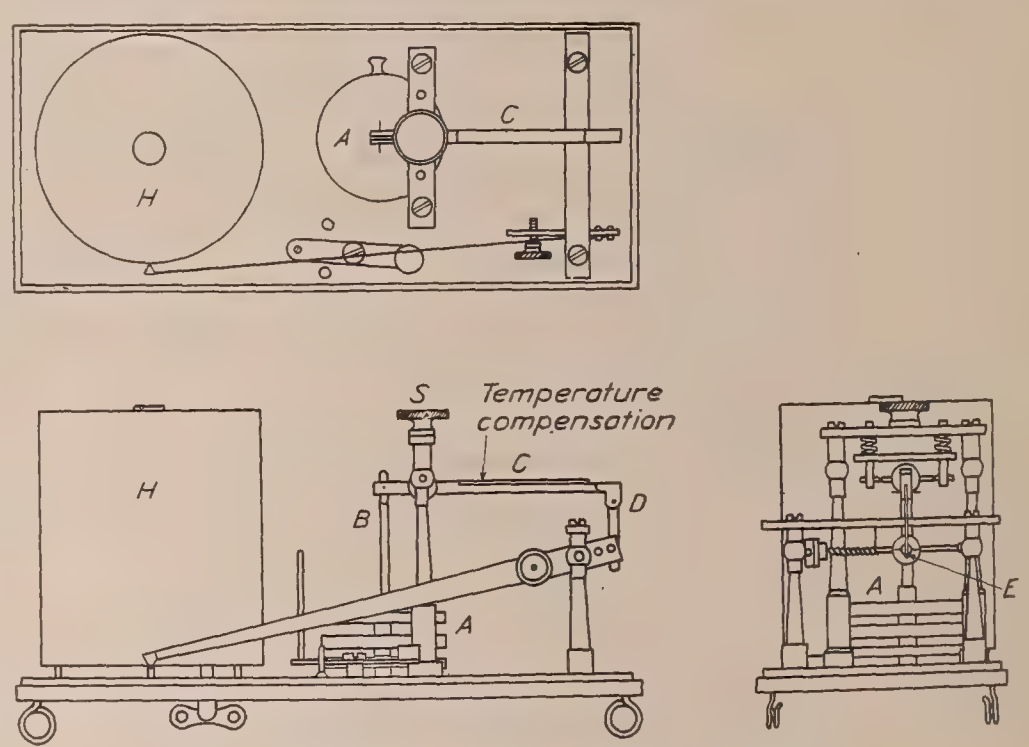


FIG. 13.—Short & Mason 10,000-feet altigraph Cover removed.

SCHNEIDER.

The pressure element of this barograph is similar to the Richard.

One end of the multiplying lever is attached to the vaucum boxes by means of a steel link. The other end is attached by an adjustable pin to the arbor of the pen arm.

AGOLINI.

The pressure element consists of a battery of two internal spring vacuum boxes mounted rigidly on the base.

The uppermost vacuum box is connected to the multiplying lever by means of a steel link. The multiplying lever is a long brass arm, the axis of which is held in bearings mounted on a bridge. The height of the bridge above the base is adjustable by means of a thumbscrew, thus

providing for the zero setting. The thumbscrew is placed in a yoke mounted on two standards. The adjustable bridge is prevented from getting out of alignment by two guide pins placed through the yoke. The multiplying lever is counterweighted and actuates the pen arm by means of a steel link.

STOPPANI.

This Swiss barograph has a battery of two internal spring vacuum boxes mounted on the base plate by means of a short upright post.

The uppermost vacuum box is connected to the multiplying mechanism by a link. The multiplying lever is elaborate in construction, offering facilities for ready adjustment. There is a short arm made up of an adjustable pin threaded on the end. This pin is kept in tension by a helical spring, and it is held in a cylindrical-shaped fitting bored so as to receive the pin. The mounting for the multiplying lever is similar to that of the Short & Mason barograph. The pen arm is attached to the multiplying lever by means of a link. The multiplying lever is counterweighted.

KNUDSEN.

This instrument of Danish construction has for its pressure element a battery of three vacuum boxes with internal springs. The zero adjustment is made as in the French barographs.

The movement of the vacuum boxes is transmitted by a multiplying lever, one end of which is attached to the uppermost vacuum box. The other end of the multiplying lever is attached to the pen arm by means of a connecting link. The length of the arm of the multiplying lever can be adjusted by means of a slide which is held in place by a set screw. The length of the lever on the pen arm may be adjusted by means of a set of holes for the link pin.

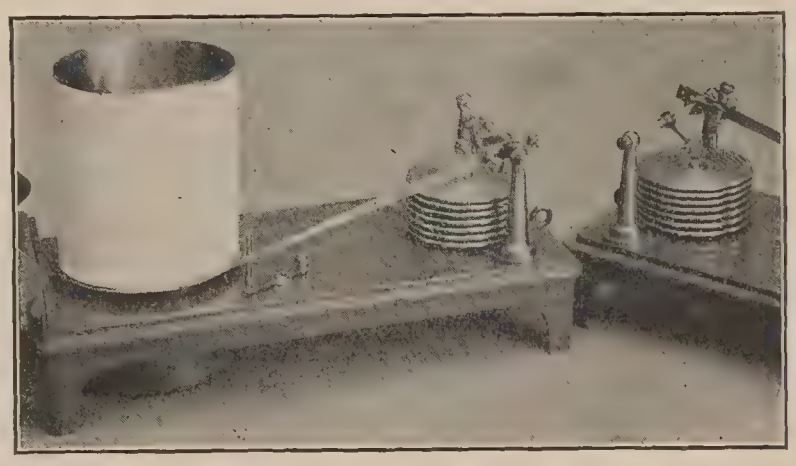


FIG. 14.—Friez sylphon barograph.

FRIEZ SYLPHON.

The pressure element of this barograph (fig. 14) is composed of a sylphon multiple capsule instead of the usual battery of separate vacuum boxes. The sylphon consists in effect of seven capsules in series, although made of one piece of metal. Instead of the usual elliptic springs there is a single internal helical spring. In laboratory tests this instrument has shown exceptionally small elastic hysteresis and after-effect errors.

The top of the sylphon is attached to the multiplying mechanism by means of a link. This link is attached to an arm projecting downward at an angle from the main spindle. The pen arm is mounted on this same spindle. The zero adjustment is accomplished by a thumbscrew adjustment on the pen arm which permits a rotation about the main spindle.

OTTO BOHNE.

This instrument, of German construction, has a battery of four internal spring vacuum boxes mounted on a flexible base so that their height may be adjusted by turning a knurled thumbscrew.

The deflection of the vacuum boxes is transmitted directly to the pen arm by means of a connection from the uppermost vacuum box to an adjustable shaft through the pen-arm arbor.

The pen arrestment is operated by a small knob which extends out through the door of the wooden instrument case. The clockwork is wound by turning the drum in the opposite direction from that in which it is driven.

EXTERNAL SPRING BAROGRAPHS.

RICHARD POCKET BAROGRAPH.

This instrument is very much smaller than the ordinary barographs. The pressure element consists of a battery of two vacuum boxes without internal springs. They are prevented from collapsing by a helical spring some distance away, which is mounted in a vertical position and which acts upward against a flat plate which serves as one of the multiplying levers. This part of the construction is somewhat similar to the helical spring altimeters. There is an adjustment for tension of the helical spring by means of a square shank screw operated by a key. This serves as a zero adjustment for the instrument. (See accompanying photograph, fig. 15.)

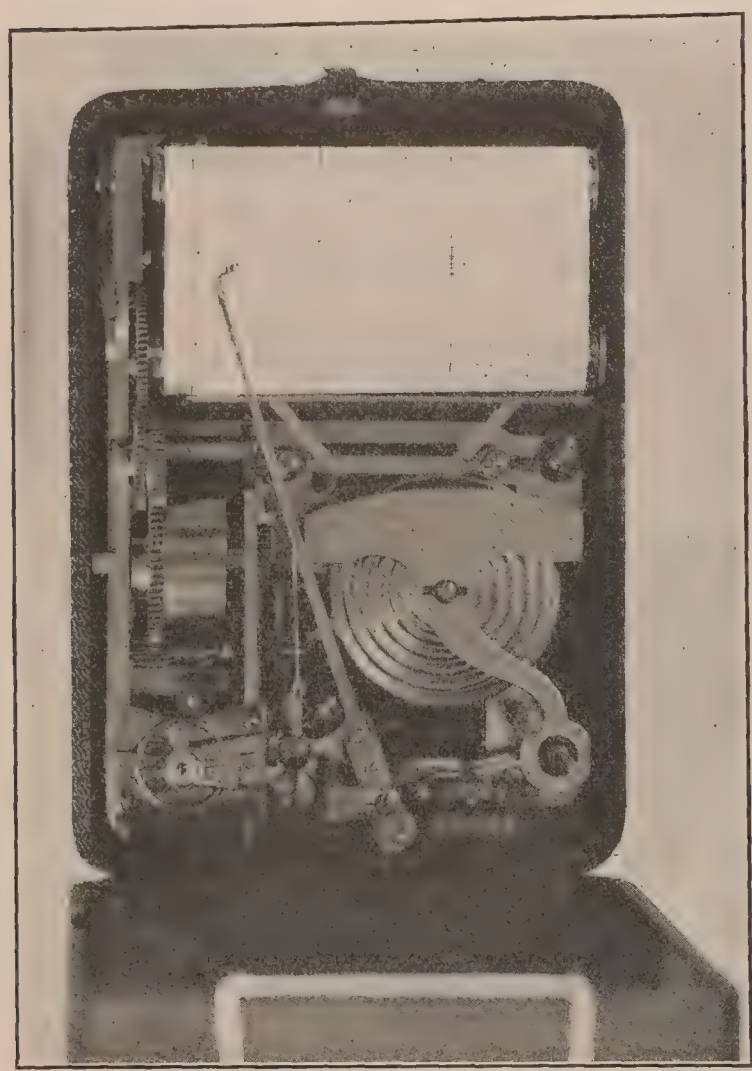


Fig. 15.—Richard pocket barograph.

Inserted into a projection from the multiplying arm or flat plate, above referred to, there is a pin which points downward. This pin acts on the curved surface of a cam. Any movement of this cam causes a spindle to rotate which carries an upright piece of wire serving as a lever. This wire rod makes a sliding contact on a fin which is mounted upon the pen arm. The back-lash of the pen arm is taken up by a hairspring.

The recording element contains a mechanism for periodically raising and lowering the pen in relation to the chart. The chart is not in contact with a single cylindrical drum as in the larger barographs, but is stretched between two small rollers so that the surface receiving the record is flat. The rollers are forced apart by a spring which can be released in order to remove the paper. The pen is raised from the chart by a slotted bar in which it is free to slide. This slotted bar is raised and lowered by clockwork; thus the record consists of a series of dots, the pen being off the paper the majority of the time. This device enables the pen arm to take up the most accurate position without restraint due to friction at all times. The necessary firmness of contact for making a legible record is secured momentarily by the periodic action of the slotted bar.

DE GIGLIO.

The pressure measuring element of this instrument (fig. 16) is very similar to that of the Richard pocket barograph. It consists of a vacuum box of the external spring type which is held from collapsing by a helical spring placed at the lower end of a T-shaped plate. This plate serves, as in the pocket barograph, as one of the multiplying levers. Any movement of the diaphragm of this altigraph is multiplied further by means of another multiplying lever (B) and the pen arm (C) which are connected together, and the multiplying lever to the T-shaped plate by means of small links. The construction of the multiplying lever makes possible con-

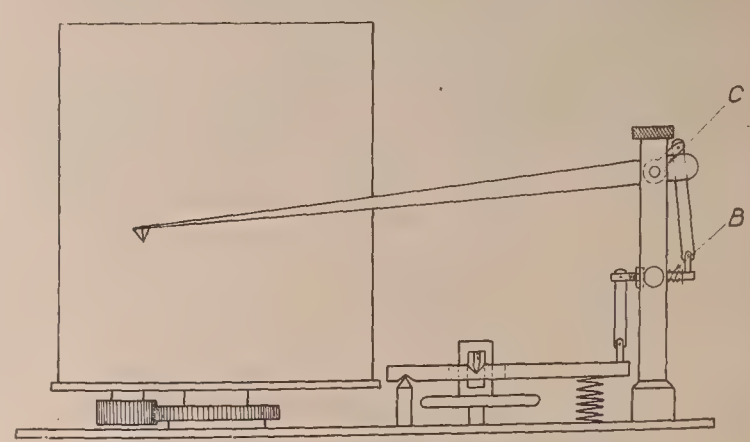


Fig. 16.—Diagram mechanism of De Giglio altigraph.

siderable adjustment of the multiplying ratio. It is really a regulator spindle, and the ratio of the length of its two arms can be varied by turning a knurled nut with the fingers. The pen and pen shaft are similar to that used in the Richard barograph.

POCKET BAROGRAPH OF THE DOUGLASS PRODUCTS CORPORATION.

Except for a slight modification of the clockwork, this instrument is similar to the Richard.

PERFORMANCE REQUIREMENTS FOR ALTIMETERS AND BAROGRAPHS.

The performance requirements of altimeters and barographs are determined, on the one hand, by the degree of accuracy necessary for the particular use contemplated and, on the other

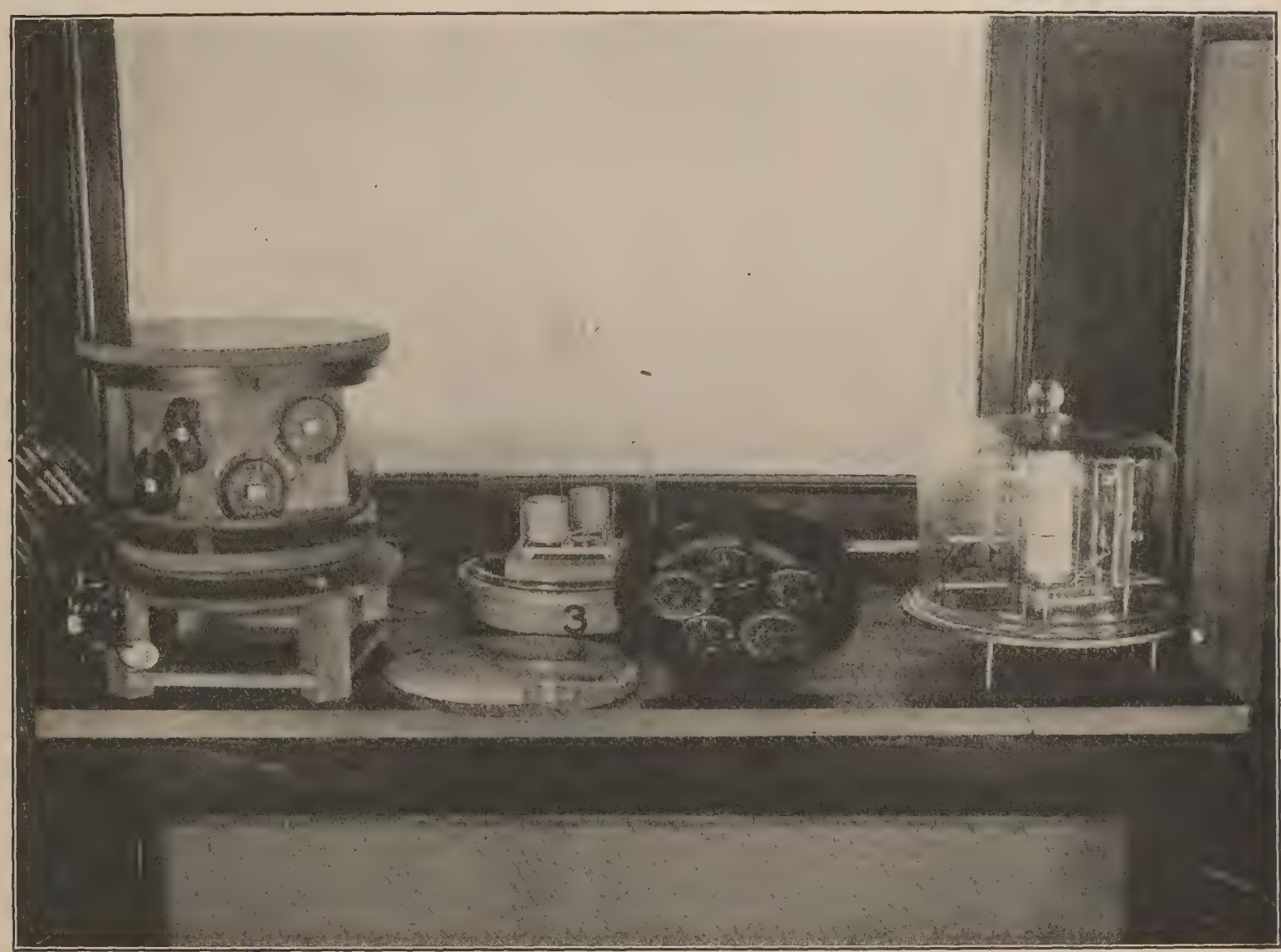


FIG. 17.—Four types of containers for aneroid testing.

hand, by the sources of error existing in the instruments under conditions corresponding to those which will be experienced in flight.

The conditions occurring in flight which may cause errors are extremes of temperature, inclination, acceleration and vibration of the instrument, and rapid change of pressure tending to cause a time lag in reading. These conditions must be reproduced in the laboratory and the instruments tested more or less completely under each condition, depending on the accuracy required and whether or not the instrument is of a new type of construction.

Schedules of permissible errors for various conditions and altitude ranges have been adopted in making tests at the Bureau of Standards. These will not be gone into numerically here, but the testing procedure will be explained and the performance characteristics to be observed in the test will be definitely stated.

METHODS OF TESTING.

The apparatus employed for the pressure and temperature tests is essentially the same for all kinds of tests on aneroids of the aviation type, and is shown in the three accompanying photographs.

Figure 17 shows various types of containers found useful in the calibration of aneroids. The bell jar to the right of figure 17 is used for experimenting on aneroids. It is equipped with electrical connections so that a small fan motor, heater, etc., may be put inside; also it is provided with small copper tubes for carbon dioxide refrigeration, so that extreme low temperatures may be maintained. Other apparatus not shown in the photograph has been used for

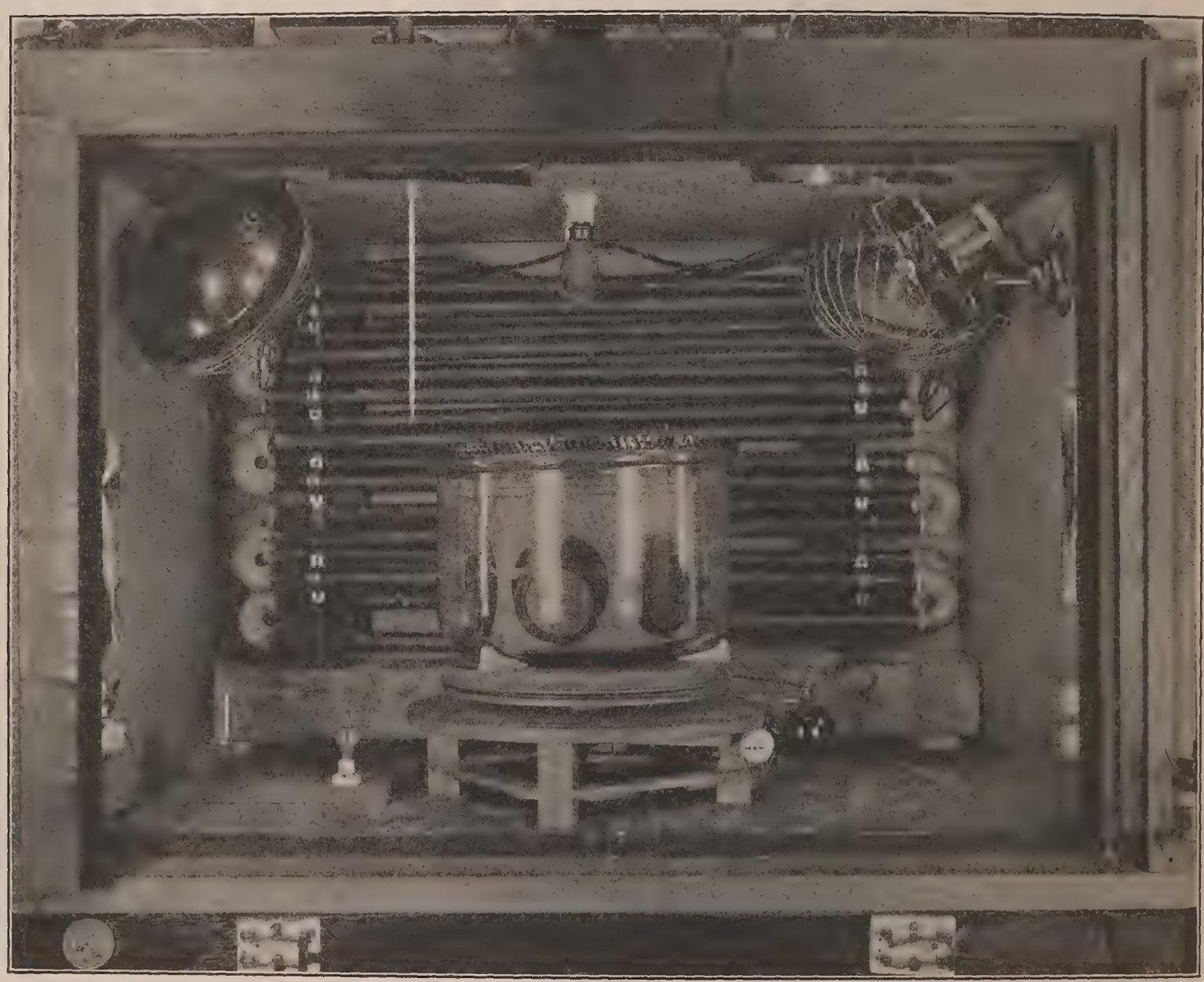


FIG. 18.—Temperature chamber with motor-driven container.

routine tests of aneroid barometers at temperatures as low as around −40° C., especially for the flight history tests.

The container in the center of figure 17 is convenient for routine testing of small groups of aneroids and especially for checking the readjustment of aneroids following repair. It may be covered either with the adjacent flat glass disk or with a small bell jar, as shown in the rear of the photograph.

The container at the left of figure 17 is one of a type designed for testing a large number of altimeters at the same time. It consists of a large bell jar, inverted, and rotated in a motor-driven support. It can be read either through the top or the side. The vacuum connection is made through a special fitting in the knob of the jar.

The next photograph (fig. 18) shows a temperature chamber in which any of the foregoing containers may be placed or two of the inverted bell jar type. By the use of the brine system

and heaters, any temperature down to —10° C. may be maintained, and slightly lower temperatures may be secured by using calcium chloride to remove the frost from the brine coils. The instruments are read through a double glass door. The containers are rotated by a motor that is controlled by the two knife switches shown in the lower foreground.

Figure 19 shows the mercurial standards and a vacuum control board of special design used in aneroid testing. Below is a large reserve vacuum tank. The tank is of value in mini-

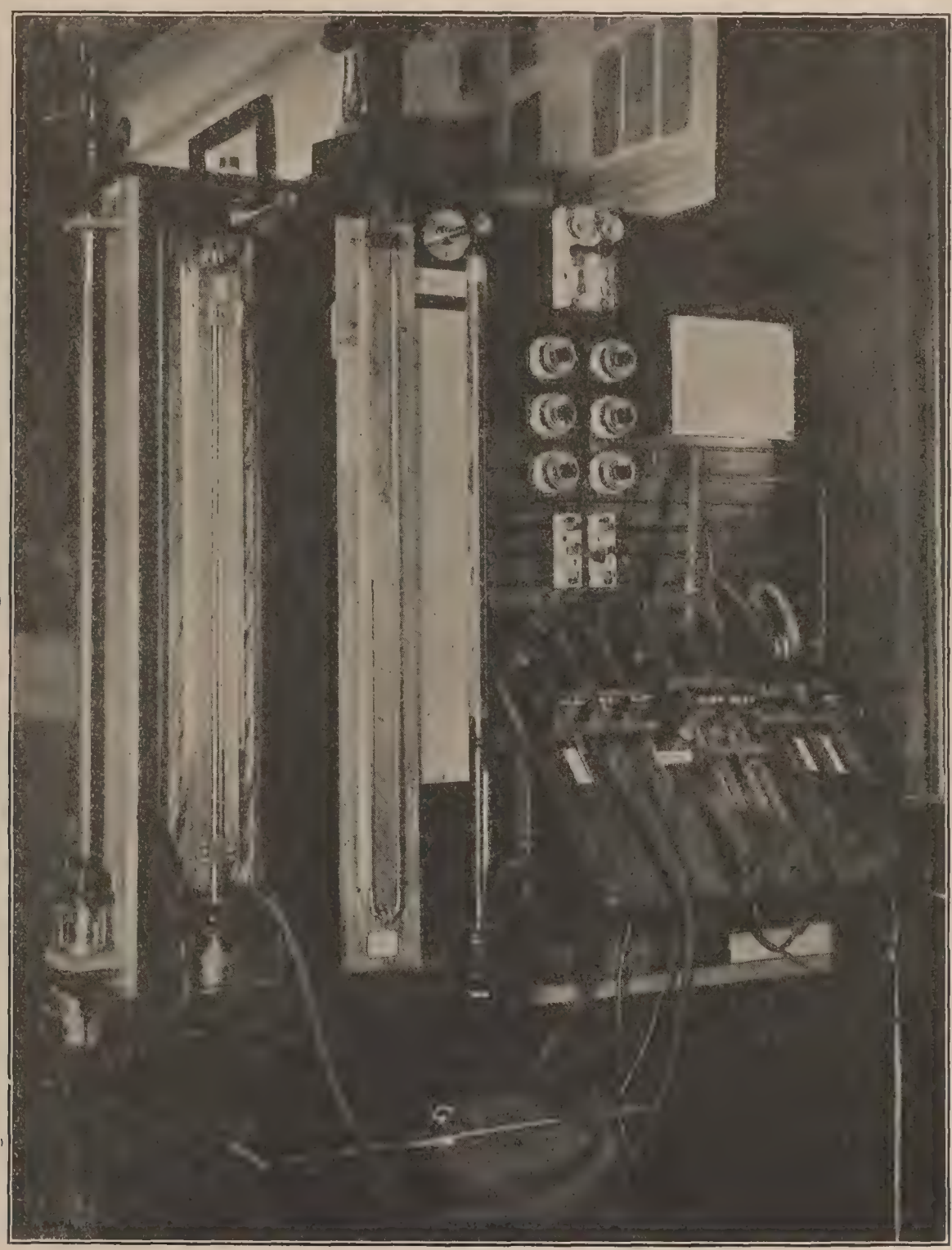


FIG. 19.—Mercurial standards and vacuum control board.

mizing the effect of slight leaks that may occur in the system. This arrangement of barometers and vacuum control board and tank makes it possible to produce different pressures in several different containers simultaneously, so that instruments of different ranges may be tested in the shortest time. Also it facilitates the calibration of the mercurial standards, which were submitted to the Bureau of Standards during the war in considerable numbers.

The vibration apparatus as shown in figure 20 is of special design in that the motor is not attached to the vibrating frame. This lengthens the life of the motor, and therefore the effi-

ciency of the vibrator. The amplitude, frequency, and direction of the vibration can be varied at will, making it possible to subject the instruments to any desired vibration.

The chain-testing device in figure 21 is shown to illustrate some of the special problems which may arise in sample instruments. Several different styles of chains were tested on this apparatus for a comparison of their performance and durability.

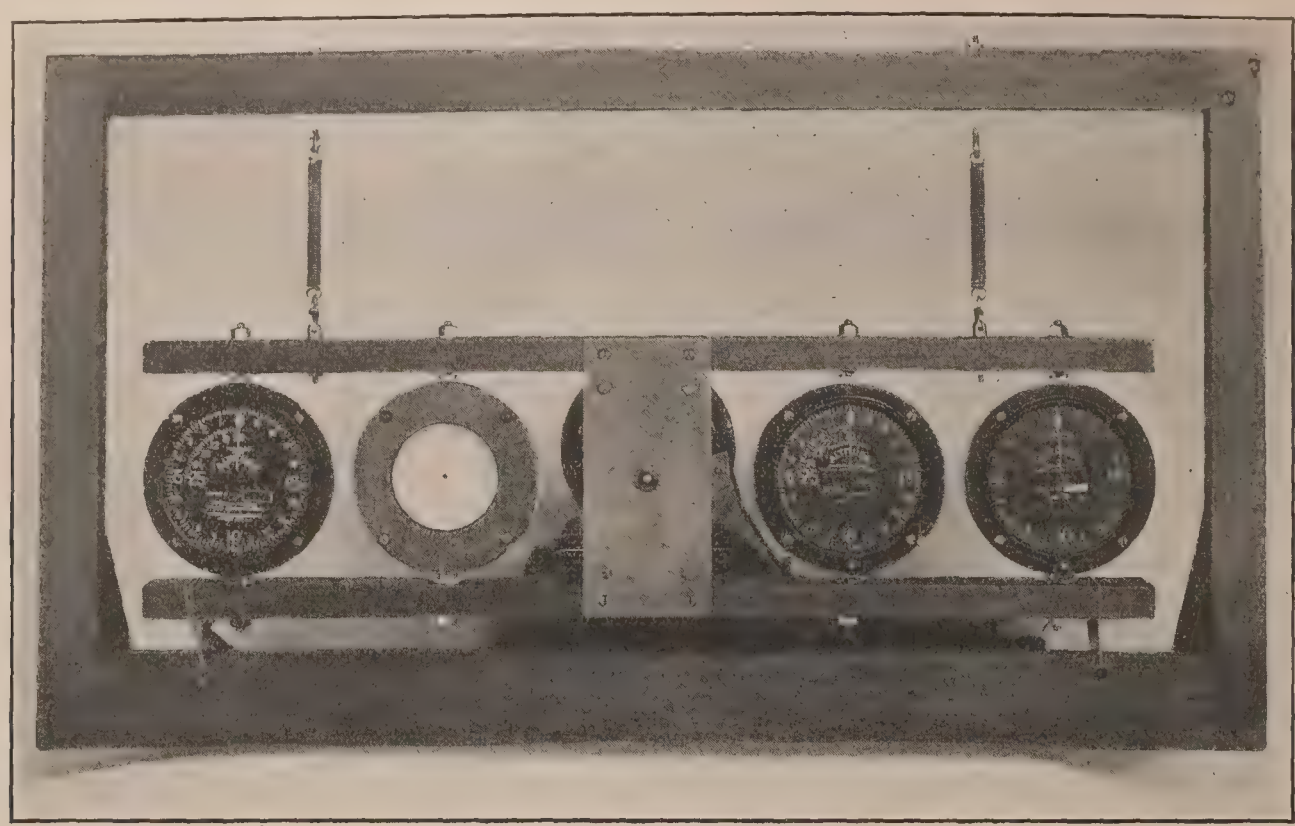


FIG. 20.—Altimeter vibrating board.

Special apparatus was also developed to test the mounting of the mainspring of the Short & Mason type of altimeter.

There are two kinds of test in common use, the general test and the special test for an experimental instrument.

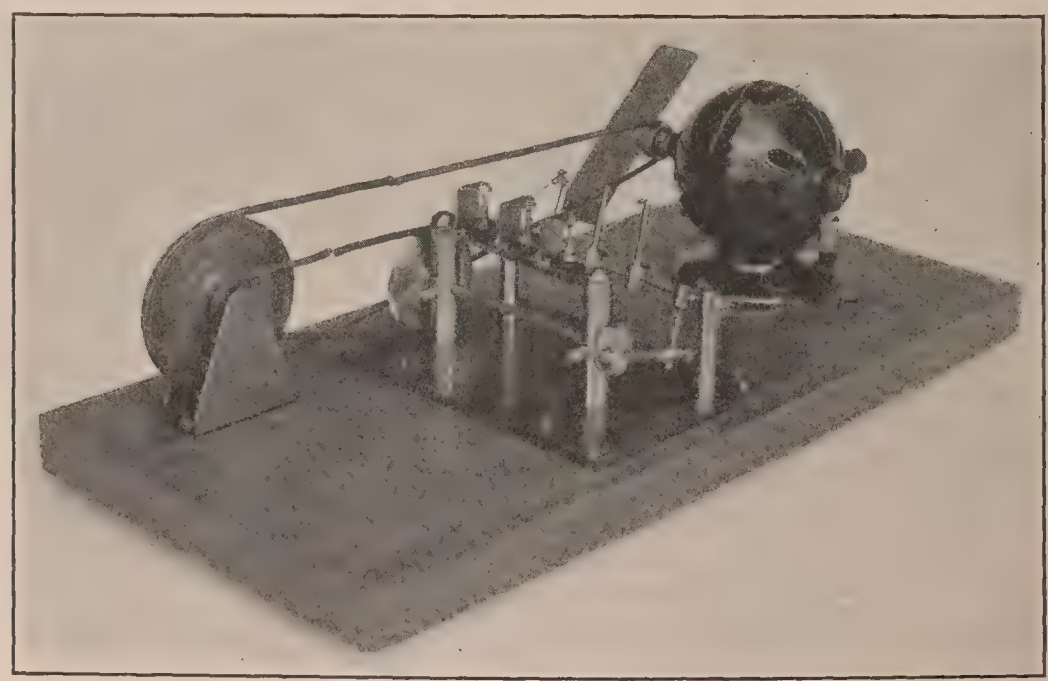


FIG. 21.—Altimeter chain-testing apparatus.

The general test is the one always given if no other is explicitly requested. It affords data for deducing approximate values of the proper correction under any given condition of use. A modification of the general test is the so-called short test on service instruments. This is an abridged form of the general test recommended only when time is not available for the

numerous stages of that test. The operations involved in the general test require a minimum of eight working days; but the short test can be finished in two days. In most cases the short test will suffice for the rejection of inaccurate instruments, but not always, and it will not provide detailed corrections. Another modification of the general test is a further extension of it in the form of supplementary tests on sample instruments. These tests include a determination of the effects of vibration, acceleration, or other aeronautic conditions not covered by the general test. Such observations are important in considering the suitability of new types or makes of instruments, but are not likely to be necessary in testing or inspecting each individual instrument.

The special test on experimental instruments consists of more precise and complete determinations of the errors for instruments of high precision to be used in experiments on aircraft. These tests may be made by extending the procedure of the general test, or, better, by taking what is known as a flight history test. This can only be done after the flight when the flight conditions are known. By a flight history test is meant one in which the actual variations of pressure and temperature experienced in the use of the instrument on a particular occasion are artificially reproduced in the laboratory.

GENERAL TEST.

The general test consists of four parts, A, B, C, and D.

TEST A (PRELIMINARY TEST).

This consists of six parts: The tapping, shift, and inclination tests; the determination of the correction at the end of the range, and of the aftereffect, by means of a pressure test; and a final examination for mechanical defects. Of course, any obvious mechanical defects that are noted upon receipt of the instrument may cause its rejection without going through any of the experimental tests.

(1) The *tapping test* is made by repeatedly tapping the instrument vigorously enough to just disturb the pointer, and noting the average deviation of the pointer reading from its mean position. Four or five taps are usually sufficient.

(2) For the *shift test* the aneroid is held in a sidewise position and struck edgewise against the palm of the hand, first with its face to the left, and then to the right. Each time it is afterwards read in a horizontal position. One half the average difference between the left and right readings is recorded, and known as the deviation by shifting, or simply the *shift*.

(3) For the *inclination test* the aneroid is held in a horizontal position, read after slightly tapping, and then again read after turning to a vertical position. The difference in readings is recorded and known as the vertical correction or inclination effect.

(4) The aneroid, having been set to read the altitude as indicated by the pressure of the mercurial standard, is placed in a container and the pressure reduced at once to the lowest point on the scale (the highest altitude). The movement of the pointer is carefully watched during this pressure change to detect jerkiness. After the pressure has been reduced, the aneroid is allowed to stand a period equal to two-thirds of the time required to ascend to that altitude at a rate of 200 feet per minute. At the end of this rest period, the readings of the standard and of the aneroid are taken and recorded. The pressure in the container is now quickly raised to atmospheric by opening the vacuum system to the air.

(5) Five minutes afterwards the aneroid and the standard are again read. The amount by which the pointer fails to come back to the true altitude is called the aftereffect. This is to be expressed in per cent of the altitude range.

(6) The mechanical defects such as jerkiness, parallax, or loose parts which have been noted during the foregoing test on which may now be found by further inspection are recorded.

The design of most aviation aneroid barometers is now such that the tapping, shift, and inclination tests are unimportant. An error of 50 feet for altimeters and 0.05 inches of mercury for aneroid barometers is excessive.

If the aneroid passes satisfactorily the six steps of test A, it is put through test B; otherwise it is rejected at this point.

TEST B (SEA-LEVEL TEMPERATURE TEST).

This test consists of two steps:

(1) After the instrument has been at the room temperature of 20° C. for at least three hours it is tapped and the pointer set to the altitude indicated by the mercurial standard and this reading recorded. It is then placed in a temperature chamber, where the temperature is lowered to −10° C. and held at this temperature for at least three hours. At the end of this period the instrument is again tapped and the position of the index observed, recorded, and compared with the reading of the standard.

(2) The aneroid is next put through the *hot test*, which is the same as the cold test except that the instrument is heated up to +40° C. and a reading taken after a four-hour interval. From the high temperature the instrument is allowed to cool down to the room temperature. After three hours it is read again and compared with the standard. This completes the temperature test at atmospheric pressure.

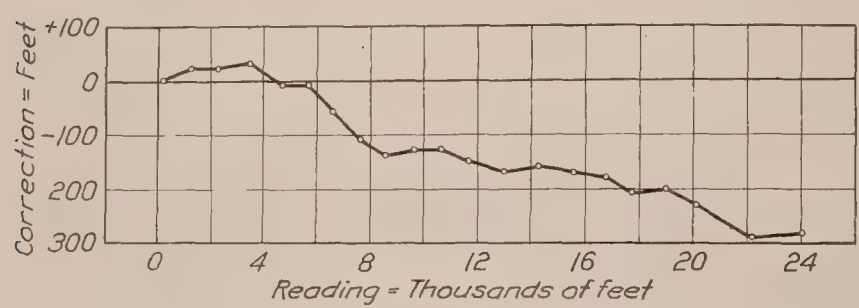


FIG. 22.—Calibration curve at 26° C. on altimeter made by S. Smith & Sons.

NOTE.—The individual steps of test B are performed in the order given to avoid the condensation of vapor within the instrument which would occur if it were first heated and then cooled, but which is avoided by heating the instrument after the cold test. Care should be taken always to tap the instrument before reading.

TEST C (CALIBRATION AND DRIFT).

(1) After setting the index to the altitude indicated by the standard, the instrument is placed in a container and the pressure decreased at an average rate of 1,000 feet for each five-minute time interval. Simultaneous readings of the instrument under test and the mercurial standard are taken at intervals of five minutes. The difference between the two readings is the correction, which is so given that the algebraic sum of the correction and the reading of the instrument under test equals the true altitude.

(2) After the pressure in the container has been reduced to correspond to the highest altitude indicated on the scale at the above rate, it is held at this value for five hours. At the end of this period the container is tapped and the instrument read, and the correction obtained as before from the mercurial standard. Extreme care must be taken that the temperature of the instrument does not change during the test, and that the pressure in the container be exactly at the same value at the beginning and the end of the five-hour period, so that the hysteresis effect is eliminated. Much fluctuation in the pressure during the five hours is decidedly to be avoided.

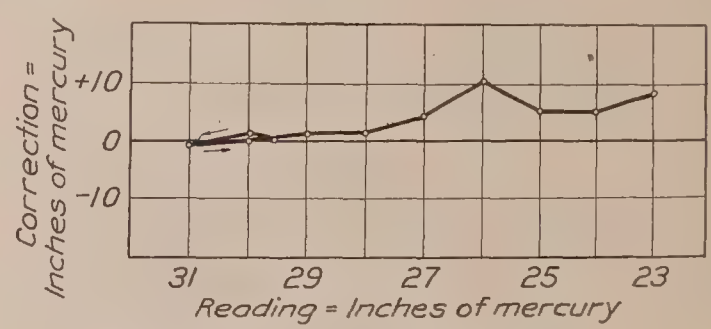


FIG. 23.—Calibration curve of Hicks pocket aneroid.

The maximum correction of the aneroid barometer reading is noted, and if desired a calibration curve can be drawn, with corrections plotted against readings, and attached to the report. Figures 22 and 23 are typical calibration curves, figure 22 for an altimeter and figure 23 for an aneroid barometer. The drift is the quotient, in per cent, of the change in the correction found after five hours at the high-altitude range divided by the true range. The average deviation is the average of the deviations of the calibration curve from the best representative straight line.

TEST D (ALTITUDE-TEMPERATURE TEST).

Test D consists of two additional calibration tests:

(1) At a temperature of $-10°$.

(2) At a temperature of $+40°$.

The corrections are found in the same manner as in test C (1). Straight lines are drawn through the calibration curve at each temperature, including the calibration at $+20°$ C. This will give three straight lines. The slope of the calibration curve at 20° C. should always lie between the other two, the slope of calibration curve at $-10°$ C. being greater and at $+40°$ C. being less. This is for curves whose coordinates are in altitude. The slopes are determined in per cent for the curve, and the difference between the slopes for the lines for $+40°$ C. and $-10°$ C. is noted as the change in scale value. This value must not exceed 5 per cent. The intermediate slope at $+20°$ C. is of value in this test in showing the regularity of the shift with temperature and as a check.

For aerobarographs or other instruments reading in inches of pressure, instead of in feet of altitude, the same methods are followed throughout tests A, B, C, and D, readings being taken at each inch of mercury pressure changes instead of each 1,000 feet. The pressure is changed at the rate of 1 inch of mercury pressure every five minutes instead of 1,000 feet every five minutes.

The four tests are purposely given in the order named. Defects due to poor workmanship can usually be discovered before the instrument is calibrated, and if the defect is serious the instrument is immediately rejected, thus saving time and labor. The temperature test B is given before the calibration because more instruments fail on this one test than on any other. Moreover the instruments are thus given a chance to rest after being strained by undergoung test A. This is an important factor which should not be overlooked. The instruments must be allowed at least 24 hours' rest after being subjected to a large pressure change, in order to obtain accurate results. Otherwise, on account of elastic fatigue, the instrument will give a false reading which may be different from one obtained after a sufficient period of rest.

The short test for service instruments differs from the general test given in the following respects: (1) Tests A and D are omitted altogether; (2) test B is made only with cooling instead of with both cooling and heating; (3) the drift observation in test C is omitted and replaced by an observation of the aftereffect. The instrument in this test is held at the low pressure for two hours only instead of for five hours.

It is inherently impossible to make the tests A, B, C, and D in less than about seven or eight days since each instrument must be allowed a period of time between each stage of the test to recover from the elastic fatigue set up in the metal by virtue of the test itself, and it is preferable that at least two days should elapse between each part of the test. The result of cutting down that time allowance would merely be to produce deceptive figures, which will not reproduce under aviation conditions.

In the supplementary test on sample instruments a vibration test lasting for one week is given. The instruments during this test are mounted on the vibration apparatus previously described. During this time observations are occasionally taken to note both the amplitude of oscillations of the pointer and whether any movement of the dial has taken place. Afterwards the instruments are again calibrated. Observations are also made on any other suspected sources of error.

In addition to the general test, additional tests are made on barographs that are to be used for competitive altitude records in order to determine the fitness of the instrument for this special purpose.

SPECIAL TESTS ON EXPERIMENTAL INSTRUMENTS.

Instruments to be used in experiments on aircraft are carefully readjusted to give the least possible errors, and calibration curves are then determined. The flight history test referred to above is also given for such instruments when the actual flight conditions are known. Only by such a test can the effects of elastic fatigue and temperature lag be properly determined.

RESULTS OF INVESTIGATION.

One of the most urgent needs of the aeronautic instrument manufacturer during the recent war was the development of reliable testing apparatus. The first problem was to design a mercurial barometer which could be easily transported and which could be read without the necessity of making numerous settings and applying corrections such as are required with ordinary laboratory barometers. Considerable experimenting was done with different types of mercurial barometers that were constructed both by the manufacturers and the Bureau of Standards, and their different characteristics were studied in detail. An interesting problem in this connection was the equipment of the fixed cistern types of mercurial barometers with an altitude scale. It was attempted to construct a fixed cistern instrument with an equally spaced altitude scale so that it would be possible to use a vernier with this scale. This idea was abandoned because the investigation showed that the advantages gained by the use of the vernier was offset by the necessity of a very much cramped scale which proved to be a decided disadvantage. The temperature corrections to fixed cistern barometers were also studied in detail. It was found that the temperature correction in altitude is approximately constant for all altitudes in the ordinary fixed cistern instrument. As a result of the above investigation, a portable standard was designed by the Bureau of Standards, satisfying the conditions that the instrument does not have to be set for the level of the mercury, and for a considerable range of temperature from room temperature it is not necessary to apply a temperature correction.

Another important problem was how and where the routine tests could be modified and still cause the rejection of all the defective instruments. In this connection an investigation was made to determine how many instruments would pass the "temperature test at sea level" (test B) and not pass "altitude-temperature test" (test D). The results showed that test D could be omitted for the ordinary service instruments, but this procedure is not recommended for an instrument that is to be used in the performance testing of aircraft. A series of tests was made to determine the shape of the curve, when the slope of the calibration curve was plotted against temperature.

The Bureau of Standards carried out an investigation on the thermometric lag of the various types of instruments for the purpose of determining how long an instrument should be held at a certain temperature during the temperature tests.

It was also necessary to devise tests to prove the fitness of the instruments to withstand the vibrations experienced under actual conditions of use in aircraft. With this in view, vibration apparatus was designed. To find out the effect of vibration the vibration of the pointer was noted; also the loosing or movement of any of the parts of the instrument and the effect of the vibration on the calibration curve, etc. Defects caused by vibration were found to be comparatively infrequent.

The most important investigation undertaken was a study of the elastic properties of aneroid diaphragm capsules. For convenience these elastic properties may be divided into three phenomena:

Drift, which is the change of displacement under a constant load.

Hysteresis, which is the excess of displacement with load decreasing over the displacement at the same load with load increasing.

Aftereffect, which is the residual displacement at any time after removal of load.

A great number of drift experiments were made to determine the shape of the drift curve, i. e., the increase of displacement plotted against time, the variation of the magnitude of the drift with the speed of loading, and the variation of drift with the load.

The improvement of the amount of drift in aneroids submitted to the Bureau of Standards from year to year has been studied, and a curve showing these results is given below (fig. 24). In this curve the average per cent of drift of the aneroids submitted during the year is plotted vertically against the year horizontally. The slight increase in the average drift during 1919

and 1920 is due to the fact that several of the instruments submitted were manufactured previous to the war.

A detailed inquiry was made of the hysteresis of various types of instruments and the relation of hysteresis to drift was investigated. A curve showing the hysteresis in several instruments will be found in Part II of this report, under precision altimeter design.

While studying the elastic properties of a D-spring type altimeter a detailed investigation of the mechanical errors of this instrument were made.

The greatest mechanical error was found to be the method of clamping the mainspring to the carriage of this instrument. A very small amount of slipping which occurred here was greatly magnified by the multiplying mechanism, so that the error became large. This error was entirely eliminated by substituting a steel support for the brass one ordinarily used and preventing the spring from slipping in this steel support by set screws.

Slipping of the knife-edge was found to be responsible for a similar error.

Still another problem in the construction of the instrument related to the delicate chain in the indicating mechanism. A device was constructed to study the endurance of various types of chain. These tests showed that both gold and steel chains would far outlast the life of any instrument, but that the gold chain showed better performance when subject to the adverse atmospheric conditions of actual use. Phosphor bronze strips showed good endurance and performance when great care was used in attaching them in the instrument.

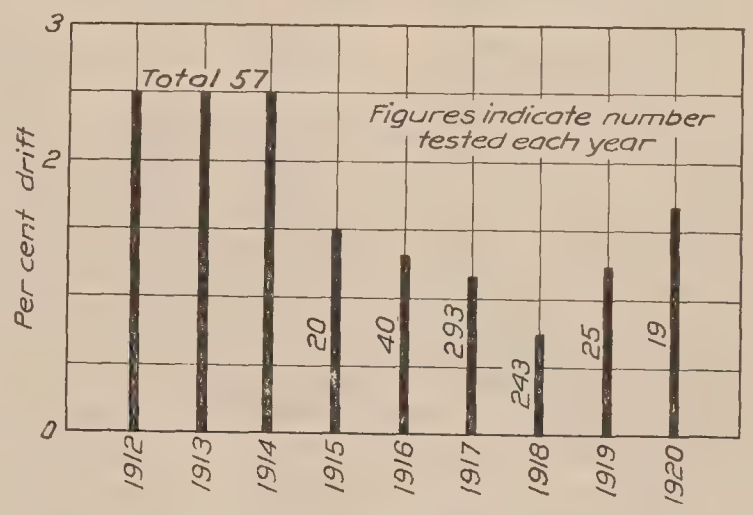


FIG. 24.—Average drift in aneroid barometers tested at the Bureau of Standards.

For the performance testing of aircraft it is often very desirable to have an instrument reading in pressure instead of altitude. Most of the aneroids reading in pressure are not suitable for aircraft. To meet this demand the bureau converted several altimeters to pressure instruments, equipping them with scales graduated in inches, millimeters, or millibars. This work led to an interesting study of the multiplying mechanisms in these instruments.

Closely connected with the conversion of altimeters to a pressure basis was the extension of the range of barographs. This was accomplished by fitting an external spring to the instrument so as to increase the stiffness of the system for a given deflection of the box. This increased the pressure range (altitude range) for the same deflection. Special charts were made for these instruments.

An optical method of testing the regularity of the motion of the drum of a barograph was developed. This consists of putting a sensitized sheet on the drum of a barograph and allowing a very fine line of light to strike the drum parallel to the axis of rotation. If the motion of the drum is uniform the paper upon development should show uniform exposure, if the motion is nonuniform the exposure will show a series of light and dark lines—light when drum is rotating fast and dark when slow.

REPORT No. 126.

ALTITUDE INSTRUMENTS.

PART II.

PRECISION ALTIMETER DESIGN.

By John B. Peterson and John R. Freeman, Jr.

SUMMARY.

In this part the general principles of altimeter design are discussed and applied to the construction of a large open-scale precision altimeter, the development of which was undertaken in accordance with a program approved by the National Advisory Committee for Aeronautics.

At the beginning of the discussion data are shown indicating the amount of error due to imperfect elasticity of ordinary aneroids, showing the great need for improvement in this particular. On the same diagram the hysteresis curve resulting from the finished precision altimeter is plotted, showing the notable improvement secured. In fact, by the use of a stiff steel spring of special quality coupled to a diaphragm made of ordinary brass but sufficiently thin to contribute practically no elastic reaction, the familiar hysteresis error has actually been made negligible in comparison with the ordinary mechanical errors of the movement.

In conclusion, suggestions are made regarding compensation of the design for minor sources of error.

Instruments of the precision altimeter class are intended primarily for use in performance testing of aircraft and as working standards for reference in laboratory tests.

INTRODUCTION.

The readings of an aneroid barometer are liable to errors as great as 4 per cent even after the readings are corrected for the temperature of the atmosphere. In several special cases where accurate determination of pressure or altitude is desirable if not indispensable, the ordinary commercial altimeter does not have the desired accuracy. It was the purpose of the authors to fulfill this need by first developing a theory of aneroid design and then checking the results by experiments on an instrument constructed accordingly.

Several cases where accurate instruments are essential are:

(1) In the performance testing of aircraft;
(2) In landing at night or in fog;
(3) In aerial mapping;
(4) In bomb dropping; and
(5) As secondary standard for laboratory or field use.

In performance tests the determination of pressure is more important than that of the altitude. Either an instrument reading in pressure units or an altimeter with a fixed dial should be used.

For landing at night, or in fog, the pilot could receive by radio from the landing station the barometer reading at the station. Knowing this pressure and having a reliable aneroid, the pilot could descend to this pressure level, i. e., ground level, as indicated by his aneroid, without danger of crashing. The necessity for accuracy here is easily seen. Too large an error may be fatal to crew and passengers.

Aerial mapping and bomb dropping are the only uses stated here where the temperature of the air need be accounted for accurately in the determination of altitudes. This being the case, temperature errors due to deviation from the altitude formula will usually overshadow errors in the pressure determination, but accurate results are desired, and instrumental errors should be eliminated when it is practical to do so.

An accurate instrument graduated in pressure units could be used to advantage as a secondary standard for the calibration and adjustment of altimeters in the laboratory and on the field. For this purpose the instrument should indicate the pressure directly with nearly the accuracy of a mercurial barometer. The ordinary commercial instrument is not satisfactory for this purpose, since its readings, owing to imperfect elasticity of the diaphragms, depend on the rate of change of pressure to which it is subjected. A secondary standard of this type is especially needed for use at aviation fields where mercurial instruments which require careful adjustment and the use of correction tables are very inconvenient.

The most difficult error to eliminate or correct for in an aneroid barometer is the elastic lag or time effect. Figure 1 shows the hysteresis in commercial altimeters. Curve B is the best instrument of a group of approximately 10 American and foreign makes which were tested. A hysteresis error of $2\frac{1}{2}$ per cent is 250 feet on a 10,000-foot instrument, or 500 feet on a 20,000-foot instrument. The cause of this hysteresis is imperfect elasticity of the spring or diaphragm, usually due to a stress too high for the material used. Also the mechanical support of the spring and the knife edge which fastens the diaphragm to the spring were found to be sources of error.

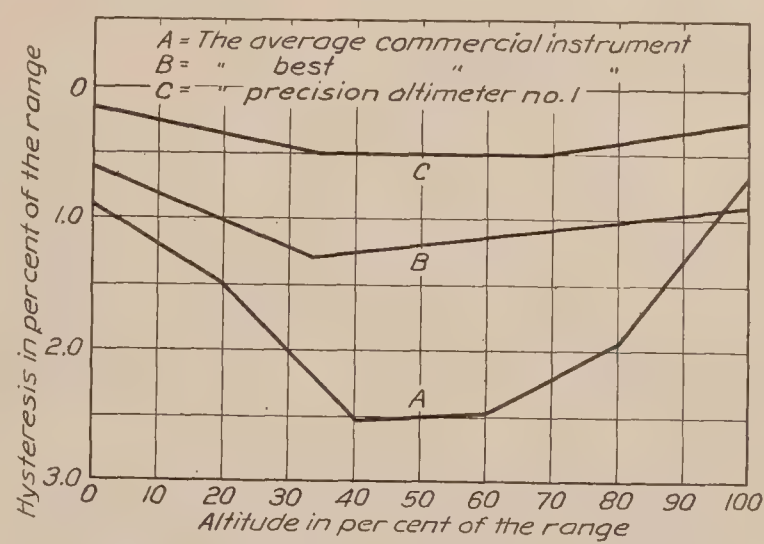


FIG. 1.—The rate of ascent and the rate of descent in the hysteresis tests were 500 feet per minute. The time at the top was 30 minutes.

ESSENTIAL PRINCIPLES OF PRECISION ALTIMETER DESIGN.

The design of an altimeter is necessarily such that the finished instrument is a compromise among the many desirable features. Some of these desirable features are:

(1) A large diaphragm, so as to get sufficient working force to operate the mechanism properly.
(2) A spring with low maximum stress, stiff relative to the diaphragm.
(3) A large deflection combined with a small multiplying ratio.

After the outside diameter of the instrument has been decided upon, the problem is to put the best possible mechanism in the space available.

DIAPHRAGM DESIGN.

EFFECTIVE AREA.

The effective area of a diaphragm is defined as the ratio of the distending force applied at the center to the pressure required to produce this force.

To calculate the effective area of a diaphragm, let us assume that a section of the diaphragm from the center to the outside rim, ABCD (fig. 2), acts as a flexible cable with a uniform stress throughout a given section.

Let r = radius of the center.
R = outside radius less the radius of the center.
$\Delta\omega$ = an angular increment including the area ABCD.
P = the atmospheric pressure in pounds per square inch.
α = angle of departure of the diaphragm from face of center plate.

Using the notation of figure 2, the area of the section ABCD is $\frac{1}{2}(R^2+2Rr)\Delta\omega$. The load on this area is $\frac{1}{2}(R^2+2Rr)\Delta\omega P$, supported partly at the center and partly at the outside rim. The moment of the load on the area ABCD about the tangent to the circle at BC is $(\frac{1}{2}R^2r\Delta\omega+\frac{1}{6}R^3\Delta\omega)P$. Divide this moment by R and we get the part of the load supported by the center, approximately $(\frac{1}{2}Rr+\frac{1}{6}R^2)\Delta\omega P$. Now the total pressure falling on the rigid center of which AOD is a sector is transmitted to the spring and has therefore to be added. The load at the center due to the pressure on the triangle BOC is $(\frac{1}{6}R^2+\frac{1}{2}Rr+\frac{1}{2}r^2)\Delta\omega P$. The total load on the center is

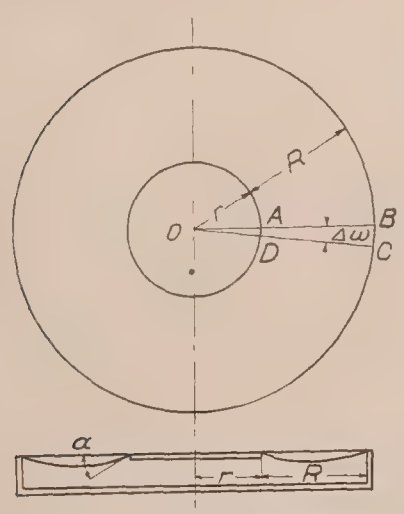


FIG. 2.—Suspension diaphragm.

$$\sum_0^{2\pi}(\tfrac{1}{6}R^2+\tfrac{1}{2}Rr+\tfrac{1}{2}r^2)\Delta\omega P=(\tfrac{1}{3}R^2+Rr+r^2)\pi P.$$

The effective area then, is, by definition,

$$(\tfrac{1}{3}R^2+Rr+r^2)\pi$$

The ratio of the effective area to the actual area is

$$\frac{\frac{1}{3}R^2+Rr+r^2}{R^2+2\,Rr+r^2},$$

which may be expressed in the form

$$\frac{\frac{1}{3}+\frac{r}{R}+\left(\frac{r}{R}\right)^2}{1+2\frac{r}{R}+\left(\frac{r}{R}\right)^2}.$$

This ratio varies from one-third when r is zero to unity when R is zero.

The foregoing analysis is an approximation satisfactory for relatively small values of the inner radius r.

FIBER STRESS.

The maximum fiber stress in the diaphragm occurs where it is fastened to the rigid center. Considering the area ABCD again (fig. 2), the vertical load on the center due to this area was found to be $(\frac{1}{2}Rr+\frac{1}{6}R^2)\Delta\omega P$. Now we denote the angle of departure from the center (fig. 2) and consider the radial tension in the diaphragm material to be the resultant of the vertical and a horizontal pull. The total radial tension in the section AD is—

$$\frac{(\frac{1}{2}Rr+\frac{R^2}{6})\Delta\omega P}{\sin\alpha}.$$

The area of the section in tension at the center is $rt\Delta\omega$, t being the thickness of the material. The fiber stress is then—

$$f=\frac{(\frac{1}{2}Rr+\frac{1}{6}R^2)\,\Delta\omega P}{rt\Delta\omega\sin\alpha}=\left(\frac{R}{2}+\frac{R^2}{6r}\right)\frac{P}{t\sin\alpha}.$$

Since the area supporting this load at any other place in the diaphragm varies directly as the distance from the center, the stress at any point is

$$\frac{r}{r_1}\left(\frac{R}{2}+\frac{R^2}{6r}\right)\frac{P}{t\sin\alpha}$$

where r_1 is the distance from the center to the point where the stress is to be calculated. This is for a smooth diaphragm.

In a corrugated diaphragm the maximum stress occurs at the bottom of the corrugation, and, using the symbols of figure 3, is

$$F=f\left(1+\frac{6m}{t}\right)$$

$$=\frac{r}{r_1}\left(\frac{R}{2}+\frac{R^2}{6r}\right)\left(1+\frac{6m}{t}\right)\frac{P}{t\sin\alpha}$$

For the proportions shown in figure 3, $1+\frac{6m}{t}=16$. In order that the maximum fiber stress shall not exceed a certain limit, a diaphragm having corrugations as in figure 3 should theoretically be 16 times as thick as a smooth diaphragm of the same radius and angle of departure from the center.

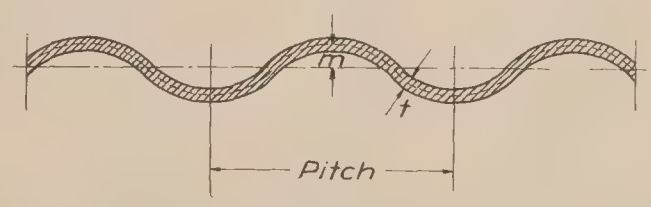


FIG. 3.—Diaphragm corrugations.

In the design of precision altimeters the diaphragm is the element which limits the general size of the mechanism. In the Bureau of Standards precision altimeter No. 1, which is described in detail later in the paper, the diaphragm diameter is about 8 inches. It is believed, however, that a diameter of 5 inches will give the proper relation of diaphragm area to spring stress and deflection. The total area of the diaphragm is then 19.6 square inches.

Substituting in the above equations, the effective area is found to be 8.6 square inches, or 44 per cent of the total area. This area is assumed to be constant for all positions of the diaphragm.

At 14.7 pounds per square inch atmospheric pressure the load which the diaphragm puts on the spring is $8.6\times14.7=126$ pounds. At 30,000 feet the pressure is about one-third of that at sea level. The load on the spring is $8.6\times4.9=42$ pounds. It is, therefore, required that the spring be designed to carry a maximum load of 126 pounds, with a working range of 42 to 126 pounds.

It is a matter of preference whether the diaphragm be corrugated or smooth. As developed above, the metal for a corrugated diaphragm should be several times the thickness of metal for a smooth diaphragm of the same size. Phosphor bronze and nickel silver are good materials for diaphragms. The stress should not exceed 12,000 pounds per square inch for either of these metals.

DRIFT COMPENSATING DIAPHRAGM.

If the altimeter diaphragm has a variable area, as shown diagrammatically in figure 4, it has two advantages over the constant area diaphragm. In the first place the area increases as the altitude increases. This tends to make a straight deflection-altitude curve, or, with a constant multiplying ratio, a uniform altitude scale. The other advantage is the drift compensating feature. If the pressure on the instrument is reduced until the pressure corresponds to a chosen altitude, and is held constant at this value, the reading then increases gradually, owing to drift in the spring and diaphragm. With the variable area diaphragm, this drift causes an increase in the effective diaphragm area, thereby augmenting the load on the spring which, in turn, tends to diminish the drift.

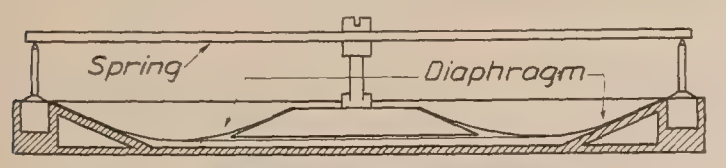


FIG. 4.—Drift compensating diaphragm.

This has been verified experimentally by the fact that the spring used in the precision altimeter, described below, shows less hysteresis when coupled with the diaphragm as used in the instrument than does the spring alone when tested by deflecting it with weights.

A possible disadvantage of the variable area diaphragm is the liability to a change in the calibration caused by stretching of the diaphragm. Further experience is necessary before the seriousness of this possibility can be determined.

SPRING DESIGN.

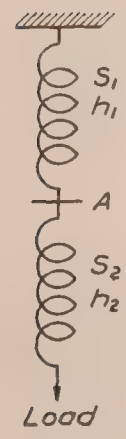


FIG. 5.—Spring system.

It can be proved that the hysteresis in an aneroid barometer may be reduced to a very low value if a spring with good elastic properties is used in conjunction with a relatively very flexible diaphragm, although the diaphragm itself may have poor elastic properties. This principle has been applied in the construction of the altimeter described in this paper and may be demonstrated as follows:[1]

If two springs with stiffness S_1 and S_2, respectively, are connected together as in figure 5, the stiffness of the system to a force applied at A is $S=S_1+S_2$. (Stiffness is defined as the amount of force applied per unit of deflection.)

Now, if h_1 and h_2 are the hysteresis values of the two springs, respectively, for a certain range of deflection, the hysteresis of the point A for the system is—

$$h=h_1\frac{S_1}{S}+h_2\frac{S_2}{S}$$

If S_1 is the stiffness of the spring and S_2 is the stiffness of the diaphragm; e. g., it is estimated that in the Bureau of Standards precision altimeter No. 1 $\frac{S_1}{S}=0.99$ and $\frac{S_2}{S}=0.01$. If the hysteresis in the spring for the maximum range is 0.02 per cent and the hysteresis in the diaphragm for the corresponding range is 2 per cent, the hysteresis of the combination is

$$h=0.02\frac{99}{100}+2.0\frac{1}{100}=0.04 \text{ per cent.}$$

COMPOSITION OF THE STEEL AND PERMISSIBLE STRESS.

Several investigators, in particular Bairstow,[2] and Smith and Wedgewood [3] have studied the possible relations existing between hysteresis in steel and the fatigue strength. They have shown that the limit of proportionality as ordinarily determined can not be taken as a criterion of the limiting stress below which there is negligible or zero hysteresis, but that the fatigue limit as determined by repeated stress is the limit below which there is no measurable hysteresis. Further, the width of the hysteresis loop increases with increasing stresses beyond the fatigue limit and for a stress range greater than the fatigue range the width of the hysteresis loop increases with increasing number of repetitions of stress.

Using these facts as a basis, it is readily seen that the permissible stress in an aneroid spring, or in any steel spring where true elastic reaction is necessary, is the fatigue limit of the steel used. This fatigue limit for good grades of commercial alloy spring steels is approximately 25,000 to 30,000 pounds per square inch fiber stress. However, since the width of the hysteresis loop is very small for stresses somewhat above the fatigue limit, and because of the relatively very low number of stress alternations an aneroid spring would undergo during its lifetime, compared with the number required to cause failure from fatigue, a maximum fiber stress of 50,000 pounds per square inch is probably not excessive for heat-treated alloy steels. This is especially true in an aneroid spring where the stress is never reversed (tension to compression) and there is a consequent tendency for the spring to adjust itself to the range over which it operates.

Extensive experimenting was done in the attempt to make a suitable spring for the precision altimeter No. 1.

The best spring obtained was of a special nickel-silicon steel having a yield point of 276,500 pounds per square inch. Increasing the maximum fiber stress of this spring from zero to 100,000 pounds per square inch and back to zero in 20 minutes, the maximum width of the hysteresis loop was 0.3 per cent of the total deflection. To be on the safe side and to permit the use of

[1] M. D. Hersey. Theory of Stiffness of Elastic Systems. Journal of the Washington Academy of Sciences, Vol. VI, p. 569, 1916. The principle of having the spring as stiff as possible compared to the diaphragm was proposed by Mr. Hersey at the outset of the present work, and may be applied to altimeter design in general.

[2] Bairstow, Leonard. Phil. Trans. 1910, v. 210, p. 35.

[3] Smith and Wedgewood. Journ. Iron Steel Inst. G. B., 1915, 1, 365.

commercial heat-treated alloy steels, 50,000 pounds per square inch is chosen as an absolute maximum permissible fiber stress. The nickel-silicon steel on a range of 50,000 pounds per square inch should give a hysteresis of only one-tenth of 1 per cent. If the range of the instrument is 20 inches of mercury, the maximum width of the hysteresis loop is 0.02 inch, or 30 feet if the instrument is an altimeter.

SHAPE, STIFFNESS, AND DEFLECTION.

The spring should be shaped so that as near as practicable there is a uniform stress over its entire length. This method decreases the stiffness but increases the deflection without increasing the maximum fiber stress.

It is desirable to have the spring straight at either the middle of its working range or at sea-level pressure. To accomplish this, the spring is made so that its free position is curved. If it is to be straight at the load which is the middle of the working range, its free shape should be the reverse of the curve taken by a similar straight spring when its total load is applied.

The following method may be used to calculate the deflection of a leaf spring the width and thickness of which varies with the position along the length of the spring.

Figure 6 represents a cantilever spring which is the same as one-half of the altimeter spring. The cantilever is fastened at A, and this corresponds to the middle of the altimeter spring. The notation used is:

R = radius of curvature of the spring.
E = modulus of elasticity of the steel.
I = the moment of inertia of the section.
M = the moment caused by the load.
P = the load, concentrated at the end.
b = the width of the section.
t = the thickness of the section.
x = the distance from the end.
D = the deflection at the end.
f = the fiber stress.
a = the length of an increment for graphical computation.

The radius of curvature at any point along the length is

$$R=\frac{EI}{M}=\frac{E\,bt^3}{12\,Px}.$$

The angle α is then a/R_1, etc.

The deflection, $d_1=\dfrac{a \sin \alpha_1}{2}$

$$d_2=a\left(\sin \alpha_1+\frac{\sin (\alpha_1+\alpha_2)}{2}\right)$$

The end deflection is $D=a\Sigma[(\sin \alpha_1)+\sin (\alpha_1+\alpha_2)+\cdots\cdots \frac{1}{2}\sin (\alpha_1+\cdots\cdots \alpha_n)]$.

This formula is used for calculating the shape of the free spring so that it will be straight at any required load, and also for calculating the stiffness and deflection. It has been found by experiment that the stiffness for such deflections as here experienced is practically constant.

Complete calculations for the spring to be used with the 5-inch diaphragm previously discussed will be given. The modulus of elasticity of the steel is taken as 29×10^6 pounds per square inch. The correct thickness for 50,000 pounds per square inch maximum fiber stress is found to be about 0.095 inch. For the shape calculation the load at the end is 42 pounds, or one-half of the diaphragm pull at the middle of the working range. The length, $3\frac{1}{4}$ inches, is divided into six parts, each 0.54 inches long.

$$R=\frac{EI}{M}=\frac{Et^3}{12p}\cdot\frac{b}{x}=\frac{29\times10^6\times.000857}{12\times42}=49.3\frac{b}{x}$$

x	b	b/x	R	α	$\Sigma\alpha$	$\sin\Sigma\alpha$	$\frac{d}{.54}$	d
2.98	2.83	0.95	46.9	0.0115	0.0115	0.0115	0.0057	0.003
2.44	2.50	1.02	50.3	.0107	.0222	.0222	.0226	.012
1.89	2.16	1.14	56.2	.0096	.0318	.0318	.0496	.027
1.35	1.83	1.36	67.1	.0080	.0398	.0398	.0854	.046
.81	1.50	1.85	91.2	.0059	.0457	.0457	.1281	.069
.27	1.17	4.33	213.5	.0025	.0482	.0482	.1751	.095

The total deflection is 0.095 inch. To get the shape dimensions given in figure 6 each deflection is subtracted from the total deflection.

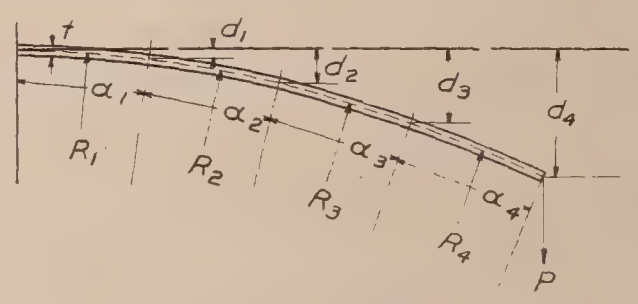


FIG. 6.—Spring deflection.

The stiffness, $s=\frac{2\times42}{.095}=890$ pounds per inch. The deflection for the working range is $\frac{84}{890}=.095$ inch. The fiber stress—

$$f=\frac{6M}{bt^2}=\frac{6\times3.25\times63}{2.75\times0.00902}=49{,}500 \text{ pounds per square inch.}$$

One-half of the maximum load of 126 pounds is used in calculating the fiber stress.

MULTIPLYING MECHANISM.

CHARACTERISTICS.

Since the area of the diaphragm and the stiffness of the spring remain constant, the diaphragm deflection will bear a linear relation to the pressure. The problem is to design a multiplying mechanism such that the movement of the end of the pointer bears a direct linear relation to the movement of the diaphragm.

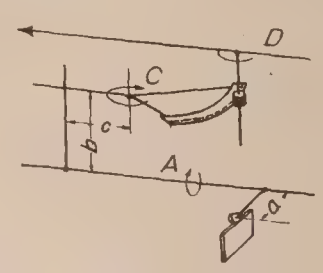


FIG. 7.—Multiplying mechanism.

Figure 7 is a representation of a lever mechanism. The sector shaft and the main shaft are perpendicular and in the same plane.

Let a, b, and c be the lengths of the first, second, and third lever arms, respectively.

Let A be the angle between a and the horizontal.

Let B be the angle between b and the vertical.

Let C be the angle between c and the plane drawn through the axes of both the main shaft and the sector shaft.

Let D be the angular deflection of the pointer.

Let x be the vertical movement of the diaphragms ($x=0$ when the arm a is horizontal).

Let R be the multiplying ratio of the sector to the pinion.

Assume that when the lever a is horizontal, b is vertical, and c is in the same plane as the sector shaft and the main shaft. That is, when $x=0$, $A=0$, $B=0$, $C=0$.

Now the locus of the point of contact between the lever arms b and c is a straight line, perpendicular to the plane in which the main shaft and the sector shaft were placed. We will use m as a symbol to denote the distance of the point of contact from this plane.

$$m=b \tan B$$

$$\text{and } m=c \tan C$$

$$\therefore c \tan C=b \tan B$$

$$\text{but } B=\sin^{-1}\frac{x}{a}$$

$$\therefore c \tan C=b \tan \sin^{-1}\frac{x}{a}$$

$$C=\tan^{-1}\left(\frac{b}{c} \tan \sin^{-1}\frac{x}{a}\right)$$

$$D=R \tan^{-1}\left(\frac{b}{c} \tan \sin^{-1}\frac{x}{a}\right)$$

If we express this equation as a series it appears that D comes very close to being exactly proportional to x when $\frac{b}{c}$ is about 1.25. As developed, the multiplying ratio is approximately constant for all positions of the diaphragm. It follows therefore that the pressure scale will be uniform.

ADJUSTMENTS.

The shape of the calibration curve may be adjusted by changing the ratio of the crossed lever arms, and the scale value, or the general direction of the calibration curve, by adjusting the length of the arm making contact with the top of the diaphragm. These adjustments should be made in the order mentioned, for the first changes the scale value but the last does not change the shape of the calibration curve.

TEMPERATURE COMPENSATION.

BIMETALLIC BARS.

It is not practicable to formulate exactly a method of temperature compensation until the nature and magnitude of the temperature errors are determined experimentally. In general, the introduction of one or more bimetallic bars somewhere in the lever mechanism will compensate the instrument.

In a bimetallic bar composed of two metals of different temperature coefficients of expansion, there is usually within each metal a neutral surface which receives no stress, the metal on one side of this surface being in tension and the metal on the other side being in compression. It can be proved that through a large range of temperatures these neutral surfaces do not change their position in the bar and, therefore, the distance between them remains constant.

When a bar, straight at one temperature, takes a curved shape at another temperature, this curve is the arc of a circle, and the radius of curvature is the same for any position along the length of the bar. Starting with these assumptions, formulæ for calculating the deflection of any bimetallic bar have been developed, leading to the conclusion that such a bar will give a maximum deflection when the steel strip is approximately two-thirds as thick as the brass strip in the usual case of the brass and steel combination.

EFFECT OF AIR IN VACUUM BOX.

Another possibility of temperature compensation is the adjustment of the amount of air left in the vacuum box. Less than one-half millimeter of air in the box has no effect on the operation of the instrument, and it is therefore never necessary to exhaust to a high vacuum. For temperature compensation 5 millimeters is the least amount which will have an appreciable effect. The box of the precision altimeter is exhausted to 0.2 millimeter.

POSITION ERROR AND BALANCING FOR VIBRATION.

The most serious vibration effect is caused by a vibration at right angles to the plane of the diaphragm. This is because it is not practical to balance the spring and diaphragm against other parts. The position error is a maximum when the face is down. To minimize position errors, the instrument should be calibrated and mounted in the airplane with the dial in the same relative position.

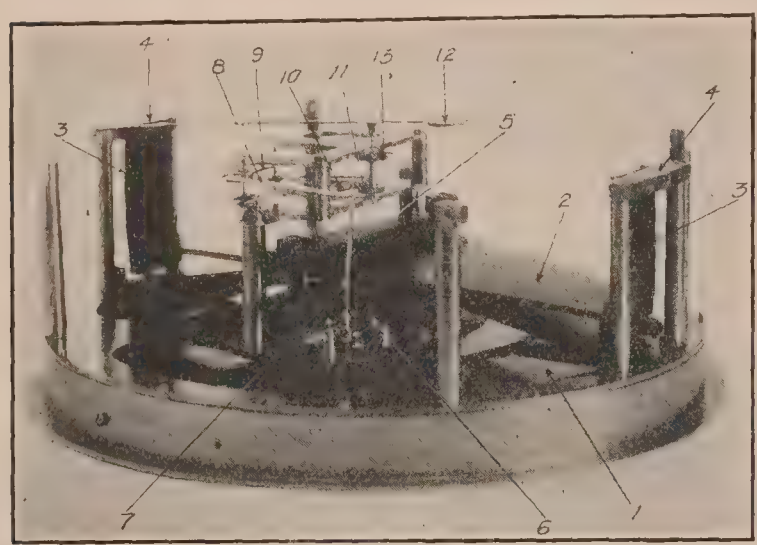


FIG. 8.—Mechanism of Bureau of Standards precision altimeter No. 1.

Both sector and pinion should be balanced about their axes. By referring to figure 7 it can be seen that an angular acceleration of the instrument in a clockwise direction tends to cause the pointer and the sector to move in a counterclockwise direction relative to the rest of the instrument. Now, they both can not move counterclockwise because they are geared together. If the moment of inertia of sector bears the same ratio to that of the pointer and pinion assembly as the ratio of the number of teeth on the sector to that on the pinion they will be balanced and the pointer will keep its position on the correct graduation. To get this ratio of the moments of inertia the sector should be made heavy and the pointer light.

THE BUREAU OF STANDARDS PRECISION ALTIMETER NO. 1.

This instrument was developed by the Bureau of Standards for the Army and the National Advisory Committee for Aeronautics, based on the theoretical considerations given above. In it are eliminated to a large extent the errors commonly found in aneroid barometers, which have also been discussed in detail in this paper.

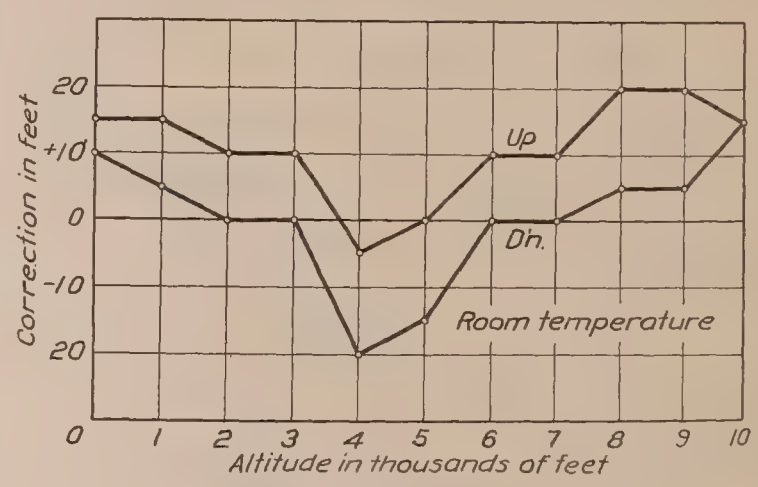


FIG. 9.—Precision altimeter calibration curve.

Referring to figure 8, a flexible diaphragm (1) is coupled to a stiff mainspring (2) by means of yoke (5), the two bolts of which screw into the center plate of the diaphragm. The mainspring is swung on two thin flexible springs (3) from spring supports (4). This method of support permits the free lateral movement of the end of the spring when deflected. A short pin mounted at the end of a short arm from the spindle (6) fits into a conical bearing soldered in the center of the diaphragm plate. Deflections of the diaphragm cause (6) to rotate. This motion is multiplied by upright (7), which is connected by rod (8) to tail of cam (9). A hairspring mounted on the same shaft as cam (9) maintains a slight pressure on the conical bearing. Cam (9) is made with a varying radius so as to facilitate the adjusting of the lever system to give an equally spaced altitude scale. The movement of this cam is communicated to a sector (10) by means of a small roller bearing on the end of arm from the sector shaft. This sector meshes with pinion (11) and rotates the hand (12). The roller bearing is held on the cam (9) by the tension of the hairspring (13).

Figure 9 is a calibration curve of this instrument. It is important to notice that the difference between the up and down readings is only 15 feet and is very small compared with that of an ordinary service instrument. The instrument embodies the drift compensating feature. As previously stated, there is some doubt as to whether an instrument with this drift compensating feature will hold its calibration, but this particular instrument has shown no change. This point is under further investigation, however. Preliminary tests indicate the temperature error of this instrument without bimetallic bars to be small.

The successful development of a precision instrument of this type is dependent on very careful workmanship. The authors were fortunate in having the assistance of Mr. F. Cordero, of the Bureau of Standards, in the construction of this instrument.

REPORT No. 126.

ALTITUDE INSTRUMENTS.

PART III.

STATOSCOPES AND RATE-OF-CLIMB INDICATORS.

By Atherton H. Mears.

INTRODUCTION.

Statoscopes are used for indicating when aircraft are maintaining a constant altitude, or for quickly determining small changes of altitude. The ordinary altitude instruments, the altimeter and the barograph, are of little use for this purpose. They are too slow in action and not sufficiently sensitive. Few altimeters or barographs will indicate with reliability a change of altitude of 20 feet. The statoscope is especially useful in the navigation of balloons and dirigibles, since it shows immediately when the aircraft is ascending or descending. This gives the navigator warning before the airship has had time to gain an appreciable vertical velocity; and thus avoids the necessity of making wasteful adjustments of either the gas or ballast. Statoscopes, of the bubble type, can also be used to indicate approximately the rate of ascent or descent of an airplane by determining the rate at which the bubble breaks.

Fig. 1.—Custer bubble statoscope.

Rate-of-climb indicators have the advantage over statoscopes of giving at once the approximate rate of climb, without time observations and in most cases without subsequent calculations. This is advantageous in helping the pilot to attain his maximum climbing speed, for example in aircraft performance tests, since he has only to observe the instrument and so manipulate the controls as to get the maximum indications. In the landing of balloons and dirigibles it is also very important to know the rate of descent.

The Bureau of Standards has recently developed a mechanical type of rate-of-climb indicator of such sensitiveness and accuracy as to fulfill the requirements of both statoscopes and rate-of-climb indicators on nearly all types of aircraft.

DESCRIPTION OF INSTRUMENTS.

STATOSCOPES.

Bubble statoscopes—American.—The indicating bubble type, as shown in the following photograph (fig. 1) and diagram (fig. 2), consists of a thermally insulated air chamber with an outlet to the external air. To this is attached a small special radium-illuminated curved glass gauge or manometer. The curvature of this gauge is varied according to the desired sensitivity to change in altitude. In this gauge is placed a very small amount of liquid, which should have low density and low vapor pressure. Its viscosity should not be excessive at a low temperature. At each end of the gauge is blown a glass trap which prevents the liquid from escaping either into the air chamber or to the outside air. When the pressure is changed in any manner the

liquid moves toward that end of the gauge which is at the lower pressure. The enlarged end of the gauge causes the liquid to form a bubble (hence the name bubble statoscope). This bubble is now pushed still farther into the enlarged section of the gauge and breaks, allowing the air to flow past, thus equalizing the pressure between the inside and outside air. The liquid now flows back into the center of the gauge and forms an indicating medium again. The above cycle of operation continues as long as there is a change in pressure.

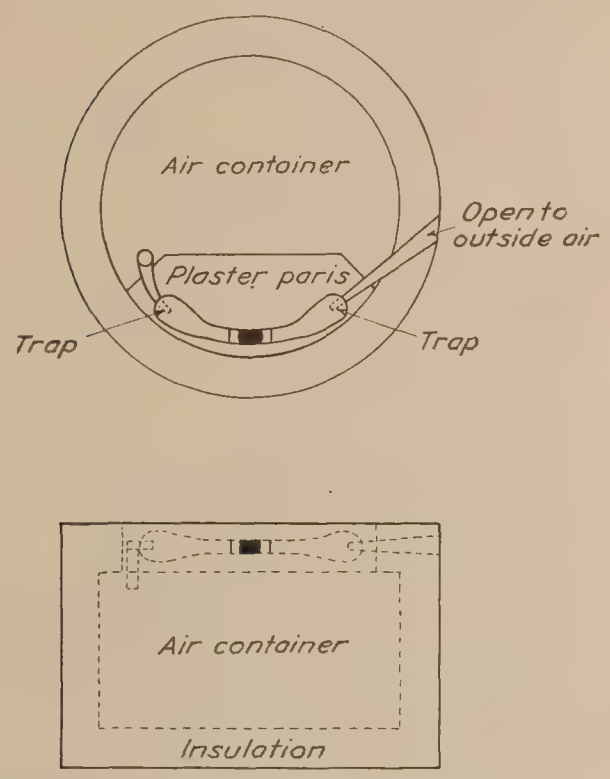


Fig. 2.—Diagram of Custer bubble statoscope.

Fig. 3.—British Wright bubble statoscope.

The sensitivity is such as to produce a movement of the bubble equal to one of the arbitrary scale divisions for a pressure change of 0.02 of an inch of mercury. This corresponds to a

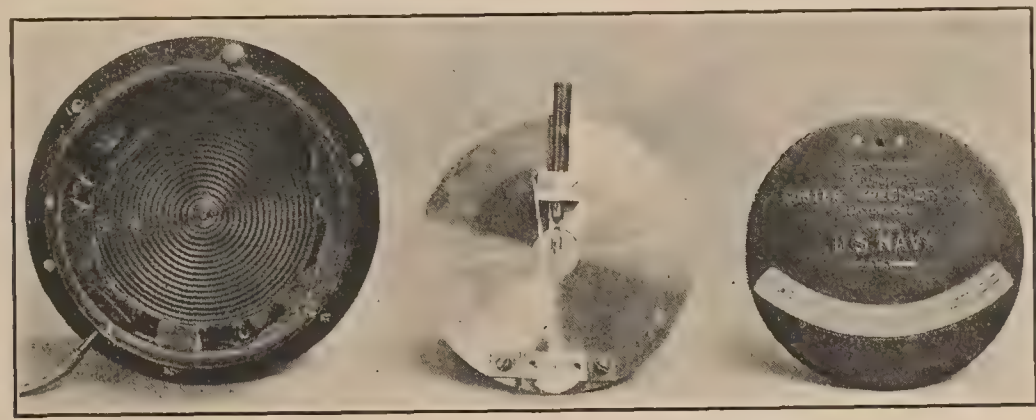


Fig. 4.—Mechanical statoscope diaphragm, multiplying mechanism and dial.

change in altitude of 20 feet at sea level or 26 feet at an altitude of 10,000 feet. This type of instrument has been the most extensively used of the various types of statoscopes.

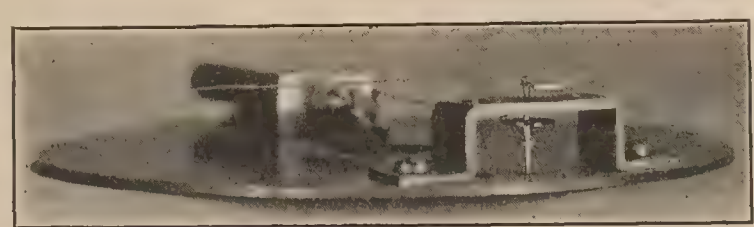


Fig. 5.—Multiplying mechanism of mechanical statoscope.

Bubble statoscopes—British.—The essential details and operations of this statoscope (fig. 3) are the same as in the American instrument. Considerable care is taken in the thermal insulation of the air chamber, which is a Dewar flask incased in wool felt. The sensitivity of the instrument is such that a change in pressure corresponding to 2 or 3 feet of altitude at sea level is indicated by this instrument.

Mechanical statoscopes—American.—The indicating pointer type shown by photographs (figs. 4 and 5) and diagrams (fig. 7), consists of a cylindrical metallic air chamber (1), one end of which is a very thin, flexible, corrugated, metallic diaphragm (2). Deflections of this diaphragm are indicated on the dial of the instrument by means of pointer (6), which is actuated by the multiplying mechanism in the following manner: The motion of the diaphragm is transmitted to the multiplying mechanism by means of the upright soldered to the center of the diaphragm. This is in contact with one arm of the bell-crank shaped lever (3) to the other arm of which is attached a connecting link (4). The other end of this connecting link is attached to a small cam (5), on the shaft of which is mounted the pointer (6). There is an outlet (8) to the air chamber to which is attached a small rubber tube. The knurled-headed screw (9) is used to adjust the zero setting of the pointer. To operate the instrument the opening in the air chamber is closed by pinching the rubber tubing. If now the external air pressure on the instrument is changed, the diaphragm deflects, due to the difference in pressure; outward if the pressure is reduced, indicating an ascent; and inward if increased, showing a descent. The dial of the instrument has a luminous arbitrarily divided scale. The value of the divisions is determined by experiment. The instrument shown in the figures 4 and 5 was not thermally insulated, hence slight variations in temperature cause it to deflect when the outlet is closed, thus making it unreliable.

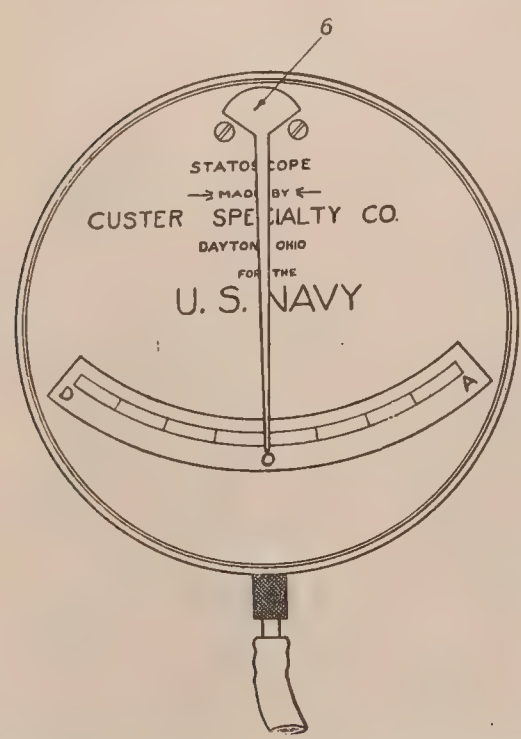


FIG. 6.—Assembly drawing mechanical statoscope.

RATE-OF-CLIMB INDICATORS.

The katanoscope, chronometric type.—This instrument is a modification of the mechanical statoscope described above. The outlet to the external air in the instrument is automatically opened and closed at regular time intervals (every 20 seconds) by means of clockwork. It has a thermally insulated air chamber the diaphragm of which is made of rubber, oiled silk, or similar materials. Deflections of this diaphragm, as in the case of the mechanical statoscope, are indicated on the dial of the instrument through the intermediary action of a multiplying mechanism.

The operation of the instrument is as follows: Suppose the pressure of the external air to be changing as in the flight of an aircraft. The clockwork controlling the automatic valve is started. This closes the automatic valve, which traps the air in the chamber. The diaphragm deflects, due to the change of external air pressure, this motion of the diaphragm being indicated by the movement of the pointer over the dial of the instrument. This deflection continues until the valve is opened by the clockwork, equalizing the internal and external pressure. The pointer now goes back to zero, indicating that the diaphragm is back to its initial position. The amplitude of this periodic movement of the pointer is a measure of the rate of change of pressure, from which the rate of climb can be computed. These instruments are furnished with an arbitrary scale, the values of its division in rate of change of pressure (rate of climb) being determined by experiment. From data obtained in the laboratory, a

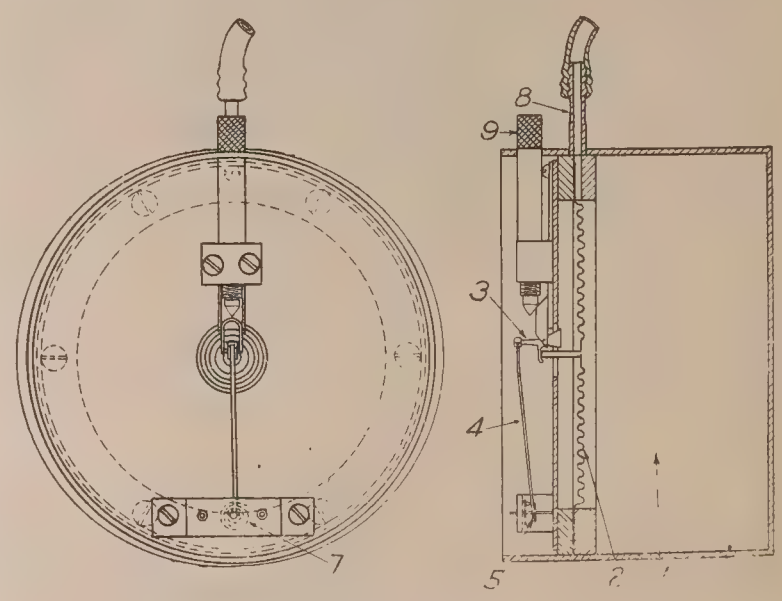


FIG. 7.—Assembly drawing mechanical statoscope.

table or chart may be constructed of the indications of the instrument at various rates of climb and altitudes. From this the readings of the instrument can be converted into rates of climb at all altitudes. Since the instrument indicates only pressure differences, the above table is necessary. A little consideration of the pressure altitude relation shows this instrument will give widely different readings for the same rate of climb at different altitudes.

Leak type of rate-of-climb indicators.—The inherent disadvantages of the katanoscope have lead to the development of rate-of-climb indicators of the capillary leak type. These instruments all operate on the same general principle. A thermally insulated air chamber is closed to the external air, except for a small opening which may consist of either a needle valve or capillary tube. If the pressure of the external air is varying in any manner, the pressure inside the air chamber will lag behind that of the external air, due to the resistance offered to the equalization of pressure by the leak opening. This causes a pressure difference which is measured

FIG. 8.—British rate-of-climb indicator and Bureau of Standards rate-of-climb indicator.

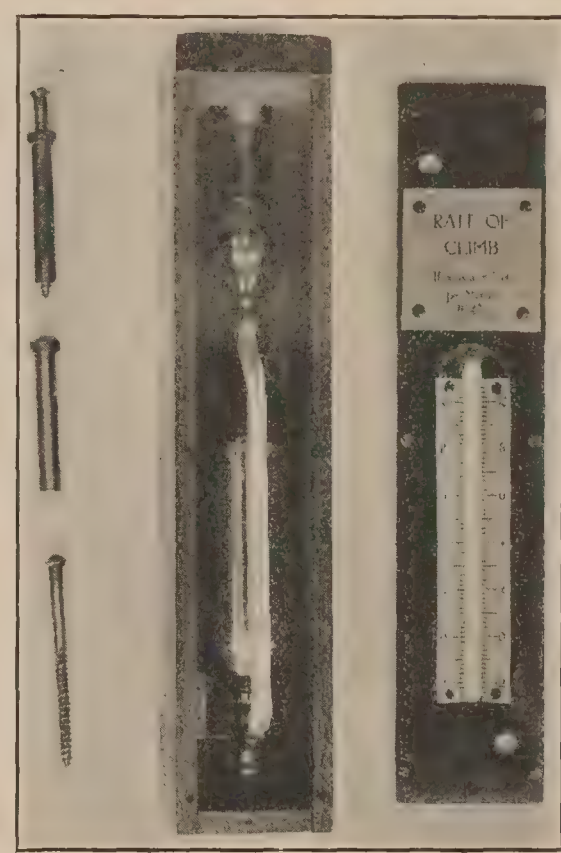


FIG. 9.—Liquid type rate-of-climb indicator.

by some type of indicating mechanism, either a liquid manometer or the deflection of a sensitive diaphragm. The nonturbulent flow through capillary tubes depends on the viscosity of the air; and since the viscosity is independent of the air density, it can be easily proved that an instrument constructed on this principle and graduated to indicate rate of climb will have a scale value which will be approximately independent of the altitude as determined by pressure. The scale value of a rate-of-climb indicator is here defined as the ratio of the true rate of climb to that indicated by the instrument.

Models using a liquid manometer—British R. A. E. model.—This instrument is typical of all liquid types of rate-of-climb indicators, which consist of an air chamber closed to the air except for a leak or vent. The pressure difference between the internal and surrounding air is measured by a liquid manometer, in most cases an ordinary U tube. This instrument, illustrated on the following pages (figs. 8, 9, and 10), consists of a thermally insulated chamber (Dewar bulb) (6) closed to the air except for a capillary leak tube (5). Any pressure difference between the two ends of the capillary tube due to variation of the external air pressure is indicated by the liquid (7) rising or falling in the specially constructed manometer tube (4). Both ends of this manometer are blown in such a manner as to prevent the liquid from spilling out no matter

how the instrument is orientated or to what pressure change it is subjected, a decided advantage over some other types of instruments.

There is attached to the trap (3), by means of a rubber tube (2), a device for flushing the manometer (1). This is operated by pressing a small knob inward, then closing the hole through

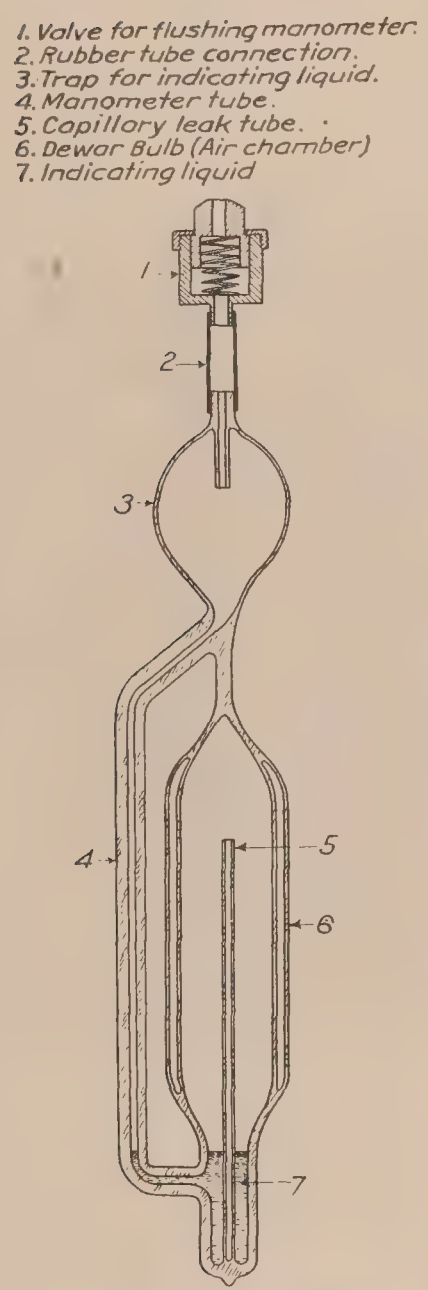


FIG. 10.—R. A. E. (British) rate-of-climb indicator.

FIG. 11.—British Wright rise-and-fall indicator and case.

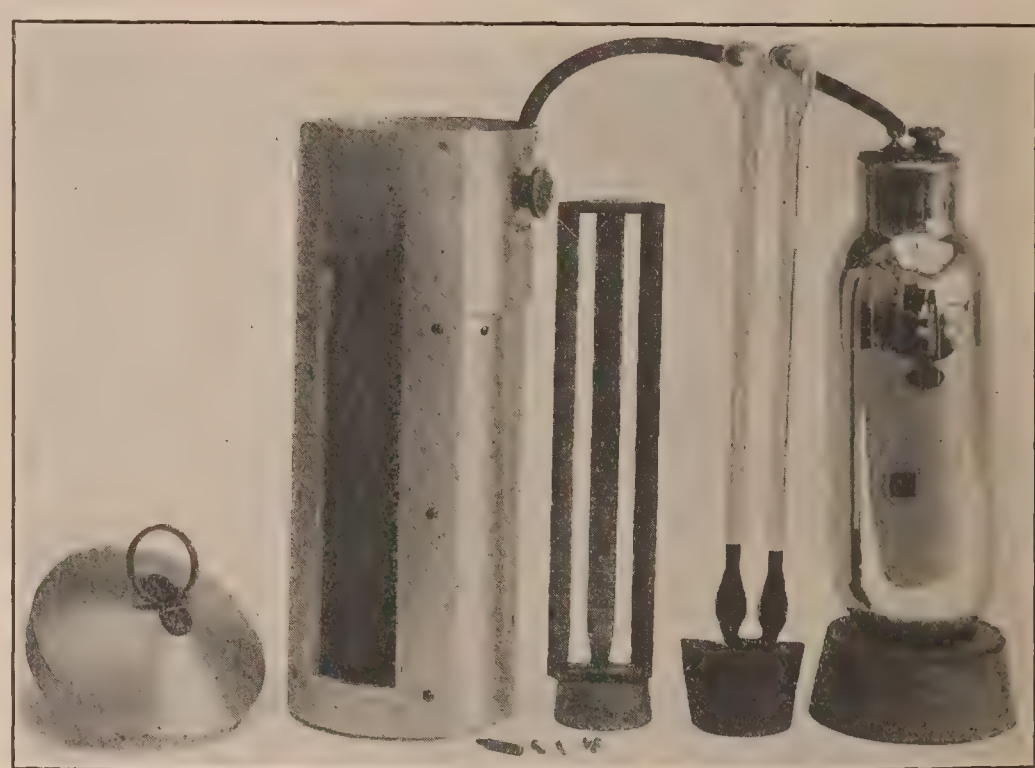


FIG. 12.—Rise-and-fall indicator and case disassembled.

this knob by the finger and allowing the knob to be pressed outward by the spring tension. The suction produced causes the liquid to rise in the manometer tube, thus flushing it.

This instrument has a range of rate of ascent up to 1,100 feet per minute and a rate of descent of 200 feet per minute. The time lag is rather large in this instrument, about 30 seconds. That is the time interval required before the instrument indicates its true rate of climb when properly calibrated.

The British Wright Co. rise and fall indicator.—The main details of this rate-of-climb indicator, as may be seen by referring to the following photographs (figs. 11 and 12), are the same as in the previously described instrument. The glass capillary is replaced by one of platinum. The specially designed manometer is replaced by a U tube. There is a cock attached to the manometer which when closed prevents the instrument from indicating. The range is from 0 to 2,000 feet per minute, both for ascent and descent.

German balloon variometer (fig. 13).—This instrument is similar to the British R. A. E. described above. The main difference is the addition of a filter and dryer for the air before it passes through the capillary leak tube. The manometer is inclined and hence requires only a small pressure difference for a given reading, thereby cutting down the time lag but at the same time making it necessary to exercise great care in maintaining the instrument level. The air chamber is a Dewar bulb surrounded with cork, which provides exceptionally good thermal insulation. This instrument is made in two ranges from 0 to 600, or 0 to 1,200 feet per minute. The time lag is small, about 5 seconds. Sometimes instruments of both ranges are mounted together, the combination being known as a double variometer.

FIG. 13.—German balloon variometer.

Vertimeter (fig. 14).—This instrument, of American manufacture, presents a unique departure from the general construction of rate-of-climb indicators. The air chamber consists of a large uninsulated stream-lined receptacle, which is mounted on the plane away from the cockpit, where it will maintain the temperature of the free air. The indicating mechanism of this instrument is shown in the following diagram (fig. 15). The air chamber is attached to the indicating mechanism by a long rubber tubing, the length and bore of which must be of the proper dimensions or the calibration of the instrument will be affected. The outlet (5) of the indicating mechanism is directly connected to the manometer (1) and a needle valve (4), the needle valve taking the place of the capillary leak tubes in the other instruments. This valve is in communication with the surrounding air through the outlet (6). When the needle valve is adjusted so that the manometer gives the proper indications of rate of climb, the valve lock nut (7) of the valve is soldered in position. The trap in this instrument does not prevent the escape of liquid, either into the external air if the rate of ascent is excessive or into the needle valve and the air chamber in the case of too rapid descent. The design of the trap is such that a loss of liquid occurs whenever the velocity of the latter is great. In the former case the instrument must be refilled, and in the latter the needle valve must be taken out and cleaned and the instrument recalibrated. Either contingency is likely to happen in flight, especially if the instrument is tilted during climb or descent. The range of the instrument is from 0 to 2,000 feet per minute for ascent and 0 to 3,000 feet per minute for descent. The time lag is small, 5 seconds, but the scale is not so open as in other instruments.

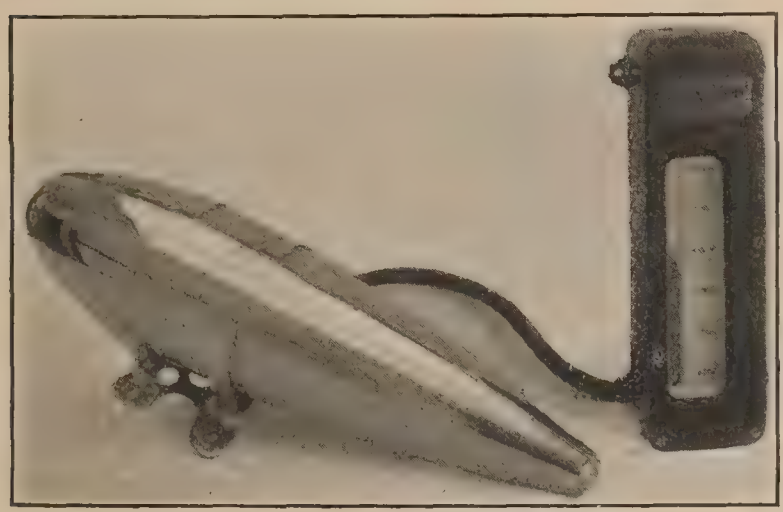


FIG. 14.—The vertimeter, an American rate-of-climb indicator.

Models using a diaphragm manometer.—Mechanical rate-of-climb indicators differ radically from the liquid type in the substitution of a sensitive diaphragm in the pressure-measuring element in place of the liquid manometer. Several models of this type have been designed and constructed at the Bureau of Standards. Two designs, known as models No. 2 and No. 3, are in practical use and are described below in detail.

Bureau of Standards, model No. 2: The operation of this instrument may be studied by referring to the following diagram and photographs (figs. 16 and 17). A bank of metallic diaphragms (1) connected together at the center by metallic rings forms the air chamber of this instrument. To this air chamber is attached a capillary leak tube (2.) The deflection of these diaphragms is communicated to the indicating hand attached at (8) through the multiplying lever (3), by the phosphor-bronze strip (4). This motion is communicated to the arbor (7) by means of another phosphor-bronze strip (4′). A

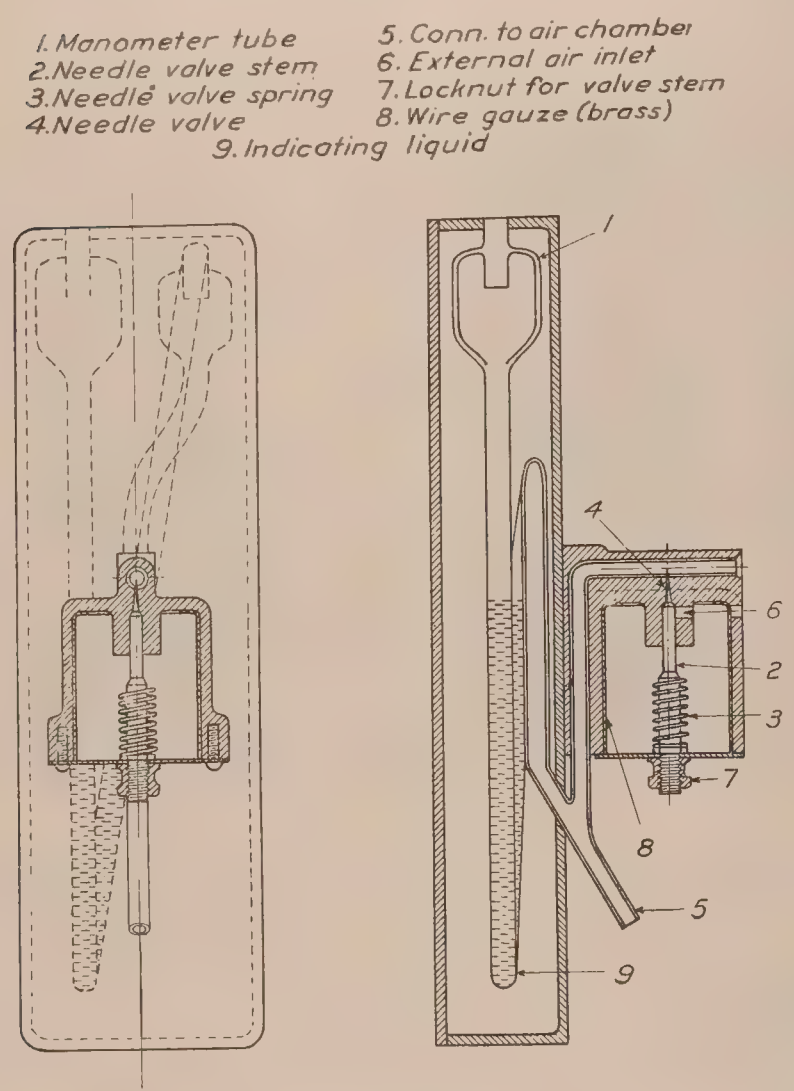
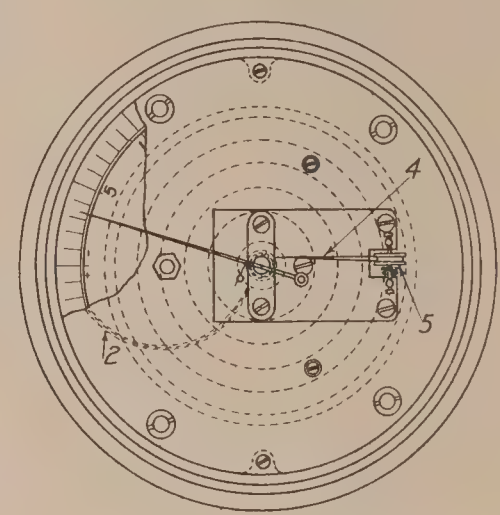


FIG. 15.—Vertimeter.



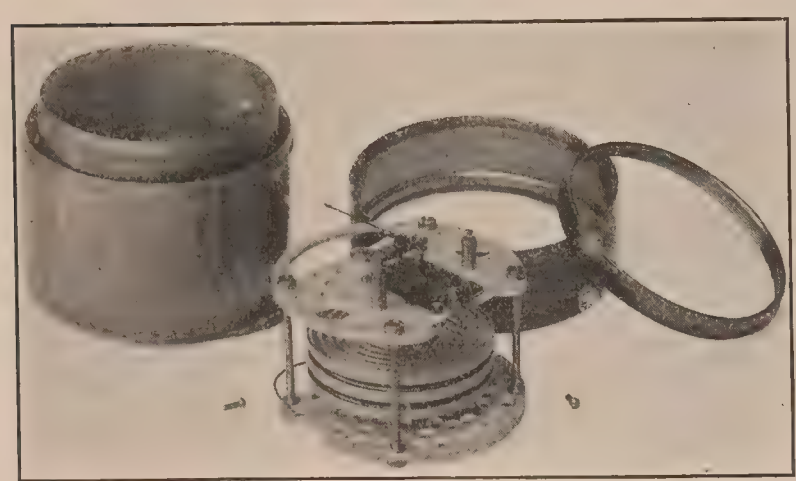


FIG. 16.—Dissasembled Bureau of Standards rate-of-climb indicator.

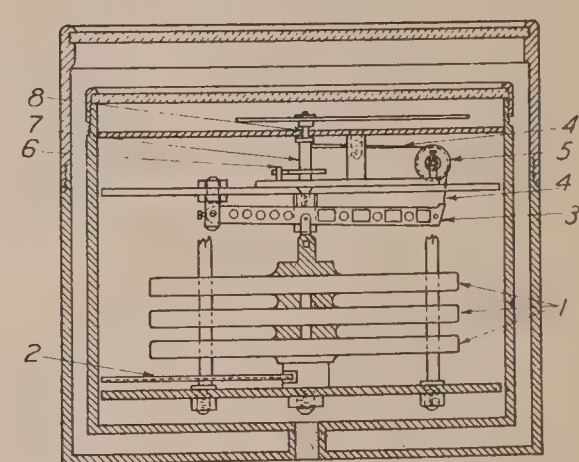


FIG. 17.—Bureau of Standards rate-of-climb indicator mechanical type. Model No. 2.

slight tension is kept on this system by the hairspring (6). Thermal insulation is provided by an air jacket around the mechanism. The range of this instrument is from 0 to 2,000 feet per minute both for ascent and descent, but it can be subjected to all rates that would be experienced in the flight of aircraft without damage. The time lag of the instrument is 30 seconds.

Bureau of Standards, model No. 3: Certain modifications of the above-described model were found desirable in order to reduce the time lag, and to diminish other sources of error.

These instruments were incorporated in the design of a new instrument for the Balloon and Airship Division, Army Air Service. Referring to diagrams and illustrations of model No. 3 (figs. 18, 19, and 20), the following are the more important changes:

First. The substitution of a large-diameter rigid type of air chamber (1) with thin diaphragm (3) in place of a bank of metallic diaphragms connected together at the center by metallic rings, which formed the air chamber in the older model. This modification makes it possible to get a sufficient deflection of the diaphragm with a smaller pressure difference, thus

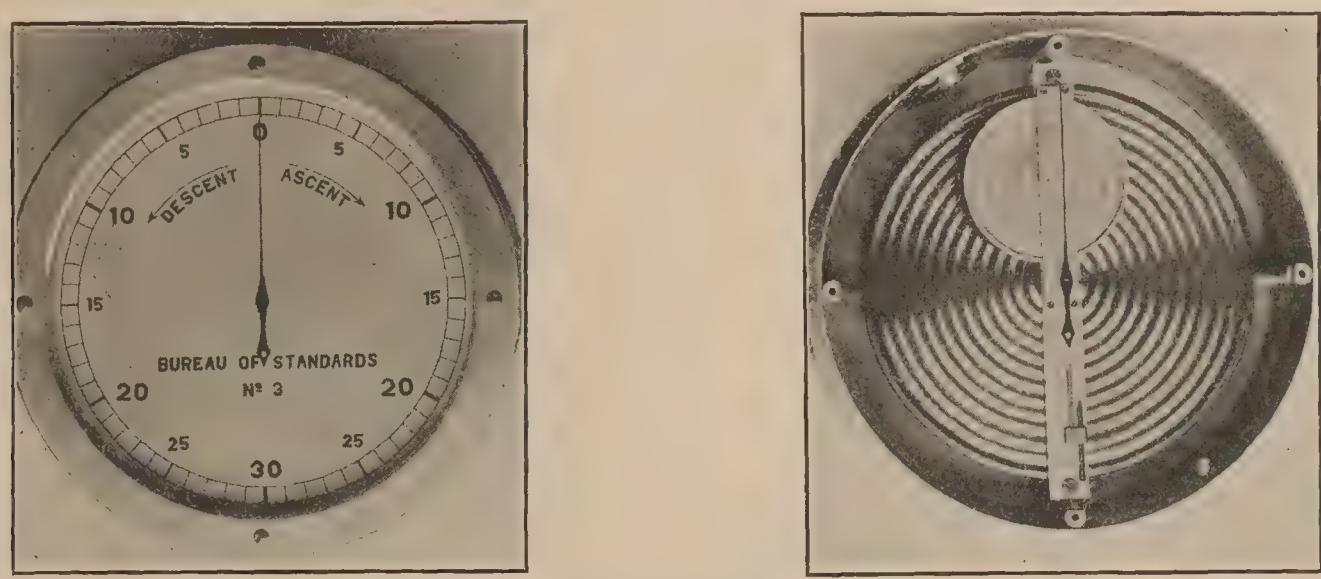


FIG. 18.—Rate-of-climb indicator, model No. 3.

cutting down the time lag to a fraction of its former value, and at the same time eliminating the large inclination error in the previous instrument, due to the mounting and mass of the diaphragms.

Second. The use of a glass capillary tube (6) instead of one of copper. This modification suggested itself when the calibration of model No. 2 was observed to change slightly, which was thought to be due to corrosion of copper tube.

Third. The elimination of all levers, facilitating the counterbalancing of the working parts of the instrument.

Fourth. The substitution of a helical spring (13) in place of an ordinary watch hairspring. This allows the addition of the zero regulator, and has also the following advantages: It make it possible to obtain a much finer adjustment of the tension on the diaphragm; it furnishes a

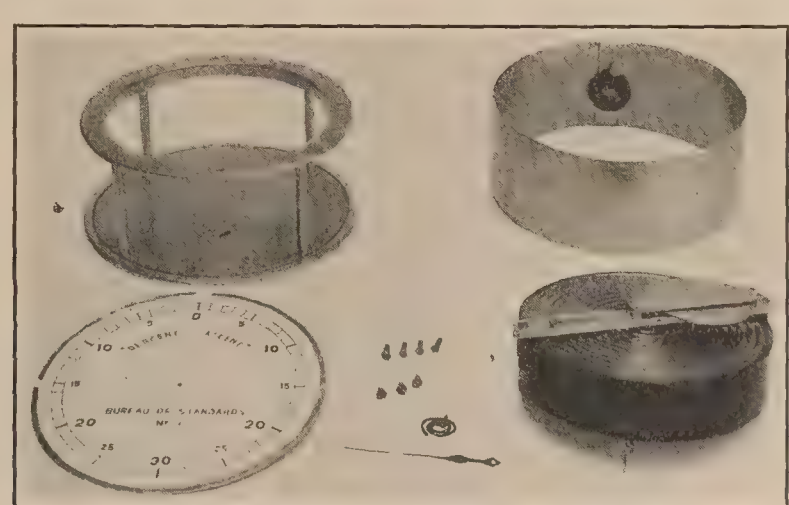


FIG. 19.—Rate-of-climb indicator, model No. 3.

means of adjusting the calibration of the instrument; and it provides a means of compensating the instrument for temperature.

The operation of the mechanism is as follows: Deflections in the diaphragm (3) are multiplied and communicated to the indicating hand (28) through the phosphor-bronze connecting strip (20), which actuates the multiplying pulley (18), which in turn transmits the motion to the pulley on the indicating arbor (26) by means of the phosphor-bronze connecting strip (21).

This motion is read on the dial by means of the indicating hand (28). The helical spring (13) serves the purpose of taking up all backlash in the instrument, and also to adjust the zero setting of the indicating hand by varying the tension on the diaphragm (3) by means of the slide (15) and the adjusting screw (16). The range of the instrument is from 0 to 3,000 feet per minute ascent to 0 to 3,000 feet per minute descent, but, like the previous model, can be subjected to all rates up to about 9,000 feet per minute ascent and to 30,000 feet per minute descent without injury, so that it is almost impossible to damage the instrument by subjecting it to a too great change in pressure. The time lag is about 5 seconds.

German mechanical rate-of-climb indicator.—This instrument (figs. 21 to 24) is similar in many respects to the Bureau of Standards model No. 3, although the two instruments were developed independently. The operation of the instrument is as follows: The movement of the diaphragm is communicated to the pointer by means of a connecting thread (3), which actuates the multiplying lever (6). This lever is counterbalanced by a counterweight. The

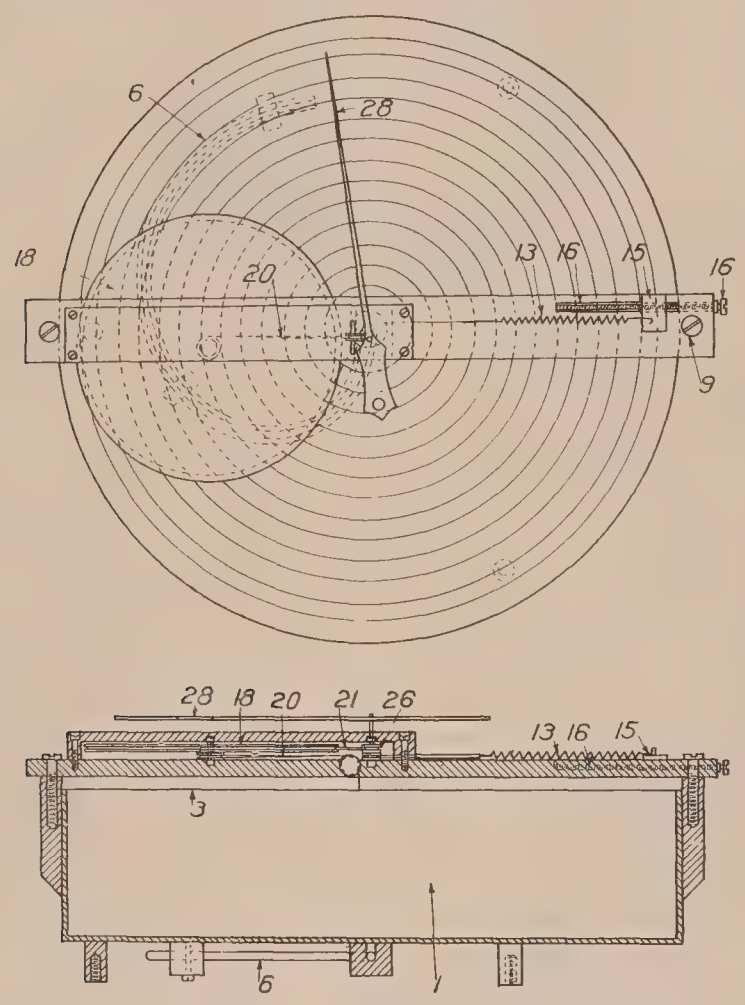


FIG. 20.—Bureau of Standards rate-of-climb indicator, mechanical type, model No. 3.

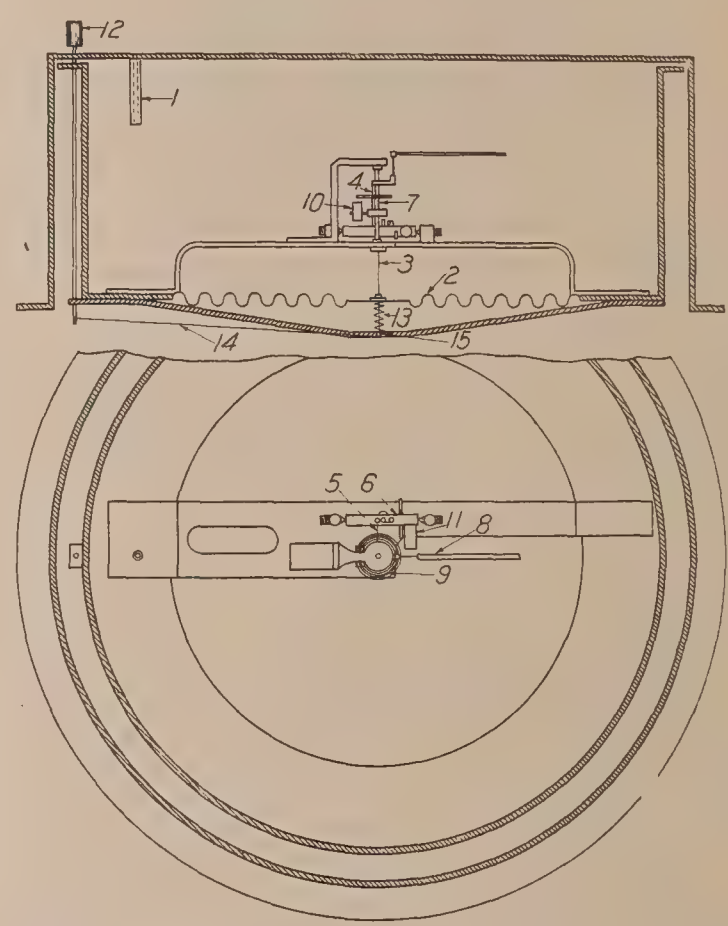


FIG. 21.—German rate-of-climb indicator, mechanical type, No. 1027.

multiplying arm of this lever communicates its motion to the indicating pulley (7) to which is attached the pointer (8) and is rotated by means of the winding and unwinding of the thread on the pulley against the tension of the hairspring (9) which is attached to the same shaft. This shaft is also counterbalanced. The motion of the above mechanism is limited by two stops inserted into the multiplying lever and bent so as to allow for proper movement of the pointer, but preventing the mechanism from being damaged by excessive pressure differences.

There is a zero regulator attached. It operates by putting tension on a light helical spring (13) which is attached to the center of the diaphragm. This is accomplished by turning the knurled screw (12) placed on the outside of the instrument case, which winds or unwinds thread (14), the other end of which passes over guide pulley (15), thus adjusting the tension on the spring (13).

The leak device of this instrument is unusual. It consists of a tube (1), into which is inserted a tapering pin. The instrument is calibrated by adjusting the position of this pin in

the tube. Thermal insulation is effected by an air jacket between the air chamber and the outer case.

The range of the instrument is from 0 to 1,000 feet per minute, both for descent and ascent. The time lag is about the same as in Bureau of Standards model No. 3.

Magnetic type.—While instruments of the leak type and automatic statoscopes represented by the katanoscope constitute the only types of rate-of-climb indicators which have found practical use, various other types of rate-of-climb indicators have been suggested. A magnetic instrument of Dutch manufacture uses a propeller-driven armature revolving in the field of a permanent magnet. This magnet swings like a pendulum and is always in a vertical position. The rate of climb is determined by the speed of rotation and orientation of the armature in the magnetic field, since the rate of climb is a function of the air speed and the position of the armature in the magnetic field.

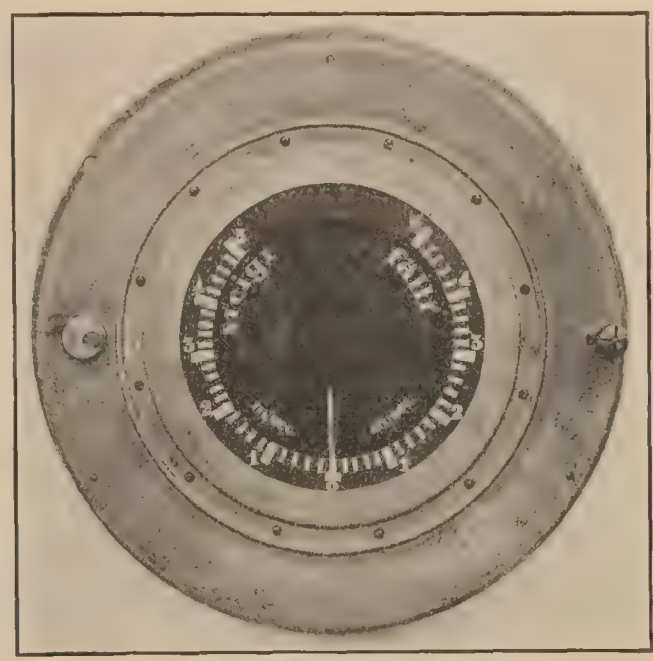


FIG. 22.—German rate-of-climb indicator.

COMBINATION STATOSCOPE AND RATE-OF-CLIMB INDICATOR.

It has been suggested that an instrument be designed to function both as a statoscope and a rate-of-climb indicator. One way of doing this would be to provide a valve to close the leak orifice, in which case the instrument can be used as a statoscope. With instruments as sensi-

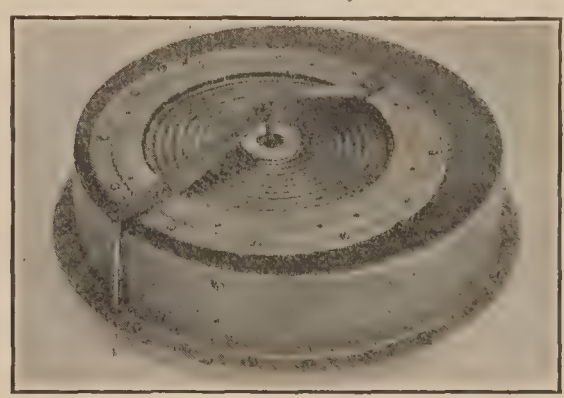
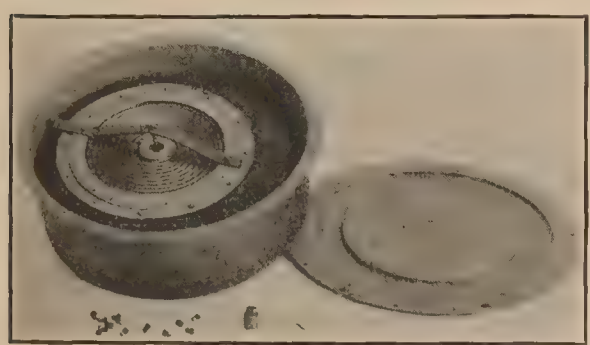


FIG. 23.—German rate-of-climb indicator.

tive as the mechanical rate-of-climb indicator this will not be necessary for heavier-than-air craft, since these instruments indicate a rate of climb as small as 20 feet per minute; however,



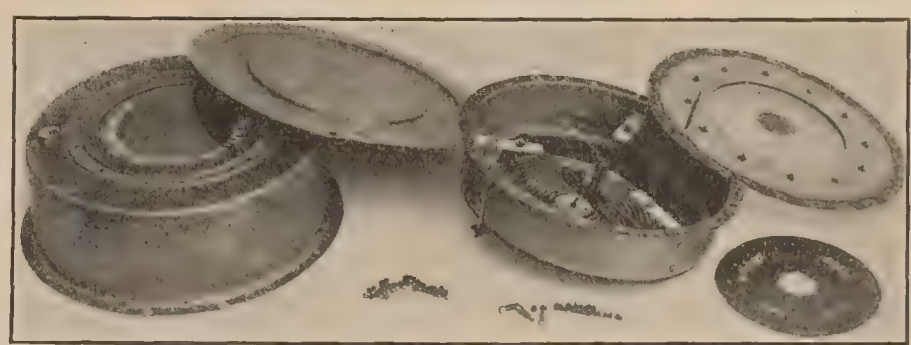


FIG. 24.—German rate-of-climb indicator, disassembled.

in the operation of lighter-than-air craft an instrument giving much more sensitive indications than is possible with a rate-of-climb indicator is required, and for this purpose a combined instrument would be advantageous.

TESTING METHODS FOR STATOSCOPES.

PRELIMINARY TESTS.

Before proceeding with the ordinary routine tests, all types of statoscopes are examined as regards workmanship and for any mechanical defects. A bubble instrument is inspected for broken and disconnected tubes. A mechanical type is tapped lightly to determine whether the pointer is loose on the shaft. It is then given a slight rotary motion, so as to deflect the pointer. If serious oscillations occur, the hairspring is too weak or a part of the lever system is disconnected. The mechanism is then tested for balance by placing the instrument first in a horizontal and then in a vertical position and noting any change in the position of the pointer.

Mechanical instruments are tested for leaks in the following manner: The instrument, with its air outlet closed, is placed in a glass bell jar or other container in which it can be observed. The pressure is reduced until the pointer indicates the maximum deflection, and the instrument is held at this pressure for one hour. If during this time the reading of the instrument decreases, a leak is indicated. Care is taken that the temperature of the instrument is kept constant throughout this test.

TEMPERATURE TESTS.

The thermal insulation of the air chamber of the instrument is tested as follows: The instrument is placed in a temperature chamber at —10° C. and kept there for three hours to

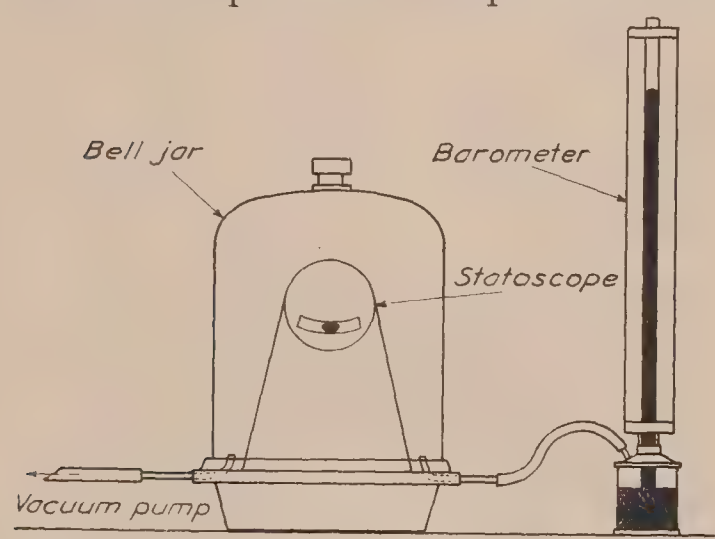


FIG. 25 —Diagram of connections for testing bubble type statoscope.

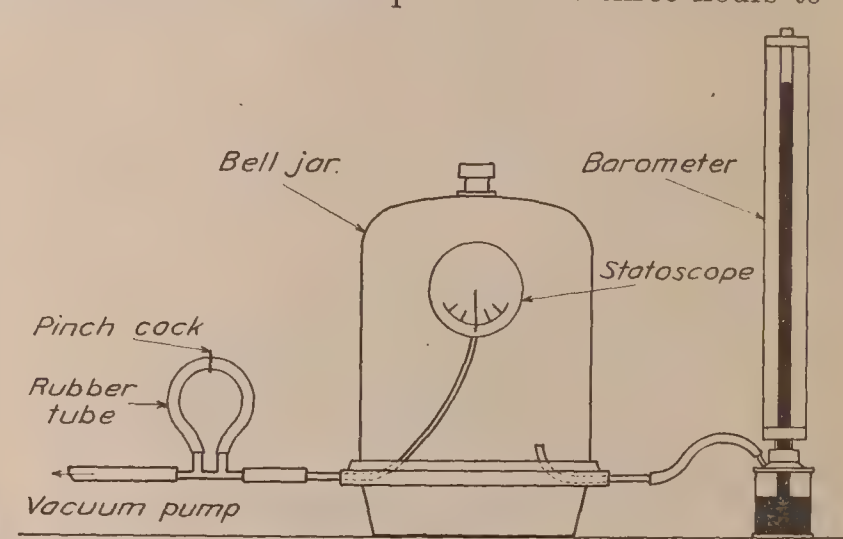


FIG. 26.—Diagram of connections for testing mechanical statoscope.

make sure that it has attained throughout the temperature of the chamber. It is then quickly placed in a second chamber, at a temperature of +40° C. and read at intervals for two hours. The barometric pressure is also noted so that corrections can be applied for change of atmospheric pressure. During this test the instrument should not give a greater indication than would be produced by a change in altitude of 1,000 feet (i. e., a change of pressure of about 1 inch of mercury), allowance having been made for any change of atmospheric pressure during the test. The instrument is also observed for any change of reading due to expansion or contraction of the mechanism, with temperature change, and the bubble statoscope for large changes in viscosity of the indicating liquid.

PRESSURE DIFFERENCE TESTS.

These tests are conducted in the following manner: In the case of the bubble type, the instrument is placed in a bell jar which is connected to the barometer and the vacuum pump as is shown in the following illustration (fig. 25). When adjusted and the apparatus is found free from leaks, the pressure in the system is reduced until one end of the bubble is brought to a certain mark on the curved gauge. After the pressure has been kept constant for a short time, the reading of the barometer is noted. The pressure is now reduced until the bubble breaks, and comes back to the same chosen position in the curved gauge. Another reading of the barometer is taken, and the change of pressure computed from the two readings of the barom-

eter. These observations of the instrument should be repeated about ten times. The average of the readings is computed, and is known as "the pressure equivalent of the bubble." This may also be expressed as a change in altitude.

The mechanical statoscope requires some method by which the outlet to the external air may be closed at any desired pressure. This was done at the Bureau of Standards by means of a specially designed hose connection as shown in the accompanying diagrams. Figure 26 shows the barometer,[4] vacuum-pump, and bell-jar connections, and figure 27 the detailed tube connection. The small rubber tube of the instrument is attached at (1). Another small rubber tube equipped with a pinchcock is attached to the copper tubes (2) and (3). After these connections are made and the system is free from leaks, the instrument is tested in the following manner: The pinchcock is removed so that the air chamber of the instrument is subjected to the pressure in the testing system, the pump is started, and the pressure reduced to the desired amount. If there is a resistance to the flow of air in the connections, the statoscope will show a small deflection, but the pointer will come to its zero setting when the pressure ceases to change. The pinchcock is now put on the rubber tube at (4), a pressure reading is taken on the barometer, and then the pressure slowly reduced until the pointer of the instrument moves to the first graduation on the dial. At this point the vacuum pump is shut off and as soon as equilibrium is reached the barometer reading is noted. This procedure is continued for each graduation until the end of the scale is reached. The pressure in the bell jar is now increased and observations made at each point on the scale of instrument through the zero point to the end of the descent scale. Check readings are taken by repeating the above procedure. From these observations a table of "pressure equivalents" for each graduation is prepared.

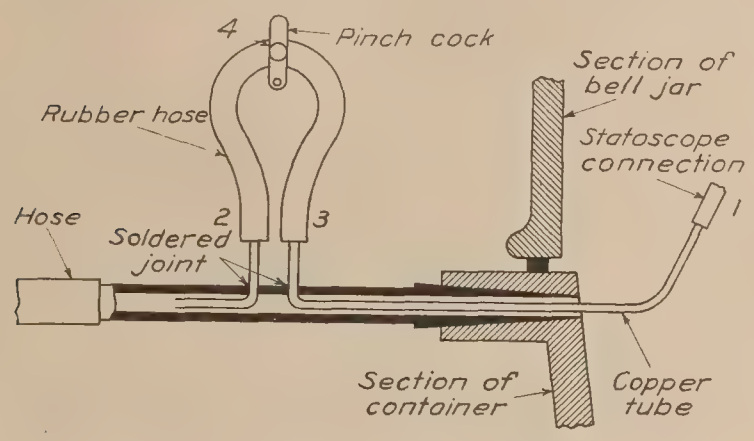


FIG. 27.—Detail of tube connections for mechanical statoscope.

TESTING METHODS FOR RATE-OF-CLIMB INDICATORS.

TESTING APPARATUS.

The set-up for testing rate-of-climb indicators is shown diagrammatically in figure 28 and consists of the following equipment:

Bell jars and stand.—The bell jars are of glass, various sizes being used to meet the needs of the instruments to be tested. The stand is made of a circular steel plate, 20 inches in diameter and one-half inch thick, mounted on three steel legs. This plate is equipped with both pressure and electrical connections. Two pressure connections are required for the testing of rate-of-climb indicators, one for an outlet to the pump, the other for the connection to the barometer, since it is necessary to minimize the friction of the flow of air between the bell jar and the barometer so that the barometer will indicate the true pressure change (rate of climb) in the bell jar. The bell jar is sealed to the stand with a soft paste made of paraffin, beeswax, vaseline, and rosin.

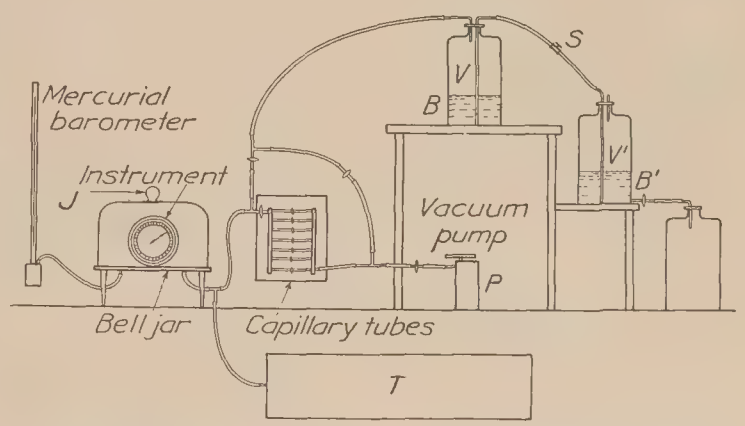


FIG. 28.—Diagram of connections for testing rate-of-climb indicators.

[4] The method of connecting the barometer to the bell jar so that it will indicate the proper pressure should be noted. This connection should be as short as possible, so as not to offer resistance to the flow of air and thus cause a time lag. This effect is extremely annoying when testing rate-of-climb indicators.

Barometer.—The barometer used was designed and constructed at the Bureau of Standards and is provided with both pressure and altitude scales. The altitude scale is read during the tests on rate-of-climb indicators; and the rate of climb determined by finding with a stop watch the time required for the mercury to fall a chosen altitude interval. This altitude interval should not be greater than 1,000 feet since the rate of climb is likely to change during larger intervals and thus give inaccurate results. By using a large-capacity pump and volumes of about 12 cubic feet, a fairly constant rate of climb can be obtained over a pressure interval corresponding to several thousand feet of altitude.

Capillary tubes.—The capillary tubes are adjusted so as to give the desired rate of climb by grinding to the proper length and by properly constricting the bore. By making up a set of these and using them in various combinations in conjunction with a variable volume, any rate of climb can be produced from 100 feet up to the maximum range of any instrument.

Variable volume.—This consists of a large glass bottle (B) of about 2 cubic feet capacity connected to the expansion tank (T) and to a second bottle (B′) of the same capacity. By varying the level of (B′) water can be forced to or from the bottle (B) thereby varying the volume of the system which included the calibrating bell jar (J). Since the pump (P) is operated at a constant speed the same rate the change of pressure (rate of climb) in the bell jar can in this manner be accurately controlled. When the required volume adjustment has been made the bottle (B′) is cut off by closing the stopcock (S).

Expansion tank.—This is a hot-water boiler (T) (fig. 28) of about 10 cubic feet capacity which serves the purpose of adding volume to the system. It is insulated so as not to cause any pressure changes in the system due to sudden changes in the room temperature.

Vacuum pump.—For ordinary testing a small pump (P) (fig. 28) connected directly to the test chamber is sufficient, but for rapid rate of ascent and for more accurate work a large pump and tank are required. The pump used has sufficient capacity to evacuate 10 cubic feet from 30 to 6 inches of mercury in 10 minutes.

TESTING OF RATE-OF-CLIMB INDICATOR.

The tests on rate-of-climb indicators used by the Bureau of Standards are arranged to determine the accuracy of the instrument under the various conditions of flight. The main factors which affect the reading of the instrument are lag, changes of temperature, and changes of air density.

Preliminary tests.—Each instrument is inspected for defects in workmanship, defective tubes, and leaks. The capillary leak tube should be clean and made of material that will not corrode.

Calibration tests.—The apparatus is assembled as indicated in figure 28. A chosen rate of climb is produced by starting the vacuum pump and opening the stopcocks connected to the capillary tube which has been selected to give the desired rate of climb in the system. If the tests require that this rate be definitely specified—for example, exactly 100 feet per minute—the levels of the liquid in the variable-volume bottles are adjusted until this rate of climb is obtained. When the apparatus gives the proper rate of climb the system is opened to the air and allowed to come back to atmospheric pressure (care of course being taken that the instrument is not damaged by being subjected to a too rapid rate of change of pressure).

The pump is now started again, and as the mercury passes a chosen altitude division on the barometer, say, the 1,000 feet mark, the stop watch is started. When the mercury passes another chosen mark on the barometer the watch is stopped and the time interval noted. During the above operation another observer simultaneously takes readings of the instrument. From the observed time interval required for the mercury to pass the two chosen altitude divisions the rate of climb is computed. This operation is repeated until the desired number of points on the scale of the rate-of-climb indicator are tested. The results of these tests are charted by plotting the rate of climb as given by the barometer against the reading of the instrument. See following curves (figs. 29 to 35) of various instruments tested at this bureau.

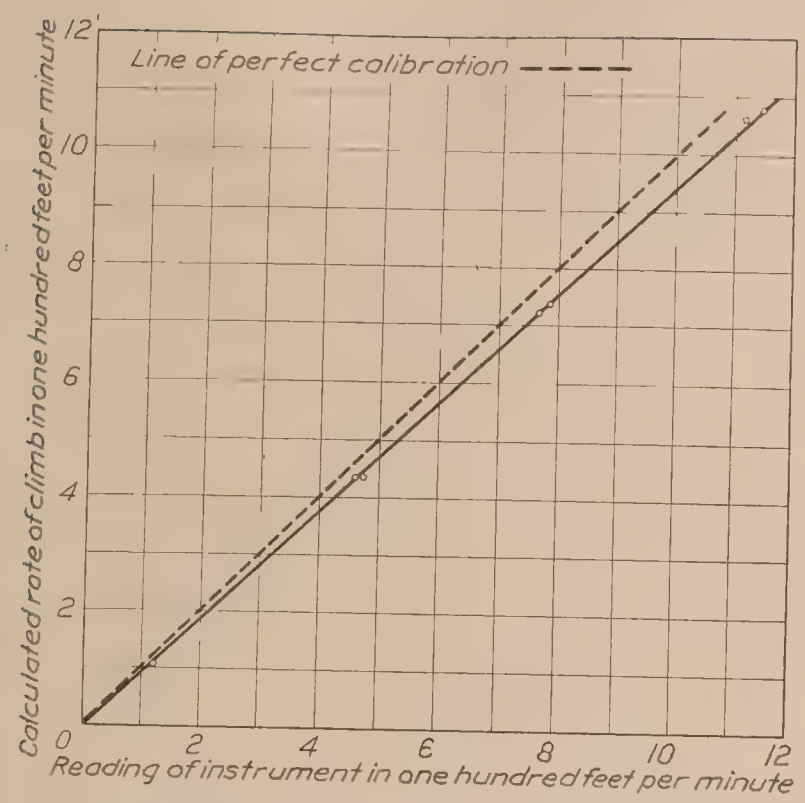


FIG. 29.—Calibration of British rate-of-climb indicator No. 22.

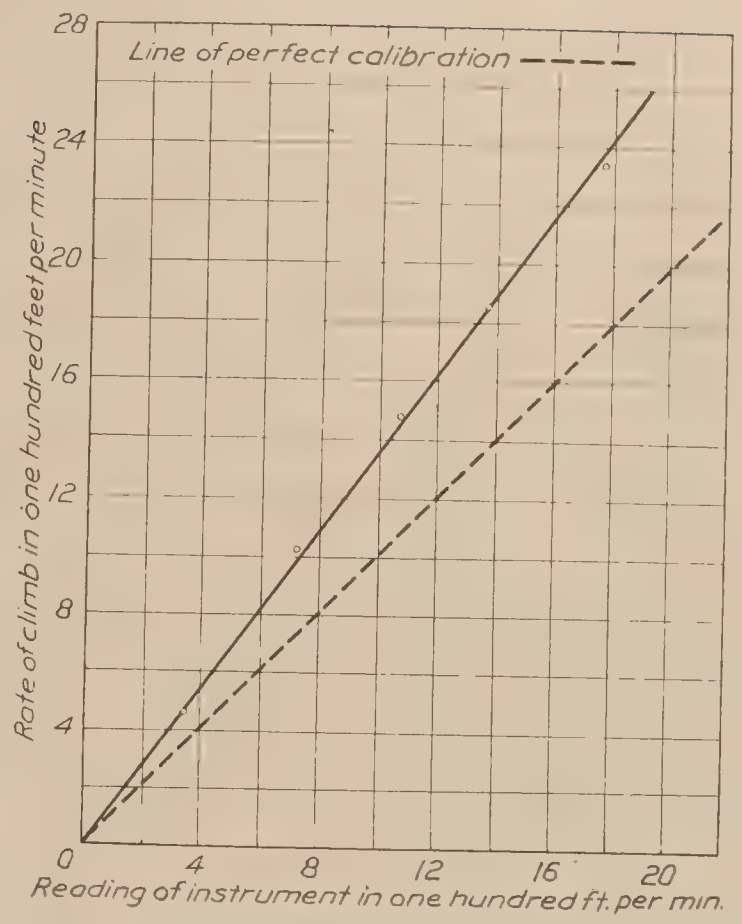


FIG. 30.—Calibration of vertimeter.

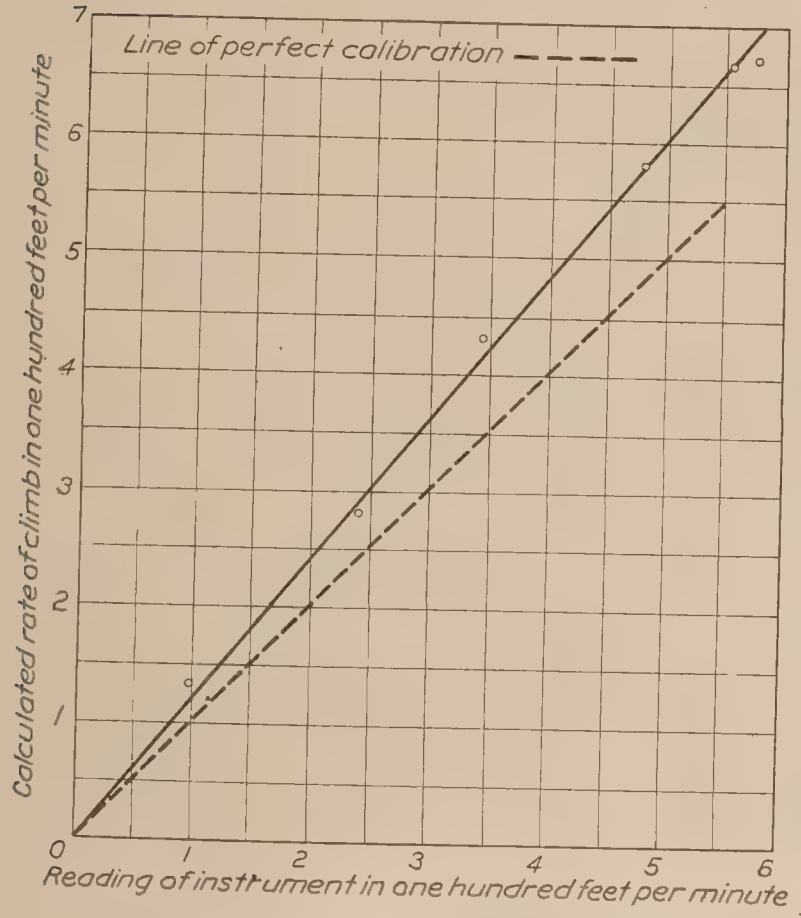


FIG. 31.—Calibration of German balloon variometer No. 1.

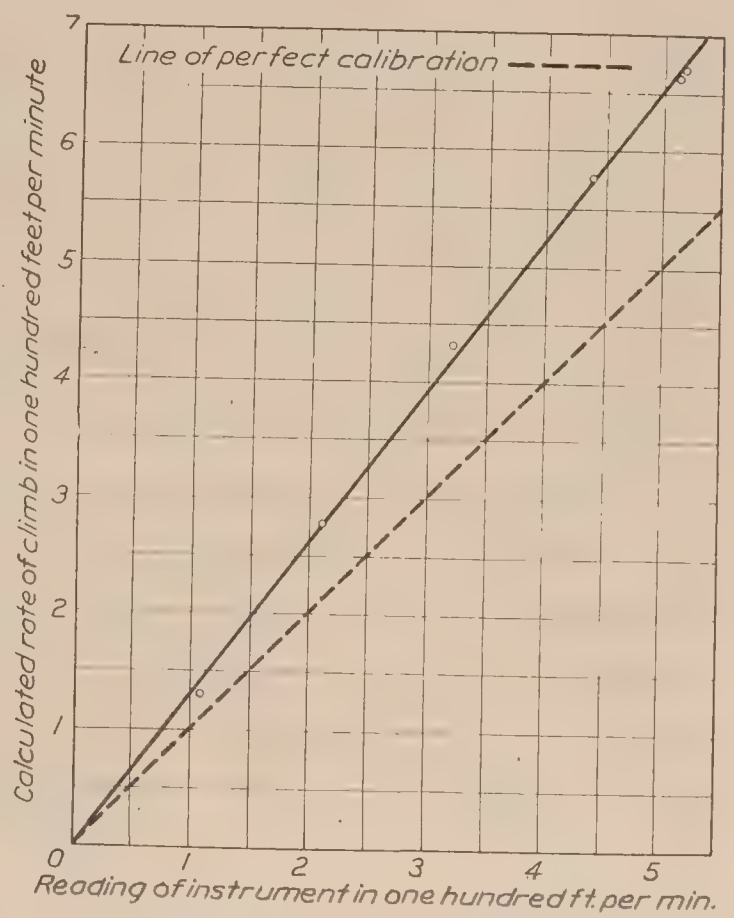


FIG. 32.—Calibration of German balloon variometer No. 2.

Tests to determine variation of scale value with altitude.—This test is conducted by maintaining a constant rate of climb in the test apparatus and making rate-of-climb observations on barometer and instrument as in the calibration test, for every 5,000 feet from sea level to 40,000 feet.

FIG. 33.—Calibration of Bureau of Standards rate-of-climb indicator No. 2.

From the data obtained in these tests the ratio of the calculated rate of climb from the barometer and stopwatch observations to the reading from the instrument are computed and plotted against the altitude.

Results of tests on various types are shown on the following graphs (figs. 36 to 40).

Lag tests.—The theory of the behavior of rate-of-climb indicators shows that the time lag depends both on the altitude and the pressure difference between the two ends of the leak tube. Therefore tests are conducted to determine the variation of the lag, both with altitude and deflection.

The lag of the instrument is determined in the following manner: A chosen rate of climb is produced in the apparatus. When this has been held constant until the pointer becomes steady (in most cases a minute is sufficient) the instrument is read. The calibration tank and pump are then suddenly shut off from the system thus stopping the pressure change in the container. At the same instant a stop watch is started and then stopped when the reading of the instrument comes to one-third of its former value. The lag is the time required in seconds. For a constant deflection these readings are repeated every 5,000 feet until 40,000 feet of altitude are reached.

The lag is also observed for a series of rates of climb, thus showing variation with deflection.

Temperature tests.—A calibration test is run at a temperature of −10° C. to determine the change in slope of the calibration curve with temperature. The instrument is placed in a temperature chamber where it is kept for four hours at the above temperature to allow the instrument to come practically to the temperature of its surroundings; then a calibration similar to that described above is obtained. The following observations are made during the

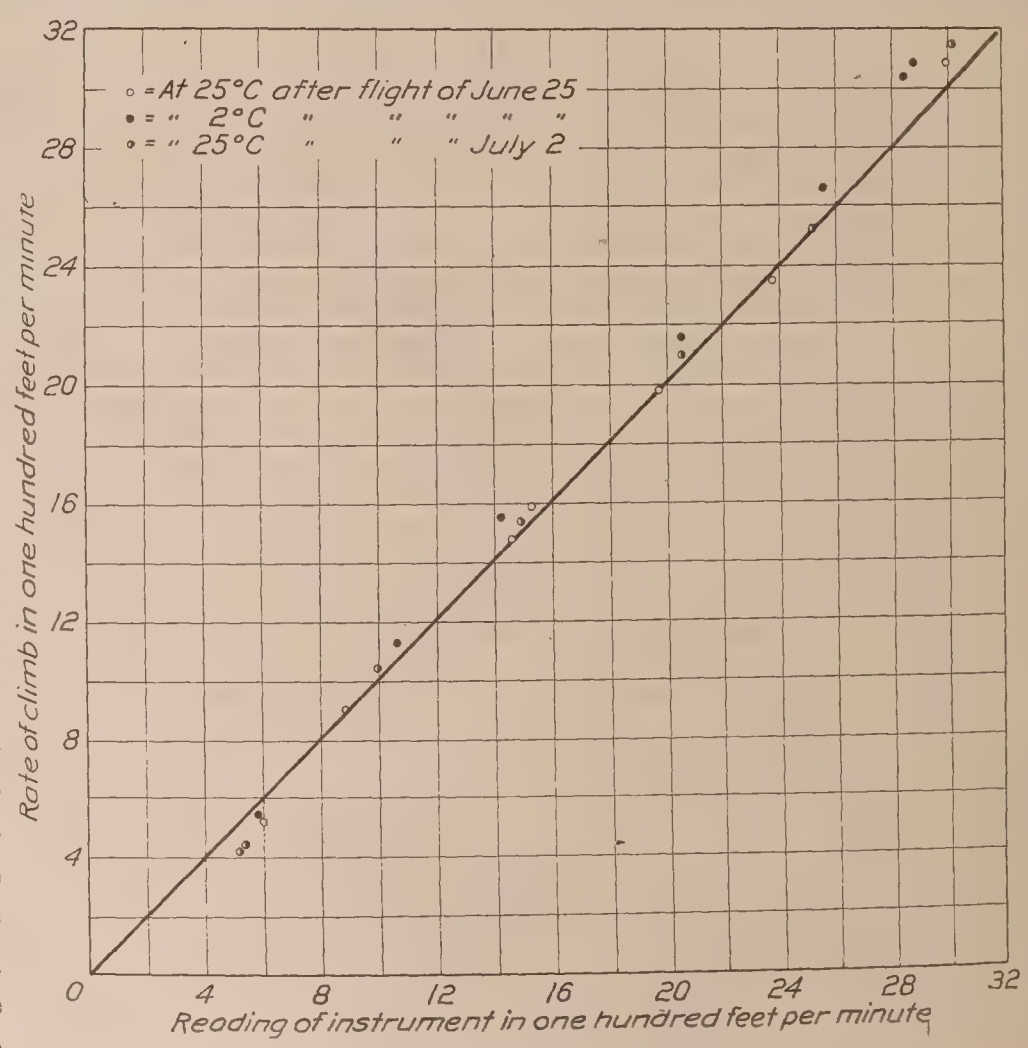


FIG. 34.—Calibration of B. S. rate-of-climb indicator No. 3 after flight tests.

progress of the above test. Just after the instruments are placed in the cold chamber they are observed to see if they indicate a descent. This shows the efficiency of the thermal insulation. After an instrument of a liquid type has been subjected to a low temperature for two hours the indicating liquid is observed for change in its viscosity, and the mechanical type for change in zero due to contractions in the mechanism.

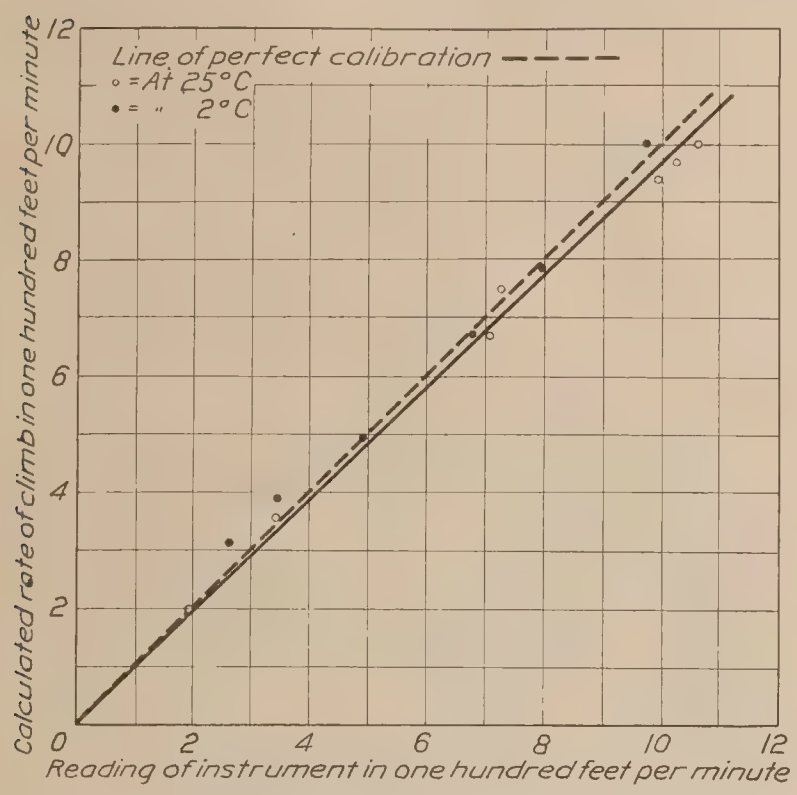


FIG. 35.—Calibration of German rate-of-climb indicator, mechanical type No. 1027.

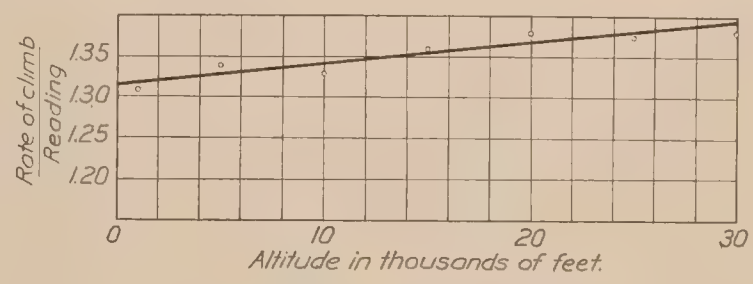


FIG. 37.—Variation of scale value with altitude. German balloon variometer No. 2 at 600 feet per minute.

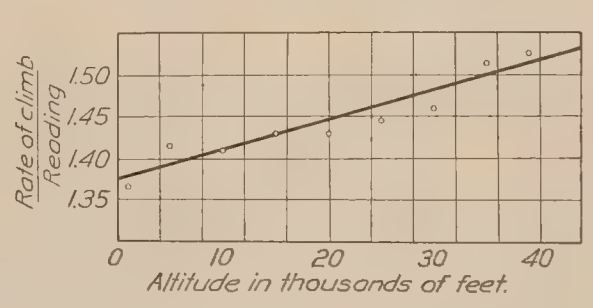


FIG. 38.—Variation of scale value with altitude vertimeter.

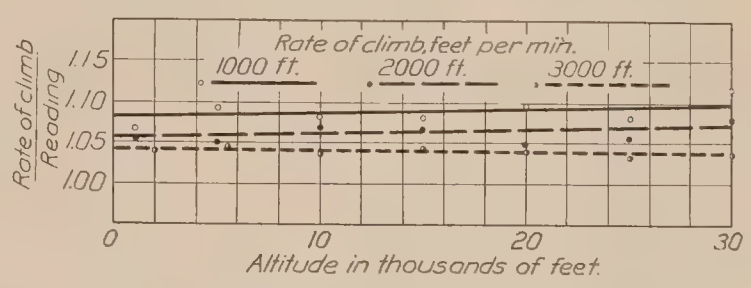


FIG. 39.—Variation of scale value with altitude and deflection. Bureau of Standards rate-of-climb indicator No. 3.

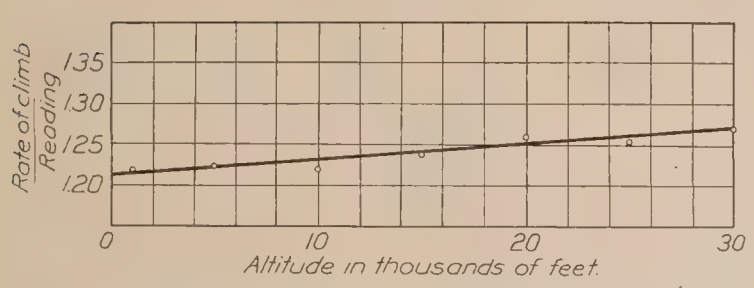


FIG. 36.—Variation of scale value with altitude. German balloon variometer No. 1 at 600 feet per minute.

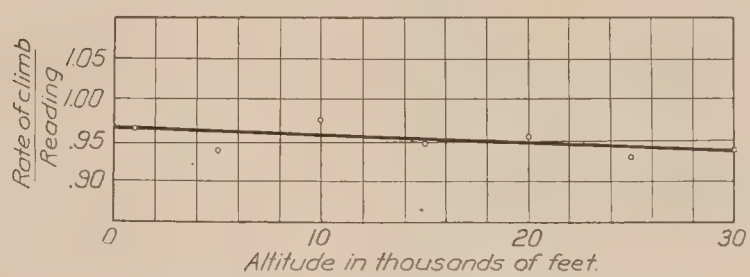


FIG. 40.—Variation of scale value with altitude. German rate-of-climb indicator No. 1027 at 600 feet per minute.

Inclination tests on mechanical instruments.—All of the above tests on the mechanical instruments are made with the dial of the instruments in a vertical position. A calibration test is repeated with the dial horizontal. If there is a considerable change in the slope of the calibration curve a complete set of tests as described above are made on the instrument with the dial in a horizontal plane.

REPORT No. 126.

ALTITUDE INSTRUMENTS.

PART IV.

AEROGRAPHS AND STRUT THERMOMETERS.

By John A. C. Warner.

SUMMARY.

This part contains a description of the principal types of thermographs and other aerographic instruments, together with a discussion of performance characteristics and methods of testing.

Direct and remote indicating strut thermometers are also included, for while these instruments are not mechanically self-recording, they are commonly employed for the purpose of preparing curves to show the temperature distribution at successive altitudes and are therefore chiefly of interest in connection with aerographic instruments.

THERMOGRAPHS.

In the field of aeronautics it is often important that a temperature-time record be secured, either in connection with performance tests of aircraft or in obtaining meteorological information of value to the flyer. The thermograph is designed for this purpose, and may be used to record cockpit temperatures to which the flight instruments are subjected, or in its meteorological capacity to record the free-air temperature at the ground or at any altitude to which it has been lifted by airplane, balloon, or kite. The records thus obtained facilitate the application of instrumental corrections made necessary by the errors of the instruments due to varying temperature conditions, or provide data for the reduction of altitude determinations as well as useful meteorological information unobtainable by other means.

All types of thermographs combine some form of temperature element and recording device with a timing or clock mechanism in such a manner that the record chart presents a curve of temperature against time over some definite period. The general principles of operation are identical, no matter what the particular duty of the instrument may be.

TEMPERATURE ELEMENT AND RECORDING DEVICE.

The commonest types of thermograph depend for temperature indications upon either the Bourdon tube or the bimetallic strip. Of these, the Bourdon tube instrument is perhaps more common and will be considered first.

Bourdon tube type.—The action of the liquid-filled Bourdon tube as a temperature element is so well understood that very little explanation will be undertaken in this paper. Suffice it to say that the effect of varying temperatures upon the curved metal tube of elliptical cross-section, completely filled with a liquid at the proper pressure, and hermetically sealed at its ends, is to cause it to change its curvature and assume varying positions as the inclosed liquid expands or contracts with temperature changes.

Referring to fig. 1, the Bourdon element will be seen at (A), pivoted at the end of a bracket which holds it outside the instrument case where it may be influenced by free-air conditions. The Bourdon element of a typical instrument has an elliptical cross section approximately

33 millimeters along its major axis by 2 millimeters along its minor axis, and has a length of 65 millimeters along its curved edge. Adjusting screw (B) acts upon the tube through the lever arm attached to the pivoted end of the tube and so allows for satisfactory adjustment. The adjusting arm is held in place by the compression spring shown at (B). The wire frame below the tube serves as a protection from mechanical injury as the instrument is moved about.

At the lower or movable end of the tube and rigidly attached to it is a connecting rod which transmits the motion of the tube through a simple linkage, pivoted near the top of post (C) to the light spring metal arm (D), which carries the recording pen at its outer extremity. A thumb nut is provided at (G) so that the pressure of the pen on the chart may be properly regulated. The vertical spindle (F) may be moved by the small lever attached at its lower end in the base of the instrument so as to raise the pen from the chart when the temperature record is not desired.

Bimetallic strip type.—The bimetallic strip principle has long been used in many types of instruments and apparatus. When two strips of metal with different thermal coefficients of expansion are firmly joined by soldering, welding, or otherwise along their entire length, a change

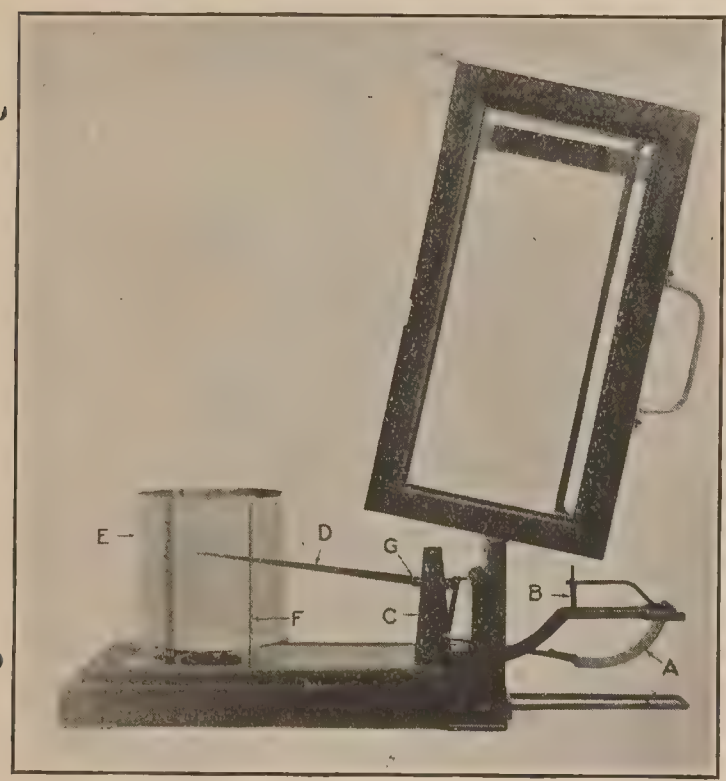


FIG. 1.—Bourdon tube thermograph.

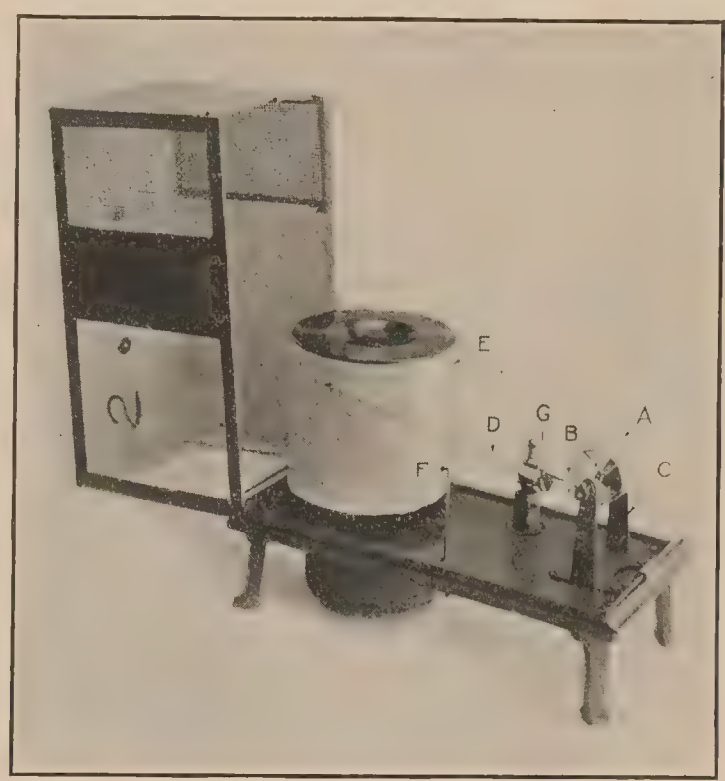


FIG. 2.—Bimetallic strip thermograph.

in temperature causes a distortion so that the bimetallic strip assumes a shape approximating an arc of a circle. When the initial temperature is restored, the strip returns to its former shape. Inasmuch as the distortion has a definite and practically fixed relation (within proper temperature limits) to the existing temperature, the bimetallic strip presents itself as a suitable element for a temperature-recording instrument.

It is possible to combine practically any two metals with different thermal coefficients of expansion to form a temperature element, but in cases where a relatively large distortion is desired it is necessary to choose metals whose coefficients differ materially. Brass or bronze with steel or invar make suitable combinations. The ratio of the coefficients of expansion at room temperature is approximately 2 in the case of ordinary brass to steel and 18 in that of brass to invar. These ratios vary over a considerable range, however, according to the alloys employed.

The strips are used in the form of helices, U-shaped members, or as straight pieces. The advantage gained by the helical or coiled form is that the distortion which takes place under varying temperature conditions produces a direct rotative motion thus making the use of linkages or levers in the recording mechanism unnecessary.

Such is the case with the American instrument shown by figure 2, in which the helical member is seen at (A). In a certain instrument of this type the strip has the following approximate dimensions: length, 355 millimeters; width, 6.5 millimeters; thickness, 1 millimeter. This forms a helix approximately 33 millimeters in length by 32 millimeters outside diameter. A spindle coincident with the axis of the helix cylinder and supported at its extremities by bearing posts (C) holds the element in position and serves to transmit its motion of rotation to the light spring metal arm (D) mounted at the end of the spindle and carrying the recording pen point at its outer end.

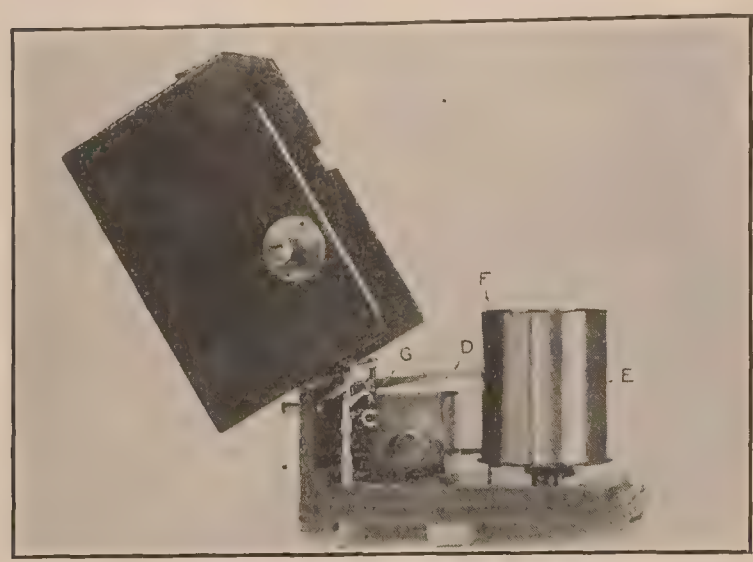


FIG. 3.—Italian thermograph.

The element is provided with two adjusting devices indicated by (B) which also connect it to its spindle. A still further adjustment may be effected by means of the set screw which holds the recording arm in place at the end of the spindle. The pen-point pressure may be varied by the adjusting thumb nut (G) and the pen raised from the record chart by movement of the vertical rod (F) as previously described.

Figure 3 shows an interesting Italian adaptation of the bimetallic strip temperature element replacing the pressure chambers of an ordinary aneroid type of barograph, thus converting it into a thermograph. This particular instrument is provided with two recording pens, the first to give temperature records and the second to mark time intervals when acted upon by an electromagnet energized at definite intervals by an intermittent current externally controlled. The record drum is caused to rotate by the original barograph clock mechanism.

FIG. 4.—German thermograph.

A metal tube mounted on the case cover provides for communication with the outer air so that a satisfactory circulation around the temperature element may be maintained. (Letters represent same elements as in previous description.)

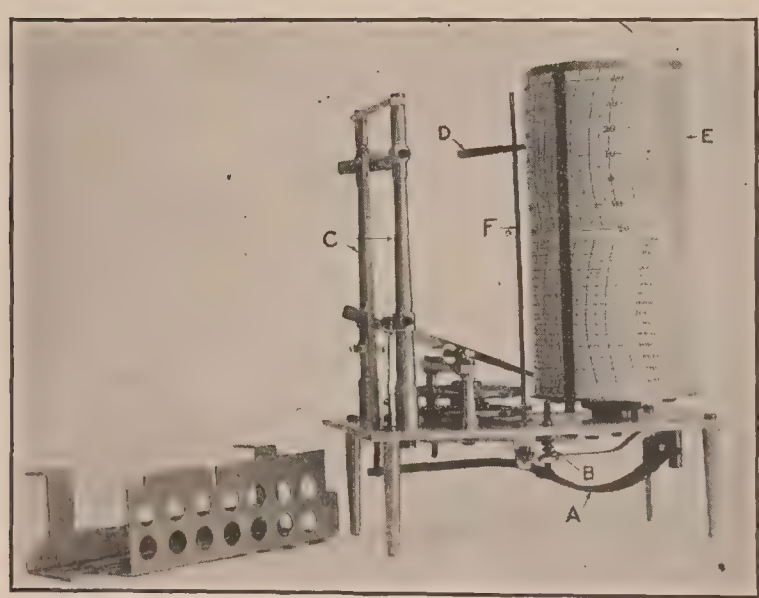


FIG. 5.—Baro-thermograph.

Figure 4 shows a small bimetallic strip (zinc-steel) thermograph of German manufacture. Several interesting features are incorporated in the mechanism, such as the straight bimetallic element, the method of balancing the linkage (see weight at end of recording arm), and the timing mechanism which is contained in one of the vertical record cylinders. The other cylinder is mounted on a movable base so that the distancebetween the two cylinders may be varied by moving a lever when it is desired to remove the chart. The instrument is constructed throughout with a view to extreme lightness and weighs but 624 grams.

RECORD CHART AND TIMING DRUM.

The record chart and timing drum common to all thermographs are very similar in design in all cases. The record chart, in the form of a paper strip suitably ruled and marked, is held in place upon the rotating timing drum either by a spring strip of metal as shown in figure 1 or by having its ends secured by means of glue. The ordinates of the chart represent temperatures and the abscissæ, time. The Bourdon tube and bimetallic strip instruments are adapted to a comparatively large range of temperatures, and so may be used with charts of varying limits. The inital adjustment is made for each type of chart by means of adjusting screws, as the indications are compared with those of a standard.

The charts generally provided cover a range of about 50° or 60° of centigrade scale, but the instruments designed for the study of conditions at very high altitudes require a greater range with a comparatively extreme lower limit. A millimeter of the temperature scale ordinarily represents about 1° or 2°. The instruments shown by figures 1 and 2 make a weekly record on a chart about 29 centimeters long, while the German thermograph and French baro-thermograph each make 21-centimeter records in 5 and 6 hours, respectively.

The clock mechanism which causes the record drum to rotate at a definite rate is usually contained within the drum itself. It may, however, be mounted in the base of the instrument as shown in figure 2. In this latter disposition the drum may be removed for the renewal of the record chart without disturbing the clock mechanism. By this arrangement the drum is also lightened considerably and friction reduced thereby. When the clock movement is inclosed within the drum, a space otherwise unused is utilized, and the simple handling of the clock mechanism without involving the whole instrument is made possible.

It is important that the driving connections between clock movement and record drum of an instrument for use on aircraft be free from lost motion. This difficulty may be overcome in most cases by careful workmanship in the construction of the parts and in their adjustment. In some instances this cause of irregular drum action has been avoided by making a direct connection between the mainspring and drum.

The construction of the clock movement as regards the time required for a complete rotation is dependent upon the kind of service for which the instrument is to be used. The common meteorological thermograph for recording temperatures at ground level is ordinarily equipped with an eight-day clock movement which carries the drum through one complete turn in a week's interval, with graduations and markings on the record chart indicating the successive days—while the instrument to be used in the upper atmosphere is designed so as to provide for a complete rotation in the maximum number of hours which the flight may occupy.

Certain of the more recent instruments are equipped with clock mechanisms provided with a gear-shifting device which allows the choice of several different rotative speeds. Records of flights of varying length may thus be made and the time scale value adjusted so as to suit the conditions.

THERMOGRAPH COMBINATIONS.

The thermograph is often combined with other instruments to form a single unit recording several distinct quantities against time, the records appearing upon a single chart. The barograph or the hygrograph is most frequently found in this combination. Figure 5 shows a baro-thermograph (without case) of French manufacture, the pressure markings being at the lower part of the chart, with temperature record above. The Bourdon tube element is underneath the base of the instrument where it is protected from mechanical injury by the cage as shown. (Letters represent same elements as in previous descriptions.)

A description of the Marvin, the Fergusson, and of a French meteorograph will be found below. These meteorographs are combination instruments incorporating the temperature element with others.

TESTING OF THERMOGRAPHS.

Two principal thermograph tests are required for the determination of calibration errors and lag constant of the instrument. The laboratory installation for this work consists of a properly constructed temperature chamber with glass observation window and equipped with heating and cooling coils of sufficient capacity to cover the temperature range of the instrument under test. A fan system is usually arranged to provide proper circulation, which insures uniform temperature conditions throughout the chamber. A standard thermometer of known calibration and characteristics is mounted in the chamber, where its readings may be compared with those of the instrument under test.

Calibration test.—The instrument is first put into proper running order with the clock mechanism functioning and the recording pen making a fine line upon the record chart. The adjusting screws are then regulated to make the reading correct for the surrounding temperature. The thermograph is next placed in the chamber, which is then closed and the test begun.

The temperature of the chamber is first raised to the upper limit of the instrument in two or three stages of approximately equal temperature intervals. At each stage the temperature is held constant for a sufficient period to allow the pen to come to rest and for the standard to reach its equilibrium temperature. Readings of the thermograph and standard are taken. The thermograph is then shaken and the reading taken again. The difference in these two readings is the friction error of the instrument.

The heating coils are then disconnected and the temperature allowed to fall. In case it is desirable to determine the magnitude of hysteresis effects, the descent from the higher temperature may also be made in stages and the "up" and "down" readings compared. The same procedure as outlined above is followed for the lower part of the scale by bringing the cooling coils into action.

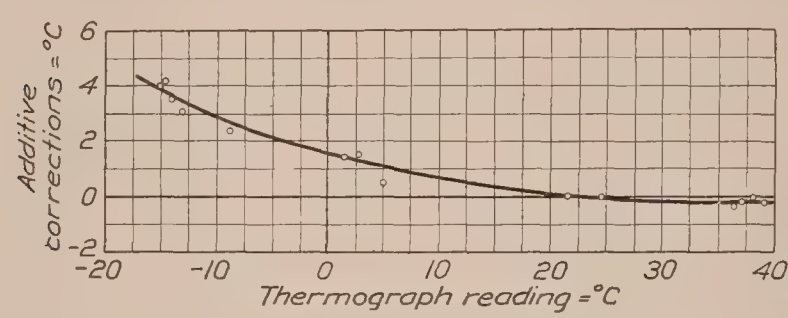


FIG. 6.—Thermograph calibration curve.

A thermograph calibration curve is shown by figure 6. The large departures at the lower end of the temperature scale are due primarily to the change in the ratio of the effective lengths of the lever arms as the deflection increases. The simplest method of overcoming this error is to determine by experiment the deflections corresponding to given temperatures, and to rule the chart accordingly. If, however, the thermograph is adjusted to cover some temperature interval other than that for which the chart is prepared, the ruling will in general show calibration errors, since the adjustment shifts the position of the lever arms with reference to the chart.

Lag test.—Considerable lag is found in even the best thermographs. In the determination of the lag constant, temperature chamber and standard thermometer are used as in the above test. The temperature of the air immediately surrounding the thermograph is first carefully determined. The heating coils in the chamber are then connected and the temperature of the chamber brought to a point 10° or 15° warmer than that of the instrument under test. The thermograph is then quickly transferred to the chamber and careful note is made of the time required for the difference between the instrument reading and the chamber temperature to be reduced to $\frac{1}{e}$ times its original value. It is in order to simplify mathematical calculations in which occurs the value e (equal to 2.718), base of the Naperian system of logarithms, that the fraction is customarily taken as $\frac{1}{e}$.[5] As an example, let us assume that the initial difference between thermograph indication and chamber temperature is 10.8° C. The lag constant would be taken as the number of seconds required for the thermograph indication to reach to within

[5] Bulletin of the Bureau of Standards, vol. 8, pp. 659–714. Thermometric Lag, by D. R. Harper, 3d.

$\frac{1}{e} \times 10.8°$ C., that is, 4° of the chamber temperature. It is well to make several observations of this time lag constant with the chamber at various temperatures, each time bringing the instrument back to its original temperature before making a new test.

Clock test.—The clock may be compared with a standard timepiece in order to investigate its proper functioning and to determine the errors in rotative speed of the timing drum. This test should be made with the record chart in place and with the pen resting thereon as in actual service.

Clock mechanism subjected in service to very low temperatures should be given low temperature tests. As is the case with other instruments, the mechanism should be warmed after these tests so as to drive out any collected moisture which might otherwise cause corrosion.

It may be advisable in some cases to conduct additional tests such as that to determine the effect of vibration, but this procedure is ordinarily unnecessary.

Additional tests.—In case the thermograph is likely to be subjected to vibration during use, as, for example, on an airplane, it is advisable to conduct a vibration test by mounting the instrument on a vibrating board and determining whether the action of the recording pen is satisfactory. It should not show excessive vibration.

A tilting test may also be conducted by placing the instrument in various positions and noting the changes in position of the pen.

It is sometimes advisable to make a pressure test on Bourdon tube thermographs. In a properly constructed instrument of this type very little effect is noticed with the decrease in atmospheric pressure which takes place with changing altitudes. In case it is desirable to investigate this point, the instrument may be placed in a vacuum chamber in which the pressure is reduced to correspond to the maximum altitude and minimum temperature for which the instrument is to be used. There should be no appreciable change in the indications of a properly filled instrument as the pressure is reduced, if the temperature is kept constant.

Care should be taken to see that the Bourdon tube contains sufficient liquid to keep the tube under pressure throughout the temperature range measured. Otherwise the apparatus may exhibit an erratic and irregular behavior.

METEOROGRAPHS.

THE MARVIN KITE METEOROGRAPH.

The instrument illustrated by figure 7 is the Marvin kite meteorograph, designed by Prof. C. F. Marvin, Chief of the United States Weather Bureau. Intended for use in exploring regions of high altitude, this instrument is very light in weight (1,138 grams) and makes simultaneous records of atmospheric pressure, temperature, relative humidity, and wind velocity upon a suitably divided record sheet mounted upon a timing drum.

In service this instrument, inclosed in its protective case, is secured in proper position on an aircraft or inside a kite having sufficient lifting power to carry it to the desired altitude. The screening tube seen above the record drum in figure 7 contains the anemometer for air velocity measurement, the temperature element for temperature measurement, and the hygrograph hair which serves as the sensitive element for humidity records. The aneroid pressure element is seen in its position between the screening tube and record drum. All of these sensitive elements connect through suitable devices and linkages with pens resting upon the record chart.

Air velocity element.—The air velocity element consists of a small anemometer fan mounted on light bearings inside the forward end of the screening tube. The rotative motion of this anemometer element is transmitted through worm gearing to a cam. A lever connected to the recording pen pivot bears against the cam so that as the latter comes to a certain period in its motion the lever is pulled down and the pen makes a mark upon the chart. The pen and lever then return to their original positions, and the action is repeated at intervals whose length depends upon the pitch at which the fan is adjusted.

Temperature element.—This instrument depends upon a bimetallic (bronze-invar) strip for the temperature records. The element, 25 millimeters in width and bent into circular form, is mounted in the screening tube, and its free end, which changes position in a definite and regular manner with temperature variations, is connected to the recording pen arm by means of a simple linkage. An adjustment is provided for changing the initial position of the pen upon the record chart, and another device makes it possible to vary the scale value as desired.

Humidity element.—Similar to most devices for recording indications of relative humidities, this meteorograph employs a series of human hairs which have the property of changing their length with varying conditions of moisture. Instead of combining the hairs in the usual bundle form, the designer has mounted them separately, which enables the hairs to come into equilibrium with the surrounding air more promptly. Two sets of hairs are mounted longitudinally in the screening tube upon suitable fixtures provided with the necessary adjustments, and the element is connected by direct linkage to the recording pen. Inasmuch as the change in length of the hairs is not linear with respect to the humidity change, it is necessary to provide a special scale for the individual element employed. Adjustments are provided for varying the scale value and for controlling the initial position of the recording pen upon the chart.

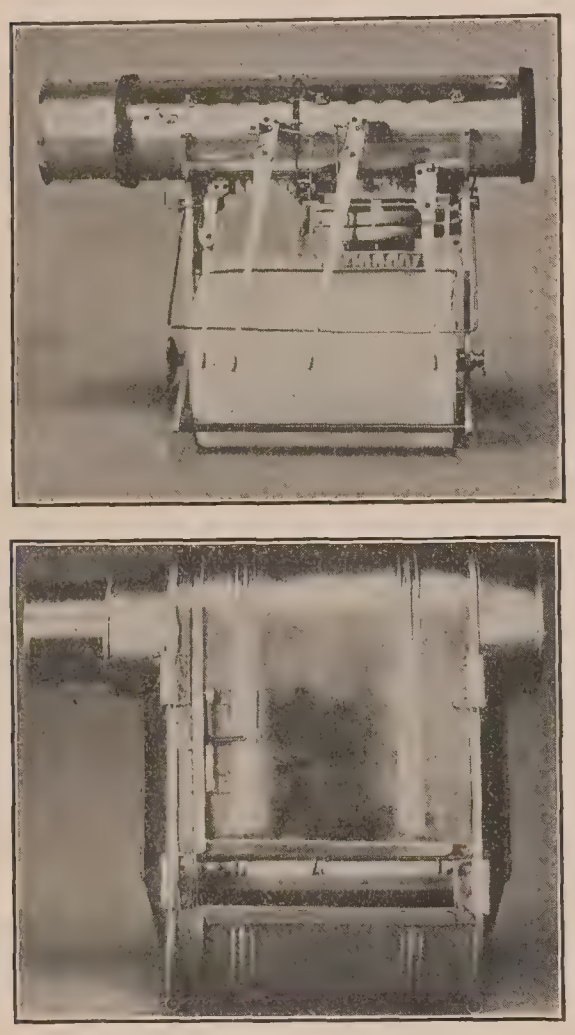


Fig. 7.—Marvin meteorograph.

Pressure element.—Two nickel-plated steel aneroid chambers provided with internal steel springs are employed as the pressure sensitive element of this instrument. In the photograph they are clearly seen mounted between the screening tube and record drum. A suitable linkage connects the aneroids with the recording pen so that the latter traces the pressure curve upon the chart with expansion or contraction of the flexible steel aneroid chambers as the external air pressure changes. A small quantity of air allowed to remain within the aneroid chambers is intended to compensate in part for deflection of the element due to temperature variation. A bimetallic strip in the connecting linkage is also provided for this purpose. The scale value may be varied by adjustment of the linkage.

Record drum and chart.—The record chart is properly ruled to receive the traces of the pens connected with the four elements. The wind curve comes at the top of the sheet, with those for temperature, pressure, and humidity occupying successive spaces over the ruled chart. The latter is held upon a removable timing drum with clock movement inclosed. The drum makes a complete rotation in an eight-hour period.

Instrument housing.—The protective housing (shown in place in the illustration) slips over the instrument so that no part is exposed excepting the screening tube which contains the several sensitive elements. Two bakelite strips insulate the tube from the case. A mica observation window is provided for the inspection of the recording elements when the housing is in place.

THE FERGUSSON METEOROGRAPH.

One of the most interesting of recent developments is the meteorograph by S. P. Fergusson, meteorologist of the United States Weather Bureau. It is essentially a modified baro-thermo-hygrograph designed with a view to extreme lightness, thus making it possible to carry it to great altitudes by means of sounding balloons. The instrument with its case (see fig. 8) weighs but 180 grams, as compared with 400 grams, the weight of the next heavier instrument of its type. In addition to its lightness, it possesses several other important advantages as noted in the brief description which follows:

Temperature element.—The bronze-invar bimetallic strip temperature element upon which this meteorograph depends for temperature records is mounted in the vertical screening tube at the right side of the instrument where it is exposed to the circulating air. The mounting is so designed and arranged as to avoid the effects of radiation and conduction from surrounding parts. The strip is bent into circular form and its free end is connected by a simple linkage to the recording stylus which rests upon the record drum. One millimeter of the temperature scale represents a change of 2° C.

Humidity element.—The humidity element is composed of six or eight series of human hairs of three strands each. They are mounted upon suitable fixtures and extend vertically inside the screening tube. Tension on the hairs is maintained by a flat spring, one end of which is connected to the recording stylus.

Pressure element.—Perhaps the most interesting feature of this instrument is the method for obtaining a comparatively open scale for the pressure records which may cover practically the entire range of atmospheric pressures. Inasmuch as the pressure scale becomes greatly contracted at extremely high altitudes, thus making the records uncertain, it is desirable that a special arrangement be made in the instrument design so as to compensate for this characteristic.

FIG. 8.—Fergusson meteorograph.

The pressure element itself is a common form of exhausted Bourdon tube with one end securely fixed to the instrument base and with its movable extremity acting upon the recording stylus through a linkage of ingenious design. It is this linkage which cares for the open-scale feature mentioned in the preceding paragraph by causing the recording stylus to make two traverses of the record chart. The first or upward traverse covers a certain range in altitudes and the second or downward trace continues the record to the upper limit. It is also possible by the adjustment of this linkage to vary the scale so that a certain upward motion of the stylus has a value equal to twice that of the downward motion in order that small pressure changes above 10,000 meters may be determined with greater precision than would be possible with a uniform scale.

Record drum and chart.—The record drum seen at the left of the illustration surrounds the clock movement which is mounted upon the base plate. The clock causes the drum to turn through one complete rotation in one hour and is so designed and connected to the latter that the clock may be rewound by turning the drum backward; the number of turns depends upon the number of rotations desired for the record.

The chart or record sheet is made of very thin sheet aluminum (0.03 millimeter in thickness) wrapped once around the drum and with the ends secured by a special lock joint. In making the instrument ready for service a layer of smoke is deposited upon the aluminum chart by holding it (in place on drum) over a camphor or kerosene flame. The recording styli rest upon this smoke film and make fine lines as the drum rotates. In addition to the recording styli mentioned above, there is a fourth one which is fixed so as to mark a base line at the bottom of the chart. A stylus lifter is provided to raise the markers from the chart when no record is desired. Finally, the record may be fixed or made fast, after it has been obtained, by applying a suitable preparation (white shellac and glycerin) which hardens the film. The record is examined and evaluated by the use of a transparent scale suitably divided.

The designer of this instrument has endeavored to avoid complication so that the parts may be easily and economically manufactured and assembled. The entire instrument, including the case, is constructed with a view to compactness and lightness without loss of rigidity. The outside length of the instrument is 210 millimeters, the height 90 millimeters,

and greatest width 85 millimeters. The clock drum is 80 millimeters in height, 57 millimeters in diameter, with a time scale of 3 millimeters per minute.

FRENCH BARO-THERMO-HYGROGRAPH.

Figure 9 shows a view of a baro-thermo-hygrograph of French design. The instrument is similar to the baro-thermograph shown in figure 5 of this paper, with the exception that in this instrument the hygrograph element is added.

Temperature element.—The Bourdon tube temperature element is mounted underneath the instrument base, where it is protected from mechanical injury by a sheet-metal framework. A simple linkage connects it to the recording pen arm resting upon the uppermost section of the chart, which is ruled to cover a temperature range from minus 30° C. to plus 30° C. in a 60 millimeter vertical direction.

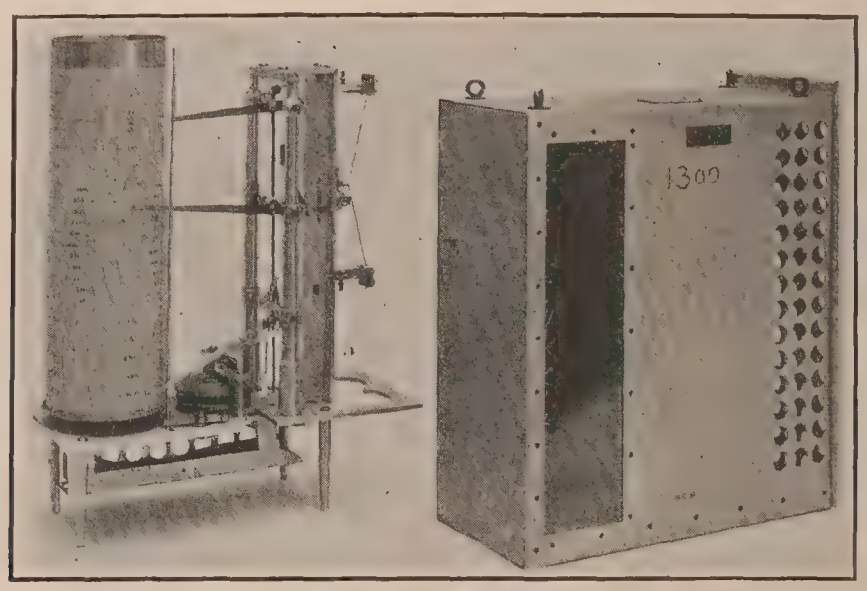


FIG. 9.—French baro-thermo-hygrograph.

Humidity element.—The usual form of hair hygrograph element is employed. A bundle of approximately 25 hairs is mounted upon fixtures upon the right-hand vertical frame section of the instrument. Tension adjustments are provided. A small hook looped over the hairs midway between the supports is connected with a pivoted lever which carries an arm shaped in the form of a cam and resting upon a second pivoted cam-shaped arm which transmits its motion to the recording pen. These cams are held in contact by a light spring in tension. This arrangement converts the changes of hair length so as to make the recording pen movement linear with respect to humidity changes. This makes possible the use of an evenly divided humidity scale, which is seen at the center of the chart. This section is 60 millimeters in height and is divided into 50 equal spaces.

Pressure element.—As in the baro-thermograph, a double aneroid element is used. This is connected through a simple linkage with the recording pen, which moves over the lowest portion of the chart and covers a range from 750 millimeters to 150 millimeters in a height of scale of 60 millimeters.

Record drum and chart.—The record drum, which contains the clock mechanism and upon which the chart is held in place by a spring clamp, has a height of 208 millimeters and a diameter of approximately 67 millimeters. It makes a complete turn in a period of eight hours. The chart is 190 millimeters in height.

Instrument dimensions.—This baro-thermo-hygrograph complete with case weighs about 1,456 grams. It stands about 30 centimeters high, has a width of 10 centimeters, and a length of approximately 22 centimeters.

STRUT THERMOMETERS.

As previously outlined in this paper, the proper reduction of aircraft performance data requires the use of figures showing free-air temperatures observed at various altitudes during flight. The strut thermometer, so called because of its usual location on the strut of an airplane where it may be affected by free-air conditions, is most often used for this purpose and for similar duty in other experimental flight. The two principal types will be considered in this paper.

REMOTE-INDICATING LIQUID-EXPANSION TYPE.

The liquid expansion principle which forms the basis of operation of one of the common types of airplane engine thermometer is also used in strut-thermometer construction. In fact the same arrangement and mechanism with slight modifications may be used interchangeably in either capacity. The liquid-filled bulb with its radiation fins (fig. 10) is mounted in a

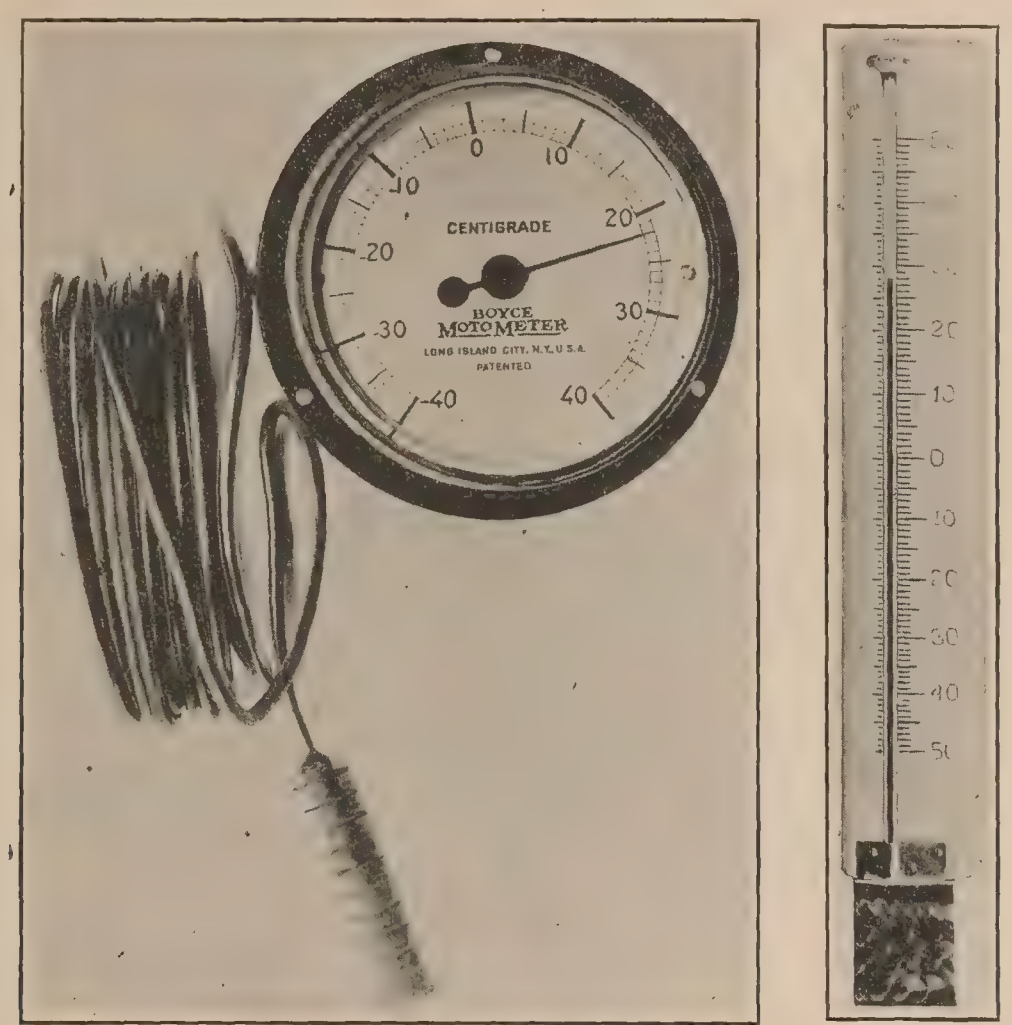


FIG. 10.—Strut thermometers.

suitable position on the aircraft, usually on a strut or on the landing gear. An increase in the temperature of the bulb causes the confined liquid to expand, which in turn produces a motion of the Bourdon element in the indicator with which the bulb is connected by means of an armored capillary tube of fine bore. The Bourdon tube is formed in several concentric helical coils, and its movable end connected to the indicating pointer through a bimetallic-strip helical coil. This bimetallic strip member is provided to compensate for the changes in the temperature of the gage and tubing. The strut bulb should be shielded from the direct rays of the sun by some arrangement which will permit satisfactory air circulation around the bulb. A complete discussion of this type of instrument may be found under the title "Thermometers for Aircraft Engines," Part III of Report No. 129.

In order to allow for a more open scale, the dials of strut thermometers have a greater diameter than those of the engine instruments, and the scale covers a range of about 80° C., with −40° or −50° as lower limit. An instrument of this type recently examined had a dial

10 centimeters in diameter, with a sufficiently open scale to allow for readings to within one or two tenths of a degree.

Suitably designed thermometers of the vapor-pressure type may also be readily adapted for use as strut thermometers.

LIQUID-COLUMN GLASS TYPE.

The second type of strut thermometer shown in the photograph is a liquid-column glass thermometer of usual form, designed for the proper temperature range and so mounted in its backing as to make it readily adaptable to use on an airplane. This instrument is ordinarily fastened to a strut of the airplane where it is subjected to an unobstructed circulation of free air and is easily observed by the test pilot. The bulb is surrounded by a brass case intended to protect it from mechanical injury and also to shield it from the direct rays of the sun.

The wooden backing of the glass thermometer tube is so shaped as to accomodate itself to the strut form, and felt strips are provided to avoid marring the strut when the instrument is strapped in place. The scale is graduated and proportioned so as to make possible the correct reading of the instrument to within about 1° C. from the pilot's position.

TESTING OF STRUT THERMOMETERS.

A detailed descriptive treatment of thermometer testing methods and apparatus may be found in Part III of Report No. 129, under the title "Thermometers for Aircraft Engines."

REPORT No. 127

AERONAUTIC INSTRUMENTS
SECTION III

AIRCRAFT SPEED INSTRUMENTS

IN THREE PARTS

By F. L. HUNT and H. O. STEARNS
Bureau of Standards

REPORT No. 127.

AIRCRAFT SPEED INSTRUMENTS.

PART I.

AIR-SPEED INDICATORS.

By Franklin L. Hunt.

INTRODUCTION.

This report is Section III of a series of reports on aeronautic instruments (Technical Reports Nos. 125 to 132, inclusive) prepared by the Aeronautic Instruments Section of the Bureau of Standards under research authorizations formulated and recommended by the Subcommittee on Aerodynamics and approved by the National Advisory Committee for Aeronautics. Much of the material contained in this report was made available through the cooperation of the War and Navy Departments.

TYPES OF AIR-SPEED INDICATORS.

The air-speed indicator, as its name implies, is an instrument which shows the speed of aircraft relative to the air. It gives the speed with reference to the ground only when no wind is blowing. The ordinary air pressure type is also used as a buoyancy meter to warn the aviator when he is approaching the stalling speed at which flight can no longer be sustained, or, in rapid descent, when the air speed is becoming excessive.

The most usual forms of air-speed indicator, the Pitot and Venturi types, depend for their action on the pressure difference developed in suitably constructed pressure nozzles by their motion relative to the air. These pressures vary with the speed of the airplane in a manner which may be definitely determined for each form of nozzle.

The complete instrument consists of two parts, the pressure nozzle, which is located ordinarily on one of the outer struts of the airplane well outside of the propeller slip stream, and the indicator, in effect a sensitive pressure gage, which is fastended to the instrument board in the body of the airplane. The pressure nozzle and indicator are connected by metal tubing, which runs along the strut and the edge of one of the wings.

Pressure nozzles commonly used are of two types, (1) Pitot nozzles in which pressures greater than those of the surrounding atmosphere are developed by the impact of the air at the open end of a straight tube pointed in the direction of motion, and (2) Venturi nozzles in which pressures less than those of the surrounding atmosphere are obtained by the rush of air through the throat of a double-coned suction nozzle. In some cases the Pitot and Venturi are combined so as to add the effects of both.

The pressure developed by Pitot tubes is expressed by the relation

$$p=\frac{1}{2}\rho V^2 \tag{1}$$

In which p is the pressure

ρ is the density of the air

and V the speed.

Numerical relationships between air speed and the pressure developed by Pitot nozzles are shown in figure 2.

Venturi nozzles are also designed to give pressures which vary as the air density and the square of the velocity over the ordinary range of airplane speeds.

The above equation may be expressed in the more general form

$$p = K\rho V^2 \tag{2}$$

The coefficient K is dimensionless, and its value is consequently independent of the system of units chosen, provided the units are consistent. In the case of Venturi tubes, however, the value of the coefficient depends upon the form of the tube, and at low speeds is also dependent upon the speed. Numerical relationships between air speed and the pressure differences developed by various types of Venturi nozzles are shown in figure 4.[1]

Since the readings of instruments of the pressure type are directly proportional to the density of the air, they read correctly for a specific density only. The standard density now commonly used in aerodynamics is .001223 gms/cm^3, corresponding to the density of dry air at a temperature of 15°.6 C. under a pressure of 760 mm. of mercury.

In practice, arrangements must also be made to determine the static pressure at the point where the Pitot or Venturi nozzle is placed. For this purpose a tube closed at its leading end and with a concentric ring of small holes or narrow slots at the side is used. This tube is pointed in the direction of motion, so that the rush of air past the openings at the side of the tube is at right angles to these openings. The static pressure within the tube is in this manner maintained the same as that of the undisturbed air without. In nozzles where both Pitot and Venturi tubes are used the static head is eliminated.

The indicator proper, or pressure-measuring element, consists ordinarily of one or more corrugated metal capsules inclosed in an air-tight case or of an air-tight case separated by a membrane of rubber or doped fabric into two air-tight chambers. The dynamic opening of the pressure nozzle is connected to the capsules, or in the rubber or doped fabric diaphragm-type to one of the air-tight chambers. The static opening is connected to the air-tight case, or in the membrane diaphragm-type to the second air-tight chamber. In Pitot-Venturi instruments the Pitot is connected to the case or one of the air-tight chambers and the Venturi to the other, no static head being used. The differential pressure developed in the pressure nozzles causes the diaphragms to expand or contract, according to the magnitude and direction of the excess pressure. This motion is carried by a suitable transfer mechanism to a pointer and indicates the corresponding speed on a dial.

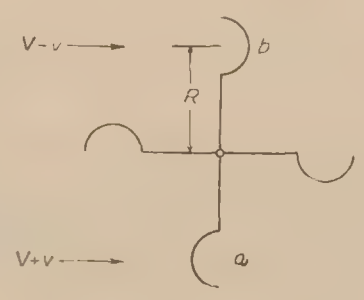
Other types of air-speed indicators have been suggested and used to a limited extent. Instruments depending on the pressure developed on a flat plate at right angles to the direction of motion have been used. In these the plate is attached to a lever whose motion is resisted by a spring. The amount of displacement of the lever, which is a measure of the air speed, is indicated by the motion of a pointer attached directly to the lever or through the intermediary of a sector and pinion. These instruments are ordinarily mounted on one of the struts of the airplane and read by the pilot at that distance. Although light and compact in arrangement and of a simple construction, they are not so accurate as the best indicators of the pressure-head type. Instruments of this type obey the ρV^2 law, for which the above-mentioned formula (2) applies.

Instruments of the anemometer type have been extensively used in Germany. In these the air speed is determined by the rate of revolution of a cup anemometer, which is attached to a tachometer or rate-of-revolution indicator. These instruments are also self-contained and are located on a strut of the airplane, from which position they are read at a distance by the pilot. Instruments of the anemometer type with distant control have also been suggested. In this form the anemometer alone is located on the strut. Wires lead from it to the indicator which is on the instrument board in the fuselage. The speed of rotation of the tachometer element of

[1] For a detailed discussion of the theory of Pitot and Venturi tubes see Annual Report of the National Advisory Committee for Aeronautics 1915, Report No. 2 by W. H. Herschel and E. Buckingham. Also see Report of National Advisory Committee for Aeronautics 1918, No. 31, on the Development of Air Speed Nozzles, by A. F. Zahm.

the indicator is maintained the same as that of the anemometer by a synchronizing commutator connected to the anemometer spindle. The energy required to drive the indicator is supplied from a battery or generator. Only the energy required to overcome friction is taken from the rotating anemometer itself.

Air-speed indicators of the anemometer type give the actual air speed at all altitudes, since their indications are practically independent of the density of the air. Their behavior may be explained in an elementary manner by assuming the motion to depend only upon the forces acting on the cups a and b, whose axes are perpendicular to the direction of motion, the effect of the cups in other positions being considered negligible.



If V is the velocity of the wind and v the linear velocity of the cups, the velocity of the air relative to cup a is evidently $V+v$ and that relative to cup b, $V-v$. Knowing the density of the air ρ, the coefficients of air resistance k_a and k_b for the cups a and b, the central cross section of the cups A, and the radius R from the axis to the center of the cups, neglecting friction, the driving couple is

$$k_b \rho A (V-v)^2 R \tag{3}$$

and the resisting couple

$$k_a \rho A (V+v)^2 R \tag{4}$$

The speed of rotation of the cups will increase until these couples are equal; that is, until

$$k_b \rho A (V-v)^2 R = k_a \rho A (V+v)^2 R \tag{5}$$

Assuming $k_a = 0.033$ and $k_b = 0.019$, as determined by Renard

$$\frac{V}{v} = 3.5 \tag{6}$$

The ratio $\frac{V}{v}$ is independent of the density of the air and hence of the altitude.

Experiments have been carried out in the Aerodynamic Laboratories at Gottingen, in Germany, which show that this ratio is practically constant over the range of velocities ordinarily experienced in airplane flight. For air densities up to an altitude of 5,000 meters and velocities above 100 kilometers per hour the variation of the ratio $\frac{V}{v}$ is considerably less than 1 per cent.

Efforts have also been made to determine air speed by the rate of flow of air through a suitably constructed flow meter. Experimental instruments of this type have been made which consist of an air-tight cylindrical chamber with a radial vane, rotatable about the axis of the cylinder. Air from a pressure nozzle similar to a Pitot nozzle is conducted into the chamber between the radial vane and a fixed radial partition so as to rotate the vane in opposition to a restraining hairspring by an amount depending on the pressure. The air flows out of the instrument by way of the clearance between the vane and the ends of the cylinder, which clearance is varied in a predetermined manner by suitable cam surfaces on the inside of the ends of the cylinder so as to give the instrument a uniform air-velocity scale.

Small propellers or helical air screws mounted on a strut of the airplane and connected to an indicating tachometer on the instrument board by electrical commutators have also been tried.

In another type the rotation of the propeller is resisted by springs, so that the resultant motion is limited to a fraction of one revolution. By a suitable transfer mechanism this motion is transmitted to the indicator, which is directly connected to the propeller support. The instrument is mounted on a strut and read by the pilot at a distance. It has also been proposed to use a propeller to operate a small magneto, the current from which is conducted by wires to an indicating galvanometer on the instrument board. The possibility of slip entering as a disturbing factor has to be considered in an instrument of this type, since an appreciable amount of energy is taken to operate the indicating element.

Hot-wire anemometers have also been designed in which the cooling effect of a stream of air on an electrically heated wire grid is taken as a measure of the air speed.[2] The amount of energy dissipated from such a grid is found to be directly proportional (1) to the temperature difference between the grid and its surroundings (2) to the square root of the velocity and (3) to the square root of the density of the air stream. Consequently if the grid is openly exposed to the full wind stream it is insensitive at the higher velocities. This difficulty has been overcome in a recently proposed device in which the grid is inclosed in a stream-line envelope provided with a perforated plug at the incident end. The velocity of the air which passes the grid is thereby greatly reduced and the sensitiveness of the instrument at high velocities mark-

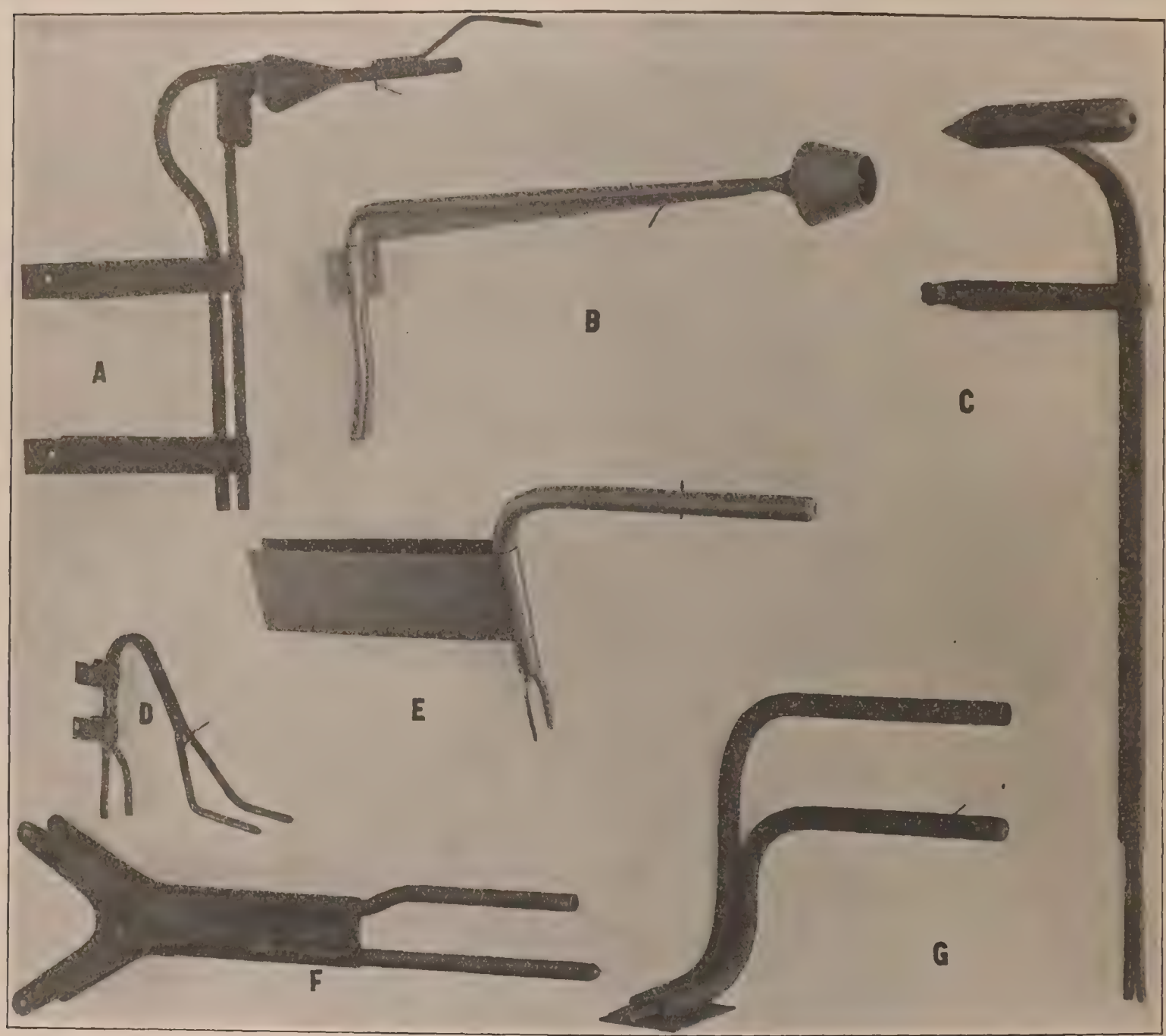


FIG. 1.—Pitot nozzles.

edly increased. By varying the size of the perforations in the plug the instrument can be made sensitive for the desired range of velocities. The grid is maintained at a constant temperature of from 200° to 300° C. above that of the air stream by balancing it on a bridge. The air velocity is found by the energy dissipated by the grid. The device is complicated and not suited for ordinary use for determining the speed of aircraft, but might be used in performance tests. Since the readings of the instrument are proportioned to the square root of the product of the density and the velocity and not of the density and the square of the velocity, the device is not a direct reading buoyancy meter as is the Pitot or Venturi meter of the ordinary type. It is also evident that a determination of the air density is necessary before the velocity can be found.

[2] See also, On some proposed electrical methods of recording gas flow in channels and pipes based on the linear hot-wire anemometer, by L. V. King, Journal Franklin Institute, August, 1916, p. 191.

DESCRIPTION OF TYPICAL INSTRUMENTS.

Nozzles.—A group of typical Pitot nozzles is shown in figure 1. A is an early type of British nozzle and B an early type nozzle of American manufacture. The dynamic openings are at the extreme outer ends of the tubes. Each is provided with a hood, behind which the static tube terminates. A slight suction is produced by these hoods, so the tubes are strictly speaking not simple Pitot tubes. C is a German Pitot nozzle. This consists of a tube about seven-eighths of an inch in diameter and 5 inches long. The dynamic opening is at the rounded end. The nozzle tapers sharply at the trailing end. The static openings are in the depressed ring or groove in the side of the tube about 1 inch back from the open end. The tubing which connects the nozzle to the indicator is closed in a stream-line casing. D is an American nozzle in which the dynamic and static heads are separate tubes about one-fourth inch in diameter. These are bent upward at a sharp angle to prevent the accumulation of water in the tubes. The static head has a series of fine concentric holes in the side of the tube. E is a Pitot nozzle of British design in which the dynamic and static tubes are concentric, the dynamic tube being inside of the static tube. The dynamic opening is at the extreme end of the tube. The concentric ring of holes in the static tube can be seen at the side about 2 inches from the end. F is a standard British Pitot nozzle. The static head in this case has 4 concentric rings of holes about one-fourth of an inch apart along the side of the tube. G is a Pitot nozzle in which an annular series of slots are provided in the static head instead of the small round holes which are usually used. Experiments indicate that small round holes are in general a more reliable means of determining the static pressure.

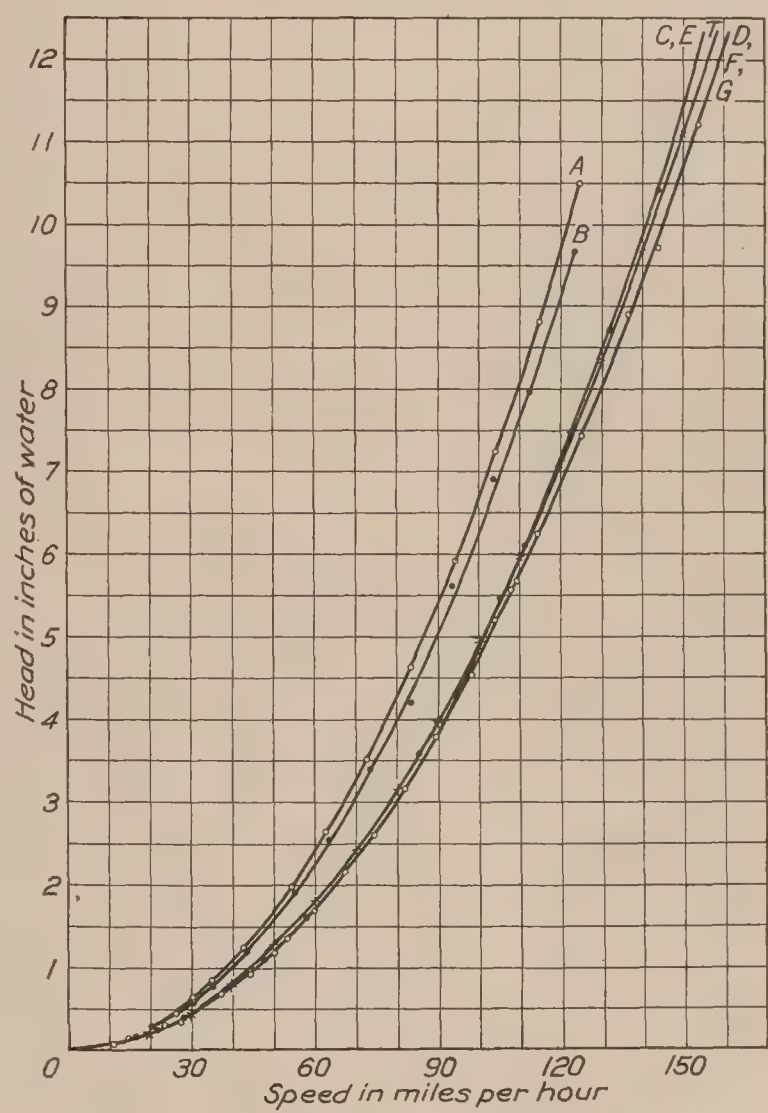


FIG. 2.—Calibration of Pitot nozzles shown in fig. 1.

The pressures developed by these nozzles for the ordinary speed range of airplanes is indicated in figure 2.[3] The curve marked T represents the theoretical values computed by the ρV^2 law. The larger differential pressures developed by nozzles A and B are due to the slight suction caused by the hoods. Nozzles C and E developed pressures which are the same within the experimental error. Nozzles D, F, and G are also practically identical in performance. With the exception of nozzles A and B the values agree closely with the theoretical curve, which shows that Pitot nozzles may differ considerably in the details of construction and still follow closely the ρV^2 law.

A series of nozzles of the Venturi and Pitot-Venturi type are shown in Fig. 3. A represents a single Venturi nozzle of French design. At the side of the tube about two-thirds of the distance from the forward end can be seen an annular row of holes which constitute the static openings. In this nozzle the suction and static tubes are concentric. B is a German double Venturi nozzle and C a French double Venturi nozzle. In both of these a small Venturi tube is located at the throat of the large Venturi tube and concentric with it. A greater differential pressure is thereby obtained than with the single nozzle. In these nozzles no static connection is used. D and G are Pitot-Venturi nozzles made from the design of Dr. A. F. Zahm for use

[3] These results and those on Venturi tubes which follows were obtained in cooperation with the Wind Tunnel Staff of the Bureau of Standards.

on naval aircraft. D is of electroplated copper and G of cast aluminum. These have both a pressure and a suction nozzle. The suction nozzles are distinguished by the long protruding cones and the Pitot nozzles by the short orifices on the supporting arm of the Venturis. By their combined action these nozzles develop a larger differential pressure than either the Pitot or Venturi alone, but they are rather large and heavy. They are attached to one of the outer struts of the airplane with the supporting arm inclined downward at an angle of 45° to prevent water choking the tubes.

Another form of Pitot-Venturi nozzle similar to the Zahm but much smaller and equally effective as regards the differential pressure developed is shown in E. This nozzle was developed

FIG. 3.—Venturi and Pitot-Venturi nozzles.

at the Bureau of Standards in collaboration with the Bureau of Aircraft Production. F is a double Venturi nozzle of French design. The Venturi tube, which is the upper of the two shown in the illustration, is stream lined on the outside. This nozzle was designed for use with the Toussaint-Lepère recording air speed indicator shown in figure 21.

Venturi nozzles have the advantage of developing much greater differential pressures than Pitot nozzles, so that more rugged indicators can be used with them, which is a consideration in view of the vibration and other unfavorable conditions to which the indicator is continuously subjected in use. On the other hand slight differences in the size and shape of Venturi nozzles introduce large changes in the pressures developed so that it is difficult to manufacture Venturi nozzles of required uniformity, while Pitot nozzles are insensitive to wide variations of dimensions and shape. At present Pitot nozzles are used almost entirely in England and Venturi

nozzles are generally used in France. During the recent war American nozzles were almost entirely of the Pitot-Venturi type. Most German nozzles are Venturis.

Calibration curves of the above Venturi nozzles are shown in figure 4. The performance of the American Pitot-Venturi nozzles D, E, and G is represented by curve D. E. G. They are all designed to develop the same differential pressures. The French Pitot-Venturi F gives slightly larger values for the differential pressure, but not quite so large values as the single French Venturi A with static connection. At corresponding speeds the two double Venturi nozzles B and C give practically three times the differential pressure developed by any of the other instruments.

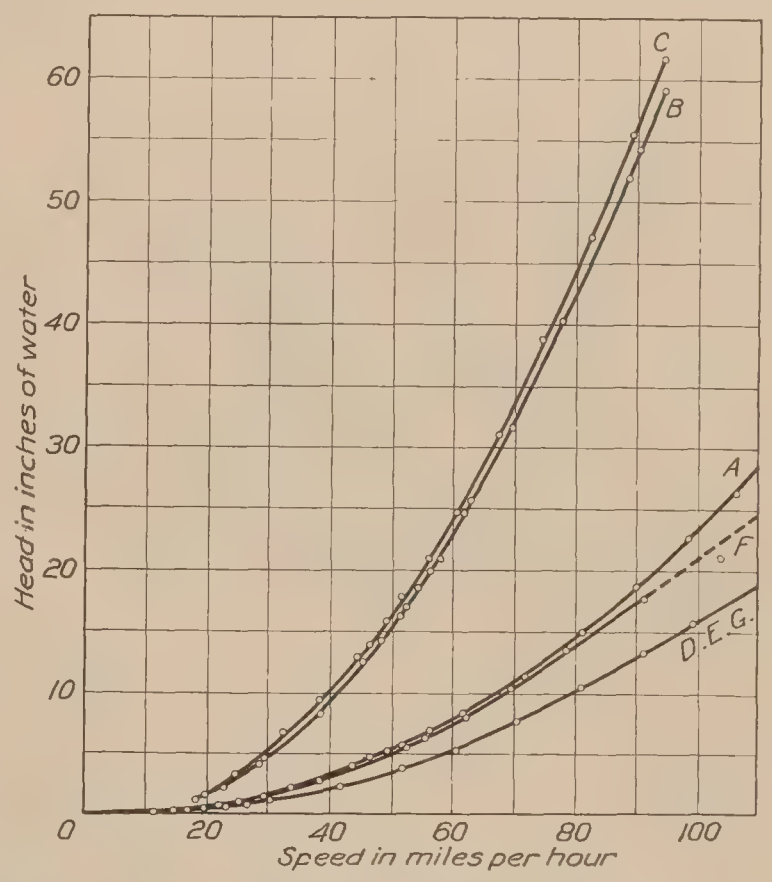


FIG. 4.—Calibration of Venturi and Pitot-Venturi nozzles shown in fig. 3.

Indicators.—The mechanisms of representative air speed indicators of the Pitot and Venturi type are shown in accompanying photographs. Figure 5 is an early Foxboro indicator of American manufacture for use with the Pitot nozzle shown in figure 1B. The nozzle is connected to two batteries of small metal diaphragms D. In addition to the elastic resistance of the diaphragms themselves to the internal pressure developed by the nozzle, small coiled springs S are provided, which connect all of the units of each battery. The motion of the diaphragms is transmitted by the levers L to the sector K which engages a pinion on the pointer shaft. The reservoir R which is inserted between the nozzle connection and the diaphragms is provided to damp the oscillations which would otherwise be set up by momentary fluctuations in the nozzle pressure. This indicator is much larger and heavier than the later types used during the recent war. The scale is also not uniform, being considerably compressed at the low speed end.

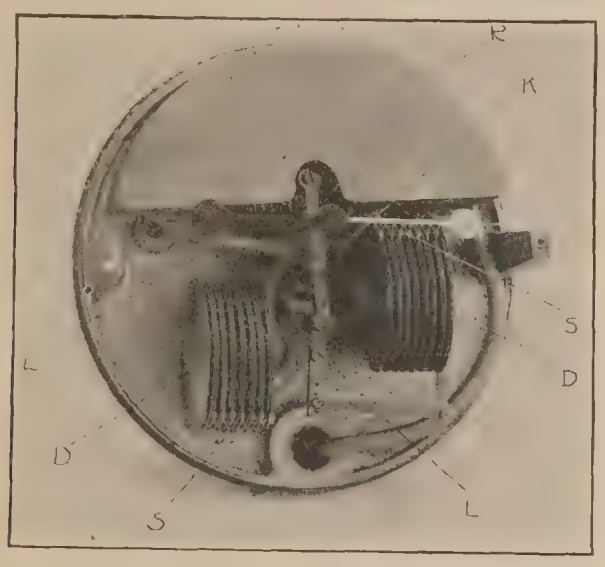


FIG. 5.—Foxboro air-speed indicator, Pitot type.

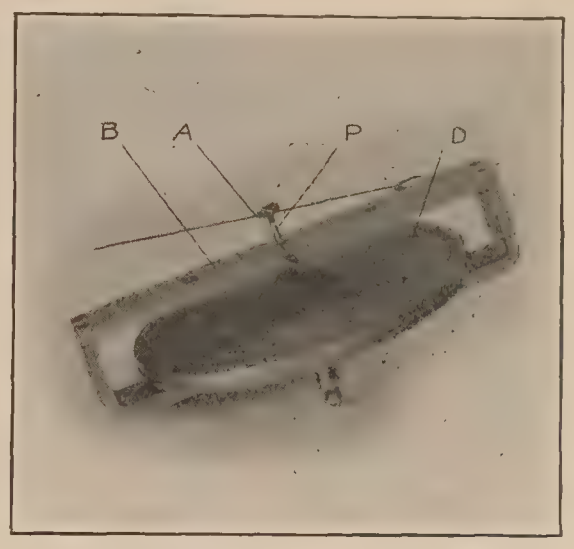


FIG. 6.—Sperry air-speed indicator.

The Sperry instrument shown in figure 6, of American manufacture, is provided with a single corrugated metal capsule D. It is used in connection with the Pitot nozzle shown in figure 1E. The diaphragm D presses against the steel pin P which has a helical groove in its outer surface. A screw A in the frame B engages the groove. The indicating pointer is fixed to the upper end of the steel pin P and as the diaphragm expands the pin P is forced out axially and rotated simultaneously by the action of the spiral groove thereby indicating through the dis-

placement of the pointer, the magnitude of the deflection of the diaphragm. This transfer mechanism is simple, but friction effects are larger than in instruments of the sector and pinion type, and, moreover, it is not practicable to vary the pitch of the spiral groove sufficiently to make the scale uniform.

Figure 7 is the mechanism of a Munroe Pitot indicator of British manufacture which has a single diaphragm made of silver. This diaphragm is connected by a short lever arm to the arbor A. At the end of the arbor is a pin P which terminates at the slot S. The expansion of the diaphragm causes the arbor A to rotate and the pin P to slide along the slot S, thereby dis-

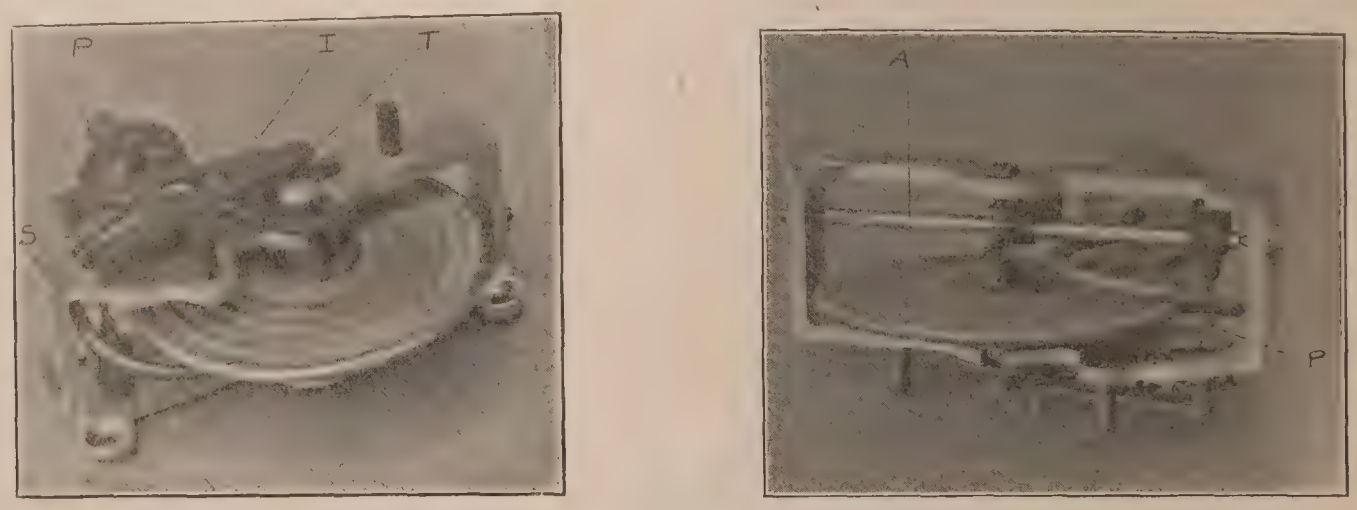


Fig. 7.—Munroe air-speed indicator.

placing the sector T. This sector meshes with a pinion on the shaft of the indicating pointer I. The sliding contact at the slot is used to make the scale more nearly uniform by regulating the direction of the slot relative to the sector so as to increase the relative deflection for the lower velocities. A hairspring is attached to the pointer shaft to prevent backlash.

Figure 8 represents a Smith indicator of the Pitot type. It is of British manufacture. The two corrugated metal diaphragms D which are offset, are connected by a lever mechanism to opposite sides of the arbor A. Otherwise the transfer mechanism is the same as that of the indicator just described. Both were designed for use with a nozzle of the type F, figure 1.

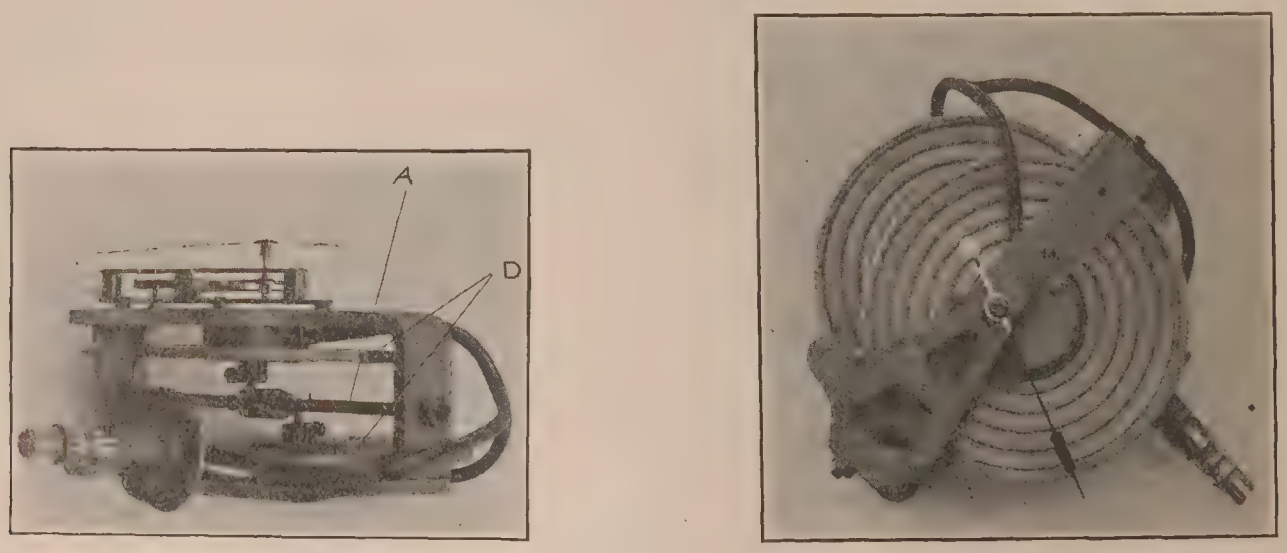


Fig. 8.—Smith air-speed indicator.

The instrument shown in figure 9, an Ogilvie instrument, also British, is provided with a diaphragm D of rubber, the tension of which is regulated to conform to the range of pressures to be measured. It is used in connection with a Pitot tube. The transfer mechanism is very simple. A silk fiber F attached to the center of the diaphragm passes over a small idler M and is wound around the pointer stake T. The pressure nozzle of the Pitot tube is connected to the chamber on one side of the rubber diaphragm and the static connection to the other. The differential pressure thus obtained causes the diaphragm to expand, thereby unwinding the fiber from the pointer stake and indicating a deflection depending upon the pressure developed. The back of the instrument is cup shaped. With increasing pressure the diaphragm expands

until it comes in contact with the back, thereby protecting the diaphragm from injury under excessive pressure. This instrument is simpler in construction than any of the others of the same type, but is difficult to readjust if thrown out of adjustment. This indicator was designed for use with nozzle E, figure 1.

Figure 10 shows a Clift instrument which has a diaphragm of doped silk, D. It is of British manufacture. The center of the diaphragm is covered by a metallic disk K at the center of which is a pin P. The pin is in contact with the spring S. A lever L attached to an arbor A rests against the spring S. At the end of the arbor is a pin which engages a slot attached to the sector T in the usual manner. In this instrument the spring S is the elastic member which takes the place of the corrugated diaphragm in the instruments previously described. The air-tight chamber on one side of the doped silk diaphragm is connected to the pressure tube of a Pitot nozzle; that on the other side, to the static tube. As a result of the differential pressure developed the pin P is pressed against the spring S which in turn operates the indicating mechanism by means of the lever L. Nozzle G, figure 1, was designed to use with this indicator.

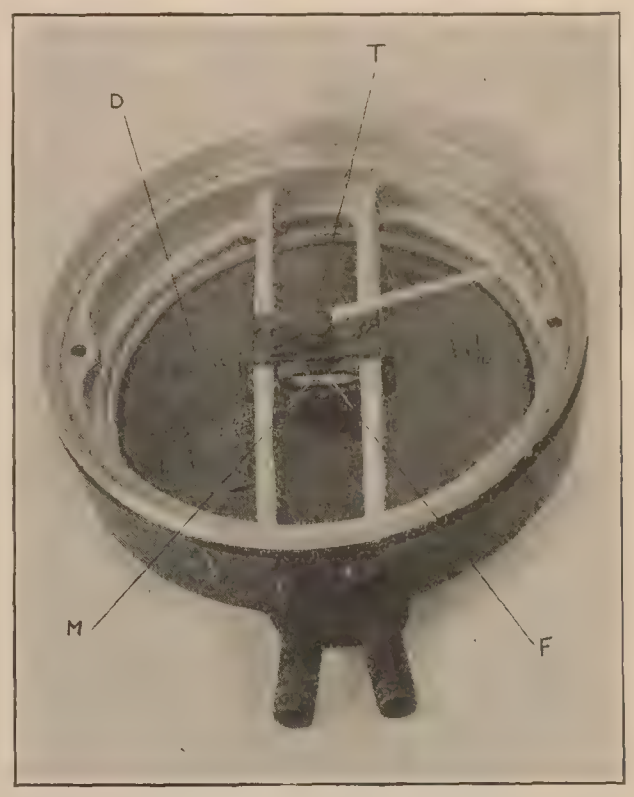


FIG. 9.—Ogilvie air-speed indicator.

An Atmos Pitot indicator of German manufacture with doped silk diaphragm is shown in figure 11. A pin attached to the plate P in the center of the diaphragm presses against the

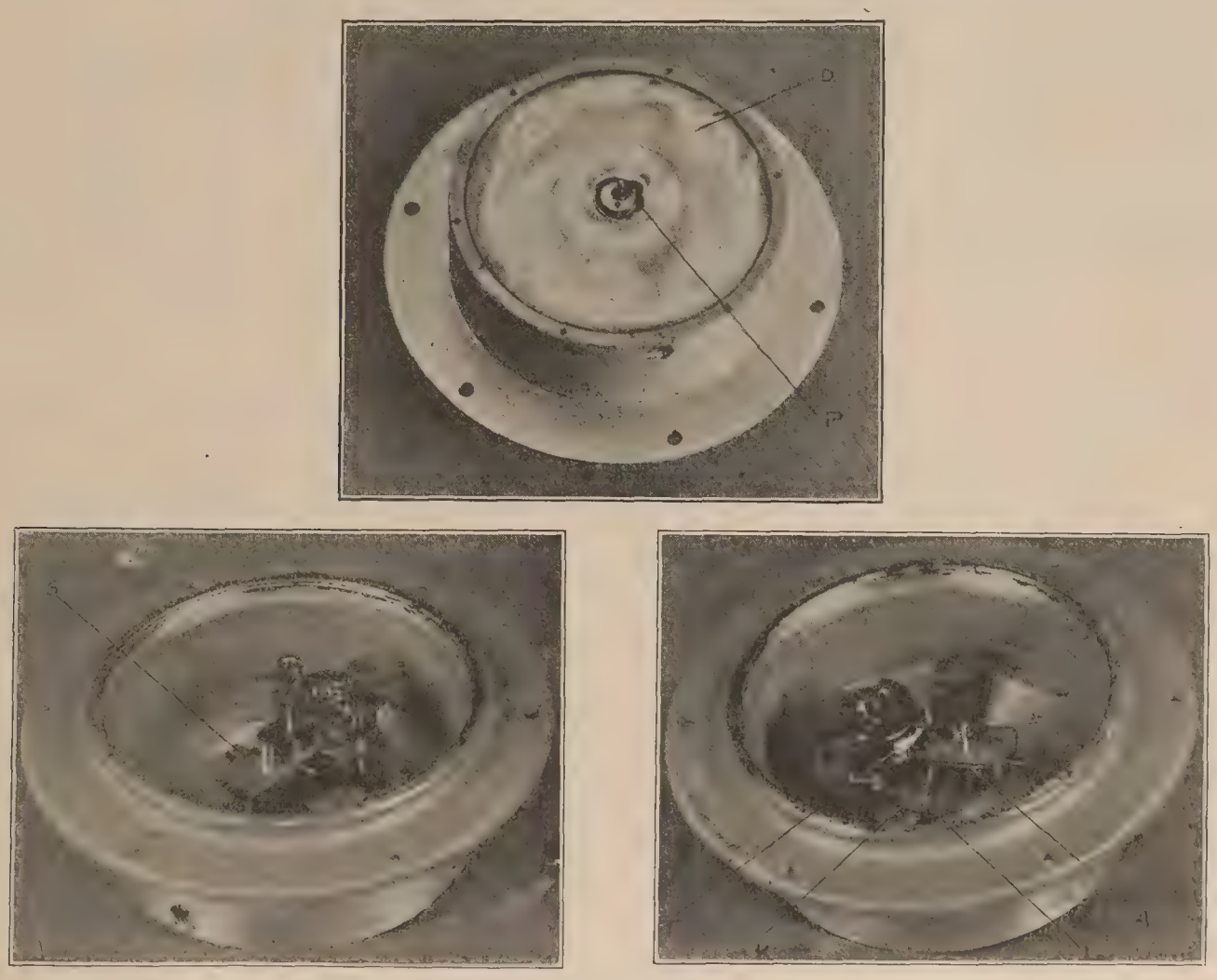


FIG. 10.—Clift air-speed indicator.

spring S which is in contact with the steel ball B at the end of the adjustable lever L which in turn is attached to the arbor A. The pin T also attached to the arbor A supports the fine

fiber F which is wound around the stake of the pointer. The expansion of the diaphragm presses the spring S against the ball lever L, thus rotating the arbor A and causing the fiber F to unwind and move the pointer. Above the spring S is a second spring S′ supported at both ends and depressed near the center by an adjustable set screw K. The lower spring S conforms to the curved surface of the upper spring S′ which is adjustable, thus making it possible to vary the openness of the scale. The lever L also provides an additional means of adjustment. This indicator was designed for use with nozzle C, figure 1.

Pitot nozzles are sometimes used with liquid manometer indicators. Figure 12 shows such an indicator, the Pioneer, of American manufacture, designed for use with the nozzle

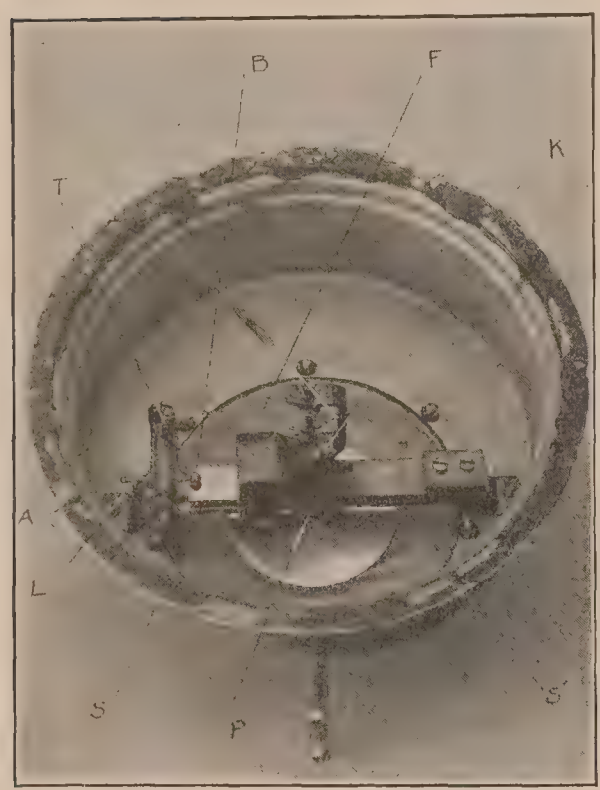


Fig. 11.—Atmos air-speed indicator.

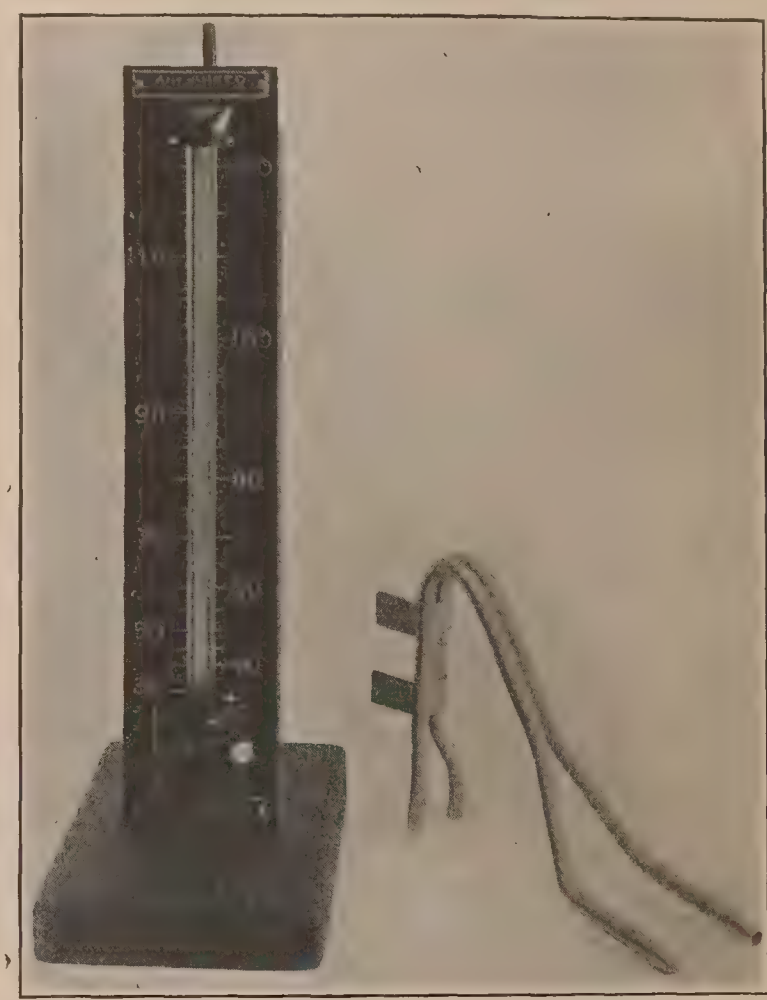


Fig. 12.—Pioneer air-speed indicator, liquid type.

shown and also in figure 1, D. The reservoir at the base is of large diameter compared with that of the column of liquid which indicates the air speed, so that practically all of the motion of the liquid is in the tube. The velocity is read directly from the scale which is graduated in miles per hour. A similar device, the Elliott, of British manufacture, is shown diagrammatically in figure 13. In this case instead of a single large reservoir at the base of the indicator tube two narrow reservoirs A and B are provided, one on each side. There are also safety traps C and D between the indicator and the nozzle connections to prevent the loss of the indicating liquid in case the instrument is inverted. The scale is adjustable for zero correction. The liquid type of indicator is not much used at present. Aside from the possibility of loss of liquid which would give erroneous readings, and also tilting errors the scale of the instrument is not uniform, being excessively compressed at the low velocity end. Moreover, it is the general opinion that a dial with an indicating pointer is much more easily read at a glance than a liquid column so that the former type of indicator is preferred by most aviators.

Figure 14 represents a Badin instrument of French manufacture designed for use with the double Venturi nozzle shown in figure 3C. A single corrugated metal capsule is used. The transfer mechanism is essentially the same as that of the Pitot instruments just described. The motion of the diaphragm D rotates the arbor A and presses the pin P against the sector T, which engages a pinion on the pointer shaft S. The indicator shown is for use with a Venturi

tube. Instruments of this design are also used with Pitot tubes. In this case a knife edge projects from the sector T beyond the pivot V. The pin P makes contact with the knife edge on the opposite side of the pivot from that shown in figure 14 so that the direction of the motion

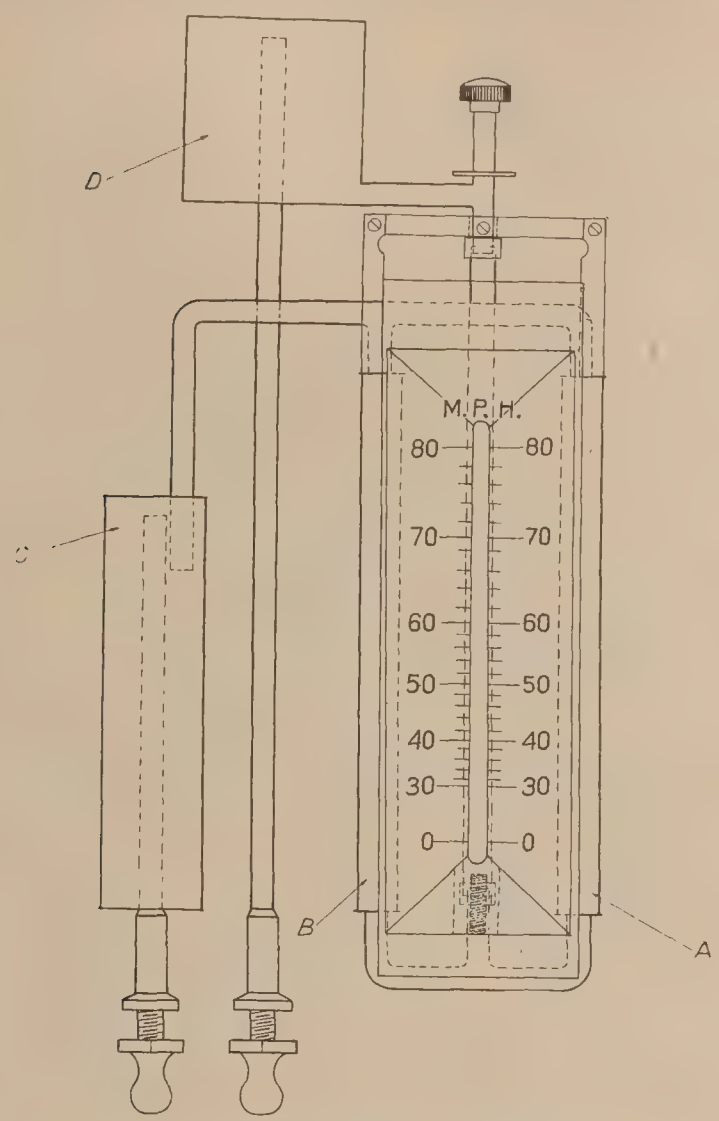


FIG. 13.—Elliott air-speed indicator, liquid type.

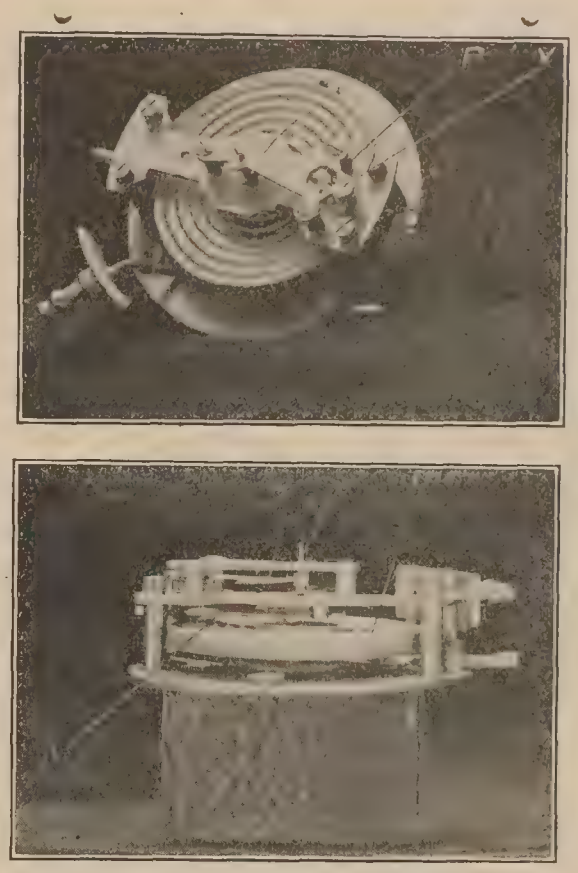


FIG. 14.—Badin air-speed indicator.

of the pointer is the same in both Pitot and Venturi instruments, although in the former case the diaphragms expand and in the latter contract.

Figure 15 shows an improved type of Badin mechanism. A small contact disk P attached to the diaphragm pushes against the jewel J at the end of the short counter balanced lever L

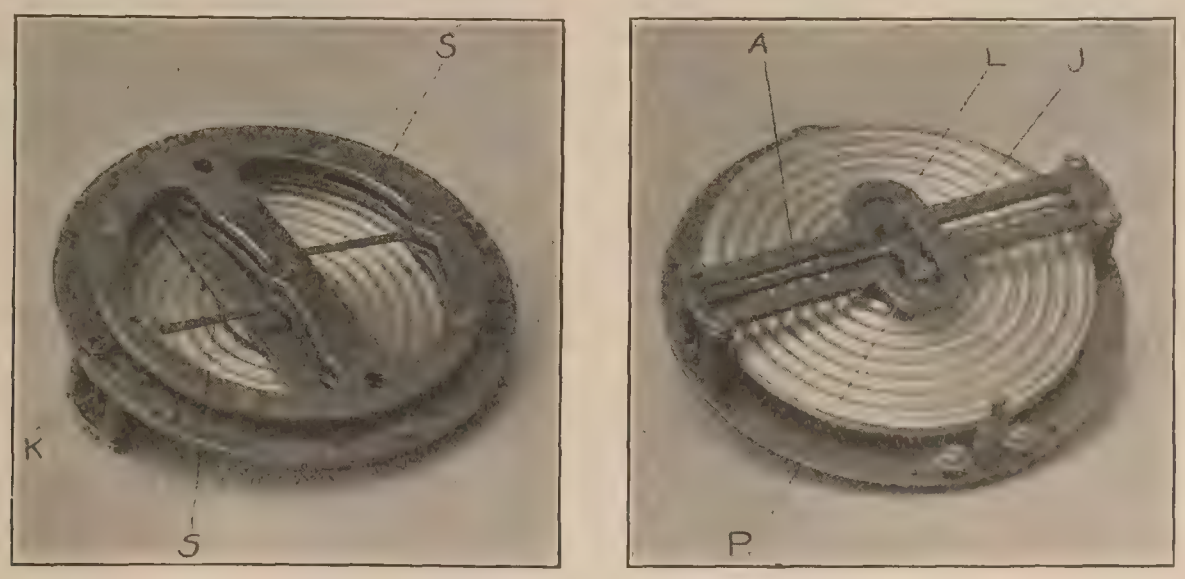


FIG. 15.—Badin air-speed indicator, improved type.

attached to the arbor A. The pin K operates the sector S which meshes with the pinion attached to the pointer. The sector S′ serves as a counterbalance for the sector S. Jeweled bearings are used throughout except those of the arbor A.

The Bruhn indicator shown in Fig. 16 is of German manufacture and was designed for use with the nozzle shown in figure 3B. The mechanism consists of a single bronze diaphragm which contracts, thereby rotating the sector S by means of the connecting strip T. The sector engages a pinion on the stake of the pointer.

Figure 17 represents a Bristol instrument of the Pitot-Venturi type of American manufacture designed for use with nozzles shown in figure 3 D, E, and G, all of which develop the same differential pressure It is provided with two superposed diaphragms. Their motion under the differential pressure of the Pitot-Venturi nozzle is transmitted by the lever L to the arbor A and thence by the pin P to a movement of the sliding contact type identical in principle to those previously described. A coil of capillary tubing C is inserted between the diaphragms and the nozzle connection to dampen the vibration of the pointer under the action of wind gusts encountered in flight. The same result is also sometimes obtained by inserting baffle plates in the connecting tubes. The baffle plates consist of disks of thin metal perforated with small holes. Still another method of damping vibration is to connect the case instead of the diaphragm to the suction nozzle, thereby increasing the volume of air displaced, which tends to neutralize momentary fluctuations in pressure.

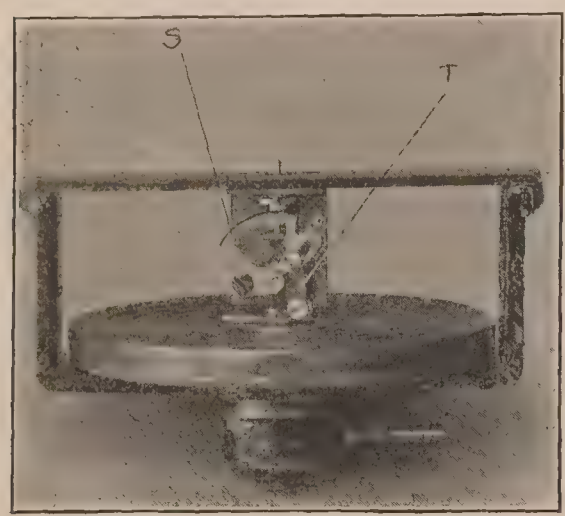
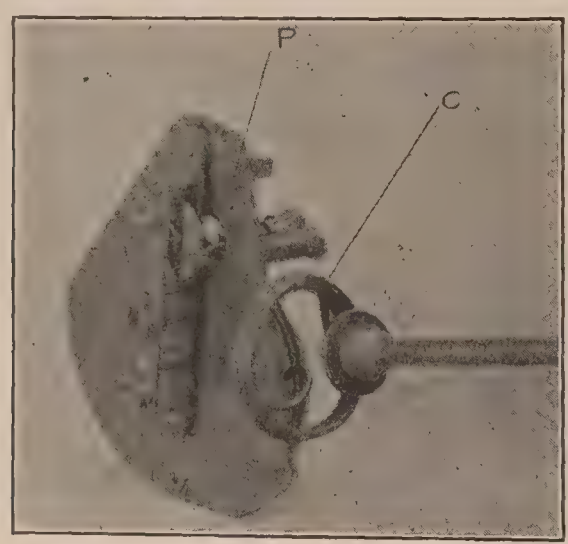


FIG. 16.—Bruhn air-speed indicator.

Figure 18 is a Foxboro instrument of the Pitot-Venturi type, which is made by the same American manufacturer as the Pitot instrument shown in figure 5. It is designed for use with the Pitot-Venturi nozzles shown in figure 3 D, E, and G. The indicator is similar in construction to the Pitot instrument except that in the Pitot-Venturi form the diaphragms contract instead of expanding under the resultant differential pressure. The levers from the diaphragms to the sector are connected so as to make the pointer rotate in the same direction in both instruments. The Pitot-Venturi instrument is also provided with adjustable springs T which are used to give a uniform scale by constraining the motion of the diaphragms when the differential pressure has become sufficient to bring the pins P in contact



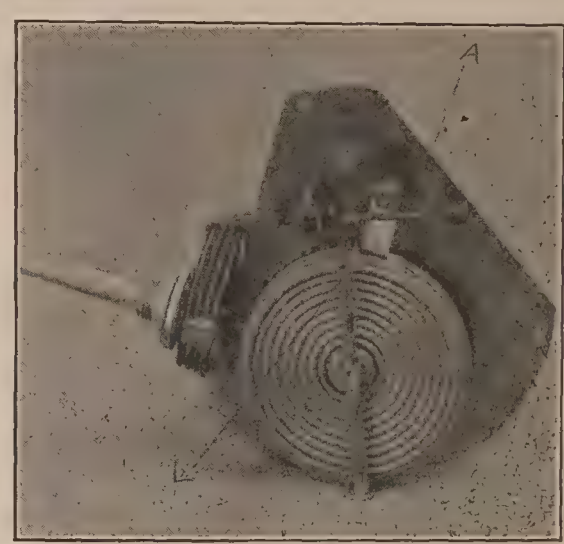


FIG. 17.—Bristol air-speed indicator.

with the springs T, thus increasing the resistance to motion for the larger velocities for which the differential pressures are proportionally greater.

The Precision instrument shown in figure 19, an American instrument of the Pitot-Venturi type designed for use with nozzles D, E, and G, figure 3, is provided with three superposed metal diaphragms. The ball and slot transmission is used but differs from those previously described in that the slot is curved instead of straight. By properly choosing the curve of the cam the scale of the instrument can be made practically uniform. The transfer mechanism is statically balanced by a small weight W attached to the arbor A. The yoke Y is provided with adjusting screws.

Figure 20 shows a King instrument, also of American manufacture, designed for use with the Pitot-Venturi nozzle shown in figure 3 D, E, and G. The motion of the diaphragms is transmitted to the arbor A, at the end of which is fastened a hairpin B of stiff wire which makes sliding contact with a second wire C attached to the sector S. The instrument is adjusted by bending the wire C and the hairpin B. A wide range of adjustment is in this way possible. On the other hand, a slight roughness or corrosion of the wire or hairpin greatly increases the friction errors and slight accidental bending of the wires seriously affects the calibration.

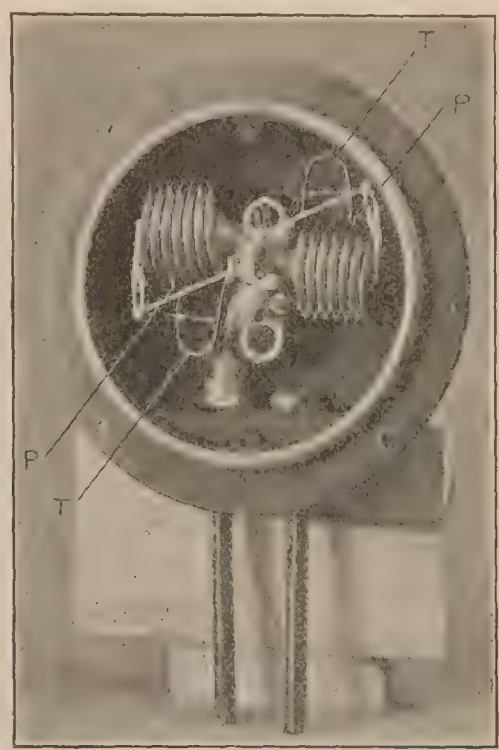


FIG. 18.—Foxboro air-speed indicator, Pitot-Venturi type.

Figure 21 is a Toussaint-Lepère recording air speed indicator of French manufacture. It is also of the Pitot-Venturi type and designed for use with the nozzle shown in figure 3 F. It is provided with two rubber diaphragms which are inclosed in the chambers C and C′. These diaphragms are built up of annular stampings of rubberized fabric which are cemented together in the form of cylindrical bellows. The diaphragms are attached to the connecting rod R which in turn actuates the lever system L and the pen P. The elastic resistance member is the spring S. This in conjunction with the lever system provides a practically uniform scale. The Pitot nozzle is connected to the chamber C and the Venturi to the chamber C′. The record is made on a graduated chart K which is fastened to a drum operated by clockwork.

Figure 22 is a Colombel instrument of French manufacture in which the combined air-speed indicator and altitude readings are obtained on a single chart. The air-speed indicator element is shown at A. It consists of a simple corrugated metal diaphragm element which is connected by a lever system L to the pointer P. It is designed for use with a Venturi nozzle.

A French device of the pressure plate type is shown in figure 23. The displacement of the square plate B under the air pressure developed is opposed by the spring S. The amount of the displacement is a measure of the air speed. This is indicated by the motion of the pointer P.

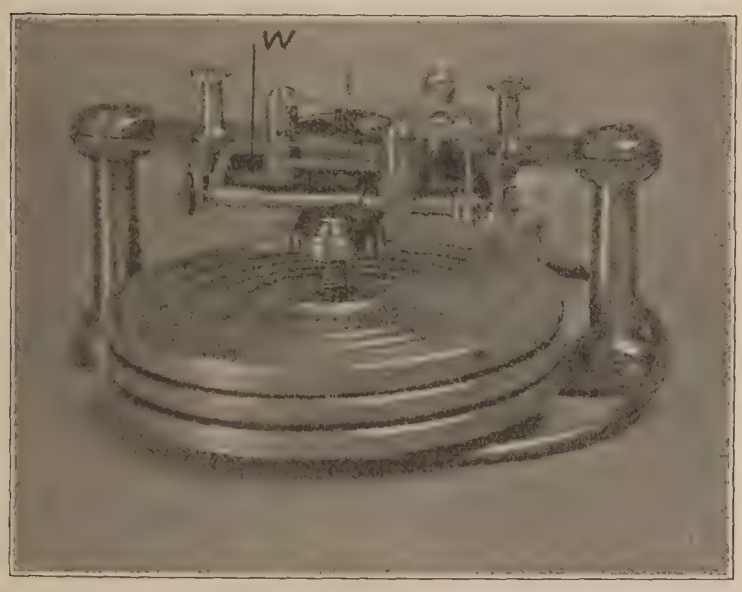


FIG. 19.—Precision air-speed indicator.

The magnitude of the excursion can be regulated by the adjustment of the leverage of the arm A and the tension of the spring at K.

Another instrument of the pressure-plate type, the Pensuti, is shown in figure 24. It is of Italian manufacture and was extensively used on Italian airplanes during the recent war. A small circular aluminum plate P is attached to the lever L. This moves the sector S which

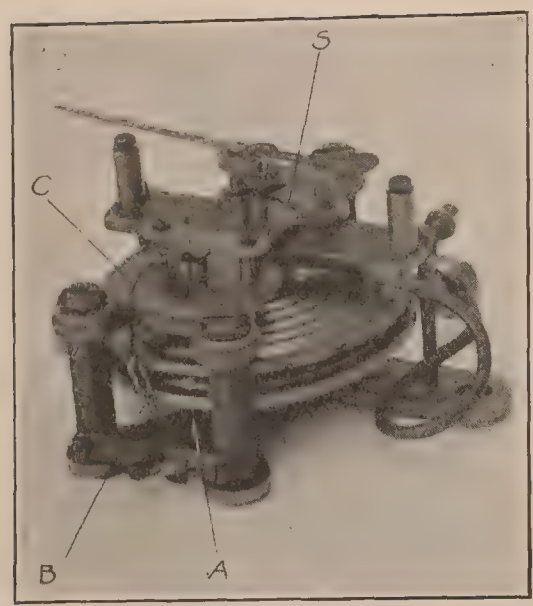


FIG. 20.—King air-speed indicator

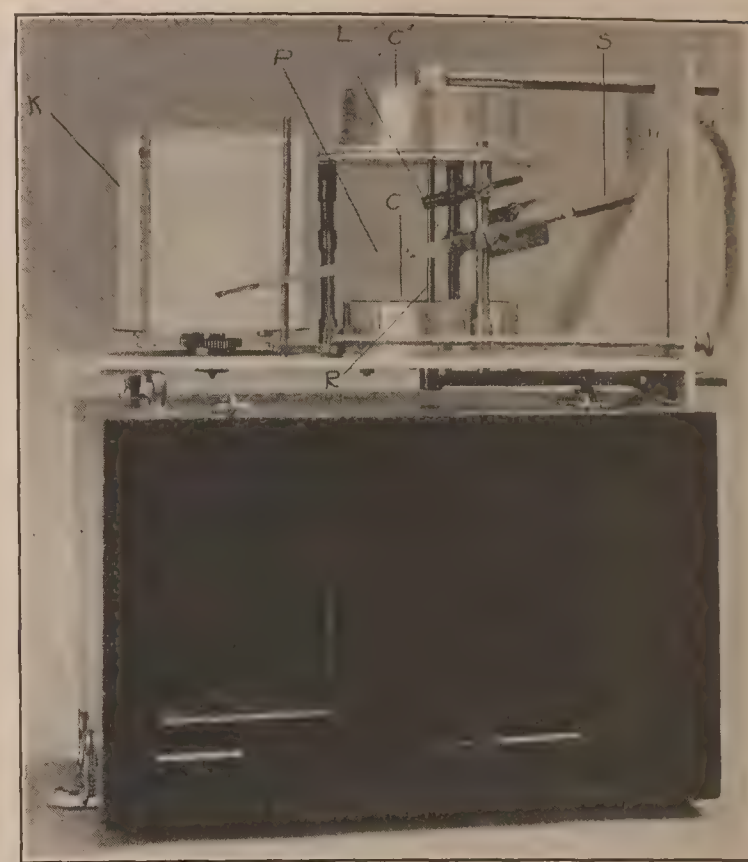


FIG. 21.—Toussant-Lepère recording air-speed indicator.

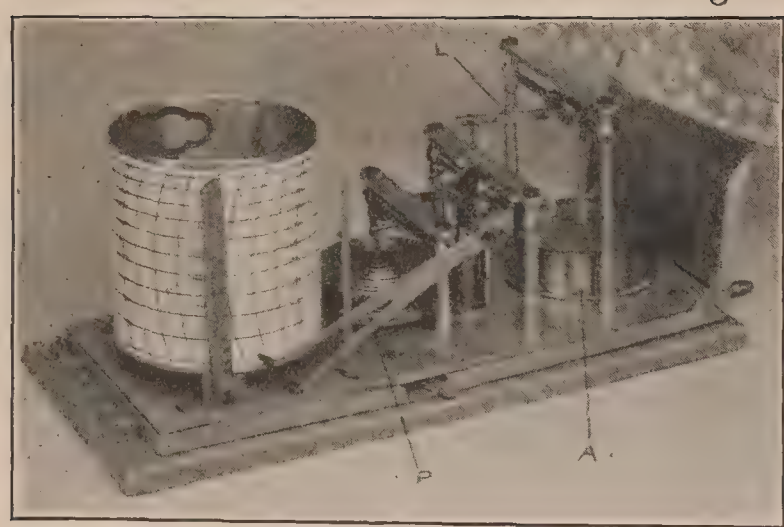
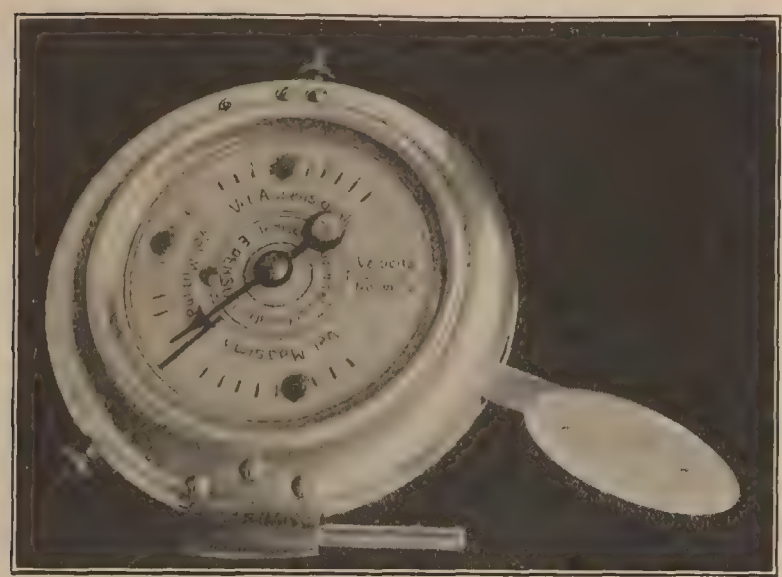


FIG. 22.—Colombel combined recording air-speed indicator and barograph.

FIG. 23.—Eteve air-speed indicator.

meshes with a pinion on the pointer shaft. The motion of the lever under the air pressure is resisted by a spring R. The amount of the displacement of the lever and the resultant motion of the pointer is a measure of the air speed of the airplane. The dial is marked to indicate minimum, maximum, and ordinary speed, but is not graduated in units of speed. The brass buttons shown on the dial are adjustable, so that the minimum, maximum, and ordinary speed can be



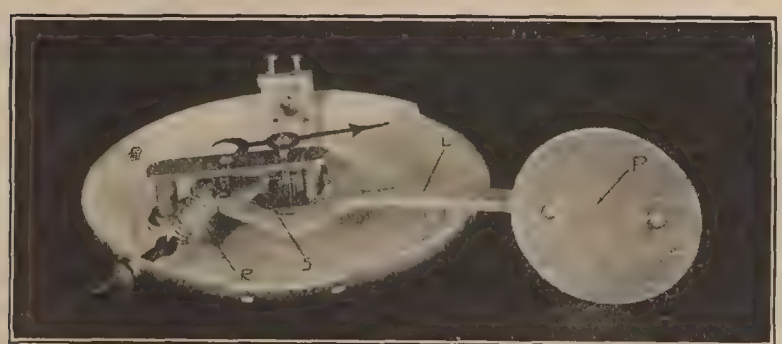


FIG. 24.—Pensuti air-speed indicator.

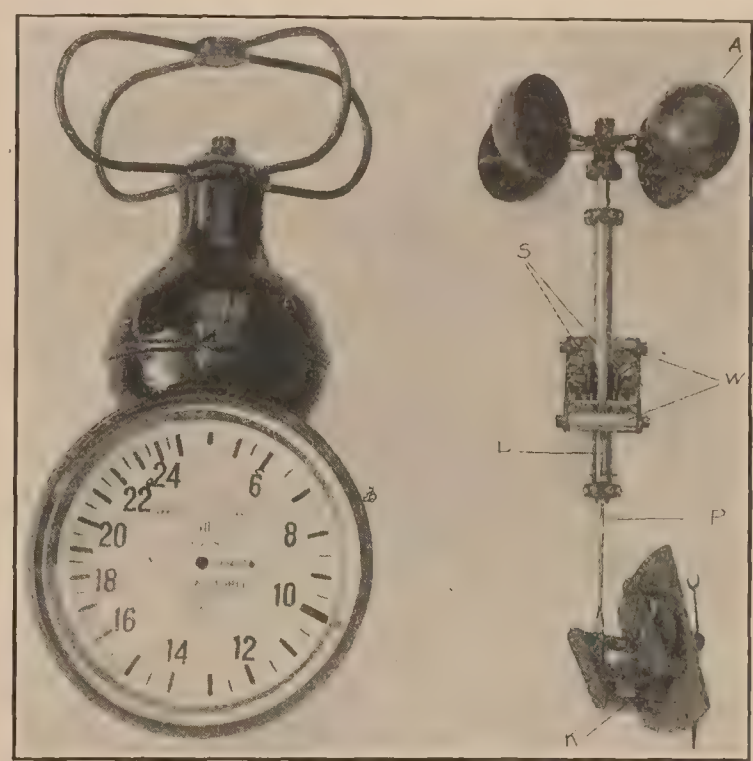


FIG. 25.—Morell air-speed indicator.

changed according to the plane on which the instrument is mounted. The indicator is located on a strut, or in planes of the pusher type near the front of the fuselage.

Figure 25 shows a Morell instrument of the anemometer type, of German manufacture. The rotation of the cup anemometer A forces the weights W of the centrifugal tachometer to which it is attached out from the axis of revolution in opposition to the springs S. This displacement lifts the pin P through the action of the lever L. The lower end of the pin is attached to the sector K which meshes with a pinion on the pointer shaft. The amount by which the pointer is displaced is proportional to the displacement of the weights W which in turn depends upon the rate of revolution of the cups and thus on the speed of the airplane. This instrument is also self-contained and is located on a strut of the airplane or elsewhere away from the propeller slip stream, from which position it is read at a distance by the pilot.

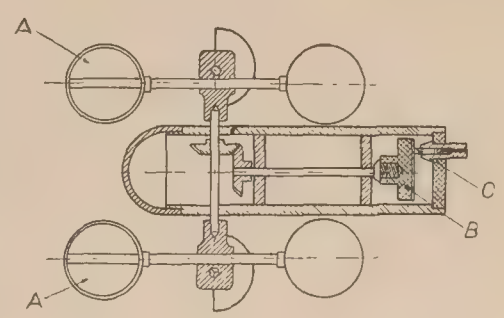


FIG. 26.—Favre-Bulle air-speed indicator.

Figure 26 shows the indicating element of a Favre-Bulle distant reading anemometer air-speed indicator. It has two pairs of cups A connected to a synchronizing commutator B. This commutator converts direct current from a battery or generator into three-phase alternating current which is supplied through bushes C to the field coils of a synchronous motor attached to the indicating element of a centrifugal tachometer on the instrument board. The frequency of the alternating current and consequently the rate of rotation of the tachometer motor depends on the rate of rotation of the commutator.

GENERAL SPECIFICATION AND PERFORMANCE REQUIREMENTS.

Under the conditions of use air-speed indicators are subjected to wide variations of temperature and pressure, to vibration and exposure to the weather. The instruments must, therefore, be carefully designed, and constructed, and made to conform to rigid specifications which from the manufacturer's point of view is difficult because of the inherent delicacy of construction.

The mechanism is ordinarily made of brass to minimize corrosion and the diaphragms, if of metal, of bronze, German silver, or similar alloys. Steel and silver diaphragms have also been used. The dial should be of metal, and adequately thick to prevent warping under service conditions. The case is generally made of aluminum or light aluminum alloy. The entire indicator need not weigh more than $1\frac{1}{2}$ pounds.

The dial graduations should preferably be concentric with the case. Important figures such as 30, 60, 90, 120, and 150 should be about 8 millimeters high to be easily read, and ordinarily are made luminous with radium paint. The scale should be approximately equally divided. The pointer should be counterbalanced, and it is ordinarily illuminated on the indicating end. The bushing should be of brass or other noncorrosive material so that the pointer can be easily removed for resetting. The pointer should be free to swing past the zero point. No stop at zero should be used. It is generally necessary to dampen the movement of the pointer of instruments of the Pitot and Venturi type by inserting a coil of fine copper tubing inside the case between the diaphragms and the suction connection or to insert in the tubing small baffle plates perforated by fine holes.

For convenience of replacement all the parts should be made with sufficient accuracy to insure interchangeability with corresponding parts of any other instrument made to the same specifications.

The relation between the pressure developed at the pressure nozzle and the corresponding air speed should be given by a table or formula, and the indicator required to be accurately calibrated in conformity therewith. For ordinary Pitot nozzles this relation is

$$V = 45.18\sqrt{h} \tag{7}$$

For the Pitot-Venturi nozzle designed by Dr. A. F. Zahm and at present used by the United States Aviation Service

$$V = 17.89\sqrt{h} \tag{8}$$

where V is the velocity in m. p. h. and h the corresponding differential pressure in inches of water.

It is practicable to make air-speed indicators so that the scale errors at room temperature do not exceed 1 per cent of the total scale range at any part of the scale. When calibrating, readings are usually taken with the dial vertical while tapping the indicator to eliminate friction errors, and with both increasing and decreasing scale readings. Indicators should be calibrated at low and high temperature as well as at room temperature since a wide variation of temperature is experienced in flight. The maximum scale errors over a temperature range from $-10°$ C to $+40°$ C should not exceed 2 per cent of the total scale range at any part of the scale, when readings are taken both with increasing and decreasing scale readings and with tapping.

To test the effect of continued vibration such as is experienced in use, the indicators are subjected in the laboratory to vibration on a suitably constructed vibration stand for several hours. During vibration they are generally maintained at approximately half scale deflection in which condition the pointer should not oscillate appreciably, certainly not more than 1 per cent of the total scale range. After vibration for at least five hours, the errors at no part of the scale should exceed 1 per cent of the total scale range.

To test the effect of changes of position, readings should be taken in laboratory tests when the indicators are changed from a vertical to a horizontal position and when rotated laterally to the right or left through an angle of 90°. The differences should not amount to more than

1 per cent of the total scale range. The pointer should move smoothly under a uniformly increasing pressure sufficient to give full scale deflection in not less than 15 seconds. As a further test of friction the difference in readings before and after tapping is taken. This should not exceed 1 per cent of the total scale range.

The elastic properties of the diaphragms may be tested by subjecting the indicators for at least one minute to a differential pressure approximately 50 per cent greater than that required to produce full scale deflection. Not more than five minutes after the release of the pressure the pointer, with tapping, should return to zero within the linear equivalent of one per cent of the total scale range. While under this pressure diaphragms and case may be tested for leaks. As a further indication of the elastic behavior of the diaphragms the indicators may be subjected for several hours to a pressure sufficient to give full scale deflection. After the release of this pressure the pointer should return, with tapping, within five minutes to approximately zero.

The pointer should have sufficient quickness of response to return to rest within one second after the release of a pressure corresponding to a deflection of approximately 100 miles per hour, and after a sudden change of pressure, corresponding to approximately 10 miles per hour, the pointer should assume its new position within one second.

RESULTS OF TESTS ON AIR-SPEED INDICATORS AT THE BUREAU OF STANDARDS.

The results [4] of tests conducted recently at the Bureau of Standards on Pitot-Venturi instruments are given in Table 1. The instruments are grouped according to the manufacturer as indicated in column 1. Column 2 gives the number of instruments subjected to the tests indicated in the succeeding columns, with the exception noted at the bottom of the table. In column 3 are the average scale errors in miles per hour computed by first finding the average of the errors without regard to signs for each instrument and then the average of these averages for each make of instrument. In column 4 are the average differences of readings with increasing and decreasing deflection computed in the same manner; in column 5, the average difference in reading before and after tapping the instruments; in column 6, the average change in reading due to maintaining the instruments at full scale deflection continuously for five hours; in column 7, the average change in reading due to subjecting the instruments momentarily to a pressure 50 per cent greater than that corresponding to full scale deflection. The instruments of groups B and C were made by the same firm, but those in group C, which represent an improved model, were so much better than the previous samples as to warrant placing them in a separate group. The improvement between groups B and C is representative of the development in air-speed indicators of American manufacture during the recent war.

TABLE 1.—*Results of tests of air-speed meters.*

BY THE BUREAU OF STANDARDS.

Manufacturer.	Number of instruments.	Errors in miles per hour.				
		Average scale error.	Average hysteresis.	Average friction.	Average drift.	Average over-pressure.
A.....	19	1.19	0.79	1.28	*0.93	0.65
B.....	18	4.03	3.78	0.94	+2.09	4.02
C.....	29	1.05	1.11	0.75	±1.10	0.44
D.....	17	2.05	1.59	°0.66	1.30	1.96

*Average of 16 instruments.
+Average of 14 instruments.
±Average of 22 instruments.
°Average of 16 instruments.

[4] The author is indebted to Mr. H. O. Stearns, formerly of the Bureau of Standards, for the compilation of these data.

ALTITUDE EFFECT ON AIR-SPEED INDICATORS.

Since the pressure developed by Pitot and Venturi nozzles [5] is proportional to the density of the medium with reference to which they are moving and the square of the speed, the pressure developed by such nozzles for a given velocity is less at high altitudes than at the surface of the earth (at 20,000 feet approximately one-half as great). Consequently an air speed indicator which depends upon dynamic pressure for its velocity indication and is calibrated for air at sea level density will show readings at high altitudes which are too low. This does not affect the use of the instrument as a buoyancy meter, however, since the buoyancy effect will be the same for a given speed reading whatever the altitude, although the airplane must be flying faster at high altitudes to produce a given reading.

If P is the pressure developed by the nozzle, V_s the speed of the airplane, V the reading of the instrument, K a constant depending upon the construction of the nozzle, ρ the density of the air, and ρ_o the standard density for which the instrument was calibrated

$$P = K\rho V_s^2 = K\rho_o V^2 \tag{9}$$

Assuming that the density of the air is proportional to the barometric pressure and the absolute temperature

$$\rho = \rho_o \frac{BT_o}{B_oT} \tag{10}$$

where B is the barometric pressure and T the absolute temperature of the air.

Substituting in Eq. (9)

$$V_s = \sqrt{\frac{B_oT}{BT_o}}\, V \tag{11}$$

or

$$V_s = FV \text{ where F is the multiplying factor} \sqrt{\frac{B_oT}{BT_o}} \tag{12}$$

The change in air-speed indicator readings at different altitudes and for different temperatures are shown in Fig. 27, from data computed at the Bureau of Standards,[6] on the basis of this formula and the assumption that the readings of the instrument are independent of compressibility and viscosity. The pressures in millimeters of mercury are plotted as abscissæ and the factor by which the observed indicator reading must be multiplied to give the correct velocity with reference to the air at various altitudes and for different temperatures as ordinates. The altitudes were computed by the altitude pressure relation:

$$H = 62900 \log_{10} \frac{759.6}{p} \tag{13}$$

Where H is the altitude in feet and p the barometer pressure in millimeters of mercury.

An instrument for computing this correction, known as the Appleyard air-speed computer, has been devised. It solves the expression

$$V_s = V \sqrt{\frac{B_o}{B}} \tag{14}$$

and hence corrects the indicated air speed V only for changes of pressure. If the air temperature is known, further correction can be made by subtracting 1 per cent of the air speed as determined by the computer for each 5.5° C. below 16° C.

The instrument consists of two concentric circular scales rotatable relative to each other. Air speeds are graduated on the inner scale and altitudes on the outer. The indicated air speed is set against the altitude and the corrected air speed is read opposite.

[5] The limitations of this statement are discussed in Report No. 110, National Advisory Committee for Aeronautics, 1920.

[6] Prepared by Mr. H. N. Eaton, of the Bureau of Standards.

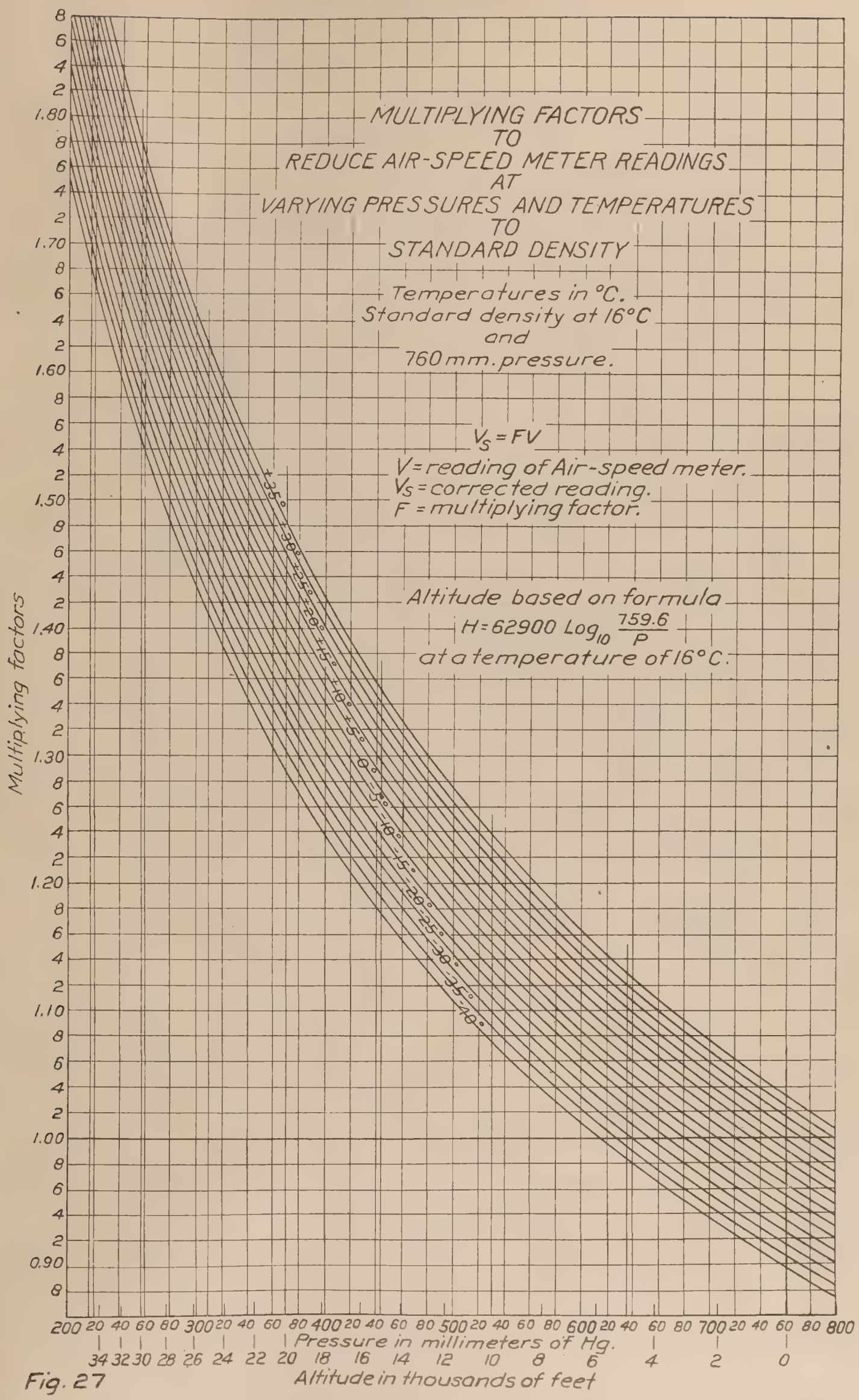

MULTIPLYING FACTORS
TO
REDUCE AIR-SPEED METER READINGS
AT
VARYING PRESSURES AND TEMPERATURES
TO
STANDARD DENSITY
Temperatures in °C.
Standard density at 16°C
and
760 mm. pressure.
$V_S = FV$
V = reading of Air-speed meter.
V_S = corrected reading.
F = multiplying factor.
Altitude based on formula
$H = 62900 \log_{10} \frac{759.6}{P}$
at a temperature of 16°C.
Multiplying factors
Pressure in millimeters of Hg.
Altitude in thousands of feet
+35° +30° +25° +20° +15° +10° +5° 0° -5° -10° -15° -20° -25° -30° -35° -40°

Fig. 27

REPORT No. 127.

AIRCRAFT SPEED INSTRUMENTS.

PART II.

TESTING OF AIR-SPEED METERS.

By H. O. Stearns.

INTRODUCTION.

There are three types of tests which may be made to determine the accuracy of air-speed meters; flight tests, wind tunnel tests, and static or laboratory tests. The flight test requires that the instrument be installed in an airplane and that a measured course be chosen. A test of this kind when properly carried out gives the most complete corrections, for it includes not only the errors in the instrument and pressure head, but also a correction due to the location of the pressure head. Ordinarily, however, owing to varying atmospheric conditions, a number of corrections must be made to the data taken to obtain the desired result. The wind tunnel tests give the correct performance of the combined indicator and pressure nozzle under conditions which can be regulated with a considerable degree of accuracy. The wind tunnel is also the most convenient means by which the errors of the pressure nozzle alone can be determined, but the error due to the position of the pressure nozzle can not be found in this manner. For an exact determination of this free flight tests are necessary. The method of combining wind tunnel and static tests will be discussed later.

The static tests are applied simply to the indicating part of the instrument, which is treated for this purpose as a delicate pressure gage. This is an accurate method of determining the errors assuming the pressures delivered by the pressure nozzle to be correct. The static tests are those to which the Aeronautic Instruments Section of the Bureau of Standards has given special attention, and a more complete description of these will be given here than of the other tests.

LABORATORY APPARATUS FOR STATIC TESTS.

The apparatus necessary for testing various types of Pitot and Venturi air-speed meters includes manometers with suitable regulating valves, a vacuum pump, or some source of reduced pressure, pressure pump, temperature chamber, and vibrating board.

For the tests a vacuum equivalent to 12 or more inches of mercury should be available. The pressure does not need to be greater than 4 to 5 inches of mercury. Since the tests are static, a large capacity is not necessary in the exhaust system. The most essential part of the equipment is the manometer and there are two types which are in general use—the U tube and the single-tube manometer. By changing the density of the liquid the lengths of these manometers can be varied over a wide range. For the manometers, tubing at least a fourth of an inch in diameter should be used in order to reduce as much as possible the effect of capillarity.

The standard manometer used at the Bureau of Standards is a single-tube water manometer with 100-inch column. It carries a scale graduated in inches and in miles per hour, also a scale in knots for calibrating Navy instruments. The miles per hour scale was taken from the calibration chart of the Zahm pressure head as determined in the Washington Navy Yard wind tunnel, for which the corresponding formula is $V = 17.89\sqrt{h}$, where V is in miles per hour and h in inches of water. Another single-tube water manometer is used for calibrating indicators to

be used with Pitot pressure heads. This has a scale graduated in cm. and another in miles per hour according to the formula $V=45.18\sqrt{h}$ for the units above mentioned. A U-tube water manometer (see fig.1) is used for most of the testing, since it is more convenient than the longer single-tube manometer.

To facilitate the rapid testing of instruments a control valve is used with the test apparatus which makes it possible to set the instruments at a predetermined reading in a comparatively

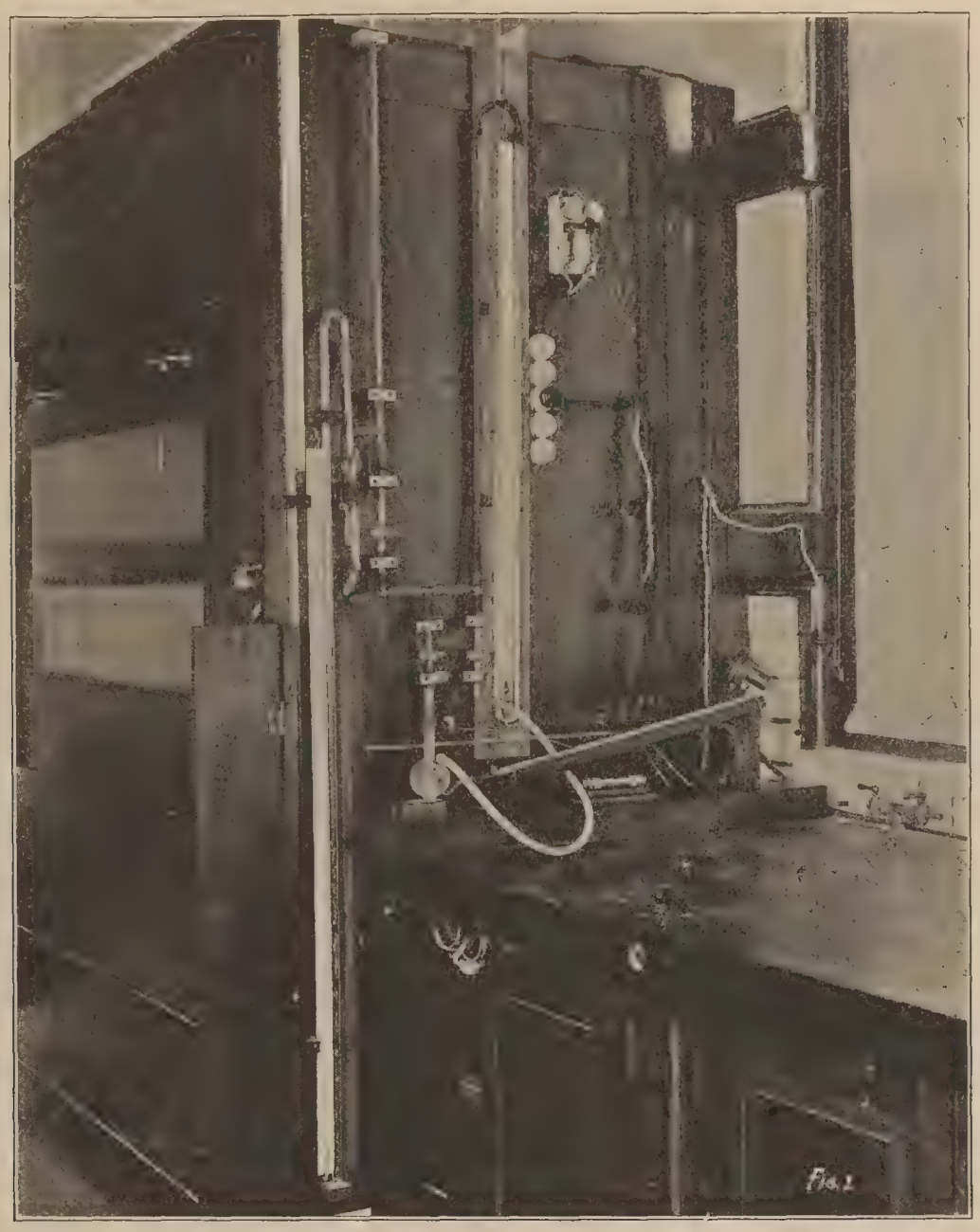


Fig. 1.

short time. This is a four-way valve which controls the flow of the air to or from the instruments at any desired rate.

Instruments are tested in a temperature chamber to determine the effects of temperature on the working parts. The chamber used at the Bureau of Standards is about 4 feet long and 3 feet deep (fig. 2). It is double-walled and has a 3-inch space between the inner and outer wall filled with heat-insulating material. The front is closed by a sliding double plate glass window so that the instruments can be read from outside. At the back of the chamber a cooling coil is located which has a radiating surface of about 26 square feet. With ammonia-cooled brine a temperature of $-10°$ C. is thus obtained. Electric heaters are attached to the upper part of the chamber, as shown in figure 2. With both of these connected a temperature

of 60° C. can be maintained. Two 8-inch variable speed fans are used to keep the air in circulation. The rack shown in figure 2 is used to hold the instruments while under test. This rack is equipped with a tapping device by which the instruments are tapped before each reading. It is operated electrically from outside of the chamber.

A vibrating board is used to determine the effect of vibration such as is experienced on an airplane, and also to see if the pointer of the instrument vibrates under conditions approximating those of actual flight. The upper part of the board is arranged to hold the instrument rigidly in place during vibration. Below the horizontal board is a motor for producing the vibrations. In figure 3 can be seen the board and an automatic device for stopping the motor at any desired time. To the motor is attached an eccentrically loaded flywheel. The position

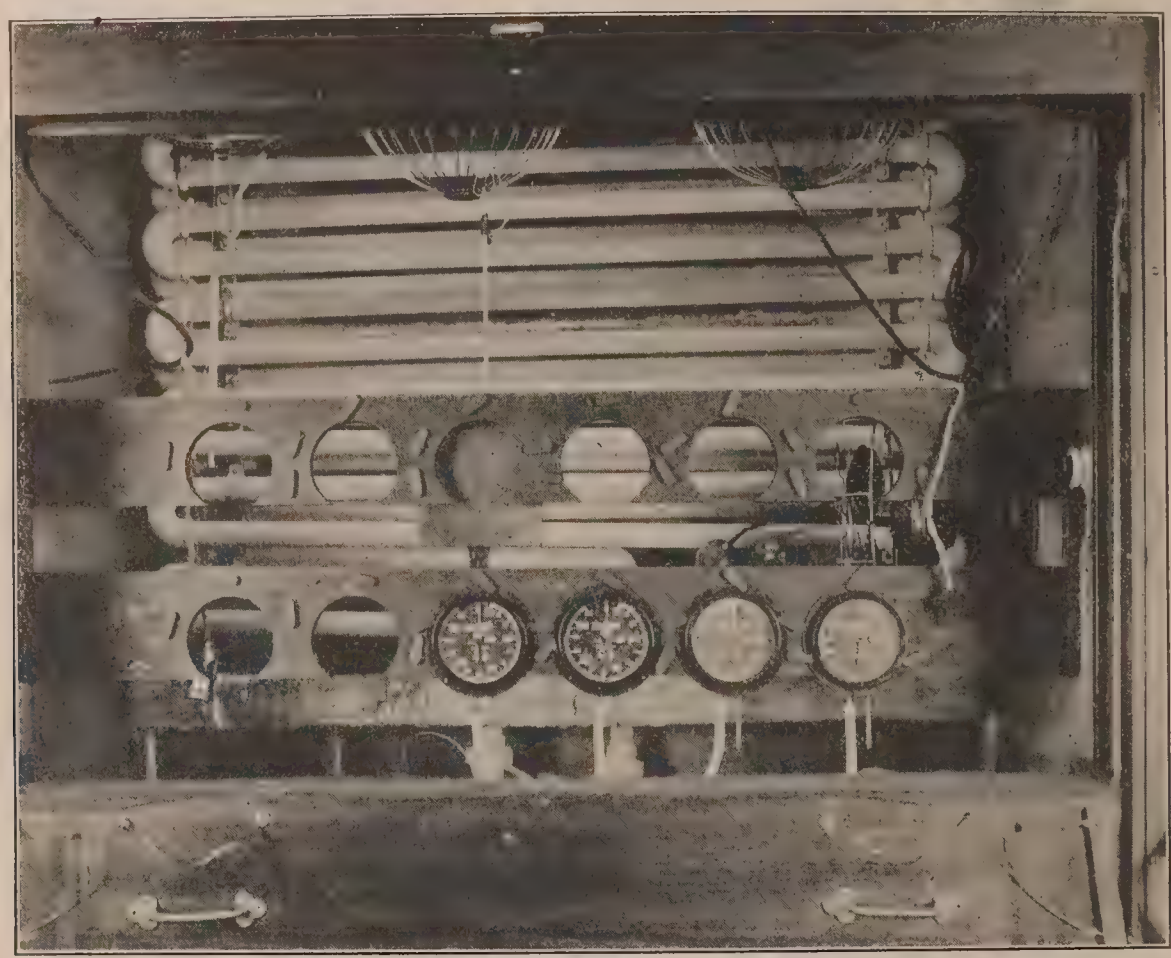


Fig. 2.

of the load can be varied, thus making it possible to change the character of the vibrations for any given voltage on the motor. Lamps and rheostats can be so arranged as to secure any required speed for the motor. For a given location of the load on the flywheel, and a given speed, the amplitude of vibration depends on the inertia of the board. Hence, the desired amplitude should be secured by making final adjustments after the instruments are in place.

PROCEDURE OF TESTS.

Before beginning a test the valves of the testing apparatus are adjusted so that both arms of the manometer are open to the air. The zero reading is then noted, and if necessary readjusted. It is advisable to run the manometer to a full scale reading and back before beginning a test. This wets the tube and prevents irregular capillary effects. Instruments of the diaphragm type are then subjected to the following tests:

Zero reading.—Before any strain has been put on the diaphragms of an instrument its zero reading should be taken.

Tests for leaks.—After all the connections have been made a differential pressure sufficient to give a 60-mile per hour deflection should be applied and held for 10 to 15 seconds to test for leaks. It has been found convenient in making connections with rubber tubing to use glycerin to insure tight joints and also to enable the connections to be made easily.

Calibration before vibration.—An initial calibration is made to determine the condition of the instruments as received and to form a basis for studying later the effect of other tests. The instruments are connected to the test apparatus previously described and the pressures required for a series of scale readings determined first with increasing scale readings to full scale deflection and then immediately afterwards with decreasing scale readings back to zero. If a single instrument is to be tested, it is often best to adjust the pressure until the pointer of the instrument stands at a predetermined mark on the scale. If, however, several instruments are to be tested, it is more convenient to set the manometer and then read the instruments. The errors during this test at no part of the scale should be greater than 1 per cent of the total scale

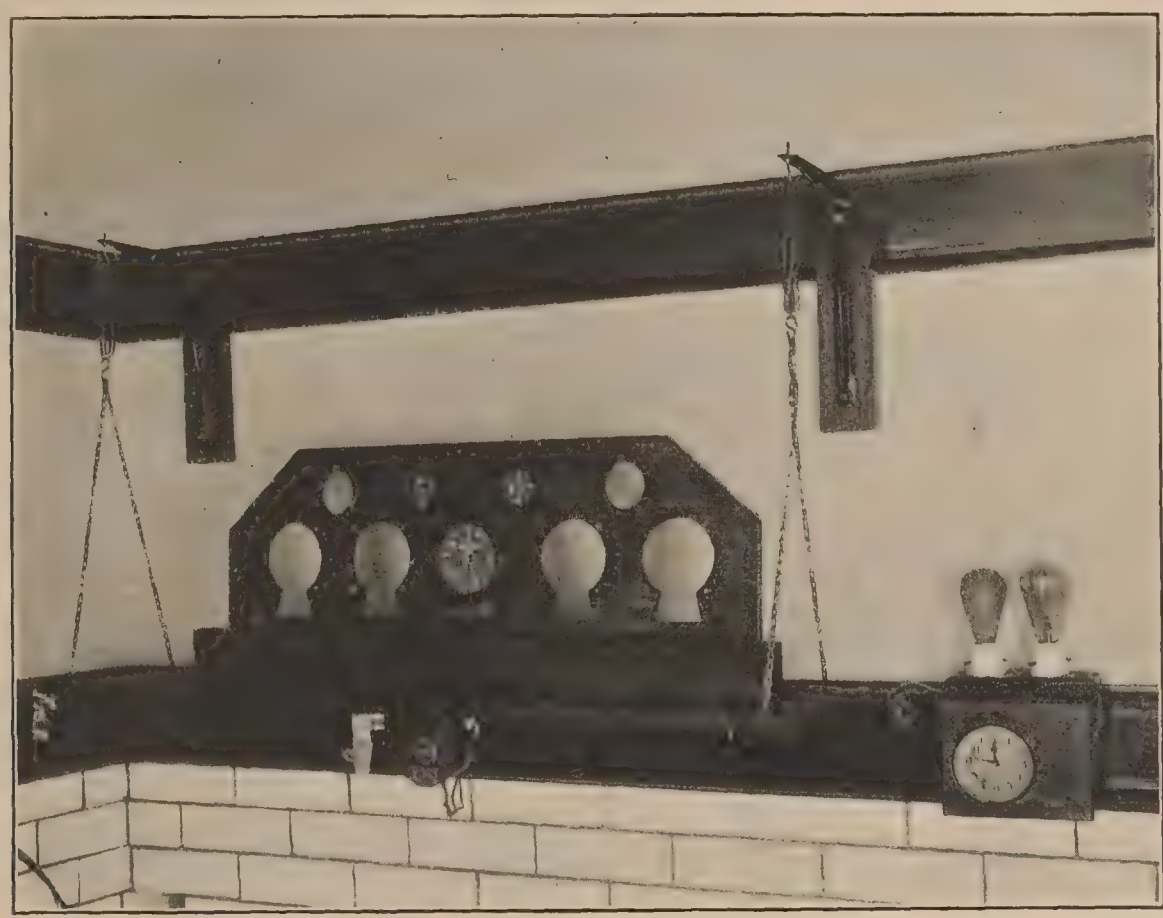


FIG. 3.

range either with increasing or decreasing readings. Instruments are always tapped immediately before reading.

Vibration.—For this test the vibrating board previously described is used. The instruments are vibrated for several hours. During vibration the pointer should not oscillate at any part of the scale more than 1 per cent of the total scale range.

Calibration after vibration.—The object of this calibration in which the procedure is the same as that made before vibration is twofold: To determine if the vibration has injured the instrument, and to give a room temperature calibration with which to compare the subsequent temperature tests. The error due to vibration for several hours should not exceed 1 per cent of the total scale range if the instrument is to perform satisfactorily under the actual conditions of use.

Temperature tests.—The instruments are placed in the temperature chamber and the temperature raised to about 50° C. After the instruments have been given time enough to come to practically the same temperature as the air in the chamber, they are calibrated as in the previous calibration. The error during this run should not at any time exceed 2 per cent of

the total scale range. This also applies to the cold run. For the cold test the procedure is the same as in the hot test, except that the temperature is lowered to −10° C.

Friction and inclination tests.—These two tests are run at the same time. The instruments are slowly brought up to a given reading which is noted. The instruments are then tapped, and the reading again noted. The difference between these two readings constitutes the friction error. After recording the second reading the instruments are turned into a horizontal plane, tapped, and the reading noted. The difference between the third and second readings is known as the inclination error. Neither of these errors should exceed 1 per cent of the total scale range.

Drift.—For this test a differential pressure sufficient to produce nearly full scale deflection is applied to the instrument for several hours. The change of reading during this time is called the drift. This should not exceed 1 per cent of the total scale range.

Overpressure.—A differential pressure 100 per cent greater than that required for full scale deflection is applied for one minute. If the pointer returns approximately to zero, it is assumed that no damage has been done. A study has been made of the effect of overpressure on the calibration curve, from which it was found that in some cases a permanent displacement occurs, thus changing the calibration of the instrument.

The results of tests on a representative instrument of Pitot-Venturi type of American manufacture are given below. It was calibrated at room temperature before and after subjection to vibration similar to that experienced in actual flight and at −7° C. and +47° C. by applying measured differential pressures to the instrument. Readings were taken at approximately equal intervals up the scale to full scale deflection and then immediately afterwards back to zero. The corresponding speeds in miles per hour were found from the pressure velocity chart previously referred to which was prepared at the Washington Navy Yard wind tunnel for use with the Zahm pressure nozzle.

In column 1 of Table I are the readings of the instrument in miles per hour. In the even-numbered columns are the additive corrections for increasing differential pressure, and in the odd-numbered columns, after column 1, the same for decreasing differential pressure.

TABLE I.—*Airspeed-Indicator Calibration.*

SERIAL NO. 137. IDENT. NO. 2.

Instruments reading, miles per hour.	Corrections at+25°C., before vibration.		Corrections at+25° C., after vibration.		Corrections at−7° C., cold run.		Corrections at+47° C., hot run.	
	Up.	Down.	Up.	Down.	Up.	Down.	Up.	Down.
1	2	3	4	5	6	7	8	9
30	+4.0	+2.0	+3.0	+1.7	+3.3	+2.3	+2.5	+0.5
60	+2.4	+1.0	+2.0	+0.8	+2.7	+1.5	+1.5	+0.2
90	+2.0	+0.3	+1.7	+0.3	+2.2	+1.4	+1.0	−0.2
120	+1.3	+0.4	+1.2	+0.4	+2.2	+1.4	+0.5	−0.6
140	+1.0	+0.3	+0.6	+0.4	+1.7	+1.4	0.0	−0.6
160	+1.0	+1.0	+1.0	+1.0	+1.5	+1.3	0.0	0.0

SUMMARY OF RESULTS.

Effect of temperature.—The readings of the instrument at −7° C. are lower than the room-temperature readings on the average by about 0.5 mile per hour, whereas the readings at +47° C. are higher by about the same amount.

Effect of vibration.—A slight decrease of the additive correction in the middle scale readings resulted from vibrating the instrument for five hours.

Friction test.—Readings taken before and after tapping in no case show differences exceeding 1 mile per hour. There was a slightly irregular motion of the pointer under gradually applied pressure.

Inclination test.—Readings taken with the instrument vertical and horizontal showed a maximum difference of 2.5 miles per hour at the 30 miles per hour point. This should be less than 1 mile per hour.

Drift.—The error caused by the elastic fatigue of the diaphragms was determined by applying for five hours a differential pressure sufficient to produce a scale reading of 155 miles per hour. This error was 1.3 miles per hour.

Overpressure.—An increase in reading amounting to about 1 mile per hour resulted from applying for 1 minute a pressure of 130 inches of water.

The results are shown graphically in figure 4.

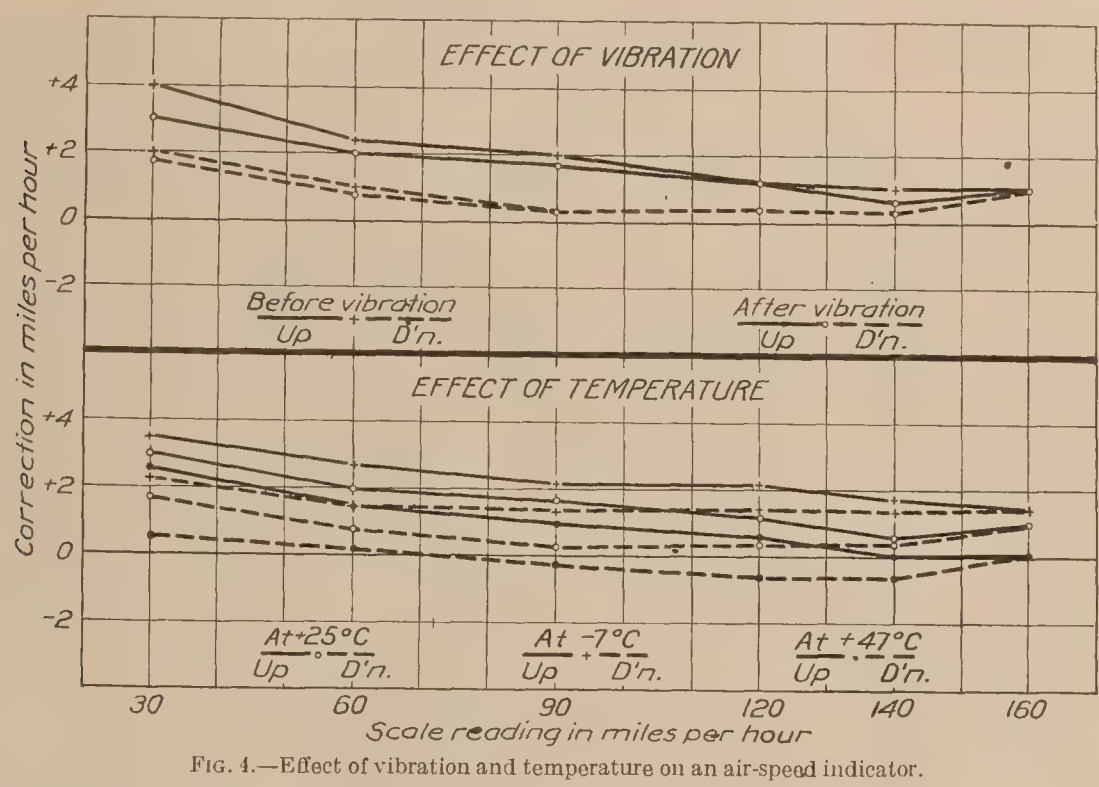


FIG. 4.—Effect of vibration and temperature on an air-speed indicator.

METHODS OF COMBINING WIND TUNNEL TESTS WITH STATIC TESTS.

The purpose in this section is to indicate briefly the method of obtaining the calibration curve of a given pressure head and indicator by combining the results of wind tunnel and static laboratory tests. In general the procedure is as follows:

A wind tunnel test is made on the pressure nozzle in which the relation between the pressure developed by the nozzle and the velocity of the corresponding air stream is directly determined. The results of this test are plotted on logarithmic paper, with the air speed as one axis and the pressure as the other. This curve will be a straight line within the ordinary range of airplane speeds if the pressure head follows the ρV^2 law. The differential pressures corresponding to a series of indicator readings are then found in the laboratory with a manometer in the usual manner. The speeds corresponding to these pressures are computed from the logarithmic plot and the corrections found by subtracting (algebraically) the instrument reading from the true reading. A plot of these points is made, and the smoothed curve drawn through these points is the calibration curve. An example of this procedure is seen below in some tests made on Venturi air-speed meters by the Bureau of Standards in cooperation with the Washington Navy Yard, where the wind tunnel tests were made.

TABLE II.—*Wind tunnel data.*

DOUBLE-THROAT VENTURI.

Speed in miles per hour.	Suction in inches of water.
20	1.89
25	3.07
30	4.87
35	7.06
40	9.10
45	12.15
50	14.90
55	18.20
60	22.30
65	26.55
70	29.60

From the above a curve was plotted on logarithmic paper. With suction in inches of water as ordinates and speeds in miles as abscissas this curve is shown in figure (5).

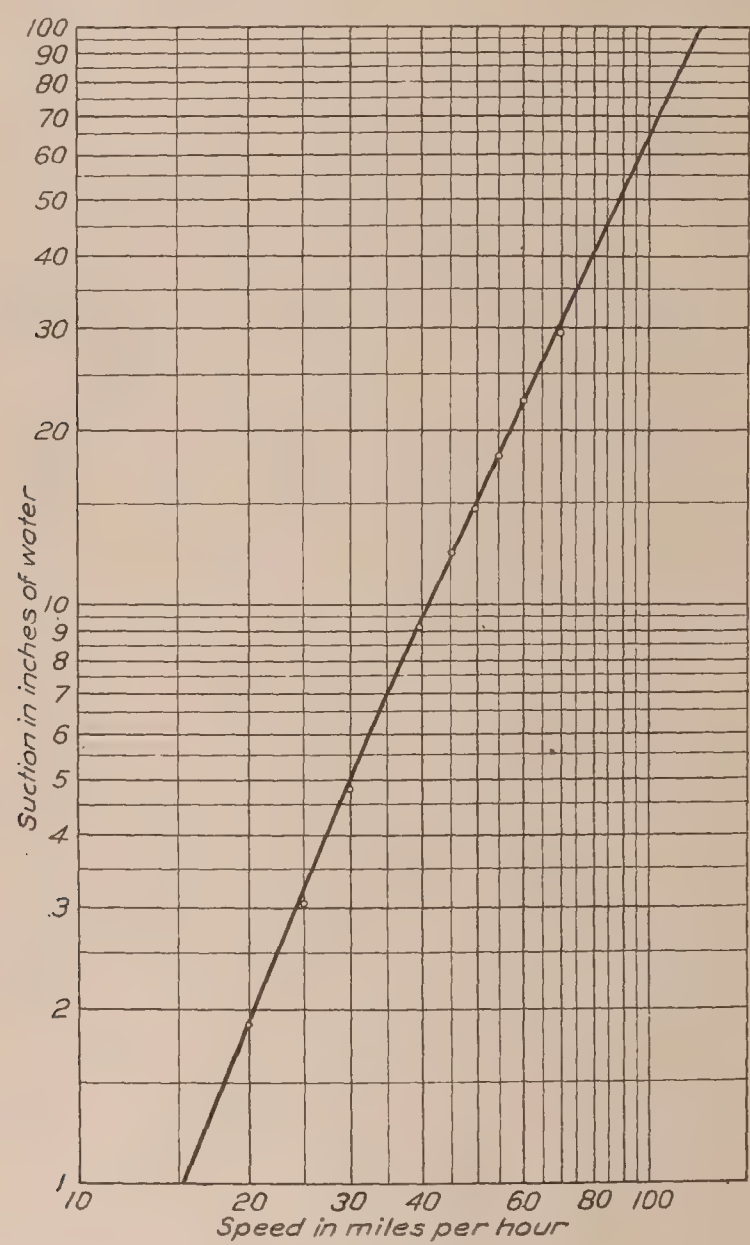


FIG. 5.

In Table III are shown the data of a static test on the indicator and the corrections computed from the above calibration of the nozzle.

TABLE III.—*Static tests.*

READING TAKEN WITH INCREASING SUCTION.

Suction in inches of water.	Reading, miles per hour.	True speed, miles per hour.	Correction, miles per hour.
7.34	39	36	−3
11.88	48	45	−3
14.88	53	52	−1
17.81	57	54	−3
21.10	61	58	−3
31.59	73	70	−3
46.20	86	83	−3
63-10	98	96	−2
82.0	111	108	−3
84.0	113	109	−4

READING TAKEN WITH DECREASING SUCTION.

Suction in inches of water.	Reading, miles per hour.	True speed, miles per hour.	Correction, miles per hour.
6.26	40	34	−6
10.80	50	44	−6
15.95	57	52	−5
22.3	64	60	−4
32.6	76	72	−4
47.5	89	85	−4
63.3	100	96	−4

In Table IV are shown the results of wind tunnel and static tests [7] on a series of instruments. The fact that the average discrepancy between the tests is less than 1 mile per hour is noteworthy, considering the errors which may enter.

In the static tests the errors are largely due to the difficulty of reading the instrument. An accuracy exceeding 0.2 mile per hour is rarely attained since the manometer can only be read to about this degree of accuracy. In wind tunnel tests the rapid fluctuations of the manometer reading introduce more serious errors. When the results of the two tests are combined there may be a further error due to interpolation.

TABLE IV.—*Comparison of wind tunnel tests with static calibrations.*

Instrument.	Instrument No.	Reading, miles per hour.	Static calibration corrections.	Corrected scale reading.	Scale reading corrected for Venturi error.[1]	Standard Pitot reading.	Difference.
F	110	75	−7.8	67.2	65.0	66.3	−1.3
F	225	76	−8.2	67.8	65.6	66.3	−0.7
F	83	77	−4.5	72.5	70.3	66.5	[2] +3.8
F	324	67	+1.2	68.2	66.0	66.6	−0.6
F	345	67.5	+1.0	68.5	66.3	67.7	−0.4
B	25	65.5	+1.8	67.3	65.1	67.3	−2.2
B	9	67.5	+0.8	68.3	66.1	67.5	−1.4
B	9	49.8	+0.2	50.0	47.8	53.6	−5.8
B	71	70.5	+0.1	70.6	68.4	67.5	+0.9
B	282	70.1	+0.5	70.6	68.4	67.5	+0.9
B	261	68	+1.7	69.7	67.5	67.4	+0.1
P	5	72	−0.9	71.1	68.9	67.5	+1.4

[1] Pitot-Venturi No. 138-B used with all instruments. At 65.6 miles per hour by Bureau of Standards Standard Pitot, 138-B reads 67.8 miles per hour, which gives a correction of −2.2 miles per hour.
[2] The difference between up and down readings at this point amounts to 3 miles per hour.

FLIGHT TESTS.

Flight tests on air-speed indicators involve determining simultaneously the readings of the instrument during flight and the speed of the airplane itself relative to the air by an independent method, which usually consists of measuring the time of flight of the airplane over a measured course. In general such observations are made from the ground, although it is also possible with suitable arrangements to determine without the use of an ordinary air-speed indicator the air speed of an airplane from the airplane itself. One of the difficulties encountered in all of these measurements of air speed is that of eliminating the effect of the wind

[7] These results were obtained in cooperation with the Wind Tunnel Staff of the Bureau of Standards.

which, in general, is not constant either in magnitude or direction for any considerable length of time.

The method most usually adopted is to conduct the experiments when no wind is blowing or to select a measured course in the direction of the wind and have the airplane fly back and forth across this course with and against the wind. At each end of the course an observer is stationed who notes with the aid of sighting wires the exact moment at which the airplane passes the designated point. The time intervening between these observations is determined either by the use of synchronized stop watches or preferably a chronograph and from this and the known length of the course the speed of the airplane can be calculated. The mean of the measurements with and against the wind gives the true air speed of the airplane. Comparison of these observations with those obtained from the readings of the air-speed indicator on the plane itself make it possible to determine the errors in the speed indicator. If stop watches are used some method of signaling between stations such as a telephone must be provided. At the exact moment when the airplane passes the first station the observer starts his stop watch and signals to the second station, who also starts his watch. When the airplane reaches the second station on signal from the second observer both stop watches are simultaneously stopped. The average of the readings of the two watches is taken as that of the time of transit of the airplane over the course. If a chronograph is used it is provided with two recording pens. One is connected to a standard clock which makes a mark on the chronograph record each second; the other, connected electrically to the stations of the measured course, records the time when the airplane passes the observers, each of whom presses a key to make a mark on the chronograph strip. The interval between the two marks as determined by the series of one-second marks on the chronograph record makes it possible to obtain within 0.1 second the true time of transit of the airplane. These observations, together with those made on the air-speed indicator during the flight, provide the necessary data for computing the corrections of the air-speed indicator itself. If the course is not exactly in the direction of the wind the magnitude and direction of the latter must be determined by meteorological methods and a suitable correction applied. Because of the variability of the wind this is ordinarily a troublesome and uncertain factor.

The speed of the airplane can also be found from the airplane itself by the use of a measured course or of two objects on the ground a known distance apart by determining with a suitable sighting device and stop watch the time required for the airplane to fly back and forth between the selected points, which should be in the direction of the wind. The air speed is determined, as in the previous case, by taking the average of the observations with and against the wind and this value compared with the average of a series of readings taken simultaneously on the airspeed indicator.

The disturbing effect of the wind can also be eliminated by flying over a selected triangular course, the distances between the three points which determine the triangle having been carefully measured. From the time required to fly over each of the known distances the effect of the wind can be determined.

It is also possible to calibrate air-speed indicators by determining the position of an airplane in flight at frequent intervals with the use of theodolites or camera obscuras located on a base line of known length. The speed of the plane at the various positions at which observations are taken can then be computed and the air-speed indicator calibrated by comparing these speeds with those read at corresponding times on the air-speed indicator carried on the airplane. These methods will not be considered here at length because they pertain to investigations relating to the performance of the airplane itself rather than to the calibration of air-speed indicators.

Additional data sufficient to determine the temperature of the air and the barometric pressure are also required for flight tests of air-speed indicators because the readings of the instruments depend on the density of the air. This is particularly true in the case of the last-mentioned method, where observations at varying altitudes are made.

REPORT No. 127.

AIRCRAFT SPEED INSTRUMENTS.

PART III.

PRINCIPLES OF GROUND-SPEED MEASUREMENT.

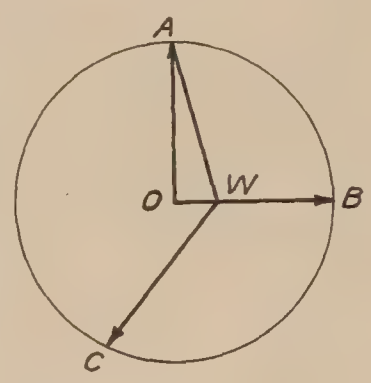
By Franklin L. Hunt.

MEASURING GROUND SPEED FROM THE GROUND.

The measurement of the speed of aircraft relative to the ground is of great importance in long-distance flying, in bombing operations, and in connection with aircraft performance tests. The problem of finding suitable methods for the purpose has been given much thought, but as yet no entirely satisfactory solution has been found.

Methods which have been considered may be divided into two classes: (1) Those applicable essentially to aircraft performance tests in which the speed of the aircraft is measured from the ground and (2) the much more important class in which the ground speed is determined from the aircraft itself.

Of the former methods, the one most commonly used is that of measuring the speed of the aircraft over a measured course. The principal difficulty in this case is that the wind may introduce errors which are difficult to eliminate. This can best be done by selecting a course exactly in the direction of the wind. By determining the time required to fly over such a course first in one direction and then in the other, the effect of the wind can be eliminated assuming that the speed of the aircraft is constant, by taking the average of the velocities with and against the wind. When ground speed is determined by this method an observer is ordinarily stationed at each end of the measured course to note with a stop watch or record on a chronograph the time of transit of the airplane. Telephonic or telegraphic communication is provided between the stations to coordinate the time data.



Another possibility is to select a triangular course and to determine the time required for the aircraft to fly over each of the three legs. The effect of the wind can then be determined graphically as follows: If WA, WB, and WC represent in magnitude and direction the three velocities attained on the triangular course the radius OA of the circle drawn through the points A, B, and C will represent the velocity of the airplane and WO the velocity and direction of the wind.

The difficulty here is that it is almost impossible if there is a strong side wind to fly exactly over the corners of the course.

Another method essentially of a military nature and applicable only where bombing areas are available is to fly twice over a chosen course, once exactly with the wind and again exactly against it and to drop each time two bombs at a definitely determined time interval. From the time intervals and the distance between the spots where the bombs strike the effect of the wind can be eliminated the speed of the aircraft found. Here again it is also possible to eliminate the effect of the wind by making three flights in different directions and from the three values of the velocity and the directions of the flights to eliminate the effect of the wind by the graphical method previously described. To obtain the necessary accuracy by this method the interval at which the bombs are dropped should be at least from 5 to 10 seconds.

It is also possible to determine the speed of aircraft from the ground by the use of theodolites. If only one instrument is available the ground speed can be found by using it in conjunction

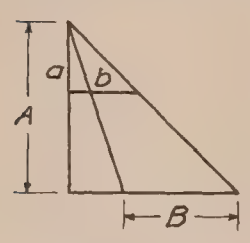
with a barograph by making a series of azimuth and altitude observations with the theodolite at known time intervals and determining from the barograph the corresponding altitudes. More accurate determinations can be made by using two theodolites located one at each end of a measured base line. This base line should if possible be at least a mile in length. In this case altitude and azimuth observations are made at known time intervals with each instrument, from which, with the length of the base line, the ground speed of the aircraft can be determined. Telegraphic and telephonic communication between the two stations is provided to coordinate the time data. In a recent instrument of this type photographic records of the readings of the instruments are obtained by the use of an especially constructed film camera attachment. The shutter of the camera is operated electrically and the film is advanced automatically after each exposure. A chronograph is also provided which records the time when each observation is made. This instrument has the advantage that an observer is not required to record the settings. Moreover, since the settings are recorded instantaneously they can be made at more frequent intervals, which increases the accuracy of the results. It is also possible by suitable electrical connections to record the settings of the instrument at both ends of the base line simultaneously, in which case the observer at each theodolite must keep his instrument trained constantly on the aircraft. A further refinement would be to provide the theodolites with telescopes having cameras attached and in this way obtain simultaneously records of the readings of both instruments and the positions of the aircraft in the telescopic fields.

Another method, the same in principle, is to place camera obscuras at the ends of a measured base line and plot the position of the aircraft at definite time intervals. The plots obtained, together with the optical constants of the cameras, and the length of the base line, provide the necessary data for computing the ground speed of the aircraft.

MEASURING GROUND SPEED FROM AIRCRAFT.

The methods of determining ground speed from the aircraft itself are also subject to classification according or not as they depend upon seeing the ground. The former are essentially optical, the latter dynamical or electrical.

If a sighting telescope is available the ground speed can be determined from the airplane by flying over either a straight or triangular measured course or between any two objects on the ground of known distance apart and noting the time required. This method is essentially one previously described where observations are made from the ground. The statements made in regard to the effect of the wind and methods of eliminating it apply equally in this case.



The type of optical ground-speed indicator most commonly used on aircraft, the type ordinarily seen on bomb sights, depends upon determining with a stop watch the time required for some object on the ground to pass between two sighting points on the ground speed indicator in a horizontal line longitudinal with reference to the aircraft. The ground-speed is calculated from (1) the ratio of the distance apart of the two sighting points, (2) the distance from the horizontal line defined by them to a third sighting point at the observer's eye, and (3) the altitude of the aircraft. The principle may be demonstrated as follows:

Let a = distance from line b to the eyepiece.
b = distance between the two sighting points.
A = the altitude.
B = the distance traversed by the aircraft while the object appears to move between the two sighting points.
t = time in seconds required.
S = speed of the aircraft.

Then by geometry

$$\frac{a}{A} = \frac{b}{B}$$

But $B = St$

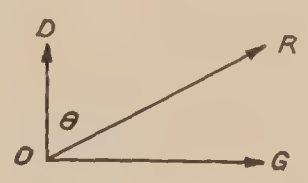
$$\therefore S = \frac{A}{a}\frac{b}{t}$$

$$= \text{Const.}\ \frac{A}{t}$$

It is also possible by a suitable optical arrangement to neutralize the apparent motion of objects on the ground as viewed through a telescope or to cause some reference point in the telescopic field to move at the same rate as the image of the object on the ground. If then the rate at which the telescope or the image in the telescopic field is moving is determined and the altitude of the aircraft is known the ground speed may be found. A number of devices of this kind have been invented.

Another method is to introduce by means of a rotating telescope or similar device an artificial drift at right angle to the actual drift relative to the ground. From the direction of the resultant apparent drift and the magnitude of the artificial drift the ground speed can be computed. The principle may be illustrated graphically as follows:

Let OG represent the ground speed of the aircraft the magnitude of which is to be found and the direction of which is shown by the use of some drift device; OD the known artificial drift introduced at right angles to the ground speed OG by the rotating telescope or other device OR will then represent the resultant apparent drift as seen through the rotating telescope and if the angle θ between the artificial drift and the resultant apparent drift is measured the magnitude of the ground speed can be calculated by the relation



$$OG = OD \tan \theta.$$

Considering now the dynamical methods, it is theoretically possible to obtain the speed of an aircraft by determining the time integral of the accelerations to which it is subjected. It has been proposed to do this by supporting a mass between springs so that it is free to move in a horizontal plane in a fore and aft direction. The displacement of the mass under these circumstances will be proportional to the acceleration of the aircraft. The time integral of this displacement may be obtained mechanically and shown on a direct reading dial, but the inherent friction of the integrating mechanism and the inevitable accumulation of errors in integration seriously limits the practicability of such a device. Moreover it is necessary that the mass move only in a horizontal plane to prevent accelerations of the mass due to gravity which can apparently be brought about only by gyroscopic stabilization.

The principle involved may be demonstrated as follows:

Let M = mass supported by the springs.

x = displacement of mass M due to the acceleration of the aircraft.

v = velocity of the aircraft.

C = constant of proportionality depending on the strength of the springs.

t = time.

The force exerted on the mass M due to the acceleration of the aircraft is

$$F = M\frac{dv}{dt}$$

This force is also proportional to the displacement of M since the reaction is due to the resistance of the springs, hence

$$F = Cx$$

Equating

$$M\frac{dv}{dt} = Cx$$

and integrating

$$v = \frac{C}{M}\int x dt$$

i. e., the velocity is proportional to the time integral of the displacement of the mass M which in turn is proportional to the acceleration of the aircraft.

Ground speed may also be determined by observing the displacement of a mass in opposition to a resistance proportional to the velocity of the mass. Such a resistance might be either of mechanical or electrical origin. An experimental model made at the Bureau of Standards [8] to illustrate this principle consists of a metal ball free to move along a horizontal glass tube filled with oil. Owing to the viscosity of the oil the ball experiences a resistance proportional to its velocity. The principle is as follows:

Let m = the mass of the ball.

v = its velocity with reference to the viscous medium.

v_o = its absolute velocity in space.

R = resistance which it experiences.

x = the displacement of the mass in time t.

c = a constant of proportionality.

Then $R = -cv$ since the resistance is proportional to the velocity.

But also $R = m\frac{dv_o}{dt}$ by the laws of motion.

Therefore

$$\frac{dv_o}{dt} = -\frac{c}{m}v$$

Integrating

$$v_o = \frac{c}{m}x$$

and since v_o does not differ appreciably from the ground speed of the aircraft, the displacement is evidently a measure of the ground speed. This device would also require stabilization, which in this case presents the special difficulty that the movement of the ball displaces permanently the center of gravity of the stabilized system.

The recent development of directional wireless telegraphy methods has presented another possibility for ground-speed measurement. With a suitable receiving apparatus, the position of the aircraft at successive intervals of time with reference to two sending stations of known distance apart may be determined and from these observations the ground speed computed.

It has also been suggested that methods of determining the potential differences set up electromagnetically in conductors on the aircraft by their motion through the earth's magnetic field might be developed and applied to the measurement of the speed of the aircraft with reference to the earth. The practical difficulty is to avoid the equal opposing electromotive force set up by the return wire of the electro-magnetic circuit. Moreover, varying static potential differences in the atmosphere encountered by the aircraft in flight might easily introduce troublesome disturbances of such magnitude as to swamp the main effect. No practical device of this kind has thus far been made.

SUMMARY.

Of the methods of determining ground speed from the ground, the measured speed course is the most exact. Of the methods from the aircraft itself the optical methods mentioned are practicable and reasonably precise but troublesome to the aviator because they require that observations be made at frequent intervals. Of the methods not depending upon the earth being visible, the wireless triangulation method appears most likely to yield practical results.

[8] Suggested by M. D. Hersey.

REPORT No. 128

AERONAUTIC INSTRUMENTS
SECTION IV

DIRECTION INSTRUMENTS

IN FOUR PARTS

AERONAUTIC INSTRUMENTS SECTION
Bureau of Standards

REPORT No. 128.

DIRECTION INSTRUMENTS.

PART I.

INCLINOMETERS AND BANKING INDICATORS.

By W. S. Franklin and M. H. Stillman.

INTRODUCTION.

This report is Section IV of a series of reports on aeronautic instruments (Technical Reports Nos. 125 to 132 inclusive) prepared by the Aeronautic Instruments Section of the Bureau of Standards under research authorizations formulated and recommended by the subcommittee on aerodynamics and approved by the National Advisory Committee for Aeronautics. Much of the material contained in this report was made available through the cooperation of the War and Navy Departments.

SUMMARY.

This part points out the adequacy of a consideration of the steady state of gyroscopic motion as a basis for the discussion of displacements of a gyroscope mounted on an airplane, and develops the simple theory on this basis.

The principal types of gyroscopic inclinometers and stabilizers are briefly described and performance requirements stated. Experimental results are given for two of the spinning top inclinometers investigated.

The various liquid and mechanical inclinometers are then described, including new developments, and the chief characteristics to be determined by laboratory tests are discussed.

Part I concludes with a brief account of the possibilities offered by the earth inductor and other methods for the measurement of aircraft inclination without gyroscopes.

1. PRINCIPLES OF GYROSCOPIC INCLINOMETERS.

In the following discussion the normal position of the gyro axis is assumed to be vertical, and the terms *true zenith*, *pendulum zenith*, and *gyro zenith* refer respectively to the points on the celestial sphere where the true vertical cuts the sphere, where a pendulum would cut the sphere, and where the gyro axis cuts the sphere.

The utility of the gyroscope as an inclinometer or stabilizer is due to the fact that a very considerable torque must act on a spinning gyro for a considerable time to produce an appreciable displacement of the gyro axis from its normal position.

Proposition (the steady state).—The effect of any torque on an airplane gyro may be determined with sufficient accuracy for all practical purposes without considering the delay in the establishment of the steady precession corresponding to the torque, or, in other words, by assuming that the gyro is at all times in what is called the steady state. This proposition is of great importance, and it is valid chiefly because the precession is always very slow, not because the disturbing torques change slowly.

Righting torque.—In every case, what we will call a "righting torque" must be provided for to bring the gyro axis back to its normal direction after it has been displaced therefrom. This righting torque causes the gyro zenith to precess toward the pendulum zenith, and it is in general

a definite function[1] of the angle ϕ between the gyro zenith and the pendulum zenith. Under these conditions the "righting torque" carries the gyro zenith away from the true zenith while the pendulum zenith is displaced during a bank. Righting torque is due to pivot friction in the case of the simple spinning top, and in the case of the gyro which is supported by and driven through a universal joint the righting torque is due in part to the rocking friction in the cross-pins of the universal joint and in part to the driving torque.

Gravity torque.—When the center of mass of the gyro is not coincident with the point of support, the force of gravity produces a torque action on the gyro when the gyro axis is inclined. This torque is neglected in the following discussion because the following discussion applies to a gyro which is always very nearly balanced and of which the axis of spin is always very nearly vertical.

Torque due to horizontal acceleration of the airplane. Centrifugal torque.—When the center of mass of the gyro is above or below the point of support the inertia reaction of the supported gyro causes a torque action about a horizontal axis when the airplane has a horizontal acceleration, and the expression for this torque action is well known.

Error of gyro zenith developed by the righting torque.—While the airplane is performing any maneuver the pendulum zenith describes a definite path on the celestial sphere and the righting torque causes the gyro zenith to follow the pendulum zenith in a curve of pursuit; that is to say, the gyro zenith is being carried toward the pendulum zenith at each instant by the righting torque at a rate which is easily expressed in terms of the righting torque and the angular momentum of the gyro. Therefore, if the righting torque is a known function of the angle ϕ between the gyro zenith and the pendulum zenith, it is evidently possible to calculate the movement of the gyro zenith during any specified movement of the pendulum zenith, using step-by-step integration. The following very simple case, however, covers the ground sufficiently for most practical purposes:

The gyro zenith G may be assumed to be always very near the true zenith Z as shown in Figs. 1 and 2. Then while the airplane is banking through any fraction of a circle, the pendulum zenith P will describe the same fraction of a circle PP' (a small circle on the celestial sphere) whose angular radius ϕ is the angular displacement of the pendulum due to centrifugal action in the banking airplane, and the gyro zenith G will describe approximately the same fraction GG' of a much smaller circle. In this statement the rate of precession p of the gyro is assumed to be constant, because ϕ is sensibly constant and consequently the righting torque which produces p is constant. It is evident, therefore, that the maximum displacement of the gyro zenith by the righting torque during banking will be produced by a half-circle bank, in which case PP' and GG' are semicircles as indicated in Fig. 2.

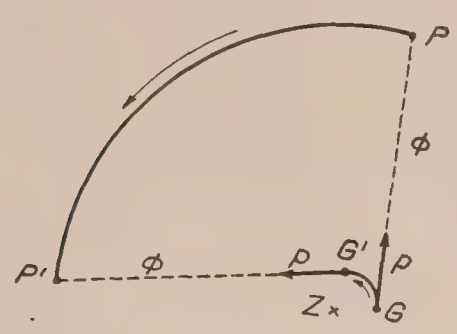


FIG. 1.—Effect of righting torque in displacing gyro zenith.

To calculate the displacement GG' of the gyro zenith which is produced by the righting torque during a half-circle bank of the airplane in a circle of radius R at velocity V, we must know, by test, the value of the righting torque which corresponds to the angle $\phi = \tan^{-1}\left(\frac{V^2}{Rg}\right)$. Let us represent this torque by T. Then the rate of precession p in Fig. 2 is $p = T/\omega K$, where ωK is the angular momentum of the spinning gyro. Multiplying p by the time of a half-circle bank, namely, $\pi R/V$, we get the length of the semicircle GG' in radians, and multiplying this by $2/\pi$ we get the arc GG' in radians. Therefore

$$\text{Displacement of gyro zenith by righting torque during a half-circle bank} = GG' = \frac{2TR}{\omega K V}$$

This calculation can be easily modified so as to give the displacement GG' in Fig. 1 due to bank in any fraction of a circle.

[1] In the Gray gyro the righting torque can be reduced to zero at any time, whatever the value of ϕ may be, and to this extent the righting torque in the Gray gyro is not a definite function of ϕ.

It must be remembered that the above equation is an approximate equation and is true only when the displacement GG' is small. In particular GG' does not increase indefinitely with R as the equation would seem to indicate. In fact, if we assume the righting torque to remain finite when ϕ is very small, the gyro zenith would catch up and remain coincident with the pendulum zenith if the radius of the banking circle is so large that the speed of the pendulum zenith along the semicircle PP' is less than the speed of the gyro zenith p.

In many cases the righting torque is pretty nearly independent of ϕ, as, for example, in the gyro which is supported and driven by a universal joint. In such a case the rate of travel p is nearly independent of ϕ and the maximum displacement GG' in Fig. 2 depends only on the time required for the semicircular bank and is proportional thereto.

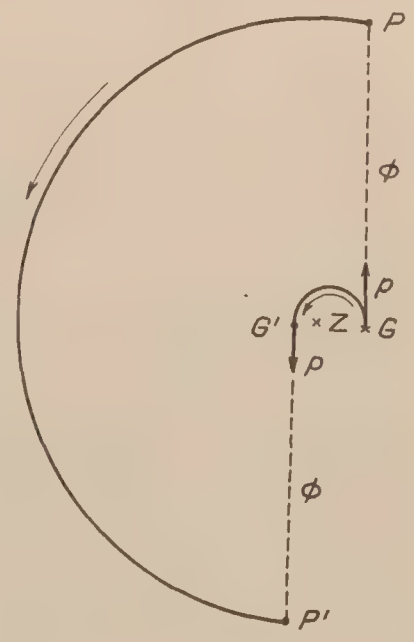


FIG. 2.—Maximum displacement of gyro zenith by righting torque.

Error of gyro zenith developed by centrifugal torque.—Here again the gyro zenith travels in a kind of curve of pursuit as the pendulum zenith describes a curve on the celestial sphere, but with this difference, namely, that G travels at each instant in a direction at right angles to the line GP (see Fig. 3). In this case also the total displacement of the gyro zenith during any prescribed maneuver could be found by stepwise integration, but the following very simple case covers the ground sufficiently for most practical purposes.

During a half-circle bank of the airplane the pendulum zenith traces the semicircle PP' (Fig. 3), whose radius is the banking angle ϕ. To calculate the displacement GG' of the gyro zenith in Fig. 3, we calculate the value of the centrifugal torque, namely $\frac{V^2}{R} \cdot mx$, where V is the velocity of the airplane, R is the radius of the banking circle, m is the mass of the gyro and stabilized structure and x is the distance of the center of mass of gyro and stabilized structure above or below the point of support. When gyro and stablilized structure are mounted separately and linked together this torque is the algebraic sum of the centrifugal torques exerted on them individually. Dividing this torque by ωK we get the rate of precession p; multiply p by the time of the half-circle bank we get the length of the semicircle GG' in radians; and multiplying this by $2/\pi$ we get the diameter GG' in radians. Therefore,

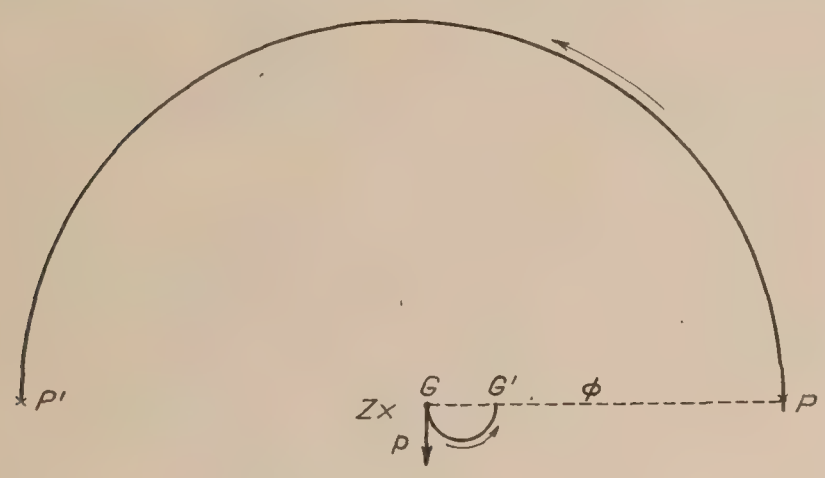


FIG. 3.—Effect of centrifugal torque.

$$\text{Displacement of gyro axis by centrifugal torque during a half-circle bank} = GG' = \frac{2\,Vmx}{\omega K}$$

This calculation can be easily modified so as to give the displacement of the gyro axis by centrifugal torque during a bank in any fraction of a circle.

It is to be noted that the displacement of gyro zenith by righting torque during a half-circle bank is always in the direction in which the airplane travels before the bank, as may be seen from Fig. 2; whereas the displacement of the gyro zenith by centrifugal torque during a half-circle bank is to right or left with reference to direction of travel of the airplane before the bank. Fig. 3 shows the displacement to the right. The displacement of gyro zenith by the combined action of righting torque and centrifugal torque is to be found by superposition, and it is numerically equal to the square root of the sum of the squares of the separate displacements.

The displacement of gyro zenith at starting of an airplane is very similar to displacement by centrifugal action. It may be either to right or left with reference to the pilot.

Proposition (stability of gyro zenith).—Leaving out of account, for the moment, the effects of centrifugal torque and the effects of the earth's rotation, it may be stated that the utmost degree of stability (fixity) of gyro zenith depends solely upon smallness of righting torque (smallness of speed of pursuit p in Figs. 1 and 2). The slower the speed of pursuit the less the gyro zenith will ever depart from the mean position of the pendulum zenith.

Therefore the design of a gyro to give a fixed zenith to any desired degree of accuracy depends upon the reduction of centrifugal torque, and it is bound up with the question of the rotation of the earth, and the latter, only, is serious because the centrifugal torque can be practically eliminated by careful balancing of the gyro and stabilized structure.

Influence of earth's rotation.—The true zenith travels to the east at 45° North latitude at a speed of about 10° per hour, and a stable-zenith gyro must follow the true zenith with negligible lag.

Ordinarily the righting torque is the only thing available for carrying the gyro zenith forward with the true zenith, and, if the righting torque is a vanishing function of ϕ (becoming zero when the angle ϕ is zero) we face a dilemma, either (*a*) enough lag must be allowed to develop to give a righting torque sufficient to carry the gyro zenith forward at a speed of 10° per hour, or (*b*) the righting torque corresponding to a very small value of ϕ must be large enough to carry the gyro zenith forward. In the first case a very large zenith error will soon develop due to the earth's rotation, and in the latter case very large displacements of gyro zenith will be produced by the righting torque during a maneuver of the airplane (see Figs. 1 and 2). Under conceivable conditions the lag error under case *a* might be sufficiently constant to be allowed for as a correction; but, in the second case, no allowance correction would be possible because irregular flight is unavoidable. This dilemma does not arise in the case of the gyro which is supported upon and driven by a universal joint *if the gyro is mounted on a nonoscillating base* because the righting torque of this gyro is *not* a vanishing function of ϕ, and because the nearly constant value of the righting torque in this gyro may be sufficient to carry the gyro zenith forward 10° per hour and yet be small enough to displace gyro zenith only a small fraction of a degree in a short-radius bank. This valuable characteristic of this gyro disappears, however, when it is mounted on an oscillating base as it must be on an airplane or on board ship. Therefore in practice we must design a gyro so that its lag behind the earth's rotation will be constant, or we must compensate for the earth's rotation, and the former is undoubtedly impracticable.

Compensation for the earth's rotation.—To compensate for the earth's rotation is to provide a constant torque which acts on the gyro about an east-west axis, and thus carries the gyro zenith eastward (by precession) at a constant angular speed, with arrangements for adjusting this torque or altering the speed of revolution of the gyro. This adjustment is necessary not only to adapt the compensation for a given latitude but also to change the compensation for change of latitude. To provide for a torque about an east-west axis means, of course, a compass of some kind to define the east-west direction.

Very simple considerations lead to the conclusion that no single combination of gyros can be devised which will inherently compensate for the earth's rotation, for, in the first place, any gyro or combination of gyros must have a very considerable resultant angular momentum if the gyro or system of gyros is to be displaced to a negligible extent by unavoidable disturbing torques, and in the second place a gyro or system of gyros with a large resultant angular momentum must be acted on by an outside torque about an east-west axis to make the momentum axis follow the plumb line.

Hand-controlled compensation.—If a gyro stabilizer is to be used for bomb sights on a large airplane one of the crew could, during the major part of the journey to the objective, devote himself to a hand-controlled compensator for the gyro stabilizer, and be relieved for other duty very shortly before the bomb dropping is to be done. It is very easy to provide a torque of the correct value by hanging a weight on an arm attached to the stabilized structure so as to give the desired gravity torque, and the helper could watch a good magnetic compass and keep this arm north or south. If the arm could be thus held so that its mean position is within 1° of

geographical north or south, then about 59/60 of the effect of the earth's rotation would be compensated. It might seem that under the specified conditions all but about 1/5000 of the effect of the earth's rotation would be compensated, but the lack of complete compensation would show itself as a northward or southward displacement of the gyro zenith.

Automatic compensation.—On board a ship which is equipped with a gyro master-compass the turning of the above-mentioned arm could be done automatically.

Displacement of gyro zenith due to rolling and pitching oscillations of the airplane.—An approximate calculation of these displacements is very easy and, no doubt, sufficiently accurate for all practical purposes inasmuch as this calculation would be always made to show that these displacements would be negligible

Of course rolling and pitching oscillations involve certain amounts of fore-and-aft and athwartship accelerations, and therefore lead to disturbing torques when the center of mass of the stabilized structure is above or below the point of support. The effects of these torques are, however, negligible if the stabilized structure has been balanced with moderate care.

The chief displacement of the gyro zenith due to pitching or rolling oscillations is that which grows out of the righting torque, and a very rough but really adequate calculation of this displacement may be made as in the following example. Suppose that the half-period of the rolling oscillations is t seconds, and that the half-amplitude is $a°$. Knowing the value of the righting torque for $\phi = a°$, we may assume that this value of righting torque acts constantly but in reverse directions during successive half-oscillations. Then the range of the to and fro movement (athwartship) of the gyro zenith must be less than the product pt, where p is the precession rate corresponding to the above-mentioned value of righting torque.

2. BEHAVIOR OF A SPINNING TOP ON AN AIRPLANE.

On account of the popularity of the spinning top inclinometer with the French Air Service, the question of its adoption for American production came up and led to a detailed study of the behavior of a spinning top in an airplane.

The complete formulation of the motion of a spinning top on an airplane is complicated by two effects which are ordinarily negligible, as follows:

1. When a force acts on a body the acceleration which corresponds to the force begins simultaneously with the force without any time lag, and when the force ceases the acceleration ceases. There is no momentum effect associated with translatory acceleration. On the other hand, when a torque acts on a spinning top or gyro, axis of torque not parallel to axis of already existing spin, the angular acceleration which is produced shows itself as a precession, there is a certain amount of angular momentum associated with this precession, and therefore some time is required for a precession to be established after a torque begins to act, and a precession does not cease instantly when the torque ceases. This lag effect is neglected in the following discussion. After the precession corresponding to a given torque is fully established we have what is called *steady gyroscopic motion,* and to neglect the above lag effect is to assume that the motion of the gyroscope or top is always in a steady state.

2. A spinning top which is inclined is nonsymmetrical with respect to the vertical axis of precession, and the precessional motion develops what may be thought of as a torque reaction which very greatly complicates the equations of motion of a precessing top. This effect is neglected in the following discussion.

As a matter of fact effects 1 and 2 are negligibly small when the rate of precession is very slow, and the precession of any well-balanced gyro or top is always very slow.

Other approximations are used in the following discussion but they refer so particularly to the arrangement of the spinning top that they are best explained after the arrangement of the top and the details of our notation are specified.

Figure 4 is a top view of the cup jewel showing the pivot of the top, and the vector ω represents, according to the usual conventions, the angular velocity of the top.

Figure 5 is a side view of the cup jewel and pivot as seen by looking in the direction of the arrow A in figure 4.

Figure 6 is a side view of the cup jewel and pivot as seen by looking in the direction of the arrow B in figure 4.

The vertex V of the cup jewel is the lowest point in the cup with respect to the pendulum vertical.

Notation.—The *top zenith* is the point where the axis of the spinning top cuts the celestial sphere.

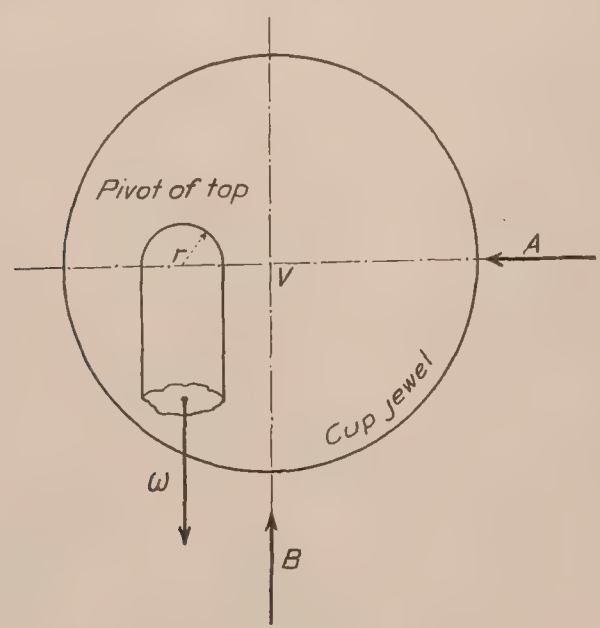


FIG. 4.—Top view of cup jewel.

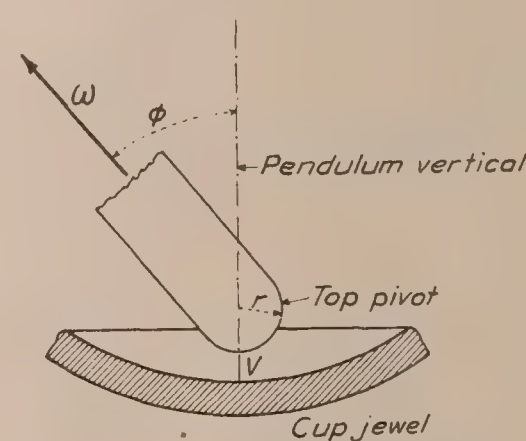


FIG. 5.—Side view—looking along A of figure 4.

The *pendulum zenith* is the point where a non-oscillating pendulum mounted on the air plane would cut the celestial sphere.

ϕ = angle between pendulum zenith and top zenith.

β = angle whose tangent is equal to μ, where μ is the coefficient of sliding friction between top pivot and cup jewel.

r = radius of the spherical end of the top pivot.

m = mass of the top.

K = moment of inertia of the top.

ω = speed of top in radians per second.

g = acceleration of gravity.

a = horizonal acceleration of the airplane.

$G = \sqrt{g^2 + a^2}$

3. Occasionally the spherical end of the pivot of the spinning top rolls round and round in the jewel cup and carries the center of mass of the top rapidly around a small circular path but this condition seems to be abnormal—it occurs when the top is suddenly disturbed, and it lasts for a short time only. This motion is therefore negligible; or, at any rate, it is neglected in this discussion.

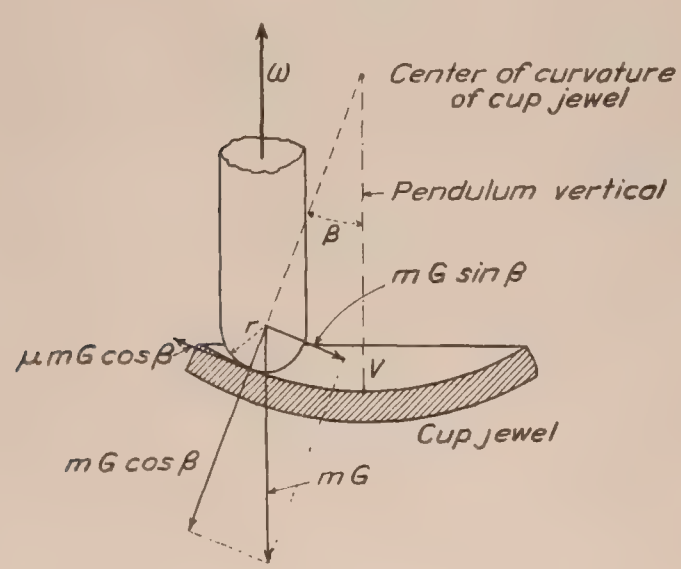


FIG. 6.—Side view—looking along B of figure 4.

4. The pivot of the inclined top rolls sidewise in the cup jewel until the down-hill component $mG \sin \beta$ in figure 6 is balanced by the up-hill frictional force $\mu mG \cos \beta$. This relation involves two approximations, namely, (*a*) The center of mass of the top travels very slowly in a very small path so that the acceleration of the center of mass of the top relative to the cup jewel is negligible and therefore the forces which act on the top are balanced. The horizontal acceleration of the airplane is not neglected in this statement because everything is referred to the pendulum vertical; and (*b*) the slow-speed sliding of the pivot in the cup jewel due to the slow motion

of the pivot in the cup as the top precesses is negligible in comparison with the high-speed sliding $\omega r \sin \phi$ which is due to the spinning motion of the top.

Righting torque.—The two equal and opposite forces $mG \sin \beta$ and $\mu mG \cos \beta$ in figure 6 constitute a pure torque whose value is $\mu mGr \cos \beta$ and whose component at right angles to the axis of spin of the top is $\mu mGr \cos \beta \cos \phi$. This is the righting torque which causes the top zenith to move towards the pendulum zenith. The value of the righting torque is given with sufficient accuracy for most purposes by taking $G = g$, $\cos \beta = 1$ and $\cos \varphi = 1$ which gives the value μmgr for the righting torque.

Motion of top zenith due to righting torque.—This motion may be calculated for any given maneuver of the airplane, or the displacement of the top zenith during any fraction of a circular bank of the airplane can be calculated as explained in section 1 (Principles of Gyroscopic Inclinometers).

Motion of top zenith due to centrifugal torque.—When the center of mass of the top is above or below its point of support a horizontal acceleration of the airplane causes a torque action on the top and this torque action is called centrifugal torque in section 1. The motion of the top zenith due to centrifugal torque can be calculated for any given maneuver of the airplane, or the displacement of the top zenith during any fraction of a circular bank of the airplane can be calculated as explained in section 1 of this paper.

3. DESCRIPTION OF SPECIFIC INSTRUMENTS.

SPINNING TOPS.

Garnier.—One of the best known spinning tops is a French type, the Garnier, shown in figure 7. This consists of an air-driven rotor, weight about half a pound, spinning with its

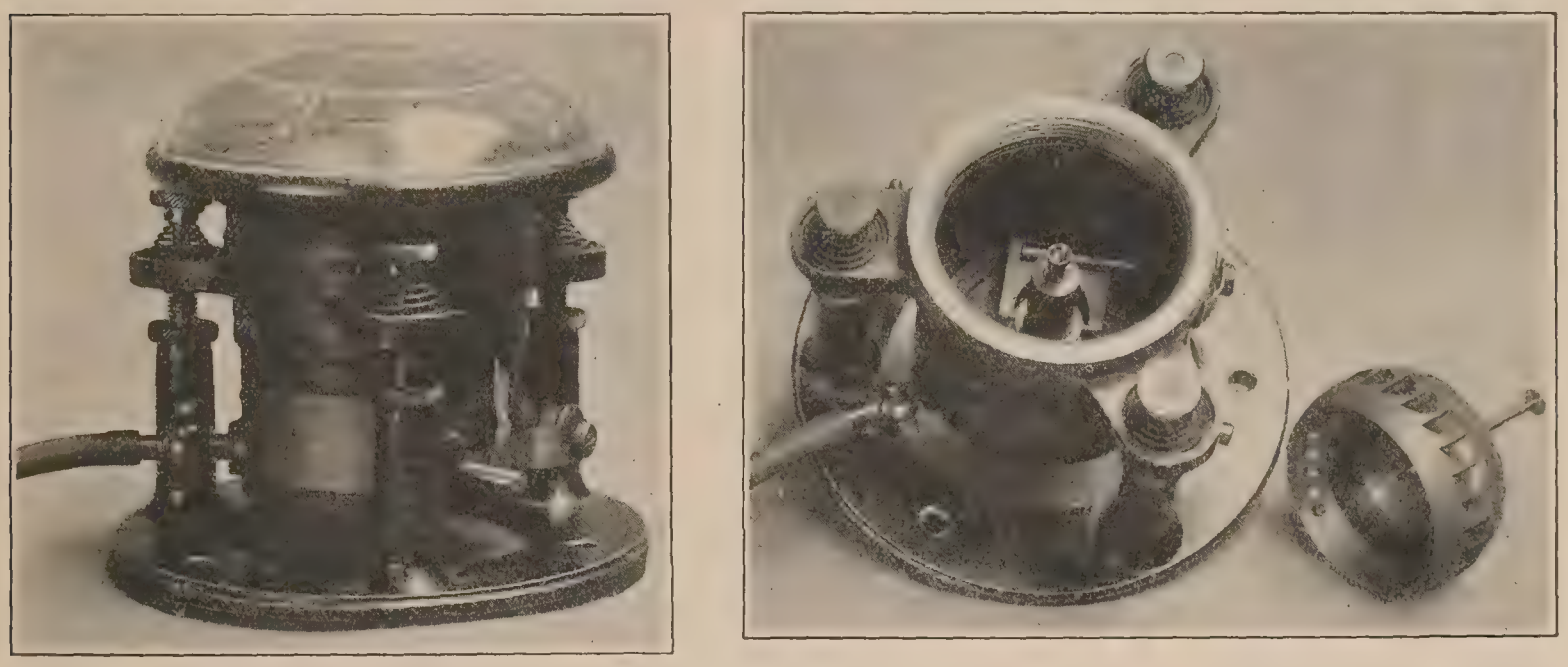


FIG. 7.—Garnier spinning top.

steel pivot in a hemispherical steel cup. The casing is covered by a glass dome graduated in zones and great circles to show angles relative to the vertical. A white spot on the top of the spindle, made self-luminous for observations at night, constitutes the indicating element, and moves about just underneath the surface of the convex dome. This top is driven by air jets like a turbine, having the appropriate blades in the form of grooves on the rotor. The air jets are actuated by suction inside the case generated by a Venturi tube mounted outside the fuselage in the air stream.

A characteristic source of error in this design is the disturbing torque produced by the air jets whenever the axis of rotation is displaced from the axis of symmetry of the casing.

Hebrard.—This instrument, also of French construction, is somewhat similar to the Garnier, but larger, more substantially constructed, having a slightly greater range of angular deflection, and driven in a different manner. Instead of having an air-drive, the Hebrard instrument is

mechanically driven through a universal joint in the bottom of the casing by means of a flexible shaft connected to an air propeller outside of the fuselage. The connection between this universal joint and the rotor itself is made through a ratchet, which permits the top to spin even if the propeller should stop. The antivibration mountings of the two instruments are also different, the Hebrard mounting consisting of three rubber disks,clearly shown in figure 8.

Construction and performance constants for the Hebrard top are given below in section 5 (experimental results).

Other top developments.—Two modifications of the spinning top suggested by one of the authors in the course of this work were, first, what may be termed a breathing top, and, second, a top spinning in hydrogen at extremely low pressure.

The breathing top is a hollow top mounted on a pivot in a glass casing. The air pressure in this casing is subjected to periodic increase and decrease, an inlet valve admits air to the interior of the top when the pressure increases, and the air thus entrapped escapes through a series of nozzles and drives the top by reaction.

The idea of the hydrogen top was to provide so small a resisting torque as to enable the top to run for several hours after being started at the beginning of a flight. This would have the

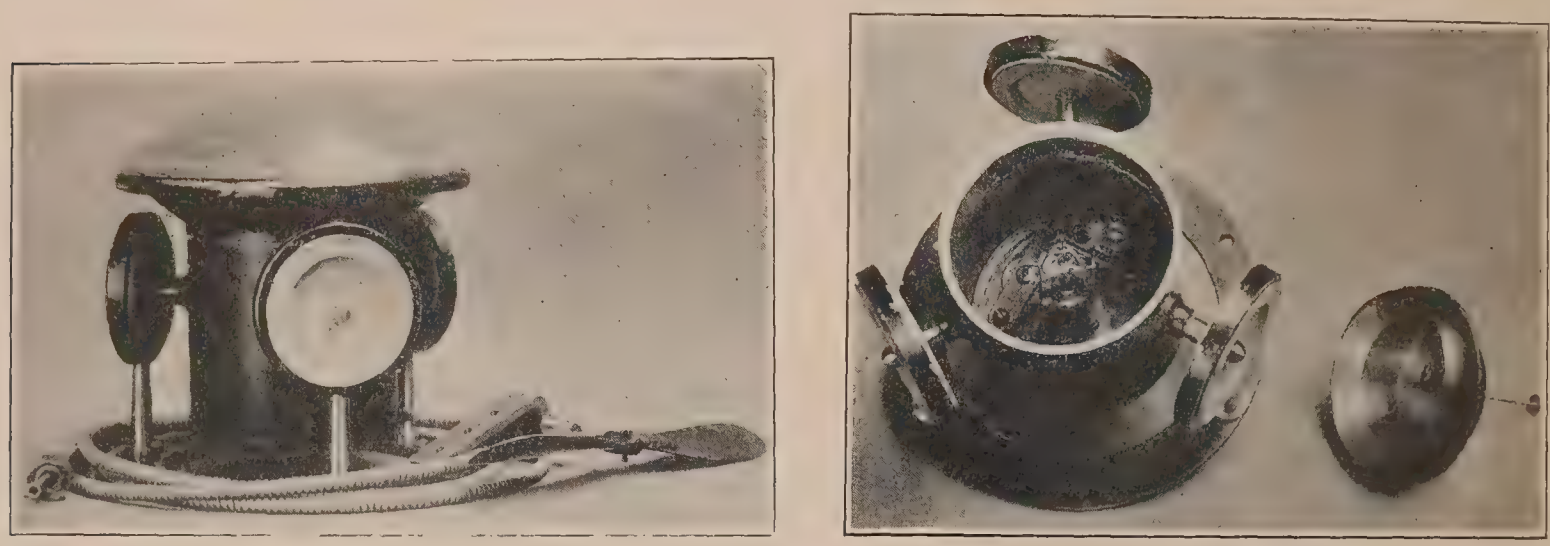


FIG. 8.—Hebrard spinning top.

advantage that the driving apparatus would not be available to the enemy in the event of a crash, and also that it would simplify the apparatus needed on the airplane. Laboratory data on the experimental model of this type will be found in the section on experimental results.

INCLINOMETERS WITH PENDULOUS GYROSCOPES.

Sperry inclinometer.—The possibilities of a single gyrostat supported in gimbals, with its axis vertical, driven by three-phase current through flexible leads and slip rings, have been developed by the Sperry Gyroscope Co., and modifications of this arrangement have also been constructed by the Royal Aircraft Establishment. Such inclinometers are satisfactory for steady flight, but subject to disturbances from acceleration, since the center of gravity must be placed below the point of support in order to utilize gravity for the righting torque. This disturbance is, however, far less than in the case of an ordinary pendulum, hence the utility of the instrument. In this type of gyroscope the gravity torque is not in itself a righting torque, but it produces precession and thereby causes the gimbal axes to rock and the friction which opposes this rocking motion is a righting torque.

Multiple gyro instruments.—Undoubtedly the best known example of this type is to be found in the Sperry automatic pilot, with its four gyro units. A self-recording inclinometer, known as the Bureau of Standards stable zenith, was developed by Prof. J. F. Hayford and Dr. L. J. Briggs for use in free flight tests by the National Advisory Committee for Aeronautics; this is briefly described in the annual report of the Director of the Bureau of Standards for 1918 (p. 128) as a two-gyro combination. The use of coupled gyros for stabilization has been discussed by one of the present authors in a paper on Gyroscopic Oscillations (Phys. Rev. 34: 48–52, 1912).

INCLINOMETERS WITH NEUTRAL GYROSCOPES.

The possibilities of the neutral or free gyro have never been fully utilized in practice because of mechanical difficulties. A neutral gyro uninfluenced by external force, would maintain its axis of rotation fixed in space. Two instruments in which this principle has been sufficiently realized for temporary observations are the Norton recording stunt indicator and the Duff-Hyde stabilizer.

Duff-Hyde.—The stabilizer (or inclinometer) developed by Prof. A. W. Duff and Lieut. W. A. Hyde [2] is shown in figure 9. It is a short period gravity pendulum, combined with an independently supported gyrostat. The oscillations of the pendulum are damped by air dashpots, and the pistons of these dashpots serve for the coupling between the pendulum and the gyroscope. The pendulum part is mounted on gimbals, so as to be free to move about two horizontal axes aa and bb at right angles. The gyro itself is of the Sperry A. C. motor type. It is mounted above the pendulum with its axis vertical, and on gimbals having two horizontal axes of rotation respectively parallel to aa and bb. Four dash pots and their pistons form links between the pendulum and the gyro. The mechanical connections are made by hinged joints. Now the axis of the spinning gyro tends to remain fixed in space and so furnishes a comparatively stationary position for the dashpot pistons. The effect of this device is to damp the quick oscillations of the pendulum, but to permit slow movements without much hindrance, thus enabling the pendulum to maintain the vertical. The stable zenith of this instrument is through the pendulum itself.

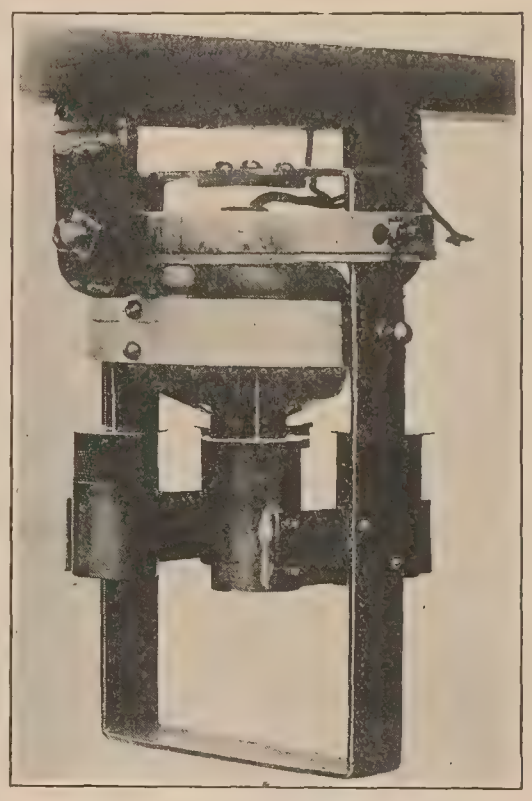


Fig. 9.—Duff-Hyde stabilizer.

Norton.—The recording inclinometer or stunt indicator under development by F. H. Norton, of the National Advisory Committee for Aeronautics, differs from ordinary gimbal instruments in that no obstacle is encountered when a complete rotation takes place, as in looping the loop. In other instruments when two of the axes become coincident, the instrument loses its freedom, it is then locked and no longer able to rotate about the third axis. The Norton gyro is wholly incased inside of a spherical shell. This shell is gripped between four rollers, symmetrically placed at the corners of a regular tetrahedron. These are like planimeter wheels in that they record the motion in one direction, while slipping freely at right angles. These wheels in turn are geared to the dials after the fashion of a polar planimeter, so as to show at a glance the total components of rotation of the aircraft about the respective three axes.

An instrument developed by the Royal Aircraft Establishment, for use in aircraft stability investigations, has somewhat the same purpose, but records angular velocity instead of angular displacement. The principle of operation of the Royal Aircraft Establishment instrument is similar to that of a gyro turn indicator.

STABILIZERS FOR BOMBING AND PHOTOGRAPHY.

Two further instruments particularly designed as stabilizers are the Gray and Lucian instruments; each carries a single gyro unit of the pendulous type, and in each the auxiliary device for producing a righting torque is unique. Attention may here be called to Report No. 131, in which is given a more detailed account of a proposed type of stabilizer with a particularly simple erecting device—namely, the friction of a universal joint, upon which the gyro is sup-

[2] Aviation and Aircraft Journ., I: 322, 1920.

ported, and through which it is driven. Such stabilizers may prove useful for a variety of purposes, including not only bomb sighting and photography, but the control of a double-pivot compass so as to eliminate the northerly turning error; the furnishing of an artificial horizon for sextant observations; and the mounting of dynamical type ground speed or distance indicators, which have to be held horizontal.

Gray.—The stabilizer developed by Dr. J. G. Gray in Scotland obtains its restoring torque by the displacement of a number of balls which are slowly rolled around a horizontal plate by means of a pair of rotating crossbars. When this plate is inclined, a restoring torque of the desired direction is produced on the vertical spinning gyro which is rigidly attached to the horizontal plate. This ingenious arrangement is effective because of the fact that the ball is guided into a sidewise displacement by the rotating arms instead of rolling directly forward when the airplane pitches downward. Also the restoring forces may be easily made inoperative at will so as to preserve the position of the gyroscope during a rapid bank.

Lucian.—The stabilizer developed by Dr. A. N. Lucian [3] utilizes electromagnetic action for the production of a suitable righting torque. This torque is brought into play when either of two simple pendulums, of short period, is displaced so as to make an electric contact. These pendulums swing about the gimbal axes of the gyro, one in a fore-and-aft plane, the other transversely. The entire construction of this outfit is comparatively light, as stabilizers go.

4. TESTING OF ANY PROPOSED TYPE OF GYRO INCLINOMETER OR STABILIZER.

The only thing that need be said here concerning the driving mechanism is that it is necessary to determine whether this mechanism exerts any torque on the gyro about an axis at right angles to the axis of spin, and if so how much. For example, the gyro which is supported on and driven through a universal joint has such a torque exerted on it.

Moment of inertia and normal running speed of gyro must be determined.

Mass and location of center of mass of gyro and stabilized structure must be determined for the purpose of calculating what is above called the gravity torque and for calculating torque action due to horizontal acceleration.

The most important thing to determine is the experimental functional relation between righting torque and angle ϕ between gyro zenith and pendulum zenith. This determination can be easily made by observing the rate p at which the gyro zenith approaches the pendulum zenith for various values of ϕ, speed of gyro and moment of inertia of gyro being known. This test would of course be made in the laboratory and the pendulum zenith would coincide with the true zenith. In this test the gyro zenith should be displaced northwards or southwards of the true zenith so as to eliminate the influence of the earth's rotation.

A separate test of the amount of lag of gyro zenith due to the earth's rotation might be advisable, although this can be calculated from the above data.

The most important calculations from these test data would be:

(*a*) Displacement of gyro zenith due to a half-circle bank of the airplane, using several airplane velocities and several radii of banking circle.

(*b*) Displacement of gyro zenith due to centrifugal action on unbalanced stabilized structure during a half-circle bank at various airplane speeds. This displacement is independent of the radius of the banking circle.

(*c*) Lag of gyro zenith due to earth's rotation at a chosen latitude and for a specified amplitude and frequency of east-west oscillation of the pendulum zenith. In general this will require tedious step-by-step integration using a tabulated set of values of righting torque at short intervals during one oscillation of the pendulum zenith.

(*d*) Upper limit of possible displacement of gyro zenith during a prescribed rolling or pitching oscillation of the airplane.

Various other simple tests such as time required to bring gyro up to normal speed, time for gyro to come to rest when driving torque ceases, will suggest themselves to anyone.

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[3] C. E. Mendenhall: Journ. Franklin Inst. 191: p. 85. An air-jet type is also referred to here.

5. EXPERIMENTAL RESULTS.

Construction and Performance Constants of the Hebrard Top.—Mass = 596 grams; moment of inertia K = 7680 gr.-cm ^{2}; pivot of hardened steel, small radius r = 0.08 cm.; distance of center of mass below point of support, x, = 0.052 cm. Cup jewel of hardened steel, radius of curvature 0.41 centimeter. Staff of top 6.8 centimeter long, disk on end of staff 0.9 centimeter diameter. Inclination of staff with respect to casing can be read to about 1°.

When tried out on an airplane (Curtis J. N.) the behavior of the top was quite satisfactory for banking angles of less than 30°, but the driving mechanism is very bumpy when the casing is inclined 30° or more with respect to the top.

The top came to rest in 13 minutes from 1,000 revolutions per minute when the driving torque ceased. At 1,500 revolutions per minute the top rose from 20° inclination to 10° inclination in 35 seconds; calculated time 30 seconds; this test was made in the laboratory.

(1) Error developed in 15 seconds by horizontal component of driving torque when casing is inclined 20° (in the laboratory) was about 2°.

(2) Calculated error due to pivot friction alone, when V, the speed of the airplane = 36 meters per second, and R, the radius of the turn = 500 meters, with top running 1,500 revolutions per minute, is 4.7°.

(3) Calculated error due to horizontal acceleration alone, when V = 36 meters per second, R = 500 meters, and speed of top = 1,500 revolutions per minute, is 2.7°.

(4) Calculated error due to horizontal component of driving torque alone, when V = 36 meters per second, R = 500 meters and speed of top = 1,500 revolutions per minute, is 0.9°.

(5) Calculated error due to combined action of pivot friction, centrifugal force and horizontal component of the driving torque T when V = 36 meters per second, R = 500 meters and speed of top = 1,500 revolutions per minute, is 6.2°.

(6) Calculated value of constant error of staff of top due to rotation of the earth is 0.12° at 45° N. latitude, when speed of top is 1,500 revolutions per minute.

Data for hydrogen top.—Taking the mass of the top 100 grams, diameter 6 centimeters, thickness of rotor at edge 1 centimeter, the radius of gyration will be 2 centimeters and the moment of inertia 400 centimeter-gram-second units. These data apply to the top constructed at the Bureau of Standards by Mr. F. Cordero.

Tests made on the first model of this type, as actually constructed, showed that it would run about one hour in hydrogen at 2 or 3 millimeters pressure, while dropping its speed from 3,000 to 500 revolutions per minute.

6. DESCRIPTION OF LIQUID AND PENDULUM INCLINOMETERS.

For practically steady flight, free from acceleration, instruments constructed so as to show the direction of the apparent gravitational field are satisfactory and have been used with good results in performance testing of aircraft. Such instruments, of course, can not take the place of gyroscopic inclinometers for absolute measurement when accelerations are present. A forward acceleration equal to gravity, which is not uncommon, throws the direction of apparent gravitational force 45° toward the rear. Inclinometers constructed on the liquid or pendulum principle will, of course, respond to this change of direction almost instantly, and this effect is indistinguishable from a true inclination of 45°.

Fore-and-aft inclinometers, French types.—The principal type of liquid fore-and-aft inclinometer is triangular in shape, as shown by figure 10, and has been extensively developed by the French. The principle involved is simple enough: The liquid seeks its proper level, so when the airplane climbs the liquid recedes down the front part of the triangular circuit, allowing the meniscus to rise along the rear part of the circuit to which a graduated scale is attached. The commercial form is not over 10 inches on each side. It is understood that Capt. Toussaint has developed a special form of this instrument in which the triangular circuit extends back a long distance through the fuselage of the airplane in order to secure extremely high sensitivity.

Drexler.—A liquid fore-and-aft indicator of the triangular type forms a part of the Drexler combination inclinometer and gyro indicator described in Part IV of this report.

Rieker.—This is an American-made instrument quite similar to the French types of fore-and-aft inclinometer.

National Advisory Committee for Aeronautics.—A liquid fore-and-aft inclinometer of exceptionally open scale was developed by Edward P. Warner of the National Advisory Committee for Aeronautics and employed with good results in free flight investigations.[4]

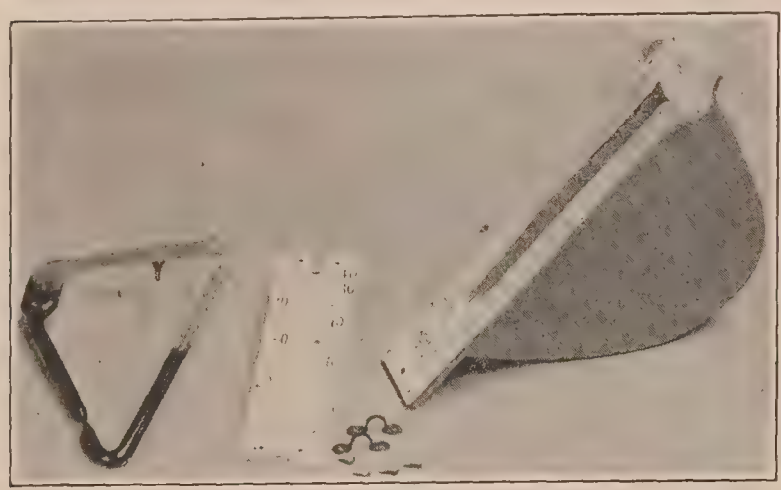

FIG. 10.—Liquid fore-and-aft inclinometer

Danish sector type.—Liquid inclinometers of the sector type are familiar on German airplanes, and in steady flight may be used either for fore-and-aft or for lateral observations. Two such instruments of Danish manufacture are shown in figure 11.

German double circuit type.—A double circuit type of liquid inclinometer found on German aircraft is shown in figure 12. While similar in principle to the triangular circuit, this arrangement is evidently more sensitive for a given size and can therefore be furnished in a more compact form.

Sperry fore-and-aft inclinometer.—Figure 13 shows a liquid damped pendulum device for fore-and-aft observations developed by the Sperry Gyroscope Co.

Richard clinometer.—This instrument of French design, is similar to the Sperry in its fundamental principle.

Russell liquid-damped pendulum.—Pendulums damped with a viscous liquid have been employed primarily for artificial horizons in connection with sextant observations. Such an instrument, developed by H. N. Russell, is further described in Report No. 131. This problem has also been investigated by Mr. E. G. Fischer, of the United States Coast and Geodetic Survey.

Pentz compass.—This compass, considerably used by the United States Air Mail Service, was designed to serve also as an inclinometer. From the description given in Part III of this report it will be noticed that the pendulum zenith at any time is indicated by looking down through the spherical glass cover, which is suitably marked.

FIG. 11.—Liquid inclinometers, sector type.

7. BANKING INDICATORS.

Banking indicators are, of course, intended only to show the departure of the airplane from the proper banking angle. It is a fallacy to suppose that a banking indicator ought to show the absolute inclination of the craft with respect to the earth, which it can not do for the reasons explained at the beginning of section 6. The instrument is not designed to show absolute inclination. The indicator should read zero when the banking angle is correct for the actual speed and radius of turn, regardless of the absolute amount of the bank relative to the horizontal.

British bubble type.—Figure 14 shows the familiar bubble type of banking indicator developed by the British. The Taylor and Rieker instruments of American manufacture are based evidently on the same principle. The banking indicator is mounted transversely on the instrument board in front of the pilot. For night flying the bubble can be illuminated by a small electric lamp placed at the end of the glass tube so that the rays are transmitted through the liquid.

[4] Report No. 70, National Advisory Committee for Aeronautics, Fifth Annual Report.

The bubble banking indicator is sometimes furnished in combination with the fore-and-aft inclinometer.

Bubble banking indicators appear to have been very popular among the British fliers and in the American Navy, but not among the French or in the American Army.

Sperry pendulum type.—In figure 15 is shown an air-damped pendulum indicator developed by the Sperry Co. This is made in a self-luminous form for use at night. The zero position is that in which the movable indicator is horizontal.

Luminous rolling ball type.—In figure 16 is shown the luminous ball banking indicator developed by one of the authors and found satisfactory for night flying. It operates essentially on the same principle as a bubble banking indicator, but deflects in the opposite direction.

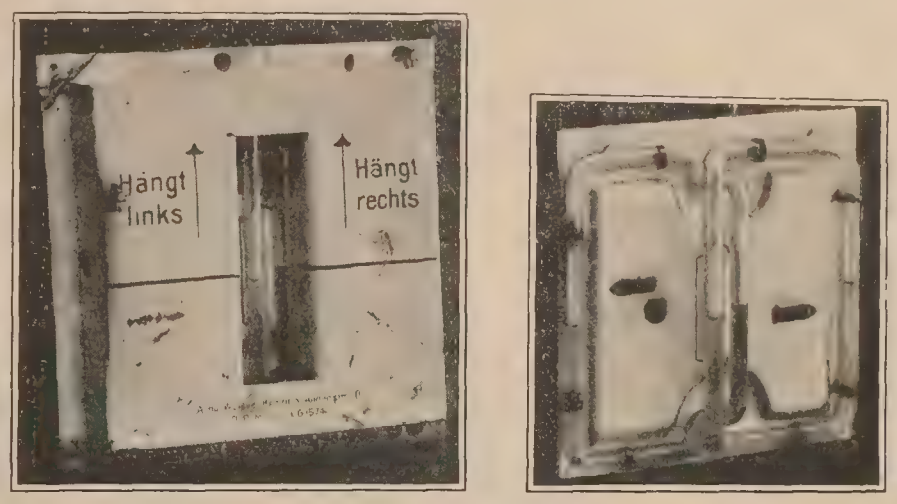

FIG. 12.—Liquid inclinometer—Double-circuit type.

Drexler.—In the new model Drexler aircraft steering gage, to be described in Report No. 131, another form of rolling ball banking indicator is employed, having a steel ball. The action of this instrument is similar to the one previously developed at the bureau, but does not have the self-luminous feature.

8. PERFORMANCE CHARACTERISTICS OF THE LIQUID AND PENDULUM TYPES.

The testing of inclinometers and banking indicators depends on the construction, rather than on the use of the instrument. Temperature tests and observations concerning the quickness of action are essentially for liquid-filled instruments, regardless whether they are to be employed as inclinometers or banking indicators. Air-damped instruments are not likely to show excessive time lag, but should be examined for friction, looseness, and general accuracy, and the damping should be sufficient to prevent oscillation.

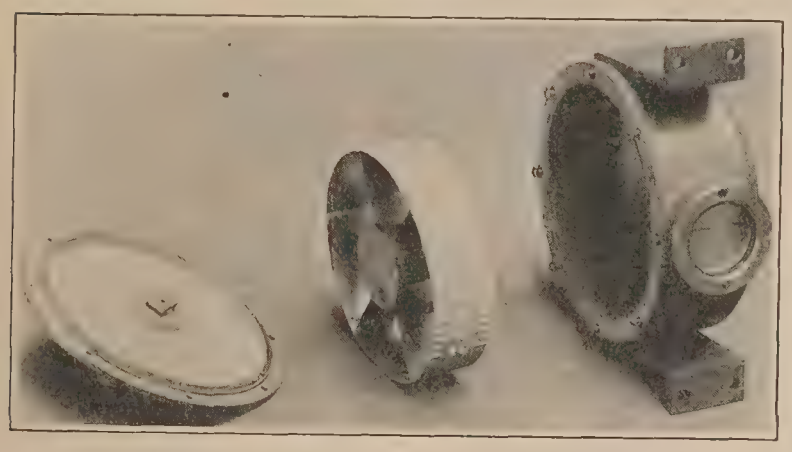

FIG. 13.—Sperry liquid damped fore-and-aft inclinometer.

Either type of instrument may be tested for time lag by pivoting the instrument with an initial upward inclination and then dropping it suddenly onto a slanting support at an equal angle below the horizon. The entire time of travel may be noted with a stop watch, and should not exceed about two seconds. It is important for this test to be repeated at temperatures fully as low as those which may be experienced in flight. Alcohol and other nonfreezing solutions have been used for filling the instruments. Inclinometers and banking indicators should also be tested under vibration. The bubble type of banking indicator has been found at times to show a systematic average displacement during vibration.

9. ABSOLUTE MEASUREMENT WITHOUT GYROSCOPES.

Both the dip needle and earth inductor have been frequently proposed for inclinometers.[5] Either the dip needle in conjunction with a magnetic compass, or the earth inductor without a compass, if arranged in multiple units, would serve as an inclinometer over any region of the earth's surface where the direction of the earth's field is known.

[5] The Earth Inductor as an Inclinometer, N. E. Dorsey, Journ. Terr. Mag. and Atoms. Elect. 18: 1–38, 1913; Induction Inclinometers, W. Uljanin ibid., 24: 113–117, 1919.

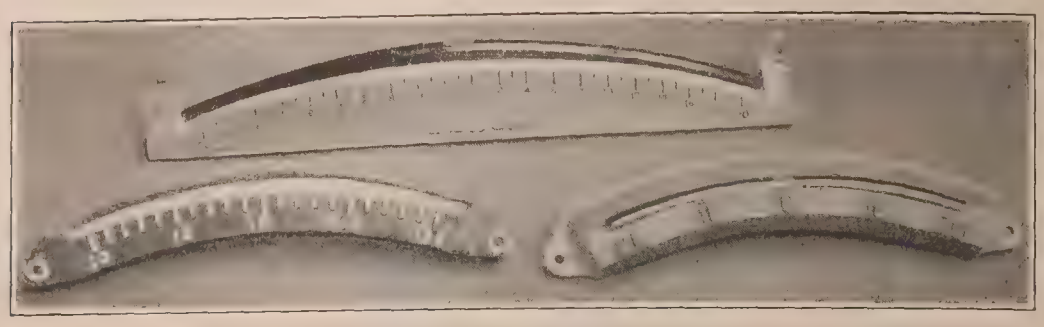

FIG. 14.—Bubble banking indicators.

FIG. 15.—Sperry pendulum banking indicator.

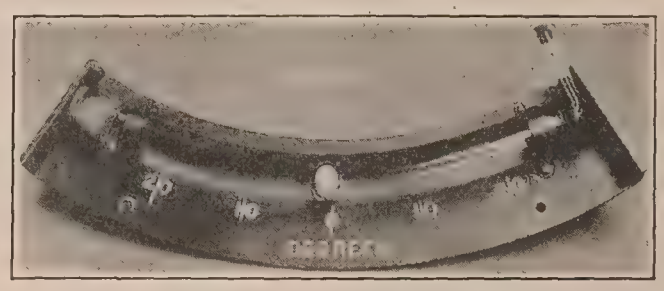

FIG. 16.—Luminous ball banking indicator.

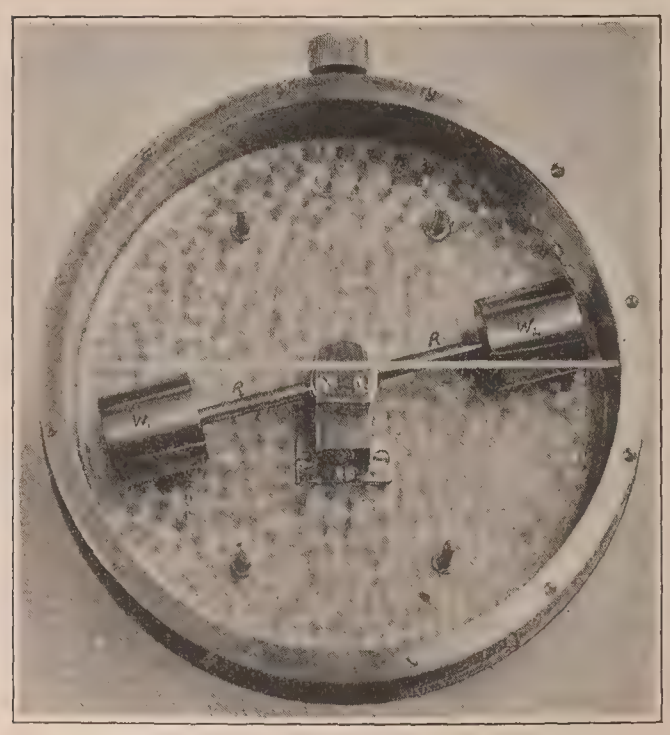

FIG. 17.—Differential lateral inclinometer.

The inductor in its simplest form consists of a flat coil of copper wire connected to a suitable millivoltmeter; commutation is provided such as to build up a continuous electromotive force due to cutting the earth's field. The ordinary equations of the dynamo apply to this instrument and show that by sufficiently increasing the speed it can be made entirely practicable.

The recording sun camera developed by the British at Martlesham Heath gives an excellent tracing of the image of the sun, thus serving as a recording inclinometer when small corrections are made for the relative movement of the sun during the time of the flight.

Both in this country and elsewhere the reflection of an airplane in a smooth body of water below has been utilized for recording photographically the angular movement of the airplane.

The observation of the trajectory of a falling object, if made visible, will give information concerning the angular position of the aircraft, provided suitable allowance is made for the initial linear velocity of the falling body and for drift of trajectory due to wind. For the trajectory of such a body would be determined solely by its initial velocity, together with the true gravitational field of the earth, barring the effect of the wind. Thus the trajectory is uninfluenced by acceleration of the aircraft. In this way the difficulties characteristic of pendulum indicators can be avoided. It has been proposed [6] to apply this to an instrument inclosed in an air-tight case somewhat after the fashion of an hourglass. In this way the effect of the wind would be eliminated, but the trajectory would still be independent of aircraft accelerations. Instead of observing the falling particles of sand, a fine jet of mercury might be used.

There remain a number of possibilities for inclination measurement in a manner free from the usual errors of pendulum indicators, although not attaining completely the status of absolute measurement owing to the necessity for some assumption regarding the path of the airplane or the motion of the atmosphere. Under ordinary conditions these assumptions are entirely legitimate. A proposal of this kind by one of the present authors has been the use of a spring pendulum. The stretching of the spring serves to measure the excess of the apparent gravitational field, caused by centrifugal force over and above the true gravitational field. The angular deflection of the pendulum shows, as usual, the direction of apparent gravity. Thus the proper interpretation of this observation would go definitely one step beyond the simple, rigid pendulum toward giving the true inclination of the aircraft with respect to the ground. A second instrument developed by one of the authors likewise based on centrifugal force is shown in figure 17. This instrument is essentially a differential lateral inclinometer, and has elsewhere been referred to as the balance banking indicator. It consists of two equal masses practically balanced on a knife-edge. The center of gravity of the system is just slightly below the knife-edge. The instrument is mounted on the instrument board with the knife-edge parallel to the fore-and-aft axis of the aircraft, so that the beam of the balance may deflect in a plane parallel to the instrument board. The case of the instrument is filled with a damping liquid. Now, when the aircraft in its flight turns about a vertical axis, the two equal masses of the system, W^1 and W^2, share the same angular velocity but are located at different distances from that axis. Hence, the centrifugal force on one would be greater than that on the other, and the balance beam tends to remain perpendicular to the axis of rotation of the aircraft—that is, approximately horizontal. Thus, the angular deflection of the balance beam relative to a reference line on the case indicates the inclination of the aircraft relative to the ground while going around a bank.

The Aveline stabilizer, a recent French development,[7] automatically operates the controls of the ship by compressed air actuated by a combination inclinometer. The indicator element is in principle a mercury-filled inclinometer which, instead of operating on the usual liquid inclinometer principle, has an auxiliary correction for centrifugal force. This correction is automatically made by a device in the nature of a turn indicator, consisting of two Venturi tubes suitably connected and located at the respective extremities of the wings.

Until gyroscopic appliances can be reduced in bulk, weight, and expense, the development of these various semiabsolute methods of measurement seems to be a desirable field for further investigation.

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[6] The suggestion is believed to have originated with Mr. Benedict, a member of the Air Service, in 1918.

[7] Aerial Age Weekly, Vol. 12: pages 656–658, 667, 1921.

REPORT No. 128.

DIRECTION INSTRUMENTS.

PART II.

THE TESTING AND USE OF MAGNETIC COMPASSES FOR AIRPLANES.

By R. L. Sanford.

INTRODUCTION.

One of the most important and least satisfactory of all aeronautic instruments is the magnetic compass. Owing to the extraordinary conditions under which a compass must operate in flying, the ordinary marine type is impossible to use, and new and radically different designs have been found to be necessary. Unfortunately, even the best types of instrument which have so far been produced are unreliable under certain conditions. This fact has led most pilots to regard the compass with suspicion, and many have come to the conclusion that a compass should not be included in the instrumental equipment of an airplane. There are many times, however, when known landmarks are not available, and it is necessary to rely upon the indications of the compass. It is essential, therefore, that the pilot should understand the characteristics of his compass in order that he may know under what conditions its indications are reliable. An understanding of the principles involved may also enable pilots to offer valuable suggestions as to design and use as the results of experience and observation.

TESTING.

Before installing a compass in an airplane it should be carefully inspected and tested to be sure that there are no defects of material or workmanship and that it is in good working order. These points can easily be determined by means of simple laboratory tests. The performance characteristics which are inherent in all instruments as distinguished from their behavior under actual flying conditions are pivot friction, calibration, period, and damping. In some cases it is desirable to measure the strength or magnetic moment of the needles but usually this is not necessary as weak needles are indicated by too long a period.

PIVOT FRICTION.

Excessive pivot friction in a compass reduces its sensitivity to small changes of direction and is generally evidence of damage or imperfection in material or workmanship. This point therefore is generally the first to be considered in the testing of a magnetic compass. The only auxiliary apparatus required for this test is a small magnet or, better, a coil by means of which the compass may be given a momentary deflection. In practice the compass under test is deflected by various small angles in each direction and the amount by which it fails to return to its original resting point noted. This procedure is repeated with the compass oriented in different directions, usually on headings corresponding to each of the cardinal points. The pivot friction as determined in this manner is rarely a constant quantity but there is generally no difficulty in deciding whether or not an instrument has an excessive amount. When released from an initial deflection of 5° in either direction a good compass should return to its original position to within 1°.

CALIBRATION.

The term calibration expresses the accuracy with which a compass indicates direction on any heading exclusive of the effect of pivot friction which can generally be removed by tapping. There are several factors which determine the accuracy of calibration of a compass, namely: (1) The orientation of the magnetic needles on the card; (2) the accuracy of the graduation on the card or scale; (3) accuracy of centering of the pivot; (4) magnetic materials in the bowl or mounting; (5) location of the lubber line.

The magnetic needles should be mounted parallel to the north-south line on the card. If this is not done there will be a constant error on all headings equal to the angle of error in mounting.

In good instruments the errors in graduation of the card are usually neglibible. Graduation errors may be in either direction and varying in amount.

If the pivot is not correctly centered on the card the resulting eccentricity error varies from zero to a maximum value depending on the heading and the amount by which the pivot is off center.

The presence of magnetic materials in the bowl or mounting may produce errors in reading depending upon the amount and location of such impurities.

While it is generally possible to separate these errors it is usually not necessary to do so except for the purpose of discovering the cause of an excessive error.

For determining the calibration errors of compasses a simple testing stand has been constructed. This stand is shown in figure 1. It consists of a rotating table graduated around its edge so that by means of a vernier index angles of rotation can be read to 0.1°. Two telescopes are carried on an adjustable support which are used for sighting on horizontal card compasses. If compasses having vertical cards are to be tested an auxiliary stand is used. This stand has upon its base a horizontal line which is at right angles to the plane of the back of the stand. It also has provision for tilting with a scale for measuring the angle of tilt. An airplane compass card should be free to turn when the compass is tilted 20° in any direction.

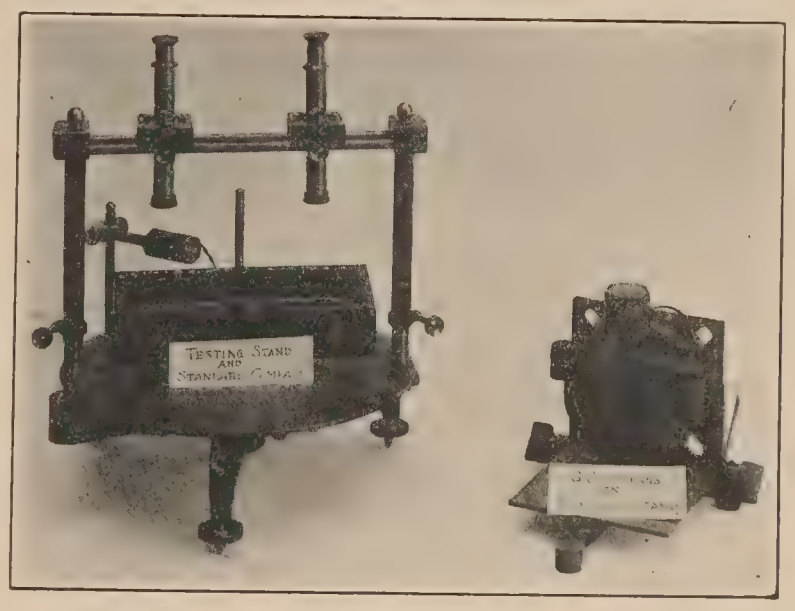


FIG. 1.—Compass testing stand.

The standard compass, also shown in the figure, consists of a single magnetic needle having a sapphire cup and suspended on a diamond pivot. The pivot friction of this combination has been found to be entirely negligible. Upon the upper side of this needle is a line which is accurately parallel to its magnetic axis. The needle is pointed and the points lie in the vertical plane passing through the index line. It is therefore possible to check the parallelism of the index line and the magnetic axis by taking observations with the needle in its normal position and inverted.

When a compass is to be tested, it is set on the stand and the telescopes focused on the north and south points of the card, or if it has a vertical card the auxiliary stand is used and the telescopes are focused on its index line. The compass is then placed at a distance and the standard compass substituted and raised or lowered on an adjustable base until the index line is in focus. The telescopes are not disturbed after the initial adjustment. When the stand is so oriented that the index line of the standard compass is focused on the cross hairs the vernier index is set to zero. The standard compass is then taken away and the other compass replaced. If the table is rotated till the north-south line is on the cross hairs of the two telescopes, the angle read on the vernier shows the error on the north heading. The error on any other heading can be determined by rotating the table through any desired angle and

noting the difference between the compass reading and the table reading. After the test has been completed a check reading on the standard compass is taken in order to be sure that the direction of the magnetic meridian has not changed while the observations are in progress. The ordinary type of airplane compass graduated with 10° division should not be in error on any heading by more than 2°.

PERIOD.

For any particular type of compass, since the moment of inertia of the moving system is practically the same for all instruments, the period, measured at the same place or at different places where the horizontal intensity of the earth's magnetic field is the same value, is a good indication of the strength of the magnetic needles. An excessively long period is an indication of weak needles. For compasses having considerable damping it is difficult to determine the complete period (the time interval between successive transits in the same direction through the position of equilibrium) and it is necessary to determine the half period. This is accomplished by setting the card to swinging by means of a coil or small magnet and noting, by the aid of a stop watch, the time interval between successive transits through the equilibrium position. The variations in the horizontal intensity of the earth's magnetic field are such that the period of the same compass will not be the same in different parts of the earth. For instance, a compass having a period of 20 seconds in Washington would have a period of 23 seconds in Bangor, Me., and 17 seconds in New Orleans. Various types of airplane compasses are designed to have widely different periods (from approximately 10 to 50 seconds) and opinion differs as to the most desirable value. One type of instrument developed during the war has a complete period of approximately 15 seconds.

DAMPING.

The liquid which is used in the majority of airplane compasses, besides taking some of the weight from the pivot, is for the purpose of damping out oscillations of the card and so enable the compass to give a steady reading. For the purpose of comparing the degree of damping of various airplane compasses the "damping constant" has been defined as the ratio of consecutive deflections on the same side of equilibrium when the card is swinging. In order to obtain truly comparable values, it is necessary to make the determination from the same value of initial swing which is usually taken as 45°. It is not proper to take the observations for damping by releasing the card from the required initial deflection. The system must be swinging freely. The degree of damping varies quite widely in instruments of different design. The specifications for one type used during the war stipulated that the damping constant should be not less than 15 nor more than 45.

INSTALLATION AND ADJUSTMENT.

The location of a compass in an airplane is of considerable importance. The instrument must be so placed that the pilot can read it at a glance without changing his position. In order to avoid parallax errors, the line of sight when reading should be in the plane through the lubber line and the pivot. Another consideration in the location of the compass is the presence in the plane of magnetic material which will cause deviations of the compass needles and consequent errors in reading. The effect of stationary metal parts can generally be neutralized by the use of small compensating magnets placed in tubes provided for the purpose. The effect of moving iron parts can not be neutralized, however. The ignition system is also a source of trouble in some planes and may cause errors amounting to as much as 10 or 15 degrees. This difficulty is so great in some cases that the possibility of placing the compass at a distance (on the tail or wings, for instance) has been seriously considered. This condition points to the necessity of carefully considering the requirements of the compass when designing an airplane.

SOURCES OF UNRELIABILITY.

Assuming that the results of laboratory tests are satisfactory and that the effect of magnetic material in the airplane has been completely neutralized by means of the adjusting magnets there are still sources of error under certain conditions in flight. The most important of these sources of unreliability are vibration and rapid accelerations and quick turns.

Because of the fact that the center of gravity of the compass card is below the point of support it acts as a pendulum. The result is that vibrations of the point of support of sufficient amplitude and certain frequencies will cause deflections or in extreme cases actual rotation or "spinning" of the card. If there is excessive pivot friction or the pivot and cup do not have the proper corresponding shapes there may also be deviations or turning due to a sort of "ratchet effect" between the pivot and cup. These are largely overcome in most instruments by proper antivibration mountings which absorb the vibrations. The ordinary gimbal mounting should never be used. It is on account of vibration effects that the pivot is on the card instead of the cup as is customary in marine instruments.

There are two possible effects due to rapid turns and straight accelerations. On quick turns the liquid in the compass may be set into rotation and drag the card with it. This is largely overcome by making the clearance between the card and the bowl large. This reduces the effect, as the liquid in contact with the sides of the bowl is most affected. Another way of preventing the liquid from being set into rotation to an appreciable extent is to make the bowl of approximately spherical shape.

The other effect of turns and acceleration is of a different nature. It is a well-known fact that the direction of the earth's magnetic field is not horizontal. The angle of inclination or "dip" varies in different parts of the world from zero at the "magnetic equator" to 90° at the magnetic pole. In a compass the vertical component is balanced by a small mass on one side of the point of support. For this reason the horizontal component is the only one that exerts a directive force on the card. When an airplane makes a quick turn the resultant effect of gravity and centrifugal force is such that the plane of the card is inclined to the horizontal. The vertical component of the earth's magnetic field then has a component in the plane of the card and this exerts a directive force. The direction in which the card will tend to turn will be the direction of the resultant of the horizontal component of the earth's field and the component of the vertical force which is in the direction of the plane of the card. This causes an error which depends upon the angle of bank and the duration of the turn. This effect is most noticeable when turning east or west from a northerly course and hence is usually termed the "northerly turning error." In this case the north end of the card is drawn down and if it turns rapidly enough may even indicate that a turn has been made in the opposite direction from that actually made. It is readily apparent that under such condition a pilot who is flying in clouds and whose airplane is turned by gusts may think he has turned in the opposite direction from what is actually the case and in attempting to correct his direction will turn still more and may eventually find himself in a spin. Pilots are generally warned against taking a northerly course in flying through clouds.

The actual direction of the card at any instant depends not only on the angle of inclination but also on its period. A compass with a very long period for instance may take so long to respond to the disturbing force that the turn may be completed before there is an appreciable error in its reading. A very long period compass has been strongly recommended by some and undoubtedly is much less affected by the "northerly turning error" than one with a short period but in practice it is so sluggish that it is unsatisfactory for use under ordinary conditions. It is generally considered preferable to use a short period instrument which can be read more quickly after any disturbance and not to rely on its indications when the conditions are known to exist which render it unreliable. A good short period compass in connection with a reliable turn indicator seems to be the best combination so far suggested.

REPORT No. 128.

DIRECTION INSTRUMENTS.

PART III.

AIRCRAFT COMPASSES—DESCRIPTION AND CLASSIFICATION.

By John A. C. Warner.

SUMMARY.

This part contains a brief general treatment of the important features of construction of aircraft compasses, and descriptions of the principal types used in America and in foreign countries. Brief mention is also made of several compasses now in process of development but not in production. At the conclusion of this part will be found a descriptive tabular classification of the various instruments included in the text.

INTRODUCTORY.

There is probably no aircraft instrument which has been the subject of more careful and serious study than has the magnetic compass. Its supreme importance to the navigator of the air accounts for the energy which has been devoted toward its perfection. But with all the attention it has received there still remains the possibility of vast improvement. The working conditions are different and much more severe for the aircraft compass than for its marine prototype and it has been necessary to make important modifications in the latter in order to adapt it to use in aviation. It is the purpose of this paper to discuss the more important characteristic features of construction of the aircraft compass and to describe the principal American and foreign types which have been put into production. In Report No. 131, Section VII of this series, under the title "Aerial Navigation and Navigating Instruments," will be found a discussion of the use of the compass and its errors. Part III of this report, entitled "The Testing and Use of Magnetic Compasses for Airplanes," contains additional material not elsewhere considered.

GENERAL FEATURES OF CONSTRUCTION.

The common types of magnetic compass used on aircraft comprise the following principal parts with elements as noted:

1. Rotating system—
   - *a.* Card.
   - *b.* Float chamber (in liquid damped type).
   - *c.* Magnetic elements.
   - *d.* Bearing member.
2. Bowl—
   - *a.* Container.
   - *b.* Damping medium.
   - *c.* Expansion chamber.
   - *d.* Lower bearing member.
   - *e.* Lubber-line and divided scale.
   - *f.* Observation openings.
   - *g.* Illuminating device.
3. Compensating device.
4. Mounting support.

ROTATING SYSTEM.

Card.—Two principal card forms are noted, the horizontal and the vertical. Not a few instruments combine the two forms and make possible either horizontal or vertical observations. Furthermore, horizontal card instruments are often provided with a prism which allows observations to be made in either a horizontal or vertical direction. When the latter device is employed it is necessary that the card have two scales, one for the direct reading and a second scale with characters inverted and reversed for prism observations. The different systems of marking are described in a following section of this paper. It is important that the characters and divisions of the card be clear and large so as not to require too great an effort on the part of the pilot in making readings. Provision is often made for the use of the compass at night by marking the principal divisions and characters with self-luminous material. This feature is of use only at times of extreme darkness in the immediate vicinity of the compass since the material is ordinarily not sufficiently brilliant to render the markings visible under conditions of dim lighting from another source.

Float.—The card is commonly carried by a hollow water-tight float chamber filled with air and properly supported and guided in a surrounding body of liquid held in the bowl container. In order to minimize the effect of liquid drag, the float chamber is invariably made in the form of a hollow body of revolution (usually a somewhat modified cylinder or ellipsoid) symmetrical about an axis perpendicular to the card. Liquid drag is caused by the tendency of the liquid, when it takes up a swirling or rotative motion with changes of heading of the aircraft, to drag the float with it. The float serves to relieve the pivot bearing, upon which it is supported, of most of the weight of the rotating system. This is important since the magnitude of the vibrational difficulties and frictional error to which the card is subject are largely dependent upon the weight on the bearing. By reducing this weight to a minimum it is also possible to prolong the life of the pivot support which tends to become misshapen under excessive vibration and shock. Several instruments, notably the Creagh-Osborne Type 5/17, have rotating systems so light as to require no float. This, of course, is also true of the dry type of compass in which no liquid is used.

Magnetic elements.—The magnetic elements upon which the compass depends for its action are usually formed of small cylindrical or flat magnetized needles or rods (2 to 12 in number) of hardened alloy steel attached either within the float chamber or upon its lower surface. In certain types they are suspended upon wires below the card. Numerous dispositions of these elements will be noted in referring to the descriptions which follow. The choice and position of these elements is an important factor governing the action of the rotating system. With compasses in which the elements are surrounded by a damping liquid having a corrosive effect upon steel, the magnets are either plated or covered by a noncorroding metal.

The moment of inertia of the rotating system depends to some extent upon the size, number, and position of these elements; the magnetic moment and, in turn, the period of the compass are also dependent upon these factors. It is necessary, then, in practice to so select and mount the magnets that the operating requirements will be satisfied as completely as possible.

Bearing member.—In most of the older types of aircraft compass the rotating system was supported upon a bearing composed of a cup or socket fixed centrally upon the rotating part and resting upon a pivot attached to the bowl. The more recent practice reverses this arrangement and we find practically all of the modern instruments with the pivot on the movable element and the cup mounted below upon a bearing post attached to the bowl. This results in an improvement in the stability of the card.

Brief consideration of the problem will show that the pivot forms a most important element of the compass and must be carefully designed. It is often subjected to violent shocks which it should withstand without breaking or becoming blunted. Otherwise, the action of the compass will become very unsatisfactory. Various materials are used for pivots, including iridium and alloys of iridium, special alloy steels, and agate. One of the most common combinations is agate for the pivot and sapphire for the cup. It is usual practice to select a pivot of

material slightly softer than the cup upon which it bears, otherwise the surface of the cup would be roughened owing to the cutting action of the pivot. The radius of curvature of the pivot point is made smaller than that of the cup.

BOWL.

Container.—The compass bowl acts as a container for the damping medium. It is usually made cylindrical in form in order to reduce as much as possible the swirling or rotation of the inclosed damping fluid when changes of heading take place. One of the instruments later described was designed to overcome this effect to a very large degree by the use of a completely spherical bowl. The usual well designed bowls do not, however, go to this extreme. A generous clearance should be allowed between the card and the wall of the bowl so that the swirling error will be relatively small.

Damping medium.—It is the function of the damping medium inclosed within the bowl and surrounding the rotating system, first (in the case of liquid damping), to reduce the weight of the moving system on the pivot bearing, thus protecting the bearing from shock, and reducing errors due to friction and vibration as described above; second, to damp excessive oscillations of the rotating system and thus improve the stability and action of the card. The amount of damping depends upon the viscosity of the damping medium as well as upon the construction of the movable parts. In air-damped compasses the action of the moving parts against the inclosed air produces the damping effect. In this case it is obviously necessary for the rotating system to present a greater surface to the action of the air than is the case with liquid-damped instruments. The Favé compass, shown in Fig. 18, is an example of this type.

It is important that the damping medium shall be such as to maintain a practically constant viscosity within the range of temperatures experienced in service. Pure alcohol is sometimes used, but it has the disadvantage of more or less rapidly dissolving practically any existing type of paint with which it comes in contact. This action destroys the permanency of the card markings and results in a formation of a deposit of sediment upon the bearing, thus introducing a friction error. Alcohol diluted with some other liquid is most often used, a common mixture being 30 per cent alcohol to 70 per cent distilled water. Colorless, acid-free kerosene is also employed in certain compasses. The liquid is introduced into the bowl through a tapped and plugged filler hole in the wall.

Expansion chamber.—The volumetric changes in the body of liquid confined within the closed bowl are cared for in two different ways. The device most frequently employed is an expansion chamber composed of one or more thin metal diaphragm boxes, similar to those used in aneroids, communicating with the interior of the bowl. In certain compasses the expansion is compensated by the use of a single corrugated diaphragm forming the base of the bowl. A second method for overcoming this difficulty is to mount a small hollow chamber at the top of the bowl so that the excess liquid may flow into it from the bowl when expansion takes place. This device also serves as an air trap to which bubbles from the liquid may rise, thus avoiding any objectionable action resulting from their presence in the liquid.

Lower bearing member.—A polished sapphire or garnet cup held in a brass socket is most often employed as the lower bearing member in which the pivot of the rotating element rests. In many instances the jewel is set in a socket which is free to move a short distance vertically against a shock-absorbing spring. A second method for relieving the shock effects to some extent is to set the jewel against a layer of rubber or cork which acts as a cushion. The bearing post which supports the cup is fixed to a bridge member attached to the base of the bowl.

Lubber-line and divided scale.—In order that accurate observations may be made of the position of the compass card relative to the bowl, a reference marking or lubber-line is usually placed inside the bowl at the side where the observations are to be made. The instrument should be mounted so that a line passing through this lubber-line and the pivot of the compass is parallel to the longitudinal axis of the aircraft. A divided scale (starting from a point above the lubber-line) is often found at the upper rim of the bowl above the observation glass. A sliding index or pointer is used in conjunction with this scale in course setting and taking bearings.

Observation openings.—Observations of the horizontal card are made through a cover-glass held by a bezel ring which joins it to the upper rim of the bowl. The joint is made tight by the use of a rubber gasket. Vertical card observations are made either through a small glass-covered opening in the side of the bowl or, in certain compasses, through the cylindrical glass bowl container itself. The latter arrangement has the advantage of magnifying the divisions and characters of the card, due to lens effect caused by the convexity of the glass.

Illuminating device.—Modern compasses are provided with a miniature electric lamp so mounted in a shielding socket as to illuminate the card. In some instances a special opening (covered by ground glass to prevent glare) is provided either in the base or side wall of the bowl so that the light rays may enter. Designs which do not allow the light to fall directly upon the card should provide for illumination either by transparency of the card or by reflection from properly painted interior walls of the bowl.

COMPENSATING DEVICE.

Owing to the presence of magnetic fields set up by the power plant and auxiliaries of the aircraft system it is necessary to provide a compensating device to minimize the disturbing influences of these fields. Small bar magnets suitably placed relative to the magnetic elements of the card are employed to neutralize the effect of objectionable extraneous fields. A common form of compensating device is that consisting of a vertical slotted or grooved rod mounted directly below the bowl and carrying two sliding collars, adjustable in a vertical direction only, thus making it possible to vary the distance between the card and the correcting bar magnets (usually two in number) which are carried by each collar. The correcting magnets attached to one of the collars are secured with their longitudinal axes parallel to the fore-and-aft line of the aircraft while the axes of the second set extend parallel to the athwartships line. The compensation is governed by the number, strength, and position of these members.

Another form of compensating device consists of a holder mounted upon the bowl directly above or below the card (and sometimes in both positions), centered relative to its axis. The holder contains a fore-and-aft and an athwartships tube in which the small correcting magnets are placed. In this case the amount of compensation is governed by the number and strength of the magnets employed.

A third device for compensation consists of an arrangement of four tubular holders so mounted on the sides of the bowl or bowl housing that the correcting elements may be properly placed in fore-and-aft and athwartships positions. This disposition as well as the preceding one have the advantage of compactness and ease of manipulation.

MOUNTING SUPPORT.

In view of the excessive vibrations existing on aircraft and their ill effect upon the compass it is advisable to provide the instrument with a mounting which will reduce the influence of vibration as much as possible. This is usually accomplished in one of three ways: first, by inclosing the bowl in a housing lined with antivibrational material, such as felt or horse hair, to overcome the vertical shocks and with flat metal springs properly placed to relieve vibration and shock in a horizontal plane; second, by mounting the bowl upon supports or cushions of felt or fibrous material and with spiral metal springs to oppose horizontal effects; third, by attaching the bowl mounting to the aircraft with rubber cushions at the points of attachment. Certain deviations from these usual arrangements will be noted by referring to the descriptions and photographs of the individual instruments. Certain of the older types and many of the present foreign compasses are swung in gimbals. This practice is not to be recommended for airplane installations but is not without merit for airship service where excessive vibration and violent accelerations are not as prevalent.

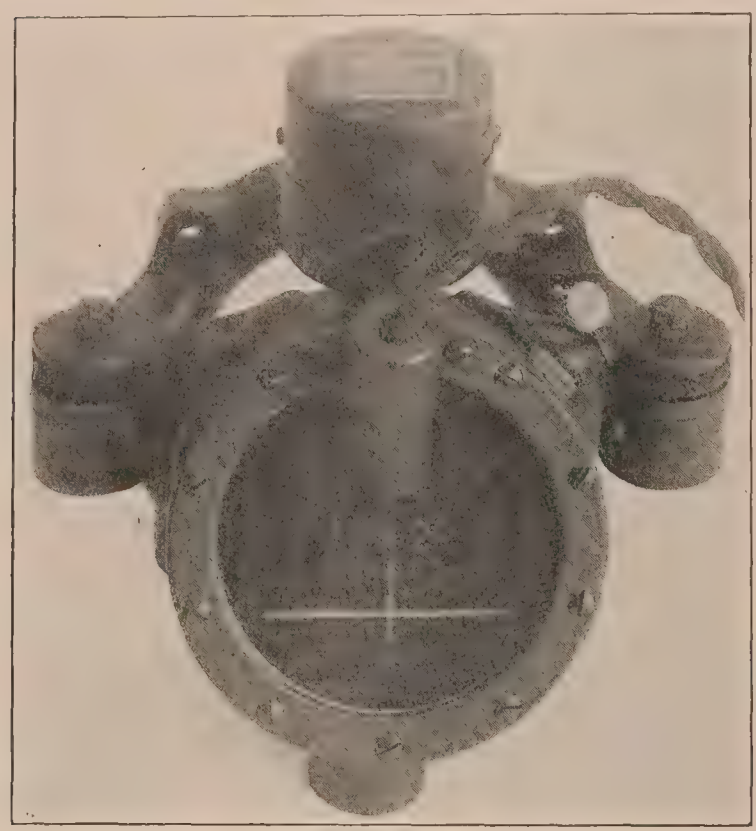
In the pages which follow will be found illustrations and descriptions of the principal types of aircraft compasses which have been used in America and abroad. Some of the most important details of the instruments described have been classified in tabular form at the conclusion of the paper.

DESCRIPTIONS OF AMERICAN COMPASSES.

GENERAL ELECTRIC AIR COMPASS TYPE B.

The liquid damped compass (acid-free kerosene damping-fluid) shown at the left of figure 1 is widely used in the military aircraft of this country. Many of its features are patterned after those of the original British Creagh-Osborne 5/17 compass, and its appearance closely resembles that of the British instrument.

The card has the form of a truncated cone with smaller diameter of 43 millimeters at the top and flaring to a diameter of approximately 54 millimeters at the bottom. (See illustration of rotating element and bearing post at right of Fig. 1.) The markings are found upon the exterior face of the cone, which has a depth of 10 millimeters along a generating element. The card is lettered at the cardinal points with luminous material and divided each 10°, with numerals marking each third division, at which luminous dots are also painted. Four light spokes, extending from the pivot mounting at the center, serve to support the card ring.



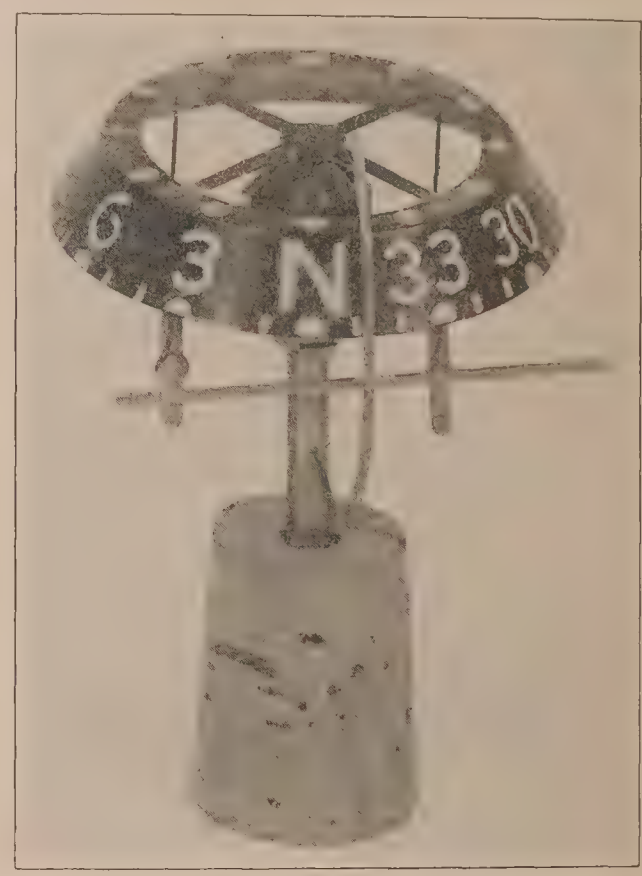


Fig. 1.—General Electric air compass, Type B.

The pivot is of special alloy and rests in a sapphire cup mounted upon a bearing post attached to the lower surface of the bowl. The pivot is prevented from leaving its socket by a split cage, the two halves of which are screwed to the bearing post, and their upper edges bent inward so as to overhang a bell-shaped hood member just above the pivot on the rotating element. The two bar magnets of tungsten steel (50 millimeters long and 29 millimeters between centers) are suspended upon four wires extending from the card ring.

A vertical wire painted with luminous material is attached near the lower extremity of the bearing post and extends upward toward the front of the bowl to serve as a lubber-line. A horizontal wire is attached to the lubber-line wire and forms a reference relative to the lower line of the card thus making it possible to use the compass as an inclinometer.

The main body of the bowl is spherical in shape and has an interior diameter of approximately 85 millimeters. The bowl projects toward the front in a cylindrical extension (with axis inclined at approximately 26° above the horizontal) which is capped by the glass observation window (68 millimeters diameter) held in place by a bezel ring and inclined backward at an angle of approximately 26° from the vertical. A miniature incandescent lamp in shielding socket is attached to the bezel ring at the top and provides illumination for the card at times when the luminous markings are not clearly visible.

Mounted at the top of the bowl and communicating with the interior is a combination air trap and expansion chamber. A filler plug is provided in this cylindrical chamber as well as in the back wall of the bowl itself. Liquid expansion is cared for by this device and any bubbles which may form in the damping fluid rise to the surface in the chamber and produce no objectionable effects. The compensation chamber surmounts this air trap and consists of two brass tubes (one along the fore-and-aft line and the other athwartships) of diameter great enough to accommodate several correcting bar magnets. The compensating device is covered by a brass cap which prevents the magnets from slipping out.

Three lugs as shown in the illustration project from the bowl and rest upon antivibrational supports projecting from the main mounting bracket. Each of the three lugs rests upon a felt washer acting as a cushion for vertical shocks, while a flat spiral spring fastened to a bolt from the lug and centered in a cylindrical cup at the under side of each bracket lug serves to relieve vibration effects in a horizontal plane.

The compass above described has a period of 12 seconds and a damping constant of 20. The card is free to swing when the compass bowl is tilted 30° from its normal position. The instrument weighs 2.5 pounds.

AMERICAN MODIFICATIONS OF THE CREAGH-OSBORNE AIR COMPASS, TYPE 5/17.

The production during the war of the type B compass previously described exceeded that of the other modifications of the Creagh-Osborne air compass, type 5/17. However, several other instruments of this same general type were produced in limited quantities.

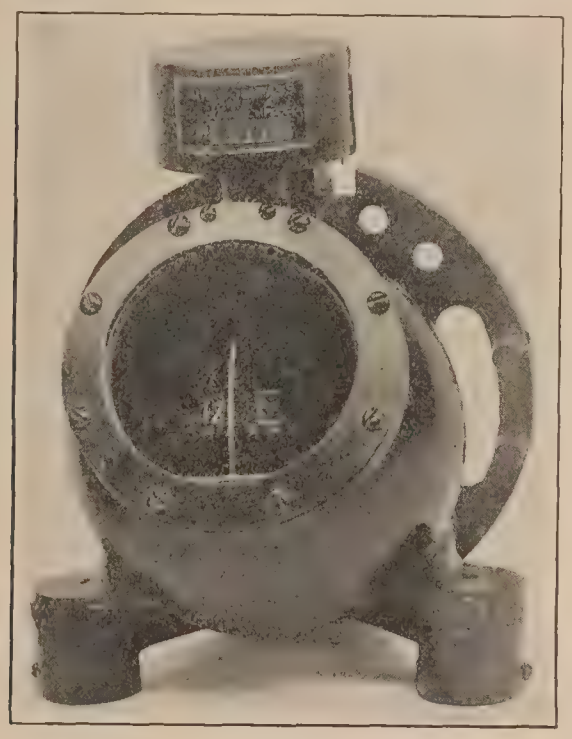


FIG. 2.—Air compass, Type A.

FIG. 3.—Navy standard compass No. 1.

A compass known as the Creagh-Osborne air compass Mark VIII, so nearly duplicates the British instrument later described that no space will be devoted to its treatment at this point.

The instrument shown by figure 2 is known as the type A. The mounting is different from that of the British instrument and it is somewhat smaller in size. The illustration shows the instrument without the incandescent lamp illuminating device in place.

Several other modified instruments of less importance than those already mentioned were manufactured in very limited numbers.

NAVY STANDARD COMPASS NO. 1.

The Sperry aircraft compass Mark XVI, also known as the Navy standard compass No. I (fig. 3), is a type widely used in aircraft. The large flying boats and military bombing airplanes ordinarily carry an instrument of this general type as standard equipment, while a somewhat smaller model, known as the Navy standard compass No. II, is used on the smaller

airplanes. The instrument to be described has a 76-millimeter card while the smaller model has a card 50 millimeters in diameter.

The card of the Navy standard compass No. I is of the combination horizontal and vertical type with divisions each 5°. Numerals mark the 30° points and the cardinal points are lettered. The top or horizontal surface has a diameter of approximately 76 millimeters, while the vertical card surface, cylindrical in form, has a diameter of 63 millimeters and a depth of 19 millimeters. These cards are carried by a float chamber with the magnetic elements inclosed within.

The alloy pivot mounted in a recessed cavity in the lower surface of the float rests upon a sapphire cup mounted upon a bearing post in a shock-absorbing spring socket. The post is attached centrally upon a bridge member at the center of the bowl base. A vertical curved lubber-line wire is provided both at the front and rear of the bowl, one for vertical and the other for horizontal reading. The markings upon the horizontal card surface start 180° from those of the vertical card so that the readings of the former are taken with reference to the back lubber-line while those of the latter are made with the forward line as reference.

A cylindrical ring of glass (109 millimeters inside diameter and 56 millimeters in height) forms the vertical walls of the bowl and rests upon the base casting where a rubber gasket is used to make a tight joint. Viewing the vertical card through the cylindrical bowl causes magnification of the card due to lens effect. The bowl ring is surmounted by an aluminum ring with four lugs extending from the sides; the top surface of this ring bears 5° divisions with numerals at each 10° point. Within this outer member is mounted a rotatable ring carrying sights which are intended for use in taking bearings and in making drift observations. The cover-glass is clamped against a rubber gasket resting upon the top surface of the cylindrical bowl glass by means of four bolts passing through the four lugs and corresponding projections of the aluminum base casting.

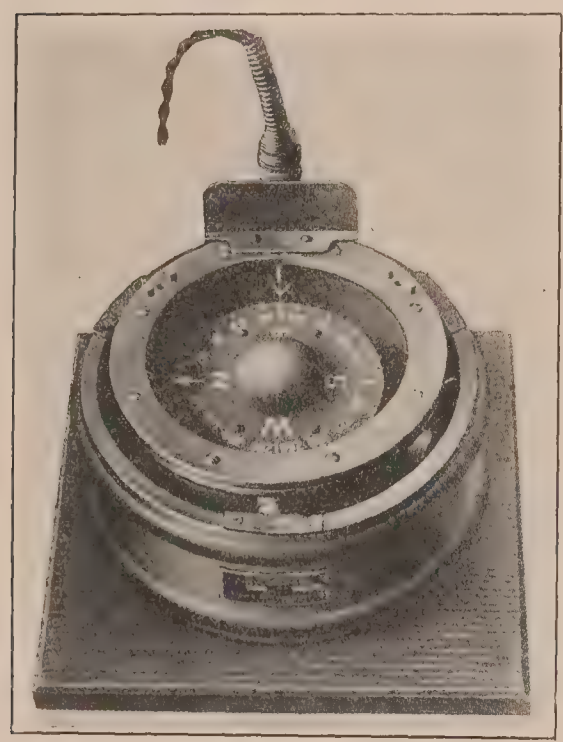


FIG. 4.—Creagh-Osborne air compass (Sperry) Mark II.

A hole through the base communicates with a single diaphragm expansion box contained within a protective housing. This housing also serves the purpose of an auxiliary base plate and carries the compensation box at the center. The latter is of the ordinary type with a fore-and-aft and an athwartships tube acting as holders for the correcting magnets. A filler hole passes through the base casting. The instrument is so designed that it can be mounted upon either a horizontal or vertical surface by using the proper mounting plates.

The period of the compass is approximately 20 seconds and the damping constant 5. The weight is approximately 3.7 pounds.

CREAGH-OSBORNE AIR COMPASS (SPERRY) MARK II.

The compass known as the Creagh-Osborne air compass (Sperry) Mark II (fig. 4) is of the liquid damped type (alcohol mixture) with a horizontal card.

The mica card (76 millimeters diameter), divided each 5° and with luminous markings and numerals at each 10° point, is carried by a float chamber of usual form. The alloy pivot is mounted in a recessed cavity in the lower surface of the float and rests upon a sapphire cup held in a cup socket. This socket in turn rests upon a shock-absorbing spring in the hollow section of the bearing post. The latter is held by a short bridge member at the center of the base of the bowl.

The bowl has an inside diameter of approximately 108 millimeters and a depth of 42 millimeters. A luminous wire lubber-line is found at the back of the bowl at the point where a window is mounted in the wall to allow for illumination from an incandescent lamp attached outside. A diaphragm expansion chamber is centrally fixed to the underside of the bowl.

The bowl rests upon three antivibrational rubber-covered rods extending from the walls and held by corresponding suspension cradles attached to the inner surface of a protective housing which surrounds the bowl. This housing is lined with horsehair so as to provide a cushion for the bowl. A filler hole extends through the side wall of the latter. The instrument is designed to be mounted upon a horizontal surface by means of four bolts in the base of the housing with rubber shock-absorbing collars attached.

Compensation of the type shown in the illustration is effected by means of a compensation block made of wood, with holes to hold the necessary correcting magnets. This block is fastened either above or below the card with its vertical axis extended coinciding with that of the bearing post. Another model of this instrument has four compensation tubes attached to the outer wall of the housing, one pair being parallel to the fore-and-aft line of the aircraft and the other athwartships. The required number of magnets are placed within these tubes.

The Mark II instrument has a period of approximately 18 seconds and a damping constant of 10. It weighs approximately 3.3 pounds.

PENTZ COMPASS.

The design of the Pentz liquid (kerosene) damped compass (fig. 5) is a departure from the usual practice. The entire compass system, including card, bearing, and bearing post, is held suspended by a float chamber in the damping liquid within the spherical bowl.

Extending downward from the float (60 millimeters diameter) and attached at points diametrically opposite each other are two light flat rods, at the lower extremities of which is attached a horizontal wire ring approximately 92 millimeters in diameter. The bearing post, centrally located and supporting a sapphire cup at its upper extremity, is suspended from two wires, extending downward from the ring to the lower end of the post. This arrangement will be understood by referring to the illustration at the left of figure 5.

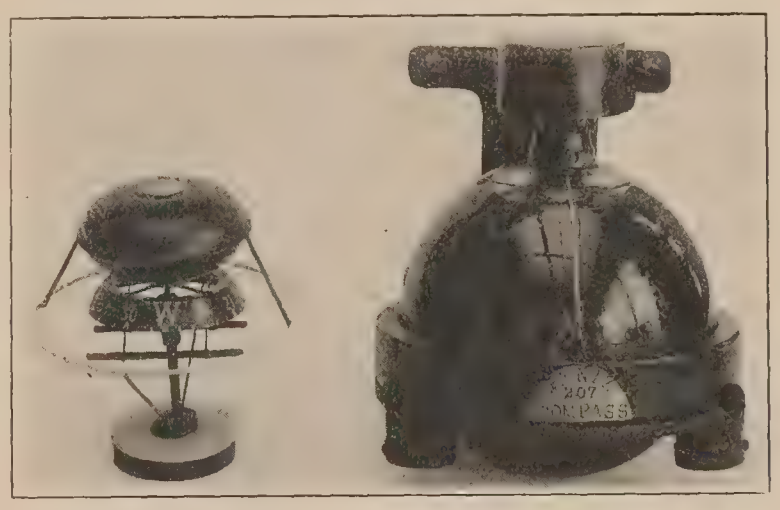


FIG. 5.—Pentz compass.

The card and magnetic element are very nearly identical with those employed in the General Electric type B compass previously described, the principal dimensions being the same. The point of difference lies in the manner of preventing the alloy pivot from leaving the cup socket. The ball and socket cage of the type B instrument is replaced in the Pentz by a shallow pan-shaped member attached to the upper surface of the card frame and restricted in vertical motion by coming in contact with the lower surface of the float before the card has lifted far enough for the pivot to leave the socket in which the jewel cup is held.

The designer of this instrument has endeavored to overcome the errors due to swirling, by inclosing the system described above in a spherical bowl the upper half of which (95 millimeters inside diameter) is of glass and the lower half of brass. The apparent size of the card when viewed through the sphere is magnified. A tight joint between the hemispherical halves is secured by means of a rubber gasket and a screw collar threaded to a shoulder forming part of the lower half. This shoulder overhangs the ring-shaped base of the mounting bracket and three felt, spiral spring, antivibrational supports similar to those used in the Creagh-Osborne compass support the bowl. A combination air trap, expansion chamber, and compensation box similar to the Creagh-Osborne device surmounts the bowl. A wire lubber-line is mounted both inside and outside the bowl, and parallax is avoided by sighting past the two lines.

This instrument, in addition to its capacity as a compass, is useful as an inclinometer. When used in this service a circular spot painted centrally upon the top float surface is observed with reference to a small circle cut upon the glass with the top pole of the sphere as center. A second circle engraved upon the glass at the equator is used with reference to the 92-millimeter wire ring which forms part of the floating system within the bowl.

The Pentz compass has a period of about 12 seconds and a damping constant of 40. It weighs about 3.3 pounds.

DESCRIPTIONS OF BRITISH COMPASSES.

CREAGH-OSBORNE AIR COMPASS, TYPE 5/17.

The Creagh-Osborne air compass, type 5/17 (fig. 6) has been one of the most widely used aircraft compasses. It is best adapted to service on scout planes where the advantages of a quick period instrument are desired. The compass is of the liquid damped type (alcohol of 0.84 specific gravity damping fluid) and is equipped with a card totally different from that employed in most compasses.

The card consists of a pan-shaped thin section of white metal (48 millimeters greatest diameter), which with the magnetic elements and pivot is so light as to require no float. The horizontal base surface of the card is cut away so as to leave four spoke members extending outward from the center to support the rim; the lower edge of the latter is inclined inward toward the pivot at an angle of 30° from the vertical. The two bar magnets (40 millimeters length) are suspended below the card (25 millimeters between centers) upon wire suspensions. The agate pivot is mounted upon a brass stem attached at the center of the card and rests in a sapphire cup held on a central post. A vertical adjustable wire extends from the inside

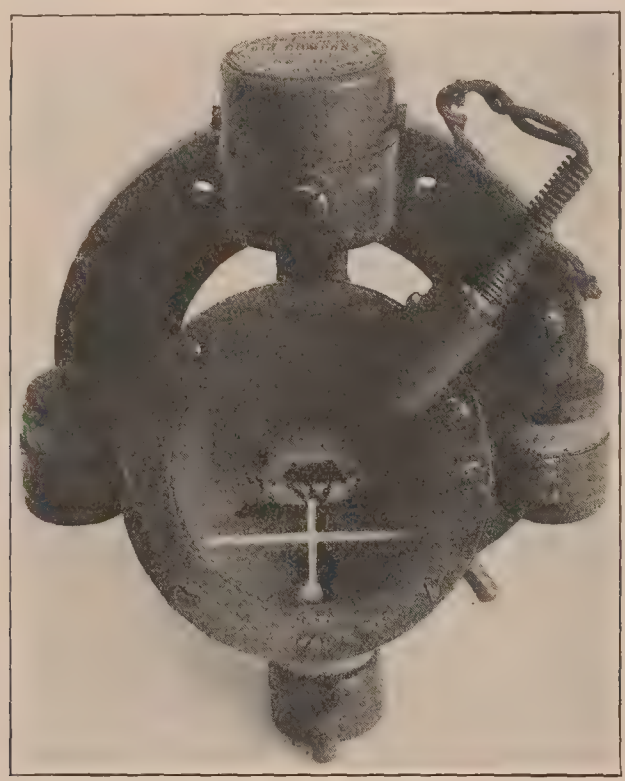


FIG. 6.—Creagh-Osborne air compass, Type 5/17.

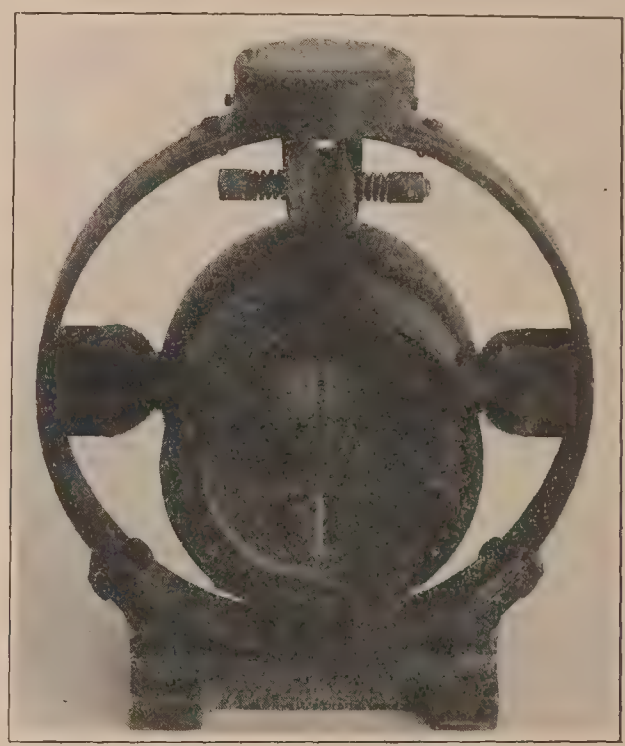


FIG. 7.—Creagh-Osborne aero compass, Type 259.

upper surface of the bowl to within a short distance from the top center of the card and prevents the latter from leaving the cup bearing. The lubber-line fixture is mounted inside the bowl, as shown by the illustration.

The bowl is approximately spherical (80 millimeters inside diameter) except at the front where a short cylindrical projection extends inclined at an angle of 26° above the horizontal. This extension is capped by the cover-glass inclined back from the lower edge at an angle of 26° from the vertical. A nonleak joint is made between cover-glass and bezel ring by the use of a rubber gasket. An air trap and also a chamber for holding the compensating magnets in proper position are mounted at the top of the bowl. The air trap is arranged so as to collect any air bubbles which may form in the liquid and to allow for liquid expansion. Two filler holes are provided, one upon the air trap and the other upon the bowl itself.

The bowl is fitted with three lugs which hold it upon the supporting members of the mounting bracket. Felt washers at the points of attachment between the bracket lugs care for vertical vibration, while flat spiral springs are provided at the points of support to relieve the horizontal vibrations. A small electric bulb mounted with suitable shield upon the verge ring provides illumination for the card.

The Creagh-Osborne air compass has a period of from 8 to 10 seconds and weighs 2.8 pounds.

CREAGH-OSBORNE AERO COMPASS, TYPE 259.

The Creagh-Osborne aero compass, type 259 (fig. 7) is of the vertical card, liquid-damped type (alcohol mixture damping liquid). The card is formed of a mica band (50 millimeters in diameter and 13 millimeters in height) divided at 10° intervals and with luminous numerals every 30°. The cardinal points are lettered with luminous material. The card is carried by a float chamber with recessed cavity in its lower surface where the agate pivot is mounted upon a brass stem. The pivot rests upon a sapphire cup held in a socket at the upper extremity of the bearing post, the latter being mounted upon the bowl base. The float is restricted in vertical motion by a wire extending from the top of the bowl with a small hood at its lower extremity. This hood comes directly over the center of the float and prevents it from lifting far enough for the pivot to slip from the cup socket.

The main bowl chamber is approximately spherical (70 millimeters inside diameter) with a forward extension of circular section to which the vertical glass is attached, and with diaphragm expansion chamber at the back in a protective housing. A luminous lubber-line and a horizontal reference line are mounted inside the bowl, as shown in the illustration. The clearance between card and lubber-line is 12 millimeters.

Four lugs attached to the bowl provide for mounting upon the surrounding bracket ring with corresponding mounting clips. Felt washers and light compression springs are used at the points of attachment to take up vibration. A cylindrical box to contain compensating magnets is attached at the top of the ring, while a pedestal properly drilled for mounting is bolted at the lower side.

The compass described above has a period of approximately 25 seconds and weighs 2.1 pounds.

A much larger model of this instrument has been constructed and is known as type 256.

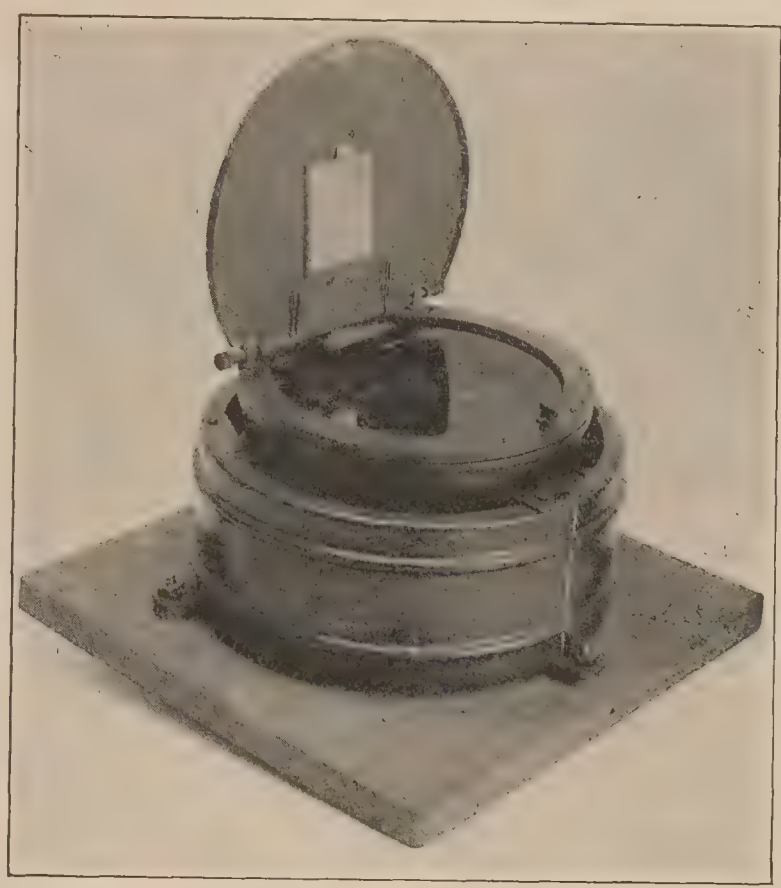


Fig. 8.—Creagh-Osborne aero compass, Type 253.

CREAGH-OSBORNE AERO COMPASS, TYPE 253.

The Creagh-Osborne aero compass, type 253 (fig. 8), is designed for use on the larger types of aircraft. This compass is liquid damped (alcohol damping fluid) and is provided with a mica horizontal card 112 millimeters in diameter mounted upon a suitable float chamber. An agate pivot in a recessed cavity in the lower float surface rests upon a sapphire cup on the bearing post.

The bowl is of brass, has an inside diameter of 150 millimeters and depth of 80 millimeters. A diaphragm expansion chamber 113 millimeters in diameter is attached at its base. The card clears the lubber-line at the back of the bowl by approximately 10 millimeters. The observation glass, held by a verge ring with rubber gasket, caps the bowl and a hinged cover serves as a protection for the glass.

The inner bowl above described is suspended upon four corrugated shock-absorbing springs held inside a surrounding brass housing-cylinder approximately 200 millimeters in diameter. The space between outer and inner bowl is packed with horsehair intended as a shock-absorbing medium. The outer bowl is provided with lugs for mounting.

The instrument has a period of approximately 25 seconds and weighs about 7 pounds.

R. A. F. PILOT'S COMPASS, MARK II.

The compass shown at the right of figure 9 is a liquid-damped vertical card instrument known as the R. A. F. Pilot's Compass, Mark II. The circular metal frame holding the vertical celluloid card ring (68 millimeters in diameter and 17 millimeters in height) is attached to the float chamber by means of four light L-shaped spokes which also act as damping vanes. The card divisions are marked upon the interior surface of the ring and "back readings" are taken with reference to a vertical curved wire attached at the back of the bowl which serves as a lubber-line. An agate pivot is mounted at the center of a recessed cavity in the lower float surface and rests upon a sapphire cup mounted upon a suitable bearing post. The pivot is prevented from leaving the cup, when the compass is in extreme positions and under conditions of excessive vibration, by means of a light wire arm extending from the interior wall of the bowl to within a very short distance from the top center of the float.

With a view to reducing errors in turns due to swirling of the compass liquid, the bowl of the R. A. F. Pilot's Compass is made as nearly spherical as possible. In order to completely carry out this feature the interior surface of the observation glass is concaved on a radius equal to that of the bowl proper. This has the disadvantage of reducing the apparent size of the card, when viewed through the glass, because of the lens effect caused by the concavity. This difficulty is overcome to some extent by the use of a damping liquid (zylol) possessing a high refractive index. The cover-glass has a diameter of 78 millimeters, slopes backward from the lower side at an angle of about 55° from the vertical, and is held in place by an ordinary bezel ring with rubber gasket at the joint. A miniature lamp mounted upon the bezel in a shielding socket illuminates the interior of the bowl. A flexible diaphragm 100 millimeters in diameter covers the expansion chamber attached below the bowl; the expansion chamber communicates with the bowl through two small holes in the spherical wall. Two filler plugs are provided at the top of the bowl.

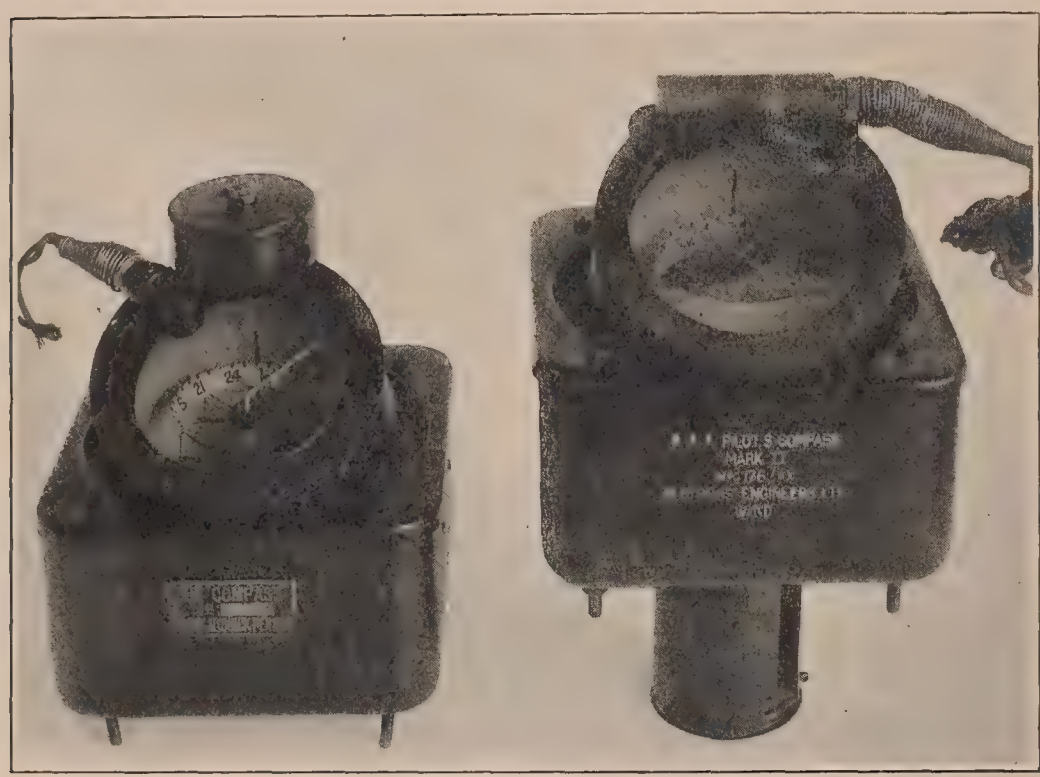


FIG. 9.—R. A. F. pilot's compass, Mark II. Air compass, Mark II (quick period).

The bowl is mounted in a protective housing by means of four lugs which rest upon felt washers to relieve vertical shocks. Four strips of phosphor bronze attached to the sides of the bowl with their extremities touching the inner surfaces of the housing case react against vibrational motion in a horizontal plane.

The compensating device extends beneath the bowl and consists of two vertical series of horizontal tubes (one series fore-and-aft and the other athwartships) in which the correcting magnets may be placed. The latter are prevented from slipping out by a cylindrical sleeve which surrounds them.

Inasmuch as this instrument was designed to be free from the northerly turning error, characteristic of quick-period instruments, it is heavily damped and has a long period of swing varying in different models between 40 and 60 seconds, 54 seconds being a common value. The weight of the compass is about 4.9 pounds.

AIR COMPASS, MARK II (QUICK PERIOD).

The air compass, Mark II (quick period) shown at the left in figure 9 is identical with the R. A. F. Pilot's Compass, Mark II, described above, insofar as the bowl and mounting are concerned. The magnetic system and compensating device differ materially, however.

The magnetic system somewhat resembles that employed in the Creagh-Osborne 5/17 instrument and is so light as to require no float. The card has the form of a truncated cone with a diameter at the bottom of 51 millimeters tapering upward to a diameter of 63 millimeters at the top. This conical surface of thin sheet brass bears the card markings and has a height of 10 millimeters measured along a generating element. It is attached to the pivot hub by four brass wires. The pivot is of agate and rests in a sapphire cup mounted upon a bearing post similar to that of the instrument above described. The two bar magnets, 50 millimeters in length, are suspended below the card (30 millimeters between centers) upon light wires. The rotating system is prevented from lifting off the bearing by a wire retaining-arm similar to that used in the R. A. F. pilot's compass.

The compensating device is mounted above the card and consists of two brass tubes (one fore-and-aft, the other athwartships) of sufficient diameter to allow several correcting magnets to be inserted. The latter are prevented from falling out by a rotating brass sleeve held in position by a thumb nut.

This instrument is rather heavily damped but has a much shorter period of swing than the pilot's compass. The disposition of the compensating device makes it better adapted to mounting on certain planes. The weight of this instrument is approximately 4.9 pounds.

CAMPBELL-BENNETT APERIODIC COMPASS, TYPE 6/18, MARK II.

Among the most interesting of the more recent compasses is that developed by G. R. C. Campbell and G. T. Bennett, of the Admiralty Compass Observatory, Slough, England, and known as the aperiodic compass (figs. 10 and 11). With instruments of the ordinary type, a deflection of the rotating system from its position of rest is followed by a motion of oscillation of the system. In cases where the damping coefficient is small, the oscillation continues for some time on either side of the equilibrium position with ever diminishing amplitude. By increasing the damping coefficient, however, it is possible to make the motion "aperiodic," i. e., the system returns to equilibrium without oscillation. The Campbell-Bennett compass is designed to possess this characteristic. The following description is intended to give an idea of the important features of the instrument:

FIG. 10.—Campbell-Bennett aperiodic compass, Type 6/18, Mark II.

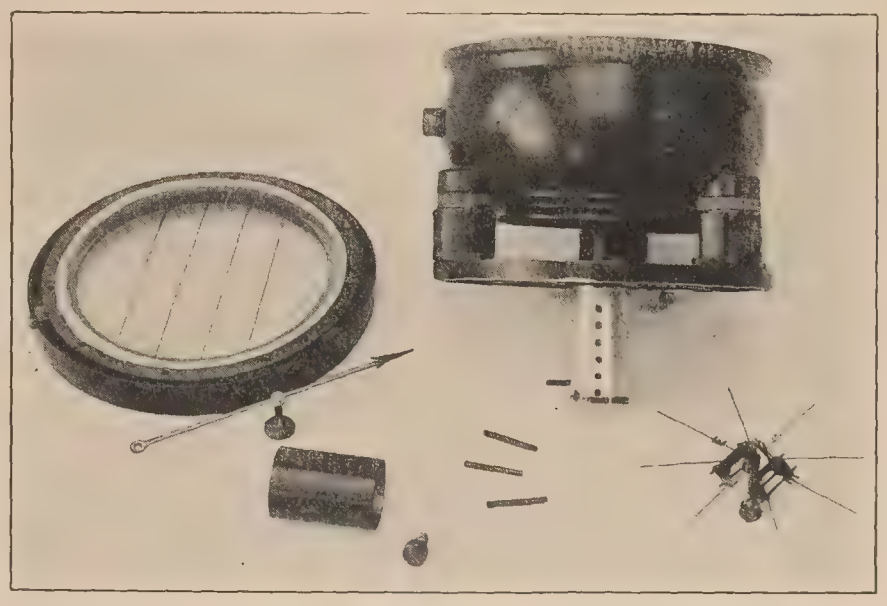


FIG. 11.—Campbell-Bennett aperiodic compass, Type 6/18, Mark II.

Referring to figure 11, the rotating element is seen to consist of a "spider" of eight wires radiating from a main hub member at the center of which the agate pivot is mounted (pivot point is in the plane of the wires). These wires are of brass or copper, have a diameter of

approximately 0.47 millimeter, and extend along the radii of a circle approximately 90 millimeters in diameter. The wires are equally spaced. Sheet-metal letters attached to the proper wires designate the cardinal points. The six bar magnets are suspended in suitable frames below the "spider," three at either side of the pivot. The end pieces of the suspension frames supporting these magnets are triangular in shape and hold each set of three magnets so that the individual bars are approximately 7 millimeters between centers. The object of a rotating element constructed in this way is obviously to bring about the condition of high damping resistance without appreciably increasing the moment of inertia of the system.

The pivot of the rotating element rests upon a sapphire cup held at the top of an adjustable bearing post, which is mounted centrally upon a bridge member spanning the diaphragm expansion-base of the bowl. The jewel cup itself rests upon a small piece of cork which acts as a cushion. In one of the models (fig. 10) the pivot is prevented from leaving the cup bearing by means of a flanged ring attached to the bearing post and overhanging a smaller ring forming part of the magnet frame. In a second model (fig. 11) a hood-shaped member extends downward directly above the center of the rotating element from a wire bridge forming the lubber-line and spanning the top of the bowl.

The bowl is of brass, cylindrical in form, and has a depth of 52 millimeters and a maximum inside diameter of approximately 136 millimeters. A miniature electric bulb is attached at one side and projects its rays through a ground-glass window in the side so as to illuminate the interior of the container. A filler plug is provided at one side. In addition to the expansion base a series of three diaphragm boxes to care for liquid expansion is attached underneath the bowl. The bowl is covered by a glass crystal 117 millimeters in diameter which is surmounted by a rotatable bearing plate with four parallel wires extending in a north-south direction and spaced approximately 20 millimeters apart. The bearing plate is graduated in 2° intervals, with each 10° interval numbered and with the cardinal points lettered.

Three lugs spaced at equal angular intervals around the base of the bowl rest upon shock-absorbing washers of fibrous material mounted upon cylindrical hollow pedestals screwed to a circular base casting of aluminum. The above-mentioned shock-absorbing washers care for vertical vibrations, while spiral brass springs inside the cylindrical pedestals are attached to bolts from the mounting lugs so as to relieve horizontal vibration. A vertical compensating pillar threaded into the base is drilled with two series of holes (one fore-and-aft, the other athwartships) in which the small compensating magnets may be placed at suitable distances from the magnetic element to provide the necessary compensation. A brass sleeve slides over the compensation tube and serves to hold the magnets in place.

The instrument as described weighs 6 pounds. It requires about 15 seconds for the magnetic system to come to rest after a deflection of 45° from the equilibrium position.

DESCRIPTIONS OF FRENCH COMPASSES.

AÉRONAUTIQUE MILITAIRE COMPASS—NONCOMPENSATED TYPE.

The Vion Aéronautique Militaire compass (fig. 12) is of the noncompensated type. This instrument is liquid damped (alcohol mixture) and has a horizontal card. The latter, in the form of a ring of composition material resembling hard rubber, has a diameter of 70 millimeters and bears luminous markings at 10° intervals, with numerals of luminous material at the 30° points. The letters marking the cardinal points are also luminous. The card is carried on a float chamber with two magnets inclosed in brass tubes attached to the under surface. The hardened alloy pivot is mounted upon a bridge member spanning the diaphragm expansion base of the bowl, while the sapphire cup is set in a recessed cavity in the lower surface of the float. The diaphragm base is protected from mechanical injury by a metal cap which covers it.

The bowl has a depth of 46 millimeters and an inside diameter of 92 millimeters, thus allowing a clearance of 11 millimeters between the card and the wall of the bowl. A lubber-line is suitably mounted in the bowl. A tapped and plugged filler hole in the side allows for replenishment of the liquid when leakage occurs or bubbles form. A nickeled rim divided in degrees and properly marked surmounts the bowl. An adjustable index sliding along the rim is provided for the convenience of the pilot in setting his course.

The suspension of this instrument is by means of gimbals and an ordinary yoke mounting bracket. The compass is constructed largely of brass and weighs about 3.3 pounds. It has a period of approximately 17 seconds and damping constant of 9.

AÉRONAUTIQUE MILITAIRE 1 COMPASS—COMPENSATED TYPE.

The Aéronautique Militaire 1 compass (fig. 13) is one of the most recent of the French instruments. It is of the liquid-damped (alcohol mixture), horizontal-card type. Vertical reading is made possible by use of the prism seen in the illustration.

The card, in the form of a mica ring 75 millimeters in diameter, bears two sets of divisions, one set erect for direct reading and the other set inverted and reversed for observations through the prism. The divisions for direct reading are spaced at 5° intervals with numerals at the 20° points, while the divisions for prism readings are spaced at 2½° intervals with numerals at the 10° points. The cardinal points are marked with luminous letters.

The card is mounted on a float chamber of usual design with two magnets attached to the lower surface as in the instrument previously described. This compass differs from the former

FIG. 12.—Aéronautique militaire compass, noncompensated type.

FIG. 13.—Aéronautique militaire 1 compass, compensated type.

Vion instrument, however, in that the pivot (jewel in brass stem) of this model is mounted in a recessed cavity in the lower float surface, while the cup of the former type was carried by the float. The jewel cup is held upon a rubber cushion in a socket upon a bearing post mounted at the center of the bowl base.

The bowl has a depth of 37 millimeters and an inside diameter of 106 millimeters. A shelf extending from the wall of the latter bears the luminous lubber-line. A nickeled rim divided in degrees and properly marked surmounts the bowl and observation glass. An adjustable index sliding inside the rim is provided for the convenience of the pilot in setting his course. A filler hole passes through the bowl base and at one side of the latter a ground-glass window is set directly above a miniature lamp held in its socket upon a protective cover plate just below the base. This plate which is screwed to a ring extension of the bowl also covers the diaphragm expansion chamber with which the instrument is equipped.

A vertical slotted compensation column fastened centrally upon the protective cover plate carries two adjustable sliding collars in which the correcting magnets are secured. This compensating device is covered by a tube threaded to a collar on the cover plate.

The compass bowl rests in a mounting ring provided with two hubs which are supported upon rubber shock-absorbing disks fixed upon the arms of a yoke mounting-bracket.

The compass as described has a period of approximately 16 seconds and a damping constant of 7.5. Its weight is approximately 6 pounds.

MAUVE COMPASS—NONCOMPENSATED TYPE.

The Mauve noncompensated compass (fig. 14) is of the liquid-damped (alcohol mixture) horizontal-card type. The card, having a diameter of 55 millimeters, marked with nonluminous material and with circumferential divisions placed at 5° intervals, is carried by a small float member. The sapphire bearing cup is set in a small recessed cavity at the under side of the float and rests upon a hardened alloy pivot mounted centrally upon a bridge member extending across the bottom of the bowl. Two bar magnets 50 millimeters in length and attached to the underside of the float serve as magnetic elements.

Fig. 14.—Mauve compass, noncompensated type.

The bowl is hemispherical (90 millimeters diameter) with an expansion diaphragm base. A tapped and plugged filler hole passes through the side of the bowl. No fixed lubber-line or graduated bowl rim is provided, but a small adjustable index slides in a circular path above the crystal to any desired setting.

The bowl is suspended by means of six coiled wire springs, three at either side of the instrument and attached at the extremities of a brass yoke-shaped mounting-bracket.

The instrument described weighs 1.7 pounds, has a period of approximately 25 seconds and a damping coefficient of 10.

MAUVE COMPASS—COMPENSATED TYPE.

The Mauve compensated compass (fig. 15) is liquid damped (alcohol mixture) and so constructed that the card may be directly read either horizontally or vertically. The top or horizontal card surface of the cylindrical float, upon which the luminous graduations are marked at 10° intervals against a black background, has a diameter of 70 millimeters. The cylindrical vertical card surface of the float 18 millimeters in height bears similar markings to be observed through the opening in the side of the bowl. The two cylindrical magnetized bars approximately 60 millimeters in length are suspended 26 millimeters between centers at the lower side of the float. The latter carries in a suitable cavity a sapphire cup and is supported by a hardened alloy pivot upon which the cup rests.

Fig. 15.—Mauve compass, compensated type.

The vertical surfaces of the sheet brass cylindrical bowl serve as a protective housing for a heavy glass ring ($3\frac{1}{2}$ millimeters in thickness, 40 millimeters in height, and with an inside diameter of 86 millimeters) which surrounds the card. This ring rests on a gasket at the bottom of the metal bowl and is closed at the top by the glass crystal which also rests on a suitable gasket rendering the joint free from leaks. The card is visible through the glass ring which is exposed to view at either side through an opening in the protective bowl housing. The corrugated expansion base of the bowl is also covered by this housing and the filler hole is tapped and plugged in the base. The bowl is provided with a rim divided in degrees along which a sliding index and index bar are adjustable as desired by the pilot.

The vertical cylindrical compensating shaft 150 millimeters long is mounted centrally at the base of the bowl and has two sliding blocks each carrying two compensating magnets vertically adjustable upon it. The compensating device is covered by a protective cap of aluminum held in place by a hex-nut threaded to the lower extremity of the compensating shaft.

The mounting bracket of aluminum is yoke shaped and is fastened to the bowl by means of four brass springs at each side.

The instrument weighs approximately 2.4 pounds, and has a period of about 24 seconds.

Fig. 16.—De Vries and Courbet compass.

DEVRIES AND COURBET COMPASS.

The DeVries and Courbet liquid-damped (alcohol mixture) compass (fig. 16) is of the combination horizontal and vertical card type. The horizontal surface of the float (55 millimeter diameter) is marked at 10° intervals with luminous material, while the cylindrical vertical surface (13 millimeters height) bears similar markings which may be viewed from either side of the bowl through the glass ring which serves as a part of the liquid container. A luminous lubber-line consisting of a wire of small diameter is placed at the center of the observation opening. The float is of the usual form for this type of compass and is mounted upon a hardened alloy pivot extending upward from a bridge member above the expansion bowl base to support the sapphire cup member of the float.

The glass crystal caps the glass ring container (70 millimeters inside diameter), the joint being made tight by a suitable gasket. A second gasket is provided at the base of the bowl to serve as a seat for this ring. Two adjustable circular sliding indices 180° apart are provided for the use of the pilot in course setting.

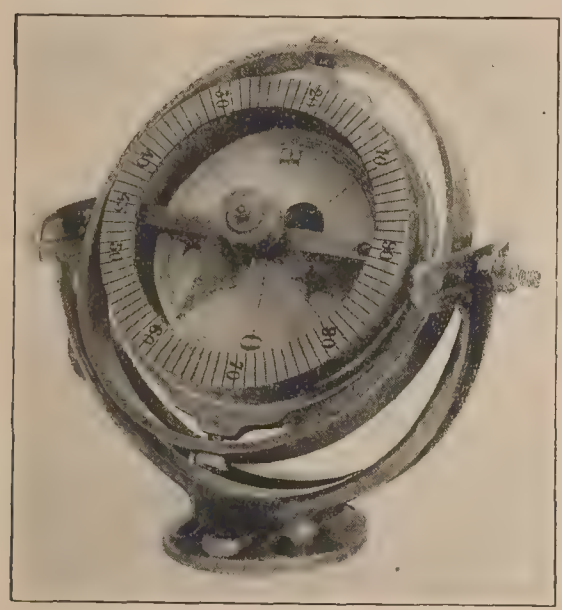


Fig. 17.—The "Monodep" compass.

The protective bowl housing surrounding the glass container ring is of sheet metal and has two hubs attached diametrically opposite each other by which the instrument is suspended. These hubs carry at their outer extremities the shock-absorbing suspension devices, each of which consists of four small compression springs radiating at 90° intervals from the hub and attached in the circular openings found at the extremities of the aluminum yoke-shaped mounting bracket.

This instrument is provided with a device intended for compensation. It consists merely of a short brass tube attached to the base of the bowl and slotted to hold the small compensation magnets in position along two axes 90° apart. The compensation tube has a threaded section at its lower extremity and a knurled cap is supposed to draw the slotted sections of the tube together and thus to clamp the magnets in place. The amount of compensation is regulated by the number, length, and location of magnets used.

This instrument weighs 1.7 pounds and has a period of approximately 15 seconds.

THE "MONODEP" COMPASS.

The "Monodep" dry or air damped compass (fig. 17) has several interesting features which deserve mention. In place of the usual cylindrical shaped magnetic elements, this instrument is equipped with two very thin magnetized plates mounted parallel to each other, with flat surfaces vertical, upon a vertical spindle with lower jewel pivot bearing. The card is also mounted a short distance below the upper extremity of the spindle and the latter is supported

near its upper end by a brass frame secured to the compass bowl. The spindle carries at its upper end above the card a small pinion which mates with the second pinion of a train of four small gears connecting with a small spindle holding a horizontal rotating arm with a black circular disk at its extremity. The gear train multiplies any motion of the magnetic element and card relative to the bowl so that the circular disk index executes a movement just four times as great as that of the card. In this manner the movements of the magnetic element are magnified so that greater precision may be obtained in the compass readings.

The card is 65 millimeters in diameter, divided into 360° and with no figures or letters except those at the cardinal points and a red star to mark the north point.

The brass frame upon which the mechanism is mounted is securely fastened into an aluminum alloy bowl with a glass top surmounted by a circle divided into 90 equal parts marked at 10° intervals from 0 to 90°. The bowl is swung in gimbals with a suitable yoke shaped mounting bracket of aluminum alloy.

Mounted at the center of the glass top is a cylindrical brass box containing the reduction train which accounts for the motion of a transparent celluloid pointer with a star at its extremity underneath the glass. This celluloid pointer moves through an angle of 90° when the brass index-blind pointer to which it is connected above the glass is moved by the pilot through an angle of 360°. The compass is so constructed that one of the cardinal points of the card lies opposite the lubber-line when the index points to the 0–90° point of the exterior divided circle of the instrument.

Fig. 18.—Favé air damped compass.

In establishing a definite compass course the brass pointer is turned until the star on the celluloid pointer holds a position directly above the place which the north marking of the card should occupy. The brass index-blind pointer will then be in a position directly over the circular disk index. Any deviation from this setting will be shown by a movement of the index equal to four times the movement of the card. It is thus seen that the pilot, in reading his compass, notes the quadrant in which the star or north point on the card is located and reads from the exterior divided circle the exact point indicated by the index and the brass marker which is directly above it.

The instrument, complete, weighs 1.2 pounds.

FAVÉ AIR-DAMPED COMPASS.

This compass (fig. 18), conceived by the French hydrographic engineer Favé, is without a doubt one of the most beautiful and delicate examples of the instrument makers' art. It is of the dry or air damped type and is designed for service on lighter-than-air craft.

The main rotating element is made up of a skillfully formed and balanced spider of slender drawn glass or quartz threads radiating from a central hub upon which the magnetic element is mounted. The threads radiate as generatrices of three different surfaces. The first is a horizontal plane surface (150 millimeters in diameter) in which the 12 bar magnets are also fixed in a suitable frame. The second surface is that of a cone with its apex near the bearing and with an apex angle of approximately 60°, while the third surface is also that of a cone with its apex similarly placed, but with an apex angle of approximately 30°. The threads of the three surfaces are held in place by other threads forming circular rings and attached to

the element threads by minute beads. This system moving in the surrounding air furnishes the damping effect.

One element of the horizontal surface lying parallel to the magnets, and thus along the meridian, is more rugged than the others and has the point and tail of an arrow attached at the respective extremities of the spine and pointing toward the north and south magnetic poles. This rotating system is mounted on a jewel bearing (jewel cup on card, alloy pivot on post) at the center bearing post, which is provided with a device similar to that found in transit compasses for lifting the rotating part when not in use from its bearing and against a guide rod extending from the crystal. This lifting device is operated by a knurled thumb nut mounted at the side of the case. The N–S element of the system extends to within about 1 millimeter from a horizontal annular disk attached to the sides of the bowl and bearing the scale divisions marked by degrees from 0 to 360.

The bowl, which is covered by a glass crystal, has an inside diameter of 178 millimeters and depth of 100 millimeters. The instrument is mounted in gimbals as shown. It weighs 5.4 pounds, has a period of 9 seconds, and a damping coefficient of 5.

DESCRIPTIONS OF GERMAN COMPASSES.

KAISERLICHE MARINE KOMPASS.

The Kaiserliche marine kompass (fig. 19) has a horizontal card (100 millimeters diameter) on a cylindrical float 65 millimeters in diameter and 19 millimeters deep. A jewel cup mounted

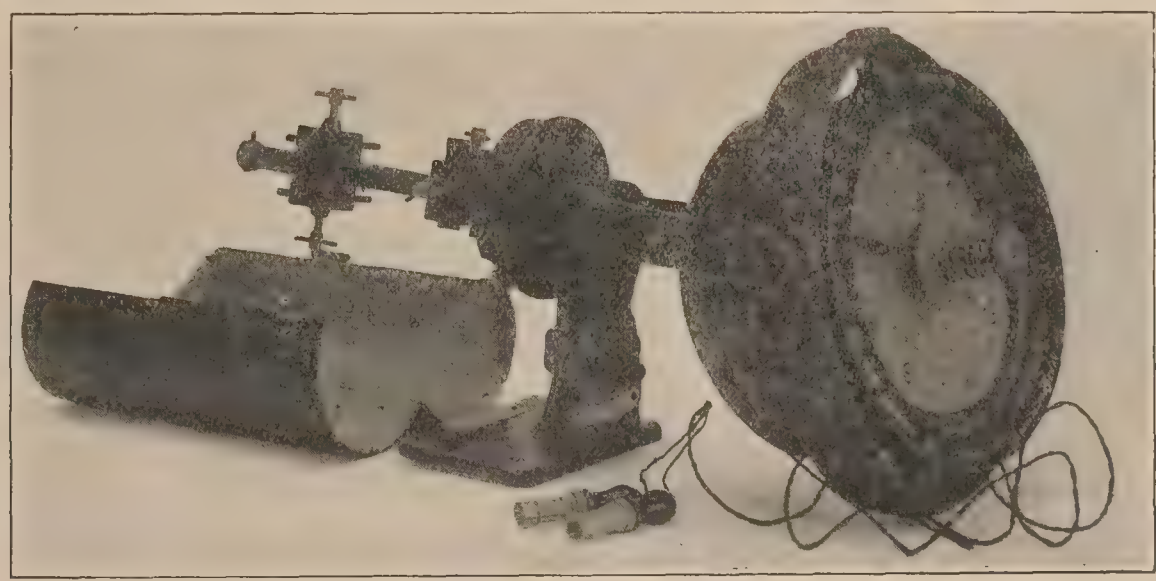


FIG. 19.—Kaiserliche marine kompass.

in a cavity in the lower surface of the float rests upon an alloy pivot on a spring supported by a bearing post extending from a bridge member at the base. Two flat bar magnets are attached at the bottom of the float. The card is graduated in 5° intervals with distinctive markings for the cardinal points. On the inner surface of the bowl are four black lines (on white) at 90° intervals. These lines are repeated as white marks on the outer ring which holds the glass in place.

The bowl is cylindrical (135 millimeters in diameter and 65 millimeters deep) and contains alcohol as a damping liquid. At the bottom is a weighted cap carrying a small electric lamp at its center. The base of the bowl is formed by a corrugated metal expansion diaphragm. In the center of this diaphragm is a circular glass window through which light from the lamp enters the bowl. Internal reflection in the bowl provides sufficient illumination for the face of the card.

A unique feature of the instrument is an index pointer mounted on the card, which may be set from the outside. A plunger, with a knob on the outside and a cogged disk on the inside end, is held in a packing box through the center of the glass. By pressing down on the knob, the cogged disk is engaged in a similar disk on the pointer. At the same time a stiff spring under the pivot is compressed and the float is lowered until a cogged rim around the cup engages

a similar ring on the pivot support. Then the pointer may be set by turning the knob while the card remains stationary.

The bowl is hung in heavy gimbals mounted in a yoke which is pivoted so as to be adjustable upon the supporting bracket. The yoke carries an index line and a clamping screw, and the bracket a scale marked from +10 through 0 to −10. The slotted compensating magnet column extends downward from the supporting bracket to which it is attached and has two sliding collars adjustable vertically for holding the correcting magnets. A detachable sheet-metal cover protects the compensating magnets.

Fig. 20.—Ludolph armee kompass I.

The period of this instrument is about 25 seconds. Its weight is 6.4 pounds.

LUDOLPH ARMEE KOMPASS I.

The Ludolph compass (fig. 20) is of the liquid-damped (alcohol mixture) type. The float is cylindrical in shape, about 70 millimeters in diameter and 35 millimeters in depth, and bears a beveled projecting rim around the lower edge. Four magnets in the form of cylindrical rods are attached at the bottom of the float. The upper face, forming the horizontal card, is graduated in 10° intervals with the cardinal points marked and lettered. The vertical face of the float bears two identical scales graduated at 5° intervals from 0 to 360. The lower scale is on the beveled rim and the upper scale is separated from it by a colored band, red for the north and blue for the south half of the scales. The pivot, which is of alloy, is supported by a small brass post at the bottom of the bowl. A sapphire cup is set in the float.

The bowl is hemispherical in shape (about 120 millimeters in diameter and 80 millimeters in depth) with a rectangular projection at the side, 70 millimeters wide and 50 millimeters deep, which is covered by a glass observation window. At the bottom of the bowl is a metal diaphragm chamber to compensate for expansion of the damping liquid. A filling hole with a threaded plug, and a small circular window over which a lamp may be attached are found on the bowl. The horizontal face bears a scale around the rim graduated at 5° intervals, and at the center of the glass is pivoted a metal pointer. The inside of the bowl is painted white and bears two lubber-lines, a black mark for the horizontal face of the card and a black wire for the vertical face.

No provision is made for attaching compensating magnets. Two small angle brackets at the top of the bowl are used for mounting the compass. The compass weighs 2.9 pounds.

The Ludolph armee kompass II is similar to the instrument just described but is somewhat larger. The bowl is about 135 millimeters in diameter and 90 millimeters in depth.

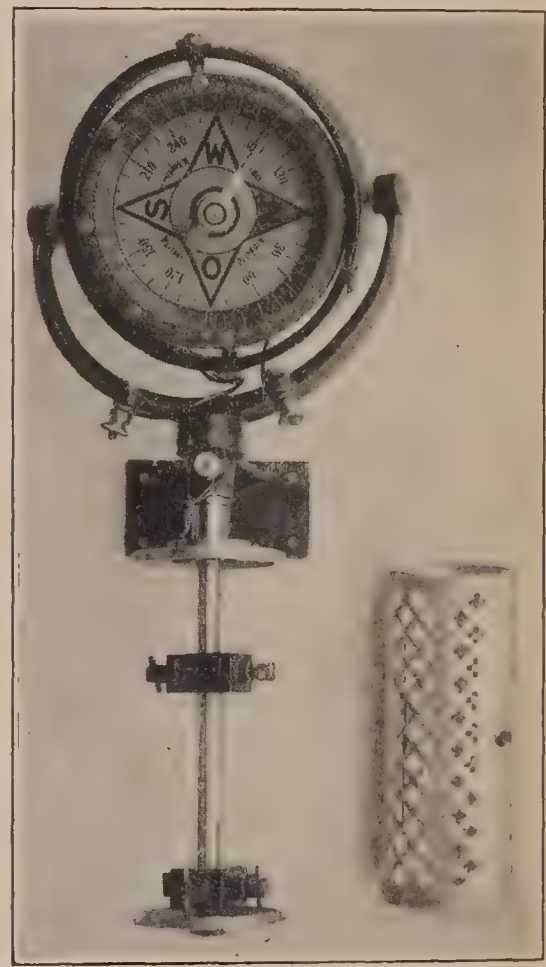


Fig. 21.—Sendtner armee kompass III.

SENDTNER ARMEE KOMPASS III.

The Sendtner armee kompass III (fig. 21) is of the liquid-damped (alcohol mixture), horizontal-card type. The card (85 millimeters diameter) graduated in 10° intervals is carried by a float of usual design. The cardinal points of the card are distinctively marked and let-

tered. Numerals are found at the 30° points. An alloy pivot is carried in an indented cavity in the lower surface of the float while the jewel cup upon which the pivot rests is supported upon a post with spring shock-absorbing cup socket at its upper end. The magnetic elements are sealed inside the float chamber.

The bowl is cylindrical, 115 millimeters in diameter and 60 millimeters deep. A metal pointer, pivoted at the center of the glass, may be set to any desired angle as indicated by a scale graduated at 10° intervals on the rim of the bowl. The interior of the bowl is painted white and bears four black lubber-lines under the 90° points of the exterior scale. The base of the bowl is formed by a metal diaphragm in the center of which is set a ground-glass window. A heavily weighted cap containing an electric lamp covers the base. Two flexible wires from the lamp lead to binding posts on the supporting yoke.

The gimbal suspension of the bowl is mounted in a yoke which is adjustable upon the bracket support. A grooved compensation column with adjustable sliders holding the correcting magnets is mounted below the bracket.

This instrument has a period of approximately 25 seconds. Its weight is 4.8 pounds.

PFADFINDER ARMEE KOMPASS III.

The Pfadfinder armee kompass III (fig. 22) is of liquid-damped (alcohol mixture) type with a horizontal card. The card (84 millimeters in diameter) is graduated at 10° intervals, with distinctive markings for the cardinal points. It is fastened to a float to which are attached two bar magnets incased in copper tubing. An alloy pivot on the float rests in a jewel cup set on the end of a bearing post attached to a spider at the bottom of the bowl.

Fig. 22.—Pfadfinder armee kompass III.

The bowl is about 120 millimeters in diameter and 70 millimeters deep. Its base consists of a metal diaphragm covered by a weighted cap. A rim graduated at 10° intervals is set around the glass face of the bowl. In this rim is also a lamp socket. At the center of the glass is pivoted a movable pointer. The interior of the bowl is painted white with four black wire lubber-lines set under the 90° points of the exterior scale.

The bowl is suspended in a gimbal ring mounted upon a yoke bracket support. To this bracket is also attached the compensation column carrying the correcting magnets in sliding collars of the usual form. A protective case surrounds the compensating device.

This instrument weighs 5.5 pounds.

PFADFINDER ARMEE KOMPASS IV.

The Pfadfinder armee kompass IV (fig. 23), a liquid-damped (alcohol mixture) compass, has a combination horizontal and vertical card. The float with card markings upon it is cylindrical and about 70 millimeters in diameter by about 23 millimeters in depth. The magnetic elements are sealed inside the float chamber. The horizontal face bears graduations at 10° intervals, with special markings and letters for the cardinal points. The vertical face is beveled from each edge inward toward the center. Upon this face are two identical scales graduated at 10° intervals from 0 to 360°. The scales are separated by a colored band, blue from the 0 to the 180° scale divisions and red for the remainder. The bearing is formed by an alloy pivot attached in a cavity in the lower float surface and resting upon a sapphire cup supported by the usual form of bearing post with spring shock-absorbing socket.

The cylindrical bowl is 105 millimeters in diameter and 85 millimeters in depth. The bottom is formed by a diaphragm expansion covering. On the curved surface of the bowl are a filler hole with threaded plug, and a window for viewing the vertical face of the card. The horizontal glass observation window is held in place by a metal ring graduated from 0° to 360° at 10° intervals. This ring also contains the lamp socket. A movable pointer is pivoted at the center of the glass. The inside of the bowl is painted white. Two black wires, 180° apart, serve as lubber-lines for both faces of the card.

The support consists of a bracket, one end of which is formed into a circular plate with a clamping ring by which the compass bowl is held in place. On the lower side of the plate

FIG. 23.—Pfadfinder armee kompass IV.

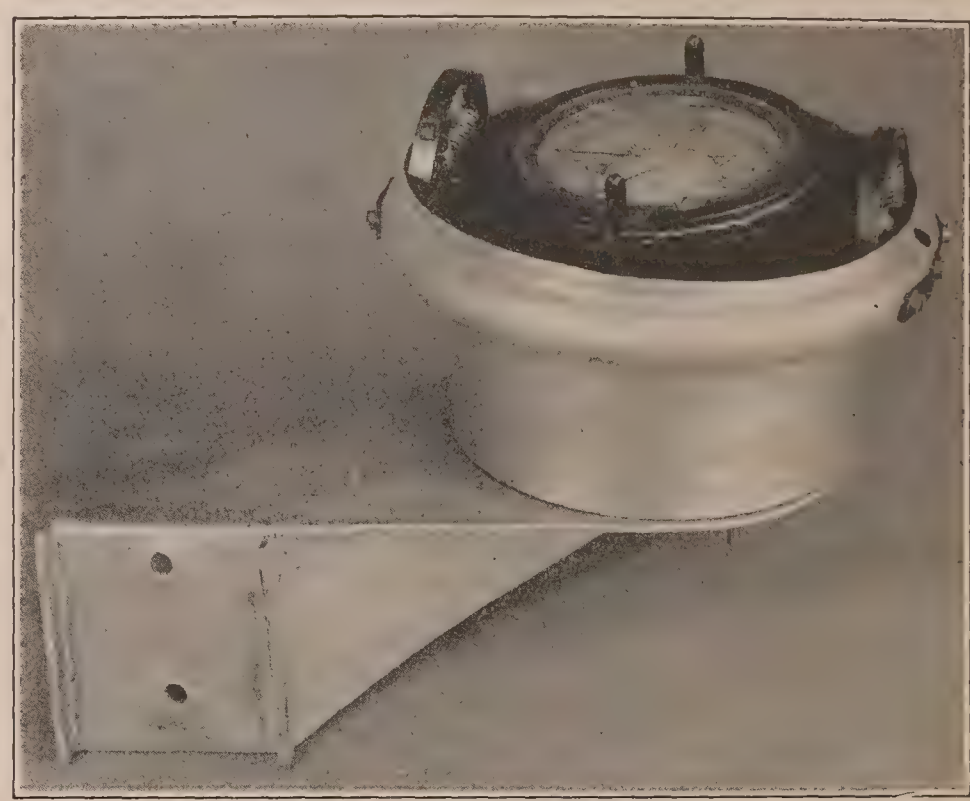


FIG. 24.—Pfadfinder kompass für aviatik.

an aluminum bar projects, holding the compensating magnets. An aluminum case protects these magnets. There are two binding posts on the bracket, connected by wires to the lamp socket.

This instrument has a period of approximately 25 seconds and weighs 3.2 pounds.

PFADFINDER KOMPASS FUR AVIATIK.

A compass of the liquid-damped type is illustrated in figure 24. The unusual form of bracket is used to mount the compass upon a vertical member of the airplane structure remote from disturbing magnetic influences. The instrument may be mounted in the wing, as shown in figure 25. The support consists of a large aluminum bracket carrying an aluminum bowl. The gimbal ring, in which the compass is swung, is hung in this bowl by means of spring suspensions, which can be clearly seen in figure 24.

The horizontal card is 73 millimeters in diameter and is graduated at 5° intervals. The cardinal and 45° points are marked by black triangles, with the exception of the north point, which is marked by a red arrow. The card is attached to the top of the float, in which are mounted two magnets. A sapphire cup on the float rests on an alloy pivot, which is supported from the base of the bowl. Alcohol is used as the damping liquid.

The compass bowl is cylindrical, about 110 millimeters in diameter and 75 millimeters deep. The base is formed by a metallic diaphragm pierced by a hole which connects the bowl with a flat expansion chamber of corrugated metal. A heavy lead disk forms a protecting cap. At one side of the bowl is a filling hole stopped by a threaded plug.

The face of the bowl shows an interesting departure from the usual practice. A fixed glass carries the lubber-line, a black radial line which is continued down on the inside of the bowl. Above the fixed glass is a second glass set in a brass ring, which is graduated in single degrees from 0 to 360. A red radial line extends from the center of this glass to the zero point of the scale. Glass and ring may be turned as a unit by means of two brass knobs on the ring. Outside of the movable ring is a metal ring, carrying an index mark directly over the lubber-line. This ring may be clamped by means of two screws with long projecting heads set 90° away from the index mark. Clamping this ring also clamps the movable ring.

This instrument has a period of about 25 seconds. It weighs 4.9 pounds with the mounting bracket.

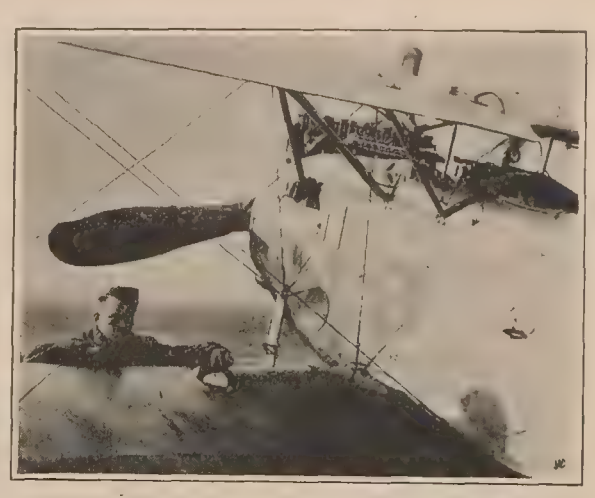


FIG. 25.—Wing mounting of compass.

REMOTE INDICATING AND REMOTE CONTROL COMPASS.

Among the most interesting of foreign aircraft instruments is the Bamberg remote indicating and remote control compass (figs. 26 to 31, inclusive). In this compass arrangement we find an ingenious design in which difficulties due to disturbing magnetic influences from the motor and elsewhere are avoided, by locating the magnetic compass element at a position remote from these conditions which ordinarily present such a serious obstacle to the proper functioning of the instrument. Intended for use on the larger types of aircraft, the Bamberg compass system serves as a means of control between the navigator or observer and the pilot whose position in the aircraft may be at some distance from the navigator's station.

The magnetic compass.—The magnetic compass upon which the system depends for its indications is a comparatively heavy liquid-filled type mounted in gimbals and having a period of 25 seconds. The compass bowl has an inside diameter of 145 millimeters and is equipped at its base with an expansion chamber consisting of two flexible metallic diaphragm boxes.

The magnetic element is of the float type, but instead of a card graduated in the ordinary manner it carries a metal disk cut in such a shape as to act as a blind in regulating the passage of rays of light projected upward from the base of the compass bowl by two 8-volt electric lamps attached diametrically opposite each other (figs. 28 and 30). The light rays from each

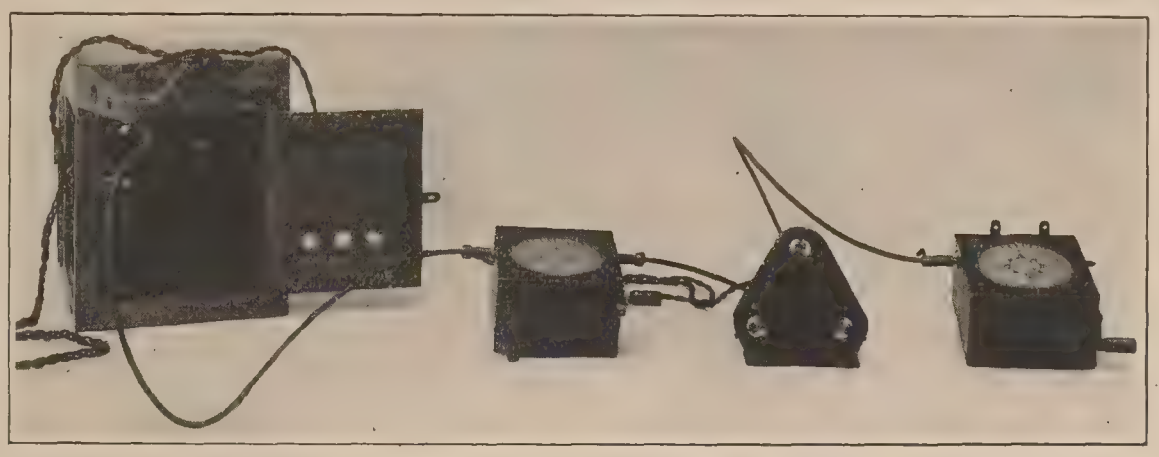


FIG. 26.—Remote indicating and remote control compass. (Complete outfit.)

lamp are focused by a lens upon a corresponding selenium cell incased in a water-tight bridge member which spans the top of the bowl (figs. 28 and 30). The lamps are made adjustable in their sockets to allow for varying their distance from the lenses.

Selenium cells.—These selenium cells have the property that their electrical resistance varies with changes in the intensity of the light which falls upon them. Thus, with the magnetic element carrying the blind in a certain position relative to the bowl, both light cones are eclipsed and the two selenium cells remain in darkness. If, however, the compass bowl is rotated

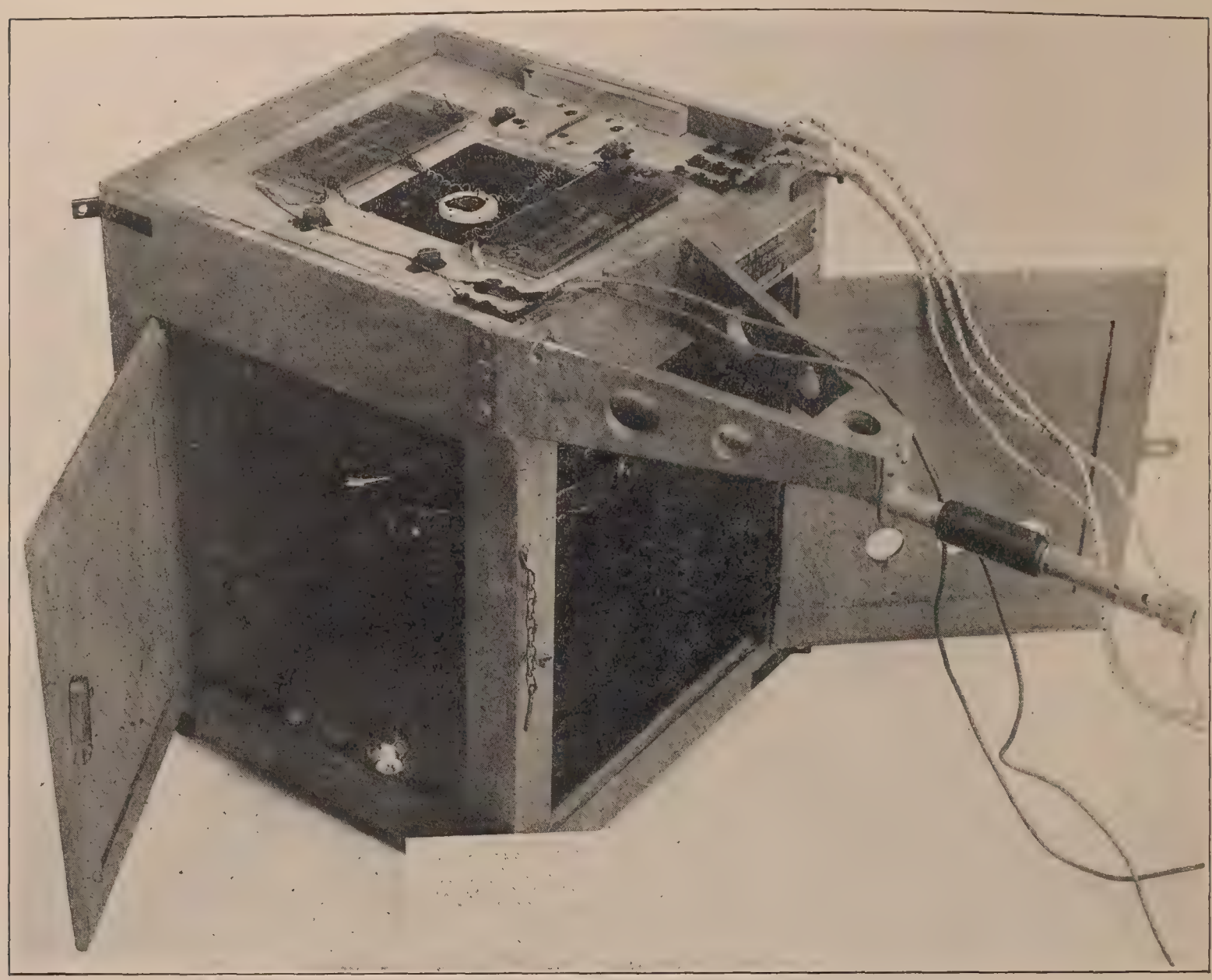


FIG. 27.—Remote indicating and remote control compass. (Magnetic compass in housing.)

FIG. 28.—Remote indicating and remote control compass. (Magnetic compass removed from housing.)

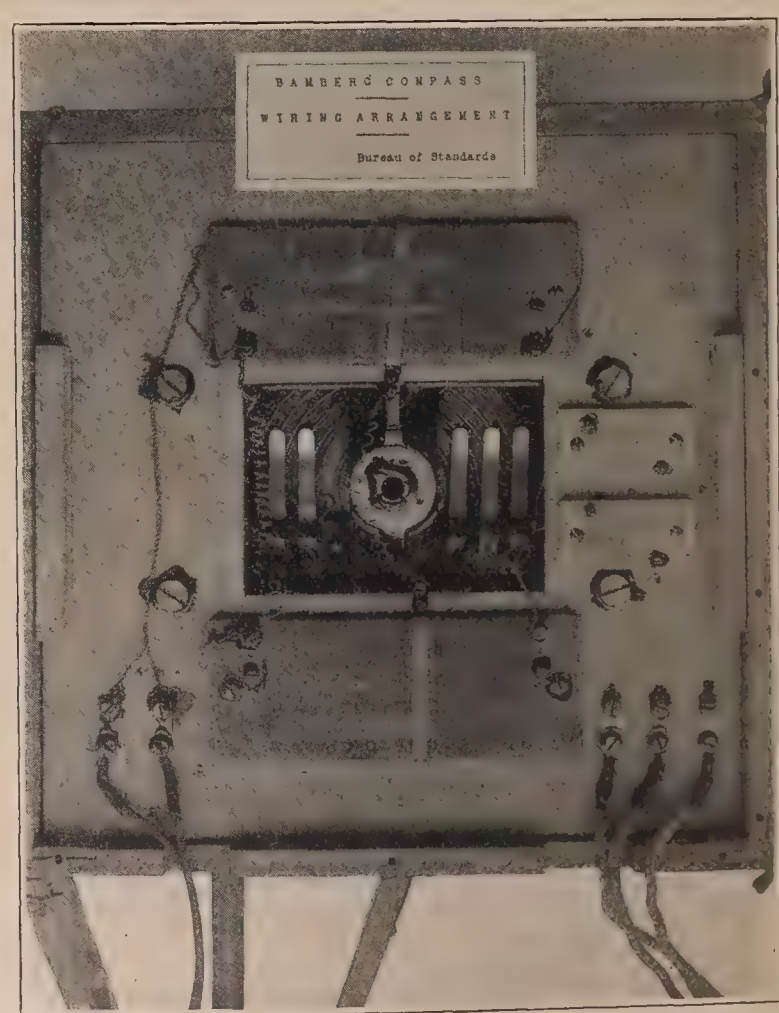


FIG. 29.—Remote indicating and remote control compass. (Wiring arrangement.)

through a certain angle, one of the light cones remains eclipsed while the other is uncovered so that its rays fall upon the selenium cell above it. This lowers the resistance of the illuminated cell by an amount which is dependent up to a certain limit (when one light becomes completely uncovered) upon the angle through which the compass bowl has been turned relative to the magnetic element.

The selenium cells form two arms of a Wheatstone bridge, the remaining arms consisting of resistances wound upon slate cards (figs. 29 and 31). Current is supplied either from a battery or a direct current wind-driven generator. A small deviation from the indicated course unbalances the bridge, which is indicated to the pilot by the deflection of the bridge galvanometer located in the pilot's cockpit.

Course indicator or control box.—The navigator is equipped with a course indicator or control box (shown in fig. 26) which he uses in controlling the direction of flight. The pilot (as well as any other occupant of the aircraft) may also be equipped with one of these indicators. The mechanism is inclosed in a small wooden box with a glass window in its upper side, through which the pilot or navigator observes a dial graduated with a scale similar to an ordinary compass card. A black lubber-line is painted on the glass for use as a reference point. The mechanism consists simply of a train of gears which connect a hand crank to the dial and also to a spindle equipped with connections for flexible shafting. The gear ratio is such that one complete turn of the crank causes the flexible shafting spindle to make two complete revolutions while the indicating dial turns through an angle of 6°.

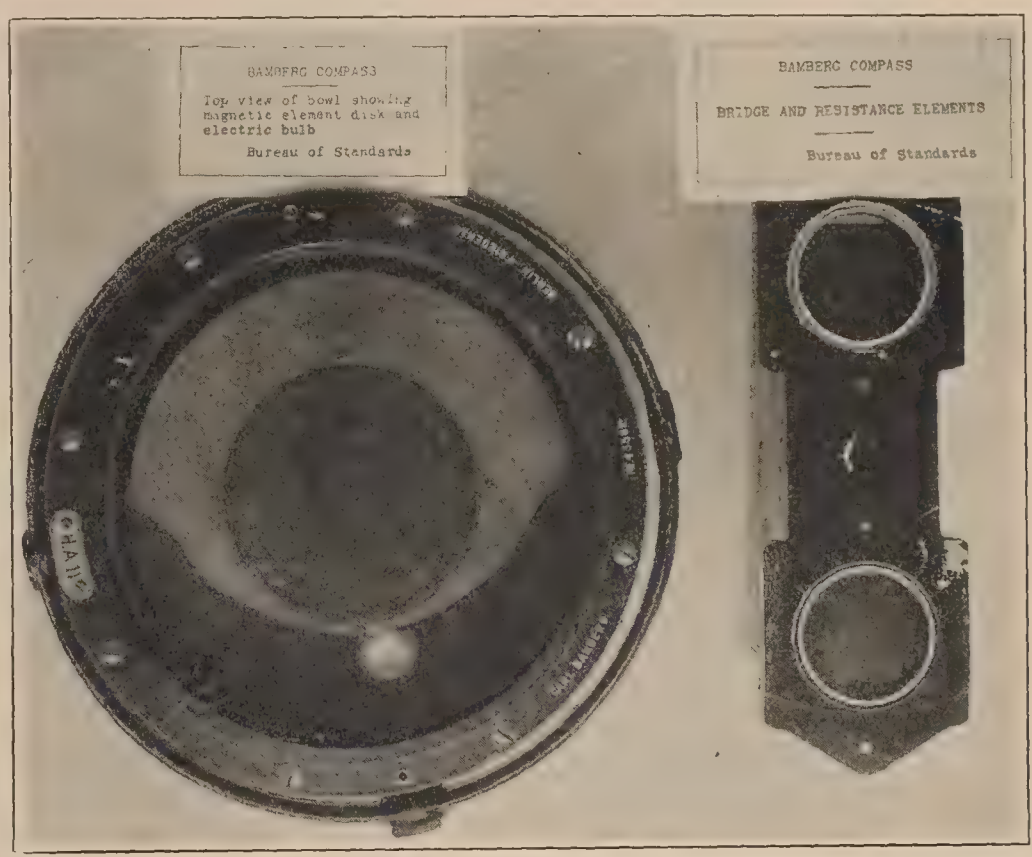


Fig. 30.—Remote indicating and remote control compass. (View of bowl, bridge, and resistance elements.)

Compass control.—The course indicators in the airplane are connected to each other and in turn to the compass itself by means of lengths of special flexible shafting, so that all the indicators are set simultaneously and in the same manner as the compass bowl itself.

The flexible shafting which extends back to the compass bowl is connected with the latter through a worm and mating gear (fig. 28), which function in such a manner as to cause the main yoke suspension of the compass bowl, which is integral with the worm gear, to rotate as the flexible shafting turns. The electrical connections from the movable elements of the compass are brought to the fixed elements through commutator rings with corresponding brushes as shown in figures 29 and 31.

Operation of the installation.—Let us first assume that the outfit is so installed in the aircraft that both selenium cells are in darkness when the indicators show the aircraft to be directed along the north and south magnetic meridian. In this position the bridge circuit will be balanced and the pointer of the pilot's galvanometer or steering gage (shown in fig. 26) will be in its neutral position. Any deviation from this north-south course will become apparent by a change in position of the pointer of the steering gage, which will turn clockwise or counterclockwise according to whether the heading of the aircraft changes to the right or to the left. Thus by watching this instrument the pilot is able to hold to the course.

Now, let us assume that the navigator wishes to change the course by an angle of 15°. By rotating the crank of his course indicator or control box until the dial shows a change of 15° in the desired direction he also turns the other indicators in the aircraft and at the same time the bowl of the compass. This causes one of the selenium cells (both cells turning with the bowl) to receive a greater illumination than the other, the balance of the bridge circuit is destroyed and the pointer of the steering gage before the pilot changes from its neutral position to a new position showing in which direction he must rudder in order to follow the new course. The amount of this turning is indicated roughly up to a certain degree (about 25° when one light is completely uncovered) by the angle through which the pointer turns.

In case an accident occurs so that the aircraft is out of control during a period long enough for it to assume a heading differing by 180° from that of the proper course, the pilot is able to

KEY TO WIRING DIAGRAM OF THE REMOTE INDICATING AND REMOTE CONTROL COMPASS.

(Refer to figure 31.)

A, B, G(3)	Connectors for battery or generator wires.
D, E	Connectors for pilot's steering gage.
F, G, H, K, L	Commutator connections leading to the rotating compass bowl so as to provide electrical connections between the illuminating elements, the selenium cells, and the stationary elements of the circuit.
R_1, R_3	Resistances for controlling sensitivity of galvanometer.
R_2, R_4	Bridge arm resistance coils.
S_1, S_2	Selenium cells with common wire lead to K.
T_1, T_2	Lamps supplying illumination to the selenium cells.

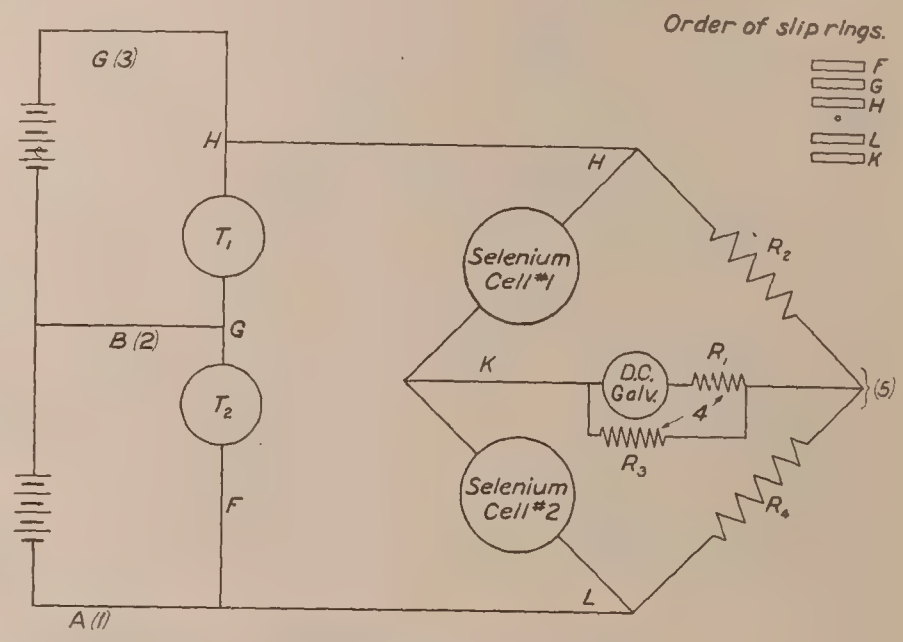


FIG. 31.—Remote indicating and remote control compass. (Wiring diagram.) See key to diagram.

recognize this fact even though the pointer of his steering gage is in a neutral position, for each turn of the aircraft to one side or the other is shown by the steering gage as a turn in just the opposite direction.

The weight of the system is as follows:

	Pounds.
Magnetic compass and bridge elements in protective housing	16.0
Two course indicators	8.4
Pilot's steering gage or galvanometer	1.8
Flexible shafting	.9
Total weight not including battery or generator	27.1

DESCRIPTIONS OF MISCELLANEOUS COMPASSES.

The foregoing descriptions of American and foreign compasses relate to representative instruments which have been developed and placed on the market for use on aircraft. In concluding this paper it may be of interest to mention briefly several of the instruments now in the process of development, as well as one or two types already produced but which do not logically come under the groupings as carried out in the first part of the paper.

EARTH INDUCTOR COMPASS.[1]

The following description relates to the earth inductor compass which was developed by Dr. Paul R. Heyl and Dr. Lyman J. Briggs of the Bureau of Standards at the request of, and with funds furnished by, the Engineering Division, Air Service, United States Army.

The application of the earth inductor to the determination of magnetic direction is not new. In magnetic survey work, use is made of both the earth inductor and the dip circle for

[1] The author is indebted to Dr. Briggs and Dr. Heyl for the above description of the Bureau of Standards earth inductor compass.

the determination of magnetic inclination or dip, with results of equal precision. A number of earlier attempts have been made to use the earth inductor as a compass, but no one of these proposed devices possessed sufficient practicability to bring it into use during the recent war.

In all previous attempts at the construction of a compass of this type the current developed in the rotating coil, amplified if necessary, was caused to pass through a galvanometer, and the course of the vessel judged from the amount of deflection produced. This instrument differs from all previous attempts in the following respects:

1. It employs a null method for its indications, and hence enjoys all the advantages of sensitivity characteristic of null methods as a class.

2. A course-setting device of a novel type is employed. By turning a movable dial at the instrument board the electrical connections of the distant revolving coil to the galvanometer are so arranged that the pointer reads zero only when the vessel lies in the desired course.

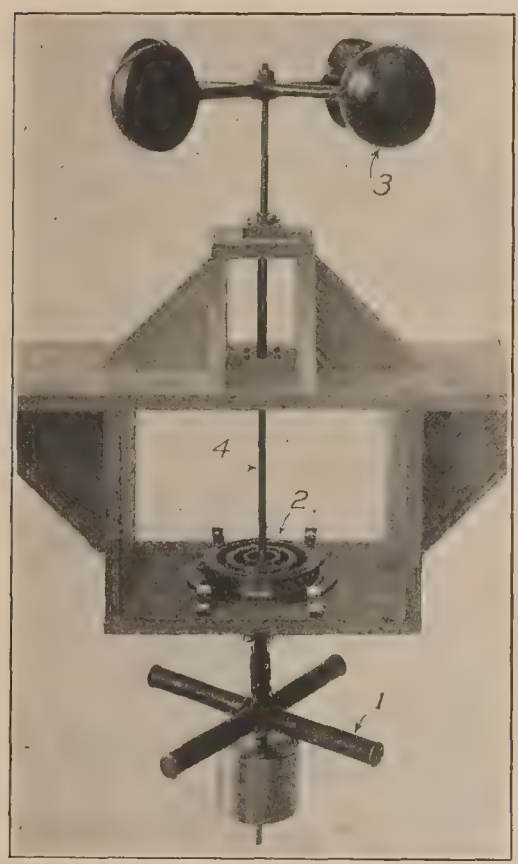


FIG. 32.—Earth inductor compass.

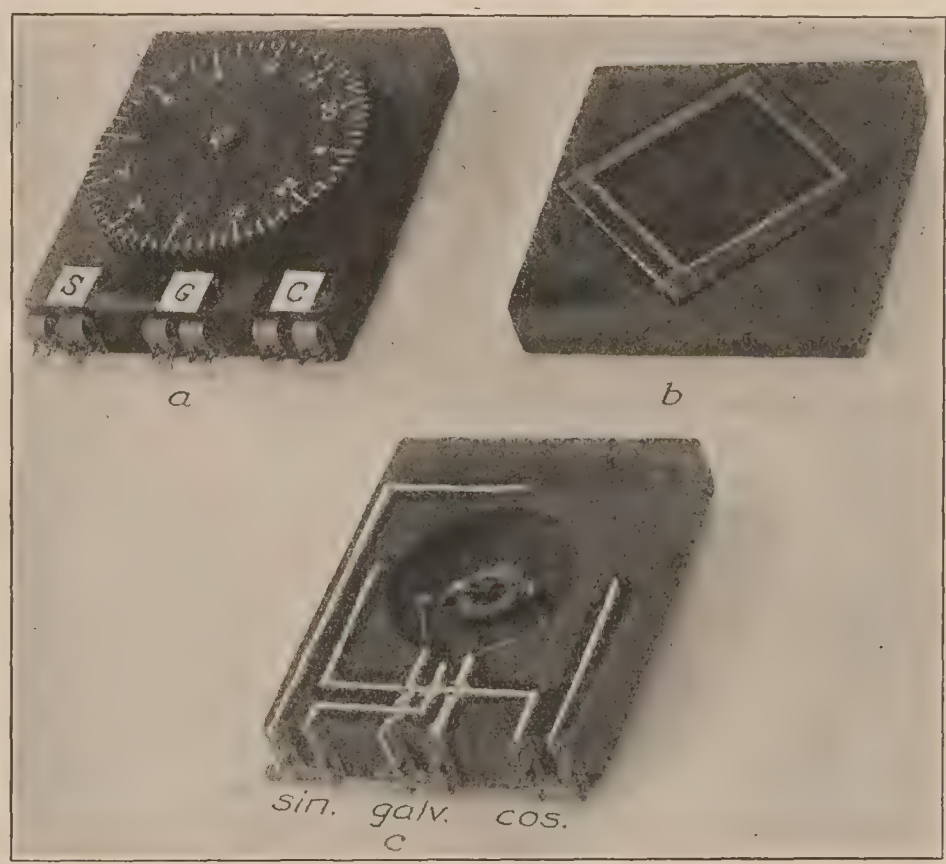


FIG. 33.—Earth inductor compass.

3. A method is provided for eliminating the errors due to rolling and pitching, arising from the action of the vertical component of the earth's field.

4. By the judicious use of iron in the core of the revolving coil the size of the apparatus may be sufficiently reduced to make it practicable of installation in an airplane.

Current is generated by the rotation in the earth's horizontal field of a cross-shaped armature 1 (fig. 32) rotating about a vertical axis. The vertical position of this axis is preserved during rolling and pitching by suspending it in the gimbal ring 2. In the case of installation in an airplane, power is applied to drive the armature by the cup propeller 3 acting through a section of flexible shaft 4.

A four-segment commutator and four collecting brushes, spaced at 90° takeoff current from the armature. The electromotive forces at the two pairs of brushes will be functions of the course followed by the ship. The setting of the brushes is so made that when the ship lies in a line making an angle θ with the magnetic meridian, the electromotive forces at the brushes are $\sin\theta$ and $\cos\theta$.

Since, in general,

$$m \sin\theta + n \cos\theta = 0$$

if $\tan\theta = -\frac{n}{m}$, the galvanometer reading may be made zero in any desired course by combining, additively or subtractively, suitable fractions or multiples of the voltages from the two pairs of brushes. This is done by the dial switchboard (fig. 33).

A movable dial (fig. 33, *a*) carrying compass graduations, has on its under side two wiping contacts, which connect with opposite diametral points of a square resistance frame (fig. 33, *b*). The sine brushes are connected to the upper and lower corners of this square and the cosine brushes to the right and left corners (fig. 33, *c*). The galvanometer leads are connected to the wiping contacts above mentioned through another pair of wiping contacts and the movable hub which carries the dial.

The mathematical theory of this device shows that if a circular resistance frame be used, the compass will be affected by an octantal error amounting at its maximum to about four degrees. By the use of a square resistance frame this error is eliminated.

The armature 1 (fig. 32) is wound on each arm with 500 turns No. 20 wire. The arms are connected in series as a closed coil winding. The brushes are of carbon. Experiment has shown that in a consecutive run of 146 hours such brushes suffer only trifling wear, and deliver the exact voltage necessary for the successful application of the null method.

The gimbal system 2 (fig. 32) is provided with frictional damping at the bearings. It is found that a short, heavy, damped pendulum of this type makes an excellent stabilizer.

A modification of this instrument has been constructed in which the rotation is produced electrically. A small 3-phase motor is mounted on the axis of the armature. The stray field of the motor, revolving at the same speed as the armature induces no e. m. f. in the latter; and the symmetrical position of the motor with respect to the armature prevents any twist of the earth's field.

GYROSCOPIC COMPASSES.

A magnetic compass surrounded by the unsatisfactory conditions found in service on aircraft is, at best, working under great disadvantages. Depending as it does upon the relatively weak horizontal component of the earth's magnetic field for its action, the instrument at the outset is not endowed with an actuating force of any appreciable power. Coupled with this disadvantage it has the disturbing influences presented by the uncertain and more or less variable magnetic fields developed by the power plant and auxiliaries of the system.

Among the interesting and promising substitutes for the magnetic compass is the gyroscopic compass. One of the obvious advantages of such an instrument is its independence of the earth's magnetic field as well as of the disturbing fields of the aircraft itself. On the other hand, the gyroscopic compass is necessarily larger and heavier than the magnetic compass, more complicated in design, and costlier to construct. In the various types of gyroscopic compasses in use on shipboard, the gyroscopic system is not neutrally balanced, use being made of a gravitational couple to keep the gyroscope precessing into the meridian. The system is consequently subject to disturbing forces whenever accelerations are present as on aircraft, and these disturbances persist for some time after the acceleration has ceased. Errors from such causes so far have proven a formidable difficulty in the development of gyroscopic compasses for aircraft.

A certain instrument, gyroscopic in principle, now under development in America consists of a neutral gyro in the form of a steel sphere resting upon an air film and rotated at an extremely high speed by a jet of air from a small compressor. The gyro element is surrounded by a suitable spherical housing, the lower half of which is mounted upon a frictional plane so as to be free to swivel about a vertical axis coincident with that of the inlet tube of the air jet. The plane is made frictional practically by the leakage of air from the inlet which forms a film. The lower half of the spherical bowl has a shallow channel cut in one side and extending toward the top. It is the air escaping from this channel which causes the sphere to rotate. It is intended that the sphere when once started spinning with its axis of rotation horizontal and pointing toward the north shall maintain that position indefinitely. The action of the channel above mentioned is such as to cause the lower hemisphere, which is free to swivel, to always turn into a position so that the plane of the channel is perpendicular to the axis of rotation of the sphere. Hence, if a compass card is mounted in a horizontal position on the lower hemisphere it will indicate the compass direction. The system above described is subject to certain inherent difficulties but presents an interesting attempt to solve the compass problem.

RECORDING COMPASS.

A foreign inventor has designed a recording compass which allows the flyer to follow the progress of the flight. A stylus is connected with the rotating system in such a manner as to record upon a suitable chart, rotated by a clock movement, the deviations from the true course both in magnitude and duration. This makes possible a subsequent proportionate correction.

DANISH AND SWISS COMPASSES.

Following are descriptions of the Knudsen compass and the Stoppani double pivot inverted compass.

KNUDSEN COMPASS.

The Knudsen compass (fig. 34) is a Danish instrument of the liquid-damped (alcohol mixture) type with a horizontal card. The card is marked with the 32 points subdivided into quarter points. It is attached to a small float chamber which also carries two magnets and

FIG. 34.—Knudsen compass.

FIG. 35.—Stoppani double pivot inverted compass.

a sapphire cup bearing on its lower surface. The pivot is supported on a spider near the bottom of the bowl. It consists of an alloy point set in a short brass rod.

The bowl is cylindrical, 106 millimeters in diameter and 60 millimeters in depth with diaphragm expansion base protected by a heavily weighted cap. A filler hole closed by a screw is set in the side of the bowl. The glass is held in place by a brass ring. Above this is a rotatable brass ring carrying a red and white index pointer for use in course setting. Another ring fixed to the first is graduated in single degrees from 0 to 360. This ring carries a small movable red index on its outer rim. The interior of the bowl is painted white and has two black wire lubber-lines set 90° apart.

The suspension consists of gimbals supported in a yoke. No provision is made for compensating magnets or for illumination.

This compass has a period of about 17 seconds. Its weight is 3.5 pounds.

STOPPANI DOUBLE PIVOT INVERTED COMPASS.

This compass (fig. 35) is a horizontal-card, liquid-filled (alcohol mixture), inverted type. It is designed to be mounted directly above the pilot in a position as remote as possible from disturbing magnetic influences. The observation glass is held in place by a ring graduated

at 10° intervals at the underside of the bowl (120 millimeters diameter, 100 millimeters depth), while a diaphragm expansion member covered by protective housing forms the upper end. The bowl is hung upon two hubs resting upon the gimbal ring of the mounting bracket. A vertical compensating rod extends above the mounting bracket and carries two adjustable sliding holders for the correcting magnets.

The bearing features of this instrument differ from those of the usual types. Instead of the single pivot arrangement common to the majority of aircraft compasses, this instrument is provided with two pivots (alloy), one mounted upon the inner surface of the cover-glass below the float and the other attached to the upper float surface. The float carries two cylindrical bar magnetic elements attached to the upper surface. The lower surface of the float is indented by a cavity holding a jewel cup while the upper jewel cup is held in a socket attached to a vertically adjustable post, mounted on a bridge member spanning the upper bowl surface and extending downward to meet the upper pivot. With the double pivot arrangement it is possible to make the buoyancy exactly neutralize the weight of the rotating system. This reduces the vibrational error which varies with the force exerted on the pivot by the vibration. The double pivots also prevent the balancing oscillations of the card with respect to the bowl.

The instrument described weighs 3.5 pounds.

TABULAR CLASSIFICATION OF COMPASSES.

Name.	Type.	Type of card.	Location of pivot.	Material of pivot.	Material of cup.	Magnetic element.	Damping fluid.	Mounting.	Compensated.	Period (complete).	Weight.	Page.
AMERICAN.												
										Seconds.	*Pounds.*	
General Electric air compass	Type B	Vertical	Card	Alloy	Sapphire	2 bars	Kerosene	Springs and felt	Yes	12	2.5	24
Navy Standard compass, No. 1	Mark XVI	Horizontal and vertical	do	do	do	do	Alcohol	Rubber (1)	Yes	20	3.7	25
(Sperry) Creagh-Osborne air compass	Mark II	Horizontal	do	do	do	do	do	Rubber and hair	Yes	18	3.3	26
Pentz compass		Vertical	do	do	do	do	Kerosene	Springs and felt	Yes	12	3.3	27
BRITISH.												
Creagh-Osborne air compass	Type 5/17	Vertical	Card	Agate	Sapphire	2 bars	Alcohol	Springs and felt	Yes	8–10	2.8	28
Creagh-Osborne aero compass	Type 259	do	do	do	do		do	do	Yes	25	2.1	29
Do	Type 253	Horizontal	do	do	do		do	Springs and hair	No	25	7.0	29
R. A. F. pilot's compass	Mark II	Vertical	do	do	do		Zylol	Springs and felt	Yes	40–60	4.9	30
Air compass (quick period)	do	do	do	do	do	6 bars	Liquid	do	Yes		4.9	30
Campbell-Bennett aperiodic compass	Type 6/18, Mark II.	(1)	Magnetic system.	do	do	do	do	do	Yes	(1)	6.0	31
FRENCH.												
Aéronautique militaire compass		Horizontal	Bowl	Alloy	Sapphire	2 bars	Alcohol	Gimbals	No	17	3.3	32
Aéronautique militaire 1 compass		do	Card	Jewel	Jewel	do	do	Rubber (1)	Yes	16	6.0	33
Mauve compass		do	Bowl	Alloy	Sapphire	do	do	Springs	No	25	1.7	34
Do		Horizontal and vertical	do	do	do	do	do	do	Yes	24	2.4	34
DeVries and Courbet compass		do	do	do	do		do	do	Yes	15	1.7	35
Monodep compass		Horizontal	Card	do	do	2 plates	Air	Gimbals	No		1.2	35
Favé air-damped compass		(1)	Magnetic system.	do	Jewel	12 bars	do	do	No	9	5.4	36
GERMAN.												
Kaiserliche marine kompass		Horizontal	Bowl	Alloy	Jewel	2 bars	Alcohol	Gimbals	Yes	25	6.4	37
Ludolph armee kompass	Type I	Horizontal and vertical	do	do	Sapphire	4 bars	do	Bracket	No		2.9	38
Sendtner armee kompass	Type III	Horizontal	Card	do	Jewel		do	Gimbals	Yes	25	4.8	38
Pfadfinder armee kompass	do	do	do	do	do	2 bars	do	do	Yes		5.5	39
Do	Type IV	Horizontal and vertical	do	do	Sapphire		do	Bracket	Yes	25	3.2	39
Pfadfinder kompass für aviatik		Horizontal	Bowl	do	do	2 bars	do	Springs and gimbals	No	25	4.9	40
Remote indicating and remote control compass.		(1)	do	do	Jewel	4 bars	Liquid	Gimbals	No	25	27.1	41
MISCELLANEOUS COMPASSES.												
Bureau of Standard Earth Indicator compass (1)												44
Knudsen compass		Horizontal	Bowl	Alloy	Sapphire	2 bars	Alcohol	Gimbals	No	17	3.5	47
Stoppani double pivot inverted compass		do	Card and bowl	do	Jewel	do	do	do	Yes		3.6	47

[1] See text.

NOTE.—The author is pleased to acknowledge his indebtedness to Mr. K. H. Beij and Mr. C. L. Seward, of the Bureau of Standards, who assisted in the preparation and checking of parts of this paper.

REPORT No. 128.

DIRECTION INSTRUMENTS.

PART IV.

TURN INDICATORS.

By R. C. Sylvander and E. W. Rounds.

SUMMARY.

This part gives a brief history of the development of airplane turn indicators, with detailed descriptions of all known types and makes. The results of laboratory and flight tests are given for the several available gyroscopic turn indicators.

INTRODUCTION.

The turn indicator, which until recently had not come into general use, was early realized to be necessary for flying when no objects outside the plane were visible.

A properly functioning turn indicator shows to the pilot whether the airplane is flying on a straight course or is turning. It is not possible even when using the turn indicator to steer a perfectly straight course under all conditions, as this depends on the smoothness of the air, the manœuverability of the airplane, and the fact that before a turn is indicated it must have already started. However, the use of the turn indicator combined with the lateral inclinometer and air speed meter makes it possible for the pilot to keep the airplane in a safe flying attitude and to make good a desired course within fairly close limits.

The use of inclinometers and pendulums of various types for indicating lateral equilibrium of the airplane have been in use practically from the time when flying was first begun. When the airplane flies on a straight course, these devices serve to indicate turns about the longitudinal axis of the machine. They do not, however, indicate a turn of the airplane from its course.

As early as 1899 a gyroscopic device for indicating angular motion was patented in England by Van Overclift. The fundamental idea of the instrument is that of the modern airplane turn indicator.

It is probable that the first turn indicator intended for aircraft was explained in principle in a memorandum by C. C. Mason and Sir Horace Darwin to the British Advisory Committee for Aeronautics in 1912. Some experiments were made at that time, but it was not until March, 1918, that patent was applied for on the static head turn indicator developed from the above principle. This instrument is now being manufactured by the British Wright Co. and is described below.

In January, 1917, two Frenchmen, J. de Lesseps and R. Courtois-Suffit, patented a differential pressure instrument, using Pitot, Venturi, or other tubes placed one on each wing and as far as possible from the plane of symmetry. This device was intended to show also the air speed of the airplane.

In May, 1917, J. B. Henderson applied for a patent on what seems to have been the first gyroscopic turn indicator. In this instrument precession of the gyro was communicated through bevel gears to a pointer moving over a scale.

At about the same time, Smith and Lindeman developed, in England, the gyroscopic instrument described below as the Royal Aircraft Establishment turn indicator.

During the war the Germans developed and used an electrically driven gyroscopic turn indicator invented by Drexler. A detailed description of this instrument is given in a later paragraph.

Two gyroscopic turn indicators, the Sperry and the Pioneer, are at present being manufactured in this country and are described below.

Apparently the latest British turn indicator is one devised by G. H. Reid. This instrument, according to available information, is of the air-driven gyroscopic type. A turn is shown to the pilot by the lighting up of electric lamps, which obtain their current through a commutating device actuated by precession of the gyro. A mercurial inclinometer is combined with the turn indicator and a series of lamps is lighted by the passage of current between contacts through the mercury in the inclinometer tube.

It is probable that other turn indicators have been constructed, since various methods have been suggested, such as the use of the apparent increase of weight on a turn and the measurement of the difference of electrostatic potential of the wing tips.

In general there are two types of turn indicators in use, the gyroscopic and the differential pressure types.

The gyroscopic turn indicator depends in principle upon the action of a gyroscope which is mounted in such a way that it may precess about only one axis. The turning movement of the plane causes precession about this axis and this precession is indicated on a dial by means of a suitable mechanism.

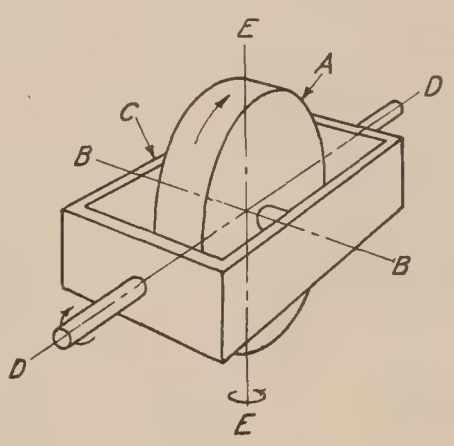


FIGURE 1.

The principle is probably best illustrated by reference to figure 1.

The gyro wheel (A) is mounted on an axis (B–B) which should be athwartships and normally horizontal. The bearings for (B–B) are in a frame (C) which is in turn mounted on the axis (D–D) on bearings in the main case of the instrument. The axis (D–D) should be in the same plane as (B–B) and should be mounted in the aircraft in a fore-and-aft position. The whole unit should be balanced about (D–D); the gyro should be carefully balanced about its axis of rotation (B–B). If mounted as described above the effects of pitching, rolling and accelerations are made negligible.

With the gyro wheel running in the direction shown by the arrow, suppose the plane carrying the instrument to make a turn to the left, as indicated by the arrow about the vertical axis (E–E). The turn will cause precession of the gyro unit about (D–D) as indicated. A turn to the right will cause precession in the opposite direction. The amount of this precession is controlled by a spring system and is usually limited by the frame striking a positive stop. Sufficient motion and power is thus obtained for actuation of the indicating mechanism.

The gyro may be driven by varied means. In one type of turn indicator it is in the form of a windmill and is driven by the air stream directly. In another type it is actuated by air which is drawn past the rotor by the suction of a Venturi tube which is mounted in the air stream, the gyro being mounted where convenient. In still another type the gyro is the rotor of a small induction motor which is driven by a fan generator mounted in the air stream.

The differential pressure turn indicator depends in principle upon the effect of centrifugal force developed on a turn and the difference in static pressure due to change in altitude of two static tubes mounted symmetrically on the extreme ends of the wings, and pivoted in such a way as to head directly into the air stream at all times. The pressures are communicated through tubing to either side of an extremely sensitive differential pressure gage.

The action is probably best explained by the following extract from a paper by Sir Horace Darwin entitled "The Static Head Turn Indicator for Aeroplanes."

"In order to make the action clear, we will assume that the aeroplane is moving in a circle, that it is not banked, and that the air tube connecting the static heads is horizontal and points along a radius of the circle. The forces acting on the air in this tube are:

"1. Gravity acting vertically downwards. As the tube is horizontal this will cause no difference of pressure at the manometer nor cause any tendency of the air to move along the tube.

"2. The atmospheric pressure at the static heads. As the tube is horizontal the pressures at the ends of the tube are equal and in opposite directions, and no effect is produced on the manometer.

"3. The pressure of the inner surface of the tube against the air; this clearly has no effect on the manometer.

"4. Centrifugal force is the one remaining force which can cause a movement of the differential manometer. The air will tend to move along the tube in an outward direction and can only be prevented from so doing by a difference of pressure on the two sides of the diaphragm in the manometer. It is this difference of pressure which is indicated on the manometer and shows a right or left hand turn.

"All turns, however, are banked and this assumption is only made to make the action clear.

"Let us now consider a banked turn and assume that the aeroplane is banked at the correct angle. By the correct angle is meant an angle which causes no side slip; that is, such an angle that the apparent direction of gravity (that is, the resultant of gravity and centrifugal force) is at right angles to the plane of the wings.

"Again consider the forces acting on the air in the tube.

"1. As the banking is at the correct angle, the resultant of gravity and centrifugal force acts at right angles to the direction of the tube and has no effect.

"2. The pressure against the inside of the tube clearly has no effect.

"3. The atmospheric pressure at the two static heads is not equal; as the aeroplane is banked, the outer end is higher up and at a place where the air is at a less pressure. The differential manometer will show this difference of pressure."

An instrument of this type is described in detail below.

AMERICAN TURN INDICATORS.

THE PIONEER TURN INDICATOR.

The Pioneer turn indicator is shown in figures 2 and 3. This instrument depends in principle upon the action, described above, of a small gyroscope which is mounted in such a way as to allow precession only about an axis parallel to the longitudinal axis of the airplane.

The gyroscope is driven by a jet of air drawn into the case through a nozzle by the suction produced in the throat of a double Venturi tube, figure 8, mounted in the air stream.

As with all turn indicators of the gyroscopic type, care must be taken to mount the instrument so that effects of pitching and rolling are negligible.

Turning of the airplane about its vertical axis is indicated by the appearance of a white sector in the triangular-shaped openings of the dial. A turn to the right brings this sector into view in the right-hand openings and similarly, a turn to the left is shown in the left-hand opening.

The indicator weighs, with the Venturi tube, about 2½ pounds.

DESCRIPTION.

Figure 3 shows the mechanism. The gyroscope (A) is mounted in the aluminum frame (B) which is inclosed in the case (C) of the same material. The brass rotor is mounted on two short steel shafts (D) held in brass bushings (E). The shafts are hardened and form the inside races of specially designed ball bearings. The outer race is contained in a recess of the rotor and consists of a steel disk taking the side thrust and a steel ring in which five three-sixteenths-inch steel balls run. A brass disk pressed into the rotor holds the outer race in place, protects

the bearings from dirt, and serves to prevent loss of oil. A felt coil is held between the two bearings and acts as an oil retainer. Four holes in the thrust disk allow the lubricating oil to pass into the bearing.

The bearings are adjusted by screwing the bushings (E) in or out in the frame (B). The clamp screws (G) hold the bushings in place after adjustment.

Oil may be added to the central reservoir through one of the steel shafts (D) and its bushing, which are drilled axially. The removal of a plug (H) in the side of the case permits access to the oil hole.

The frame (B) is supported at each end on sets of special ball bearings at (J) and (K). Each bearing consists of a steel disk and a flanged ring which serve as thrust members, the latter also serving as the inner ball race, and a steel ring which forms the outer race. Twelve one-eighth-inch steel balls complete the bearing unit. All bearing surfaces are hardened.

Fig. 2.—Pioneer turn indicator.

Both bearings are held in position by brass plates, the one at (J) being screwed to the frame (B), while the other at (K) is fastened to the cast aluminum frame (L) which is secured to the forward end of the case (C).

The inner race of the bearing (J) fits over a brass pin or pivot (M) in the rear of the case (C). This pivot is drilled to connect with the intake port in the case and acts as the nozzle directing the jet of air downward onto the buckets (N) cut in the periphery of the rotor.

Precession of the gyroscope about the axis through bearings (J) and (K) is transmitted to the indicating disk (O) by a brass shaft on the end of which the disk screws. A second brass disk (P) is secured to one end of the frame (B). The gyro unit is balanced about the precession

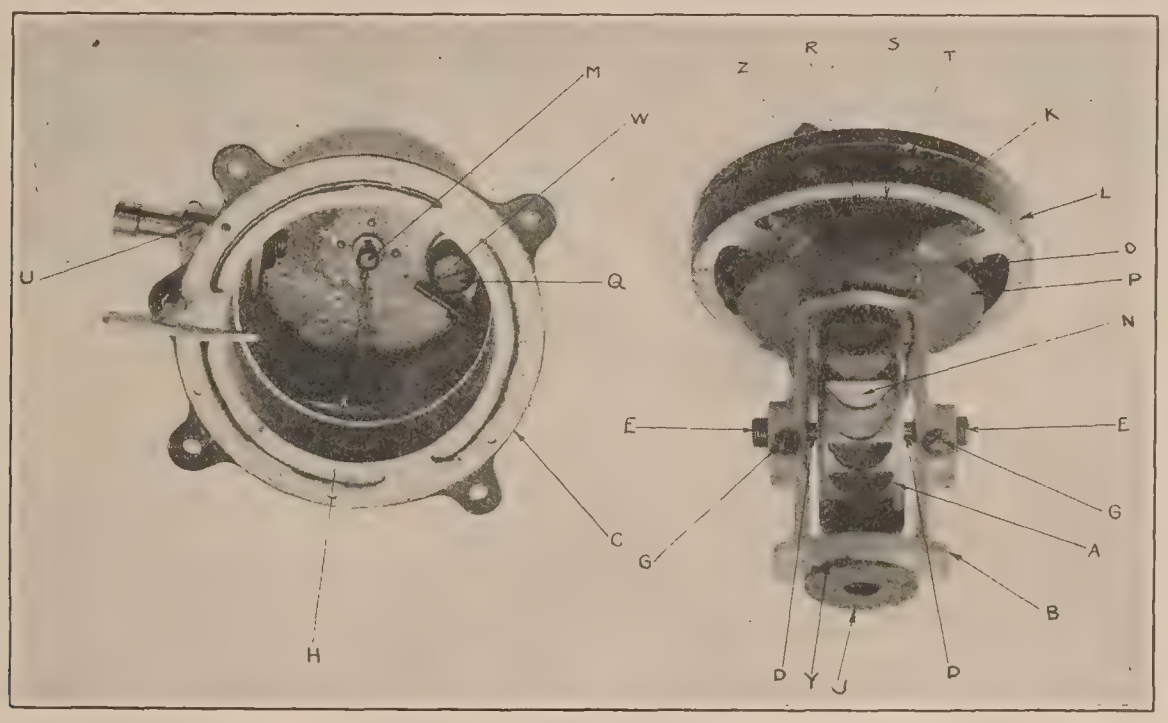


Fig. 3.—Pioneer turn indicator.

axis by means of solder placed on this disk. In an earlier type, instead of the disk (P), a circular reservoir partly filled with liquid was used as a damping device.

A rubber-covered stud (Q) screwed into the case (C) limits the angle of precession.

The gyro unit and indicator are normally centered by a bronze spring (R) which is fixed to the gyro unit at one end, eccentric to the axis of precession, by means of a swivel and at the other by a flat bronze spring (S) attached to the ring (L). A screw (T) changes the position of the strip (S) thus changing the tension of the spring (R) and the sensitivity of the indicator.

The main adjustment of sensitivity is provided by varying the opening of the exhaust valve (U). For convenience, seven positions are indicated on the dial. With the valve wide open maximum air flow and hence maximum speed of the rotor is obtained. Since the precessional force depends on the speed of the rotor, this setting gives maximum sensitivity.

The case is made air-tight by the use of shellac, so that all air flow is through the nozzle. The air intake is covered by a screen held by a cap which may also clamp in place a flexible metal tube for drawing dry air from any desired location in the airplane.

Lubrication of the rotor is described above. The precession bearing (J) may be oiled through a small copper tube (W) after the removal of a screw. The oil from the tube drops into a recess (Y) and flows into the bearing. Removal of the screw (Z) permits oiling of the other bearing (K).

TEST DATA.

Venturi suctions.—The suctions obtained from wind tunnel tests on the Pioneer Venturi tube with the indicator connected and its valve adjusted for maximum air flow are given below. All values are reduced to standard density (15.6° C. temperature and 760 millimeters mercury pressure).

Air speed in miles per hour.	Suction in inches of water.
40	8.4
50	13.2
60	19.5
70	27.1
80	35.9
90	45.0
100	54.9

Indicator calibration.—The calibration of a Pioneer turn indicator with an impressed suction of 19.5 inches of water, which corresponds to that obtained at 60 miles per hour air speed, is given below. The exhaust valve was wide open during the tests.

Per cent of full scale deflection.	Complete turns (360°) per minute.
2	0.05
25	0.16
50	0.38
75	0.68
100	1.15

Calculations show that, the air speed being 60 miles per hour, turns having a radius greater than 3.2 miles will not be shown.

SPERRY TURN INDICATOR.

The Sperry Mark I, Model A, turn indicator is shown in figures 4, 5, 6. The instrument is of the gyroscopic type which is described in principle above.

The gyro of the Sperry instrument is driven by a jet of air impinging upon cups or teeth cut in its periphery. The air enters through a hole in the top of the case and passes by the gyro rotor and out of the case through a tube directly below. This tube is connected to the throat of a double Venturi, shown in figure 7, which is mounted preferably in the slip stream of the propeller. The suction of the Venturi partially evacuates the case which is air-tight, air rushes through the jet, impinges upon the gyro, rotates it and passes out through the tube to the Venturi. The flow of air is sufficient to drive the gyro at a high rate of speed.

The mechanism of the turn indicator is arranged in such a way that when a turn to the right is made a white sector moves into view at the left of the dial. This indicates that the pilot should apply left rudder to bring the plane back to a straight course.

Means are provided for preventing excessive speed of the gyro. At a certain value of the suction a ball valve in the case opens and allows air to enter, reducing the vacuum and lessening the flow of air through the jet.

The instrument is designed to fit into a hole in the airplane instrument board, being held in place by four screws. The weight of the indicator and the Venturi combined is about 2¼ pounds.

DESCRIPTION.

As shown in figure 6 the turn indicator mechanism consists of a brass rotor or gyro (A) mounted in a cast aluminum frame (B). The frame carries a disk (C) on the upper part of which is painted a white sector. The movement of this sector past either of two diametrically opposite openings in the dial shows the direction and roughly the magnitude of the turn. When the white does not show the airplane is supposed to be on a straight course. The instrument does not indicate directly the direction of turn but the direction in which the pilot should steer to correct for the turn; i. e., a turn to the right causes a left deflection which is made zero by moving the rudder to the left.

The gyro is carried on radial ball bearings which are lubricated by oil-soaked cotton waste packed in the aluminum housings (D), holding the special gyro shaft ball bearings. The gyro rotor is built up of three parts riveted together. The gyro shaft, which is in one piece, is of

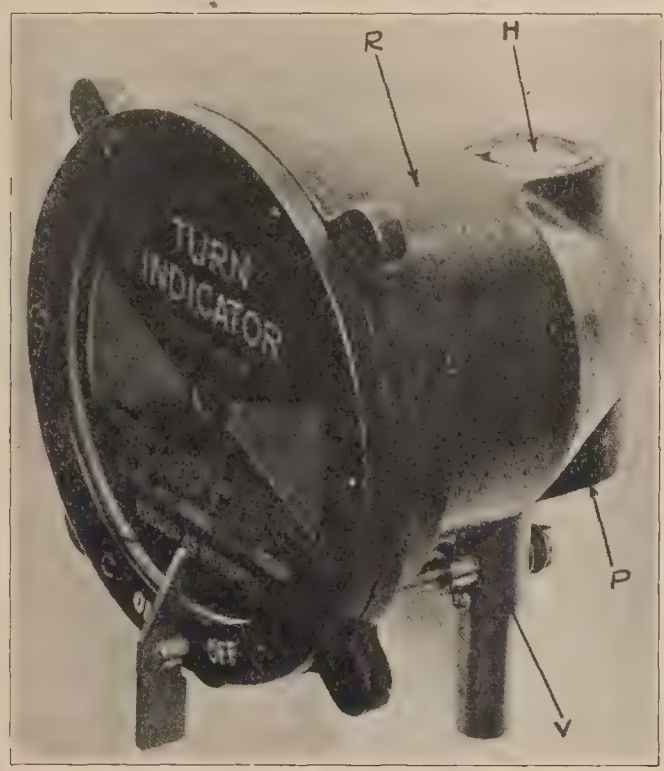


FIG. 4.—Sperry turn indicator.

FIG. 5.—Sperry mechanism.

steel with bearings one-eighth inch diameter. The side play of the gyro is adjusted by means of screws (E) with lock nuts (F), one on each side, the ends of which bear against the ends of the gyro shaft. The aluminum caps (G) are threaded for these adjusting screws and also serve to retain the oil and keep out dirt and moisture.

The gyro has buckets cut in its periphery and is driven by the impact of a jet of air which enters through the screened hole (H) and the nozzle (L) and passes out of the case through the hole (M) and the valve (V) to the throat of the Venturi tube, figure 7. Adjustment of the valve (V) regulates the flow of air and consequently the sensitiveness of the instrument. The valve may be closed and the instrument shut off if desired.

The gyro frame is carried fore-and-aft on steel pivot and cup precession bearings, the pivot (K) being fixed to the back of the case (R) and the pivot (N) to an aluminum bridge (O) which is placed across the front of the case opening. (N) is adjustable and is locked in place by a check nut. The gyro mechanism can easily be removed from the case as a unit, as shown in figure 6.

The gyro is normally held in position by the spring (S), which is so mounted that its tension increases as precession takes place. Too great a motion is prevented by the rubber-covered arms (T) striking the bridge (O).

The sensitivity is not adjustable except by installing a spring of different characteristics.

Protection against suction which would cause excessive gyro speed is provided by a ball valve at (P), figure 4, which automatically allows air to enter the case through the hole (Y) when its evacuation reaches a certain predetermined value, thereby maintaining a steady flow of air and preventing racing of the gyro. The steel ball is held against its seat by a small helical compression spring. As soon as the difference between atmospheric pressure and that inside the case exceeds the strength of the spring the ball moves upward, air rushes in and the difference in pressure tends to reduce to its former value.

TEST DATA.

Venturi suctions.—The suctions obtained from the wind tunnel tests on the Sperry Venturi tube with the indicator connected and running at full flow, are given below. All values are reduced to standard density (15.6° C. temperature and 760 millimeters mercury pressure).

Air speed in miles per hour.	Suction in inches of water.
40	6.0
50	9.7
60	13.8
70	18.0
80	22.4
90	27.6
100	32.8
110	38.5

Indicator calibration.—The calibration of a Sperry turn indicator with an impressed suction of 13.8 inches of water, which corresponds to that obtained from the Venturi at 60 miles per hour air speed, is given below. The exhaust valve was wide open during the tests.

Per cent of full scale deflection.	Complete turns (360°) per minute.
2	0.06
5	0.31
50	0.86
75	2.72
100	4.65

Calculations show that, the air speed being 60 miles per hour, turns greater than 2.7 miles radius would not be indicated.

PIONEER TURN AND PITCHING INDICATOR SYSTEM.

The Pioneer turn and pitching indicator system consists of a turn indicator, a pitching indicator and a power unit for driving the indicators. It is intended for use on dirigibles and is designed to function at air speeds much lower than is possible with the Venturi-driven type of turn indicator.

The turn indicator, shown in the left of figure 9, is the same as the Pioneer turn indicator described in detail above.

The pitching indicator, shown in the right of figure 9 is very similar to the turn indicator except that the sensitive element is so mounted in the case as to indicate departures from a horizontal plane instead of departures from a vertical plane. A downward deviation of the course causes the luminous part of the disk to show in the lower opening of the dial and vice versa. The instrument is provided with similar adjustments and means for oiling as are found in the turn indicator.

Two power plants or suction pumps are shown. The earlier type, figure 10, consists of a laminated wooden propeller (A) mounted in ball bearings. Channels (H) extend from the tips of the blade to the propeller axle which is hollow and connects through a port hole with the hollow tube (C). The tubes (C), (B), and (D) are arranged so as to form a rigid mounting for the propeller; (C) has two connections for flexible tubing, which lead to the indicators. The

air in the channels (H) is thrown outward by centrifugal force as the fan rotates, drawing more air in through the tube (C) and the indicators.

The propeller pump is 24 inches in diameter and weighs approximately 4½ pounds.

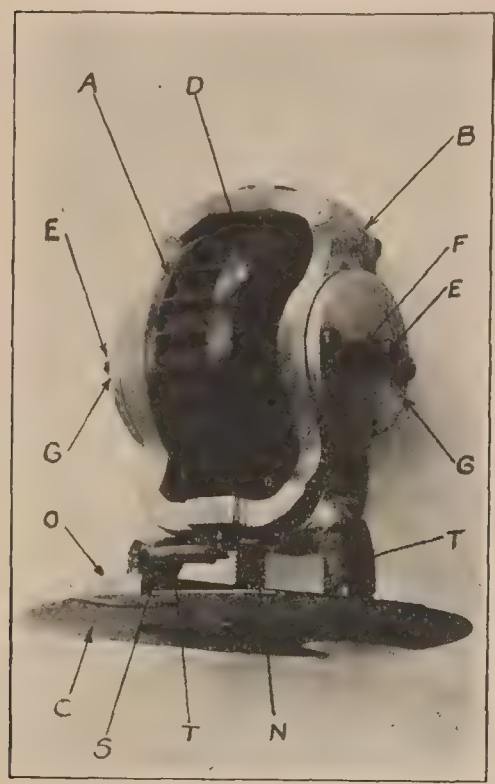


FIG. 6.—Sperry mechanism.

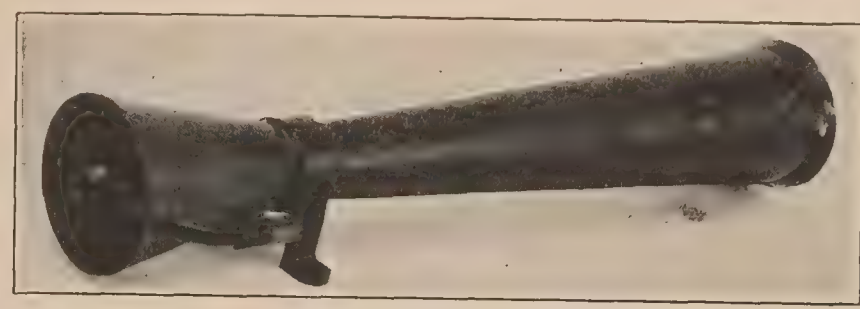


FIG. 7.—Venturi tube for Sperry turn indicator.

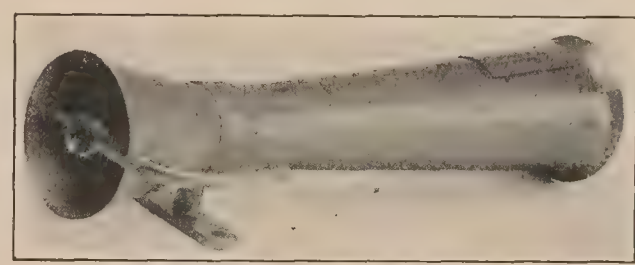


FIG. 8.—Venturi tube for pioneer turn indicator.

The later type, figure 11, consists of an electric pump. A 12-volt direct current series-wound motor in an aluminum case (A) drives a brass four-bladed paddle (B) fixed to its shaft. The paddle (B) forces air out of the case through the groove (C). The flow of air is as follows: through the indicators, flexible tubing to the connections (E) which are fixed to the brass cap (D), and past the motor through the four holes (F) into the paddle case, where it is forced out

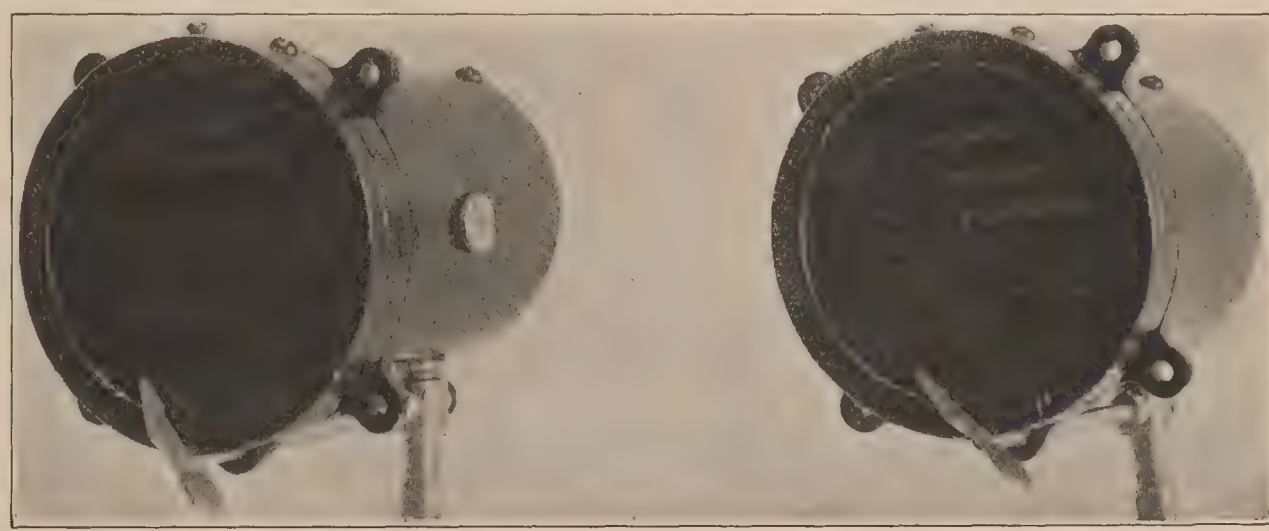


FIG. 9.—Pioneer turn and pitching indicators.

by centrifugal force as described above. The aluminum cover (G) forms the case of the instrument.

The pump stands about 5¾ inches high and the diameter of its base is 7 inches. The outfit weighs about 4½ pounds.

TEST DATA.

Propeller pump.—Preliminary wind tunnel tests showed that the design of the propeller is such that it would rotate at speeds considered unsafe even when the air speed is as low as 25 or 30 miles per hour. The results given below, therefore, are for low air speeds only.

The suctions in inches of water and rotational speeds in revolutions per minute are tabulated below for various air speeds. The air speeds are reduced to standard conditions. During

the tests a turn indicator and a pitching indicator were connected with their valves adjusted for maximum air flow.

Air speed, miles per hour.	Propeller speed, revolutions per minute.	Suction, inches of water.
7.3	610	0.90
9.7	940	2.23
10.9	1,220	3.55
12.5	1,380	4.87
14.4	1,580	6.75
17.0	1,940	9.80
19.6	2,250	13.10
22.7	2,600	17.80

The suction obtained at an air speed of 19.2 miles per hour was sufficient to operate the indicators in a satisfactory manner.

The data indicates that at 60 miles per hour air speed the revolutions per minute of the propeller would be about 7,500.

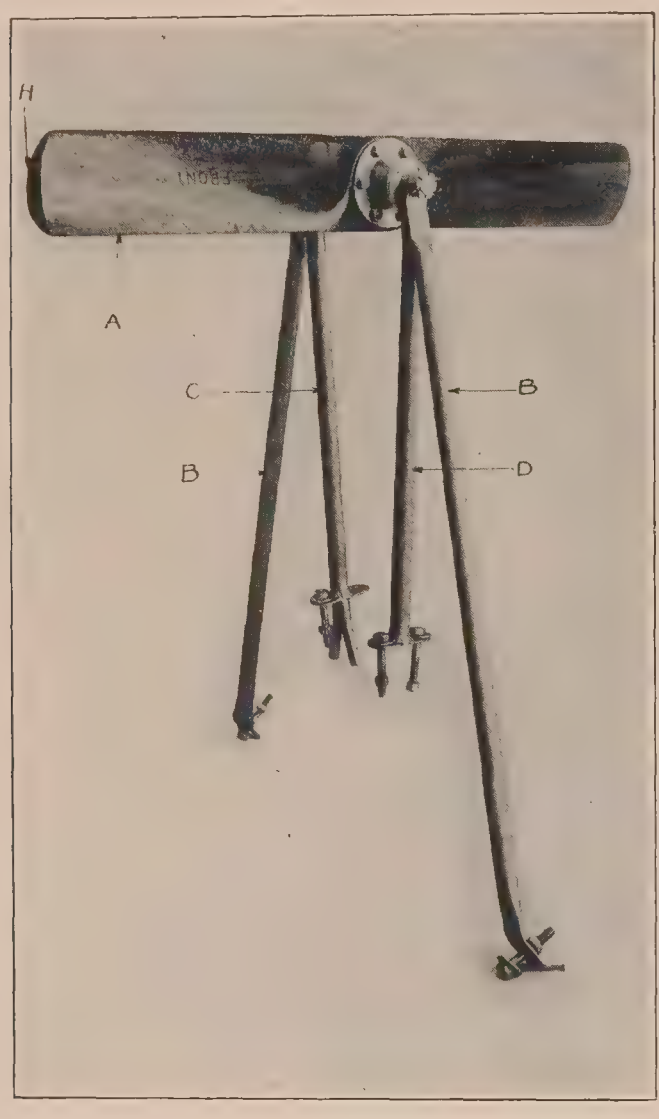


FIG. 10.—Pioneer propeller pump.

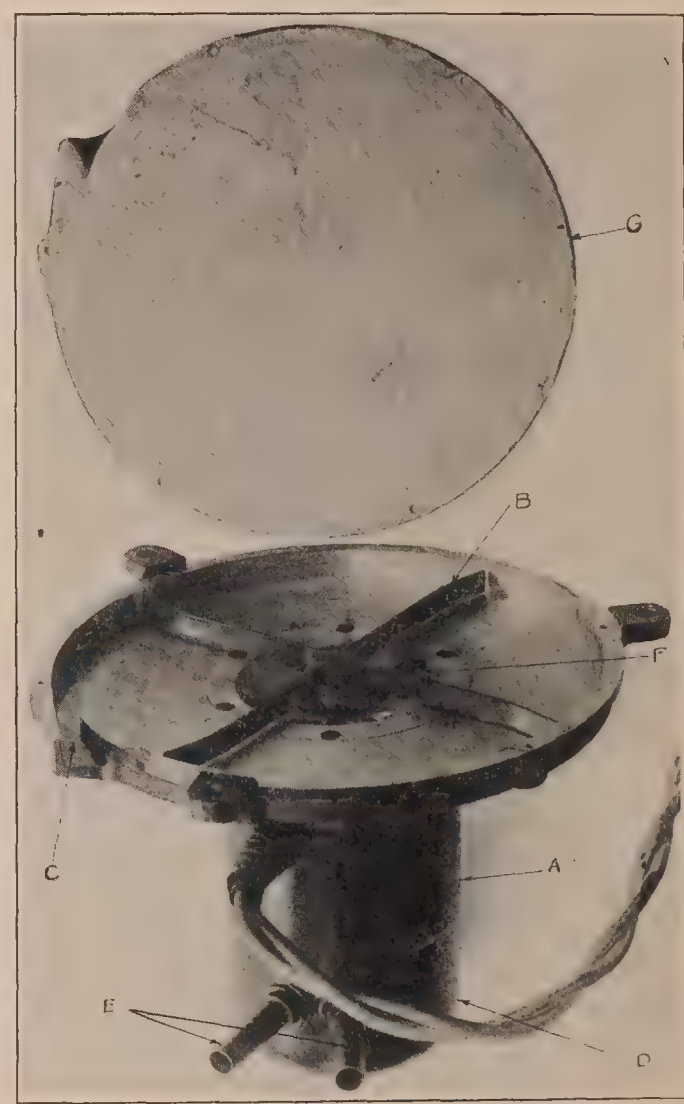


FIG. 11.—Pioneer electric pump.

In a destructive whirling test the propeller broke at a speed of 5,800 revolutions per minute.

Electric pump.—The suctions in inches of water at 26° C. for various voltages are tabulated below, together with the amount of current required. Both turn and pitching indicators were connected and running with their valves adjusted for maximum flow of air.

Volts.	Amperes.	Suction, inches of water.
6.2	6.4	5.9
7.5	6.2	7.1
8.8	6.3	8.3
11.8	6.3	10.6

For best performance of the system the impressed voltage should be about 12 volts.

BUREAU OF CONSTRUCTION AND REPAIR STATIC HEAD.

Figure 12 shows an experimental head developed by the Bureau of Construction and Repair, United States Navy, for use with a differential type turn indicator. Three types of head were made, a Pitot tube, a static tube, and a closed tube. At about the same time the British Wright turn indicator which is based on similar principles was produced in England and consequently the work was abandoned.

BRITISH TURN INDICATORS.

R. A. E. TURN INDICATOR.

The R. A. E. Mark V turn indicator, manufactured by the Royal Aircraft Establishment, is shown in figures 13 and 14.

This instrument is a single unit type, deriving its motive power from the air stream acting directly on the inclined sides of holes in the gyroscope. This necessitates its being mounted close to the side of an airplane, a disadvantage in some types.

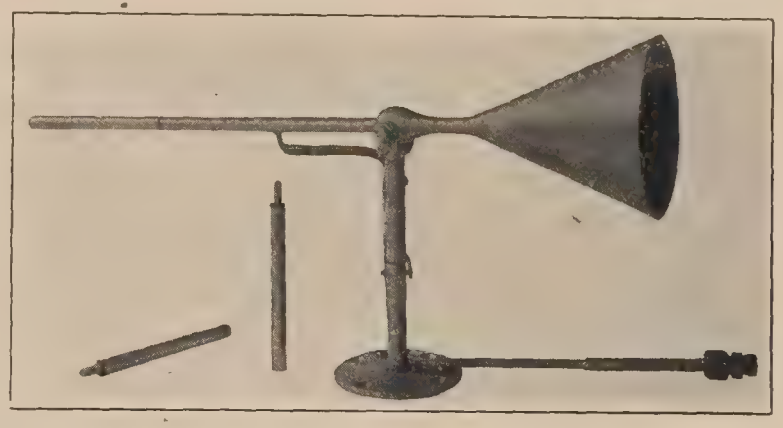


FIG. 12.—Bureau of C. & R. static head for experimental differential pressure type turn indicator.

The overall length of the turn indicator is slightly over 14 inches and its weight is $3\frac{1}{2}$ pounds. A bracket for mounting is clamped around the frame and its weight adds perhaps another half pound to the complete installation.

In mounting the indicator care must be taken to mount it with the tubular portion of the case parallel to the transverse axis of the airplane. If this is not done pitching of the airplane will be shown as a turn on the indicator although with the indicator horizontal and set nearly athwartships the effect of pitching will be small. There is usually no reason for not setting the instrument properly and in service no trouble should be experienced from this source.

Looking forward the gyroscope rotates counter-clockwise, a turn to the right causing precession which turns the pointer to the right of the zero position. A turn to the left similarly moves the pointer to the left of the zero.

A lever, the knob of which appears at the left of the dial, is capable of being set in any one of 11 different positions, giving 11 different degrees of sensitivity. In the notch labeled zero the sensitivity is a maximum, while in notch No. 10 it is a minimum. The zero point is adjustable by turning the bezel ring.

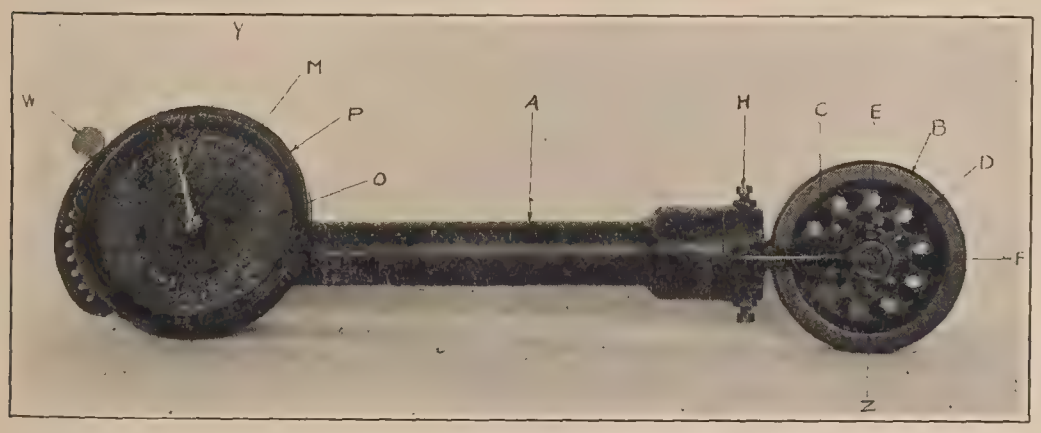


FIG. 13.—R. A. E. turn indicator. Mark V.

DESCRIPTION.

As shown in figures 13 and 14 the instrument consists of a main frame of cast aluminum in which most of the mechanism is housed.

The gyroscope (B) is mounted on two sets of double annular self-aligning ball bearings. These are in turn mounted on a small steel shaft (Z) which is supported in the yoke (C) by two threaded bronze bushings (D). Set screws (E) hold the bushings after being adjusted to the proper position. A brass disk or washer (F) is placed next to the bearings on each side to protect them from the weather.

Two rows of 10 holes each are drilled in the web of the rotor at an angle of 45° with the plane of rotation. It is the action of the air stream on the sides of these holes which produces motion.

The yoke (C), which is of bronze, is held in position on a steel shaft (G) which runs to the head of the instrument, by two set screws (H). The yoke is adjustable about the axis of the shaft.

This steel shaft is mounted at each end in double annular self-aligning ball bearings. The bearing at the yoke end is held in a bronze sleeve fitted into the aluminum case. The bearing is held in place by the position of the yoke. The head end of the shaft is mounted in a similar ball bearing of smaller dimensions, held in position by a bronze plate screwed to the aluminum housing.

A stud (J) is screwed to the plate and is used to limit the angle of precession, as noted below.

On the end of the shaft is screwed a brass flanged disk (K) which positions the shaft in the bearings. This disk gear is held in place by a washer cut to fit the shaft which is flattened on two sides. The washer is pinned to the disk and a brass nut (L) holds it in place.

A hole in the disk fits over the stud mentioned above. The angle of precession is limited to approximately 40° by the sides of the hole striking on the stud (J). The flanged disk (K) has teeth cut in about a quarter of its flange, forming a crown sector, which meshes with a 10-tooth pinion on a shaft (M) carrying the pointer (Y).

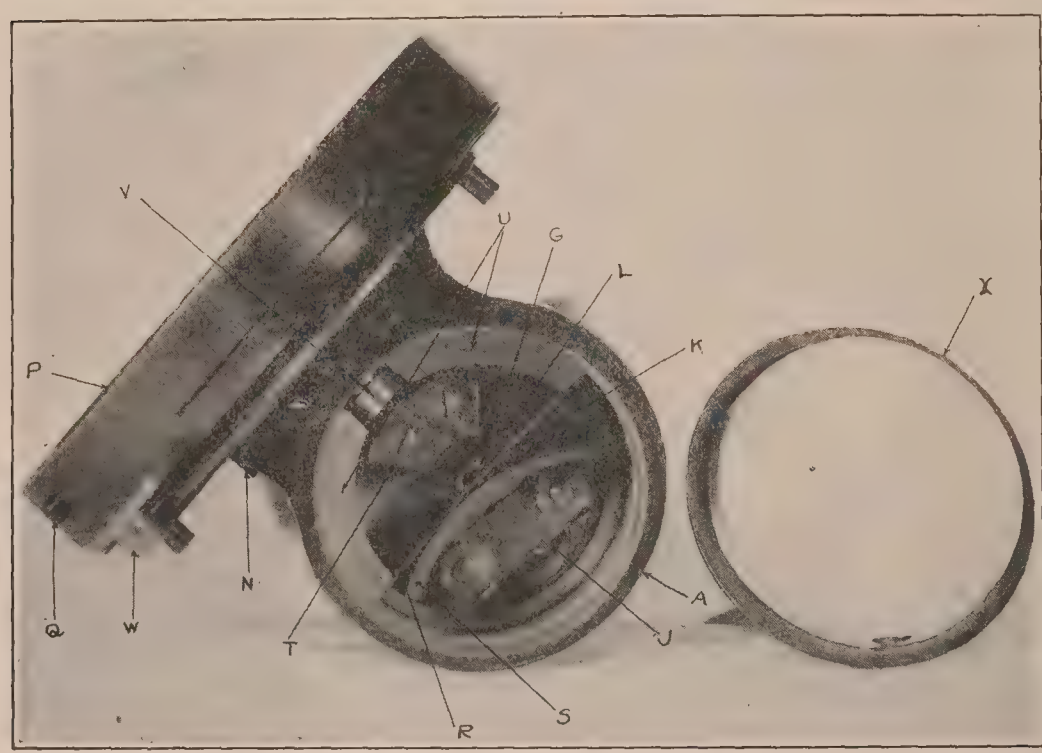


FIG. 14.—R. A. E. turn indicator. Mark V.

The shaft (M), and with it the pointer, rotates in a bronze bushing held in place by a set screw (N). A light wire spring bearing against the shaft tends to prevent chattering. The dial (O) is connected to the bezel ring (P) by a small screw (Q) so that turning the bezel changes the position of the zero.

A flat steel spring (R), shown inside the head of the instrument, acts as a centralizing device and is also used to adjust the sensitivity. The spring bears against a brass sector (S) bolted to the crown gear. The flange of this sector is the arc of a circle whose circumference passes through the axis of precessional rotation. As precession takes place, the line of contact moves off center and a couple is set up tending to retard further precession and return the parts to zero position.

To balance the weight of the flange bearing against the spring, about the center of rotation, a brass counterweight (T) is mounted with its center of gravity diametrically opposite that of the flange. This device neutralizes the effect of acceleration along the longitudinal and vertical axes of the airplane.

The spring is supported at each end by a brass sector (U) sliding circumferentially in a groove in the aluminum case. A symmetrical cam (V) moved by the lever (W) shown at the left of the dial spreads the sectors so that the ends of the spring subtend a greater arc, thus increasing the pressure of the spring against the flange. This action decreases the sensitivity by supplying a greater couple to retard precession.

A light aluminum cover (X) protects the mechanism from the weather.

The mechanism of the instrument is such that when a right turn is made the pointer (Y) moves to the right and vice versa. During the first tests it was found that the position of the pointer changed when the sensitivity was changed. Upon examination the spring which is

used to center the pointer was found to have received a permanent set. This was remedied after which the position of the pointer was unchanged by varying the sensitivity.

All bearing surfaces are either bronze or SKF ball bearings. No particular means for lubrication is supplied but all ball bearings are packed with light grease or vaseline and other bearings oiled.

THE BRITISH WRIGHT TURN INDICATOR.

This aero turn indicator originally known as the Darwin turn indicator, consists of two static heads mounted symmetrically, one on each wing tip, in such a way that they always tend to head into the direction of the relative wind, together with an Ogilvie differential pressure gage mounted in the pilot's cockpit and connected to the static heads by tubing.

An adjustable scale is provided which should be set to the zero mark during straight flight as soon as possible after leaving the ground, as slight changes in the position of the pointer are likely, due to the variation in construction of the static heads and the likelihood that the pressures at the two static heads will not be equal. The variation in the zero position with a properly functioning gage is probably less than 10°.

The action of the static head turn indicator depends on the effect of centrifugal force developed on a turn and the difference in pressure due to change in altitude of the static heads,

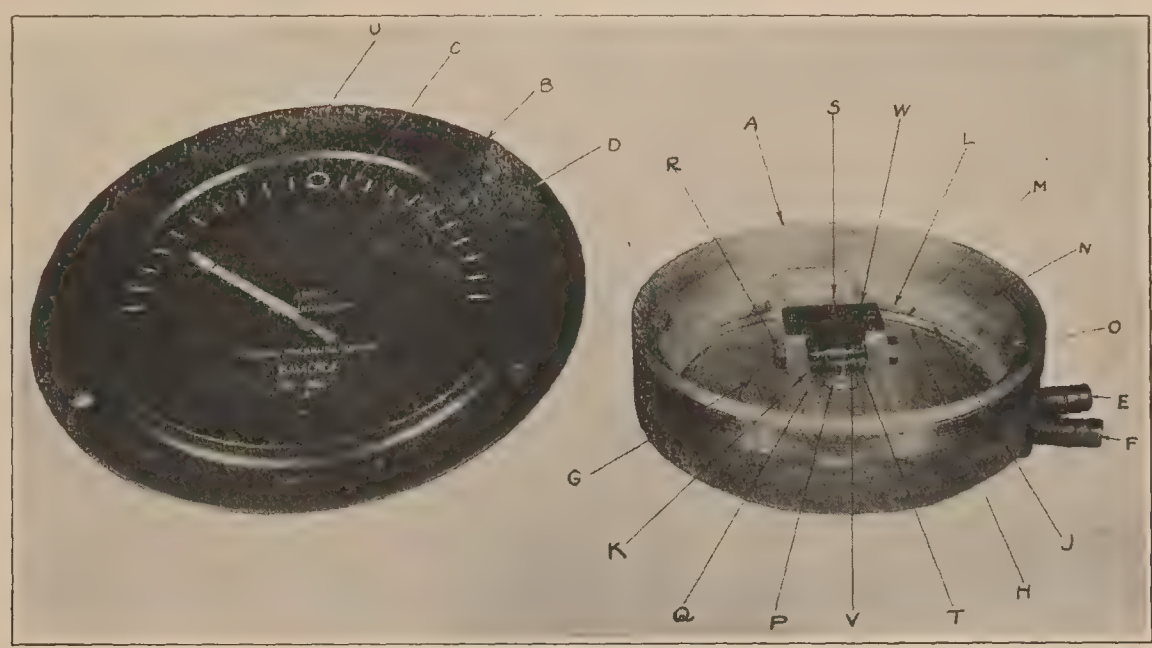


Fig. 15.—British—Wright turn indicator.

as described above. The pressures are transmitted to the gage in such a way that a turn to the right causes the pointer to move to the right of the zero.

The pressure gage is approximately 5⅝ inches in diameter and weighs about 12 ounces while the two static heads with their supports have an overall length of nearly 3 feet and weigh about 3 pounds. To these weights must be added that of the tubing connecting the gage with the static heads.

DESCRIPTION.

The differential pressure gage shown in figure 15 is very similar in construction to the gage used with the Ogilvie air speed indicator. The case consists of a cast aluminum back (A), to which is threaded an aluminum flange (B), in which is held the adjustable scale (C), and the aluminum dial (D) protected by a glass face. A small set screw prevents the flange from unscrewing due to vibration. A rubber gasket between the face, a brass ring and the back (A) makes the case air-tight. Two brass nipples (E) and (F), screwed into the case provide connections for the lines from the static heads.

The sensitive element consists of an india rubber diaphragm (G), held securely against a shoulder in the case by a rubber gasket (H) and aluminum diaphragm ring (J). The ring (J) is in turn held by the cast aluminum frame (K) which is separated from the ring by a rubber

gasket (L) and a paper gasket (M). The frame is held firmly in place by an aluminum ring (N) which screws into the case (A). Two holes (O) are sockets for a special wrench used in turning the ring.

The pressure of the ring (N) is transmitted to the edges of the rubber diaphragm (G) and makes it air-tight. The case in front of the diaphragm is made air-tight by the tightening of the flange (B) against the rubber gasket mentioned above.

Thus the case is divided into two air-tight chambers separated by the rubber diaphragm (G). The nipple (E) leads into the chamber behind the diaphragm and is connected to the static head mounted on the starboard wing tip. (F) leads through a hole in the rim of frame (K) into the chamber forward of the diaphragms and is connected to the port static head.

Cemented to the diaphragm near its center is a small aluminum disk (P) with a small steel hook attached to it. Over this hook is looped one end of a silk thread which runs over a guide roller (Q) supported in two brass bushings (R) screwed into the aluminum cross members of

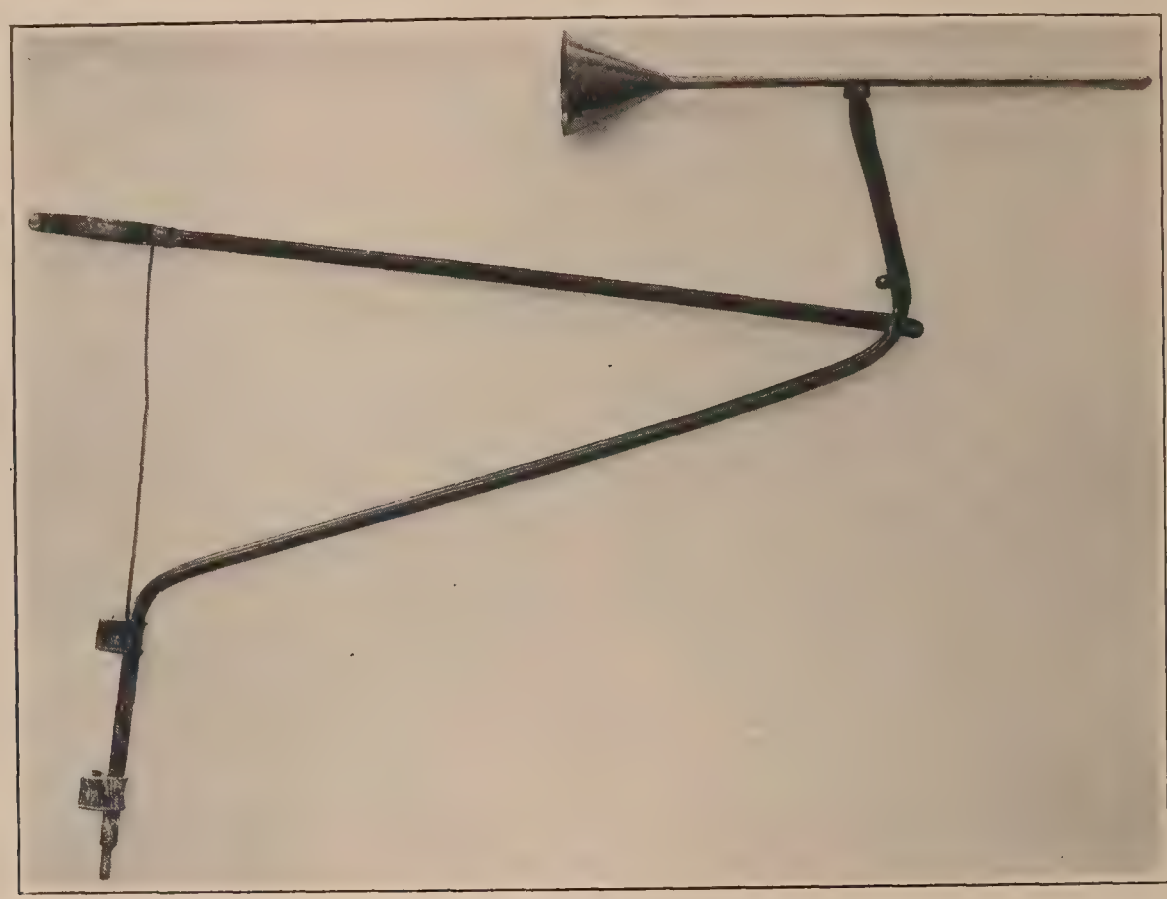


FIG. 16.—British—Wright static head.

the frame (K). The other end of the thread is knotted through a hole in the pointer spindle (S). A small hairspring (T) resists rotation of the spindle with enough force to keep the thread taut.

Motion of the rubber diaphragm is communicated to the pointer (U) by means of the silk thread. When the diaphragm moves forward the slack in the thread is taken up by the tension of the hairspring.

The pointer spindle (S) is carried in two bearings which consist of a brass bushing screwed into the aluminum cross member (V) which is part of frame (K), and the brass plate (W) screwed to the frame (K).

No lubrication of the instrument is necessary as the bearings have plenty of clearance, the hairspring taking care of any possible back lash. Oil would, of course, tend to spoil the rubber diaphragm.

Two static heads such as are shown in figure 16 are mounted, one on each side of the airplane axis, on an A-shaped bracket which fits on the outboard forward strut of the airplane, The head itself consists of a three-eighths inch brass tube closed at the forward end with a stream line plug.

Twenty-four holes of one-thirty-second inch diameter are grouped in four rows around the circumference of the tube. The rows are one-quarter inch apart and start 3 inches from the end of the tube, the air flow here being parallel to the sides of the tube.

The rear of the tube is also plugged and a copper cone with its base aft furnishes directional stability, so that the tube tends to always head into the direction of the relative wind.

The head is hinged at its center of gravity to a square bronze shaft which pivots in the end of a bronze spindle attached to the bracket supporting the head, allowing through certain limits a universal joint action.

Around the square shaft and attached to the head is secured a copper flange on which a flexible rubber tube is made fast extending down and around the supporting spindle to a copper tube which transmits the static pressure to the gage on the instrument board. The rubber tube serves two purposes, providing a flexible connection holding the pivot in its place and carrying the pressure to the gage.

The brackets holding the static heads are of triangular form, the copper transmission tube forming the lower side, a steel tube the upper, and a brass strip parallel to the strut making up the base. Metal straps and saddles hold the brackets in place on the struts.

GERMAN TURN INDICATOR.

DREXLER AIRCRAFT STEERING GAGE.

The Drexler aircraft steering gage is shown in figures 17 and 18. It consists essentially of an electrically driven gyroscope connected with the pointer in such a way that when a turn is made to the right the precession of the gyro moves the pointer to the right of its zero position. Similarly, when a turn is made to the left, the pointer moves to the left.

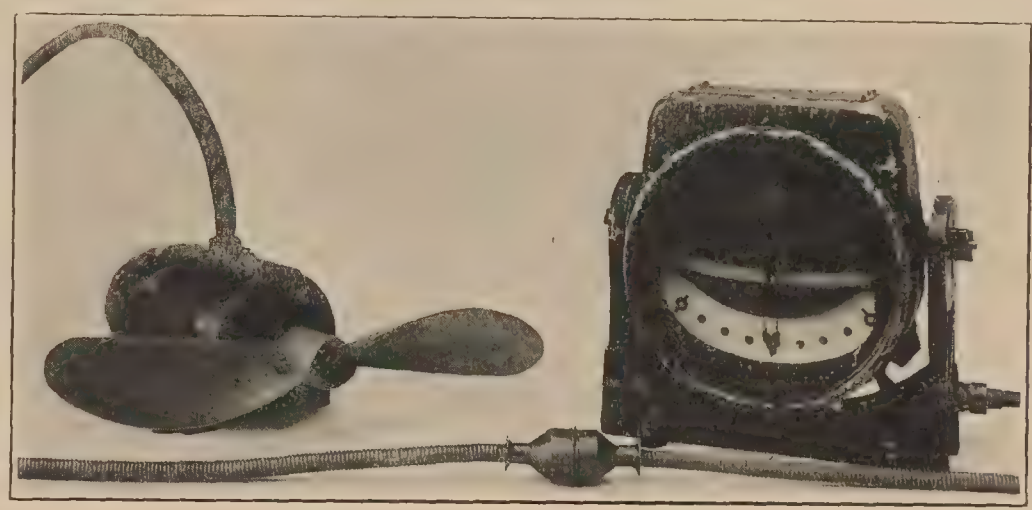


FIG. 17.—Drexler steering gauge-assembly.

An inclinometer, consisting of a glass tube which contains a steel ball whose motion is damped by a liquid, is mounted on the face of the instrument above the scale. The inclinometer is interconnected with the gyroscope in such a way that precession of the gyro may also tilt it.

As in the usual types of inclinometers, the ball is in the center of the tube when the plane is in lateral equilibrium.

It is the claim of the manufacturer that "many flying machines, according to their construction, should in turns not be inclined at the actual angle corresponding to the theoretical deflection of a liquid or solid pendulum, which, when flying in curves, will adjust itself according to the resultant of the centrifugal force and gravity."

Since, for all practical purposes, the proper angle of bank depends only on the speed and radius of the turn, and not on the weight or type of airplane, the interconnection of the gyro and inclinometer is a needless complication.

Further information seems to indicate that the tilting of the inclinometer with precession of the gyro was intended to be used in training students. Thus, if a student tended always to underbank his machine, the inclinometer would be set to indicate a less bank then actually was being made, influencing the flyer to bank his plane still more. This could readily lead to dangerous positions, and its practice should not be tolerated.

No need or good reason for interconnecting the gyroscope and inclinometer can be seen. However, mounting the inclinometer close to the turn indicator is of great advantage.

The gyroscope is a three-phase induction motor with short-circuited rotor, the current being supplied from a streamlined generator driven by a windmill in the air stream.

The sensitivity is not adjustable, but the motion of the pointer is damped hydraulically, as described below.

A light is mounted inside the case and derives its current from the motor circuit.

The indicator itself stands with its bracket about 8½ inches high and is of about the same width. Its weight is 8½ pounds. The generator weighs 5¼ pounds, making the total installation weight with windmill and connecting cable about 15¾ pounds.

Springs are provided in the bracket to absorb vibration.

Modifications of this turn indicator are made combining in one unit an altimeter, an air speed meter, an inclinometer, and the turn indicator. The altimeter is of the usual aneroid type.

The air speed meter, however, depends for its indication on the number of revolutions made by the generator, and consists of a frequency meter of the vibrating reed type. As the air speed changes the rate of rotation of the generator windmill follows, and thus the frequency of the current driving the gyroscope changes with the speed of the plane. The frequency meter

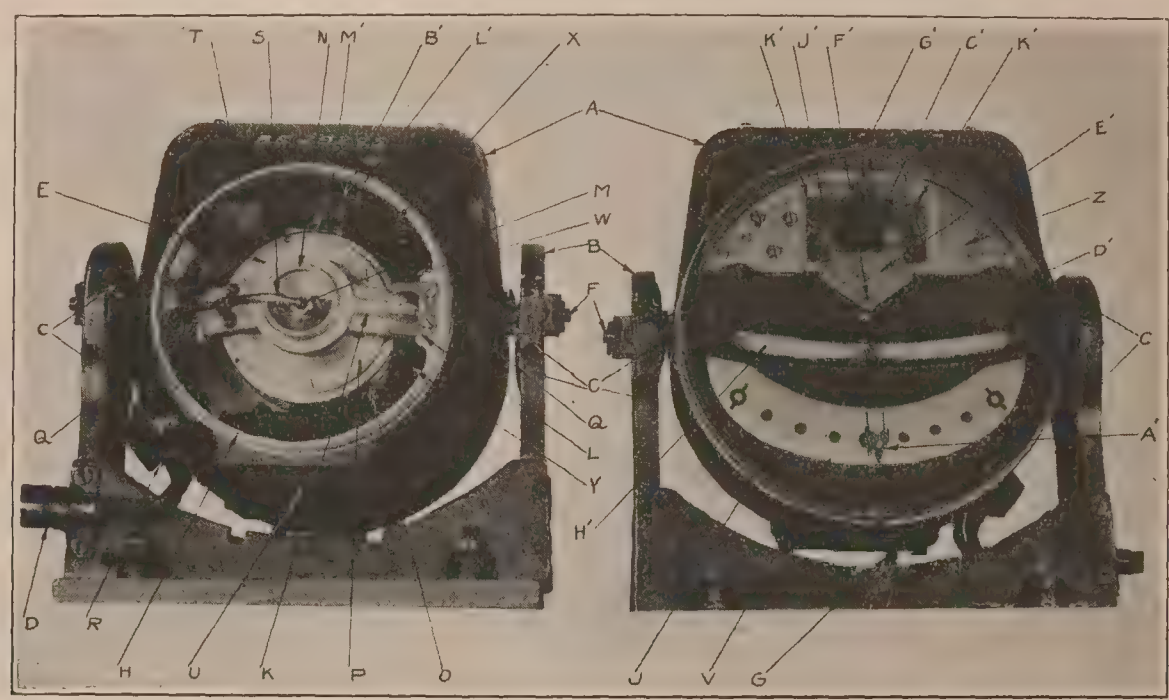


FIG. 18.—Drexler steering gauge.

is calibrated in kilometers per hour. Considerable lag is to be expected in a speed indicator of this type. The frequency obtained in normal flight should be about 300 cycles per second at the normal speed of the plane. This is obtained by adjusting the pitch of the windmill blades or changing propellers until the rotational speed is about 4,500 revolutions per minute.

When the generator is used merely to drive the turn indicator, it may be mounted either inside or outside the slip stream, but when used as a speed indicator it should always be mounted away from the effect of the slip stream.

DESCRIPTION OF TYPE KB.

The turn indicator mechanism shown in figure 18 is inclosed in a cast aluminum case (A) and is mounted in brackets (B), also of cast aluminum. A system of steel springs (C) tends to absorb small vibrations of the airplane. Four lugs with bolt holes are provided for securing the instrument. The armored cable leading to the generator is rigidly fastened by a nut (D) to the base, from which flexible wires run to the internal mechanism.

The case is carried on trunnions (F). A projecting fork (G) at the bottom engages a rod which acts as a pivot for the support holding the trunnions, thus preventing rotation of the case.

A pressed aluminum cover is screwed to a flange (H) at the rear of the case, while the front is closed by the glass held in place by an aluminum ring. The glass is packed by a tubular rubber gasket (J), which prevents breakage by vibration.

The case, with back and front covered, is practically tight against the weather, although water could enter if the instrument were submerged.

The main element inside the case is the gyroscope (E), which is mounted with its axis parallel to the longitudinal axis of the plane in a frame (K) free to turn only about an axis parallel to the transverse axis of the airplane. The angle through which this rotation or precession can take place is limited to less than 10° by a steel stop (L) screwed to the frame.

The gyroscope is the rotor of a three-phase alternating current induction motor revolving about a stationary field. The rotor is mounted on two annular ball bearings supported by a fixed steel shaft (M) through the hollow end of which pass the leads of the motor circuit. Two steel bushings (N), which screw into the aluminum frame (K), hold the shaft in position. Lock nuts prevent the shaft from turning in the bushings.

The gyroscope rotor (E) consists of a heavy steel rim (O) in which is secured a copper ring concentric with it and next to the pole faces of the stator.

Two metal disks, one on each side, are screwed to the rim and hold it in place. The outer ball races are mounted in the disks. All surfaces are finished and polished.

The precession bearings (Q) consist of steel bushings in the gyro frame bearing on hardened pivots with ball and point ends.

These pivots screw through the outer case and are held in position by lock nuts.

The motor leads enter through the outer case at (R), pass with a small amount of slack into a brass tube (S) screwed to the gyro frame which guides them to the hollow shaft (M) mentioned above.

Leads from one of the phases of the motor lines run to the primary coils of a small transformer (T) mounted in top of the case. The secondary coils of the transformer are connected through a small lamp bulb set in a recess below the gyro, supplying light on the scale and inclinometer tube. The lamp is reached through a small plate (U) screwed to the bottom of the case and is held in place between two spring contacts. A semicircular shaped piece of white translucent glass (V) is mounted directly in front of the lamp. The turn indicator scale is painted on the glass, and consists merely of three symbols denoting the center or zero and the two extremes, each side being divided into four sections by three round dots.

Precession of the gyro is transmitted to the pointer through the following mechanism: A vertical steel rod (W) connects the gyro frame to a system of diaphragms (X) which are mounted in the yoke (B′). A small steel plate screwed to B′ at (L′) has a slot which engages a hardened steel pin projecting from the pointer hub (M′). This pin is eccentric to the pointer bearings which are of the pivot type. Motion of the yoke (B′) thus causes a rotation of the pointer (A′), the whole mechanism being arranged in such a way that a turn to the right moves the pointer to the right.

The diaphragms (X), which are six in number, serve through their spring action as a centralizing device for the gyroscope. The system is supported on a brass angle which is inserted between the third and fourth cells and which is fixed to the back of the cast aluminum plate (Z). All six diaphragms are apparently filled with liquid, the three disks on each side of the support acting as reservoirs connected together by a small opening. When precession takes place the liquid flows from one reservoir to another giving a damping action similar to that of a dash pot.

The frame (K) is positioned on the rod (W) by two nuts each bearing on the frame through two springs (X) and hemispherical washers which fit into recesses. When the nuts are properly adjusted the gyro frame is centered in the stop (L). In one model the pointer may be set to zero position by lateral adjustment of the precession bearings. In model K B I, the latest available, the pointer is adjustable from outside the case by a screw which raises or lowers the diaphragm unit.

An arm attached to the pointer is fitted with a vertical slot whose center is on the pivot axis. A pin which holds the inclinometer bracket arm (C′) in place fits in this slot. When the pin is on the pivot axis no tilting of the inclinometer as mentioned above can take place. However, with the pin raised or lowered in the slot the inclinometer bracket (D′), which is pivoted

to the brass slide (E′) at (F′), is forced to tilt one way or the other. The up-and-down motion is produced by turning a screw (G′) from above. The thrust of the screw is taken by a flange bearing in a recess in the case. A stud (J′) on the slide which is held in place by two flat steel springs (K′) limits the vertical adjustment. Up or down motion of the slide (E′) changes the position of the pin with respect to the axis of the pointer. The inclinometer (H′) which is held to the bracket (D′) by two steel spring clips is a liquid-filled tube of glass, curved concave upward with a steel ball in it. When the plane is in lateral equilibrium the ball should be in the center. The action is such that when the pin is above the center of rotation of the pointer the pilot must overbank the airplane to keep the ball in the center of the tube.

No provision is made for oiling any of the bearings, but oil should be used sparingly on all contact surfaces and the ball bearings should be packed in light grease or vaseline.

Looking forward at the turn indicator, the gyro rotates counter-clockwise at a speed of 15,000 to 20,000 revolutions per minute.

The instrument should be mounted with the axis of precession horizontal, as otherwise pitching of the plane will show as a turn.

The wind-driven generator used for driving the turn indicator is of the three-phase rotating field type. The rotor, which is of rugged construction suitable for high speeds, consists of a cyclindrical iron core coaxial with the shaft, surrounded by a single circular magnetizing coil. At each end of the coil is a four-armed iron spider. The eight arms of the spider extend over the coil and intermesh so as to form alternate poles of opposite sign. Surrounding the rotor is the laminated stator wound with 12 coils from which run the 3 wires contained in the flexible armored lead to the indicator.

On the same shaft with the rotating field structure above described is the winding forming the armature of a direct-current generator which excites the field of the alternator. The field of the direct-current generator is bipolar and is carried on the same structure as the alternator stator.

The shaft also carries the commutator of the exciter and two collector rings to take the direct current to the rotating field of the alternator. The current flows in series through the latter and through the field circuit of the exciter.

The generator is driven under normal conditions about 4,500 revolutions per minute by a wooden fan or windmill whose pitch depends on the speed of the airplane. Generator speeds and voltages at various air speeds are tabulated below for windmills of 300 and 400 millimeters pitch.

TEST DATA.

Generator characteristics.—The generator speeds in revolutions per minute and voltages are tabulated for various air speeds. Two propellers were used, No. 1 of 300 millimeters and No. 2 of 400 millimeters pitch. The indicator was connected and running during the tests. On account of the rapid rate at which the voltage increased with the increasing air speed no tests were run at generator speeds over 5,000 revolutions per minute.

Propeller No. 1.			Propeller No. 2.		
Air speed.	Generator speed.	Voltage.	Air speed.	Generator speed.	Voltage.
12.7	1,000	.....	18.6	1,090	2
17.2	1,460	2	27.9	1,860	6
23.1	2,125	7	38.5	2,625	12.5
32.4	2,910	14	46.0	3,190	17.0
38.4	3,405	20	51.1	3,450	23.4
41.5	3,640	26	54.4	3,750	33.0
42.6	3,700	28	58.1	3,990	53.0
45.4	3,875	48	58.3	4,050	69.0
46.3	3,960	62	60.4	4,225	.....
49.3	4,200	73	63.0	4,325	.....
51.9	4,385	87	63.2	4,335	76.0
53.9	4,525	.....	64.7	4,460	81.5
.....	.....	.....	67.4	4,630	.....
.....	.....	.....	69.1	4,760	96.5

As seen, the speed of the generator when driven by propeller No. 1 is about 31 per cent greater for the same air speeds than when driven by propeller No. 2. The air speeds corresponding to generator speeds of 5,000 revolutions per minute are 56 miles per hour and 73 miles per hour for propellers Nos. 1 and 2, respectively.

Calibration of turn indicator.—Laboratory tests showed that with a generator speed of 3,880 revolutions per minute the corresponding air speeds being 43.0 miles per hour for propeller No. 1 and 56.5 miles per hour for propeller No. 2, a rate of turn of 0.027 revolutions per minute gave a sensible deflection of the pointer. However, a rate of turn of 0.078 revolutions per minute was required to give a deflection sufficiently great for certainty of observation under practical conditions.

The necessary rates of turn for various rate deflections are tabulated below. The generator was driven at a speed of 4,100 revolutions per minute, which corresponds to that obtained at 60 miles per hour air speed.

Per cent scale deflection.	Complete turns (360°) per minute.
0	0.027 (or less)
25	0.51
50	0.89
75	1.29
100	1.70

Briefly, the air speed being 60 miles per hour, turns greater than 5.9 miles radius are not indicated, and turns of 0.31, 0.18, 0.12, and 0.094 miles radius correspond to pointer deflections of 25, 50, 75, and 100 per cent full scale, respectively.

No tests were made on the banking indicator.

FLIGHT TESTS.

Flight tests were made on the Sperry, Pioneer, R. A. E., and Drexler turn indicators.

The R. A. E. indicator has an unnecessarily large range of adjustment, that from points 6 to 10, inclusive, being of little use, due to the small movement of the pointer and the high centralizing force. Points 5 to 0, inclusive, give sufficient adjustment for all purposes. The Drexler has no sensitivity adjustment, but its damping action is such that its initial setting is satisfactory for all types of aircraft.

It was found desirable to mount the Venturi tube which drives the Sperry and Pioneer in the slip stream of aircraft of low speed range, while on faster machines, mounting in the air stream out of the propeller blast was satisfactory.

The R. A. E. should be mounted to get free air flow at the rotor, although the instrument functions when placed in disturbed flow behind a strut.

The generator of the Drexler instrument should be mounted outside of the slip stream particularly if it is also to be used as a speed indicator.

The Venturi-driven gyroscopes may also be connected to the intake manifold of the engine close to the intake port. This installation was tried on a Hispano-Suiza 180 horsepower and on a Liberty 400-horsepower engine. No interference in the functioning of the engines at speeds over 600 revolutions per minute was noticeable, although the Liberty at idling speeds of 200–300 revolutions per minute tended to fire irregularly in the three cylinders affected. If this installation is used a valve for closing the tube to the instrument should be provided for use in starting.

The degree of sensitivity desirable seems to depend more on the manœuverability of the plane than on its size and weight. It is probable that a turn indicator satisfactory for a small fast manœuvering scout will be satisfactory for all other types,

REPORT No. 129

AERONAUTIC INSTRUMENTS

SECTION V

POWER PLANT INSTRUMENTS

IN FIVE PARTS

AERONAUTIC INSTRUMENTS SECTION

Bureau of Standards

REPORT No. 129.

POWER PLANT INSTRUMENTS.

PART I.

AIRPLANE TACHOMETERS.

By G. E. Washburn.

INTRODUCTION.

This report is Section V of a series of reports on aeronautic instruments (Technical Reports Nos. 125 to 132, inclusive) prepared by the Aeronautic Instruments Section of the Bureau of Standards under research authorizations formulated and recommended by the Subcommittee on Aerodynamics and approved by the National Advisory Committee for Aeronautics. Much of the material contained in this report was made available through the cooperation of the War and Navy Departments.

SUMMARY.

This part gives a fairly complete discussion of all the various types of airplane tachometers studied at the Bureau of Standards. French, German, and American chronometric tachometers are described in detail; also several types of foreign and domestic centrifugal tachometers. These types, chronometric and centrifugal, were those most extensively used by the Allied and Entente forces. In addition, a description is given of various tachometers of the electric, air viscosity, air-leak, magnetic, mercury viscosity, and liquid centrifugal types which have been used to some extent on aircraft.

USES OF AIRPLANE TACHOMETERS.

The airplane tachometer shows how fast the crank or propeller shaft of the engine is revolving. As a rule it indicates the revolutions per minute, or revolutions per minute of the shaft. It is driven usually by a flexible cable running from the engine to the instrument board.

The tachometer is often spoken of as the "revolution indicator," or "rev. indicator." Revolution indicators, however, show revolutions only, whereas tachometers show speed or rate of revolution.

Tachometers should be distinguished also from speedometers. The latter, though the same in principle, are used for a different purpose; namely, to show the speed of automobiles over the ground in miles per hour.

The principal use of the tachometer is as a detector of engine trouble. Engine trouble of any kind results in a slowing down of the engine. The tachometer, therefore, shows at all times, quickly and surely, whether or not the engine is working properly. The importance of knowing this is apparent, since an airplane depends on its engine, not only for propulsion but also for actual support or maintenance of level.

Tachometers are also used in adjusting the engine to its speed of greatest efficiency, and in performance tests.

Experienced aviators occasionally dispense with the tachometer as well as other instruments. To take the air, however, without first consulting the tachometer is to neglect a simple precaution and is foolhardy. In times of emergency the tachometer may be a great help, and, if working properly, is more reliable in all cases than the senses.

GENERAL INVESTIGATION OF AIRPLANE TACHOMETERS.

Apparatus was designed for investigating and testing tachometers under airplane conditions, such as vibration, change of temperature, and reduced air pressure. Detailed descriptions of apparatus and test methods are given in a later section of this report.

Type tests were made on six types of tachometers with the following results:

1. Chronometric tachometers, which measure speed by recording the amount of motion in a fixed-time interval showed exceedingly small errors throughout but had relatively low durability.

2. Centrifugals, in which the amount of deformation of a spring by centrifugal force indicates the speed, were not at all affected by reduced air pressure and not seriously by change of temperature. The calibration error and the lag were rather large and increased with continued running. Complete breakdowns, however, rarely occurred.

3. Air viscosity tachometers, which act by the viscosity of a thin air film between two concentric cylinders, were not much affected by reduced air pressure, but the temperature error was high. Heat caused an increase, cold a decrease in the reading. The calibration and lag errors were moderate.

4. Air pump tachometers, in which the speed is indicated by the pressure generated by an air pump operated at a speed proportional to the driving speed, were very seriously affected by reduced air pressure. The effect was nearly linear and about 20 per cent at one-half atmosphere (20,000 feet).

5. The magnetic tachometers, depending on the electromagnetic induction between a revolving magnet and a conducting drum or disk, were unaffected by reduced air pressure. They were, however, inaccurate in calibration, strongly affected by change in temperature and inconsistent with running.

6. The electric tachometers, consisting of a magneto used with a voltmeter graduated in revolutions per minute, had fair accuracy, but showed irregular fluctuations and a rather large temperature effect, besides being rather heavy.

Acceptance tests were made on about 300 instruments of the chronometric and centrifugal types, adoped by the Army and Navy, and taken from quantity production. As in the type tests, the chronometrics were found much superior in numerical errors, but inferior in endurance.

An experimental and theoretical investigation was made to improve the centrifugal type which, in view of its simplicity and freedom from breakdowns, seemed especially suitable for military use. A study was made also of master tachometers, of both old and new types, for accurate quantity testing and of special apparatus for rapidly calibrating the same.

TYPES OF AIRPLANE TACHOMETERS.

Airplane tachometers are the same in principle and construction as automobile speedometers; for the speedometer of an automobile, being connected to the forward axle of the machine, records primarily the speed of revolution of this axle and so is really a tachometer. However, because of the more severe conditions and requirements, there are fewer satisfactory types of airplane tachometers than of automobile speedometers.

Tachometers and speedometers are based on simple and well-known principles. Following is a classification of the various types of tachometers together with a brief statement of the principle on which each depends. Afterwards detailed descriptions of individual makes are given.

Chronometric or escapement tachometers.—The speed is measured by the motion of a gear (or toothed rack) in equal intervals of time during which it is connected with the main drive. Since the time intervals are equal, being regulated by an escapement mechanism, the motion of the gear during each interval is proportional to and, therefore, measures the average speed during the interval. The motion of the gear is shown by a pointer moving over a dial graduated in equivalent speeds of revolution.

CHRONOMETRIC TACHOMETERS.

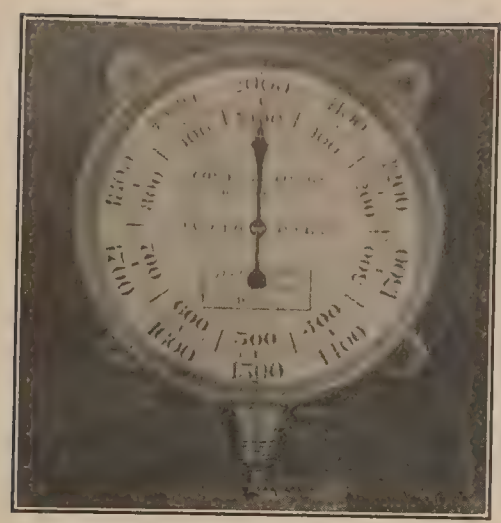


FIG. 1.—Jaeger.

FIG. 2.—Van Sicklen.

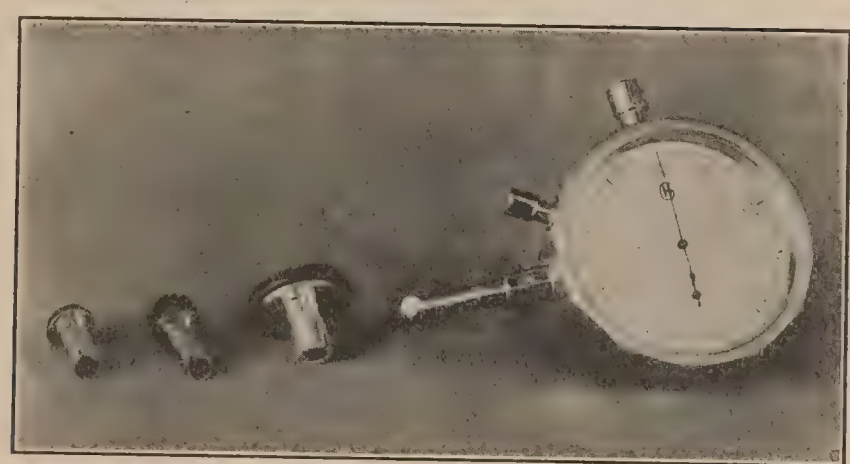


FIG 3.—Hasler.

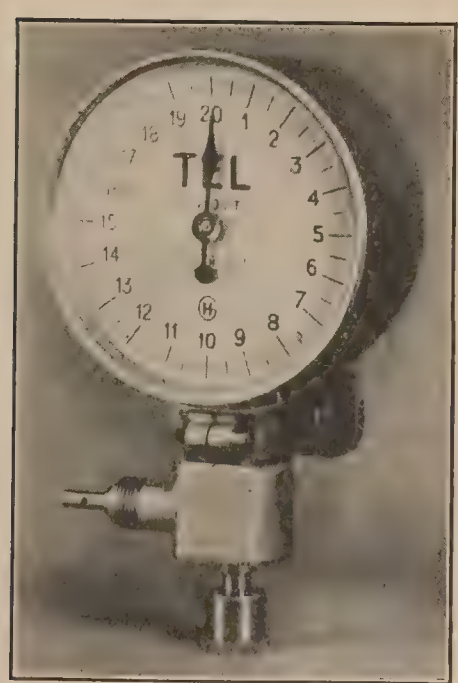


FIG. 4.—French Tel.

FIG. 5.—American Tel.

The pointer in instruments of this type is locked in position most of the time, changes in speed being indicated by sudden jumps at the ends of the equal time periods. This and the beating of the escapement mechanism, used to regulate the length of the time periods, are the distinguishing marks of this type of tachometer. Figures 1 to 5 show the exteriors of a few chronometric tachometers. Several instruments of this type are described in detail in the section entitled "Chronometric Tachometers."

Centrifugal tachometers.—Centrifugal force or the tendency of a body to fly away from the axis of rotation, which depends on the speed of rotation, acts against the elastic force of a spring. The amount of deformation of the spring determines the motion of the pointer and thus indicates the speed. The deflection, as distinguished from that of the chronometric instruments, is continuous, but in existing types, is not proportional to the speed. In liquid centrifugal tachometers the centrifugal pressure is balanced against hydrostatic pressure. Descriptions of a number of centrifugal tachometers are given in detail below in the section entitled "Descriptions of Centrifugal Tachometers."

Air drag or viscosity tachometers.—A cylinder geared to the main drive exerts a turning force on another cylinder concentric with the first through the viscosity of the thin air film between them. This force acts against a control spring deflecting the pointer by an amount depending on the speed. The deflection, like that of the centrifugals, is continuous, but not proportional to the speed. The Waltham type of air viscosity tachometer is described near the end of this part under the title "Air Drag or Viscosity Tachometers."

Air-pump or air-leak tachometers.—A pump, connected to the main drive, forces air into a chamber with a leak orifice. The pressure thus generated deflects a vane controlled by a spring. The deflection is read off on a scale graduated in corresponding speeds of revolution. A detailed description of the Van Sicklen speedometer is given later in this part under the title "Air-Pump or Air-Leak Tachometers."

Magneto or electric tachometers.—The electro-motive force or voltage of a magneto depends on the speed of revolution of the armature. Hence, a magneto used with a properly graduated milli-volt meter will show speeds of rotation. The Tetco electric tachometer is described in the section entitled "Magneto or Electric Tachometers," near the end of this part.

Magnetic tachometers.—A permanent magnet is revolved near an electrically conducting disk or drum mounted on the same spindle with the pointer and controlled by a spring. In virtue of the electric currents induced in the disk or drum a turning force is exerted on it which deflects it by an amount dependent on the speed. The deflection is read as usual on a scale suitably graduated in speeds of revolution. Two instruments of this type are described in detail near the end of this part under the title "Magnetic Tachometers."

Mercury viscosity tachometers.—The viscous drag of mercury rotating in a steel cylinder tends to carry with it a concentrically mounted steel disk and pointer. The force is balanced by means of one or more control springs. The deflection of the pointer is read on a scale suitably graduated in speeds of revolution. A description of the Atmo type of mercury viscosity tachometer is given later in this part under the title "Mercury Viscosity Tachometers."

Liquid centrifugal tachometers.—A paddle wheel is rotated in a liquid forcing it by means of centrifugal force through a valve into a system of vertical glass tubes. The height of the liquid column indicates the speed of revolution. A description of the Veeder type of liquid centrifugal tachometer may be found under "Liquid Centrifugal Tachometers" at the end of this part.

AMERICAN MILITARY AIRPLANE TACHOMETERS.

These are of the chronometric and centrifugal types. The magnetic type, used to a certain extent before the war, has been abandoned for the present.

These instruments, of whatever type or make, are required to be driven directly without adapter from the cam shaft of the airplane motor. They indicate, however, as stated above, the speed of the crank shaft or double the cam shaft speed. As a rule, therefore, American military airplane tachometers, if driven at a given speed, will indicate twice that speed.

VAN SICKLEN CHRONOMETRIC TACHOMETER.

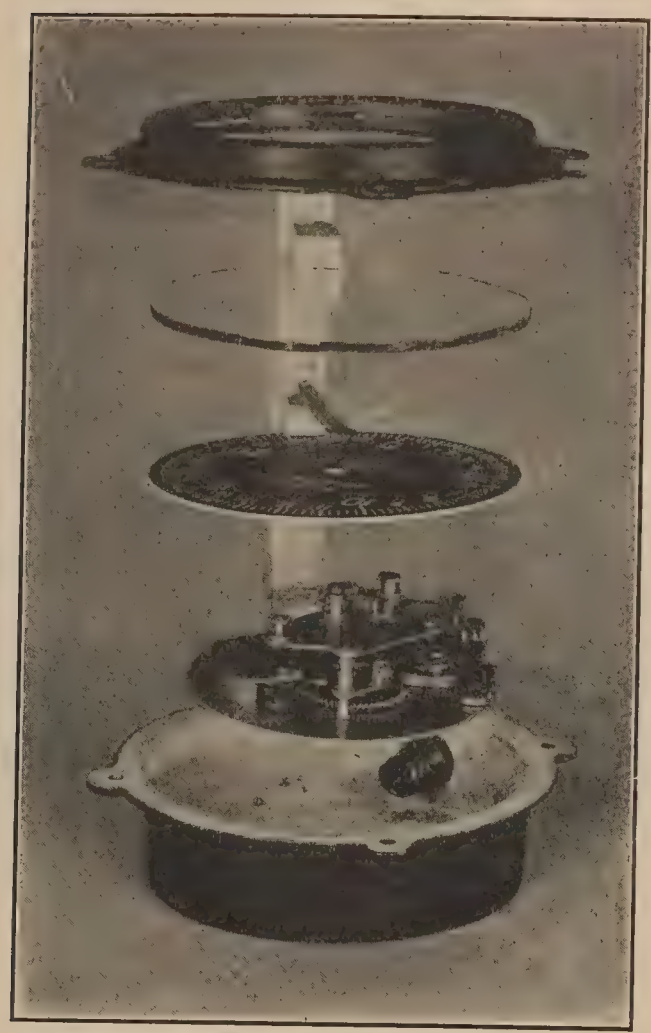


FIG. 6.—Case and Mechanism.

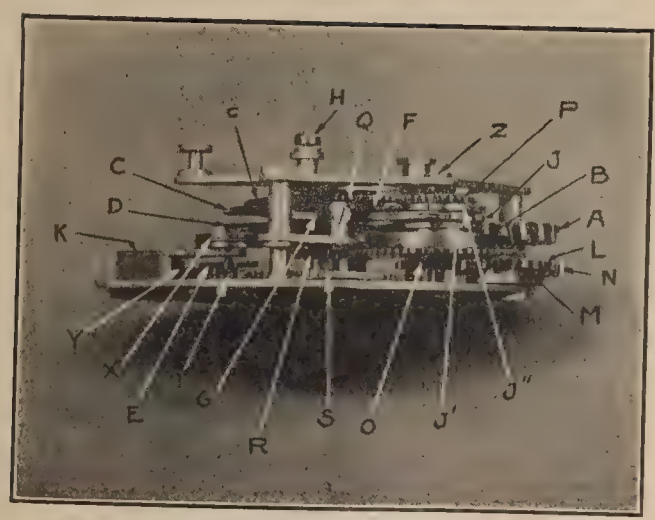


FIG. 7.—Mechanism—Side View.

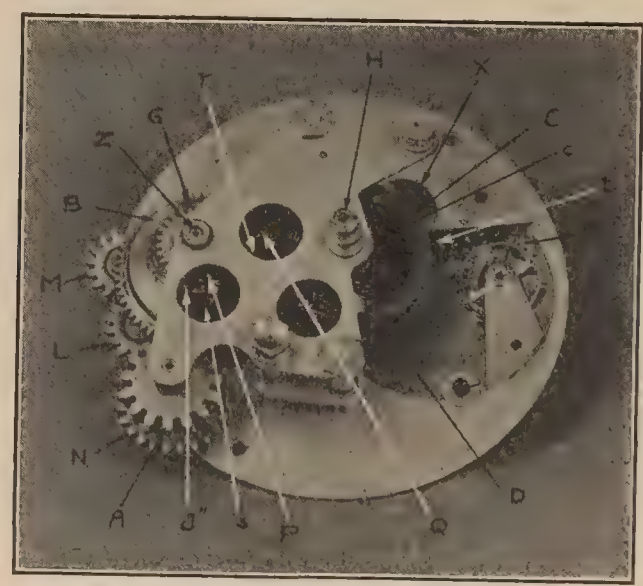


FIG. 8.—Mechanism—Perspective.

Nevertheless, in certain cases, this requirement has been waived and the use of a gear box on the end of the cam shaft, between the cam shaft and the flexible drive, allowed. This arrangement gives greater steadiness, but the adapter is an added complication and undesirable.

The bezels and flexible shaft connections are for the most part standard, independent of type or make, so that instruments are interchangeable. The dials are graduated from 0, 300, 400, or 500 to 2,400, 2,500, or 2,600 revolutions per minute in intervals of 20 or 50 revolutions per minute. The figures on the dial denote hundreds of revolutions per minute. The 0, 5, 10, 15, and 20, as well as the tip of the pointer, are made luminous and the dial plate blackened for night reading.

In the following descriptions of individual makes those used on American airplanes are treated first and in greatest detail.

VAN SICKLEN CHRONOMETRIC TACHOMETER.

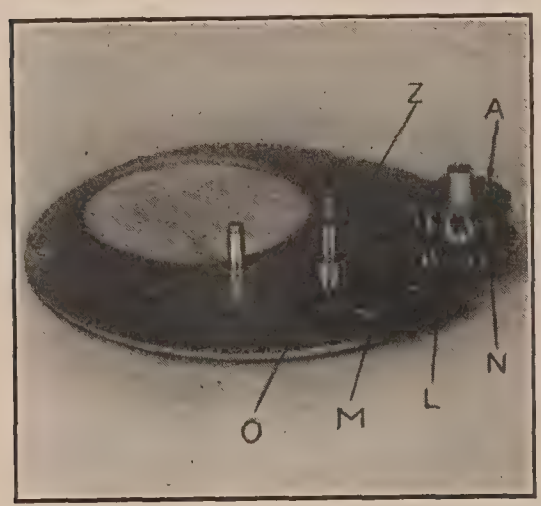


FIG. 9.—Drive System.

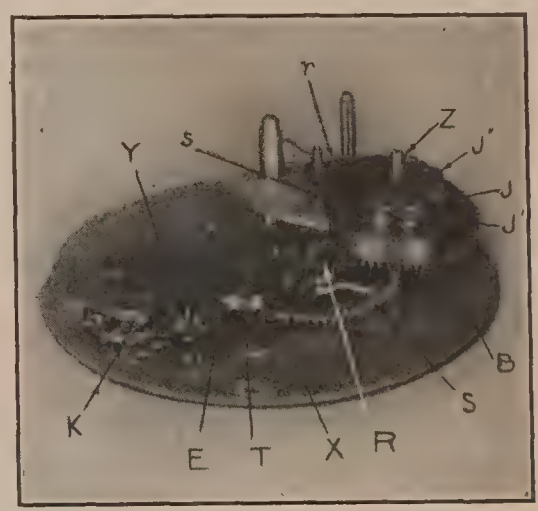


FIG. 10.—Escapement—Cam System.

FIG. 11.—Counting System.

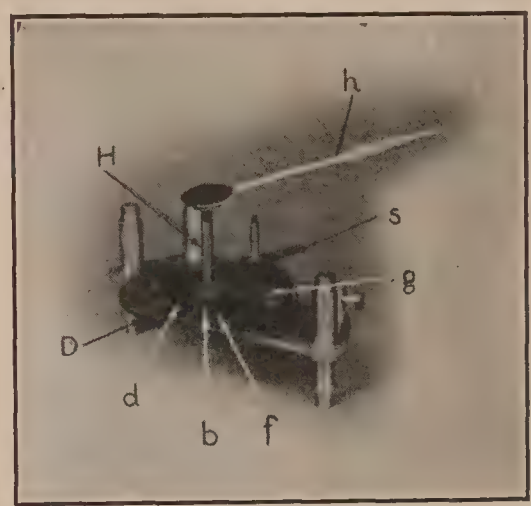


FIG. 12.—Indicating System.

CHRONOMETRIC TACHOMETERS.

Van Sicklen.—This instrument, "Type C" of the American military airplane tachometers, is a simplification and, in some respects, an improvement over the Jaeger chronometric tachometer described below. The complete instrument has already been shown in figure 2. Figures 6 to 8 show the case and mechanism. The gear referred to above, which is connected with the engine for equal intervals of time and the motion of which measures the speed of the engine, is the gear (C) called the counter gear. (K) is the escapement which regulates the length of time intervals.

The mechanism may be divided into the (1) drive, (2) escapement-cam, (3) counting, and (4) indicating systems (figs. 9 to 12). The drive system drives the counting and escapement-cam system. The escapement-cam system controls the operation of the counting and indicating systems. The motion of the counter gear is registered on the dial by the indicating system.

The drive system, figures 7, 8, and 9, begins with the main drive gear (A). It is composed of the gears (N), (L), (M), and (O). Its immediate purpose is to drive the spindle (Z) to which (O) is fixed and from which the counting and escapement-cam systems receive their motive power.

The gears (L) and (M) form a reversing mechanism which automatically provides for a rotation of (Z) in the same direction regardless of the direction of drive. This is necessary because the counting and escapement-cam systems operate in one direction only. The method of reversing is as follows: Gears (L) and (M) are mounted on a rocker arm pinned in the center in such a way that when gear (A), and consequently (N), rotates in one direction clockwise as seen in figure 8, (L) meshes with (O) and (M) idles without meshing with (O). When, however, (A) rotates in the opposite direction, (M) meshes with (O) and (L) merely drives (M) idly. (O) and (Z) rotate in the same direction in either case.

The escapement-cam system, figures 7, 8, and 10, is composed of the cams (J), (J′), and (J″), the toothed barrel (B) to which the cams are fixed, the gears (R), (S), and (T), the fly (E), and the double-roller escapement (K).

(B) receives from (Z), as described below, the motive power for the system. (R), (S), (T), and (E) transmit this power to the escapement which is driven thereby and allows the entire system, including the cams, to move suddenly at regular intervals. The operations of the counting and indicating systems, which are controlled directly by the cams, thus occur in a definite time order.

The releasing is done through (E), which engages a star pinion (Y) on the same pivot with the escape wheel. (E) has two arms and rotates through 180° at each release. The force of impact of the fly is lessened by the inertia wheel (X), which grips by friction the pivot to which (E) is attached.

The motion of the system being fixed, and that of (Z) variable, a slip drive must be used. This is in the form of a spring, called the mainspring, coiled up tightly inside of (B). The inner end of the spring is fastened to (Z). (B), on the other hand, idles on (Z) and also has no rigid connection with the spring. Consequently, as (Z) revolves, the spring slips around in (B). It exerts, however, on (B), through the friction caused by its tendency to uncoil, a turning force which drives the system. This force acts instantly when the instrument is started and continues at all driving speeds. The counting system, figures 7, 8, and 11, consists of the gear (Q), the fine toothed pinion (F) fixed to (Q), and the counter gear (C). The system is driven by the gear (P) fixed to (Z) and meshing with (Q). (F) and (Q) are mounted on a rocker arm (G), which the cam (J) causes to oscillate about a pivot (W) toward and away from (C). (F) is thus thrown alternately into and out of mesh with (C). (Q), however, remains in mesh with (P). The effect is, therefore, to put (C) successively into and out of connection with (Q) and hence with the main drive (A).

This occurs at regular intervals, in fact every second, since (J) is equally spaced and moves, as pointed out above, at regular intervals. The angle, through which (C) is rotated during each second is, therefore, proportional to the speed during the second. (C) is provided with a control spring (*c*) and a projecting stud (*a*) on its upper and under sides respectively. It is locked and released by the toothed pawl (*r*) operated by the cam (J″).

The indicating system (figs. 7, 8, and 12) is formed by the so-called pointer gear (D), similar to and below (C), the pointer spindle (H), to which (D) is fixed but on which (C) idles, the floating arms (*f*) and (*g*) pivoted on (H) between (C) and (D), and the pointer (*h*). Locking and releasing of (D) is accomplished by a toothed pawl (*s*) similar to and directly below (*r*) and operated by the cam (J). Unlike (C), (D) has ratchet instead of V teeth and can move forward while in contact with (*s*). It is provided with a control spring (*d*) similar to (*c*) and a projecting stud (*b*) on its upper side.

(*b*), (*f*), (*a*), (*g*), and a fixed stop (*m*) on (G) are arranged so as to engage each other in the order named. The engaging of (*a*), (*g*), and (*m*) stops (C) in a certain position. The engaging of (*b*), (*f*), and (*a*) holds (C) and (D) fixed with reference to each other.

The use of the floating arms (*f*) and (*g*), instead of direct contact of (*a*), (*b*), and (*m*), enables nearly two complete revolutions of (D), thus making a full circumference dial possible and insuring against injury in case of overspeeding.

When the instrument is idle, the control springs (*c*) and (*d*) cause (*b*), (*f*), (*a*), (*g*), and (*m*) to engage in the above manner. (C) and (D) thus assume definite zero positions.

In the operation of the instrument (C) and (D) are rotated away from (*m*) against the force of their control springs by (F) which drives (C) and hence (D) through the engaging of (*a*), (*f*), and (*b*). They are rotated back toward (*m*) by their control springs. By means of (*r*) (C) may be held when out of mesh with (F). Similarly (*s*) serves to hold (D) independently of (C). (C) turns back and forth continuously, returning to its zero position at regular intervals. (D), however, turns only when a change in speed occurs, forward for an increase, backward for a decrease.

If (D) is not in the zero position with reference to (C), that is, the position in which (*a*) and (*b*) are in contact with (*f*), it will always move back into that position, when free to do so, by the action of its control spring. The angular deflection of (D) and (*h*) from their zero positions is then equal to that of (C), which, as seen, is proportional to the speed. Accordingly, so long as the period of the escapement does not vary, the scale of this instrument is uniform.

The cycle of operations is determined by the cams (J), (J′), and (J″). At the start (C) is in its zero position and unlocked. (D) and (*h*) are locked in the position which they assumed in the preceding cycle. (F) meshes with (C) for one second, turning it through a certain angle. According as the speed is (1) the same as (2) greater than or (3) less than in the last cycle, (C) (1) just reaches (D) or (2) engages (D) and pushes it forward or (3) stops short of (D). In any case (*r*) locks (C) in its extreme position, (*s*) then releases (D), allowing it in case (3) to assume the zero position relative to (C). Next (C) is again released and returns to its zero position. The cycle then repeats itself.

The pointer (*h*) follows the motions of (D). It is locked at a reading equal to the speed during a given second of mesh of (F) with (C) from the end of that second to the end of the next. It then moves forward or backward suddenly by an amount equal to the change in speed. This instrument, like others of the chronometric type, therefore, deflects intermittently, indicating the average speed over an interval of time rather than the speed at each instant.

The scale is closed and graduated from 0 to 2,500 revolutions per minute in intervals of 20 revolutions per minute. The instrument is driven directly from the cam shaft without adapter.

Compared with foreign chronometrics, the Van Sicklen has only one counter gear, the reversing mechanism is simpler and the escapement of considerably smaller size. The first is a simplification, but necessitates the instrument remaining idle for part of the time. Worth mentioning is the method of fastening the pointer which is driven onto a square boss and held by a spring washer to prevent slippage from the sudden jumping of the pointer in tachometers of the chronometric type.

Tel.—This instrument, used by the American military forces and known as "Type A," is shown in figure 5. It is a copy of the French instrument shown in figure 4. Figures 13 to 16 show the mechanism. It contains drive, escapement, counting and indicating systems which perform the same functions as in the Van Sicklen. However, the counting member is a toothed rack instead of a gear. Also, the driving and locking devices are not brought into connection with the counter, but the counter with them by a motion at right angles to its counting motion.

The drive system, figure 17, consists of the main drive gear (A), fastened to the arbor (S) and meshing with a pinion on the drive spindle of the instrument, the gear (N), also attached to (S), and the gears (L), (M), (O) and (P). Its function is to turn the spindle (Z), figures 18 and 19, to which (P) is fixed and from which the counting and escapement systems are driven.

(Z) must revolve in the same direction independent of the direction of drive. This is accomplished by means of a reversing mechanism formed by the gears (N), (L), (M), and (O). (L) and (M) are mounted on a rocker (T), idling on (S), and are in permanent mesh with (N). (O) is pivoted on a stud fastened to the frame and is in mesh with (P). According as (A)

rotates clockwise or counterclockwise, the friction between (S) and (T), which is increased by the slip spring (D), rotates (T) slightly one way or the other and causes (L) to mesh with (P), (M), and (O) idling, or (M) with (O), (L) idling. The direction of rotation of (P) is the same in either case. The screw head (E), playing in the slot (G), serves as a stop for (T).

The escapement system (fig. 18) is composed of the toothed barrel (B), the shaft (J), and the double-roller escapement (K). (J) has fastened to it at one end the gear (R) meshing with (B), at the other end the escape wheel (U). The motive power for the system, which is applied, as in the Van Sicklen, by means of a slip spring, called the main spring, fastened to (Z) and coiled up inside of (B), is transmitted through (P) and (J) to (K), which releases it and allows the system, including (J), to move suddenly at regular intervals. The time for one

TEL CHRONOMETRIC TACHOMETER.

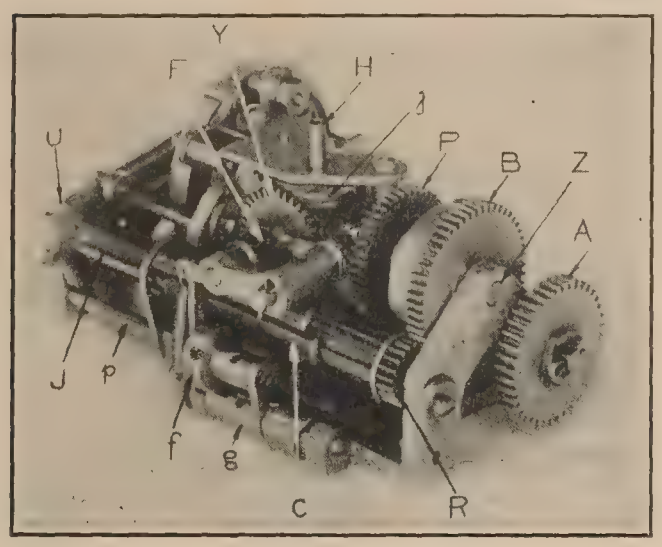


Fig. 13.—Perspective.

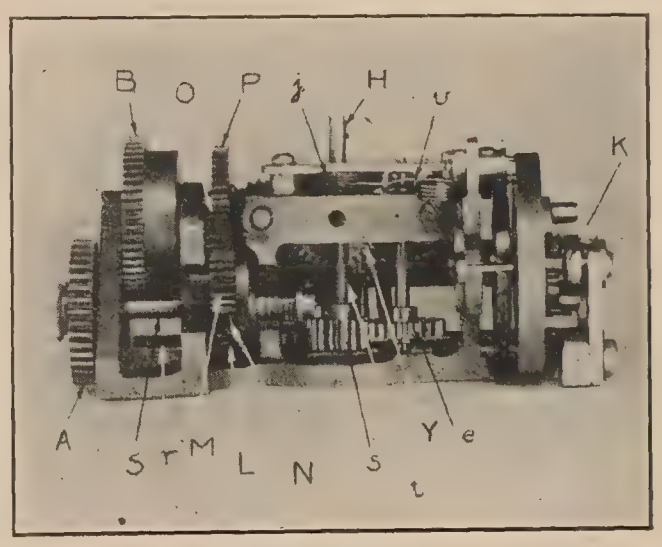


Fig. 14.—Side View.

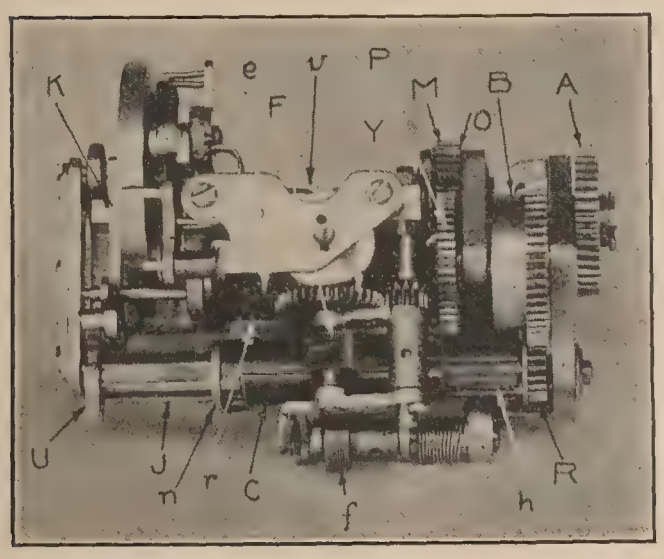


Fig. 15.—Top View.

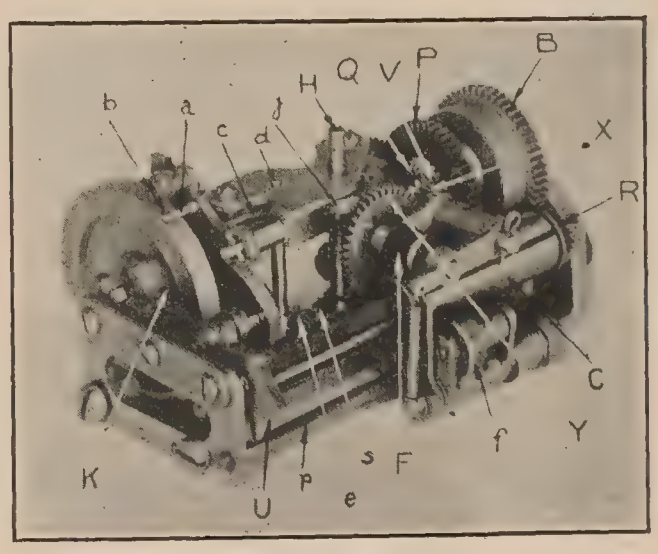


Fig. 16.—Perspective.

swing of the balance wheel is one-quarter second and 12 swings are required for a complete revolution of (U). (J) thus makes a complete revolution in three seconds in steps of one-twelfth of a revolution every quarter second.

The escapement, as seen, is very heavy. To facilitate starting, it is fitted with an auxiliary device which stops it shortly after the instrument is stopped, thus preventing the mainspring from unwinding completely, and also stops it off center with tension in the escapement spring. The arrangement is such that, when the pointer returns to its zero position, a pin (*a*) falls automatically, engaging a stud (*b*) on the rim of the balance wheel and stops the escapement. The movement of (*a*) is accomplished through a pin (*c*) which drops into a slot in the upper end of the arbor (*d*) connected to the pointer staff (H) through the gears (*e*) and (*s*).

The counting system, figure 19, is made up of the fine-toothed pinion (F), connected with (Z) through the gear train (Y–X–V–Q), the toothed racks (C), and the cylindrical pawl (*f*). The racks, in the form of three identical cylindrical arcs of 120° each, envelop the shaft (J) along which they are free to slide in grooves. Their zero position is against (R), which position they tend to assume by the action of the helical control springs (*h*) lying in grooves and fastened respectively to (J) and to the racks. (F) and (*f*) are held in contact with the racks by the springs (*k*) and (*m*) attached to the rockers (*q*) and (*g*) in which they are mounted.

As (J) rotates, clockwise viewed from the escapement end, each rack is successively (1) engaged by (F) and carried, by its rotation, along (J) from (R) toward (U) against the force of its control spring; (2) caught and held by (*f*) in the position in which it is left by (F); (3) disengaged from (*f*) and drawn back to the zero position by its control spring.

TEL CHRONOMETRIC TACHOMETER.

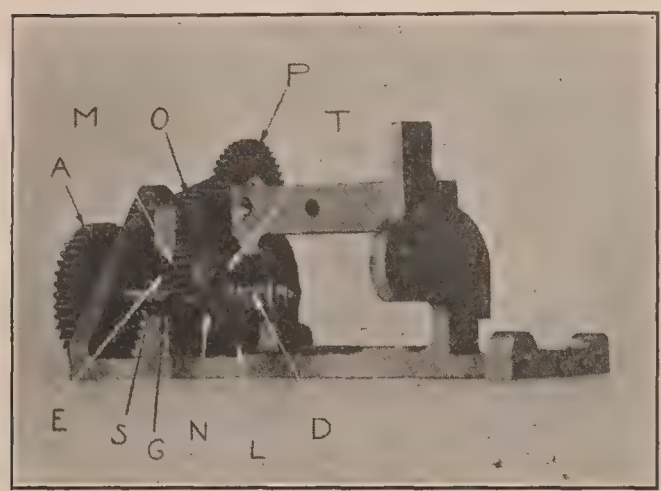


FIG. 17.—Drive system.

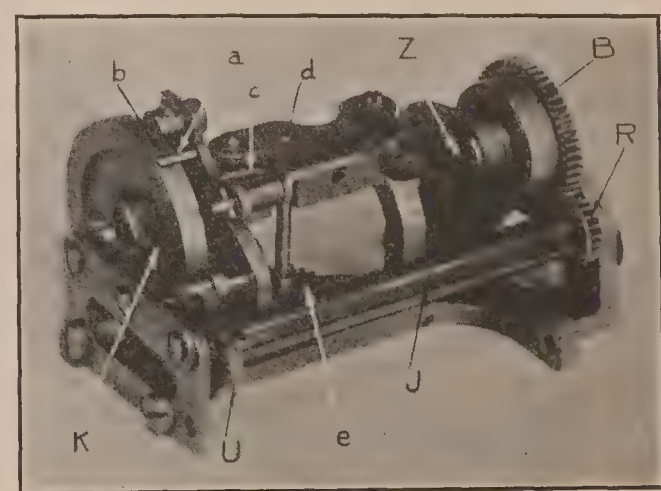


FIG. 18.—Escapement system.

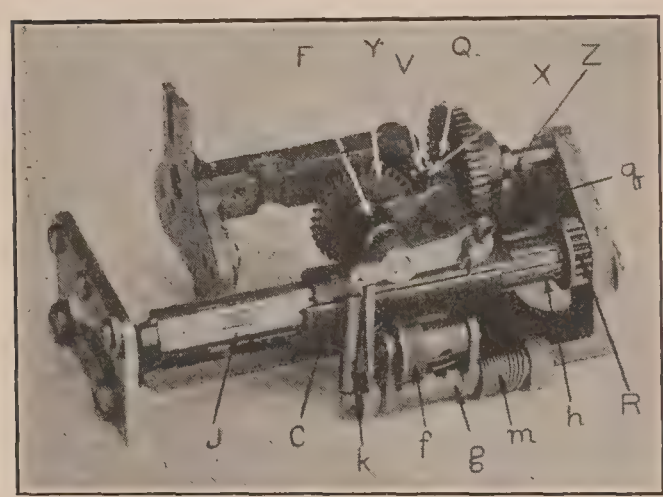


FIG. 19.—Counting system.

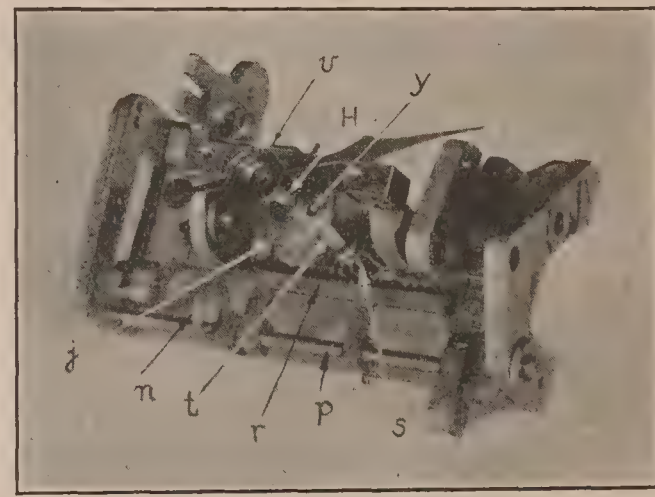


FIG. 20.—Indicating system.

Since the distances between the points of contact of (F) and (*f*) and the width of the racks are each 120°, the above operations follow each other without interruption or overlapping, have each one second, a third of the period of revolution of (J), allotted to them and are performed by the racks with a successive phase difference of one second. Consequently one of the three racks is engaged in each of the three operations at every instant and the instrument is never idle.

Now (F), being geared directly to (Z), and hence to the main drive (A), rotates at a speed proportional to the driving speed. Also, as seen, the period of mesh of the racks with (F) is constant and equal to one second. Therefore, the distances the racks are moved along (J) are proportional, so long as the escapement is unaltered, to the speed during successive seconds. Thus, the rack which is held by (*f*) during each second is at a distance from the zero position of the racks proportional to the speed during the preceding second.

The indicating system, figure 20, is formed by a collar (n) encircling (J) loosely, a toothed rack (r), which slides on a rod (p) and to which (n) is attached, and a gear (s) fixed to the pointer staff and meshing with (r). The racks, as they travel along (J), engage (n) and thus move (r), (s), (H) and the pointer. A control spring (t) opposes the motion taking up the backlash between (r) and (s) and keeping (n) in contact with the racks.

(n) rests, in its zero position, against all three racks in their zero position and, during each second, as the instrument operates against the rack held by (f) in that second. The displacement of (n) and (r) from their zero position is, therefore, the same as that of the rack and hence proportional to the speed during the preceding second. (r) and (s), however, constitute a simple rack and pinion, so that the angular displacement of (s) and of the pointer from their zero positions is proportional to the linear displacement of (r) and thus proportional to the speed. This instrument, therefore, has a uniform scale.

It does not, however, show the speed at each instant, but the average speed for periods of one second. It indicates throughout a given second the average speed during the preceding second. Then at, or very near, the end of the second the reading changes suddenly to the value for that second. This reading is maintained for the next second and so forth.

Changes in reading take place at the ends of the second periods because it is then that (n) shifts from one rack to another. If an alteration in speed occurs, the succeeding rack either (1) stops short of the preceding rack, which is in mesh with (f) and against which (n) is resting (decreasing speed), so that, when the latter is released, (n) is drawn backward by (t) into contact with the former; or (2) engages (n) a little before the close of the second (increasing speed), lifts it off the end of the preceding rack and pushes it forward suddenly, holding it on coming to rest. In both cases the change in reading is abrupt and occurs practically at the end of the second period. If the speed is constant, each succeeding rack stops just abreast of the preceding one and no change in the position of (n) or the pointer occurs.

A loose pin-and-hole connection (y) (figure 20) inserted between (H) and the pointer serves to remove fluctuations of the latter due to imperfect mesh of the racks with (F) and (f) at the expense, however, of accuracy and sensitivity. The spring (y) bearing in the threaded rim of the disk (j) acts as a damper for this arrangement and, by the dropping of its curved end into a slot in the disk, as a zero lock for the pointer. The threads prevent this action at full scale deflection.

The maximum possible lag in this tachometer, between a change in speed and its indication on the dial, is seen to be one second.

The dial is graduated in identical manner with that of the Van Sicklen and the instrument also runs without adapter.

Jaeger.—This instrument has already been shown in figure 1, and in figures 21 to 23 are perspective, top, and side views of the mechanism.

The Jaeger, widely used on French airplanes, is an intricate and beautifully made chronometric of the gear type, to which the Van Sicklen is closely related. Unlike the latter, however, it has two counting gears (C) and (C′) and thus operates continuously. The mechanism conforms to the usual chronometric type. The drive is through a crown wheel (A) engaged by a pinion bearing in the case of the instrument. From (A) through a reversing mechanism, considerably more complicated and delicate than that of the Van Sicklen or TEL, the rotation is transmitted to the spindle (Z) and to the fine-toothed pinions (F) and (F′) which are thrown into and out of mesh respectively with (C) and (C′) at regular intervals. From (Z) the escapement (K) and the cams (J) are driven through the usual spring and barrel (S.) The pointer gear (D) is midway between (C) and (C′) and these gears have corresponding to them the three locking arms (L). Between (D) and the pointer staff is a loose pin and hole connection similar to that in the TEL and serving the same purpose.

The method of meshing and unmeshing (F) and (F′) with (C) and (C′) is somewhat different from that employed in the Van Sicklen. Namely, the spindles to which (F) and (F′) are fastened bear at their upper ends only in the oscillating rocker (G). The lower ends have fixed

JAEGER CHRONOMETRIC TACHOMETER

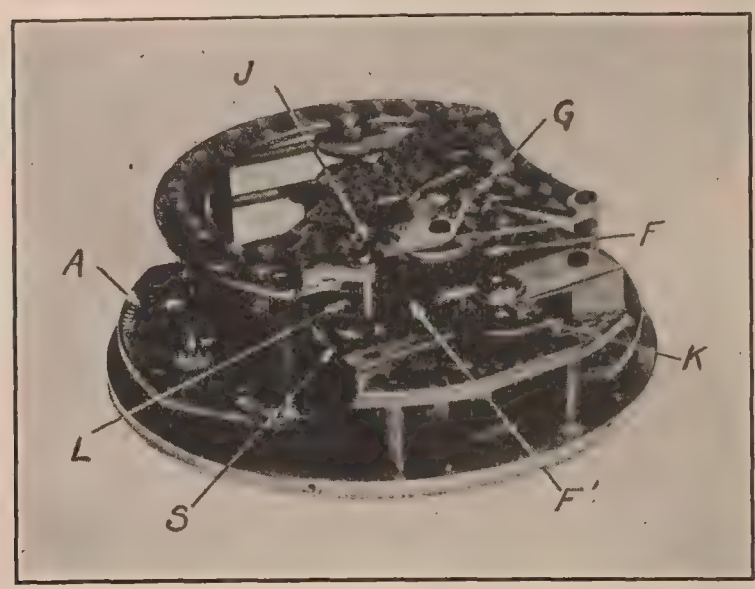


FIG. 21.

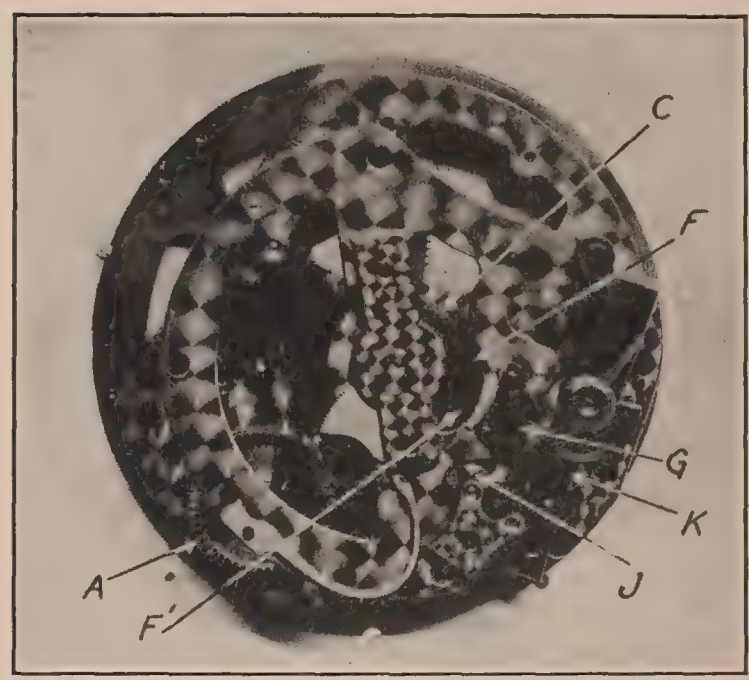


FIG. 22.

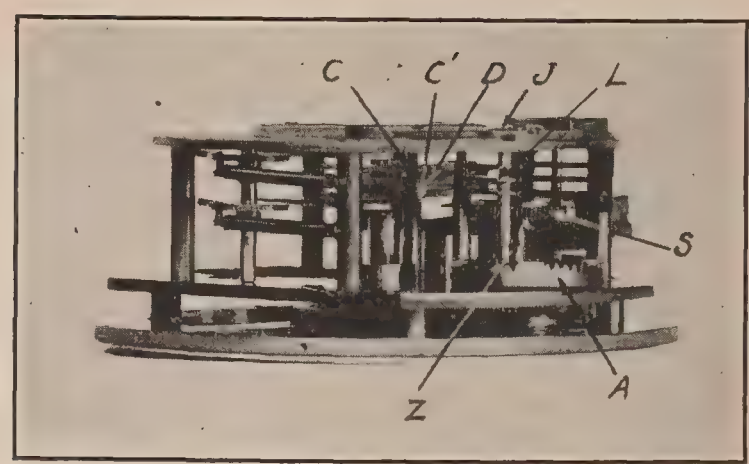


FIG. 23.

bearings in the frame of the instrument, about which as centers the spindles and gears swing, due to the motion of (G). Thus (F) and (F′) are thrown into and out of mesh with (C) and (C′). The driving pinions are fastened to the spindles close to their lower ends where the swinging motion is small, whereas (F) and (F′) are attached near the upper ends and so have considerable motion.

The escapement is about as heavy as in an ordinary alarm clock and of the double-roller type. The balance wheel bearings are jeweled. The escape wheel spindle has a light spring bearing against it to prevent rotation backward during the interval between release of one fly arm and contract with the other arm, which would cause irregularities in the counting periods.

The control springs in this instrument are helical or flat plate springs and act indirectly through toothed sectors and pinions on their respective spindles. The counter gear sectors have slender flat springs lying in slots in their faces. These are provided at their free ends with teeth which project beyond the sector teeth at the point of mesh with the pinion on the counter-gear spindle when the instrument is at rest. By the action of the springs these teeth mesh tightly with the pinions and thus eliminate backlash which, owing to the resulting uncertainty in the zero position of the counter gear, may cause a considerable error in reading of the instrument.

Stover-Lang.—The counting element in this tachometer is a radial arm which is lowered at regular intervals into mesh with a rotating crown wheel geared to the main drive spindle. An auxilliary arm, interposed between the counter and the pointer system, is arranged to drop back into contact with the counter shortly before the end of the counting period. It is then carried forward by the counter during the remaining motion of the latter and is held in the extreme position of the counter by ratchet action. The pointer system is then released and assumes the position of the auxiliary arm. The counter itself is not locked at all. The escapement is of alarm-clock size and unjeweled.

The instrument has been arranged for use with an air drive consisting of a diaphragm operated by an eccentric fixed to the rotating shaft. This diaphragm gives puffs of air with a frequency equal to that of the rotation which, being transmitted to the instrument through a tube, operates a ratchet engaging the main drive gear of the mechanism.

This form of drive is free from many defects of the flexible cable, but is intermittent in its action, and as yet has been adapted to chronometric tachometers only.

Hasler.—This instrument, shown in figure 3, is a hand chronometric suitable for tests on airplane motors and other tachometers.

The escapement functions only when one of the two push buttons seen projecting from the edge of the case is pressed and released. Furthermore, it marks off but one period, three seconds in length, during which the counter is connected with the main drive and the pointer carried through a certain angle proportional, as usual, to the speed. The pointer is locked automatically in its final position and the speed is read off. Resetting of the pointer is accomplished by means of the other push button. Ten revolutions of the pointer, giving a total range of 10,000 revolutions per minute, are provided for.

Bruhn.—This instrument, shown in figures 24 to 27, is a German chronometric of the gear type. It is very similar in design to the French Jaeger and the American Van Sicklen.

Like the Van Sicklen, it has only one counting gear; the counting period, however, is one-half second, thus giving a resetting of the pointer at the end of each second. The drive is through a bevel gear meshing with the gears (A), figures 25 and 26, which are arranged in such a way as to idle when driven in one direction and drive through to the gear fixed to the spring barrel (B) when driven in the opposite direction. The barrel (B) is always driven in the same direction for either rotation of the drive gear.

The drive continues from (B) through a coiled slip spring to the staff (S), escapement (K), and back to the cams (J), which control the meshing of the counting pinion (F) with the counting gear (C), and also the operation of the holding pawls (P) for the counting gear (C), and (M) for the pointer gear (C′). There are three pawls (P) and two (M), the pawls (P) being spaced one-

third of a tooth out of phase with each other and the pawls (M) one-half of a tooth out of phase. The escapement (K) is of the double-roller type, unjeweled, and much more rugged than that of either the Van Sicklen or the Jaeger.

A noteworthy feature is the addition of the two odometers (L) and (O). (L) is driven through a train of gears from the staff on which the cams (J) are fixed and shows the number of hours which the instrument, and therefore the engine, has run. (O) is connected to the drive gear, and its readings, multiplied by 100, give the total number of revolutions which the crank shaft of the engine has made.

BRUHN CHRONOMETRIC TACHOMETER.

FIG. 24.

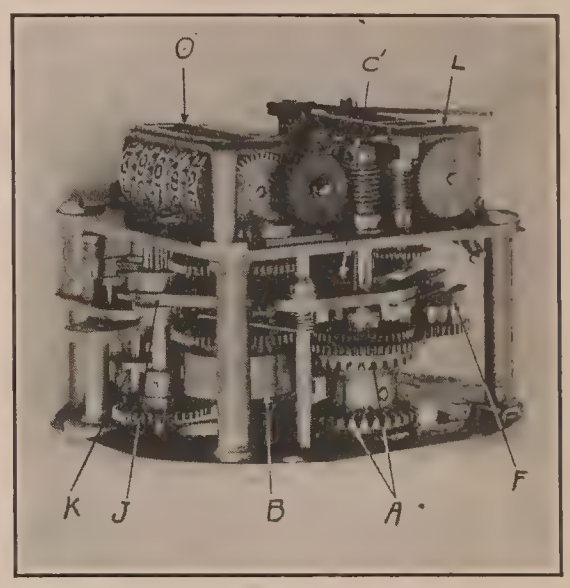


FIG. 25.

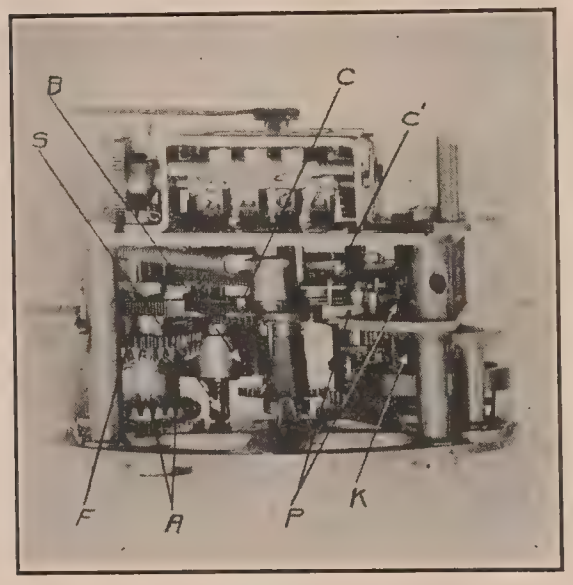


FIG. 26.

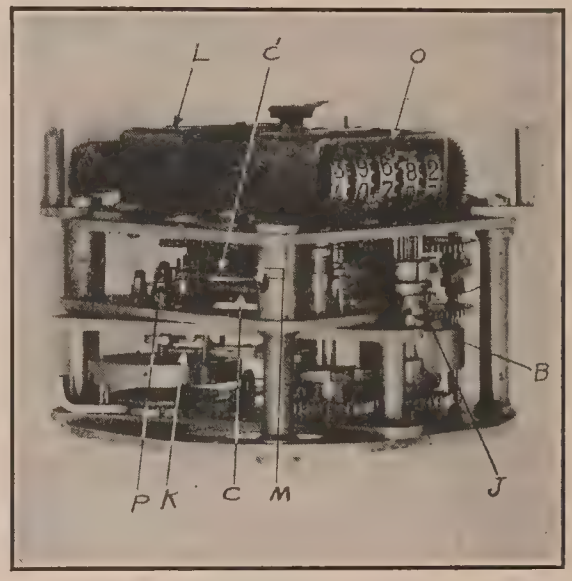


FIG. 27.

CENTRIFUGAL TACHOMETERS.

TYPES OF CENTRIFUGAL TACHOMETERS.

The most common types of centrifugal tachometers are the so-called governor and oblique-weight types.

The governor type is thus named because of the resemblance of the rotating part to an ordinary engine governor. It consists, namely, of a shaft (A), (fig. 28), with a set of weights (B) grouped about it which, as the shaft rotates, act by centrifugal force on a control spring (E). The shaft (A) is hardened and polished, and is mounted vertically and centrally in ball bearings either in the case of the instrument itself or in a separate frame. The weights (B) are attached by links above to a spider (C) fixed to the shaft and below to a grooved collar (D), free to slide along the shaft. The spring E, which is helical, encircles the shaft between (C) and (D).

When the system rotates, the weights (B) pull outward on the links. (D) is thus drawn up the shaft and (E) is compressed. Ultimately, for a fixed speed, the weights and (D) assume a definite position in which the centrifugal force is just counterbalanced by the elastic force of the spring. The motion of (D), which is small ($\frac{1}{4}$ to $\frac{3}{8}$ inch), is magnified and changed into a pointer motion by the indicating train, consisting of the pin (F) bearing on (D), the toothed sector (G), and the pinion (H). The spring (J) on the pointer staff keeps (F) in contact with (D) and also takes up the backlash between the sector and pinion.

In the oblique-weight type, shown diagrammatically in figure 29, the rotating element consists of a weight or frame (B) encircling the main shaft (A) and free to rotate about a spindle (C) fixed perpendicularly to (A). In the position of rest (B) is oblique to (A), but when rotating tends to rotate about (C) into a position perpendicular to (A). This motion is opposed by a control spring (E), so that at each speed a definite position is reached in which the centrifugal couple exerted by (B) is just balanced by the restoring torque of (E). Thus the motion of (B), imparted to the pointer through a suitable indicating train, such as shown in figure 30, gives an indication of the speed. The double oblique-weight type has two crossed weights pivoted on the same axis and placed symmetrically about the shaft.

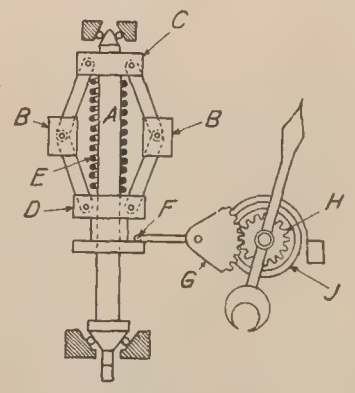


FIG. 28.—Governor type.

Other forms of centrifugal tachometers not belonging to the above types will be pointed out in the discussion of individual makes, together with various modifications of the control spring and indicating train.

Centrifugal tachometers used on American airplanes are all of the governor type, but the governor weights differ in size. This necessitates a difference in the running speed, since the control springs and indicating trains are practically alike. The light governors, therefore, must run at higher speed than the heavy ones to give the same deflection. In all cases, however, the speed of the governor is equal to or greater than the indicated or crank shaft speed; that is, equal to or greater than twice the driving or cam shaft speed. Consequently gears giving an increase in speed of one to two or more are necessary. The speed of the governor relative to the indicated speed and the location of the gears, whether within the instrument or in a special adapter on the end of the cam shaft, are pointed out in each case below.

The oblique-weight type, though extensively used abroad, especially in England and Germany, has not yet been adopted in America for use on airplanes.

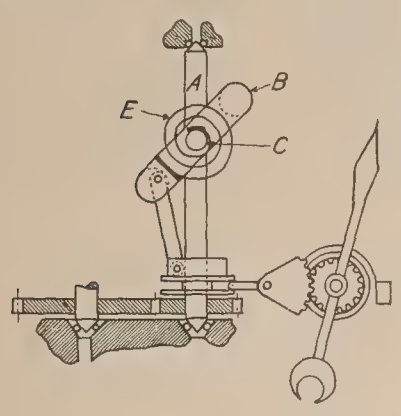


FIG. 29.—Oblique weight type.

Johns-Manville.—This instrument, shown in figure 30, is "Type B" of the American military airplane tachometers. It is of the heavyweight slow-speed type, the governor running at indicated or crank shaft speed. The speed ratio between the cam shaft and the governor is, therefore, one to two. The gears for accomplishing the change in speed are in a separate adapter on the end of the cam shaft, since it was thought, from experiments on sample instruments, that too great unsteadiness would result if they were placed in the instrument itself. Later, however, a model was designed with the gears in the instrument, but this was not produced extensively.

The governor has three weights of special shape which touch when at rest, forming a continuous girdle around the shaft. The ball bearings are contained in the case itself, the upper one being adjustable, the lower one fixed. This is because the weight of the governor falls on the lower bearing.

The indicating train, seen at the left in figure 30, is of the type shown in figure 28 and is carried by a bridge spanning the front of the case. The contact piece which bears on the sliding collar is a pivoted shoe of hard fiber. This is believed to wear better than steel and to be less apt to scratch the surface of the collar. The arm carrying the fiber shoe is adjustable on the spindle on which it rotates, but is not adjustable in length. The pointer is driven on to the tapered end of the pointer staff, and so is also adjustable. Calibration is accomplished by resetting and bending the shoe arm, adjusting the pointer, and deforming the control spring.

The scale is graduated from 400 or 500 to 2,500 or 2,600 revolutions per minute, respectively, in intervals of 50 revolutions per minute and has an angular length of about 190°.

Jones "Victometer."—This instrument, shown in figure 31, is used almost exclusively by the Navy. It has very light weights and the governor runs at twice indicated or four times driving speed. The gears having this latter ratio are contained in the instrument itself as seen in the figure.

The ball bearings are set in a ringlike frame of which the bridge carrying the indicating train is an integral part and to which the dial plate and bezel are fastened. The case proper is a thin detachable metal cup which slips on over the back of the frame. This arrangement makes the governor and indicating train very accessible and permits calibration with the dial in place.

AMERICAN CENTRIFUGAL TACHOMETER.

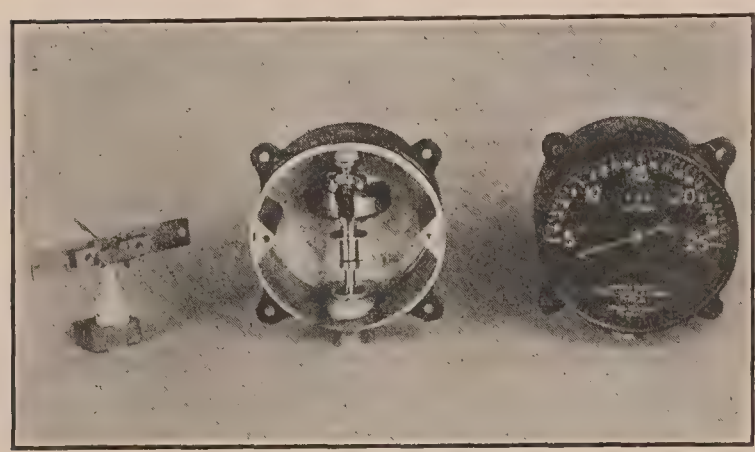


FIG. 30.—Johns—Manville.

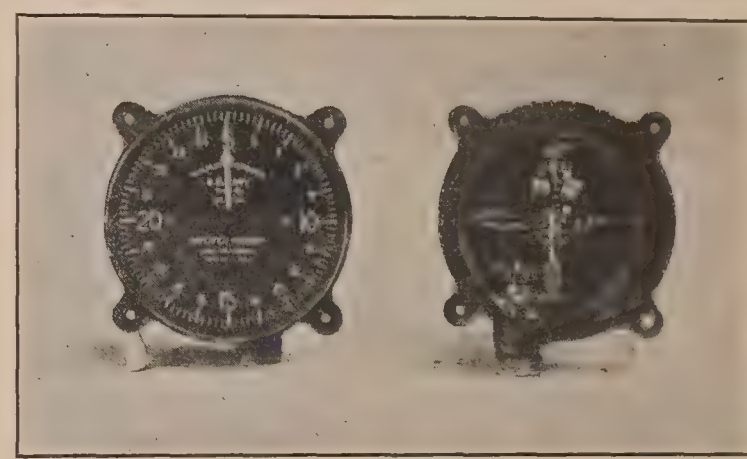


FIG. 31.—Jones.

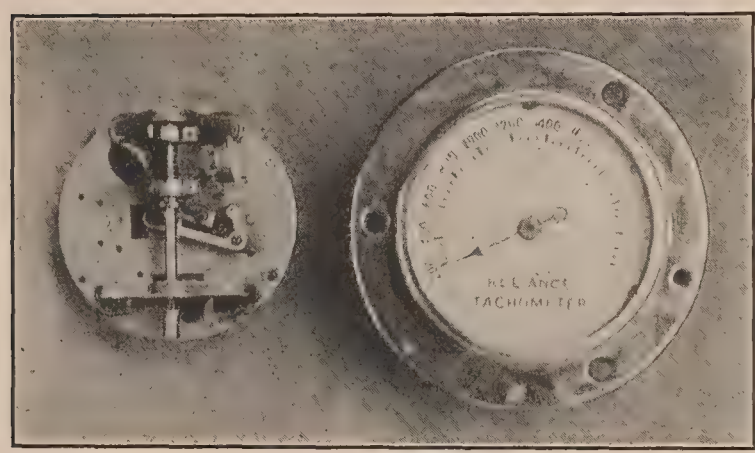


FIG. 32.—Reliance.

FIG. 33.—Hoffecker.

There are three weights. The spider to which the links are attached at the top is adjustable, being held by a set screw.

The indicating train is of the type shown in figure 28. The lever carrying the contact pin is, however, adjustable in length. The contact pin is of steel. The pointer is driven into a tapered staff, as in the Johns Manville.

Reliance.—This tachometer, used by the Navy, appears in figure 32. It also is of the light-weight high-speed type. The governor speed is about one and a half times indicated speed or three times cam-shaft speed. A special adapter is used on the cam shaft, there being no gears in the instrument itself.

The indicating train is also of the type shown in figure 28.

The contact pin is of hardened steel, the sliding collar being of brass. An adjustable contact pin lever and an adjustable zero stop are provided for use in calibration.

The governor bearings, indicating train, and dial are mounted on a detachable frame, so that the calibration may be done before inserting in the case with good access to the mechanism.

This instrument has not been standardized. The shaft connection as made is provided with a sliding element which a compression spring keeps in mesh with the flexible shaft. Thus also the end thrust in the flexible shaft, which may cause unsteadiness, is taken up.

The dial is white and the scale, determined separately for each instrument, extends from 200 to 2,400 revolutions per minute in intervals of 50 revolutions per minute over an angle of about 225°.

The case and mechanism are both of heavy construction.

Hoffecker.—This instrument, shown in figure 33, is very similar to the Reliance. The weight and speed of the governor are about the same as well as the means of calibration.

The frame carrying the governor and indicating train is of rigid though light construction.

A unique feature of the indicating train is that it has two contact pins of hardened steel which form the tines of a fork swiveled on the end of the usual contact pin lever. These pins bear on opposite sides of the sliding collar and are intended to eliminate fluctuations due to lack of perpendicularity of the surface of the collar to the shaft or roughness of the surface.

The instrument has not been standardized. The flexible shaft provided has a ball bearing end and a wedge-shaped tip which facilitates meshing at high speed.

The scale is graduated similarly to that of the Reliance and likewise is determined separately for each instrument.

Jones speedometer.—This instrument, shown in figure 34, is of the oblique weight type with sliding collar, the weight being in the form of a ring. The connection between the governor and the sliding collar is formed by a lug projecting from the upper end of the collar sleeve and bearing on a hardened steel pin embedded in the oblique weight. There are two helical control springs—one, a weak spring which is compressed by the collar, the other, a strong clip-like spring which is coiled up as the oblique ring deflects.

The indicating mechanism is unusual in that it contains a cam plate, the proper shaping of which gives a uniform scale.

Schaeffer and Budenberg.—Figure 35 shows the exterior of the instrument, which is of the same type as the Jones centrifugal speedometer described above. The motion of the weight is transmitted to the collar through two parallel links pinned at their ends. A unique feature is that the contact pin bears against the under surface of the sliding collar.

The instrument has three drive spindles and is fitted with a gear box, so that full scale deflection may be had for three different speeds, such as 200, 1,000, and 2,000 revolutions per minute, by using the proper spindle.

This type of instrument is suitable for tests on airplane motors.

Elliott.—This is a British instrument, also of the oblique weight type, and is shown in figure 36. The governor is in the form of a dumb-bell pivoted in a boxlike frame inserted in the governor shaft. The control spring is a helical spring fixed to the frame at one end and to the governor at the other.

The indicating mechanism is unusually simple in that the lever carrying the contact pin is fastened directly to the pointer staff. The pointer is pivoted in the lower part of the dial and the angular motion is only about 90°. The scale is graduated from 800 to 2,000 revolutions per minute in intervals of 50 revolutions per minute.

Smith.—This instrument, the mechanism of which is shown in figure 37, is also a British tachometer of the oblique weight type. The governor is a flat link-like casting. A hemispherical casting, fixed to the lower end of the shaft, supports the ends of the governor pin and also serves as a flywheel. There are two exactly similar flat coiled control springs. The connection between the oblique weight and the sliding collar is formed by a fork fixed rigidly to the collar and bearing on a hardened pin, as in the Jones speedometer above.

The indicating mechanism is of the ordinary construction, shown in figure 28. The scale is graduated from 600 to 2,000 revolutions per minute in intervals of 20 revolutions per minute and has an angular length of about 300°.

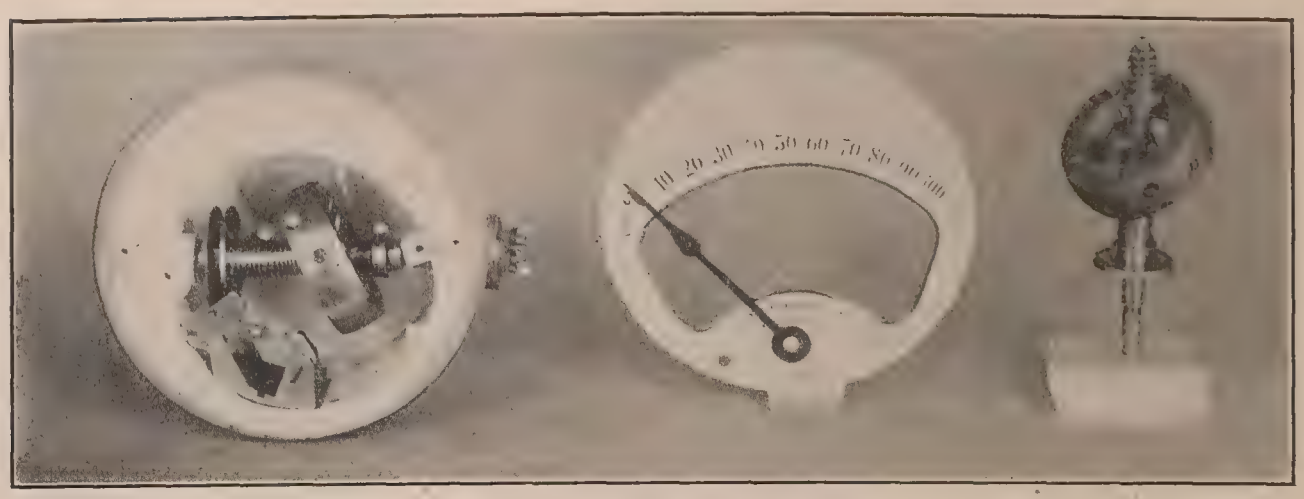


FIG. 34.—Jones Centrifugal Tachometer.

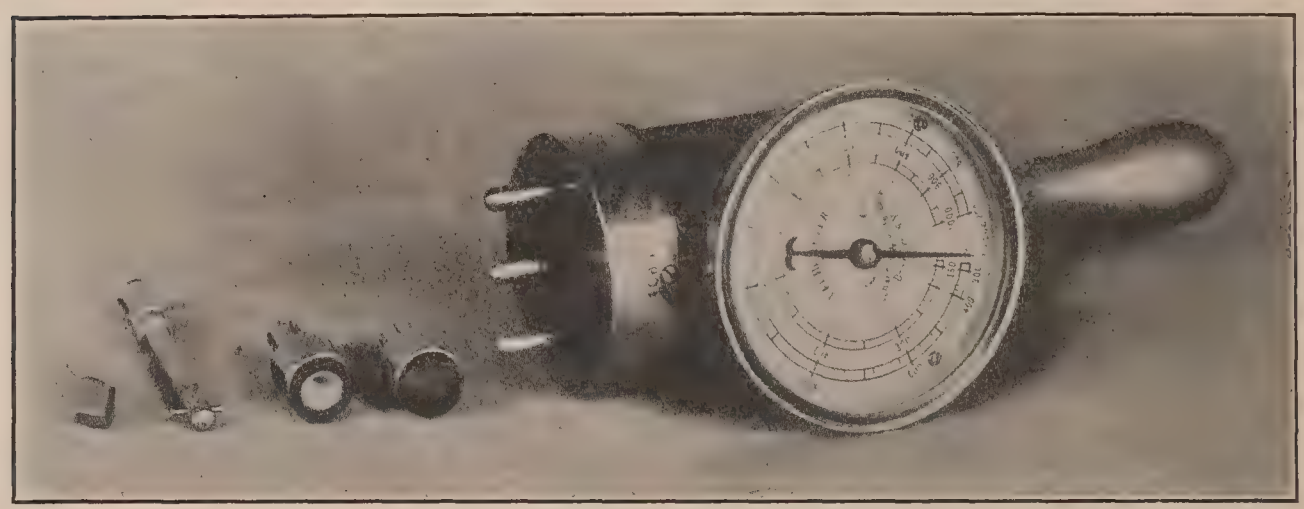


FIG. 35.—Schaefer and Budenberg.

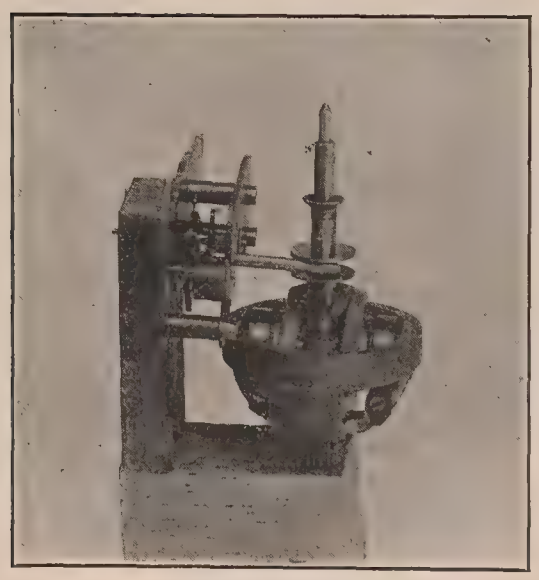


FIG. 36.—Elliott.

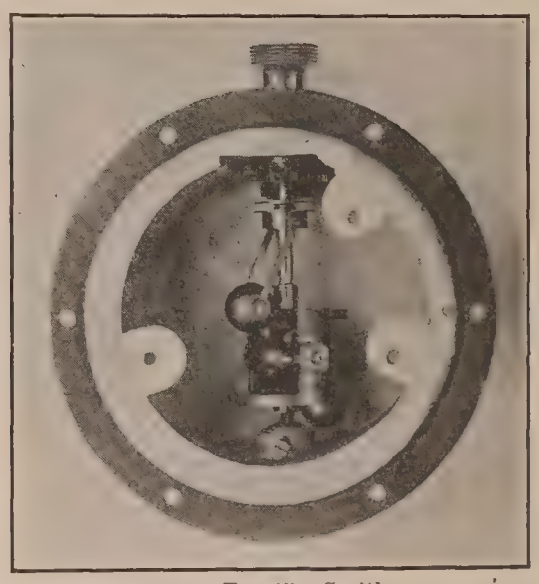


FIG. 37.—Smith.

Oliver.—This tachometer, the mechanism of which is shown in figure 38, is of the oblique weight type, but has certain peculiar features.

Uniformity of scale is obtained by means of a cam plate in the governor. This cam is attached to the oblique weight and deflects with the latter. Connection between the cam and the indicating train is made by means of a rod sliding in a boring in the shaft. The contact point with the indicating train is in the center of the rod where there is no motion and thus wear is avoided. The contact point with the cam is near the axis of rotation of the oblique weight. This tends to minimize disturbances due to a reaction of the rod on the oblique weight.

The governor control consists of two pairs of helical tension springs. The points of attachment of these springs are so placed that the lines of action of the springs pass through the axis of rotation of the oblique weight in the position of rest. As the oblique weight deflects, however, they get farther and farther away from this axis. Thus a restoring torque increasing

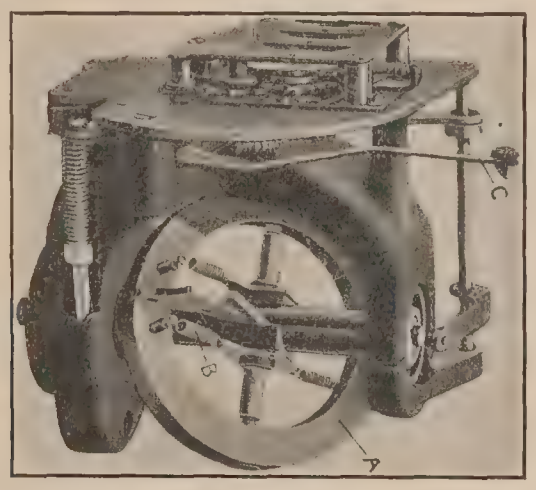


FIG. 38.—Oliver centrifugal mechanism.

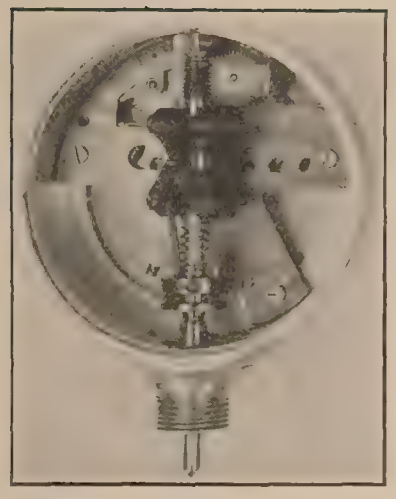


FIG. 39.—Olhovsky mechanism.

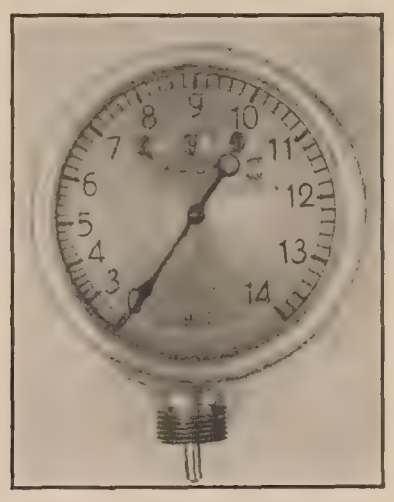


FIG. 40.—Olhovsky case.

more rapidly than the first power of deflection is obtained, which, it is claimed, facilitates the attainment of a uniform scale.

Olhovsky.—The Olhovsky tachometer shown in figures 39 and 40, is a Russian instrument of the double oblique weight type with sliding collar. The weights are in the form of rectangular frames consisting of cylindrical rods with flat connecting pieces and are in rotational balance. Each acts separately on the collar through a link. The control spring is helical and is located between the sliding collar and a ball-like shoulder on the shaft, being compressed as the collar moves upward.

A two-tined steel fork with swivel joint, similar to that in the Hoffecker described above, is used to connect the governor and indicating mechanism.

A noteworthy feature of this instrument is that it is compensated for the effect of tilting with reference to the vertical and also for the effect of external shocks. This is accomplished

by unbalancing the governor weights by an amount equal to one-half of the combined weight of the collar and floating pin and by pivoting the rollers at a distance from the centers of rotation of the weights equal to that of their centers of gravity. Thus the movement of the weights about their pivots, due either to gravity or sudden acceleration in any direction, is just counterbalanced by that of the collar and pin.

The indicating mechanism is novel. A nut with a hardened steel point bears on the float mentioned above. Longitudinal motion of this nut causes the threaded pointer spindle, which is screwed into it, to rotate. A coiled spring on the pointer staff serves as a control spring for the governor as well as to take up backlash.

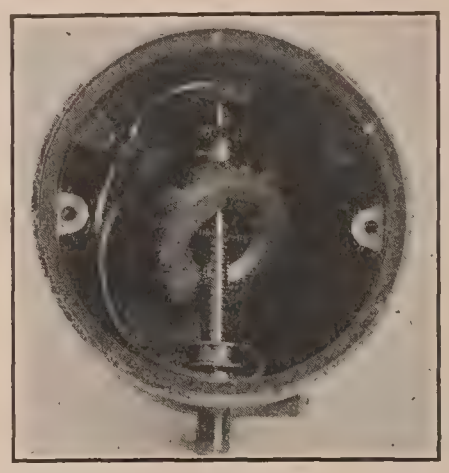


FIG. 41.—Morell "Phylax."

The instrument is very compact, the whole mechanism being contained in a cylindrical case $2\frac{1}{4}$ inches diameter by 2 inches high.

Morell "Phylax."—This instrument, a single oblique ring type of German design, is shown in figure 41. It is of the usual link and sliding collar construction. As in the Jones speedometer there are two control springs, one compressed by the collar and the other a clip-like spring between the governor and the shaft.

The indicating train is unique in that it contains a spring which serves to take up sudden shocks or changes in speed. It is also provided with an air damping device consisting of a small vane geared up so as to make many revolutions for one of the pointer pinion.

Oil tubes are provided running from a well in the top of the case to the shaft bearings and to the steel pin which bears on the collar.

Jacquet.—This tachometer, the exterior of which is shown in figure 42, is a hand tachometer of the oblique weight type with a single coiled control spring contained in a slot in the governor shaft. A small steel plunger sliding in a boring in the shaft is connected with the oblique weight by a link passing through a slot in the side of the shaft. Connection with the indicating train is through a ball-and-socket joint in the end of the plunger. The indicating train is of the ordinary sector and pinion type.

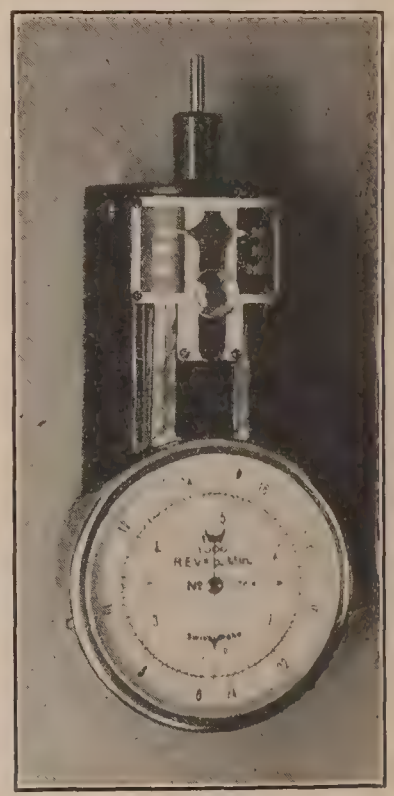


FIG. 42.—Jacquet.

The instrument has only a single spindle, although it provides for three ranges of speed. Change of gears is effected by moving the button seen in the neck of the instrument.

Horn.—This is a German double oblique weight type. The governor is similar to that of the Olhovsky and the indicating train the same as in the Jacquet. Likewise it is a single spindle hand instrument.

Standard.—In this tachometer two weights, sliding on pins fixed at right angles to the shaft, fly out when the system is rotated. Their sides each bear against pins on pivoted sectors which mesh with a circular sleeve rack sliding on the governor shaft. As the weights move outward the sectors are rotated and the sleeve raised. A pinion geared to the pointer staff also meshes with the lower part of this circular rack, the motion of which is thus communicated to the pointer.

The governor control is by means of two helical springs, which are put in tension by the rotation of the sector about its pivot.

Loring.—The Loring instrument is a modification of the double oblique weight type. The governor consists of two light weights pivoted outside the shaft in a relatively heavy casting. Rollers on the inner ends of these weights run in slots in the sliding collar, so that as the weights turn the collar is moved. A steel pin driven into the collar slides in the shaft, which is hollow and slotted. On this pin rests a floating hardened steel pin which is raised or lowered with the collar.

Calibration is accomplished by moving the upper collar, adjusting the length of the contact pin arm and bending the control spring.

The scale is graduated uniformly from 500 to 2,500 revolutions per minute in intervals of 20 revolutions per minute and extends over a complete circumference.

The instrument weighs only 14 ounces complete.

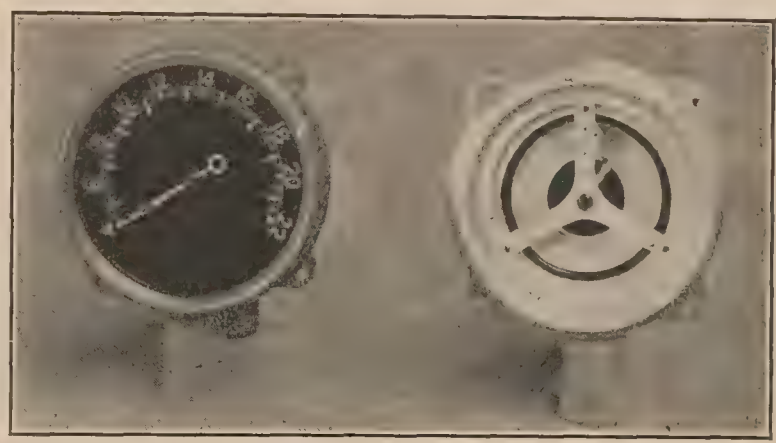


FIG. 43.—Waltham air viscosity tachometer.

AIR DRAG OR VISCOSITY TACHOMETERS.

Waltham.—The Waltham tachometer shown in figure 43 is of the air viscosity type described in principle above. There are two concentric cylinders, geared to the main drive, of which the outer one only is shown in the right-hand view. Between them, and separated from them by a thin air film, is placed the inverted cuplike cylinder, mounted in jeweled bearings, to which the pointer is fixed.

The dial is graduated to show speeds from 400 to 2,200 revolutions per minute, the deflection of the pointer for this speed range being about 240 degrees.

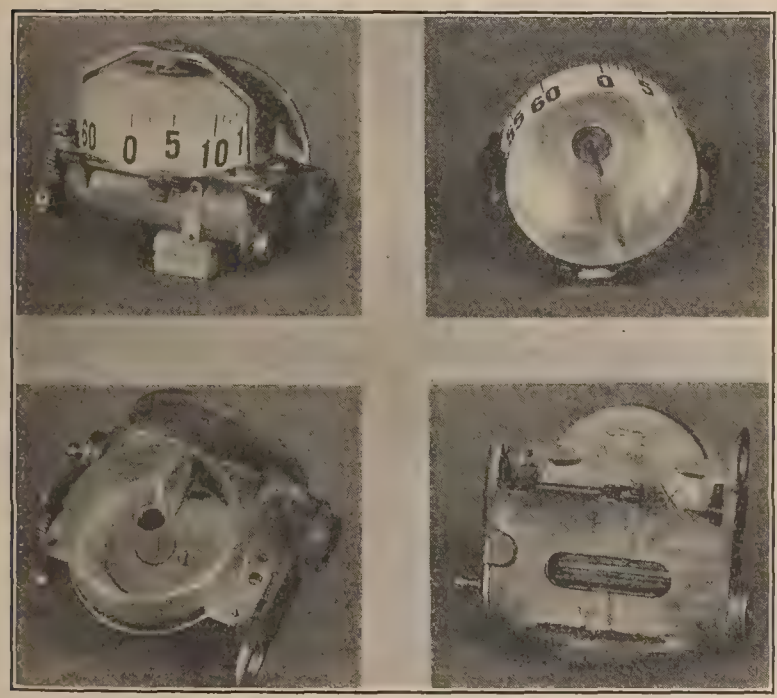


FIG. 44.—Van Sicklen air pump speedometer.

AIR PUMP OR AIR LEAK TACHOMETERS.

Van Sicklen speedometer.—This instrument, shown in figure 44, is typical of the air pump or air leak tachometer mechanism. The lower right-hand view shows the centrifugal air pump and the admission port; the lower left-hand view the discharge orifice and air chamber. Special attention is called to the variable width groove through which the air leaks beneath the vane which is shown with the indicating drum in the upper right-hand view. The width of this orifice is made such that a deflection of the vane is obtained proportional to the driving speed. The upper left-hand view shows the assembled mechanism.

MAGNETO OR ELECTRIC TACHOMETERS.

Tetco.—Figure 45 shows the Tetco electric tachometer. The magneto is of the ordinary bi-polar construction with a single permanent horseshoe magnet. The indicator consists of a suitable range millivolt-meter calibrated to read speeds from 0 to 2,000 revolutions per minute in intervals of 20 revolutions per minute.

MAGNETIC TACHOMETERS.

Warner.—Figure 46 shows a cartridge type of magnetic instrument. The permanent magnet, running in ball bearings, is in the form of a split ring. The drum, in which currents are induced, is held a definite distance from the top of the magnet. The leakage field of the magnet is utilized; consequently the magnet is comparatively strong for a given torque on the disk.

A device for compensating for the effect of temperature consists of an iron ring mounted on three bimetallic levers. Changes in temperature cause this ring to move nearer or away from the magnet, thus distorting the magnetic field and changing, by a suitable amount, the strength of the field utilized for torque.

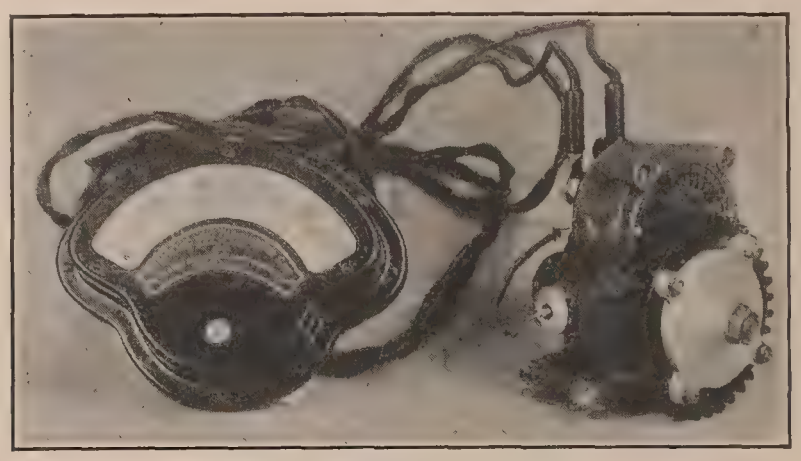


FIG. 45.—Tetco electric tachometer.

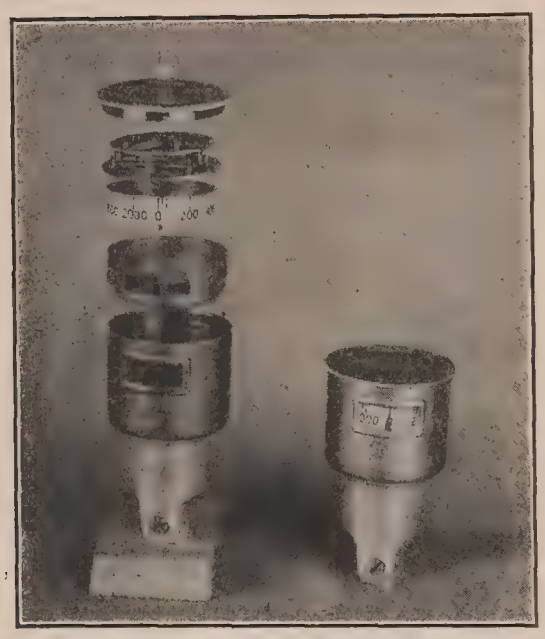


FIG. 46.—Warner magnetic tachometer, cartridge type.

FIG. 47.—Warner magnetic tachometer, airplane type.

The instrument is calibrated by moving the magnet nearer to or away from the indicating disk.

The scale, which is on the side of the drum, is graduated from 0 to 2,000 revolutions per minute in intervals of 25 revolutions per minute.

Figure 47 shows an airplane model of the same instrument. The magnet is geared to the main drive. A disk with pointer attached is used instead of a drum.

The same type of temperature compensator is used as in the cartridge model. It is not shown in the figure.

Attention is called to the pointer damping device which consists merely of a permanent magnet placed near and over the edge of the indicating disk.

The scale extends from 0 to 2,600 revolutions per minute in intervals of 50 revolutions per minute.

Deuta.—Figure 48 shows a cartridge type of German make. The permanent magnet, running in ball bearings, is in the form of a split ring. The inverted drum, in which currents are induced, is mounted in jeweled bearings coaxially with the magnet. Calibration is effected by adjusting the position of a truncated cylinder and thus varying the strength of the magnetic field.

The instrument has no temperature compensator.

The scale extends from 0 to 1,600 revolutions per minute over about 300° and is nonluminous.

FIG. 48.—Deuta magnetic tachometer.

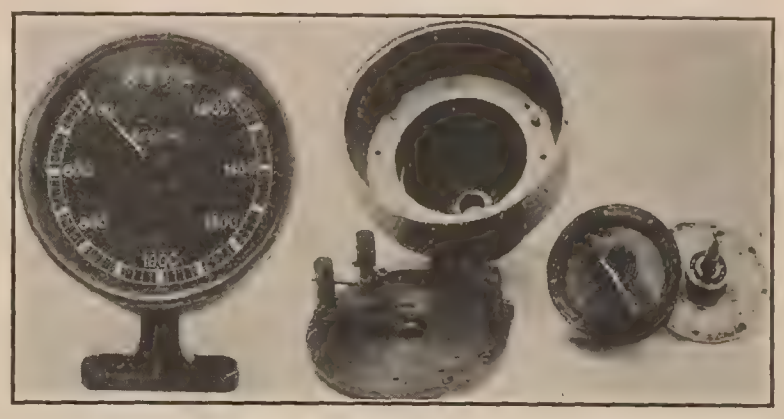


FIG. 49.—Atmo mercury viscosity tachometer.

MERCURY VISCOSITY TACHOMETERS.

Atmo.—This tachometer, shown in figure 49, is a French instrument of the mercury viscosity type. Mercury contained in a steel cylinder is rotated and tends to drag with it a disk fixed to the pointer staff and mounted concentrically with the cylinder. The deflection of the pointer is controlled by two flat coiled springs, one of which comes into action later than the other, thus tending to make the scale more nearly linear than if only one spring were used. The tachometer dial reads from 400 to 1,600 revolutions per minute, and is nonluminous. The weight of the instrument is approximately 3 pounds.

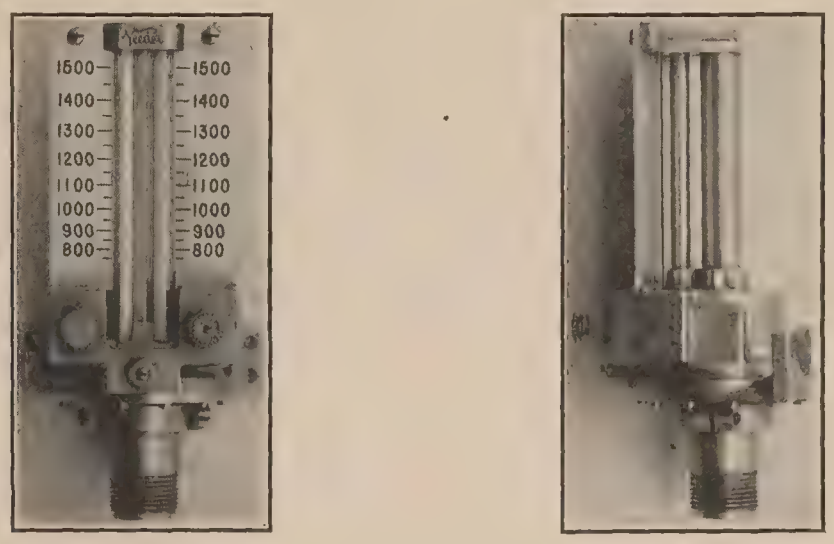


FIG. 50.—Veeder liquid centrifugal tachometer.

LIQUID CENTRIFUGAL TACHOMETERS.

Veeder.—This tachometer, shown in figure 50, is an American instrument of the liquid centrifugal type adapted for airplane use. A paddle wheel rotates in a liquid and forces it through a throttle valve up a system of glass tubes. The height of the liquid indicates the speed. The scale extends from 750 to 1,500 revolutions per minute. The weight of the instrument is approximately 1¾ pounds.

REPORT No. 129.

POWER PLANT INSTRUMENTS.

PART II.

TESTING OF AIRPLANE TACHOMETERS.

By R. C. Sylvander.

SUMMARY.

This part describes in detail the apparatus and methods of testing airplane tachometers at the United States Bureau of Standards. Also, the average results of tests on many instruments of the chronometric, centrifugal, magnetic, and air viscosity type are given and are discussed.

INTRODUCTION.

The principal tests made on airplane tachometers are for calibration error, lag, effect of reversing the direction of rotation of the drive shaft, and the effect of various conditions encountered in airplane flights, such as change of temperature, vibration, continued running, tilting, and reduced air pressure. The usual method of determining the error or effect is to compare the reading of the tachometer with that of a master instrument driven from the same shaft. In the following the apparatus and methods for the various tests, as well as the procedure in calibrating the master tachometer itself, are described.

DESCRIPTION OF TESTING APPARATUS.

DRIVING APPARATUS.

The driving apparatus, shown in figure 1, consists of a one-fourth horsepower, direct-current shunt motor (M) and a 14-inch flywheel (O) mounted on a base plate. (M) and (O) run in separate bearings, but the Oldham connection (U) makes perfect alignment between them unnecessary.

Variation in speed is obtained by means of the rheostat (R), which can be connected in series with either the armature or the field of the motor by means of the switch (A). By removing the resistance from the armature circuit and then, after reversing the switch, inserting it in the field circuit, a gradual increase in speed is obtained from zero to 3,000 revolutions per minute. In lowering the speed the operations are reversed.

By this method of varying the speed, power is wasted in heating and the rotation tends to be unsteady at high and low speeds. The latter objection is practically overcome by the flywheel, which, at the same time, allows the speed to be varied with sufficient rapidity from point to point. The method has the advantage that the drive is direct without possibility of slippage.

Fine speed regulation is obtained by friction of the hand on the rim of the flywheel, using the master tachometer (T), figure 1, as an indicator. Special tests have shown that, with proper precautions, the speed may thus be held constant within a few revolutions per minute for tests either on instruments or on the master itself.

MULTIPLE CONNECTION STANDS.

Four forms of multiple drive for connecting instruments to the driving apparatus are used. The connections are designed from the standard instrument, flexible shaft, and engine connections used on American airplanes during the war and shown in sketch No. 75 issued October 11, 1917, by the United States Signal Corps.

The first of these (V) (fig. 2) consists of a series of five right-angle joints connected in series and fitted with vertical extensions of one-fourth inch extra-heavy pipe, approximately 6 inches and 1 foot long, to which the instruments are attached in staggered arrangement by means of standard couplings. The pipe extensions serve both as casings for the flexible cables by which the instruments are driven from the right-angle joints and as rigid supports for the instruments themselves. This form of multiple connection gives good results with centrifugal tachometers, which are frequently unsteady with a rigid drive.

The unit is attached either directly to the shaft of the flywheel by means of an Oldham coupling, as in figure 2, or to one of the sockets of the double drive (X) (fig. 1), either by an Oldham coupling or a flexible shaft, as the stand (Y), figure 1.

(W) (fig. 2) is a form of multiple drive, very similar to (V) (fig. 2). The right-angle joints in this case are the same as those of (V) except that they are made with the standard engine

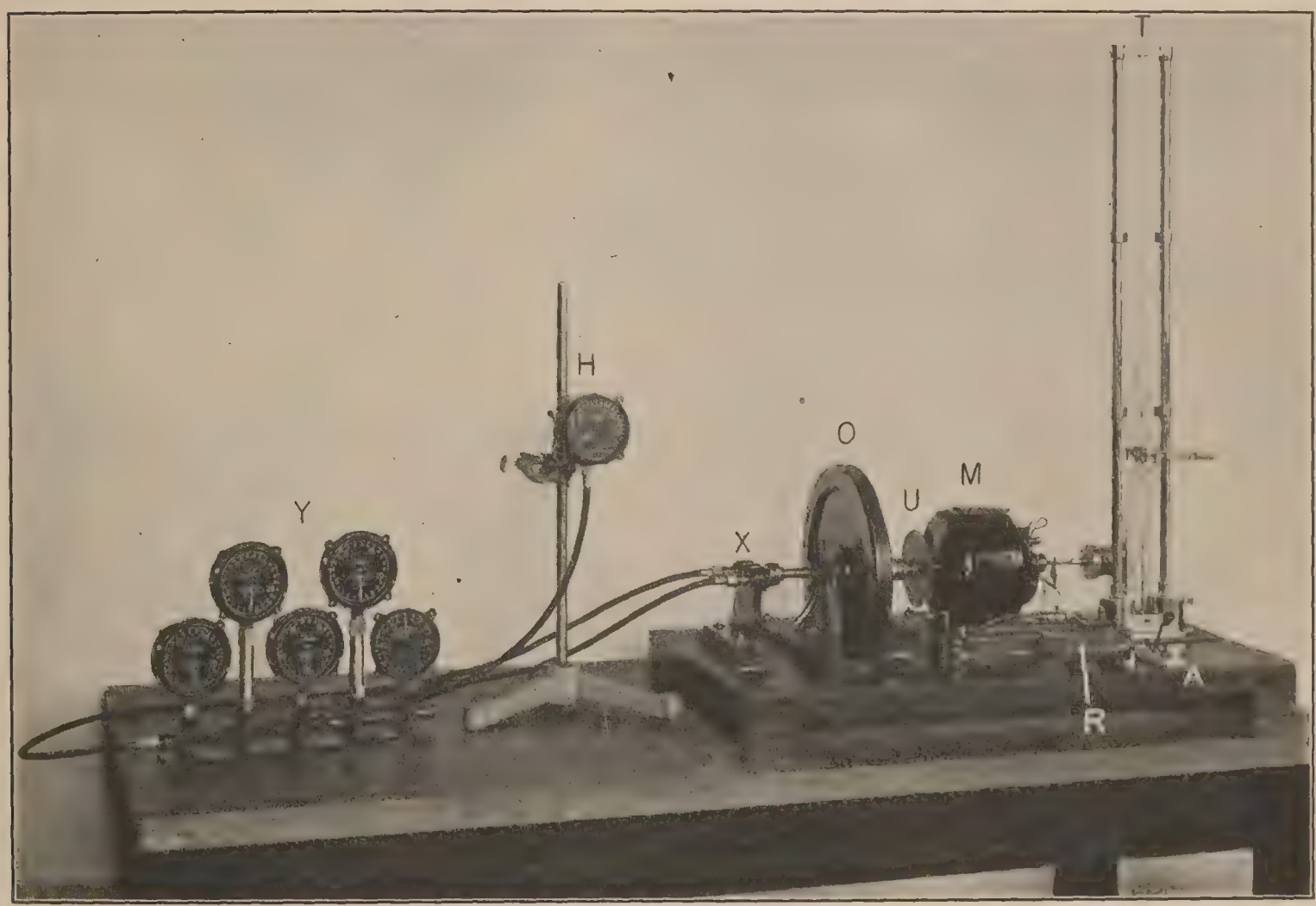


FIG. 1.—Testing apparatus for airplane tachometers.

camshaft connection, so that instruments can be driven from them by means of the regular flexible shaft. This also works well with centrifugals, but requires a separate support for the instruments.

A two-way drive of this kind is (X), shown in figure 1, attached to the driving apparatus. This is used to drive instruments individually, as (H), figure 1, or one or two of the five-way stands, as (Y), figure 1.

(Y) is a rigid multiple drive. The right-angle joints are the same as those of (V), but the instruments are connected rigidly to them—three directly, the other two by means of rods contained in pipe extensions to which the instruments are attached. The right-angle joints are provided, on the ends from which the instruments are driven, with universal tips, giving lateral play, the same as in the regular flexible shaft. Nevertheless this form of drive is apt to cause unsteadiness in the case of centrifugal tachometers, especially in the case of the instruments farthest from the driving end.

MASTER TACHOMETER.

The master tachometer (T), figure 1, is connected to the motor shaft by a rigid drive, so that its running speed is equal to that of the instruments. A comparison of the readings of the two, corrected for the errors of the master itself, thus gives the errors of the instruments.

The master tachometer used at present is a 36-inch Veeder, Form H–4, liquid centrifugal tachometer. This instrument contains colored kerosene and has a paddle wheel in the base which, as it rotates, drives the liquid up in a vertical glass tube. The speed is determined by the position of the meniscus on a scale graduated in revolutions per minute.

The instrument has a very open scale, is direct reading, sensitive, and quick in operation. Investigations made thus far indicate that the secular changes and the errors due to lag, temperature, leakage, and inaccuracy in the setting of the zero, though not negligible, are of no serious consequence in the testing of airplane tachometers, which are read ordinarily to five revolutions per minute only.

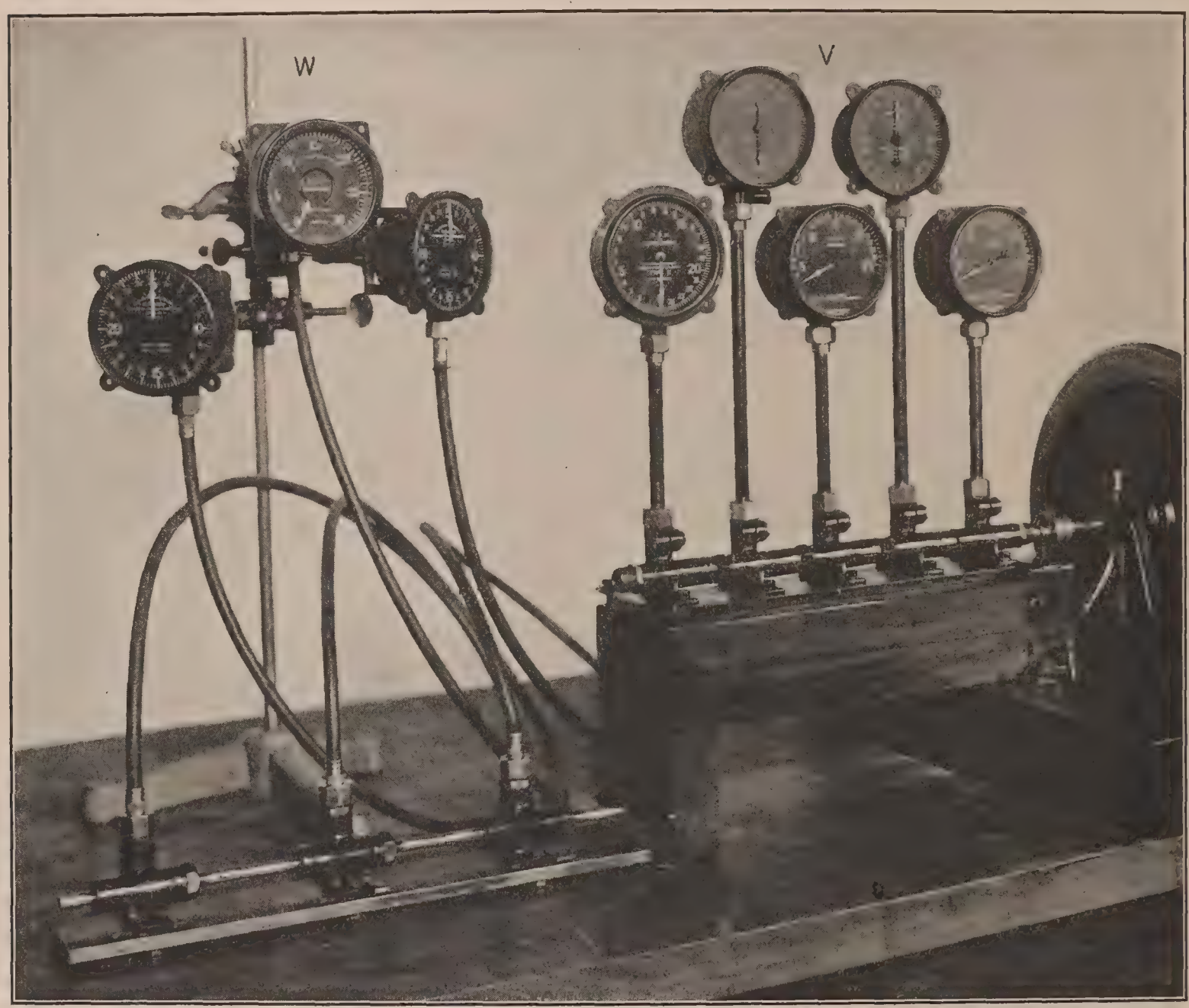


Fig. 2.—Multiple connection stands.

OTHER APPARATUS.

Another method of testing tachometers is by means of a Tinsley stroboscopic apparatus. A so-called observation disk, on which are certain regular geometric designs, is rotated at the same speed as the tachometer and is illuminated 50 times a second by the instantaneous action of a Neon tube. The lighting of this tube is effected by means of a tuning fork which, as it vibrates, makes and breaks the primary circuit of an induction coil in series with a 2-volt battery. The secondary of the induction coil then gives a discharge through the Neon tube.

It is seen that when the rate of illumination of the tube synchronizes with the sides of any figure on the rotating observation disk that figure will appear to be stationary. If the figure appears to slowly rotate backward, the rotating machine is slow of the tuning fork, and fast when it appears to rotate forward, being exactly at the initial speed when the figure is stationary. Standard speeds are easily picked out by reference to a table.

TESTING METHODS.

DETERMINATION OF CALIBRATION ERROR.

The procedure in determining the calibration error differs somewhat, according to the presence or absence of lag and whether the tachometer is continuously recording, like the centrifugals, or discontinuous in its action, like the chronometrics.

By lag is meant the failure of a tachometer to respond immediately to changes in speed, so that readings taken during or after an increase in the speed are too low; those taken at the time of or subsequent to a decrease in speed too high.

WHEN LAG IS ABSENT.

If lag is known to be absent or negligible, as in the Van Sicklen chronometric and most electric tachometers, the calibration error is determined as follows: One observer holds the reading of the master constant at the speed at which the error is desired. Another observer simultaneously takes a number of readings of the tachometer. The average of these readings minus the fixed reading of the master, the latter corrected for its own error, gives the error of the instrument in revolutions per minute. If the tachometer is a chronometric, it is well to have the readings cover several counting periods, the latter being usually one or two seconds in length. In this way the effect of the fluctuations, which occur under certain conditions even at constant speed in this type of instrument, may be partially eliminated.

WHEN LAG IS PRESENT.

If the instrument has lag, failure to take account of the same may lead to serious inaccuracy. This is because the lag lasts sometimes several minutes after a variation in speed, even when the instrument is tapped or vibrated, so that the observed error is not the true calibration error but the resultant of it and the lag.

In this case the errors with increasing and decreasing speed are observed and their algebraic mean taken as the true calibration error. Thus the effect of the lag, not only of the tachometer but of the master instrument as well, is eliminated. The procedure varies somewhat, according to the type of instrument.

If the tachometer is of the discontinuously recording chronometric type, like the TEL, in which case the lag is caused by lost motion alone, the speed is brought slowly up to, but not beyond, the point at which the error is to be determined and the error noted. The speed is then lowered 50 to 100 revolutions per minute, brought up again, and another reading taken. A few readings are made in this way and an equal number in the same manner, except that the point in question is approached from a higher instead of a lower speed. The algebraic average of these errors is the calibration error of the instrument. The procedure is similar to that employed in the case of screws and other apparatus subject to lost motion.

On the other hand, if the tachometer is of the continuously indicating type, such as the centrifugals, air viscosity tachometers, and magnetics, the speed is raised continuously from the lowest to the highest point of the scale and then lowered again continuously at the rate of about 50 revolutions per minute in 10 seconds. The throttle of the Veeder is kept open in this test by about five turns of the throttling screw, to minimize the lag in the master itself. One observer varies the speed and watches the master, signaling quickly, as the meniscus passes the respective points on the scale, to another observer, who records the error of the tachometer.

Usually a check run is made. In any case an equal number of readings with increasing and decreasing speed are taken at each point. The errors with increasing speed, or "up errors," are recorded in a column opposite the corresponding speed and the errors with decreasing speed,

or "down errors," in a parallel column. With a view to the later calculation of the lag, the up errors at each point are averaged by themselves, and likewise the down errors. The algebraic mean of the two averages gives the calibration error at the point. The values are tabulated in a third column parallel with the columns of up and down errors.

LAG.

The lag is measured numerically by the algebraic excess of the down over the up error or reading. It depends frequently on the rate of variation of the speed and on other conditions, such as vibration, and, in liquid tachometers, on the degree of throttling.

In all measurements of lag care should be taken that the lag of the master tachometer itself is negligible; otherwise the observed values will be too low. The use as a master of a tachometer of the same type as the one under test or of a type, such as the centrifugal, commonly known to be subject to lag, is especially open to question. The lag of the Veeder instrument, with the precautions as to throttling and rate of variation of speed noted above, is five revolutions per minute or less.

The lag is calculated by taking the algebraic difference of the average of the up and down readings already found. The values are recorded in a fourth column parallel with the three error columns. It is customary to omit the sign.

EFFECT OF REVERSAL.

For use in multiple-engined planes where some motors run clockwise and others counterclockwise it is necessary that the tachometer should read the same for either direction of rotation of its drive shaft. A comparison of the calibrations for both directions of drive gives this effect.

EFFECT OF VIBRATION.

Airplane instruments are always subject to more or less vibration from the engine. This, on the one hand, facilitates the movement of the parts and thus reduces the lag. On the other hand, it tends to cause unsteadiness of the pointer, looseness, and wear. The latter effects will be treated in another paragraph.

To test the effect of vibration on the lag, tachometers are mounted on a special table, as shown in figure 3. The vibration is produced by means of an unbalanced weight (W), fastened to the shaft of a small motor (M), screwed to the under side of the table. The lamps (H) regulate the speed of the motor and hence the frequency of the vibration. The legs of the table are provided with rubber tips and fit into holes in the baseboard to keep them in place. In this test the instruments are usually driven by flexible shafts, as in figure 3.

EFFECT OF TIPPING AND ACCELERATION.

Some types of tachometers, for example, the centrifugal, show an error when inclined to the vertical and also when subjected to linear acceleration, as in banking, climbing, and acrobatic flying. For this reason tests are ordinarily made with the instrument in the normal vertical position.

Tipping and acceleration produce the same effect; namely, a change in the effective force of gravity. For example, tipping an instrument upside down is the same as giving it a downward acceleration equal to twice that of free fall. Therefore the acceleration test is omitted and the tipping error taken as a measure of the acceleration effect.

The tipping error is determined by simply tilting the instrument on a ring stand or pivoted board and comparing the error in the inclined position with that in the vertical position.

EFFECT OF TEMPERATURE.

The changes in temperature which occur in airplane flights affect the reading of some types of tachometers. Determinations of the error from this cause are made by means of the thermally insulated chamber shown in figure 4. The walls of this chamber, including the

door which forms the front side, are composed of 4 inches of cork board faced with wood. Lateral windows, consisting of two sheets of plate glass separated by an air space and covered by doors having 1 inch of cork board, give visual access to the chamber. The tachometers are mounted in front of the windows either on the multiple connection stands previously described or on detachable instrument boards fastened to the wall of the chamber. The rigid or flexible connections with the driving apparatus pass through a hole in the wall or door of the chamber. In the case of the instrument boards the tachometers are held in place simply by pins and wooden buttons to facilitate attachment and removal.

Tests are made in the neighborhood of +40° C. and −10° C., a range of 50° C., or about the maximum encountered in airplane flights. The higher temperature is produced by the

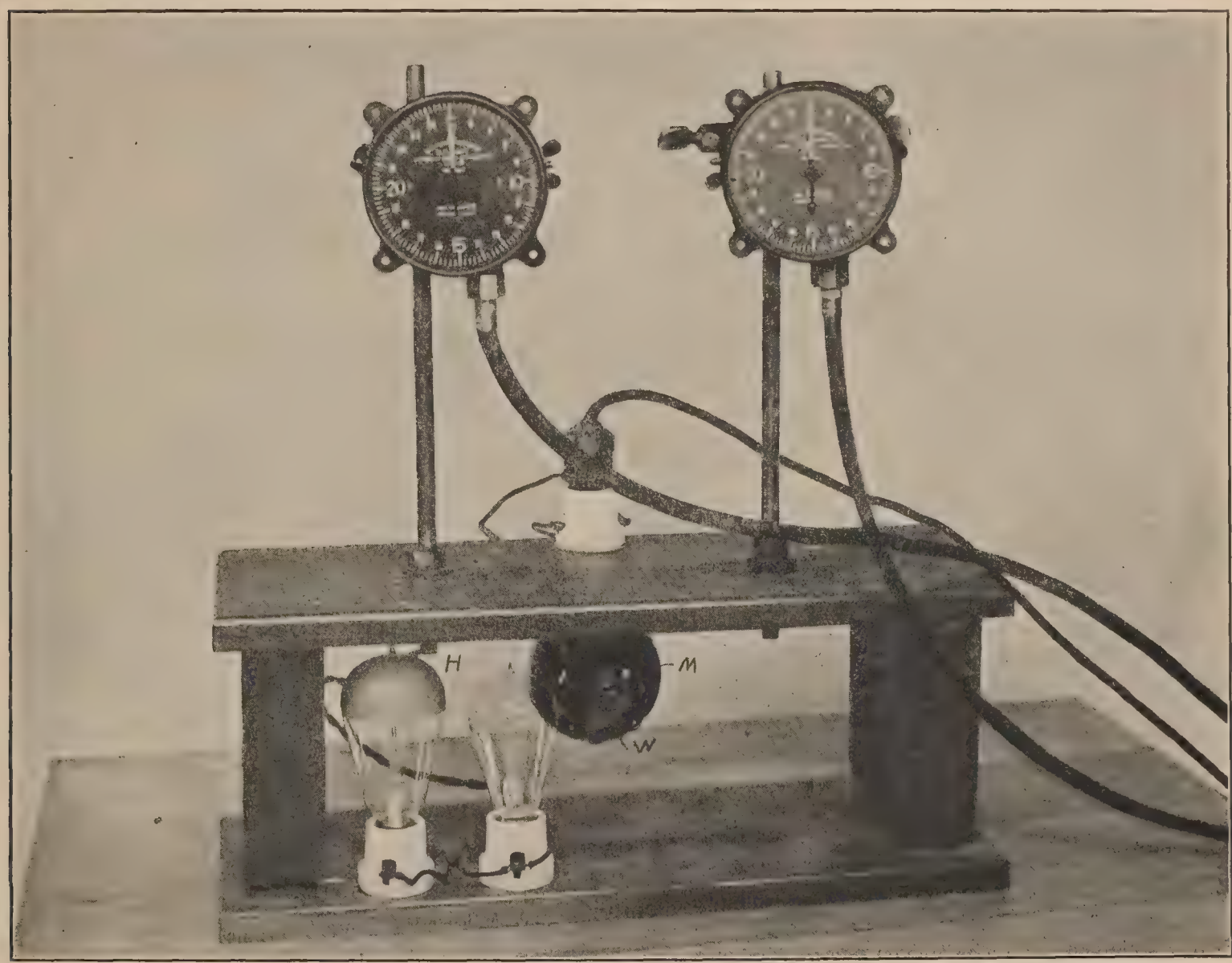


FIG. 3.—Apparatus used in vibration tests.

electric heater shown in figure 4, the lower temperature by brine circulating in the radiator. A fan insures reasonable uniformity of temperature throughout the chamber.

The procedure in determining the error is exactly the same as at room temperature. A comparison of the errors with those at room temperature gives the temperature effect. It is customary to state, for each speed, the maximum effect observed; that is, the greatest difference (algebraic) between the "hot," "cold," and room temperature errors. It is well to note also the effect on the lag, which may be considerable, due possibly to differential expansion or thickening of lubricant.

ENDURANCE AND VIBRATION.

The endurance tests and the test for the effect of long-continued vibration are performed simultaneously, partly to save time, partly to reproduce actual conditions as closely as possible. The apparatus used, shown in figure 5, is contained in a double wooden box to deaden the noise.

The outside of the inner box and the inside of the outer one, as well as the corresponding doors, are completely covered with 1 inch of hair felt, and, in addition, there is an air space of about 1 inch between the boxes on all sides. The outer box is approximately 2 by 2½ by 7½ feet.

The instruments are mounted on multiple connection stands, usually of type (Y), clamped to a table similar to that shown in figure 3 and also having attached to its under side a small eccentrically loaded motor which keeps it and the instruments in constant vibration. Motive power is furnished by one-eighth to one-fourth horsepower motors placed at the ends of the

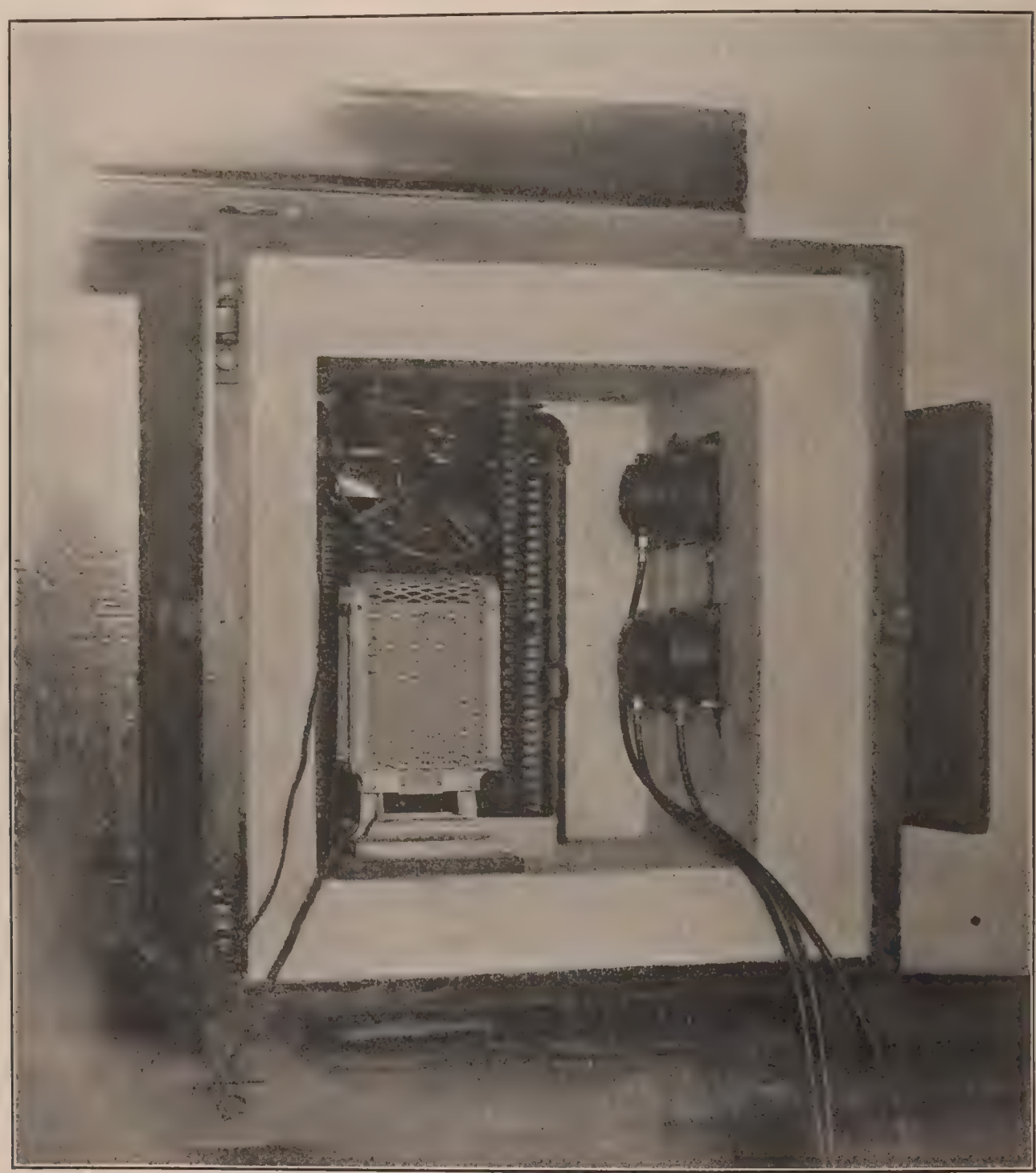


FIG. 4.—Temperature chamber.

table. Connection between the motors and the multiple drives is made though a friction pulley, which allows slippage and thus prevents serious injury to the multiple drive or instruments in case of stoppage; and, also, through a universal joint which gives the flexibility required from the fact that the multiple drive is vibrating, whereas the motor is fixed. An automatic cutout disconnects the motor, in case of stoppage or serious reduction in speed, to prevent burning out of the same, and also stops a clock so as to record the time.

Two different arrangements appear in figure 5. One connected with the left-hand motor, consists of a small horizontal centrifugal governor which collapses as the speed diminishes,

opening a contact gap in the motor circuit and pushing a plunger up against the balance wheel of the clock.

The other, connected by a flexible shaft with one of the multiple drives is seen in the upper left-hand corner of the chamber. A small fan, driven through the flexible shaft, blows air against a light aluminum wind cup pivoted in horizontal bearings. Above a certain speed the wind cup is held over away from the fan maintaining the connection with the motor (right hand). Below the speed the wind cup, being suitably unbalanced, drops back toward the fan, breaking the motor connection, but making connection between a battery and the electromagnet seen directly back of the right-hand clock. The magnet, being excited, releases a plunger which springs up against the balance wheel of the clock, at the same time again breaking the connection with the battery in order not to exhaust the latter.

Neither of the cut-outs described will function in the case of stoppage of individual instruments, unless accompanied by stoppage or considerable slowing down of the entire apparatus.

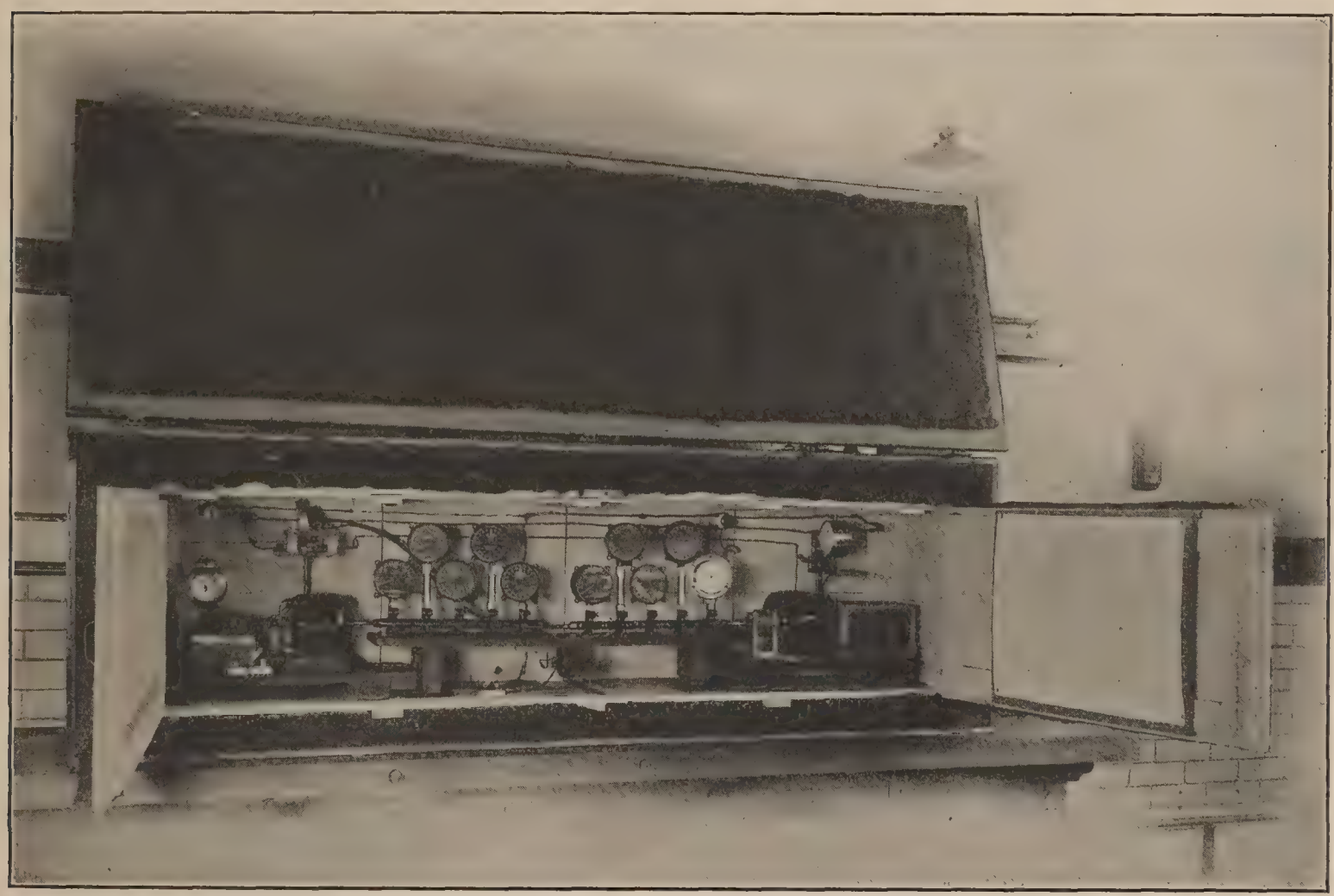


FIG. 5.—Endurance—vibration apparatus.

When this occurs, however, they serve the double purpose of protecting the motor and of recording the time, so that the duration of the run may be known.

The normal length of run is 150 hours. After the run the instruments, which have suffered no breakdown or serious mechanical defect, are recalibrated and the change in calibration at each point as a result of the run tabulated. All instruments are then dismantled and examined for wear, looseness, slippage, lubrication difficulties, or other mechanical troubles.

EFFECT OF REDUCED AIR PRESSURE.

Some tachometers, requiring air for their action, are tested for the effect of the diminution in air pressure above the surface of the earth.

The tachometer is driven inside a partially evacuated chamber through a mercury seal. The use of a packed bearing and the temperature changes, when the driving motor itself is placed within the chamber, are thus avoided. Figure 6 shows the apparatus.

(A), (B), (C), and (D), (fig. 6), are, respectively, the drive motor, master tachometer, mercury seal, and vacuum chamber. The chamber consists of a glass bell jar (E) inverted on a steel plate (F). The plate and jar are held together partly by the weight of the latter and partly after evacuation, by the pressure of the outer air. The joint between them is hermetically sealed by the rubber gasket (K), which is cemented to the edge of the jar with shellac, the surface of the plate beneath being smeared with vaseline. The tachometer (T) is supported on a stand (J) fastened to (F).

The mercury seal (C) is of peculiar construction. It consists of a heavy walled glass manometer tube about 30 inches high. The tube is connected at one end with the air chamber through the packed coupling (G). The other end is open to the air. A standard flexible cable and casing, the latter perforated at several points, are placed in the manometer tube. Connection is made at (H) with the motor shaft, at (L) with the tachometer. The tube is partly filled with mercury. Air is exhausted from (D) through the pump connection (O). As this is done mercury rises in one arm of (C) and falls in the other, as in an ordinary manometer, until the difference in pressure in the chamber and the outer air is counterbalanced. Mercury at the same time runs in around the cable through the perforations in the casing, forming an air-tight seal about the cable without sensibly impeding its rotation. A thermometer (M) and an aneroid barometer (N) show, respectively, the temperature and pressure in the chamber.

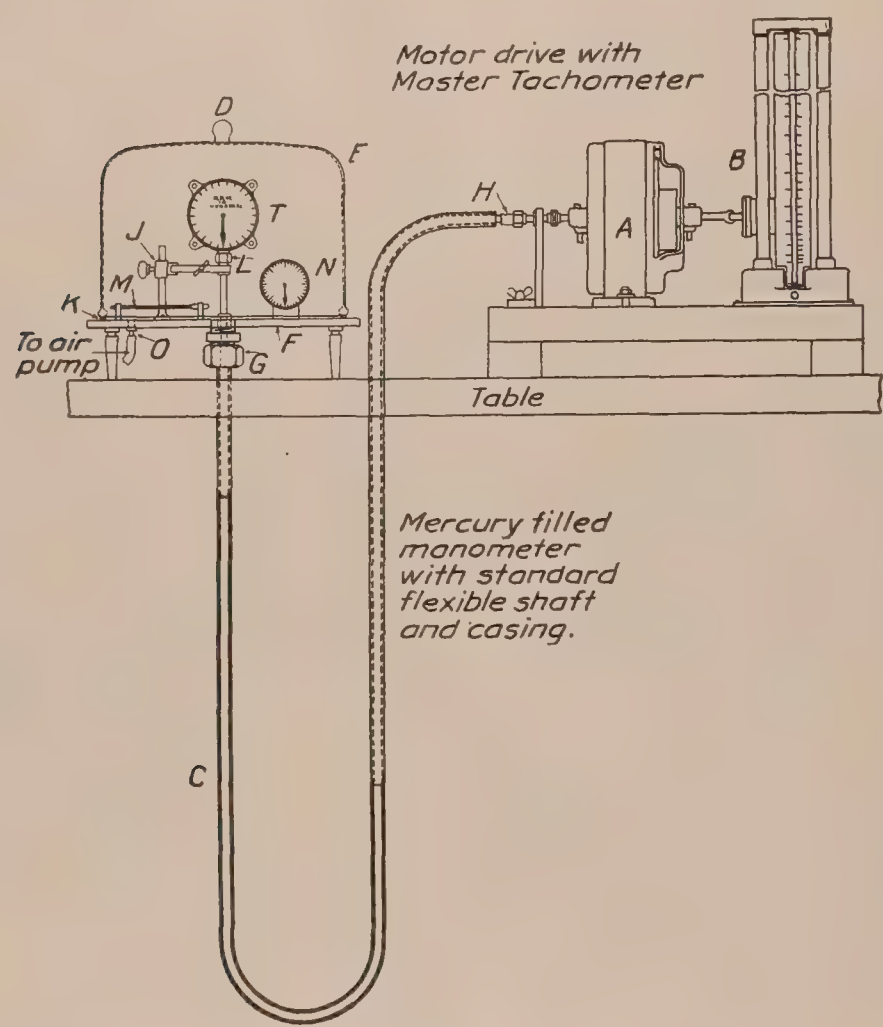


FIG. 6.—Apparatus for testing tachometers under reduced pressure.

A comparison is made of the calibration error at low pressures with that at normal atmospheric pressure. This may be determined at different speeds for the same pressure or for different pressures at the same speed. Or the pressure effect may be observed directly by holding the speed constant and either exhausting or readmitting the air to the chamber and noticing the change in reading.

The apparatus has been used at speeds up to 2,500 revolutions per minute and at pressures as low as one-half atmosphere, corresponding to an altitude of 20,000 feet.

CALIBRATION OF MASTER TACHOMETER.

Calibration and investigation of the master tachometer itself are done by an absolute method, by which the revolutions and the time are measured directly. Two different appliances are in use, both of the kind in which a revolution counter is operated automatically by means of electric time signals.

The first is a slight modification of an apparatus employed by the Veeder Manufacturing Co., and described by Amasa Trowbridge on pages 1221–1223 of the Transactions of the American Society of Mechanical Engineers for 1908.

It consists, in brief, of a relay, electromagnets, battery, and switches. The time signal current is passed through the relay, which, in turn, operates the electromagnets, and these throw the counter into and out of connection with the shaft whose speed is to be determined.

A possible advantage over some apparatus of this kind consists in the fact that the counter is engaged and disengaged by identical operations. Any error, due to a difference in the lag of the counter behind the time signal on connecting and that on disconecting, is thus avoided. The electromagnets are arranged so as to give directly, without the use of links or pivoted joints, the necessary rectilinear motion for engaging and disengaging the counter.

Figures 7 and 8 show the apparatus. The wiring diagram (fig. 8) shows that the push buttons (P_1) and (P_2) are in series with the contact gap (A) of the relay, the battery (B) and

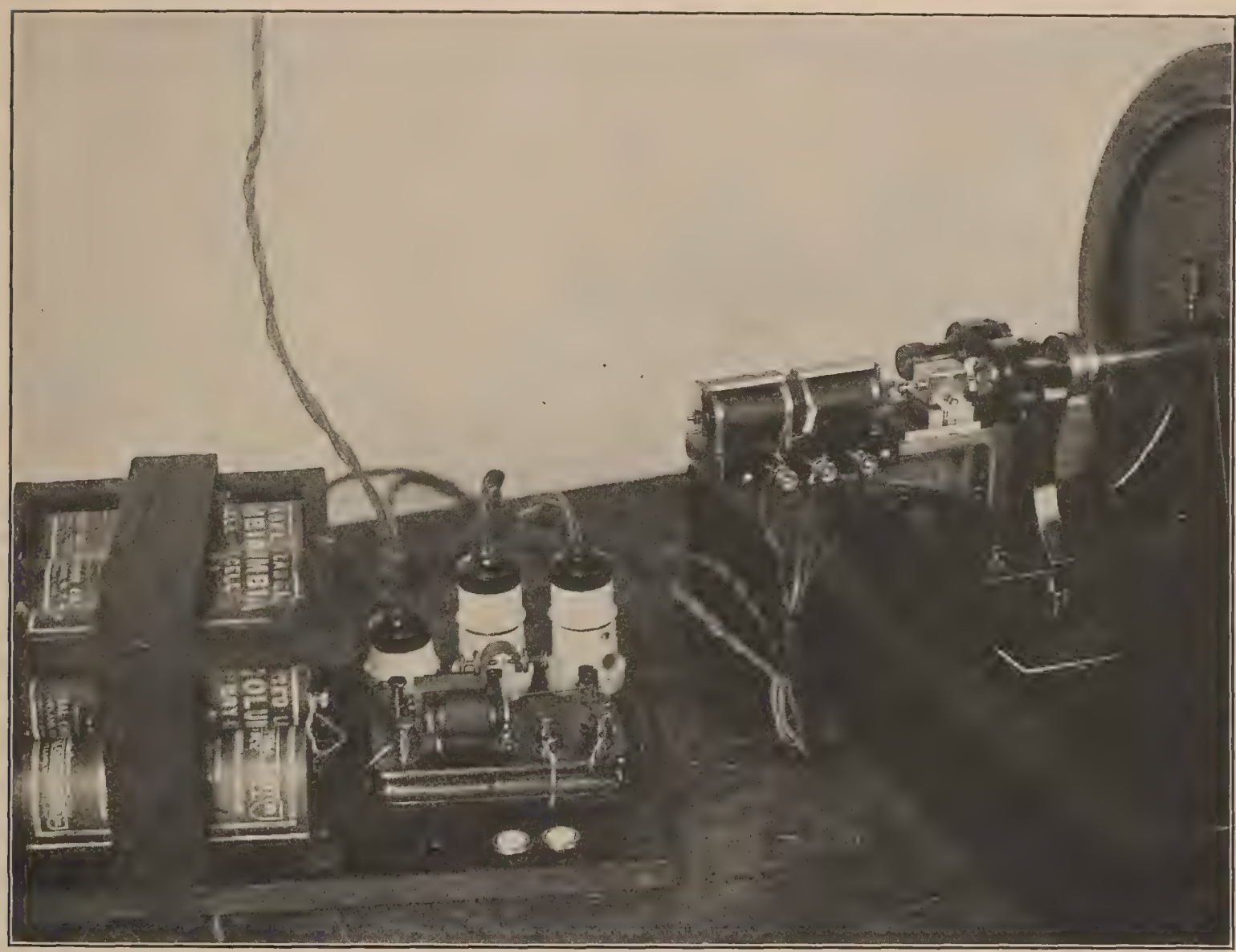


FIG. 7.—Time signal apparatus for calibrating master tachometer.

the electromagnets (M_1) and (M_2), respectively, through the connection sockets (S_1) and (S_2). Therefore, if either of the push buttons is held down, the corresponding electromagnet is actuated by the time signals. Now the magnets and counter (C) are coaxial and, by their attraction of the steel armatures (D_1) and (D_2), move the vane (L) which slides endwise in a slit in the end of the counter spindle and with which the spindle rotates. The movement of (L) is accomplished through the rod (E), which slides in holes in the cores of the magnets and to which (D_1) and (D_2) are fixed, the slip connection (G) and the rod (K), which is free to slide in a hole in the counter spindle and to which (L) is fastened. The end plates (R_1) and (R_2) are of brass. The plates (W_1) and (W_2), however, are of steel, and form, with the cores of the magnets, the return bars (H_1) and (H_2) and the base plate (T), also of steel, the partial magnetic circuits which are completed by (D_1) and (D_2). (Y) is a guiding pin to prevent rotation of (E), (D_1), and (D_2).

According as (M_1) or (M_2) is excited, (L) is thrown into mesh with the rotating clutch (N) on the end of the shaft (F) whose speed is to be determined or with the stationary clutch (O). It is held in either position by residual magnetism. Furthermore, the width of (L) is made slightly less than, but as nearly as possible equal to, the distance between (N) and (O), so that (L) goes into mesh with the one as soon as possible after going out of mesh with the other, but can not be in mesh with both clutches simultaneously.

The above form of revolution counter has the advantage that it may be connected and disconnected without moving the counter as a whole,

Suppose that, to start with, both push buttons (P_1) and (P_2) are up and that (D_2) is against (M_2). (L) is then out of connection with (F) and the counter is at rest. The reading of the counter is taken to a fraction of a revolution corresponding to the number of segments or pockets into which the clutches (N) and (O) are divided. Then just before a given signal the button (P_1) is pushed and held until after the passage of the signal. From the foregoing it follows that at the instant at which the signal is received, (M_1) is actuated, (D_1) is attracted up to (M_1), (L) is thrown into mesh with (N), and the counter begins to record the revolutions of (F). (P_1) is now released. Next, just previous to another stated time signal, preferably an even number of minutes after the first, (P_2) is pressed and held until after the signal occurs. Obviously the effect will now be, at the instant at which the signal arrives, to excite (M_2) attract (D_2) up to (M_2), throw (L) out of mesh with (N), and stop the counter. This stoppage is immediate, "coasting" of the counter being prevented by the mesh of (L) with (O). The counter is then read again. The difference between the initial and final readings gives the revolutions during the interval between the time signals, and hence the speed of (F).

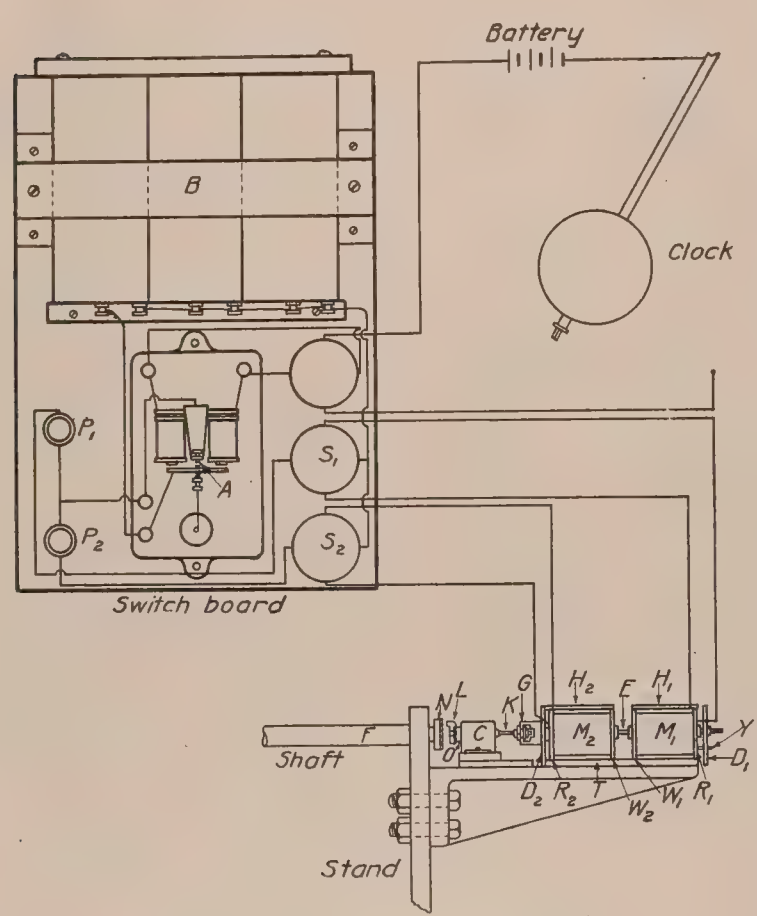


FIG. 8.—Time signal apparatus.

The time signals at the Bureau of Standards occur every second, the minutes being recognized by the omission of the fifty-ninth signal in each minute. The operation of the apparatus and the regulation of the speed are easily performed by a single observer. Further simplification, however, is obtained when the closures of the circuit, as by (P_1) and (P_2), are made automatically at intervals of one or more minutes by a clock-driven commutator. Also it is proposed to construct an apparatus with two counters arranged to operate alternately. The two-second pause may then be utilized without the necessity of waiting over every other minute.

The magnets (M_1) and (M_2) being practically alike, as stated above, the operations of connecting and disconnecting the counter are sensibly identical and the effect of inductive lag is eliminated. Also if (L) meshes to equal depth with the clutches (N) and (O), the errors due to the time required for (L) to free itself from the clutches at the beginning and end counterbalance each other. An error caused by the revolutions lost while (L) is traversing the clearance distance between it and the clutches is, however, present. This error is determined by the amount of clearance and the velocity of (L), the latter depending in turn on the pull of the magnets, the mass of the moving system, and the frictional resistance. Though as yet no separate study of this source of error has been made, the close agreement, within a few revolutions, of the results with those obtained by other methods and elsewhere indicate that the

apparatus is sufficiently accurate for the purpose for which it is intended. The separate readings themselves agree, under the best conditions, to within one-fourth to one-half a revolution.

A second, more complicated, and theoretically less accurate apparatus appears in figure 9. In this apparatus the counter is thrown in and held in by magnetic attraction, but is disengaged by a spring. The operations are timed automatically by electric signals from a clock, the observer having only to press a button twice and throw a switch once.

The counter is mounted on a sliding rod fixed to the armature of an electromagnet, seen at the right of the figure, and is fitted with a star wheel which engages a pin in the end of the shaft whose speed is to be determined. A helical compression spring encircling the sliding rod holds the counter normally out of connection with the shaft.

The arrangement is such that current begins to flow through the electromagnet at the instant of a given time signal, and continues to flow, thus putting the counter into and holding it in connection with the shaft. Another signal, exactly a minute later, breaks the circuit of

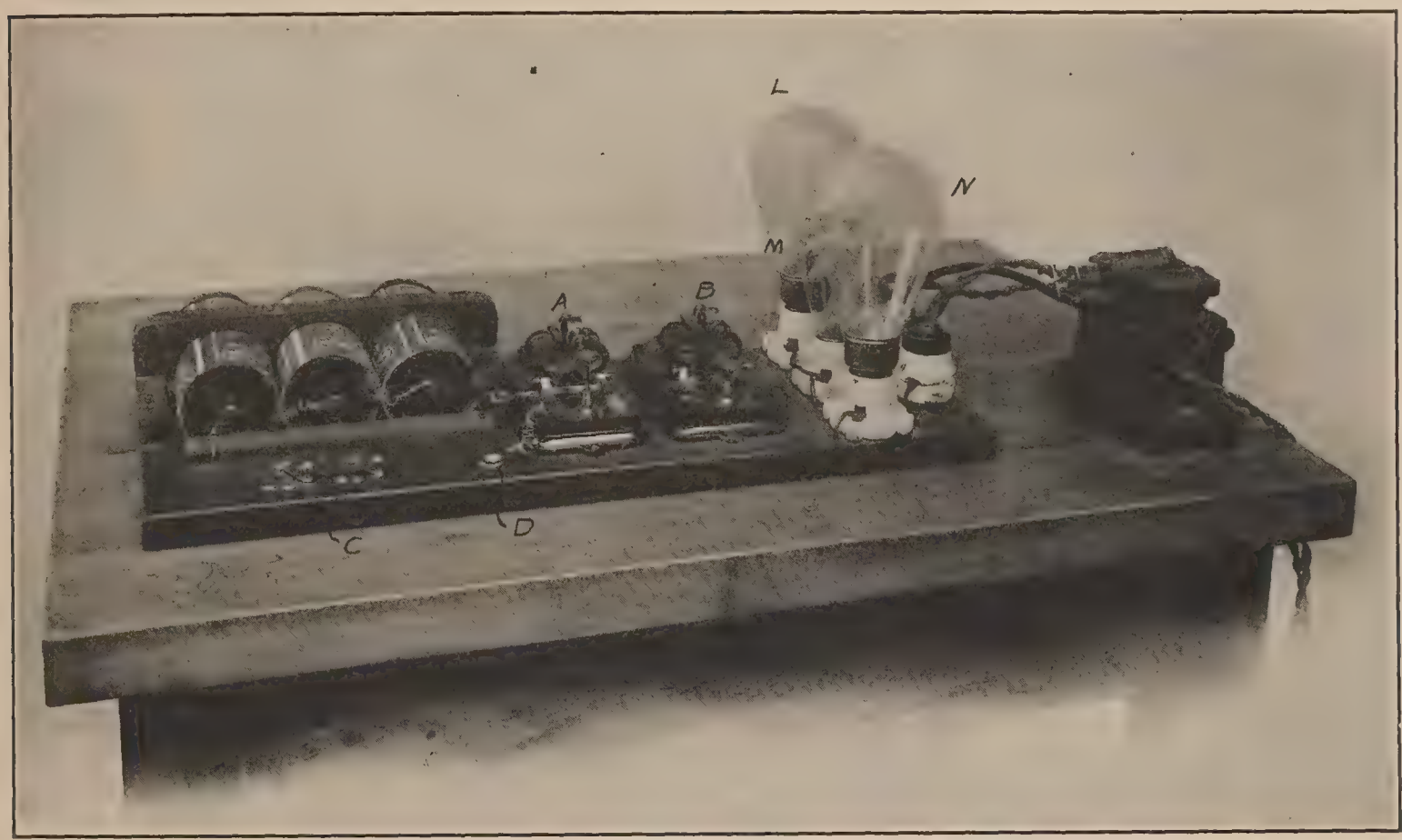


FIG. 9.—Time signal apparatus used in calibrating master tachometer.

the electromagnet and the counter is disconnected. The necessary electrical connections for these operations are made by means of the switchboard seen in the center of figure 9.

Ordinarily only the relay (A) is actuated by the time signal. If the push button (D) is pressed, however, the relay (B) is also operated by the signal. (B) is provided with an insulated stud on the inner side of its armature, connected as shown, and when actuated puts itself and the counter magnet in connection with the 110-volt mains through the lamps (L), (M), and (N) in parallel. At the instant of the first signal occurring after (D) is pressed, therefore, the counter magnet is excited and the counter begins to record revolutions. Moreover, since the exciting of (B) itself maintains the connections, the current continues through the counter magnet. The counter is thus held in connection with the shaft and continues to count revolutions.

The switch (C) is now moved to the left. The contact with the central studs, which is made before that with the right-hand studs is broken, disconnects (B) from the mains but maintains the connection with the counter magnet through the lamps (L) and (M), so that counting still proceeds. The contact with the left-hand buttons merely rearrange the connections so that actuation of (B) will now disconnect the counter magnet from the mains

instead of connecting it as before. Accordingly, (D) being pressed, at the time of the next following signal, the counter magnet is disconnected. The counter is thus disconnected and ceases to record revolutions. The time and the revolutions both being known, the speed is readily calculated.

This apparatus has the relative disadvantages of complexity, comparative uncertainty in action, and, theoretically, of error due to the fact that the counter is connected and disconnected by different means—by magnetic attraction and by spring force, respectively. It is, therefore, not considered, on the whole, as desirable as the first apparatus described.

RESULTS OF TESTS ON AMERICAN AIRPLANE TACHOMETERS.

Detailed results of various tests on several types of American airplane tachometers are given below. A brief summary of these results has already been given in an earlier section of this report.

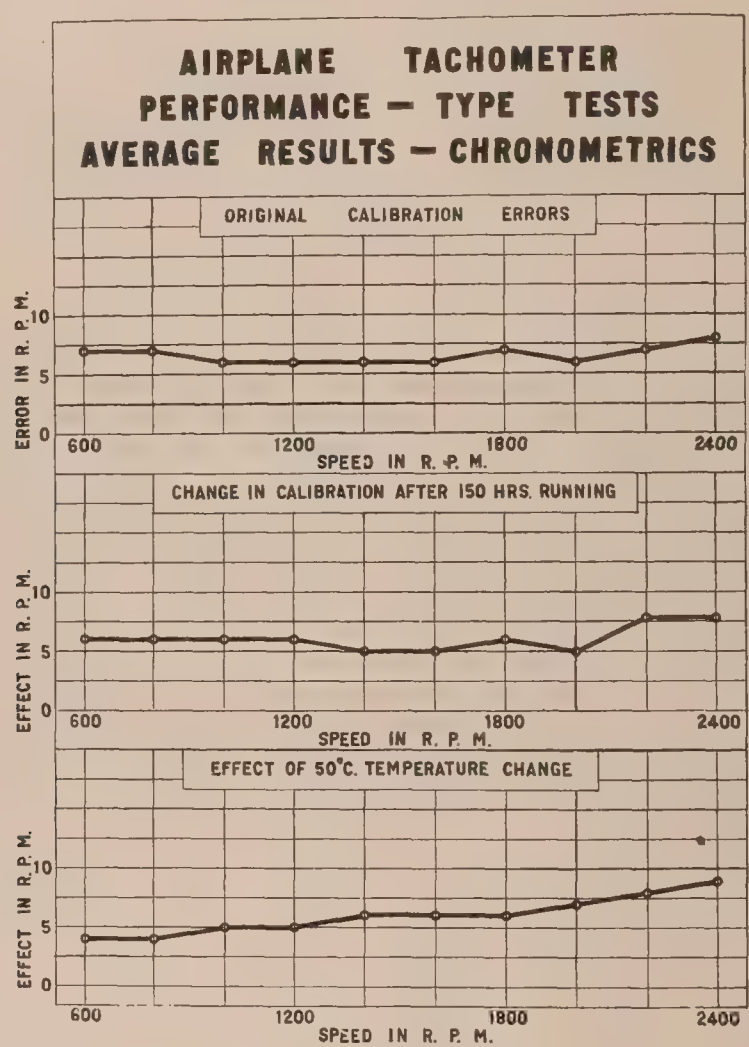


FIG. 10.

The following types were tested thoroughly: Chronometric, centrifugal, and magnetic. Other types, such as air viscosity and electric, were tested less completely but sufficiently to prove that they were not at present well adapted to airplane use. For this reason the second group of instruments were not given certain tests which they could probably pass successfully.

The performance of each of the chronometric, centrifugal, and magnetic types is first discussed separately and then compared with that of the other types.

CHRONOMETRIC TACHOMETERS.

Chronometric tachometers were studied for calibration errors, lag, effect of temperature change, tilting, and running on the calibration. Two makes, based upon the same principle but differing greatly in detail, were tested. In the following discussion these two makes are identified by the letters "A" and "B". A weighted mean of the results for the two makes is tabulated as the performance of the type, and is plotted in figure 10.

Calibration.—The calibration errors of a new instrument of this type are usually small. The average errors of 204 instruments, of which 79 were of make "A" and 125 of make "B," are tabulated below in Table I, together with the average errors of instruments of each make.

TABLE I.

Speed in revolutions per minute.	Average errors in revolutions per minute.		
	Make "A."	Make "B."	Type.
600	9	5	7
800	9	5	7
1,000	8	4	6
1,200	7	5	6
1,400	6	5	6
1,600	6	5	6
1,800	6	7	7
2,000	6	6	6
2,200	7	7	7
2,400	9	7	8

The average error for all speeds was about 7 revolutions per minute for make "A" and 6 revolutions per minute for make "B."

The percentage of instruments having errors between the following limits—namely, greater than 20, 20 to 10, and 10 to 5, respectively—was 11, 69, and 20 for make "A" and 5, 46, and 49 for make "B."

Effect of running on calibration.—Table II shows the effect of 150 hours running and vibration on the calibration for 35 instruments of make "A" and for 51 of make "B" and also for the type.

TABLE II.

Speed in revolutions per minute.	Average effect in revolutions per minute.		
	Make "A."	Make "B."	Type.
600	7	5	6
800	6	6	6
1,000	6	6	6
1,200	7	5	6
1,400	5	5	5
1,600	6	5	5
1,800	6	6	6
2,000	5	5	5
2,200	8	5	7
2,400	8	6	7

It is seen that the average effect in each make is practically that of the type, or 6 revolutions per minute.

Of 35 instruments of make "A" 14 per cent showed maximum changes of 20 revolutions per minute or over, 57 per cent changes between 10 and 20 revolutions per minute, and 29 per cent showed changes less than 10 revolutions per minute.

Of 51 instruments of make "B," 8 per cent showed changes over 20 revolutions per minute, 45 per cent between 10 and 20 revolutions per minute, and 47 per cent less than 10 revolutions per minute.

Temperature.—The average effect of 50° C. change in temperature is tabulated below in Table III for 36 instruments of make "A" and 34 of make "B" and for the type.

TABLE III.

Speed in revolutions per minute.	Average effect in revolutions per minute.		
	Make "A."	Make "B."	Type.
600	7	0	4
800	8	0	4
1,000	9	1	5
1,200	9	1	5
1,400	10	2	6
1,600	11	1	6
1,800	11	1	6
2,000	13	2	7
2,200	13	2	8
2,400	15	3	9

It is seen that the effect is greater for instruments of make "A" than for those of make "B." The average effect for the type is small, the maximum being 9 revolutions per minute. It will be noticed that the temperature effect increases slightly with the speed.

Of a group of 36 instruments of make "A" tested for the effect of temperature change 14 per cent had effects equal to or greater than 25 revolutions per minute, 70 per cent between 15 and 25 revolutions per minute, and 16 per cent less than 15 revolutions per minute. Of 34 instruments of make "B" only one instrument showed an effect at any point of 25 revolutions per minute, the remaining 97 per cent having the maximum effect less than this amount.

Tilting.—Instruments of the chronometric type read the same for all positions of the axis with respect to the vertical.

Lag.—The chronometric tachometer has no lag due to friction and inertia such as is found in the centrifugal. A time lag is, however, characteristic, owing to the fact that the pointer is locked in position during each counting period. Consequently, any change in speed during this period, usually one second, will not be shown until its end, and may be considerable. But, if the speed is increased to a given value and held constant for a few seconds, the reading will be practically the same as if the speed had been decreased to the same value and held there.

A pointer-steadying device used in instruments of the type denoted make "A," causes a difference between up and down readings of 20 revolutions per minute or more, due to lost motion. This lost motion is purposely inserted, a steady pointer evidently being deemed by the manufacturers as more desirable than freedom from lag.

If chronometric tachometers do not have this special pointer-steadying device sudden slight jumps of the pointer are usually present. This is due to the fact that the teeth of the fine-toothed pinion do not always mesh correctly with those of the rack or counter gear, and slippage or sliding of one tooth upon its mate until proper mesh is made must occur before the mechanism functions properly. The maximum jump which could occur from this cause is in the neighborhood of 15 revolutions per minute. The jump observed in practice is usually less than 5 revolutions per minute.

Durability.—An idea of the durability of the chronometric tachometer is given by the following data: Of 50 instruments of make "A" tested for endurance 14, or about 28 per cent, failed before the test was completed. Of 69 instruments of make "B," 14 or 20 per cent failed. The length of the test was 150 hours.

It is only fair to say, however, that of the failures of make "B" the larger number occurred in instruments of the earlier production. The later instruments were improved so that of the latter third of the instruments submitted to the tests the failures were few.

Conclusions.—It is seen that the performance of the chronometric type is very good as regards accuracy under different conditions of temperature, altitude, etc. Owing to the fact, however, that the readings are intermittent, it is not so satisfactory as an indicator for speeds which are changing. For instead of indicating the change in speed gradually as it occurs, it

shows all at once at the end of a given time interval the total change in speed which occurred during that interval. This lack of sensitivity is a serious defect for certain kinds of work and certainly is not a desirable feature for use in airplanes.

CENTRIFUGAL TACHOMETERS.

Centrifugal tachometers were studied for calibration errors, effect of running, tilting, lag, effect of temperature change and durability. Two makes, "C" and "D," both of the so-called governor type, were tested. The results of these tests are given below for instruments of each make, and a weighted mean of these results is tabulated as the average error for the type, and is plotted in figure 11.

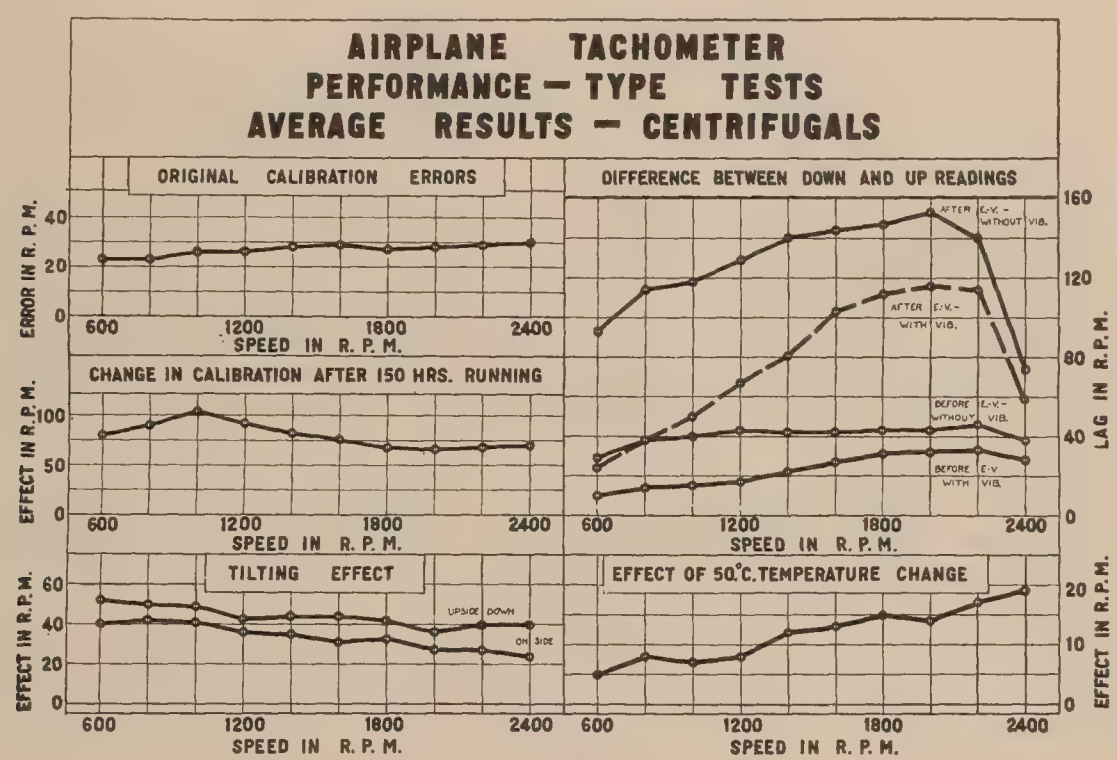


FIG. 11.

Calibration.—The average calibration errors of new instruments of the centrifugal type are somewhat higher than those of the chronometric type. The average errors of 73 instruments, of which 9 were of make "C" and 64 of make "D," are tabulated below, together with the average errors of each make.

TABLE IV.

Speed in revolutions per minute.	Average error in revolutions per minute.		
	Make "C."	Make "D."	Type.
600	24	23	23
800	20	25	23
1,000	20	31	26
1,200	23	31	26
1,400	25	30	28
1,600	28	29	29
1,800	27	28	27
2,000	27	28	28
2,200	30	28	29
2,400	31	29	30

The average error for all speeds was 26 revolutions per minute for make "C," 28 revolutions per minute for make "D," and 27 revolutions per minute for the type. Of make "C," 89 per cent had errors at some point of 20 revolutions per minute or more and of make "D," 71 per cent.

Effect of running on the calibration.—The average effect of 150 hours running and vibration on the calibration is shown below for 33 instruments of which 5 were of make "C" and 28 of make "D."

TABLE V.

Speed in revolutions per minute.	Average effect in revolutions per minute.		
	Make "C."	Make "D."	Type.
600	26	96	86
800	24	90	81
1,000	20	104	92
1,200	22	92	82
1,400	20	82	73
1,600	20	75	67
1,800	16	68	61
2,000	12	66	58
2,200	17	69	62
2,400	27	70	64

The average change in calibration caused by running is 20 revolutions per minute for make "C" and 81 revolutions per minute for make "D." The average effect for the type is 73 revolutions per minute.

Of make "C," the greatest effect observed at any point was 58 revolutions per minute; 80 per cent of the instruments tested showed an effect at one or more points of over 20 revolutions per minute.

Of make "D," 65 per cent had errors at some point of 50 revolutions per minute or more, 15 per cent between 50 and 20 revolutions per minute, and 20 per cent less than 20 revolutions per minute.

Tilting.—Table VI shows the effect of the position of the instrument with respect to the vertical. The error for each point was determined with the instrument (1) upside down and (2) on its side. In all cases the reading in these positions was higher than in the vertical position.

TABLE VI.

Speed in revolutions per minute.	Average change in reading due to tilting in revolutions per minute.					
	Upside down.			On side.		
	Make "C."	Make "D."	Type.	Make "C."	Make "D."	Type.
600	30	63	52	15	53	40
800	35	57	50	25	50	42
1,000	38	55	49	23	50	41
1,200	28	51	43	25	41	36
1,400	25	53	44	25	40	35
1,600	22	56	44	14	39	31
1,800	15	56	42	13	44	33
2,000	13	47	36	6	38	27
2,200	13	54	40	6	38	27
2,400	15	53	40	10	31	24

The average errors for make "C" are 23 and 16 revolutions per minute for the upside down and on side positions, respectively. For make "D" the corresponding average errors are 55 and 42 revolutions per minute.

Lag.—The differences between readings taken with decreasing speed and with increasing speed are tabulated below. Four tables are given showing the lag (1) before the endurance-vibration run without vibration, (2) the same with the instrument being vibrated while calibrated, (3) after the endurance-vibration run without vibration, and (4) after the endurance-vibration run with vibration. These values are shown graphically in the upper right-hand plot of figure 11. The effects of vibrating the instrument during the calibrations before and after the endurance run are evident from the data and plots; also the effect of the run itself.

TABLE VII.

Speed in revolutions per minute.	Average lag in revolutions per minute.											
	Before endurance-vibration run.						After endurance-vibration run.					
	(1) Without vibration.			(2) With vibration.			(3) Without vibration.			(4) With vibration.		
	Make "C."	Make "D."	Type.	Make "C."	Make "D."	Type.	Make "C."	Make "D."	Type.	Make "C."	Make "D."	Type.
600	2	36	29	4	13	10	27	129	93	10	29	24
800	9	46	38	6	20	14	35	158	114	18	44	38
1,000	11	48	40	7	20	15	40	162	118	16	61	50
1,200	15	50	43	8	23	17	41	178	129	25	81	67
1,400	14	49	42	10	29	22	42	195	140	22	100	81
1,600	19	48	42	12	36	27	41	201	144	27	128	103
1,800	20	49	43	14	42	31	45	204	147	31	139	112
2,000	22	49	43	16	43	32	50	211	153	27	146	116
2,200	25	52	46	13	46	33	54	187	140	30	142	114
2,400	23	42	38	15	36	28	39	93	74	29	69	59
Number of instruments tested	9	34	43	9	13	22	5	8	13	2	6	8

For make "C" the average lag before the endurance-vibration run is 16 revolutions per minute without vibration, and 11 revolutions per minute when vibrated. The corresponding values after the endurance run are 41 revolutions per minute and 23 revolutions per minute respectively. The lag was increased by from 100 to 150 per cent as a result of the 150 hours run.

For make "D" the average lag before the endurance-vibration run is 47 revolutions per minute without vibration and 29 revolutions per minute when vibrated. The corresponding values after the endurance run are 172 revolutions per minute and 94 revolutions per minute. For this make the effect of running was to increase the lag by from 200 to 250 per cent.

The averages for the type are (1) 40 revolutions per minute before the run and without vibration, (2) 23 revolutions per minute with vibration, (3) 125 revolutions per minute after the run without vibration, and (4) 76 revolutions per minute with vibration. The average effect of vibration during the calibration is to reduce the lag by about 40 per cent; the average effect of the endurance run is to increase the lag by over 200 per cent.

Temperature.—The average effect of 50° C. change in temperature is tabulated below for 3 instruments of make "C" and 11 of make "D" and for the type.

TABLE VIII.

Speed in revolutions per minute.	Average effect in revolutions per minute.		
	Make "C."	Make "D."	Type.
600	1	9	5
800	5	10	8
1,000	6	8	7
1,200	5	10	8
1,400	8	16	12
1,600	7	19	13
1,800	11	18	15
2,000	9	19	14
2,200	10	24	17
2,400	11	26	19

The average effect for make "C" is 7 revolutions per minute, for make "D" 16 revolutions per minute, and for the type 12 revolutions per minute.

Durability.—The centrifugal tachometer is not likely to break down until badly worn. A few instances of the pointer being loosened on its staff were, however, noticed and in one or two instruments defective steel balls in the end bearings caused an unsteady pointer. In all other cases the instruments continued to indicate, although in many cases the reading was in error by several hundred revolutions.

Conclusions.—It is seen that the average performance of instruments of make "C," although poor, is much better in most respects than that of instruments of make "D." This is explained

in part by details of design, make "C" having a light, high speed governor while make "D" has a heavy and comparatively slow speed governor. Most of the difference, however, is due to superior quality of workmanship and finish of critical parts found in make "C."

The average performance of the centrifugal type tachometer is far from satisfactory especially in regard to lag and change of calibration with use. The average errors or effects for all speeds are as follows: Calibration error, 2 per cent; effect of 150 hours running on calibration, 5 per cent; original lag, 3 per cent without vibration and 2 per cent with, and after 150 hours' running 9 per cent without vibration and 5 per cent with; effect of 50° C. temperature change, 1 per cent; effect of tilting, 4 per cent for the "inverted" position and 3 per cent for the "on side" position.

MAGNETIC TACHOMETERS.

Magnetic tachometers were studied for calibration errors, effect of running and vibration, tilting, lag, effect of temperature change, and durability. All instruments tested were of the same make and design. The results are shown graphically in figure 12.

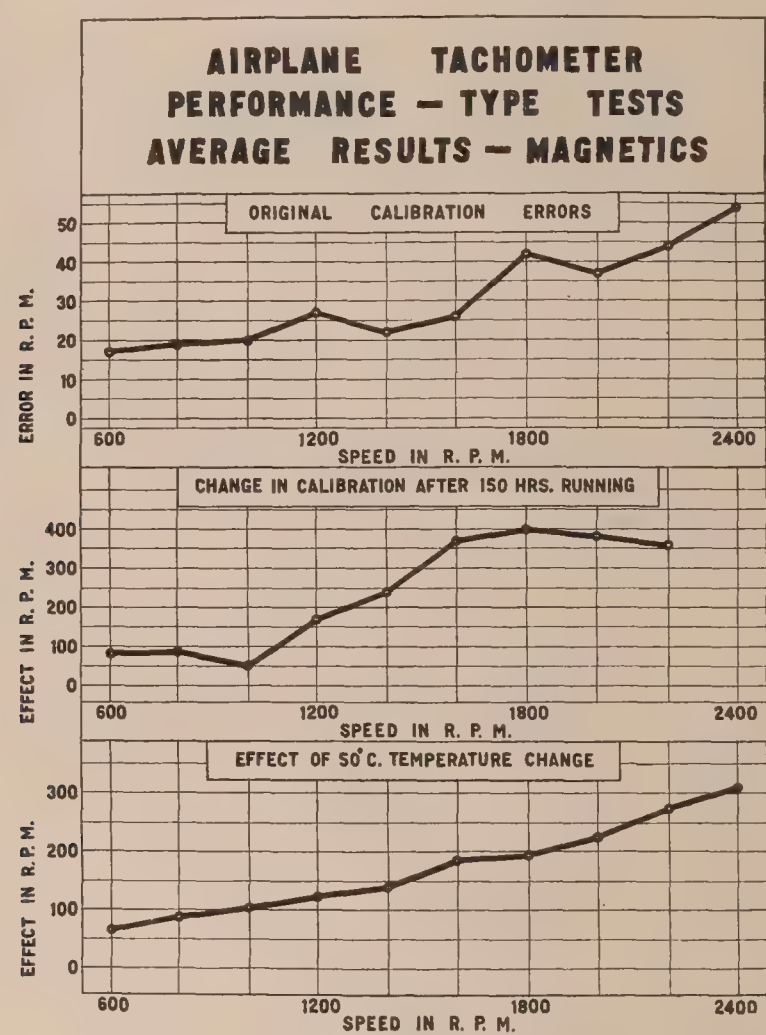


Fig. 12.

Calibration.—The average calibration errors of six magnetic tachometers are tabulated below.

TABLE IX.

Speed in revolutions per minute.	Average error in revolutions per minute.
600	17
800	19
1,000	20
1,200	27
1,400	22
1,600	26
1,800	42
2,000	37
2,200	44
2,400	54

The average error for all speeds is 31 revolutions per minute.

No instrument had all errors less than 20 revolutions per minute; 60 per cent had some error over 50 revolutions per minute, one instrument having errors as high as 375 revolutions per minute. The errors of this instrument are not included in the data tabulated above, as they were considered anomalous.

Effect of running on calibration.—The effect of running and vibrating for 150 hours on the calibration of one airplane model magnetic tachometer is shown in the table below.

TABLE X.

Speed in revolutions per minute.	Average effect in revolutions per minute.
600	81
800	88
1,000	50
1,200	170
1,400	239
1,600	367
1,800	400
2,000	384
2,200	360

The average effect is 237 revolutions per minute; the maximum effect is 400 revolutions per minute at a speed of 1,800, or an error of about 22 per cent.

This is considered very poor performance.

Tilting.—The magnetic tachometer reads practically the same for all positions of its axis.

Lag.—The lag in the magnetic tachometer rarely exceeded 20 revolutions per minute. If the speed were held constant after being either decreased or increased the reading would be the same after a second or two, the lag being caused wholly by sluggishness of action.

Temperature.—The average effect of about 50° C. change in temperature is tabulated below for five instruments.

TABLE XI.

Speed in revolutions per minute.	Average effect in revolutions per minute.
600	64
800	86
1,000	103
1,200	122
1,400	139
1,600	184
1,800	194
2,000	223
2,200	273
2,400	308

In general the effect of lowering the temperature is to increase the reading for a given speed, that of raising the temperature to decrease it.

The average effect is 170 revolutions per minute. The maximum effect is 308 revolutions per minute at a speed of 2,400, or about 13 per cent.

Durability.—The magnetic tachometers tested for endurance all showed some serious defect as a result of the run. The usual and most serious was a pitting of the lower jewelled bearing in which the electrically conducting drum or disc was mounted. This allowed it to move into a stronger magnetic field and hence give a greater deflection for a given speed.

Conclusions.—It is seen that the performance of the magnetic airplane tachometers was not very satisfactory. The average calibration error was about 2½ per cent for full scale reading; the effect of running was 16 per cent and the effect of temperature was 11 per cent. Also the instruments seemed to be of too delicate construction.

COMPARISON OF THE TYPES.

Data showing the performance of the different types of airplane tachometers under various conditions are given in Table XII. The tests are far from being complete; nevertheless a discussion of such results as are given may be of interest.

TABLE XII.—*Airplane tachometer performance—type tests.*

Speed in revolutions per minute.	Error or effect in revolutions per minute.																		
	Calibration error.					Effect of 50° C. temperature change.					Effect of 150 hours running.			Lag error.			Tilting error.		Change at 16,000 feet altitude.
														With vibration.		Without.	Inverted.	On side.	
	Chronometric.	Centrifugal.	Magnetic.	Air viscosity.	Electric.	Chronometric.	Centrifugal.	Magnetic.	Air viscosity.	Electric.	Chronometric.	Centrifugal.	Magnetic.	Centrifugal.	Air viscosity.	Centrifugal.	Centrifugal.	Centrifugal.	Air viscosity.
600	7	23	17	29	22	4	5	64	50	24	6	86	81	10	58	29	52	40	.....
800	7	23	19	31	29	4	8	86	60	12	6	81	88	14	30	38	50	42	10
1,000	6	26	20	50	37	5	7	103	83	9	6	92	50	15	26	40	49	41	.....
1,200	6	26	27	48	41	5	8	122	94	24	6	82	170	17	13	43	43	36	8
1,400	6	28	22	49	44	6	12	139	111	29	5	73	239	22	11	42	44	35	.....
1,600	6	29	26	50	43	6	13	184	126	.....	5	67	367	27	13	42	44	31	.....
1,800	7	27	42	45	40	6	15	194	135	.....	6	61	400	31	19	43	42	33	.....
2,000	6	28	37	54	.....	7	14	223	155	.....	5	58	384	32	18	43	36	27	20
2,200	7	29	44	.....	.....	8	17	273	.....	.....	7	62	360	33	25	46	40	27	.....
2,400	8	30	54	.....	.....	9	19	308	.....	.....	7	64	.....	28	.....	36	40	24	.....

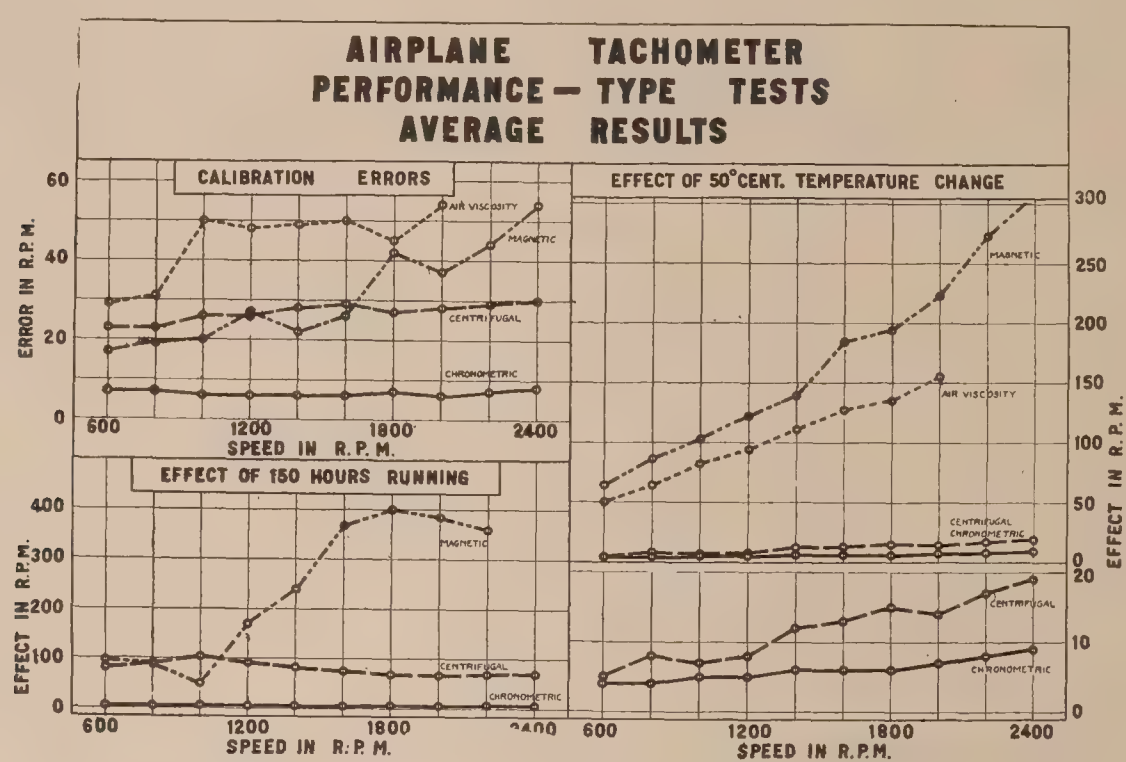


FIG. 13.

The plot, figure 13, shows graphically for comparison (1) the average calibration errors of the chronometric, centrifugal, magnetic, and air viscosity types, (2) the average effect of 150 hours running on the calibration of the chronometric, centrifugal, and magnetic types, and (3) the average effect on the calibration of 50° C. temperature change for the chronometric, centrifugal, magnetic, and air viscosity types. The numerical data for these plots is tabulated in Table XII, mentioned above. The results of the different tests are discussed below.

Calibration.—The type of instrument, number of instruments tested, average calibration error for all speeds in revolutions per minute, average percentage error at speeds of 600 and 2,400, and the percentage of instruments having their average errors in excess of the specification limit of 20 revolutions per minute, are shown below. The percentage error of most instru-

ments is usually less at full scale deflection than at lower speeds, as is evident from the fact that the error in revolutions per minute is more or less constant throughout the scale.

TABLE XIII.

Type.	Number of instruments tested.	Average error.			Percentage instruments having average error over 20 revolutions per minute.
		Revolutions per minute.	Per cent.		
			600	2,400	
Chronometric	204	7	1.2	0.4	7
Centrifugal	73	27	3.8	1.3	63
Magnetic	6	31	2.8	2.3	100
Electric	1	37	3.7	2.2	100
Air viscosity	2	45	4.8	2.7	100

The instruments are arranged in the order of their apparent reliability for calibration. Allowance must be made, however, for the fact that relatively few instruments of the magnetic and air viscosity type and only one of the electric type were tested.

Temperature.—Table XIV is a summary of data showing the average effect in revolutions per minute for all speeds of 50° C. temperature change and in per cent at speeds of 600 and 2,400 revolutions per minute for five types of airplane tachometers. The last column shows the percentage of the instruments tested which had average effects exceeding 20 revolutions per minute.[1]

TABLE XIV.

Type.	Number of instruments tested.	Average effect of 50° C change.			Percentage instruments having average effect over 20 revolutions per minute.
		Revolutions per minute.	Per cent.		
			600	2,400	
Chronometric	70	6	0.7	0.4	9
Centrifugal	14	12	.8	1.3	15
Electric	1	20	4.0	2.1	0
Air viscosity	2	101	8.3	7.8	100
Magnetic	5	170	11.0	12.8	100

The instruments are arranged in the order of their apparent reliability for temperature effect.

The performance of the air viscosity and magnetic types for different conditions of tempreature is very poor, so poor, in fact, as to eliminate the types as they exist from further consideration with those suitable for use in airplanes.

Effect of running on calibration.—Table XV is a summary of data showing the average effect at all speeds, of 150 hours running and vibration in revolutions per minute, and in per cent for three types of airplane tachometers. The last column shows the percentage of instruments tested which had average effects exceeding 20 revolutions per minute.[2] The averages for the different speeds are given in Table XII.

[1] Error allowed by Air Service specifications.

[2] Error allowed by Air Service specifications.

TABLE XV.

Type.	Number instruments tested.	Average effect of 150 hours' running.			Percentage instruments having average effect over 20 revolutions per minute.
		Revolutions per minute.	Per cent.		
			600	2,400	
Chronometric	86	6	1.0	0.3	10
Centrifugal	33	73	14.3	2.7	80
Magnetic	1	237	13.5	15.0	100

It is seen that the performance of the centrifugal and magnetic types is very poor as regards consistency of calibration with use.

Lag.—Table XVI shows the average difference in readings for all speeds taken at the same speed with decreasing and increasing speeds. The instruments were vibrated while being tested and had not previously been run. Table XII gives the complete data and also data showing the average lag in the centrifugal type without the vibration.

TABLE XVI.

Type.	Number of instruments tested.	Average lag with vibration.		
		Revolutions per minute.	Per cent.	
			600	2,400
Centrifugal	22	23	1.7	1.2
Air viscosity	2	24	9.7	1.1
Magnetic [1]				
Electric [1]				
Chronometric [2]				

[1] Small; not observed quantitatively. [2] See discussion.

At slow speeds the lag in the air viscosity type is much greater than in the centrifugal. However, if the speed be held constant after being raised or lowered the lag will practically disappear in the air viscosity type while in the centrifugal type it will not. Hence actually the lag effect in the air viscosity type is less serious than in the centrifugal.

Tilting.—The centrifugal type is the only one tested which read differently for different positions of its axis. Data showing the average change at all speeds in the reading at normal or vertical position for two other positions are given in Table XVII and complete data for the different speeds in Table XII.

TABLE XVII.

Type.	Average effect of tilting.					
	In revolutions per minute.		In per cent.			
			Upside down.		On side.	
	Upside down.	On side.	600	2,400	600	2,400
Centrifugal	44	34	8.7	1.7	6.7	1.0
Chronometric [1]						
Magnetic [1]						
Air viscosity [1]						
Electric [2]						

[1] No effect observed. [2] Not tested; believed small.

It is seen that in the centrifugal type the effect of position of the axis of the instrument with respect to the vertical is by no means negligible.

Effect of air-pressure change.—The air viscosity type is the only one of those tested affected by altitude change. Data taken at a barometric pressure corresponding to that at 16,000 feet altitude is given in Table XII.

It is seen that the average effect observed at any speed did not exceed 20 revolutions per minute, the maximum average percentage error being 1.0 per cent.

Durability.—Of the four types, chronometric, centrifugal, magnetic, and air viscosity, the centrifugal type gives the best performance as regards durability. The percentage of breakdowns, complete or partial, so that readings can not easily be taken is small. However, the reading may be considerably different from the true speed as is seen from the foregoing discussion of the characteristics of the type.

About 20 per cent of the instruments of the chronometric type broke down so as to necessitate repairs and replacement of parts. Also, in several cases, the escapement failed to start unless the instrument was shaken. In use on a plane this would necessitate dismantling.

Instruments of the magnetic type did not break down completely, but the pointer gradually became more and more unsteady until finally readings could not be taken.

The air viscosity type was not tested for endurance, but it is believed that its behavior under airplane conditions would be similar to that of the magnetic type.

Weights.—The average weights for the different types are given in Table XVIII.

TABLE XVIII.

Type.	Weight in pounds.
Chronometric	1.3
Centrifugal	1.0
Magnetic	2.0
Air viscosity	1.4
Electric	4.8

Attention is called to the relatively large weight of the electric type.

REPORT No. 129.

POWER-PLANT INSTRUMENTS.

PART III.

THERMOMETERS FOR AIRCRAFT ENGINES.

By E. F. Mueller and R. M. Wilhelm.

SUMMARY.

This part describes the principal types of distance-reading thermometers for aircraft engines, including an explanation of the physical principles involved in the functioning of the instruments and the proper filling of the bulbs. Performance requirements and testing methods are then given, concluding with a discussion of the sources of error and results of tests.

INTRODUCTION.

The term "airplane thermometer" is usually used to designate an instrument, which indicates the temperature of the water or oil at some point in the respective circulating systems of these liquids in an aircraft engine. Thermometers may be used for experimental purposes, to measure tempratures existing in various other regions about the engine or aircraft, but such thermometers will not be discussed in this article.

Thermometers designed for use in the water-cooling system have been used to measure the temperature of the lubricating oil. The use of thermometers for this purpose is not general, although oil thermometers were specified as part of the standard equipment of several types of American airplanes. The discussion which follows will apply more especially to instruments used to measure the temperature of the circulating water.

The bulb of the thermometer is usually placed in the top of the radiator and for this reason these instruments have sometimes been called "radiator" thermometers.

FUNCTIONS.

A temperature indicator for aeronautic engines may serve one or more of the following purposes: To indicate engine trouble, possibly before any of the other instruments would show this; to assist in operating the engine at maximum efficiency; to indicate whether the water is at or near its boiling point, or the oil is at such a temperature that its lubricating properties are impaired; to warn that the engine is becoming too cold to start up again after having been cut off in gliding; to warn that the water is in danger of freezing or that the oil is too thick to flow. The last condition may be encountered in cold climates when the engine is allowed to stand idle for a sufficient period, or on long glides from high altitudes.

REQUIREMENTS.

The requirements for an airplane engine thermometer may be summarized as follows:

1. The indicator must be at a distance from the bulb. This precludes the use of the liquid in glass or other nondistance-reading type of thermometer.

2. The thermometer must indicate temperatures in the range 0° to 100° C. but since the operating temperature of the water in an airplane engine is in the neighborhood of 80° under normal conditions, it is obvious that the instrument should be most reliable over this part of

the scale. Readings in the neighborhood of 0° C. may also be of importance since in long glides or occasionally under other conditions the temperature of the water may be reduced to such an extent that the danger of freezing becomes serious.

3. At high altitude the instrument will be subjected to external pressures much lower than normal atmospheric, and the indications should not be significantly affected by such changes.

4. The gage and connecting tubing may be at various temperatures, since that part of the tubing passing close to the engine may be heated considerably above the other parts of the tubing and gage, while at high altitudes and in winter some parts may be cooled much below the temperature met with at lower altitudes or in summer. The error from this source should be reduced to negligible proportions.

5. The instrument should be light in construction and as small in size as is compatible with strength and visibility. Furthermore, nonferrous materials have been preferred.

POSSIBLE TYPES OF THERMOMETERS.

The types and forms of distance reading thermometers available for practical use may be classified under two general heads; namely, electrical and pressure. A consideration of the relative advantages of these two general types led to the adoption in this country of the pressure instrument for airplanes, on account of smallness in size and weight, ruggedness of construction and immediate availability. This last factor was important since the pressure thermometers on the market could be easily adapted for airplane work, while it would have been necessary to evolve practically a new type of electrical indicator to fulfill the conditions imposed.

A type of electrical thermometer taken from captured German planes consists essentially of a resistance thermometer connected to a small ohmmeter and a battery. It indicates the temperature only at the time when a push button switch is operated.

The advantage of this type of instrument over the pressure thermometers are sensitivity to rapid changes in temperature, absence of errors due to change in atmospheric pressure, or change in temperature of connections. However, its greater complexity, costliness, weight, and inconvenience in that the pilot must operate the push button to observe the reading, offset the advantages above enumerated.

PRESSURE THERMOMETERS.

Pressure thermometers comprise a bulb containing a liquid or gas or both, and connected by means of capillary tubing, to some form of pressure gage.

The pressure thermometers which have been used up to the present time in this country on airplanes are either of the vapor pressure type, with a free surface of the liquid in the bulb, or the liquid filled type, with the liquid completely filling the thermometer.

Two distinct types of gages have also been used, one employing an ordinary Bourdon tube with sector and pinion multiplying mechanism, the other, a long Bourdon tube of many turns which is connected through a bimetallic temperature compensator to the pointer.

Either type of gage could be used for either of these two types of pressure thermometers. The ordinary Bourdon gage, however, has been used up to the present time only for vapor pressure instruments, while the multiple turn Bourdon tube has been used for the liquid filled instruments.

The photograph, figure 1, shows a liquid filled thermometer (A), and two types of vapor pressure thermometers (B and C) used on American airplanes. Interior views of the bulbs and gages of these instruments are also shown.

PRINCIPLES UNDERLYING ACTION OF VAPOR PRESSURE THERMOMETERS.

The action of the vapor pressure thermometer depends upon the fact that the pressure inside the thermometer is determined solely by the temperature of the free surface of the liquid. It follows that the thermometer must be so constructed that one free surface is always in the bulb. If this condition is fulfilled the readings of the instrument will not be sensibly affected by changes in the temperature of the gage and capillary. It must be remembered, however,

that the liquid will always accumulate in the cooler parts if possible, i. e., if these parts are not already filled with liquid (or with noncondensible gas). The two extreme cases to be considered, therefore, are: (1) The bulb is the hottest part of the system. (2) The bulb is the coldest part of the system. The former is the more important and more usual condition. Assuming that the thermometer contains no noncondensible gas, the gage and capillary will fill with liquid and there must still be liquid in the bulb if the thermometer is to indicate properly. It follows that *the volume of liquid must be greater than that of the gage and capillary combined.* An interesting variation of this case is that in which the volume of liquid is barely sufficient when gage and capillary are at room temperature, but when these are cooled, the liquid in them contracts, so that all the liquid in the system is insufficient to fill the gage and capillary as shown in figure 2 (*a*). The thermometer under such conditions indicates properly when the parts other than the bulb are at room temperature but fails when these are cooled. The second condition, in which the bulb is coldest, may occur occasionally. In this case, the liquid goes to the bulb, but if the volume of liquid is more than sufficient to fill the bulb, the free surface will be in the capillary and the thermometer will fail to indicate properly as shown in figure 2 (*b*). A less extreme case is that in which the capillary or a part of the capillary alone is heated to a higher temperature than either bulb or gage. In this case the liquid will be driven out of the heated parts of the capillary. A thermometer containing more than enough liquid to fill the bulb will fail on the simplest test, namely, it will indicate the temperature of either the capillary or the gage when the bulb is put in ice. It is evident, from the above, that *the volume of the liquid should be insufficient to fill the bulb at any temperature.* From the above two conditions in regard to amount of liquid required it can be seen that *the combined volumes of the capillary and gage must be less than that of the bulb.* It is evident that if a relatively small bulb is required the capillary must be very fine in order that the last-named condition may be fulfilled.

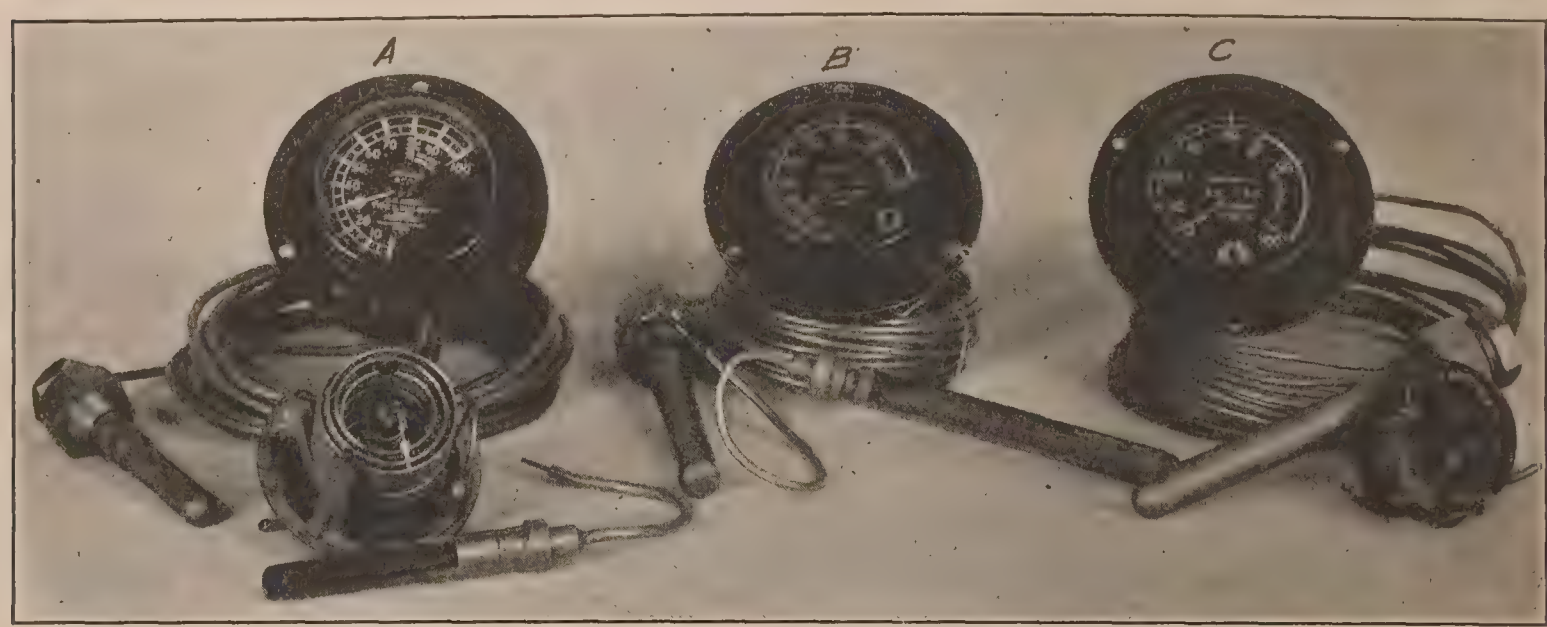


FIG. 1.—Types of thermometers.
A. Liquid filled thermometer with long Bourdon tube.
B. Methyl-chloride vapor-pressure thermometer.
C. Ether vapor-pressure thermometer with ordinary Bourdon gage.

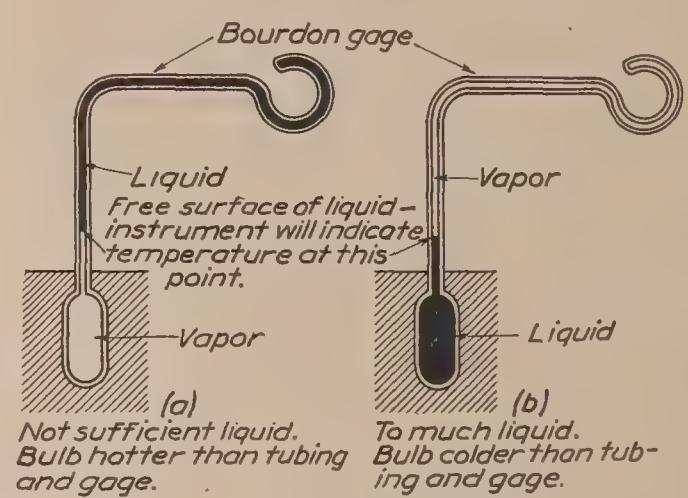


FIG. 2.—Defective vapor pressure thermometers.

The problem of obtaining proper volume relations may be simplified by the use of a transmitting liquid in the tubing and gage. Such a liquid must have a high boiling point, low freezing point, be noncorrosive, and must neither react with nor mix with the volatile liquid. If such a liquid is used to fill the gage and capillary, the effective volume of these is reduced to practically zero, and the necessary volume of the bulb and of the volatile liquid are greatly reduced.

The transmitting liquid type of thermometer has not come into use in this country, although some experimental instruments have been made and submitted for test. The transmitting liquid was a mixture of glycerin and water, the volatile liquid being methyl chloride.

Either ethyl ether or methyl chloride is used as the volatile liquid in the majority of the vapor pressure thermometers in use at the present time on American airplanes. A brief consideration of some of the liquids that could be used may be of interest.

The following table gives the properties of some of the liquids which could be considered for use in a vapor pressure thermometer for an airplane engine, and also the number of degrees centigrade that a thermometer using such a liquid would read high at an altitude of approximately 19,000 feet (one-half atmospheric pressure):

Liquid.	Boiling point.	Vapor pressure at 100° C.	Error at 19,000 feet altitude.	
			Bulb temperature 0° C.	Bulb temperature 80° C.
	° C.	*Atmospheres.*	*° C.*	*° C.*
Alcohol (ethyl)	78.3	2.2	62	10
Ether (ethyl)	34.6	6.4	27	5
Sulphur dioxide	−10	27.8	7	1.3
Methyl ether	−24	30 ?	6	1
Methyl chloride	−24	30 ?	5	(1)
Ammonia	−33.4	61	2.5	.5

[1] Less than 1° C.

The volatile liquid used must be stable, readily obtained and purified, must not act on the metals with which it will be in contact, must have a sufficiently low freezing point, and its critical temperature must be above 100° C. The table shows that the effects of variations in external atmospheric pressure occurring at different altitudes cause smaller variations in the indications of thermometers filled with liquids having lower boiling points, and consequently higher vapor pressures in the working temperature range. The high-pressure gages used with these liquids are also more robust than the low-pressure gages would be. Methyl chloride, methyl ether, and sulphur dioxide fulfill the requirements as regards physical properties. However, on account of availability and ease in handling the liquid, ethyl ether filled thermometers were the only thermometers of the vapor pressure type produced in quantity up to near the termination of the recent war.

PRINCIPLES UNDERLYING ACTION OF LIQUID-FILLED THERMOMETERS.

The liquid-filled thermometers utilize the thermal expansion of a liquid. The increase of pressure with temperature is nearly linear in the range 0° to 100° C. for the liquids used. Alcohol has been used in these instruments, which are made to have an internal pressure at 0° of about 100 pounds per square inch, the pressure increasing to 700 or 800 pounds at 100° C. This large pressure range requires the use of a rugged gage mechanism and makes the indications of the instrument practically independent of the variations of atmospheric pressure with altitude. Since changes in the temperature of the gage and capillary tubing affect the internal pressure, some form of compensator must be used if these parts contain sufficient volume to make the error from this source appreciable, since they are subjected to considerable changes of temperature in use.

A bimetallic compensating helical coil has been employed. This coil is connected at one end to the pointer spindle and at the other end to the Bourdon tube. It may be designed to compensate for changes in the temperature of the gage alone or of the gage and capillary together. This compensator can not eliminate errors due to changes in the temperature of the capillary alone. It is therefore necessary that the volume of the bulb be large relative to the volume of the capillary.

CHOICE OF TYPE.

Since the vapor pressure of a liquid does not change linearly with changes in temperature but increases much more rapidly at higher temperatures, vapor-pressure thermometers have a very open scale in the upper part of the temperature range and a relatively contracted scale in the lower part of the range. If the upper part of the range is the most important this is an advantage, but there is a corresponding disadvantage if readings in the lower part of the scale are of importance. The liquid-filled type has an equally divided scale.

It has been previously pointed out that vapor-pressure thermometers are subject to error if used at high altitudes and that this error is greater the lower the vapor pressure of the liquid used and the lower on the scale the readings are taken. This error for liquid-filled thermometers may be reduced to almost negligible proportions, and is the same regardless of the part of the scale on which the reading is taken.

It may therefore be said that if it is important that the thermometer shall give accurate indications at lower temperatures, the liquid-filled type is to be preferred.

When correctly filled the readings of the vapor-pressure thermometers are sensibly independent of the temperature of the gage and capillary. The liquid-filled thermometer is subject to error if the temperature of the gage and capillary differs from that at which it was calibrated. This error may in some cases be eliminated by employing a compensator as previously mentioned, but in general it may be said that if the most important consideration were that the indications of the thermometer should be independent of the temperature of the capillary, then the vapor-pressure type should be chosen.

SPECIFICATIONS FOR AND TESTING OF AIRPLANE THERMOMETERS.

The discussion of airplane thermometers which has been given is based on data obtained as the result of a considerable amount of investigation extending over about a year and a half, during which time many tests were made for the military branches of the Government and specifications written in cooperation with the War Department.

These specifications, which were revised from time to time, include, apart from the actual details of construction, a series of tests which were devised for the purpose of inspection.

These tests included primarily determinations of (*a*) the scale errors at various temperatures under ordinary laboratory conditions, (*b*) the error that would be introduced if the tubing or gage either together or separately were heated or cooled to temperatures corresponding to those that might be encountered under actual conditions of use, and (*c*) the error caused by the reduction of the external pressure on the gage, the condition met with at high altitudes.

Other special tests were made from time to time including vibration tests to ascertain the comparative effect of vibration on different instruments, tests to determine the mechanical hysteresis due to poor gage mechanism, the lag in the reading when the temperature was changed quickly, tests to determine whether the indications changed with time or in shipment, and chemical analyses of the liquids taken from the instruments.

APPARATUS USED IN TESTS.

Very little special apparatus was required for these tests, since the thermometry laboratory was already equipped for similar work.

A well-stirred water bath, heated by a bunsen burner, served, with the aid of a standard thermometer, for the determination of the scale errors, the mechanical hysteresis in readings due to faulty gage mechanism, and the lag of the readings.

For cooling the tubing and gage, use was made of a low temperature bath cooled by the expansion of carbon dioxide.

For determining the error caused by the reduction of the external pressure on the gage an ordinary vacuum desiccator evacuated to the required extent was used. The pressures were read on a mercury manometer.

RESULTS OF TESTS.

As a result of the testing, the following conclusions have been reached in regard to the accuracy of the instruments examined and the magnitude of the errors that may be expected under various conditions:

In general it was found that well-made instruments of either the ether vapor-pressure or the liquid-filled types would indicate the temperature of the bulb to within an accuracy of from 1° to 3° C. under ordinary laboratory conditions.

Although methyl chloride and methyl ether vapor-pressure thermometers were produced in quantity toward the end of the war, only a small number were received at the Bureau of Standards for test and some of these were experimental instruments. The results of tests showed that it was possible to construct methyl chloride or methyl ether thermometers possessing an accuracy comparable with that of the liquid-filled types, but in general the readings, especially in the neighborhood of 0° C., were less reliable in the vapor-pressure thermometers.

Mechanical hysteresis errors can be attributed to backlash and friction in the moving parts of the gage mechanism. The advantages of the directly connected gage as used on the liquid-filled thermometer over that of the sector and pinion type, especially if the latter is poorly constructed, was very clearly demonstrated.

The readings of the vapor-pressure type of instrument are theoretically independent of the temperature of the gage and capillary tubing. The heating and cooling tests previously

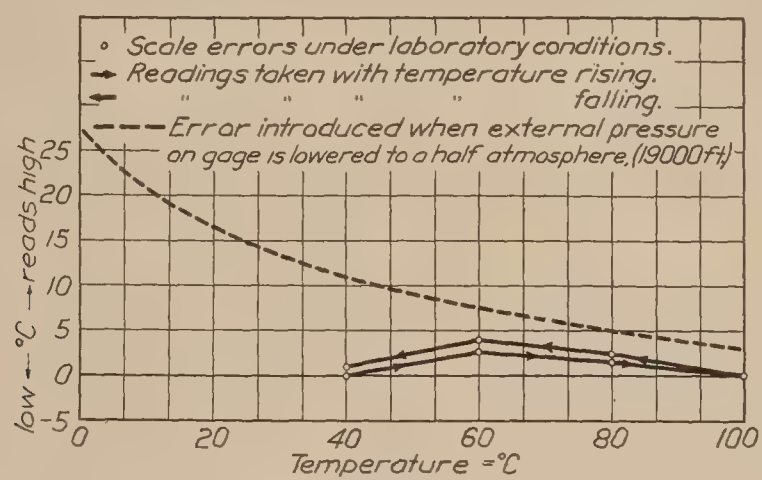


FIG. 3.—Results of tests of ether vapor-pressure thermometers.

mentioned were therefore intended as checks on the filling of the instruments, i. e., to determine whether the proper amount of liquid was contained with reference to the volume of the various parts. Although some of the first thermometers submitted as samples were found not to be properly filled, this defect did not appear in the thermometers furnished under the specifications.

The liquid-filled thermometer is subject to error when the capillary is heated separately from the gage. For lengths up to 12 feet it was found that the capillaries could be made small enough in diameter to permit moderate heating or cooling without introducing excessive error. For longer capillaries the error due to this cause was appreciable, as, for example, a 23-foot tube cooled to −9° C., introduced an error of about 9° C. in the reading. It has been previously stated that a compensator is employed on the liquid-filled thermometers to allow for the expansion or contraction of the liquid in the gage. This compensator may also be so designed as to allow for the expansion and contraction of the liquid in the tubing provided both gage and tubing are heated and cooled at the same time and the same amount. Tests of instruments so designed indicated that it was possible to compensate very accurately for these temperature changes.

The errors caused by decreasing the external pressure on the gage of airplane thermometers depend on the pressure range of the instruments.

The liquid-filled instruments if properly designed will be practically free from error from this source, since the pressure range can be varied at will. Tests of the instruments of the type submitted showed that the error had been reduced to within 1° C. by the use of a suitable gage and sufficient pressure range.

The theoretical errors due to change in external pressure on the gage of vapor pressure instruments have been previously indicated. The errors actually found agreed very closely with those predicted.

Impurities in the liquids used in vapor pressure thermometers may clog up the capillary tubing and render the instruments inoperative. A number of methyl chloride and methyl ether thermometers were found defective from this source. Difficulty in securing pure methyl chloride or methyl ether in sufficient quantities interfered with the manufacture of satisfactory instruments of this type.

Figures 3, 4, and 5 show characteristic calibration curves obtained from observations taken for ether vapor pressure, methyl chloride vapor pressure, and liquid-filled thermometers.

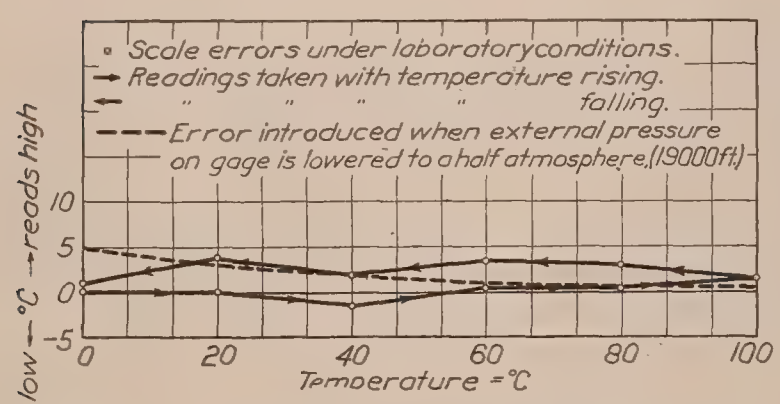


FIG. 4. Results of tests of methyl chloride vapor-pressure thermometer.

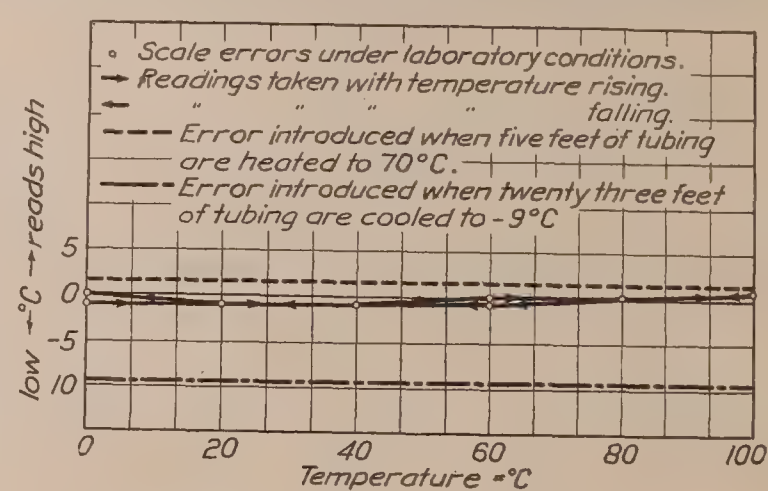


FIG 5.—Results of tests of liquid filled thermometers.

The charts also indicate the magnitude of the error due to reducing the external pressure on the vapor pressure thermometers and to either heating or cooling the capillary of a liquid-filled instrument.

Vibration tests made in the laboratory failed to indicate the superiority of one instrument over the other from the standpoint of ability to withstand hard usage.

Other reasons for failure of the instruments submitted were loose pointers and the breaking of the capillaries. These are frequent causes of failure of the instruments in the field.

It is unfortunate that very little data could be obtained in regard to the behavior of the instruments under actual conditions of use, although it is understood that those passing the tests proved satisfactory.

REPORT No. 129.

POWER PLANT INSTRUMENTS.

PART IV.

AIR PRESSURE AND OIL PRESSURE GAGES.

By H. N. Eaton.

SUMMARY.

This part discusses briefly the use of air and oil pressure gages on aircraft, and describes the construction of various American, British, and German gages. Methods of testing these instruments at the United States Bureau of Standards are described and sample reports are given.

DESCRIPTION OF AIRCRAFT PRESSURE GAGES.

Air and oil pressure gages are used on aircraft to measure the pressure of the air in the gasoline tank and the pressure in the oil system of the engine. The importance of keeping the air pressure in the gasoline tank at the proper value is obvious when it is remembered that it is this pressure which forces the gasoline to the carbureter. The usual pressure is about 3 pounds per square inch, although a value as great as 5 pounds per square inch may sometimes be reached. A safety valve is used to prevent the pressure from increasing beyond this limit. Since the life of the engine depends upon its receiving a sufficient supply of oil, the gage which indicates the oil pressure is also of very great importance. It shows not only whether the pressure is maintained within the proper range but also any stoppage which may prevent the flow of oil.

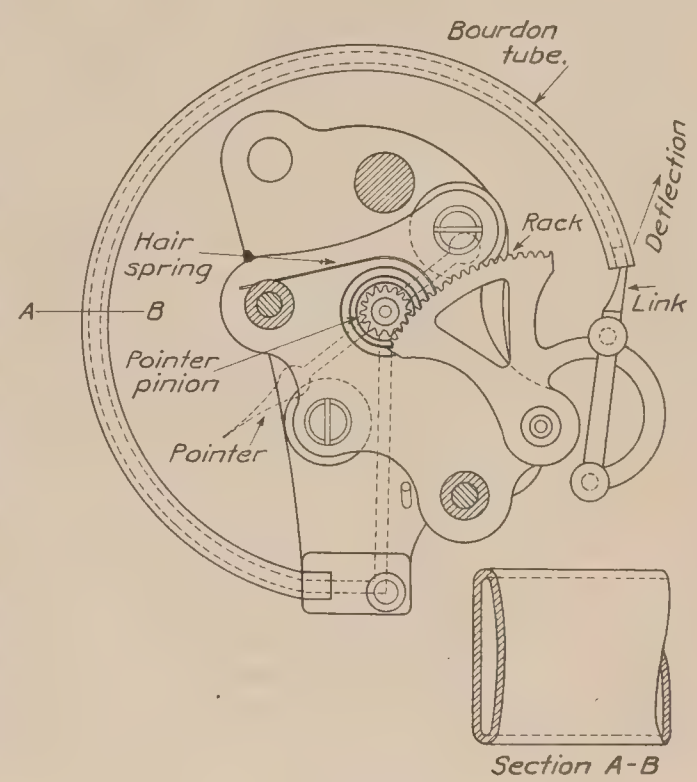


Fig. 1.—Pressure gage.

The pressure gages just mentioned are of the Bourdon tube type in the great majority of cases. A Bourdon tube is constructed by flattening a circular tube, bending the flattened tube longitudinally to the arc of a circle, and sealing the ends. (See fig. 1.) Now if the pressure is introduced into the tube, the cross section tends to increase in area and this change in cross section tends to straighten the tube longitudinally. When a Bourdon tube is used in an instrument, one end is mounted rigidly to the case while the other end is left free to deflect and operate the mechanism. Bourdon tubes possess the advantage that the deflection produced is closely proportional to the pressure applied, but they have very little power when utilized as pressure indicators and so are not adapted to use in instruments where the pressure element is called upon to exert an appreciable force. To change the range of pressure for which the gage is adapted, it is simply necessary to use a Bourdon tube of different stiffness.

Figure 2 shows a group of American, British, and German pressure gages. All of these gages are of the Bourdon tube type except the Prerauer and Scholz gage, figure 2(F), which has a corrugated diaphragm element.

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Figure 2(A) shows an American oil-pressure gage whose construction is typical of that used in American aircraft oil and air pressure gages. This gage is of the concentric type; that is, the axis of rotation of the hand is at the center of the dial. A number of American airplane pressure gages have been built with the hand eccentric with respect to the dial, but the mechanism is not materially different from that shown in figure 2(A).

The pressure is applied to the inside of the Bourdon tube through a screw connection under the main casting. The free end of the tube deflects as indicated in figure 1, thus operating the rack through a connecting link. The rack in turn operates the pointer pinion on which the pointer is mounted. A hairspring is attached to the shaft carrying the pointer pinion so as to eliminate the effect of backlash. The Bourdon tube is usually made of seamless drawn bronze.

FIG. 2.

The graduations on the dial of this particular instrument are not luminous, but it is customary to have the pointer and the principal graduations and figures finished with luminous paint.

A British air-pressure gage is shown in figure 2(B). In this instrument the hand is mounted eccentrically with respect to the dial. Owing to the small angular motion required of the pointer, the rack and pinion are replaced by a linkage which is simpler to construct and will give practically a uniform scale. Stops are arranged for the zero position and for preventing the Bourdon tube from deflecting so far as to overstrain the metal in case of an accidental application of overpressure. The tube is made of seamless phosphor bronze tubing. Neither the pointer nor the graduations on the dial are luminous. A conspicuous red line is printed at the point of the scale corresponding to 2.5 pounds per square inch indicating the proper pressure to be maintained.

Figure 2(C) shows a British oil-pressure gage with a range of 0 to 60 pounds per square inch. A rack and pinion are used to transmit the motion of the free end of the Bourdon tube to the pointer. No hairspring is used to eliminate backlash, but instead the teeth are made V shaped, so that there is practically no lost motion between the rack and the pinion. No stop is provided to prevent the Bourdon tube from being deflected too far. The specifications, however, require this gage to sustain a total pressure of 180 pounds per square inch without damage; consequently the stop is hardly needed.

Figure 2(D) shows a Benz air-pressure gage of German manufacture having a range of from 0 to 5 pounds per square inch. Here, as in the British air-pressure gage, only a small angular motion of the pointer is utilized, and a linkage is used to transmit the motion of the free end of the Bourdon to the pointer.

A German oil-pressure gage is shown in figure 2(E). The instrument has a range of from 0 to 4 kilograms per square centimeter (approximately 0 to 57 pounds per square inch). A rack and pinion are used and a hairspring eliminates backlash in the mechanism. A stop curved to fit the outer surface of the Bourdon tube is mounted on the case. If the Bourdon tube is subjected to overpressure, it comes in contact with this stop for a considerable portion of its length and so overstrain of the metal comprising the thin walls of the tube is prevented.

Figure 2(F) shows a German air-pressure gage of the diaphragm type. As the diaphragm deflects it raises with it a helical surface shown above the diaphragm in the figure. The pointer is mounted on a shaft which carries an arm with a small wheel to provide rolling contact with the helical surface. Consequently, as the surface rises it rotates the arm and the pointer shaft. A hairspring takes up backlash. Owing to the fact that the relation between the applied pressure and the deflection of the diaphragm is not linear, this instrument has a scale whose graduations are far from uniform.

TESTS OF AIRCRAFT PRESSURE GAGES.

The tests to which airplane pressure gages are subjected are designed to bring out clearly any characteristics of an instrument which would make it unsuitable for use under the conditions peculiar to aeronautics. In particular, the gages must work satisfactorily under severe vibration and at low temperatures. The tests specified are as follows:

1. Calibration.
2. Vibration.
3. Temperature.
4. Friction.
5. Endurance.
6. Interchangeability.

CALIBRATION TEST.

When the gages are received they are calibrated at room temperature in order that their scale errors under normal laboratory conditions may be ascertained. The air-pressure gages are calibrated directly against a mercury column, since the range of the instruments (usually 0 to 10 pounds per square inch) is sufficiently small to permit the use of this method. Air pressure is applied sufficient to deflect the pointer to the 2-pound division while the instrument is tapped, and the true pressure is read on the manometer which is graduated in pounds per square inch. This process is repeated for successive 2-pound intervals over the scale, first with increasing pressure, then immediately afterwards with decreasing pressure. The difference in the manometer readings with increasing and with decreasing pressure is a measure of the elastic hysteresis of the instrument (if backlash and friction are eliminated).

Since oil-pressure gages have a much greater range than the air gages, usually from 0 to 120 pounds per square inch, it is not convenient to use a mercury column as the standard for measuring the pressures. Instead a dead weight oil gage tester is used. This tester consists of a vertical cylinder in which a piston of known cross-sectional area floats on oil. An oil-filled tube connects this cylinder with the gage under test. Weights can be placed upon the piston, and since the cross-sectional area of the latter is known, the pressure thus set up in the oil system is

also known for any given weight. For purposes of testing, the weights are designed to alter the oil pressure as they are applied by successive increments of 5, 10, or 20 pounds per square inch.

The calibration of the oil gages is carried out just as described for the air gages. Readings are taken for pressure changes of 20 pounds per square inch over the range of the instrument with both increasing and decreasing pressure.

VIBRATION TEST.

This test is made to determine the effect of prolonged vibration upon the instrument, such as may occur under flight conditions. The result may be simply to change the calibration of the instrument slightly or to loosen parts of the mechanism.

After the calibration test just described has been completed, the gages are fastened rigidly to a vibrating stand and for five hours are subjected to vibration similar to that experienced in actual service. At the end of this time they are recalibrated. A comparison of the two calibrations, one before, the other after, vibration, serves to determine any change produced by this treatment, and a brief examination of the instrument suffices to detect any looseness which may have been produced in the mechanism.

TEMPERATURE TEST.

The effect of temperature changes upon the calibration of each type of gage is determined by tests at approximately $-10°$ C and $+40°$ C. For this purpose the gages are mounted in a chamber in which the temperature can be varied over the necessary range. The air in the chamber is brought to the desired temperature and maintained at that point until the gages have acquired the temperature of the air. The calibration is then made in a manner similar to that already described.

FRICTION TEST.

Pressure is applied sufficient to deflect the pointer over a small portion of its range and a reading is taken without tapping the instrument. The instrument is then tapped and a second reading is taken while the pressure is held constant. The difference in the two readings is a measure of the friction at this point of the scale. The process is repeated at several other points of the scale. The average change in reading due to tapping the instrument is taken as the friction error. Friction is rarely serious in pressure gages, as the vibration of the airplane tends to eliminate its effect. Instruments are rarely rejected because of friction unless there is some defect of the mechanism which causes the pointer to stick and move in a jerky manner when the pressure is varied uniformly.

ENDURANCE TESTS.

Three distinct endurance tests are given airplane pressure gages at the Bureau of Standards:

(*a*) A *drift* test to determine the effect of prolonged application of pressure.

(*b*) A *seasoning* test to determine the effect of repeated applications of pressure.

(*c*) An *overpressure* test.

(*a*) *Drift.*—The increase in reading of an instrument when subjected to a given pressure for a prolonged period is called its "drift" or "creep." This effect is due to the yielding under stress of the metal which makes up the pressure element. The gage is subjected for five hours to sufficient pressure to produce approximately one-half of full scale deflection. A reading is taken when the pressure is first applied and another at the end of the five-hour interval. The difference in the two readings is taken as the drift.

(*b*) *Seasoning.*—A calibration is given immediately after 200 applications of pressure sufficient to produce full-scale deflection have been given the gage. A comparison of this calibration with the room temperature calibration following the five-hour vibration of the instrument shows the effect of repetition of pressure.

(*c*) *Overpressure.*—In order to test the ability of the gages to withstand the occasional accidental excess pressures which may be given them, both types of gage are subjected to a momentary overpressure. After the application of this overpressure, the gages are rested for

a few minutes so that the worst of the temporary elastic effect caused by the overpressure may disappear; then a final calibration is given. Since the overpressure often causes a permanent change in the calibration of the gage, this final calibration should be taken as characteristic of the instrument at the conclusion of the tests. In the overpressure test a pressure of 25 pounds per square inch is applied to air gages of 10-pound range, while a pressure of 180 pounds per square inch is applied to oil gages of 120-pound range.

It is desirable to rest the instruments for several hours between each two consecutive tests in order to afford time for the disappearance of the temporary elastic effects produced by the last test.

INTERCHANGEABILITY TEST.

Since both the air and oil pressure gages used by the Air Service are of the same size and general construction, certain parts can be made standard for all. Consequently it is required that cases, connections, bezels, and cover glasses shall be interchangeable. On this account the dimensions of such parts are checked at the conclusion of the performance tests of the instruments.

TOLERANCES FOR AIRCRAFT PRESSURE GAGES.

The following table summarizes the tolerances specified for aircraft oil and air pressure gages used by the United States Air Service. The values are in pounds per square inch.

Table of tolerances for aircraft pressure gages.

Gage.	Range.	Vibration.	Scale error −10° C to +40° C.	Friction.	Drift.	Over-pressure (amount).
	Pounds per square inch.					
Air	0–6	0.3	0.3	0.3	0.3	15
Do	0–10	0.3	0.3	0.3	0.3	25
Oil	0–120	3.0	3.0	3.0	3.0	180

The following are typical reports for two American pressure gages, one air and one oil.

REPORT ON AIR PRESSURE GAGE, SERIAL NO. 156.

Range of instrument, 0 to 10 pounds per square inch.

The results of the tests applied to this instrument follow. Corrections are in pounds per square inch and are to be added algebraically to the instrument reading.

Instrument reading.	Corrections at +25° C.		Corrections at +25° C.		Corrections at −8.5° C.		Corrections at +46° C.	
	Before vibration.		After vibration.		Cold run.		Hot run.	
	Up.	Down.	Up.	Down.	Up.	Down.	Up.	Down.
Pounds per square inch.								
2	+0.1	0.0	+0.1	0.0	+0.1	+0.1	0.0	0.0
4	+0.1	+0.1	+0.1	+0.1	+0.1	+0.2	+0.1	0.0
6	0.0	−0.1	0.0	0.0	+0.1	+0.1	0.0	−0.1
8	0.0	0.0	0.0	−0.1	+0.2	0.0	−0.1	−0.2
10	0.0	.....	+0.1	.....	+0.1	.....	−0.1	.....

Change in reading due to—	Average.
	Pounds per square inch.
Vibration	0.0
High temperature	+0.05
Low temperature	−0.10
Repetition	0.0
Drift (at 5 pounds per square inch)	0.1
Overpressure	.....

REPORT ON OIL PRESSURE GAGE, SERIAL NO. 157.

Range of instrument, 0 to 120 pounds per square inch.

The results of tests applied to this instrument are as follows:

Instrument reading.	Corrections at +25° C.		Corrections at +25° C.		Corrections at −6° C.		Corrections at +42° C.	
	Before vibration.		After vibration.		Cold run.		Hot run.	
	Up.	Down.	Up.	Down.	Up.	Down.	Up.	Down.
Pounds per square inch.								
20	+2.0	0.0	+1.5	0.0	+3.0	0.0	+1.5	0.0
40	−1.0	−2.0	0.0	−1.5	0.0	−1.0	−0.5	−2.5
60	0.0	−3.0	−1.0	−2.5	0.0	−1.5	−1.5	−2.5
80	0.0	−2.0	−0.5	−1.5	0.0	0.0	−1.5	−2.5
100	+1.0	0.0	+0.5	0.0	+2.0	+1.5	−0.5	−1.5
120	+3.0	.....	+2.5	.....	+4.0	.....	+0.5	.....

Change in reading due to—	Average.
	Pounds per square inch.
Vibration	+0.05
High temperature	+0.75
Low temperature	−0.95
Repetition	+0.05
Drift (at 60 pounds per square inch)	0.5
Overpressure	+0.2

REPORT No. 129.

POWER PLANT INSTRUMENTS.

PART V.

GASOLINE DEPTH GAGES AND FLOWMETERS FOR AIRCRAFT.

By John A. C. Warner.

GASOLINE DEPTH GAGES.

The development of a satisfactory gage for indicating the depth of fuel in the reservoirs of aircraft has received much attention from instrument designers both in America and in foreign countries. As yet, however; their efforts have not met with unqualified success owing to the unfavorable conditions under which these instruments function.

Such an instrument must be so designed as to be compact in form, light in weight, easily attached to the tank and removed therefrom, and so mounted as to be accessible to the pilot. Furthermore, it is desirable that the instrument be easily adaptable to tanks of different depths, and that the system allow for mounting the indicating gage at a distance from the tank; this latter feature is necessary in cases where the reservoir is at some distance from the pilot. The ideal instrument should be as simple as is consistent with good operation characteristics; it should have as few working parts as possible, and should function properly under the varying conditions of vibration, temperature, pressure, etc., which are encountered in service.

It is the object of this paper to consider the principal types now in use, to make a general summary of the advantages and disadvantages of each type, and to outline the methods employed in their testing and calibration.

FLOAT TYPE.

Several of the most common and most satisfactory gages incorporate the float principle in their construction (see figs. 1 and 2). In general, this type has the advantage of simplicity of construction and operation, while its greatest disadvantages are found in the tendency of moving parts to stick, and in the structural difficulties which ordinarily result in a more or less cumbersome arrangement not easily adapted to satisfactory installation on aircraft. In spite of the disadvantages, however, the instruments of the float type present a promising basis for the future development of depth gages.

Float-and-swinging-rod gage.—Figures 1 (A) and 2 (A) show assembled and disassembled a gage which combines the float with a swinging rod and suitable indicating head to show the fuel depth. Three vertical stationary rods extend from the base of the indicating head to a stay-plate to which they are rigidly attached at their lower extremities near the bottom of the tank. A fourth rod with a rigid angle-arm at either end is free to swing about a center pin mounted on the stay-plate and fitting loosely through a hole in the extremity of the lower angle-arm. The upper angle-arm is attached at the lower extremity of a vertical indicator spindle extending upward through a protective housing to the indicating head.

The two halves of the split cylindrical shellacked cork float, seen in its lowest position in figure 1 (A), are held apart by an aluminum spacer so as to admit one of the stationary vertical rods at one side of the slotted opening, and the swinging rod at the other. These rods are held in a definite position relative to the float by small guide rollers which also provide a bearing with little friction as the float rises and falls with changes in gasoline level. The float and rods are

so designed and mounted that the free rod is caused to swing in a definite path as the float changes its position. This motion is transmitted through the upper angle-arm to the indicator spindle above mentioned.

At the upper end of this spindle is a toothed sector, figure 2 (A), which engages a pinion mounted upon the vertical center axle of a cylindrical rotating scale, the divisions of which are marked upon the curved surface of the cylinder. A cylindrical glass ring with a diameter slightly larger than that of the rotating scale surrounds the latter and provides an observation window at one side where a section of the indicator housing is cut away.

Inasmuch as this instrument is adapted to use on tanks under pressure it is necessary that the glass ring be mounted upon a gasket which rests upon the inner base surface of the indicator housing. A screw top, also provided with a cork gasket resting upon the upper surface of the glass, covers the indicator and makes it air-tight. An air-tight connection is likewise made at the joint between tank and gage by means of a fitting mounted on the tank and threaded to accommodate a ring-clamp, shown in the illustration. A cork gasket serves to make the joint free from leaks.

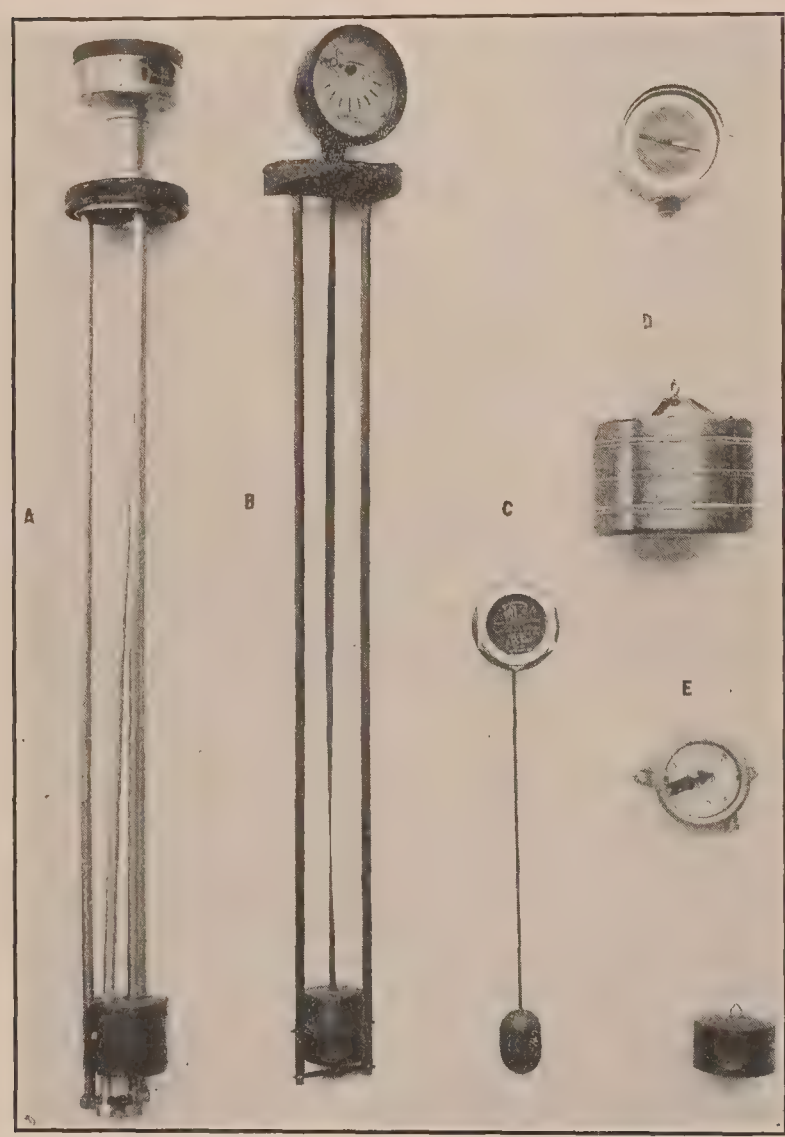

FIG. 1.—Float Type Gasoline Depth Gages—Assembled.

This type of gage generally operates in a reasonably satisfactory manner when properly mounted and handled, and is widely used. However, inasmuch as a very small component of the buoyant force of the float is effectively used in causing the rod to swing, there is great possibility of sticking, with resulting incorrect indications. This type also has the disadvantage of being more or less cumbersome and not well adapted to distant indication.

Float twisted-strip gage.—An older type of gage than the one described above is shown assembled and disassembled by figures 1 (B) and 2 (B). The hollow metal float is seen at its extreme lower position. Two vertical guide rods extending from indicator base to stay-plate at the bottom of the tank restrict the float motion to a vertical path as the float-guide pulleys rest upon the rods. A spiral twisted metal strip passes through a slotted opening through the center of the float so that as the latter moves along its vertical path, the twisted strip is caused to turn like a loose fitting screw of very long pitch in the corresponding nut.

The rotation of the strip which extends upward into the indicator housing is there transmitted through a pair of bevel gears to the indicating pointer. The cover-glass is held in place by a bezel ring. This joint is made air-tight by means of a cork gasket.

The advantages and disadvantages of this gage are practically the same as those of the swinging-rod instrument. The frictional difficulties are usually greater in this type.

Float-and-lever gage.—The float-and-lever type of gage as illustrated assembled and disassembled by figures 1 (C) and 2 (C) is little used on American airplanes. However, it deserves a brief description.

The float composed of balsa wood is attached at the end of a wire lever which extends directly to the gage head, where it is bent at right angle to pass through a brass sleeve which screws into the back of the case. The indicating device with which this instrument is equipped

is one of the simplest. As is clearly shown in figure 2 (C) the float lever carries a magnetized steel bar at the indicator end. This bar rotates with changes of float position and acts magnetically upon the steel pointer through a thin air-tight separating wall cast integral with the case. The scale of the instrument shown is graduated to give indications of the last 20 gallons in the tank only. The pointer is centered on the dial by means of a pivot point centrally mounted thereon. A cover-glass held by a bezel ring covers the gage.

It is seen that this type of indicator has the advantage of simplicity, and because of the absence of the necessarily air-tight joint usually found between glass and case is freer from the possibility of leakage. The case is threaded so as to be easily mounted upon a brass tank-fitting shown in the illustration. This gage has the disadvantage of being adapted to comparatively few installations and does not lend itself well to the requirements of distant indication.

Float-and-cord gage.—One of the simplest float gages is that which consists of a cork or sheet metal float, usually in the form of an air-and-water-tight cylindrical or spherical chamber, connected to an indicator by means of a light braided silk cord in such a manner as to give indications

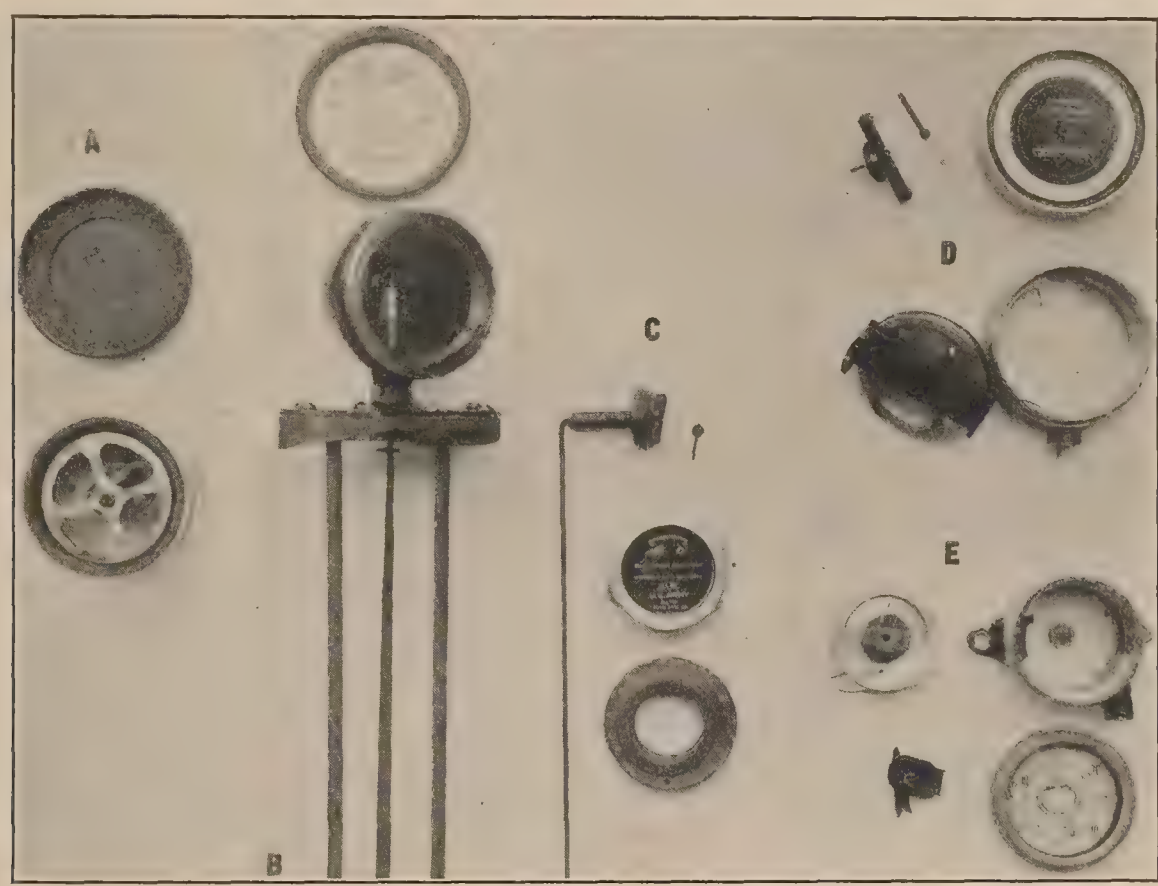

Fig. 2.—Float Type Gasoline Depth Gages—Disassembled.

of the depth of fuel in the tank. Figures 1 (D) and 2 (D) illustrate an instrument of this type recently designed by the Engineering Division, United States Air Service.

The cylindrical brass float has a diameter of 134 millimeters, a depth of 93 millimeters, and weighs approximately 600 grams. When installed on the tank this float is restricted in its motion to a vertical path by means of a fixed sheet metal tube, of diameter slightly greater than that of the float, and reaching from the top of the tank to a point near the bottom. A braided silk cord connects the float with the indicator gage, and the design is such that the gage may be mounted at a distance from the tank. In this case the cord passes through a tube of small diameter with suitable roller fittings at the bends.

At the point of entrance into the case of the gage the cord passes over a roller and thence to the main pulley member, upon which it is wound by the action of a coiled flat metal spring. The motion of this pulley is transmitted to the magnetized pointer bar (see illustration) through a train of gears which are interchangeable so as to make the gage readily adaptable to tanks of different depths. The first set of gears provides a pointer movement of 330° for a 50-inch change in float level, while the second combination gives a pointer movement

of 120° for a 10-inch change of level. The pointer is acted upon magnetically through the dial plate by the magnetized pointer bar mentioned above and moves over the scale marked upon the dial plate as the pointer bar changes its angular position. The case is covered by a glass held in place by a suitable bezel ring, making a tight joint with the case.

A float-and-cord gage of German manufacture is pictured in figures 1 (E) and 2 (E). The cylindrical metal float member has a diameter of 67 millimeters, a depth of 44 millimeters, and weighs approximately 92 grams. As in the design above described this float moves in a vertical path within a guide tube whose diameter is slightly greater than that of the float, and in a similar way connects by means of a cord with the main pulley member of the indicator.

The pointer is attached to a bronze rack (see fig. 2 (E)), which is mounted on the face of the rotating pulley, so as to engage a pinion fixed to the end of the pulley bearing. The rack is free to move radially. When the pulley is rotated by the cord attached to the float, the rack is rotated with it and forced outward radially by the fixed pinion, thereby causing the end of the pointer, which is attached to the rack, to describe a spiral path on the dial. In this way the pointer may make several revolutions without confusing the readings.

Float-resistance gage.—Several of the foreign gages, such as the French Electro-Jauge, depend upon electrical means to show the position of the float in the tank. The float is held in the usual manner by guide rods. An additional vertical column parallel to the guide rods and passing through the cork float has a bare wire resistance winding along its entire length; contact with this resistance element is made by a brush mounted on the float.

As the float changes its position with changes in gasoline level, the amount of resistance cut into the circuit by the float contact varies. A galvanometer, properly graduated and mounted upon the instrument board, shows the pilot the amount of fuel in the reservoir by giving an indication of the current flow through the resistance in the circuit of which the galvanometer forms a part.

This type of gage is well adapted to remote indication. It is too complicated in construction, however, does not hold its adjustment well, and is likely to get out of order.

PRESSURE TYPE.

Gasoline depth gages of the pressure type involve the use of a totally different principle from that of the float type described above. (See figs. 3 and 4.) The principle upon which this design is based is that of the pressure difference between the top and bottom of the gasoline reservoir caused by the head of gasoline. In general, this type is well adapted to distant reading, and the indicator is usually mounted on the instrument board at some distance from the reservoir to which it is connected by means of metal tubing of small bore.

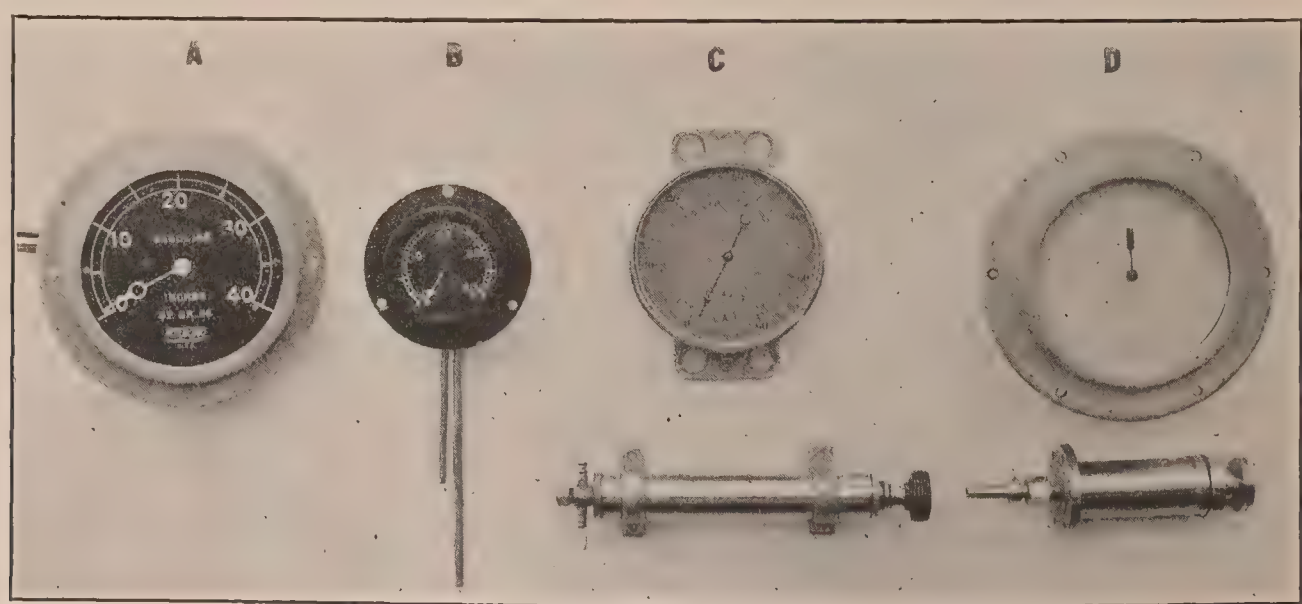

FIG. 3.—Pressure Type Gasoline Depth Gages—Assembled.

This arrangement possesses the disadvantage of being more liable to leakage, and consequent hazard to the aircraft and passengers, a particularly important point with fighting machines, when the tubes are in constant danger of breakage from gunfire.

Aneroid gage.—Figures 3 (A) and (B), 4 (A) and (B) show views of assembled and disassembled American gasoline depth gages of the pressure type which depend on a stack of nine aneroid chambers for pressure indications. Two metal tubes of small diameter lead from the gasoline reservoir to the indicator. One of these tubes extends into the tank to within about one-quarter of an inch of the bottom, while the second tube simply connects to the top of the reservoir above the surface of the liquid. The former then transmits a pressure, equivalent to the air pressure above the gasoline plus that due to the head of gasoline, to the space surrounding the aneroid chambers in the air-tight indicator case to which it is connected. The latter tube transmits the air pressure only to the space inside the aneroid boxes with which it communicates. Thus it will be seen that the elastic aneroid system will assume its position corresponding to the differential pressure existing between the top and bottom of the reservoir—i. e., a pressure equal to that of the hydrostatic head of gasoline.

Two wire coils soldered to the edges of the aneroid chambers at one side prevent that side from moving when pressure is applied, so that the expansion is taken up by a tilting action of

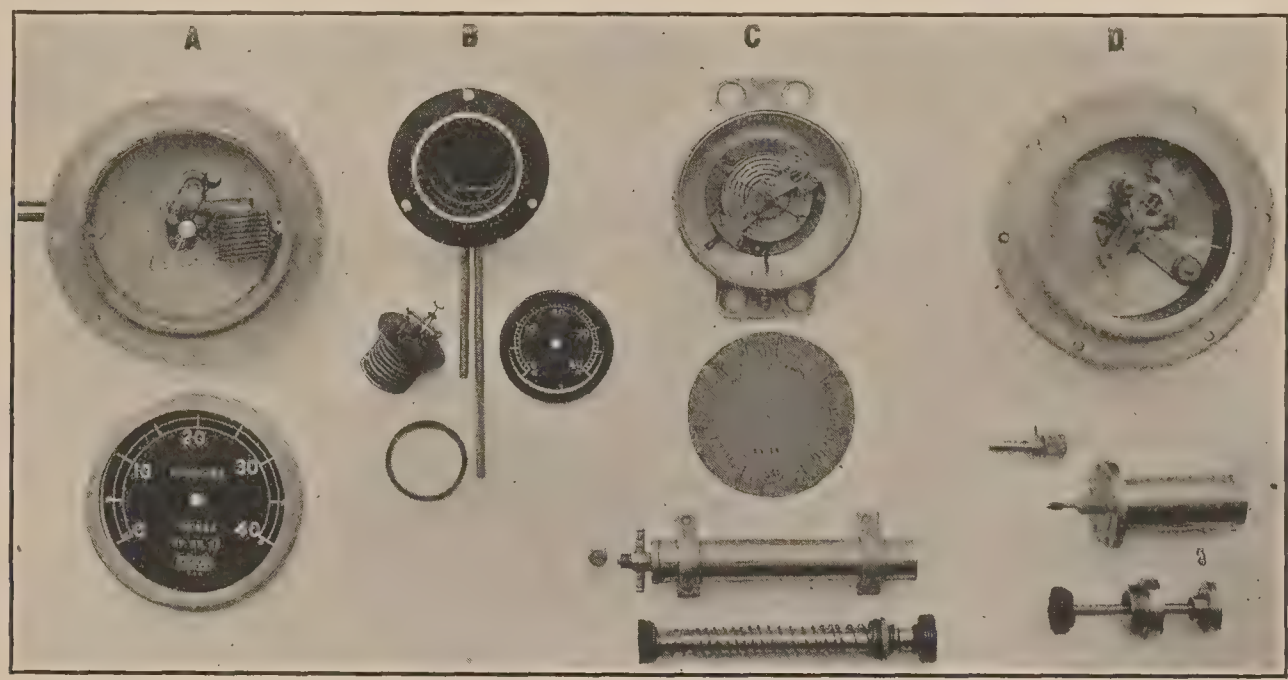

FIG. 4.—Pressure Type Gasoline Depth Gages—Disassembled.

the stack. This motion of the aneroids is preferable to the straight expansion, such as would take place without the restraining wires; for by the tilting action the connecting lever is given a proper motion which it transmits, through a link, to a toothed sector. The sector in turn transmits the motion to the pinion mounted on the pointer arbor. A flat spiral spring holds the pointer in equilibrium and takes up the backlash.

Figures 3 (C) and 4 (C), 3 (D) and 4 (D) show two foreign types of gasoline depth gage for use on reservoirs which are not under pressure. A small hand pump connected to a tube extending from the indicator to the bottom of the fuel reservoir serves to supply enough air to equalize the pressure due to the head of gasoline in the tank. When the pump is operated so as to supply a pressure equal to that of the head of gasoline, bubbles of air are formed at the lower end of the tube and rise to the surface of the liquid. This equalizing pressure is transmitted through a tube to the indicator which is properly graduated to show the depth of fuel in the tank.

The French instrument shown with its pump assembled and disassembled in figures 3 (C) and 4 (C) uses a single aneroid chamber for the pressure element. The action of this type is similar to that described above and will be readily understood after examination of the illustration.

The British type of indicator shown assembled and disassembled in figures 3 (D) and 4 (D) uses, as its pressure element, a specially treated fabric diaphragm approximately 90 mm. in diameter. The air pressure from the hand pump exerts a force upon one side of this diaphragm, which then becomes distended. The resulting motion of the diaphragm is greater or smaller according as the necessary equilizing pressure for the head of gasoline is large or small. A rod supported at the center of the diaphragm transmits its motion to the indicating mechanism which needs no detailed explanation.

As mentioned above, instruments of the pressure type lend themselves readily to remote indication. They are, however, subject to various errors due to several causes, chief among them being leakage, friction, temperature changes, vibration, imperfect elastic properties of pressure element.

TESTING OF GASOLINE DEPTH GAGES.

Tests of float type gages.—Gages of the float type are tested in the laboratory to determine their calibration and operation characteristics, by properly mounting them on a gasoline tank equipped with a water-glass type of gage and provided with means for rapidly filling and emptying.

A careful preliminary inspection of the gage will usually disclose any important causes for poor operation, such as bent parts, etc. The gage which appears to be in good working condition, is first mounted on the tank and a calibration is made by comparing its indications with those of the tank gage as the gasoline level is changed. This is first done with a rising liquid surface and then with a descending surface. Differences in the two readings are usually due to lost motion in the mechanism and should not be excessive. Readings should be taken both with and without tapping so as to determine the frictional error. In case the float or other part sticks so badly that slight jarring will not move it, the instrument should be readjusted before final calibration.

It is sometimes advisable to conduct rough tests of the tightness of metal floats and also of the buoyancy of cork floats, although any difficulties of the kind would usually be noticed in the calibration tests. In making tests of a new type of gasoline depth gage it is advisable to investigate, in addition to the above characteristics, its behavior in flight or in a perturbed liquid.

Tests of pressure type gages.—The apparatus required for the testing of pressure type gages consists of a simple liquid manometer of large bore with suitable scale and with connections and valves to control the air pressure from a source of supply. The scale may be divided to read directly in inches of gasoline, or by the use of gasoline or liquid of equal specific gravity in the manometer the unmodified inch graduations may be used. In testing, the gage is connected with the manometer and the source of air supply. Various pressures are then applied and comparative indications of gage and manometer noted.

The temperature tests are conducted in a thermally insulated chamber equipped with heating and refrigerating coils by means of which the temperature may be varied as desired. Vibration tests are made by mounting the instruments upon a board which is caused to vibrate by an electric motor mounted thereon and with an unbalanced weight upon its shaft. By varying the motor speed the frequency of vibration may be changed and brought to approximately that which the instrument would experience when installed on aircraft. Additional tests which require no explanation will be noted in the report below. This specimen report will afford an understanding of the various tests, and the results will give a notion as to the performance of gages of the aneroid type.

REPORT ON TWO ANEROID TYPE GASOLINE DEPTH GAGES.

Calibration test.—The two gages were calibrated at 22°C, −3°C, +50°C, and also at room temperature before and after being subjected to vibration, repeated stress, and over pressure. In each case the pressures corresponding to a series of scale readings were determined, with increasing pressures to full scale deflection and immediately afterwards with decreasing pressure back to zero.

Numerical results, additive corrections computed for gasoline of specific gravity 0.68, are given in the following tables and the accompanying graphs:

INSTRUMENT No. 13.

Observed reading, inches of gasoline.	Correction at 22° C., reading—		Correction at −3° C., reading—		Correction at 50° C., reading—	
	Up (1).	Down (2).	Up (3).	Down (4).	Up (5).	Down (6).
0	−1.2	−1.2	−0.8	−1.2	+0.1	−0.5
5	−0.4	−0.5	−0.3	−0.4	+0.7	−0.2
10	−0.4	−0.5	−0.1	+0.5	+0.8	−0.2
15	−0.2	−0.5	0	+0.4	+0.5	−0.6
20	+0.2	−0.3	+0.5	+1.1	+0.6	−0.1
25	+0.5	+0.1	+0.6	+1.3	+0.6	−0.3
30	+0.1	+1.1	+0.5	+1.2	+0.4	−0.5
35	+0.3	+0.4	+0.4	+1.4	−0.2	−0.5
40	0	0	+0.7	+0.7	−0.5	−0.5

INSTRUMENT No. 14.

Observed reading, inches of gasoline.	Correction at 22° C., reading—		Correction at −3° C., reading—		Correction at 50° C., reading—	
	Up (1).	Down (2).	Up (3).	Down (4).	Up (5).	Down (6).
0	−0.6	−0.5	−0.8	−0.3	−0.1	−0.2
5	−0.1	−0.4	−0.1	−0.2	+0.8	+0.1
10	+0.3	+0.1	+0.8	+0.4	+1.1	+0.4
15	+0.7	+0.6	+0.5	+0.9	+0.9	+0.1
20	+0.9	+0.6	+0.7	+1.1	+0.8	+0.1
25	+0.8	+0.4	+0.6	+0.9	+0.6	−0.1
30	+1.0	+0.7	+0.7	+1.4	+0.4	−0.4
35	+0.4	+0.2	+0.4	+1.4	−0.1	−0.4
40	−0.7	−0.7	+0.8	+0.8	−0.2	−0.2

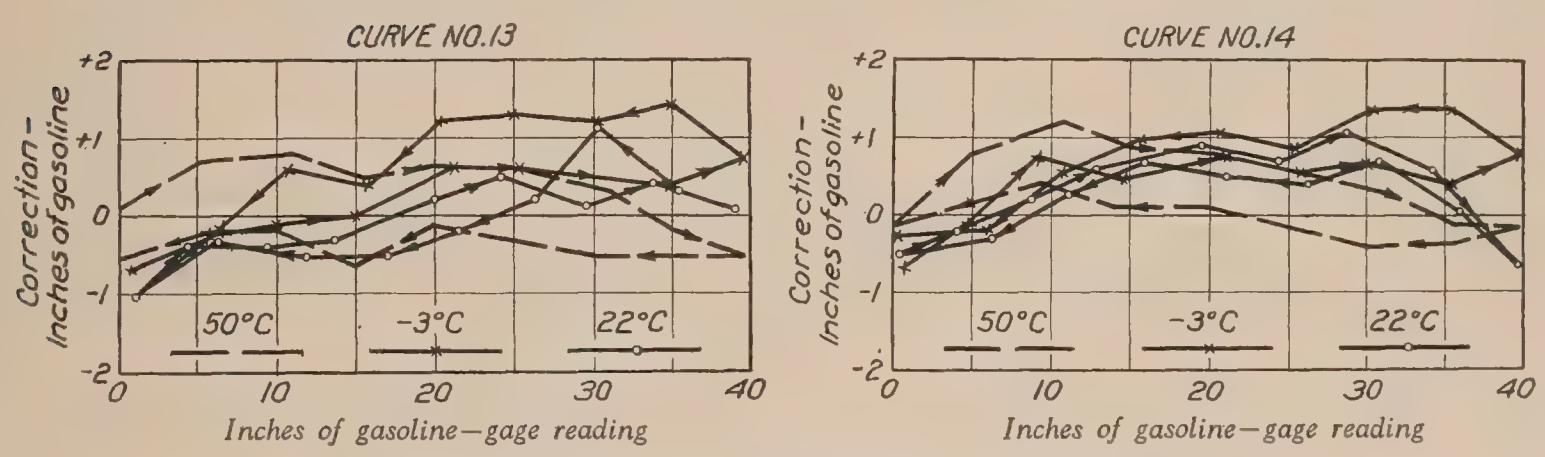

FIG. 5.—Calibration curves for aneroid gasoline depth gages. Temperature tests.

Friction test.—Readings taken with and without tapping with the dials of the instruments vertical showed the following differences:

Instrument Nos.	Differences, inches of gasoline.
13	0.0 to 0.7
14	0.0 to 0.6

Both gages were slightly irregular in action, but not sufficiently so to warrant rejection.

Inclination test.—The differences of readings taken with the dials first horizontal and then vertical for a series of pressures up to full scale deflection were as follows:

Instrument Nos.	Differences, inches of gasoline.
13	0.2 to 0.9
14	0.0 to 0.8

Drift and fatigue test.—The error caused by the elastic fatigue of the diaphragms under continuous pressure was determined by maintaining the instruments at a constant pressure equivalent to a scale reading of 20 for five and one-half hours. The drift or increase in readings at this pressure was as follows:

Instrument Nos.	Drift, inches of gasoline.
13	+0.9
14	+0.9

After subjection to 500 successive applications of a pressure equivalent to half scale deflection, the calibration varied from previous values by not more than 0.7 inch of gasoline in any case. Instrument No. 14 showed after this test an increase in the error of approximately 0.2 inch of gasoline throughout the scale.

Effect of vibration.—The effect of vibration was determined by calibrating the instruments before and after vibration for 18 hours on a machine which simulated the vibrations experienced in an airplane in flight. In general, a slight increase in the errors which reached a maximum of 0.9 inch of gasoline was obtained (see plots). The pointer of No. 13 vibrated excessively—from four to five divisions.

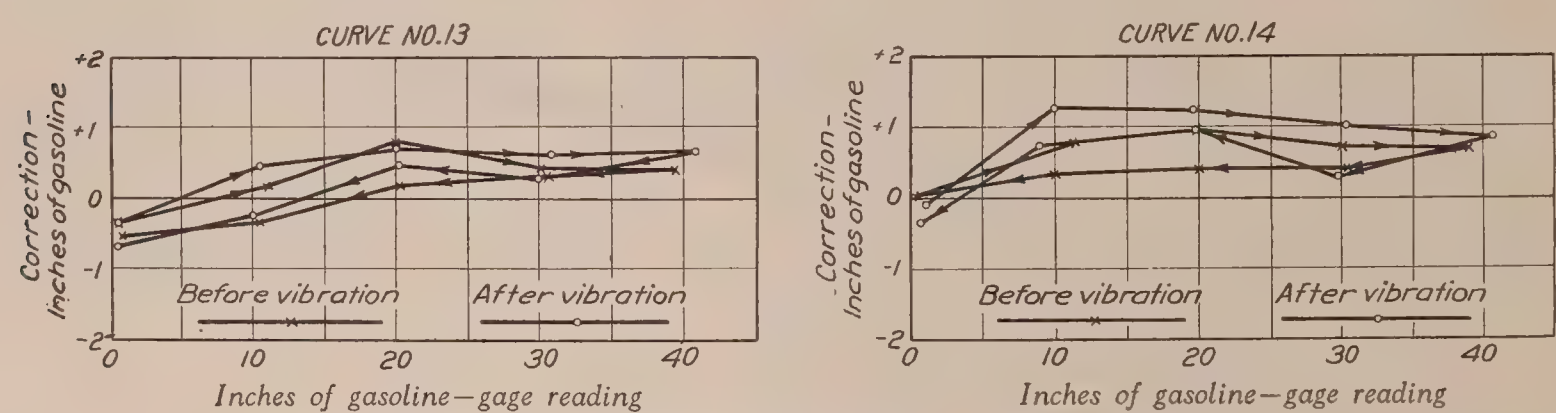

FIG. 6.—Calibration curves for aneroid gasoline depth gages. Vibration tests. (Room temperature.)

Excess pressure test.—The calibrations of the instruments before and after subjection to a momentary pressure of twice the maximum scale value differed in no case by more than 0.4 inch of gasoline.

FLOWMETERS.

In connection with performance tests of aircraft engines it is necessary that the rate of fuel flow be ascertained. It is also often desirable that similar indications be available to the aviator during long flights. Several types of instruments have been designed for this purpose and the following description refers briefly to two typical ones.

SCHROEDER FLOWMETER. (ECONOMETER.)

The instrument shown assembled and disassembled at the right of figs. 7 and 8 is the flowmeter developed by Maj. R. W. Schroeder, of the United States Air Service. The base casting is designed so that the inlet pipe from the fuel reservoir connects directly to the vertical meter tube threaded into the base and held concentrically within a surrounding tube of glass. The latter is firmly held in place upon a cork nonleak gasket seat in the base casting by means of a screw cap threaded at the top of the meter tube and extending over the glass. The cap is also provided with a gasket to make the joint free from leaks. A small adjustable screw threaded centrally into the cap and with hollow shank connecting with the atmosphere through a small radial hole is provided at the top for venting the meter.

The inner vertical tube to which the feed line is connected has an inside diameter of approximately 9½ millimeters and a narrow longitudinal flow-controlling slit cut at one side to allow the entering liquid to flow from it into the annular space included between the inner and outer tubes. From this annular space the gasoline passes through the exit opening in the base and thence to the motor.

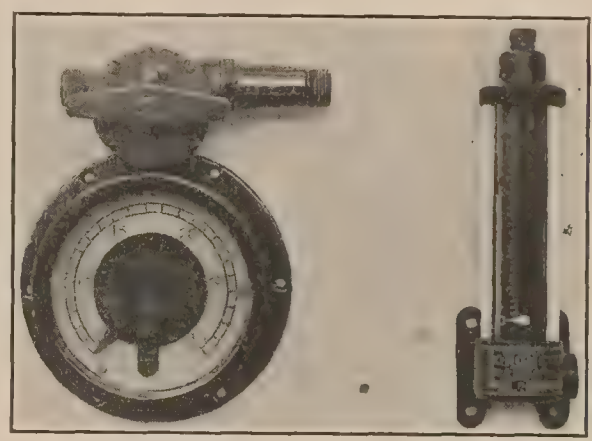

FIG 7.—Flowmeters—Assembled.

A light brass plunger fitting loosely in the central tube has mounted upon it an index pointer extending through the slit and moving over a vertical scale as the flow varies. This scale is fastened to the front flat milled surface of the tube. When the meter is in action the gasoline flowing into the main tube exerts a force upon the lower surface of the plunger sufficient to overcome its weight and thus lifts it to a definite position in the tube. The height to which it rises depends directly upon the rate of flow.

The instrument shown in the illustration weighs 575 grams and has an over-all height of approximately 160 millimeters. The glass tube has an internal diameter of approximately 26 millimeters.

R. A. E. MARK II FLOWMETER.

The instrument shown assembled and disassembled at the left of figures 7 and 8 is the flowmeter developed by the Royal Aircraft Establishment, of Farnborough, England. It is of the vane type and gives indications of rate of flow between the limits of 5 and 30 gallons per hour. Its action may be described as follows:

The gasoline from the fuel reservoir enters the meter case through a two-way valve which may be turned so as to by-pass the gasoline when, for any reason such as breakage of the meter, this procedure becomes desirable. Referring to the detail illustration at the left of figure 8 a fixed guide or baffle plate is seen projecting from the circumference of the case to the center. The gasoline enters the meter through an opening directly at the right of this guide plate and leaves it through an exit opening directly at the left of the guide. In passing from the entrance to the exit side the gasoline impinges upon the surface of a movable vane mounted upon the central pointer spindle. Sufficient clearance is left between the vane and the surrounding parts to allow the liquid to pass, but in so doing it exerts sufficient force upon the vane to move it through a certain angle, the magnitude of which depends upon the amount of flow. A helical coiled spring holds the pointer with the required force against the action of flow.

Inasmuch as the displacement of the vane does not bear a linear relation to the rate of flow when a leakage space of constant area is left around the vane, it is necessary to provide means for compensating for this characteristic. This is effected by having the space between the side wall and the end of the vane vary in depth, thus varying the leakage area at the vane extremity. When the pointer is at its minimum indication the vane occupies a position directly opposite the entrance opening. At this position the space between the wall and the end of the vane is

smallest. From this position it increases uniformly to a point opposite the exit opening, thus giving the instrument a uniform scale. The cover-glass is held in place by a bezel ring, which clamps it tightly against a nonleakable gasket joint at the case rim.

The vane described above has a length from center to end of approximately 36 millimeters and a depth of 13½ millimeters. The wall surrounding the vane has a maximum height of 15 millimeters, an inside diameter of 74 millimeters and an outside diameter of 80 millimeters. The case has an outside diameter of 90 millimeters and depth of approximately 34 millimeters. The instrument complete weighs about 1 kilogram.

FIG. 8.—Flowmeters—Disassembled.

An older form of flowmeter designed and used in Great Britain consisted of a suitably mounted vertical glass tube through which the gasoline flowed. The tube was ground internally so that the inner surface was conical and with the smaller end at the bottom. A phosphor-bronze ball within the tube assumed a position of equilibrium at a height where the rate of flow through the annular space between the ball and the walls of the tube was such that the upward force on the ball was equal to the weight of the latter in gasoline. The scale fitted beside the glass tube was graduated experimentally to show the different rates of flow. A by-pass valve was provided so that the gasoline could be diverted from the tube in case of breakage.

REPORT No. 130

AERONAUTIC INSTRUMENTS
SECTION VI

OXYGEN INSTRUMENTS

By F. L. HUNT
Bureau of Standards

REPORT No. 130.

OXYGEN INSTRUMENTS.

By Franklin L. Hunt.

INTRODUCTION.

This report is Section VI of a series of reports on aeronautic instruments (Technical Reports Nos. 125 to 132, inclusive) prepared by the Aeronautic Instruments Section of the Bureau of Standards under research authorizations formulated and recommended by the Subcommittee on Aerodynamics and approved by the National Advisory Committee for Aeronautics. Much of the material contained in this report was made available through the cooperation of the War and Navy Departments.

It has been shown that at altitudes above 15,000 feet the physical condition of aviators is seriously affected from lack of oxygen, unless artificial means is provided to supply the deficiency. The physiological symptoms are headache, which is usually the first symptom noted, palpitation, fatigue, numbness of the limbs, pains in one or both ears, which may persist for several hours, weakening of the attention, diminished sense of stability, vertigo, faintness, and finally loss of consciousness, with consequently disastrous results unless consciousness is regained before the earth is reached. It has been definitely determined that flying at altitudes of 20,000 feet or more for extended periods can not be undertaken without serious injury to the central nervous system of the aviator. These physiological symptoms can be almost entirely avoided by supplying to the aviator artificial oxygen during flight.

The normal man at full atmospheric pressure breathes approximately 16 times per minute. The volume of each inspiration is about one-half liter, so that 8 liters of air is breathed per minute, of which 1.6 liters is oxygen. This, however, is the minimum required. During flight the aviator is continually in a state of physical activity and, therefore, needs more oxygen. It has been found that 4 liters per minute is not an excessive amount to supply, allowing for inevitable losses at the mask.

The pressure of the air at sea level is approximately 14.7 pounds per square inch. It decreases at the rate of approximately one-half pound per square inch for every 1,000 feet of ascent for the first 10,000 feet and more rapidly at the higher altitudes. At 20,000 feet the pressure, and consequently the density, is approximately one-half of that at the surface of the earth; i. e., at each inhalation at this altitude the aviator would receive only one-half of the oxygen that he would get at full atmospheric pressure. The conditions for altitudes up to 30,000 feet on the assumption that the aviator requires 4 liters of oxygen per minute at full atmospheric pressure and breathes at the normal rate at high altitudes, are shown in Table I.

In column 1 is the altitude in feet computed from the formula

$$H=62900 \log_{10} \frac{759.6}{P}$$

where P is the barometric pressure in millimeters. A constant temperature of 10° C. is assumed. Column 2 gives the amount of oxygen in liters per minute which one man would receive, computed on the assumption that 4 liters per minute are required, by the relation $V=4 \frac{P}{760}$ where V is the volume in liters and P the atmospheric pressure in millimeters of mercury. In column 3 is the deficit which must be supplied in order that he may receive the normal amount of oxygen. Column 4 shows the amount specified by the British and the United States Bureau of Aircraft Production. French specifications require a delivery of 0.5 liter per minute at 3,000 meters and 2.5 liters per minute at 8,000 meters.

TABLE I.

Altitude (feet).	Oxygen available (liters/min.).	Oxygen deficit (liters/min.)	Supplied British and United States.
30,000	1.3	2.7	2.7
25,000	1.6	2.4	2.4
20,000	1.9	2.1	2.1
15,000	2.3	1.7	1.7
10,000	2.8	1.2	1.2
5,000	3.3	0.7	0.7
0	4	0	0

It thus appears that for high altitude flights approximately 150 liters of artificial oxygen must be supplied per person per hour, and under average conditions a supply for a 2½-hour flight is carried. Enough space is available in the fuselage of an airplane to store in bags the 350 or more liters required. Such bags have been made by the French weighing 1.2 kilograms and carrying 360 liters of oxygen under atmospheric pressure with a total capacity of 720 liters, which is necessary in order that the bags shall not burst during the ascent. Means are also provided to maintain a constant pressure in the bags. This is accomplished by the use of a smaller compensating bag in which the pressure is maintained constant by the speed of the airplane. Incendiary bullets set fire to the envelope and the use of such bags has consequently been discontinued.

The two methods in practical use for storing the oxygen supply are (1) to compress the required amount of gas into steel cylinders and (2) to liquefy the oxygen and carry it in a suitably constructed receptacle in liquid form. The French have used for compressed oxygen a cylinder of steel 3 millimeters thick of 360 liters capacity, weighing from 4 to 4½ kilograms, with a volume of 2,000 to 2,400 cubic centimeters and an inside diameter of 6.5 centimeters. The gas is under a pressure of 175 atmospheres. The British aviation service has used cylinders of chrome steel of 500 liters capacity, weighing 3 kilograms, which are filled with gas at 150 atmospheres. Specifications of the United States Bureau of Aircraft Production required seamless steel cylinders of 500 liters capacity at a gage pressure of 2,250 pounds at 60° F. The tanks are approximately 4 inches outside diameter and 20 inches long and weigh 12 pounds each.

The Germans use liquid oxygen exclusively. It is carried in vacuum-jacketed receptacles considerably lighter than the steel cylinders required for compressed oxygen.

Special containers for storing and transporting liquid oxygen have been developed by the British. These consist of a double-walled spherical vessel with inner and outer surfaces of spun or pressed metal 14 and 15 inches in diameter, respectively. They hold 4½ gallons of liquid which corresponds to about 15,000 liters of gaseous oxygen. The neck of the inner vessel consists of a long metal tube approximately three-eighths of an inch in diameter which is surrounded by a larger neck attached to the outer vessel. An air-tight collar is provided at the top between the two necks. The space between the inner and outer vessels including that between the inner and outer necks is exhausted to a pressure of approximately one-thousandth of a millimeter of mercury and the surfaces facing the evacuated space are highly polished to reduce the heat radiation. About 300 grams of prepared wood charcoal are placed in a small cup attached to the outside of the lower part of the inner vessel. This is to absorb residual gases in the evacuated space. A lead tube protected by a detachable cap is provided for exhausting and sealing off the space between the two walls of the container. The loss of oxygen per 24 hours by evaporation from such vessels does not exceed 7 per cent of the normal capacity of the container. Details of the German liquid oxygen apparatus, also similar ones of British, French and American design, will be considered later.

Another essential feature of the oxygen supply apparatus is a device for controlling the amount of oxygen delivered to the aviator. This consisted in the earliest apparatus merely of a hand-controlled reducing valve attached to the oxygen supply tank which the aviator

operated to deliver oxygen according to his need. In later forms an automatic pressure regulator which maintains the pressure of the oxygen supplied independent of the pressure in the tank and an automatic control for regulating the amount of oxygen supplied to the aviator according to his altitude, is provided. In addition, when compressed oxygen is used a pressure gage is added to indicate the pressure of oxygen in the supply tank and a flow indicator to show when oxygen is passing through the apparatus. Devices which have been designed for this purpose are described in the following section.

CONTROL APPARATUS - COMPRESSED-OXYGEN TYPE.

DREYER APPARATUS.

A type of oxygen-control apparatus extensively used during the war was invented by Col. Dreyer of the Medical Corps of the British Army. This apparatus is shown in figure 1 and in detail in figure 2. It uses compressed oxygen. The pressure tank is connected at C. The oxygen passes to the pressure gage A which indicates the tank pressure in kilograms per square centimeter. Beneath the pressure gage is a reduced pressure chamber B, which reduces the tank pressure of from 40 to 150 atmospheres to a pressure of approximately 15 pounds per square inch above atmospheric pressure. From the reduced pressure chamber the gas passes through copper pressure tubing to the shut-off valve V′, the purpose of which is to regulate the supply of oxygen for one or two men and to cut the supply off entirely if desired. From the shut-off valve the oxygen is conducted by copper pressure tubing to the regulating valve W, which is controlled by the battery of aneroid diaphragms G, so that as the aviator climbs, the aneroid diaphragms under the reduced pressure gradually increase the flow of oxygen through the regulating valve W to supply the deficiency of oxygen in the atmosphere. From the regulating valve the oxygen passes through copper pressure tubing to the flow indicator F, which indicates that oxygen is passing through the apparatus, thence by tubing connected at M to the mask of the aviator. These parts are all mounted on a brass base. A case of brass as indicated in figure 1 is provided to protect the instrument. When two men are supplied a Y connection is inserted beyond M so that tubes can be run to the masks of both aviators. A fine wire-mesh filter is provided at the tank connection to prevent dirt from entering the apparatus. The pressure gage A is of the Bourdon type and is connected directly to the tank connection. Beneath the pressure gage is the reducing valve, which automatically reduces the pressure of the oxygen from tank pressure to a value slightly above atmospheric pressure. This is accomplished by the action of the corrugated German-silver diaphragm D (fig. 2), which forms the top of the chamber B, into which the oxygen flows from the tank through the valve V. Diaphragm D is connected to a lever L pivoted at P and pressing upon a spring S with a German-silver tip at V, which presses upon a German-silver valve seat at V. The pressure of the oxygen which enters V from the tank causes the diaphragm D to expand, thus pressing the spring S on the seat of the valve V through the

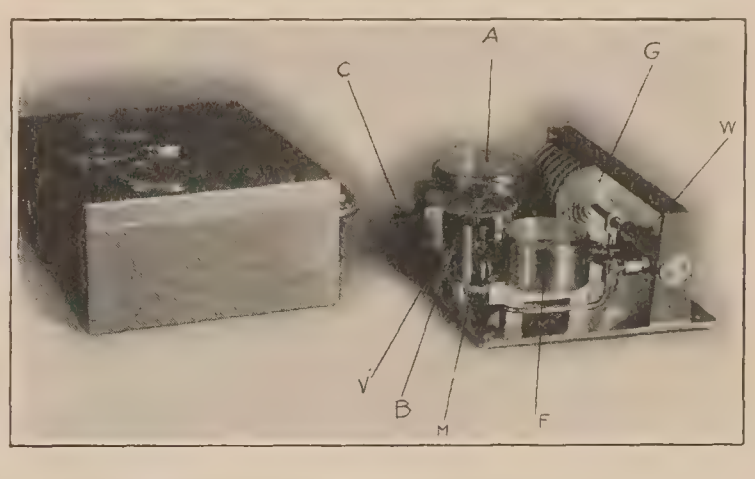

FIG. 1.—Dreyer Oxygen Apparatus.

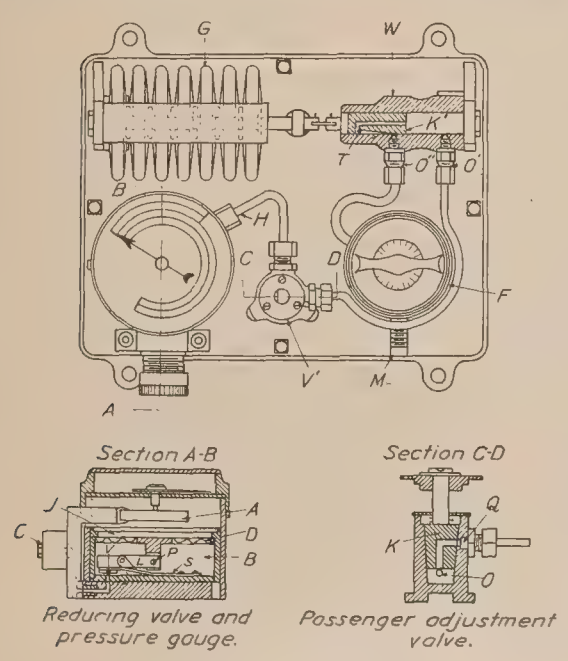

FIG. 2.—Dreyer Oxygen Apparatus.

action of the lever L. Oxygen passes from the reducing chamber B at H, thereby reducing the pressure in B, which causes the diaphragm to fall and the valve V to open. In this manner the pressure in the chamber B is automatically maintained at a nearly constant value. Above the diaphragm D is a hermetically sealed chamber J, filled with air at atmospheric pressure. The object of this is to make the action of the valve V independent of the external atmospheric pressure, which decreases as the altitude increases. It has the inherent disadvantage that the pressures of the confined air changes with the temperature; hence, at the low temperatures which obtain at high altitudes the contraction of the confined air causes the diaphragm D to partially close the valve V, thereby reducing the pressure in the chamber B, and consequently the flow of oxygen from the apparatus. If properly made this, however, should not decrease the flow more than 10 per cent. The shut-off valve V′ consists of a German-silver plug K accurately fitted to the valve case. The oxygen enters at O (section CD) and passes up through the center of the plug and thence to the outer surface at Q, communicating with a groove of variable depth cut laterally in the surface of the plug. By turning the valve the flow can be controlled through the throttling action of the groove and, if desired, entirely cut off. A graduated head is provided to operate the valve.

The aneroid control valve is shown in section at W. This consists of a German-silver plug accurately fitted to a German-silver casing. The oxygen enters at O′, flows through a passage along the axis of the plug K′, thence laterally to the surface of the plug to a groove T of variable depth cut longitudinally in the plug. This slot is located opposite the outlet O″. The aneroid diaphragms G (shown in figs. 1 and 2) are connected to the stem of the plug K′. These consist of corrugated boxes of German silver from which the air has been exhausted and which are maintained distended by internal springs. As the external atmospheric pressure decreases, the diaphragms expand through the action of the internal springs. This causes the valve plug K′ to move so as to bring a deeper portion of the slot opposite the outlet O″, thereby automatically increasing the flow of oxygen from the apparatus. By properly regulating the depth and cross section of this slot the supply of oxygen can be made just sufficient for the aviator's needs at all altitudes. The flow indicator is merely a turbine of light construction against which the oxygen from the aneroid control valve impinges in passing to the masks. The rate of rotation of the turbine gives a rough indication of the rate of flow of oxygen through the apparatus.

CLARK-DREYER AND KING-DREYER APPARATUS.

During the war instruments of the Dreyer design were produced in this country, as shown in figure 3. No essential changes were made from the British prototype; the base, however,

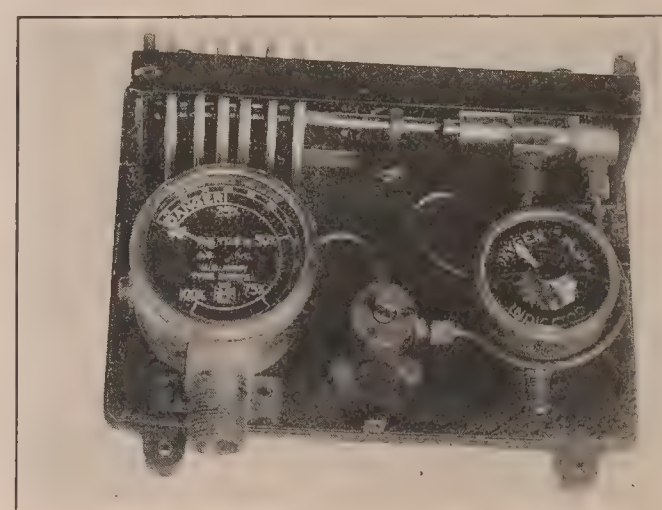

FIG. 3.—American-Dreyer Oxygen Apparatus.

is of cast aluminum instead of brass and provided with lugs with which to fasten the instrument to the airplane. The case is of copper-plated pressed steel and fits tightly over the base, to which it is fastened by screws instead of being a complete box of brass as in the British design. The

pressure gage is graduated to read in atmospheres instead of kilograms per square centimeter. The turbine is mounted on double instead of single jeweled bearings. The Y connection, used when two aviators are supplied, is connected directly to the outlet of the flow indicator, and in one of the legs of the Y is a shut-off which restricts the flow of oxygen to one mask in case one aviator only is operating the airplane.

The disadvantages of apparatus of the Dreyer type are: (1) the decrease of flow of oxygen at low temperatures, due to the contraction of the air above the graduated diaphragm of the reduced pressure chamber; (2) the tendency for the aneroid control valve to stick if the least particle of dirt or other foreign matter lodges between the piston and the walls of the valve; (3) in case one or more of the aneroid diaphragms become punctured the slot in the piston of the aneroid valve may pass beyond the outlet, thereby cutting off entirely the flow of oxygen;[1] (4) since the reducing valve is not a balanced valve the back pressure of the oxygen on the end of the piston, which acts in opposition to the tendency of the aneroid diaphragms to expand, may under certain circumstances be sufficient to prevent the valve from opening at all with decreasing atmospheric pressure.

MUNERELLE APPARATUS.

A form of apparatus very similar to that of Col. Dreyer, designed by Munerelle, is shown in figure 4. As in the Dreyer apparatus, the essential parts are a pressure gage, reducing

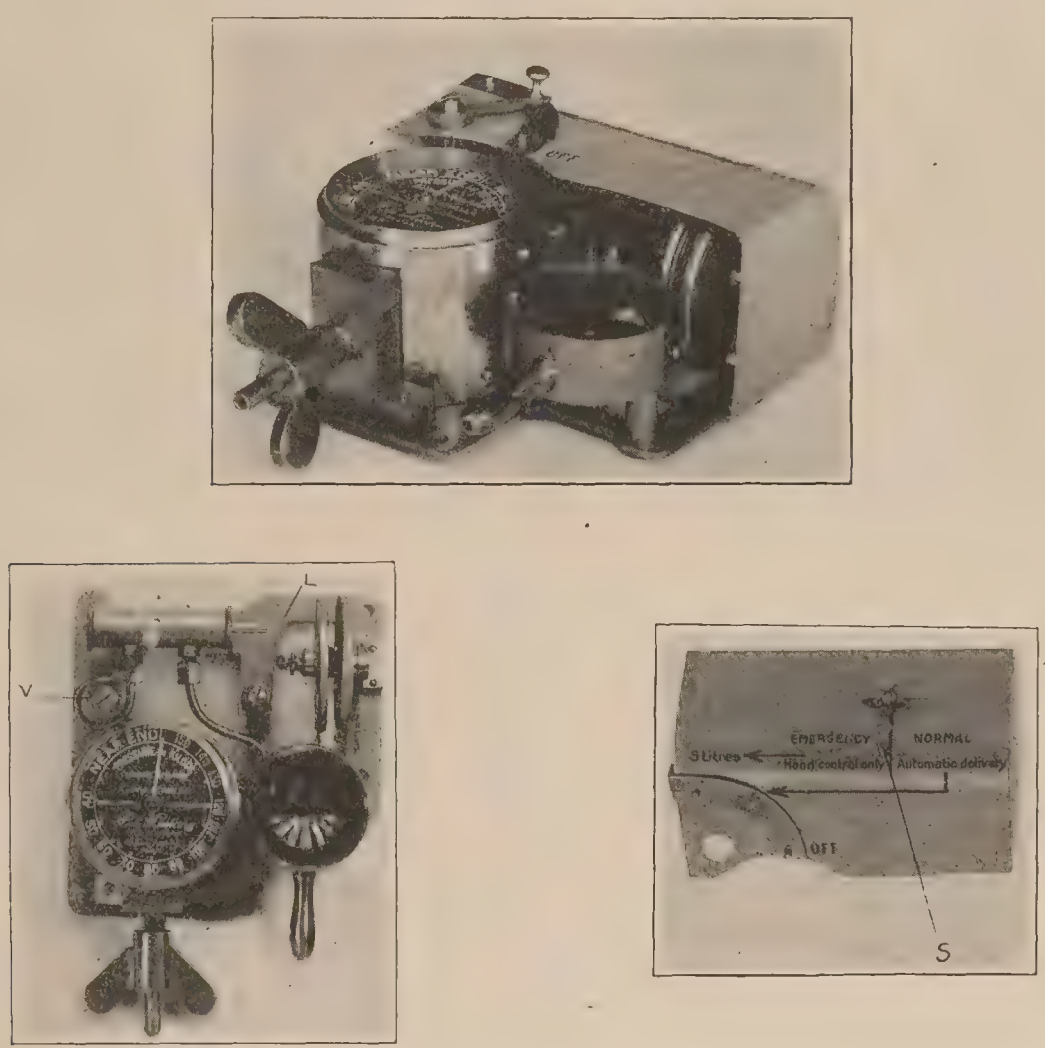

FIG. 4.—Munerelle Oxygen Apparatus.

valve, shut-off valve, aneroid control valve, and flow indicator. The pressure gage and reducing chamber and shut-off valve are practically identical with those in the Dreyer apparatus. The essential difference is that the aneroid diaphragms operate the aneroid control valve through an intermediary lever L, which lever can also be operated independently by hand by the stop S, so that in case the aneroids cease to function the aviator still is able to obtain an adequate supply

[1] This difficulty was overcome in the later models of American manufacture by providing a stop on the support of the valve stem to limit ts excursion in the direction of the cut-off.

of oxygen by bringing the stop S in contact with the lever L which in turn presses against the piston of the control valve and thereby opens the valve. Two aneroids instead of seven are provided to operate the aneroid control valve. The necessary linear motion of this valve is obtained by the multiplying action of the lever L. The result is that a tendency of the aneroid valve to stick would be more likely to make the valve inoperable than in the case of the Dreyer instrument. The emergency control of the aneroid valve is a desirable feature. A regulating screw is provided at the end of the aneroids to adjust the aneroid valve. The rate of delivery is controlled by the lever S, which operates the cut-off valve V. The pointer of the flow indicator is dispensed with, the rotation of the turbine itself, which is partially radium painted, being used to indicate the flow of gas. The instrument is more compact and lighter than the Dreyer type.

CLARK APPARATUS.

To overcome some of the disadvantages of the Dreyer instrument, the oxygen apparatus shown in figures 5 and 6 was designed. Its essential parts are a pressure gage, reduced-pressure chamber, shut-off valve, aneroid control valve, and flow indicator. A by-pass or full-flow valve is also provided to allow the oxygen to pass directly from the reduced-pressure chamber to the flow-indicator chamber in case of emergency. The parts are mounted on a

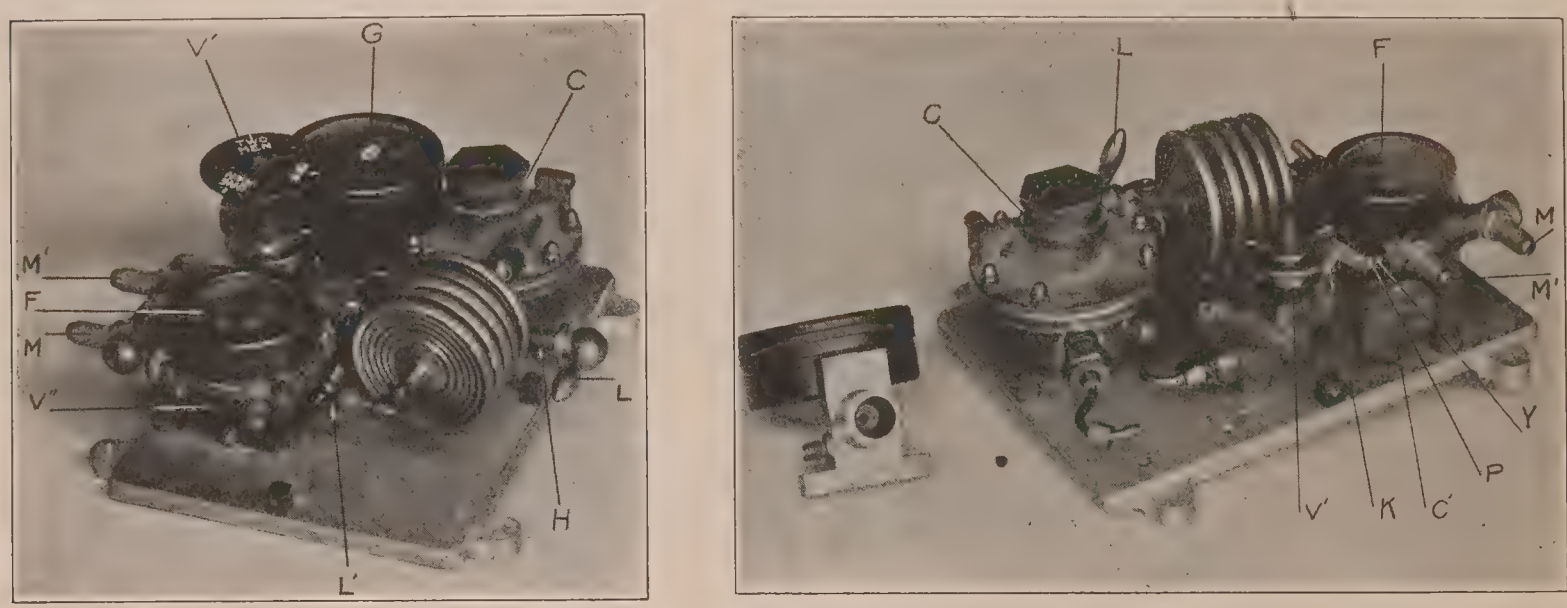

Fig. 5.—Clark Oxygen Apparatus.

cast aluminum base of the same size as that of the American Dreyer type, and a pressed steel cover like that of the latter instrument is also provided. The oxygen passes from the tank to the pressure gage G and the reduced pressure chamber C, thence to the by-pass and shut-off valve V′, which are cast in a single casing, thence to the aneroid valve V″, which is located under the flow indicator F, and finally through the flow indicator to the mask connections M M′.

Details of the reduced pressure chamber and aneroid control valve are shown in figure 6. The oxygen enters the reduced pressure chamber through the connection at O and passes to the control valve V, thence to the chamber C, causing the diaphragm D to expand, thereby bringing the bakelite valve cap B, which is attached directly to the diaphragm by a stirrup S, in contact with the German-silver valve seat. Above the diaphragm is a hermetically sealed chamber containing air at atmospheric pressure. The spring R under compression maintains the valve V normally open. An adjusting screw W permits the regulation of the flow from outside the pressure chamber. The one and two man control valve V′ (fig. 5) is of the plug type similar to that used in the Dreyer apparatus. The by-pass valve H (fig. 5), which is also of the plug type, is operated by a lever L from outside of the case. It is normally closed, so that the oxygen must pass through the one and two man control valve, thence to the aneroid control valve, and then through the flow indicator to the masks. If either of these valves fails

to function, by operating the cut-out valve oxygen is permitted to pass directly to the flow indicator and thence to the masks.

Details of the aneroid control valve are shown in figure 6. A stack of five aneroid diaphragms are connected to the lever L′, which in turn is connected to the valve stem W′. As the aneroids expand, the valve stem W′, which is threaded, is rotated by the lever L′ and raised from its seat, permitting oxygen which enters at O′ and passes to the valve seat V″ to flow in amounts depending on the excursion of the aneroids. From the control valve the oxygen passes to the flow indicator chamber immediately above, thence to the mask connections.

On the stem of the shut-off valve a cam K (fig. 5) is provided to operate the lever P, which presses against the piston Y of a small valve, which is opened by the action of the cam when the shut-off valve is set for two passengers, thereby obviating the necessity of using a Y connection in the tubes to the masks, as in the Dreyer apparatus. The flow indicator is of the turbine type. The pointer is dispensed with, and instead an aluminum disk with concentric slots, which appear as concentrated circles upon rotation, is provided.

To prevent moisture and other impurities, which are found in commercial oxygen, from interfering with the action of the apparatus, two filters are provided, one in the passage between the supply tank and the reducing valve and a second in the chamber C′ (fig. 5) between the hand valve and the regulating valve.

The advantages of this apparatus over the Dreyer type are that the aneroid-control valve is less likely to become clogged by particles of dirt or other impurities than is the piston valve of the Dreyer instrument. Moreover, in case the aneroids are punctured there is no danger of the supply of oxygen being cut off, since under these circumstances the valve will remain open. The reducing valve can be regulated without disassembling the apparatus. The by-pass or full-flow valve permits the pilot to receive an adequate supply of oxygen in case the hand valve or aneroid valve ceases to operate. Greater precautions are taken to filter the oxygen and thus remove possible contamination. On the other hand, the apparatus is rather heavy. It weighs 4.6 pounds, which is more than a pound more than the regular Dreyer apparatus. It has more parts than the Dreyer type and the design is complicated.

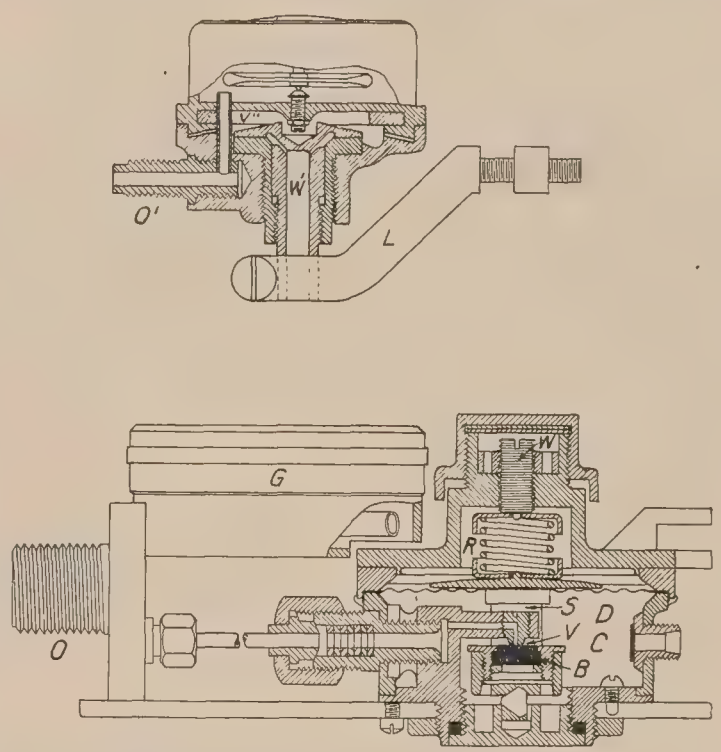

FIG. 6.—Reducing and Control Valves of Clark Oxygen Apparatus.

VAN SICKLEN APPARATUS.

A device of radically different design, invented by Prouty, is shown in figures 7 and 8. Essential features are a high-pressure chamber and reducing valve and a low-pressure chamber and reducing valve, the latter controlled by an aneroid capsule. A pressure gage and flow indicator are also provided. This apparatus differs from the Dreyer in that the required amount of oxygen is obtained by forcing the oxygen through an orifice of constant area by varying the pressure of the oxygen instead of maintaining the driving pressure effectively constant and varying the orifice in the aneroid-control valve, as in the Dreyer type. It is the lightest and most compact of the instruments thus far produced.

Referring to figure 8, the oxygen enters the high-pressure chamber H through the tank connection C. At one end of this chamber is a German-silver diaphragm D, which expands under the pressure of the oxygen, thereby closing the valve V, which is connected by the lever system L to the diaphragm. From the high-pressure chamber the oxygen passes through the

jewel needle valve V′ into the low-pressure chamber P, which is also provided with an aneroid diaphragm D′ and lever system L′ similar to that of the high-pressure side. In addition, an aneroid capsule N, backed by a spring S, presses against the center of the diaphragm D′. As the external atmospheric pressure decreases, the aneroid capsule expands, thereby exerting

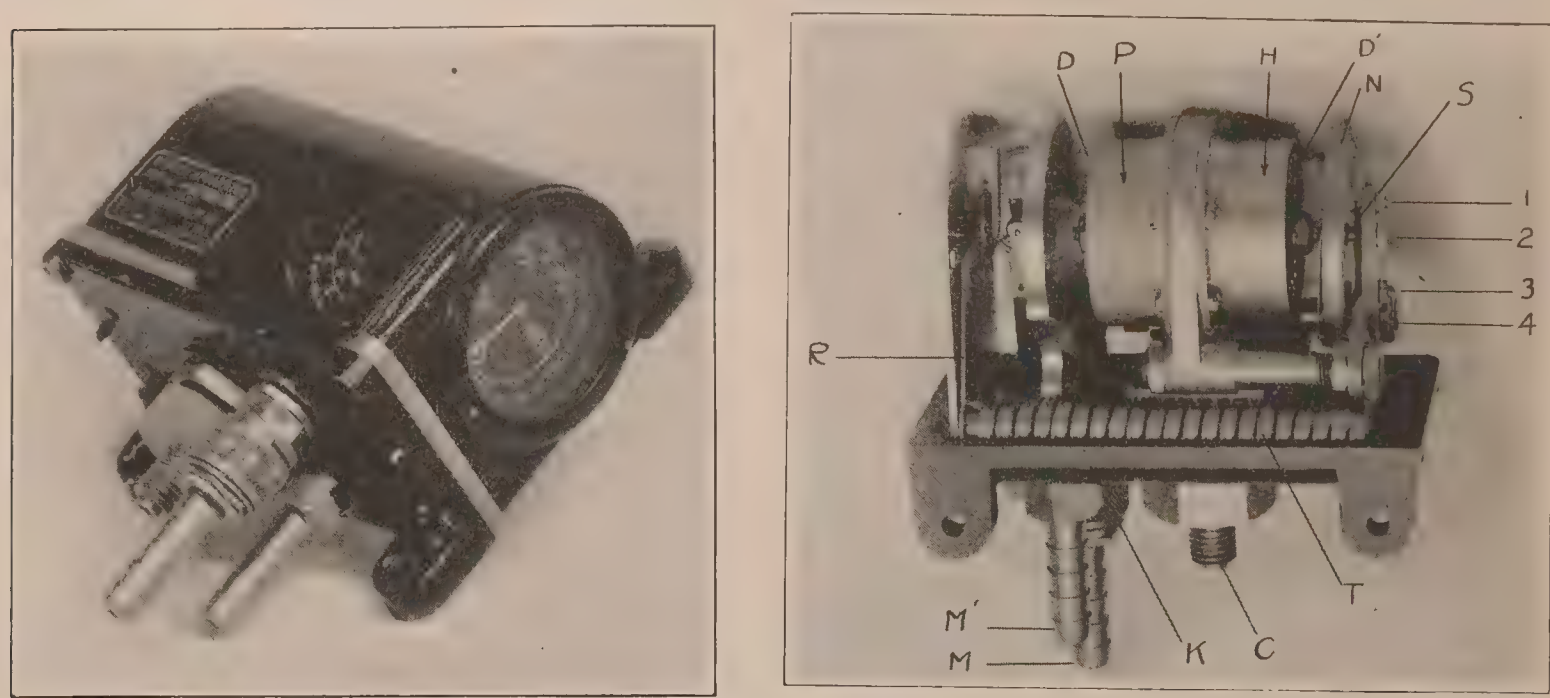

Fig. 7.—Van Sicklen-Prouty Oxygen Apparatus.

pressure through the diaphragm D′ on the lever L′ and regulating the amount of oxygen which enters through the needle valve V′.

The supply at different altitudes is regulated to the correct amount by the contact screws 2, 3, and 4, against which the steel spring S presses as the aneroid expands. Temperature compensation is secured by copper plating the spring on the inside. From the reduced-pressure chamber the oxygen passes to the mask connections M and M′ (fig. 7). In one, M, a shut-off valve K, with knurled ring, is provided to cut off the supply to one mask in case the machine is being operated by one aviator.

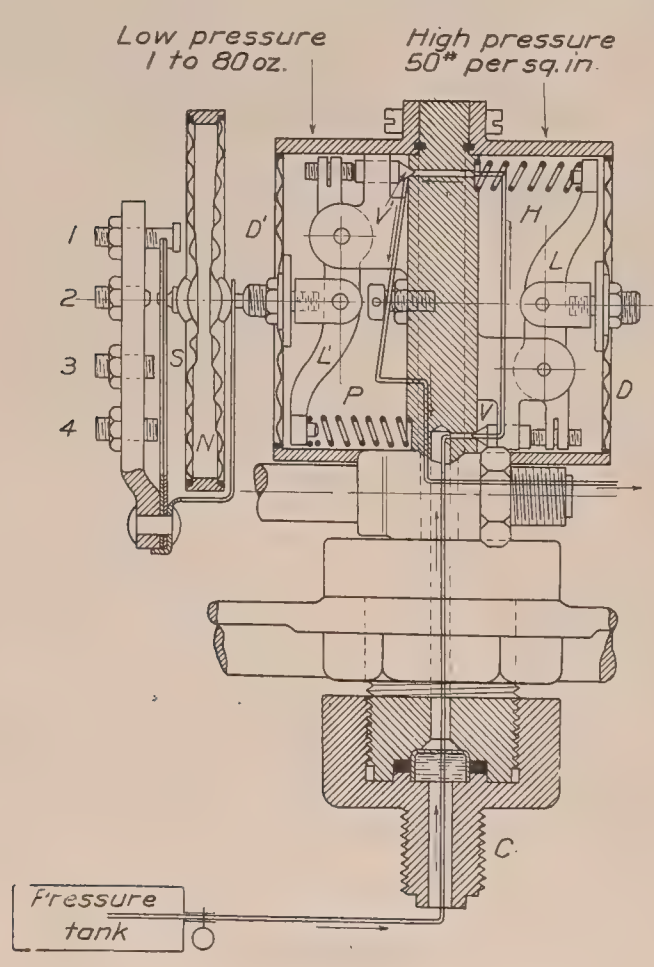

Fig. 8.—Van Sicklen-Prouty Oxygen Apparatus.

The high-pressure gage consists of the helical tube T (fig. 7), connected to the sector R, which in turn operates an indicating pointer. The coil is made to sustain a pressure of 2,500 pounds per square inch. The dial, which is equally spaced, is graduated from 0 to 200 atmospheres and also indicates the per cent of the full supply available. The flow indicator depends for its action on the static pressure of the oxygen developed on the low-pressure side of the apparatus. It consists essentially of a capsule B, one face of which is a sensitive German-silver diaphragm, the expansion of which under the pressure of the oxygen flowing from the apparatus indicates the flow roughly on the dial E through the intermediary action of a suitable lever system. It is essentially a pressure gage. No oxygen flows through the flow indicator. The parts are mounted on an aluminum base and are covered with a thin cast aluminum case provided with glass-covered openings to expose the dials of the pressure gage and flow indicator. The design is remarkably compact. The base is approximately 2 inches wide and $3\frac{3}{4}$ inches long and the cover is approximately $2\frac{3}{8}$ inches high. The weight, exclusive of external connections, is about $1\frac{3}{4}$ pounds.

This instrument has shown exceptionally satisfactory performance in laboratory tests and under service conditions to the limited extent to which it has been used.

GARSAUX APPARATUS.

An apparatus invented by Dr. Garsaux in France is shown in figure 9 and in detail in figure 10. The essential parts are a reduced-pressure chamber C_2 and an altitude-control device C_1, which operates a needle valve V.

The pressure-control chamber C_2 is divided into two parts by the rubber diaphragm D_1. The upper part communicates with the external atmosphere at A. The lower is connected to the oxygen-supply tank at T. The rubber diaphragm is held in place between the springs S_1 and S_2, of which S_1 is the stronger, and thus exerts an unbalanced pressure on the diaphragm. The oxygen which enters from the supply tank forces the diaphragm upward in opposition to the external pressure above the diaphragm and the unbalanced pressure of the springs S_1 and S_2 until the diaphragm is sufficiently displaced to close the valve T through the action of the lever system L_1. The oxygen in the chamber C_1 is thus maintained at a pressure which decreases with the altitude, but is maintained constantly in excess of the external atmospheric pressure by an amount depending upon the unbalanced pressure exerted on the diaphragm D_1 by the spring S_1, which is adjustable. A safety valve V_1 is provided to prevent the pressure in the chamber C_2 from becoming excessive in case the valve fails to close.

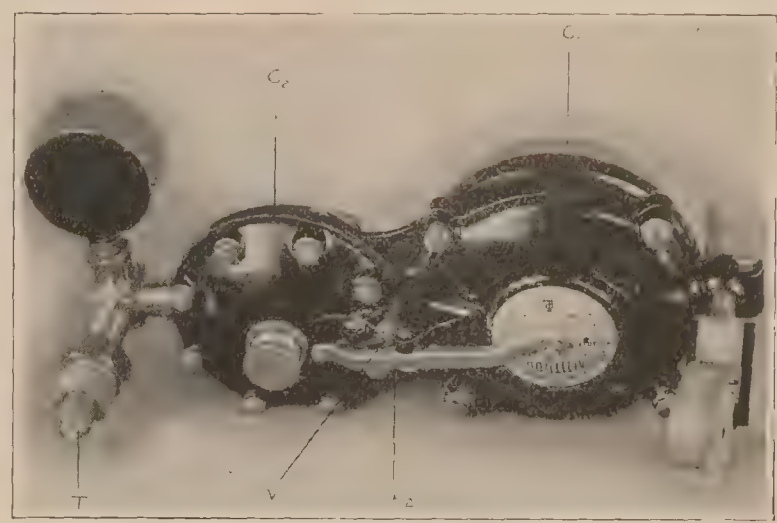

FIG. 9.—Garsaux Oxygen Apparatus.

The altitude control is effected by the expansion with increase of altitude and consequent decrease of pressure and temperature of the gas confined in the receptacle C_1 by the rubber diaphragm D_2, which acts in opposition to the spring S_3, forcing the rod T against the lever L_2 which in turn actuates the needle valve V, located between the outlet of the reduced pressure chamber and the mask connection. The supply of oxygen delivered is regulated by the size and shape of the needle valve in conjunction with the area of the diaphragm surface D_1 and the pressure of the opposing adjustable spring S_1. The pressure within the chamber C_2 is initially adjusted by means of the valve V_1.

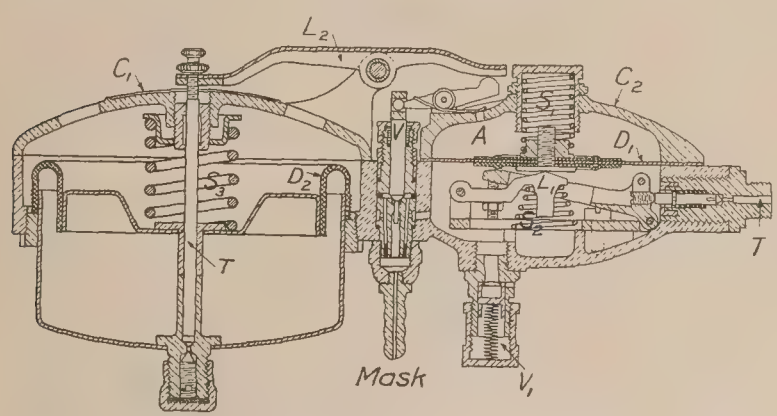

Fig. 10.—Reducing and Control Valves of Garsaux Oxygen Apparatus.

The principal disadvantage of this type of apparatus, aside from the lack of durability of the rubber diaphragms, is the temperature effect resulting from the change in volume of the air confined in the chamber C_1, which regulates the flow. This is in part due to the lag of the temperature of the confined air behind its surroundings and also because of the deviations under the conditions of use from the temperature altitude law of Radau which is assumed in calibrating the instrument.

GARSAUX APPARATUS—MODIFIED TYPE.

A simplified form of Garsaux apparatus is shown in figure 11. The amount of oxygen delivered is controlled by driving the gas under a suitably varying pressure through an orifice of constant size. A single reducing valve P is used. It is operated by the lever system L under the action of the two rubber diaphragms D_1 and D_2, which are situated between two chambers

C_1 and C_2. Chamber C_1 is hermetically sealed and contains air at atmospheric pressure. The diaphragm D_1, which rests against a metal plate, constitutes the upper part of this chamber. It is separated from the diaphragm D_2, to which it is connected by a small chamber C_3, exposed to the external atmospheric pressure through the opening H. The diaphragm D_2 constitutes the lower part of the reduced-pressure chamber C_2, into which the oxygen from the supply tank passes through the needle valve P. The diaphragms act in opposition to the upward motion of a spring. The air in C_1 expands with decrease of external pressure, thereby opening the needle valve and increasing the pressure of the oxygen in the chamber C_2, which in turn increases the amount delivered by the apparatus. From the chamber C_2 the oxygen passes through a suitable opening of constant size to the masks. The amount of oxygen delivered at any two chosen altitudes can be regulated by properly proportioning the area of the diaphragms D_1 and D_2, the stiffness of the spring S, and the size of the outlet orifice. The flow at intervening altitudes is thereby determined. A pressure gage G is provided to indicate the pressure in the supply tank. The pressure of the air in the hermetically sealed chamber C_1

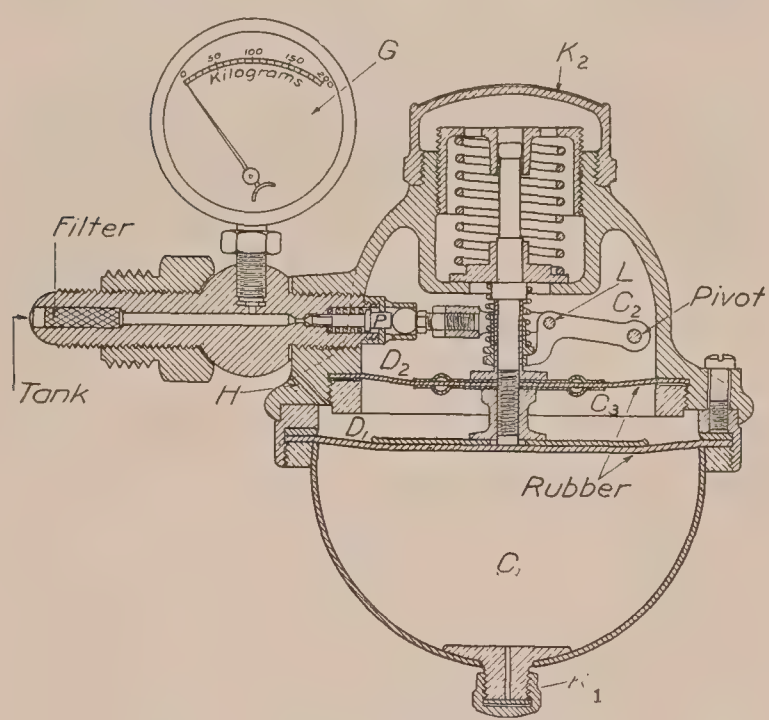

FIG. 11.—Garsaux Oxygen Apparatus, Improved Type.

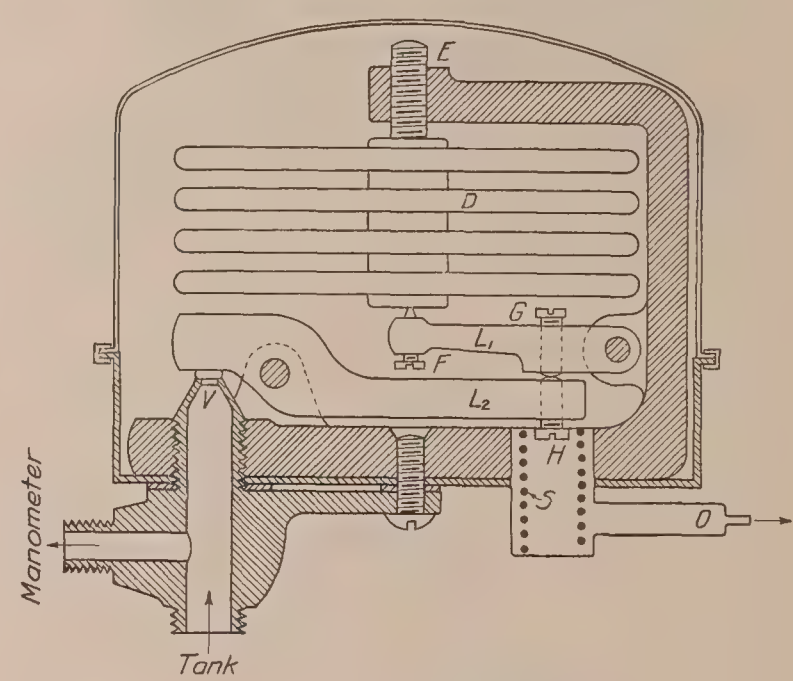

FIG. 12.—Gourdon Oxygen Apparatus.

may be regulated through the capped opening K_1. The adjustment of the diaphragms may be changed by removing the cap K_2.

GOURDOU APPARATUS.

Efforts have also been made to obtain a simple form of apparatus by allowing the oxygen to flow from a chamber at constant pressure through a suitable small orifice directly to the masks and depending on the decrease of the atmospheric pressure with increase of altitude to provide the necessary difference in pressure to force the required amount of oxygen through the orifice. If the pressure in the regulator is P_1, the external presure P_2, the quantity Q delivered will be given by the expression $Q = K(P_1 - P_2)$, where K is a constant depending on the dimensions of the orifice. Hence the ratio of the amount delivered at two altitudes will be given by the expression

$$\frac{Q_2}{Q_1} = \frac{P_0 - P_2}{P_0 - P_1}$$

from which, knowing the desired ratio of flow $\frac{Q_2}{Q_1}$ and the pressures P_2 and P_1 at the corresponding altitudes the pressure P_0 at which flow of oxygen begins is defined. An apparatus of this type (shown in figure 12) has been suggested by Capt. Gourdou. Oxygen from the supply tank enters the chamber C through the valve V and passes from this chamber through the orifice O to the masks. The pressure in the chamber is regulated by the expansion and con-

traction of the aneroid diaphragms D, which are connected to the lever system L_1 L_2, which opens and closes the valve V. A spring is provided to insure the closing of the valves when the aneroids are not sufficiently contracted to do so. The pressure of the oxygen in the chambers is regulated by the adjustment screws E, F, G, and H. The delivery curves are roughly parabolic. A limitation of both this and the previous apparatus is that the delivery can be exactly defined at two altitudes only, the assumption being made that at other points the delivery curve will not deviate from the theoretically correct amount enough to appreciably affect the aviator's breathing.

GIBBS APPARATUS.

An apparatus invented by W. E. Gibbs, formerly of the United States Bureau of Mines, is shown in figures 13 and 14. This model is made to supply one person. The distinguishing

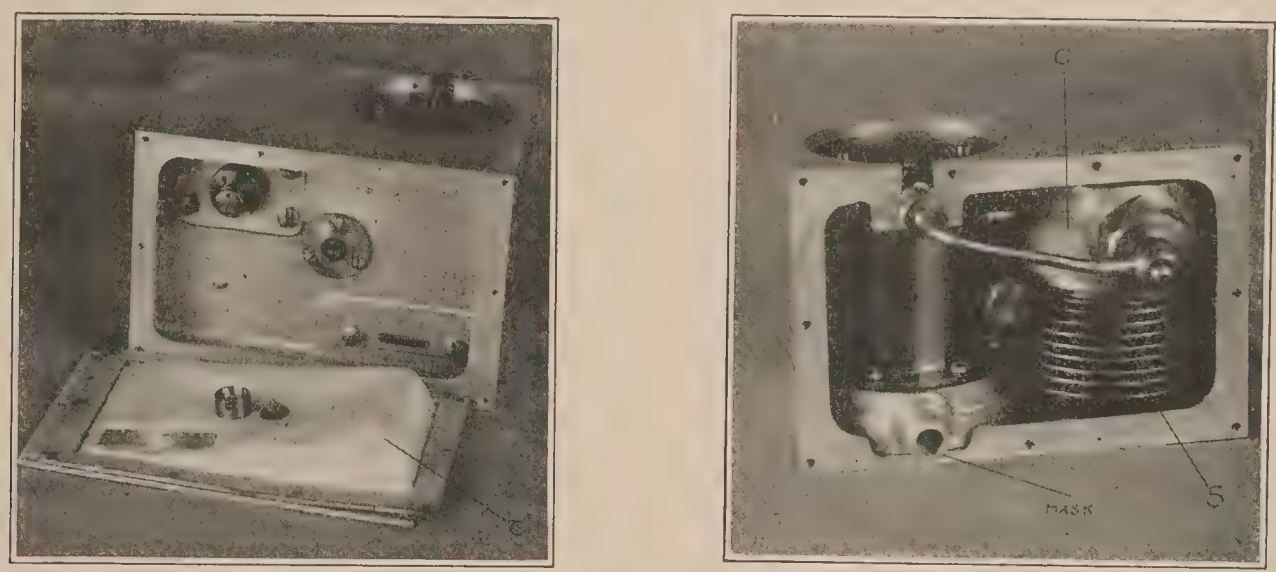

FIG. 13.—Gibbs Oxygen Apparatus.

characteristics of this apparatus are that the pressure is reduced in two stages before entering the aneroid-control valve and that a valve is provided near the mask connection, which permits oxygen to flow only when the aviator is inhaling. The pressure in the supply tank is indicated by a Bourdon gage of the usual type.

Referring to figure 14, oxygen from the supply tank enters the high-pressure reducing chamber C, the inlet of which is controlled by a valve operated by the multiple cell bronze "sylphon" valve S. As the oxygen enters the first reducing chamber and the sylphon valve S, the latter expands, thereby closing the inlet valve to the oxygen supply. The pressure is in this manner maintained in the first reducing chamber at approximately 1.5 pounds per square inch in excess of atmospheric pressure. From the first reducing chamber the oxygen passes into the second chamber C′, which is provided with a bellows B, controlled by the spring T, thereby automatically maintaining the pressure in the chamber at approximately 1 inch of water in excess of atmospheric pressure through the action of the valve V, which is operated by the bellows.

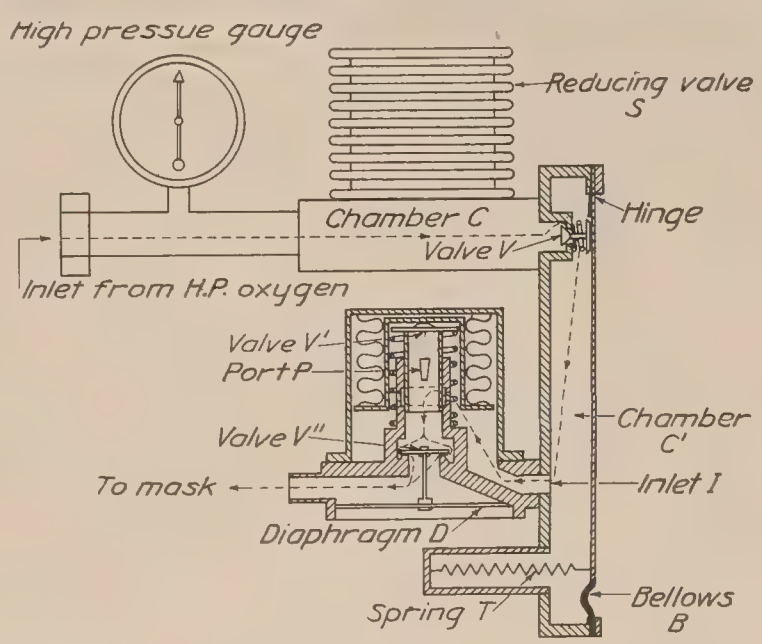

FIG. 14.—Gibbs Oxygen Apparatus.

From the second pressure chamber the oxygen passes through the inlet I to the aneroid-control valve V′, also of the multiple-cell bronze sylphon type. The air has been exhausted from this sylphon and as the external pressure decreases it expands, thereby opening the port P and permitting oxygen to pass in predetermined amount depending upon the altitude. From the aneroid-control valve the oxygen passes through the valve V″ to the mask. This valve is opened

and closed by the motion of the diaphragm D, which expands and contracts under the slight increase and decrease of pressure resulting from the aviators breathing. In its normal condition this valve is closed. The slight decrease of pressure, due to the aviator's inhaling, is required to open it and allow the oxygen to flow.

This apparatus is inclosed in an aluminum case $3\frac{1}{2}$ by $3\frac{3}{4}$ by $5\frac{3}{8}$ inches and weighs approximately 6 pounds 6 ounces. It has not been extensively used in practice. Effort is required by the aviator to operate the valve which permits the oxygen to flow only during inhalation. Although this effort is slight, it is noticeable, and it is important that the aviator experience not even the slightest sensation of obstruction or added effort in breathing. A later model of this apparatus, which is made to supply two aviators, more nearly resembles the Dreyer type. The valve operated by the breathing of the aviator has been eliminated. A hand-control or shut-off valve, similar to that used in the Dreyer apparatus, has been added to adjust the supply for one or two aviators. A flow indicator has also been added. Outside dimensions of the apparatus are approximately 7 inches square and 3 inches high; weight about 4 pounds.

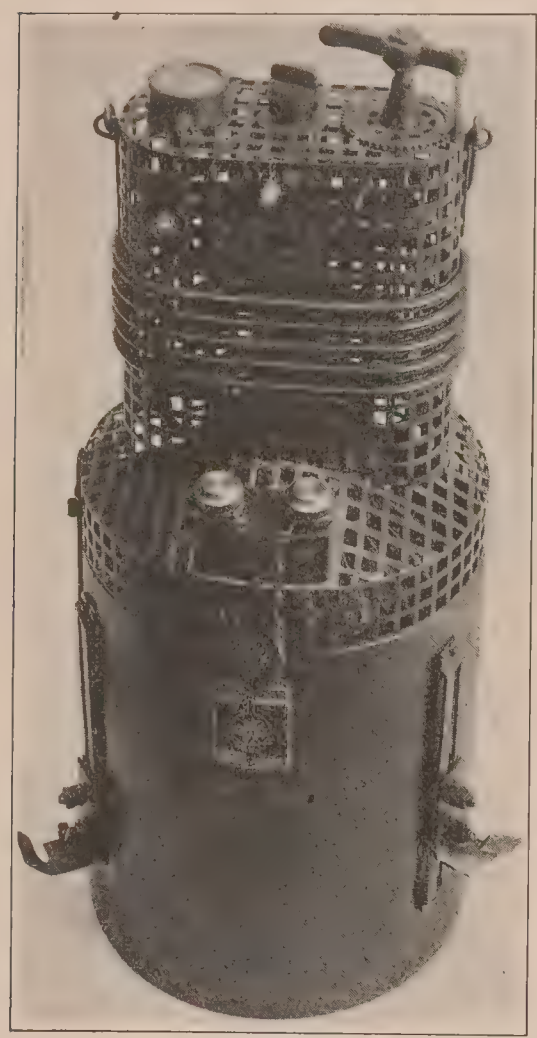

Fig. 15.—German Liquid Oxygen Apparatus.

CONTROL APPARATUS - LIQUID-OXYGEN TYPE.

The apparatus thus far considered have been designed to use compressed oxygen. It is also possible to carry a supply of oxygen in liquid form and to regulate the amount evaporated according to the needs of the aviator. The apparatus required has the advantage that it is considerably lighter and more compact than the complete equipment necessary when compressed oxygen is used. On the other hand, there is an inevitable loss of gas due to evaporation with the liquid type when the gas is not actually being used. It is therefore desirable that the apparatus be filled when about to be used and that the liquefying plant be reasonably near the base of operations. The liquid-oxygen type of apparatus is especially useful for long flights, since a large amount of oxygen can be carried with comparatively small volume and weight.

GERMAN TYPE.

An apparatus for liquid oxygen which has been extensively used by the Germans is shown in figures 15 and 16. The supply of liquid gas is contained in the double-walled spherical copper vacuum bottle B. Connected to the bottle is the tube T, in which the oxygen which evaporates is confined. A pressure gage P and a safety valve S are attached to this tube. The pressure developed in the tube T forces liquid oxygen up through the tube *t* and out through a system of coiled tubes and evaporation chambers, C_1 C_2 C_3 C_4 to the needle valve N, thence to the breathing bag R (not used in the model shown in fig. 15) and to the masks. The safety valve S prevents the pressure in the tube T, due to the constant evaporation of the liquid oxygen, from becoming excessively great in case oxygen is not being delivered. The amount of oxygen delivered by the apparatus depends upon the rate at which liquid oxygen passes through the system of coils and evaporating chambers. This is determined by adjusting the needle valve N. The heat required to evaporate the gas is absorbed from the surrounding atmosphere, which is practicable since the temperature of the latter is always far above that of the liquid oxygen (−185° C.). In the evacuated space between the two walls of the container B, on the underside of the inner wall, a layer of charcoal is provided, which is held in place by paper and metal gauze. This serves to absorb small amounts of gas which may leak into the evacuated

space. A stopper is used to insert in one of the mask connections M M_1 when one aviator only is to be supplied. The amount of oxygen delivered by this apparatus at different altitudes for one and two passengers is shown in figure 17. These curves refer to a surrounding temperature of 0° C. They indicate that the supply actually decreases above 4,000 meters when one person is being supplied and continually throughout the range of altitude when adjusted for two persons. This is in part because the amount of heat absorbed from the atmosphere, due to its decrease of density with increasing altitude, falls off more rapidly than the lowering of the boiling point of the liquid oxygen owing to the decreased external pressure. Moreover, the temperature at high altitudes is usually below 0° C., and the supply would therefore in general be even less than that indicated by the curves. However, the amount delivered is still more than enough for the aviator's needs. The weight of the apparatus just described when empty is $3\frac{8}{10}$ kilograms. It has a capacity of 2,000 cubic centimeters of liquid oxygen.

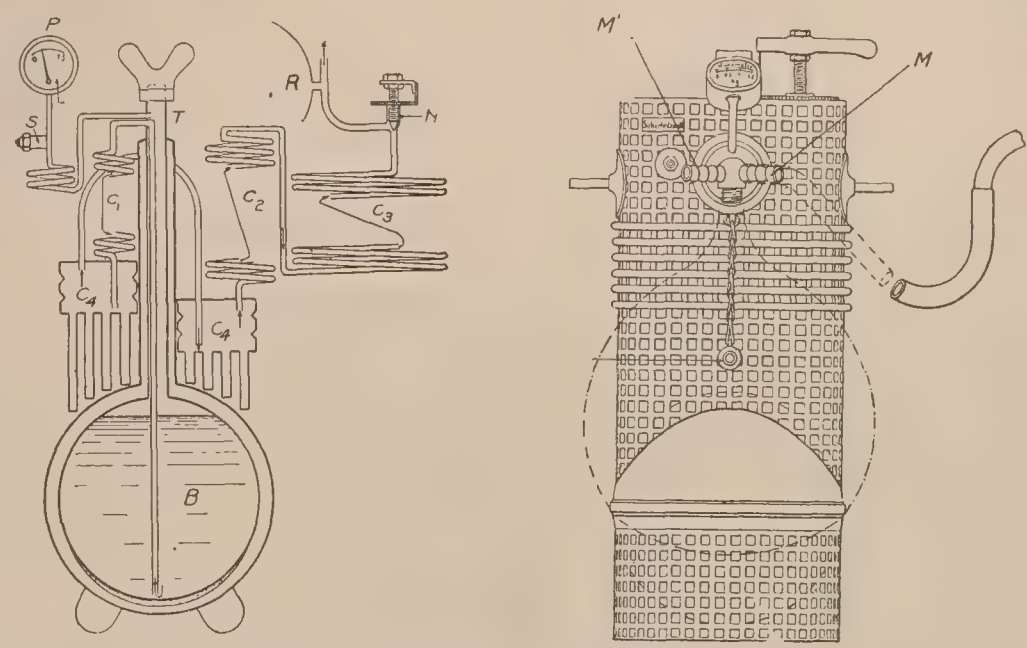

FIG. 16.—Details German Liquid Oxygen Apparatus.

To reduce the loss of gas through evaporation when the instrument is not in use, experiments have been conducted in France and England with metal containers of stamped metal silvered or galvanized and also with double-wall glass bottles. It has been found that glass bottles with a capacity of one liter of liquid oxygen, which corresponds to approximately 900 liters of gas, lose through evaporation only 12 liters per hour at 15° C. Reasonable care must be taken, however, in filling glass bottles that the temperature is lowered gradually before the liquid oxygen comes in contact with the glass; otherwise they are likely to crack. This difficulty could be avoided by using vessels of fuzed quartz.

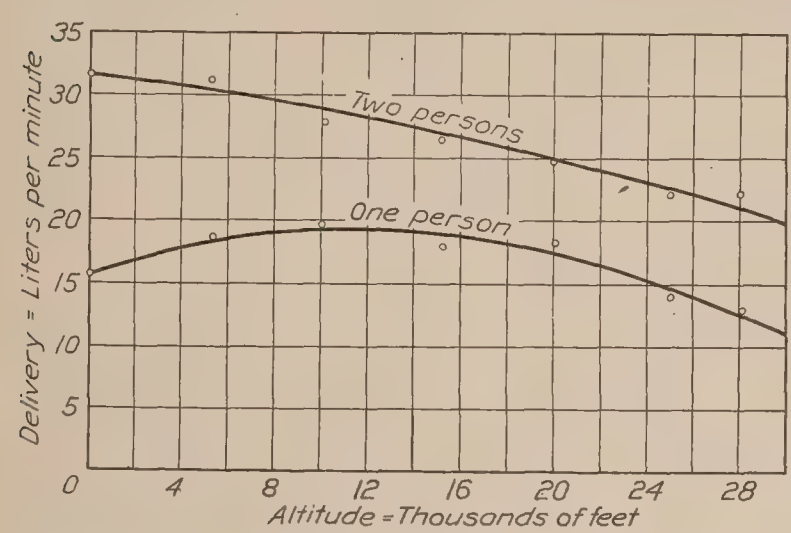

FIG. 17.—Altitude Delivery Curves for German Oxygen Apparatus, Liquid Oxygen Type.

BRITISH TYPE.

An apparatus of British manufacture similar to the German apparatus just described is shown in figure 18. The container consists of two spun-metal cylindrical vessels, one within the other, with an exhausted space between and polished surfaces facing the evacuated space to reduce radiation. There is an indentation in the lower part of the inner vessel for a charcoal pocket, which is covered by metal gauze to hold the charcoal in place.

The apparatus is made in two sizes, which have a capacity of $2\frac{3}{4}$ and $4\frac{1}{2}$ liters of liquid oxygen, corresponding, respectively, to approximately, 2,500 and 4,000 liters of oxygen gas. The container of the small model is spherical; that of the larger is elongated and rounded at the top and bottom. The arrangements for evaporating the liquid oxygen and controlling the delivery are mounted around the neck of the container. Evaporation is caused by absorption

of heat from the surrounding atmosphere. The pressure of the evaporated gas which is confined above the liquid is measured by a pressure gage D, which is connected to the inner neck. A spiral coil of $\frac{1}{4}$-inch copper tubing A forms part of the gage connection. Beyond the gage and connected to it is a safety valve V, which can be adjusted to regulate the pressure at which the oxygen blows off. That which escapes from the valve is led to the main gas-delivery outlets. The coil A prevents oxygen from reaching the gage in liquid form and warms it before it passes through the safety valve into the main delivery pipe. The main supply of oxygen is delivered through the coil C. The pressure of the evaporated oxygen above the liquid forces the liquid up a siphon tube which reaches to the bottom of the inner vessel. When the liquid has reached the top of the siphon it continues to flow under the combined action of the gas pressure and the siphon. From the siphon tube the liquid passes into the annular boiler B, provided with ventilating tubes to increase the radiating surface. The liquid is evaporated here, and the gas produced passes through a third coil around the neck to the regulating valve R, which controls the rate of delivery. At this point two delivery tubes are provided, which connect to the flow indicators and thence to the breathing masks.

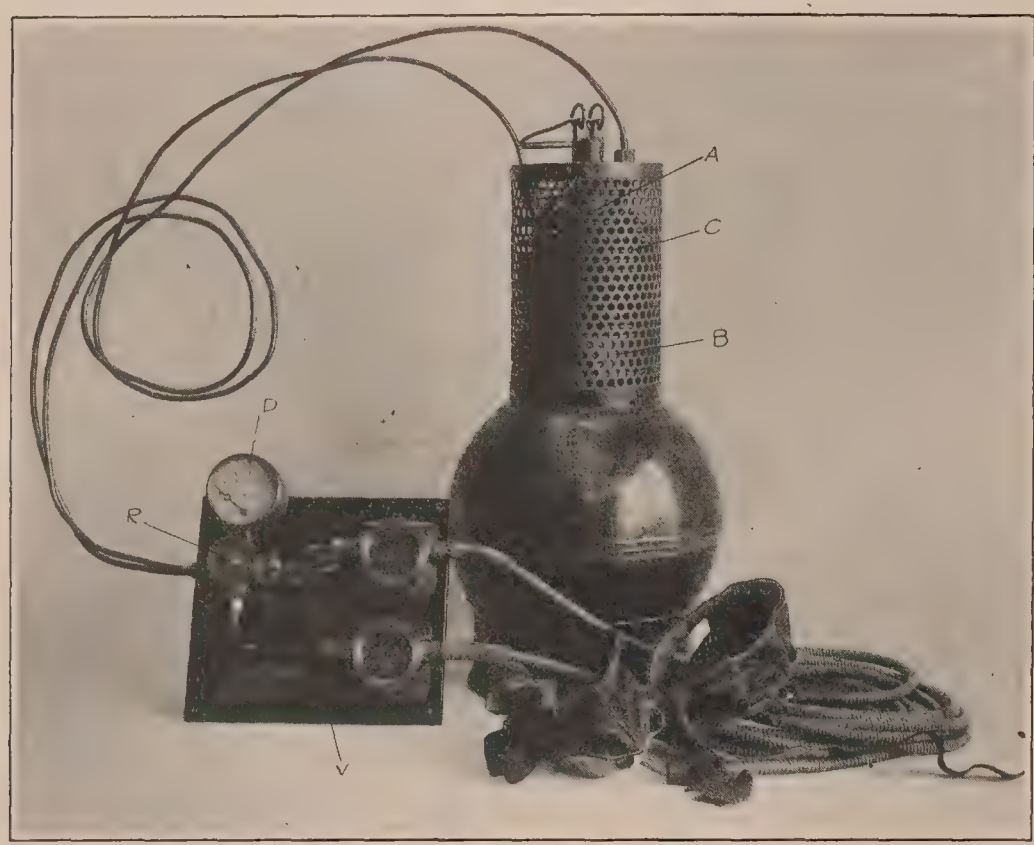

FIG. 18.—British Liquid Oxygen Apparatus.

A disadvantage of these types of apparatus, aside from the inevitable loss of oxygen due to evaporation and the inadequate control of the gas delivered, is that in case the apparatus is inverted the rate of delivery can not be controlled. The French claim that the charcoal in the evacuated space between the walls of the bottle is likely to be ignited by the oxygen in case an incendiary bullet strikes the apparatus. This is denied by the British.

FRENCH TYPE.

To overcome these difficulties and provide more satisfactory control of the amount of gas delivered, the French have proposed to make a device which utilizes a double-walled glass vessel to hold the liquid oxygen. In the latter is a quartz tube, which incloses at its lower end a tungsten-filament lamp. The amount of current supplied to this lamp, and consequently the amount of heat delivered and oxygen evaporated, depends upon a variable resistance, which is automatically regulated according to the altitude by a barometric control device. The vessel is partially filled with glass wool to prevent the oxygen from flowing suddenly in large excess from the apparatus in case it is inverted. The capacity of the container is thereby decreased about one-fifth. A small vent tube, communicating with the external atmosphere, is provided to prevent the pressure in the bottle due to evaporation from becoming excessive. The pressure thus never exceeds 25 pounds per square inch even when the lamp is at its highest temperature.

PERFORMANCE REQUIREMENTS AND SPECIFICATIONS.

Oxygen-control apparatus has been found difficult to manufacture owing to its complexity and delicacy of the adjustments required. It is essential that the apparatus be of durable construction, reliable in operation, and as light as is consistent with durability and reliability. It must not be affected appreciably in its performance by the wide fluctuations in temperature to which it will be subject in use. The vibrations of the airplane must not affect its operation, and it must be insensitive to such positional changes as will be experienced in maneuvering the airplane. The amount of oxygen delivered at all altitudes must always be at least sufficient for the aviator's requirements, and it is important that the required amount be not greatly exceeded for the sake of economy.

To fulfill these requirements it is essential that construction and performance specifications be formulated and that altitude-delivery curves with tolerances which define the allowable variation in flow at different altitudes be provided. Detailed working drawings with dimensions and tolerances are necessary. The material should be of the best quality throughout and free from defects. The workmanship should be of the highest grade in view of the delicate adjustments required. It is highly desirable that a large degree of interchangeability of parts be provided. Gages, indicators, aneroids, reducing valves, unit castings, cases, tubes, screws, and connections should be interchangeable with like parts of other instruments of the same make and type.

Material and workmanship should be carefully inspected during manufacture. This should include an inspection of individual parts and performance tests on assembled parts and on the complete instrument in accordance with adequate performance specifications. Samples of new instruments should be submitted. The right of rejection of parts and of the complete instrument should be provided.

LABORATORY TESTS.

TESTING APPARATUS.

Oxygen-control apparatus under the conditions of use is subjected to variable reduced pressures which may be as low as one-third of an atmosphere at the highest altitudes attained. It may also be simultaneously subjected to temperatures as low as $-50°$ C. The oxygen is discharged against the reduced pressure of the surrounding atmosphere.

In order that the laboratory tests may be carried out under circumstances similar to those obtaining in actual use, it is essential that the above-mentioned pressure and temperature conditions be fulfilled. This has been done at the Bureau of Standards by placing the apparatus to be tested under a bell jar, and exhausting the air from the bell jar until the pressure is as low as that at the highest altitudes at which the apparatus is designed to function (33,000 feet). Connection to an oxygen-supply tank is made through suitable connections in the metallic base on which the bell jar rests. The bell jar is simultaneously cooled by refrigeration for the low-temperature tests.

The pressure is determined by connecting a mercury barometer to the bell jar. As the air in the bell jar is exhausted the aneroid-control valve of the apparatus opens and allows oxygen to flow through the apparatus into the bell jar. It is desirable, therefore, that the pump have sufficient capacity to exhaust the bell jar with the oxygen flowing at its maximum rate (5 liters per minute)—otherwise the oxygen must be shut off at the tank while the bell jar is being exhausted. When the pressure has been reduced to the required minimum, the pump is cut off from the bell jar. The oxygen continues to flow, however, thereby increasing the pressure in the bell jar and causing the mercury in the attached barometer to fall. As the oxygen continues to flow and the pressure in the bell jar continually increases, the aneroid-valve gradually closes until the oxygen is again completely cut off as the pressure in the bell jar again approaches atmospheric pressure. The rate of delivery of oxygen can be found while the oxygen is flowing into the bell jar if the volume of the bell jar is known, and the rate at which the mercury column of the barometer falls is determined at a series of points along the scale.

The apparatus[2] used at the Bureau of Standards is shown in figures 19 and schematically in figure 20. Referring to figures 19 and 20, at the left is the bell jar B, of approximately 30 liters capacity, resting upon a heavy cast-iron plate through which the connections to the oxygen tank T, the pump P, and the manometer C are made. In the figure three oxygen tank connections are shown. This is so that three apparatus can be connected simultaneously under the bell jar and tested successively, thereby avoiding the necessity of removing the bell jar between tests and breaking the seal.

The bell jar rests upon a rubber gasket fastened to the base plate with shellac. Glycerine is used to insure an air-tight connection between the bell jar and the gasket. The mercury manometer M, which is connected to the bell jar, is of the closed cistern type, designed to read at low pressures. The scale is graduated on the left in millimeters, and on the right are the corresponding altitudes in feet. This barometer has the advantage over an open manometer

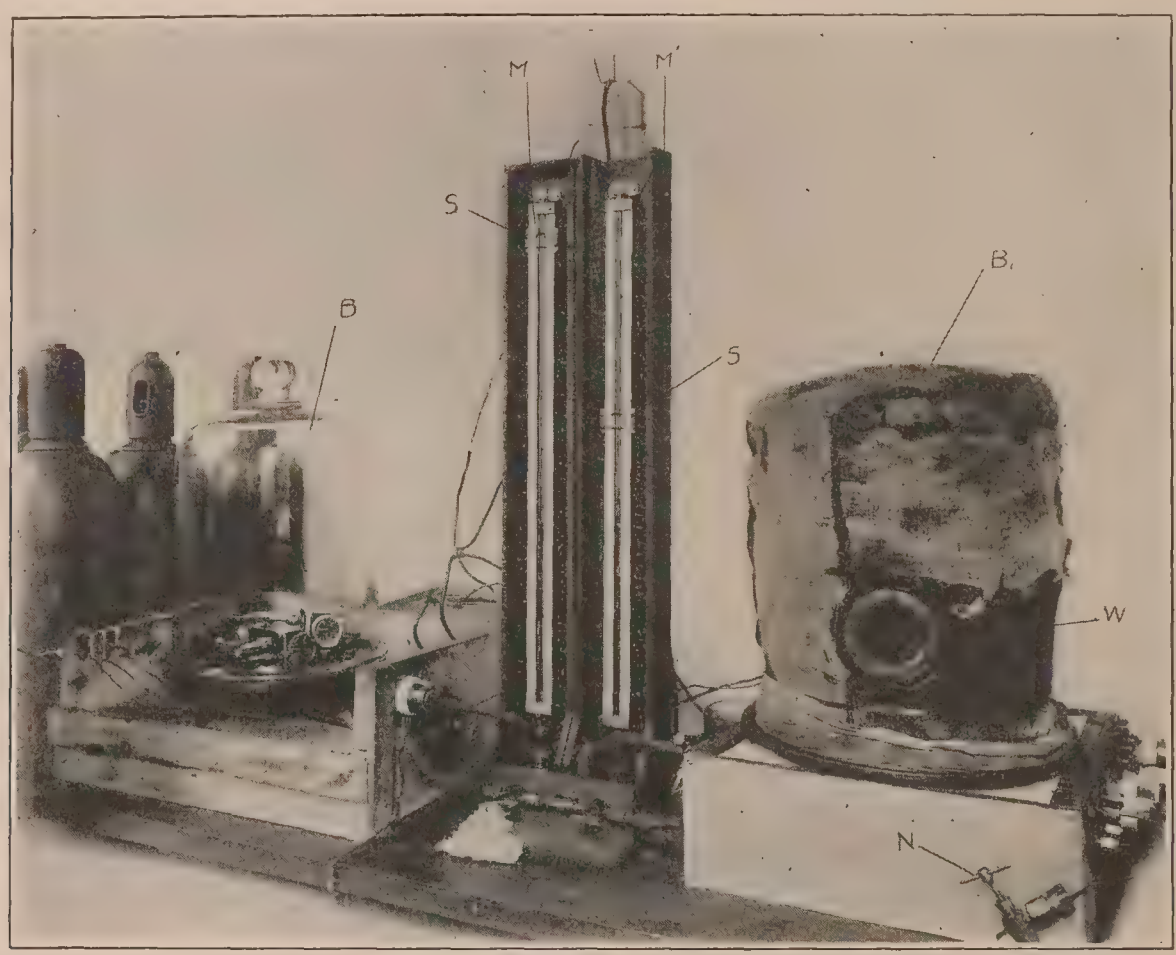

FIG. 19.—Bureau of Standards Oxygen Apparatus Testing Equipment.

that it indicates the absolute pressure in the bell jar irrespective of the fluctuation in the external atmospheric pressure. The slider S (fig. 19), which can be moved throughout the range of the scale, is provided with two horizontal knife-edges, which are in front of and close to the manometer tube and exactly two centimeters apart. The rate at which the pressure in the bell jar is increasing is found by determining with a stop watch the time required for the top of the mercury column to fall from the upper to the lower knife edge.

The barometer M′ at the right, which is identical with the barometer just described, is connected to the bell jar B′, in which the temperature tests are carried out. This bell jar consists of a dome of welded sheet steel provided with a cast-steel rim. It is heavily lagged with felt and has a plate-glass window W (fig. 19) for observations. The dome rests upon a base like that under the glass bell jar, and, as in the other case, a rubber gasket is provided and glycerine is used as a seal. It was found that vaseline and tallow solidified at the low temperatures used and afterwards on removing the dome loosened the gasket from the base. The dome is connected to the manometer and exhaust pump at C′ and P′. To obtain the low temperatures required, a

[2] Designed and constructed under the direction of Mr. L. A. Hoffman of the Aeronautic Instruments Section, Bureau of Standards.

carbon-dioxide refrigerating system is employed. This includes an expansion coil X (fig. 20), an interchanger I, and a precooler O with connections as shown. Carbon dioxide enters from the compressor at A, passes through the precooler O, thence to the interchanger I, thence to the needle valve N, where expansion takes place, then directly to the expansion coil X, thence to the return coil of the interchanger, and back to the compressor. The precooler consists of a coil of copper pressure tubing surrounded by an ice-salt freezing mixture. The interchanger is made of a double coil of copper pressure tubing. The expansion coil, which is also of heavy copper tubing, has a radiating surface of approximately 100 square inches. A small motor-driven fan F is used to secure a uniform temperature inside of the dome, and an electric light L is provided for illumination and for temperature regulation. Current for the motor and electric lamp enters through insulated binding posts in the metal base. Control switches are provided. A toluene or pentane thermometer is used to indicate temperature. The manometer, pump, and oxygen-supply connections are shown diagrammatically at the left in figure 20. The stop cocks S_1 S_2 S_3 S_4 are to cut off one or both bell jars from the pump and the oxygen supply. With this device no difficulty is encountered in maintaining continuously temperatures between $-30°$ C. and $-40°$ C.

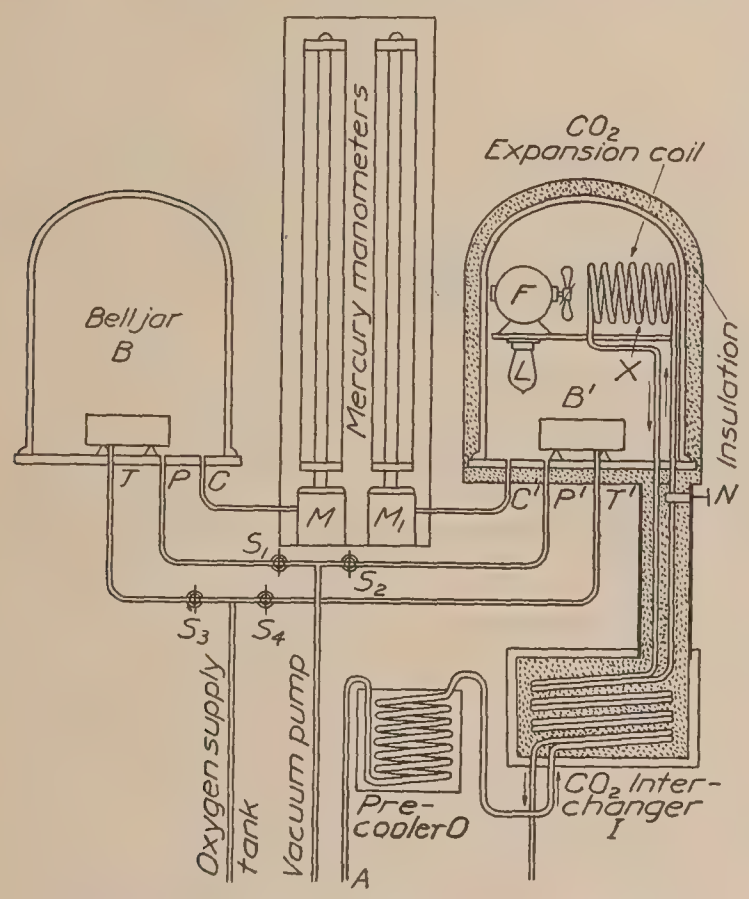

FIG. 20.—Testing Apparatus for Oxygen Control Apparatus.

PROCEDURE FOR TESTS.

After an initial inspection for obvious mechanical defects, the instrument to be tested is placed on the bell-jar base, connected to the oxygen supply, and all valves, connections and tubes tested for leaks with a smoldering wick. The oxygen is then cut off, the hand valve for one or two passengers adjusted, and the bell jar placed over the apparatus. The supply-tank pressure and the temperature of the barometer is recorded, the bell jar then exhausted to an absolute pressure of approximately 65 centimeters, the pump cut-off, and the mercury column observed to detect leaks around the base of the bell jar. The oxygen is next turned on, the pump connected, and the pressure noted when the turbine-flow indicator starts to rotate (this should not be less than 63 centimeters). The bell jar is exhausted to approximately 15 centimeters absolute, the pump cut off, and the center of the barometer slider set at 25 centimeters. Since oxygen is flowing through the apparatus into the bell jar, the barometer falls. When the meniscus reaches the upper knife-edge of the slider, the stop watch is started and stopped when it reaches the lower knife-edge. The time is recorded to the nearest fifth of a second and the mean barometric reading at the two knife-edges (25 centimeters), the slider is raised from 5 to 10 centimeters, and the time observations repeated for other 2-centimeter intervals up to approximately 70 centimeters pressure or until the oxygen ceases to flow. The pressure when the flow indicator stops is observed, then the bell jar is again exhausted to approximately 15 centimeters, and a check run made.

If both room and low temperature tests for the effect of low-tank pressure are to be made, they should be carried out without resetting the one and two passenger valve, since slight changes in setting make an appreciable difference in the amount of oxygen delivered. The one and two passenger valve may be tested for leaks by setting it at "off" while connected to the supply tank and noting if the manometer remains constant after the pressure in the bell jar has been reduced to approximately 15 centimeters.

CALCULATIONS.

The altitude corresponding to the mean pressure of the several 2-centimeter intervals may be found from pressure-altitude tables prepared by the Bureau of Standards. The capacity of the bell jar must be known, also the displacment of the oxygen-control apparatus (approximately 330 cubic centimeters for the Dreyer), and the temperature of the air in the bell jar. The rate of flow of oxygen through the control apparatus in liters per minute in terms of the effective volume, V, of the bell jar (capacity minus volume of apparatus under test), the absolute temperature, T, of the gas in the bell jar (centigrade temperature plus 273), and the time in seconds, t, for the pressure in the bell jar to change two centimeters may be found by the following formula:

$$\text{Liters/min.} = \frac{463\ V}{Tt}$$

If T does not differ appreciably from 293° absolute (20° C.) the expression becomes:

$$\text{Liters/min.} = \frac{1.58\ V}{t}$$

Derivation of formulas:

Let V be the effective volume of the bell jar in liters (capacity − volume of apparatus under test).

P_1, pressure in centimeters of mercury when stop watch started.

P_2, pressure in centimeters of mercury when stop watch stopped.

T, absolute temperature of gas in bell jar (centigrade + 273).

t, time in seconds for pressure to change 2 centimeters.

V_1, volume of gas in jar when stop watch started, reduced to pressure of 76 centimeters and temperature 20° C.

V_2, volume of gas in bell jar when stop watch stopped, reduced to pressure of 76 centimeters and temperature 20° C.

K_1 and K_2, constants.

Then by the gas law

$$P_1 V = K_1 T \qquad (1)$$

$$P_2 V = K_2 T \qquad (2)$$

Reducing to a pressure of 76 centimeters and temperature 20° C. (293° absolute) and substituting from (1) and (2).

$$V_1 = \frac{(293)}{(76)} \frac{P_1 V}{T} \qquad (3)$$

$$V_2 = \frac{(293)}{(76)} \frac{P_2 V}{T} \qquad (4)$$

Substracting (3) from (4), dividing by $\frac{t}{60}$, and remembering that in the experiment $P_2 - P_1$ is always 2 centimeters.

$$\text{Liters/min.} = \frac{60(V_2 - V_1)}{t} = \frac{(2)(60)(293)\,V}{(76)\,Tt} \qquad (5)$$

$$= \frac{463\,V}{tT}$$

If $T = 293$

$$\text{Liters/min.} = \frac{(2)(60)\,V}{(76)t} \quad \frac{1.58\,V}{t}$$

THE RESULTS OF TESTS.

The results of tests by the Bureau of Standards on representative instruments of the American-made Dreyer, the Clark, and the Prouty types and tests on the single available specimens of the French Dreyer, Munerelle, Garsaux, and the Gibbs instruments are shown graphically in figures 21–28, inclusive. Tests were carried out in conformity with the procedure described in the preceding section. Rates of delivery of oxygen by the apparatus when subjected to external pressures corresponding to altitudes up to 30,000 feet are given. The tank pressures indicated in parenthesis in the legends on the plots are in pounds per square inch or kilograms per square centimeter. The altitudes were computed from the pressure in the bell jar in accordance with pressure-altitude tables prepared by the Bureau of Standards.[3] The rates of delivery are given in liters/min. (reduced to 20° C. and 760 millimeters pressure) at the temperatures, tank pressures, and passenger adjustments indicated by the legends. The correct deliveries for one and two passengers are those prescribed by the United States Bureau of Aircraft Production specifications. For one passenger they are computed from the relation $V=4\left(1-\frac{P}{760}\right)$ where V is the delivery in liters/min. and P is the atmospheric pressure in millimeters of mercury. The amount for two passengers is only 60 per cent greater than that for one passenger at corresponding altitudes. This is because it is assumed that both aviators are in general not likely to inhale simultaneously. Duplicate runs were made in most cases to indicate the degree of reproducibility of performance.

The effect of temperature is indicated by a comparison of the curves at room temperature and at low temperature. On the average the instruments were found to deliver approximately 20 per cent less at −30° C. than at room temperature.

The effect of low-tank pressure is shown by comparing the room-temperature observations at high and low-tank pressure. This in general caused a decrease of delivery of approximately 15 per cent when the tank pressure was reduced to 100 pounds per square inch.

The static pressure on the low-pressure side of the reducing valve of the Dreyer-type instruments was measured when no oxygen was flowing by the use of a Bourdon pressure gage as a test of the adjustment of the reducing valve. This pressure varied from 4 pounds per square inch to 12 pounds per square inch in excess of atmospheric pressure. On the average it was 8 pounds per square inch.

The sensitiveness of the flow indicator of instruments of the Dreyer type was found in terms of the altitude at which the indicator started to revolve with decrease of pressure (ascent) and stopped revolving on increase of pressure (descent). On the average the indicator started at 5,000 feet and stopped at 3,000 feet.

Tests for leaks were made with a smoldering wick.

It will be noted that the two-passenger adjustment of the Dreyer-type instruments conforms much more nearly to the correct flow than the adjustment for one passenger. This is because the groove in the aneroid piston valve was cut to give correct delivery for two passengers. Under these circumstances the delivery for one passenger is larger than is necessary at low altitudes. This does not harm the aviator, but wastes a little oxygen. In the Van Sicklen type of instrument as the plot shows, this difficulty is not encountered, since the delivery is controlled by regulating the pressure of the oxygen in the reducing chamber instead of varying the size of the delivery orifice, as is done in the Dreyer type by the action of the groove in the aneroid valve. The results on the Garsaux instrument indicate that the valve was poorly adjusted. At approximately 10,000 feet it opened suddenly to the maximum rate of delivery. There was actually a slight decrease in delivery above 15,000 feet. The results at −40° C. show that the cut-off valve failed to function. The curves obtained for the Gibbs instrument also show a decrease in delivery above 20,000 feet. This is due to faulty adjustment of the ports of the delivery valve. It will be also noted that in this case the delivery at low temperatures was practically the same as that at room temperature, actually slightly greater at −29° C.

[3] Bureau of Standards Aeronautic Instruments Circular No. 3.

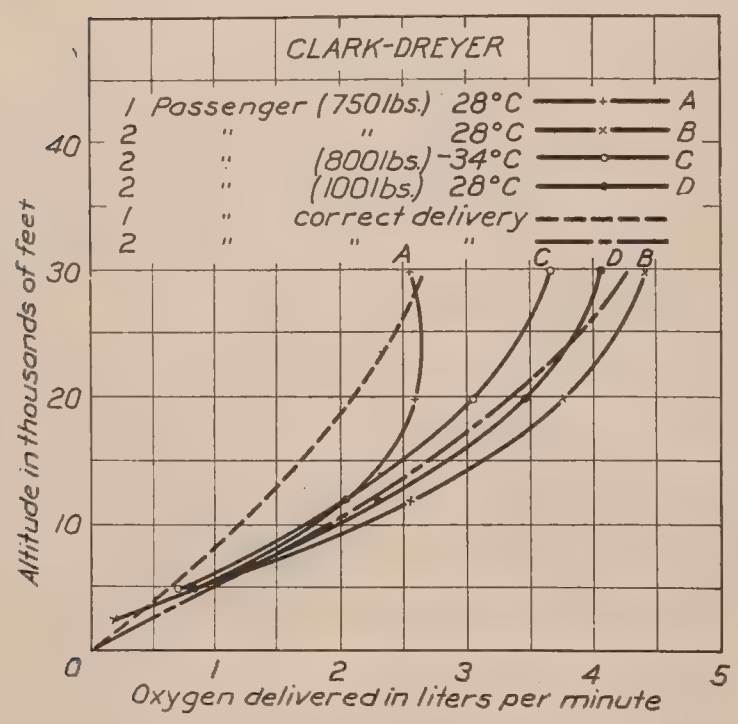

FIG. 21.—Clark-Dreyer Type.

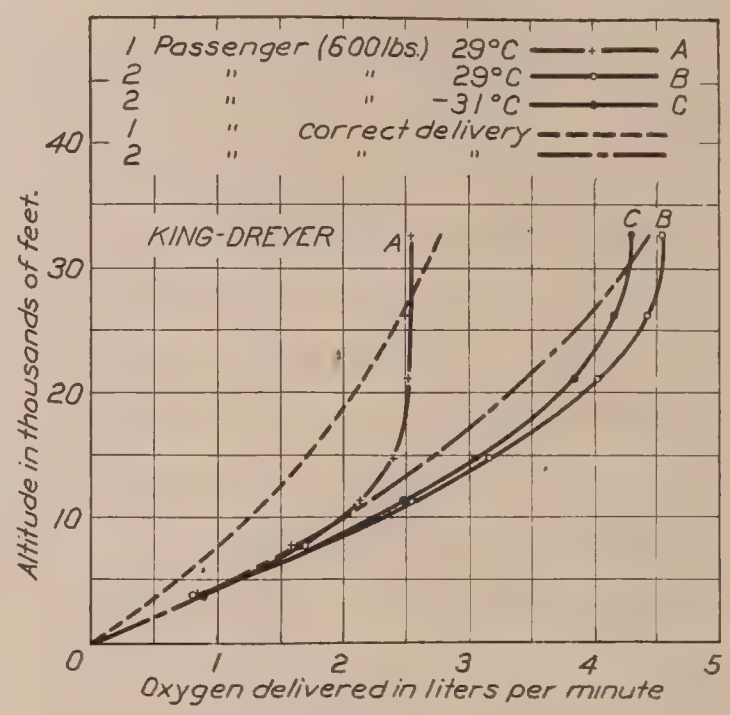

FIG. 22.—King-Dreyer Type.

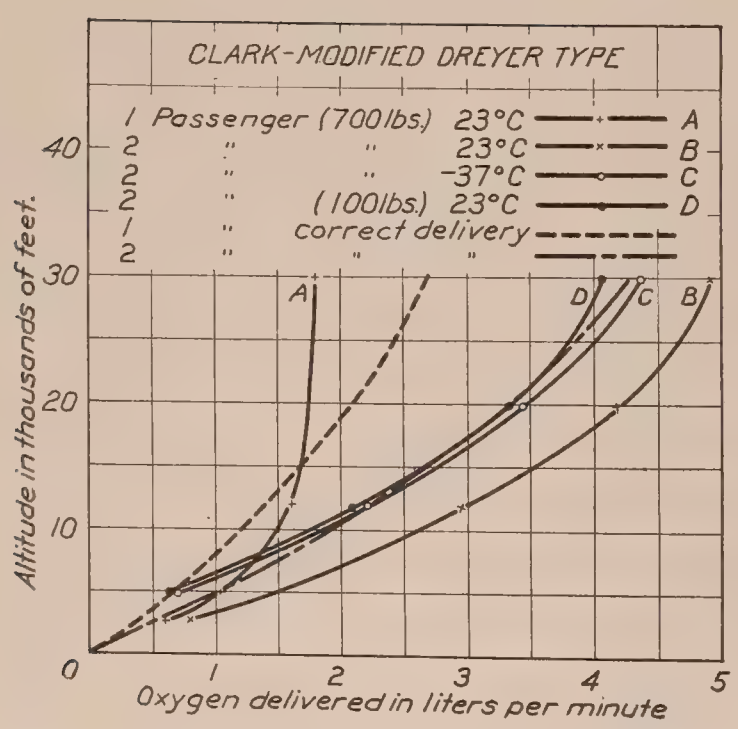

FIG. 23.—Clark modified Dreyer Type.

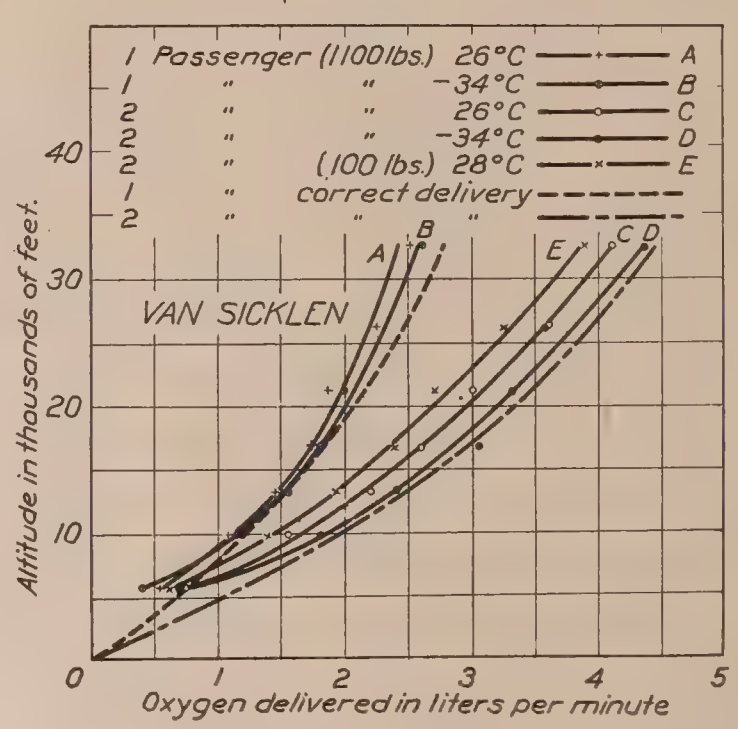

FIG. 24.—Van Sicklen Type.

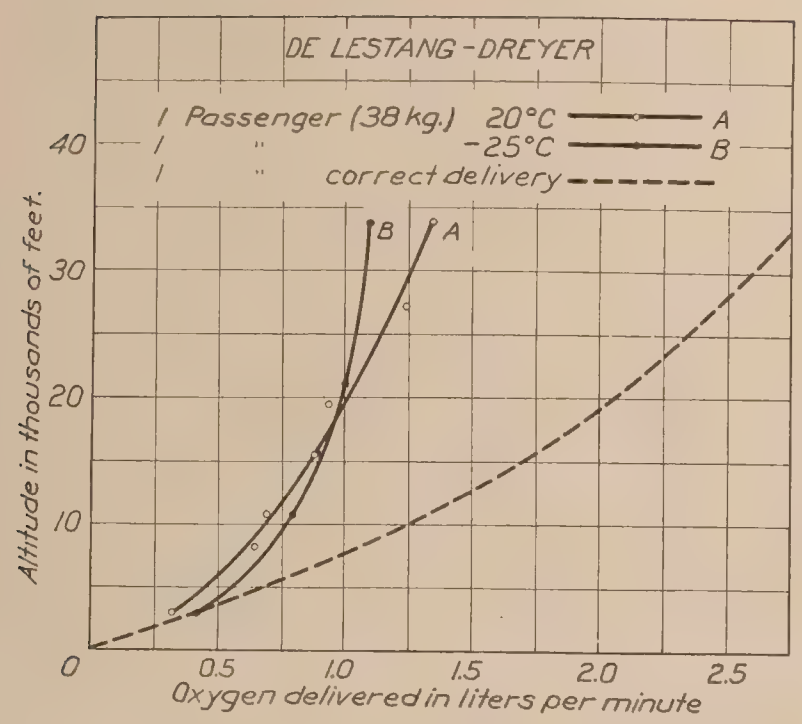

FIG. 25.—De Lestang-Dreyer Type

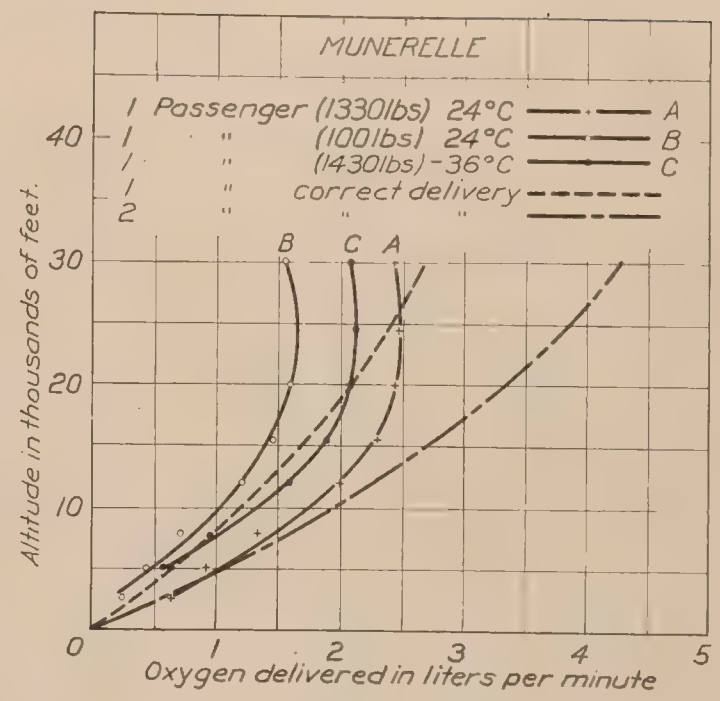

FIG. 26.—Munerelle Type.

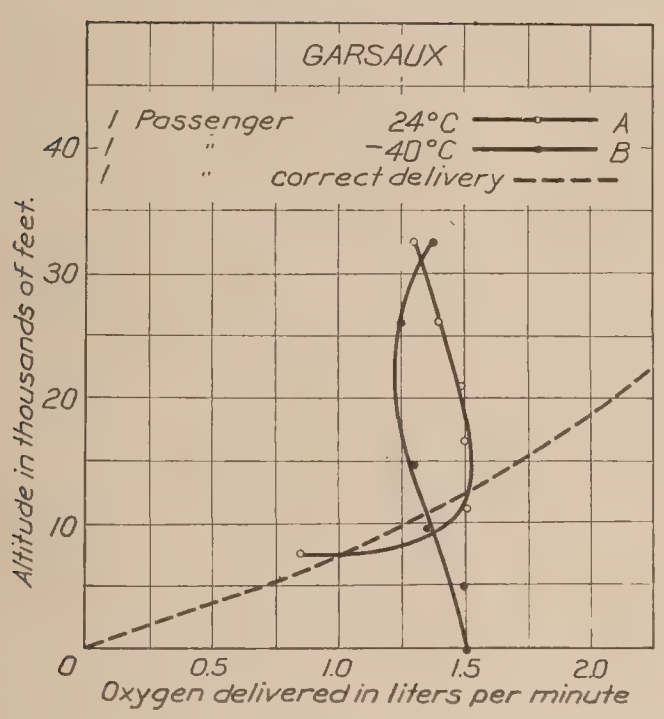

FIG. 27.—Garsaux Type.

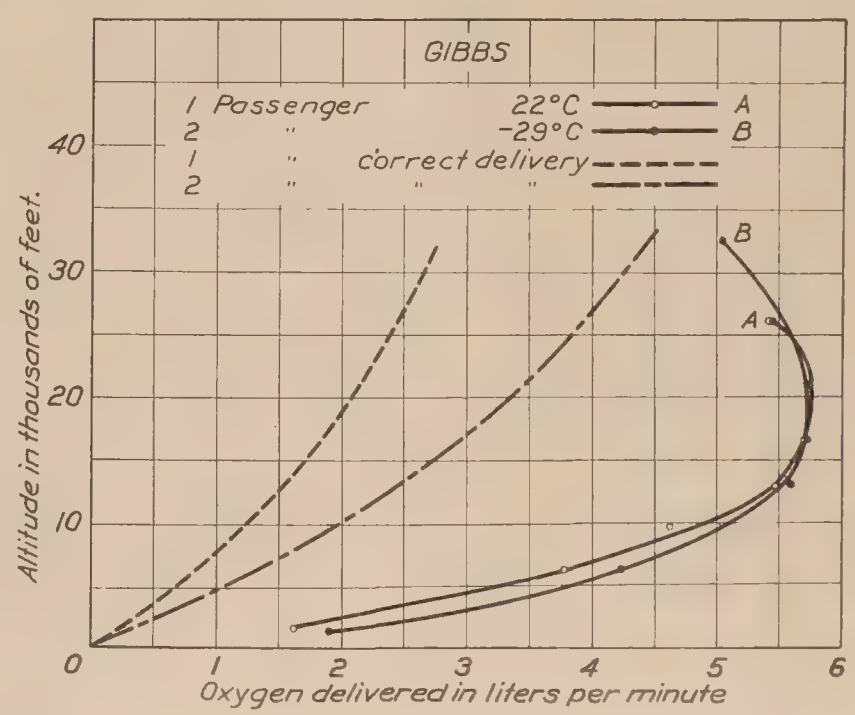

FIG. 28.—Gibbs Type.

REPORT No. 131

AERONAUTIC INSTRUMENTS
SECTION VII

AERIAL NAVIGATION AND NAVIGATING INSTRUMENTS

By H. N. EATON
Bureau of Standards

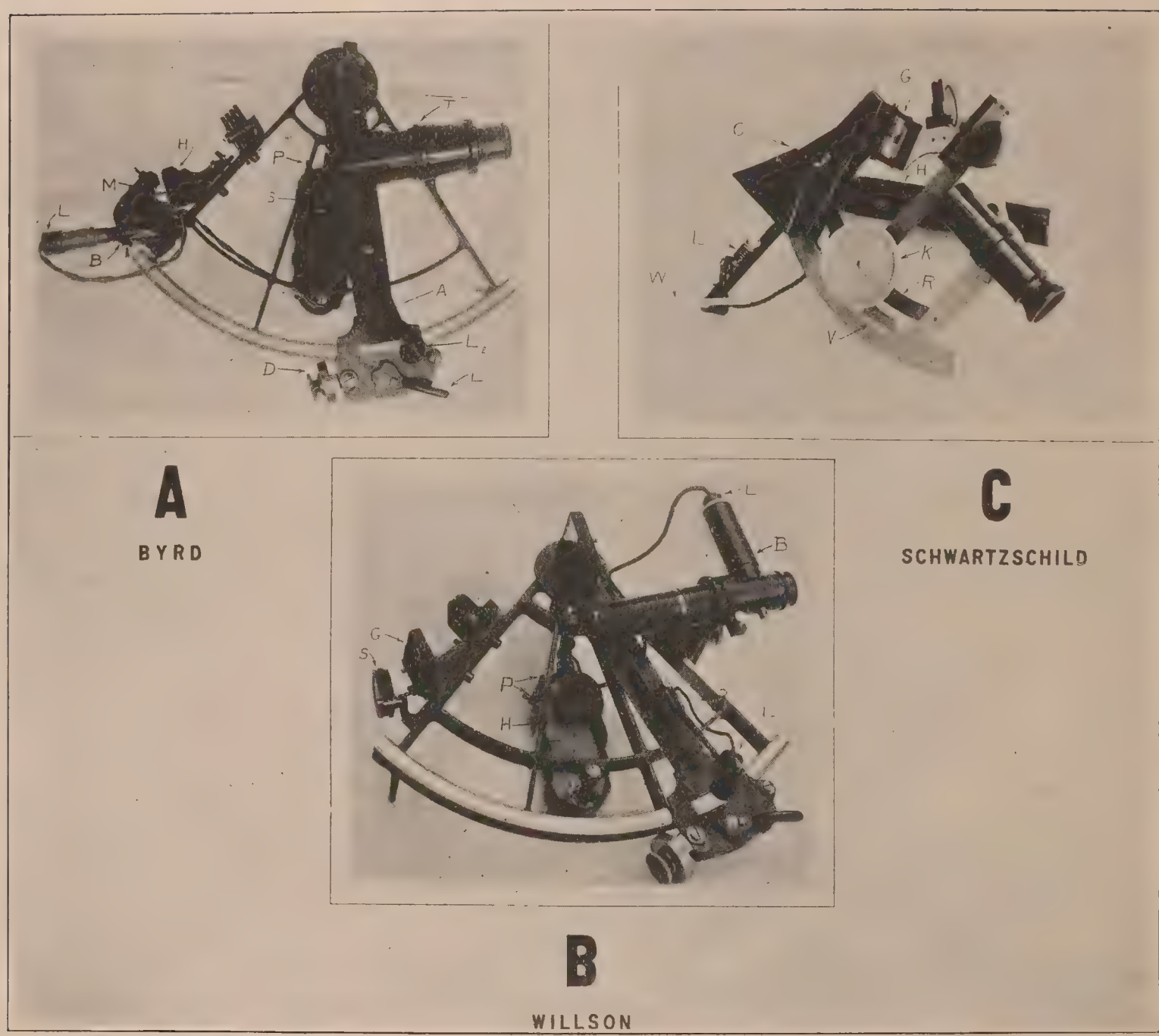

FIG. 16.—Bubble sextants.

REPORT No. 131.

AERIAL NAVIGATION AND NAVIGATING INSTRUMENTS.

By H. N. Eaton.

SUMMARY.

This report is Section VII of a series of reports on aeronautical instruments (Technical Reports Nos. 125 to 132, inclusive) prepared by the Aeronautic Instruments Section of the Bureau of Standards under research authorizations formulated and recommended by the Subcommittee on Aerodynamics and approved by the National Advisory Committee for Aeronautics. Much of the material contained in this report was made available through the cooperation of the War and Navy Departments.

The methods of marine and aerial navigation are fundamentally the same and the development of the latter has involved no new general principles. Differences in the conditions encountered on the ocean and in the air and the widely different characteristics of the craft involved in the two cases have necessitated the modification of the methods and instruments used in marine navigation, as well as the development of special instruments, to meet the requirements of aeronautics.

This paper outlines briefly the methods of aerial navigation which have been evolved during the past few years and describes the instruments used.

Dead reckoning, the most universal method of navigation, is first discussed. Then follows an outline of the principles of navigation by astronomical observation; a discussion of the practical use of natural horizons, such as sea, land, cloud, and haze, in making sextant observations; the use of artificial horizons, including the bubble, pendulum, and gyroscopic types; and a description of the rapid methods and the special instruments which have been devised to expedite the reduction of observations and the plotting of position. Lastly, the comparatively recent development of the radio direction finder and its application to navigation are discussed.

INTRODUCTION.

Until the beginning of the recent European war, aerial navigation in a scientific sense had received comparatively little attention. A limited amount of work along these lines had been done, but comprehensive methods for overcoming the difficulties peculiar to the navigation of aircraft had not been developed.

Perhaps the most conspicuous example of the use of this new art prior to 1914 is to be found in the celebrated attempt of Wellman and Vaniman to cross the Atlantic Ocean with the dirigible *America* in 1910. On this voyage a sextant with an artificial horizon was used for the absolute determination of position, while methods of measuring the air speed, ground speed, compass course, and angle of drift provided the necessary data for computing the dead reckoning position of the ship.[1] The methods of navigation used on this flight were very similar to those which are coming into use to-day. The most noticeable difference was the absence of the radio compass and the turn indicator.

During the war the art of navigating aircraft was developed somewhat, principally owing to the attempts at long-distance bombing. The British found it possible, after considerable

[1] "The Aerial Age," Wellman.

work at Oxfordness, to arrive within 5 or 6 miles of their objective after flying above the clouds for approximately 100 miles. The Germans in their Zeppelin raids on England were but partly successful in their navigation, for they managed to reach their objective yet often failed to return to their base, owing to the almost insuperable difficulties encountered in flying long distances at night with uncertain information as to weather conditions.

The war demonstrated the value of aerial supremacy beyond all question. The development of commercial aviation is the most promising means by which the heavy cost of aeronautical progress can be at least partially defrayed. Commercial aviation in turn depends to a great extent upon the solution of the difficulties encountered in aerial navigation. Not until aircraft are independent of all but the most extreme weather conditions, not until they can be flown for hundreds of miles without sight of the ground and still come out within 15 or 20 minutes flying time of their objective, not until forced landings in fog cease to be a source of serious danger will this means of transportation be suitable for commercial use.

Unfortunately, only a beginning has been made toward solving the problems of flying and landing in fog. The turn indicator has relieved cloud flying of its greatest danger, but landing from an uncertain altitude on a spot which is invisible from the air is still a dangerous undertaking. In all probability the solution will be found not only in additional instrumental aids to navigation but also in the utilization of information transmitted by wireless from a well-organized and widely distributed meteorological service provided for the assistance of the aerial fleets.

AERIAL NAVIGATION METHODS.

The reason that present-day methods of aerial navigation were so well established during the earliest days of aeronautics is probably due to the close analogy between aerial and marine navigation. A ship travels through the water on a given course at a known speed, while at the same time the ocean currents and the wind carry it from its course by an amount which is usually quite accurately known also. The position of the ship at any instant can be computed with a considerable degree of accuracy by dead reckoning and checked occasionally by astronomical observation. The aircraft flies at a known speed on a given course through the air. At the same time the winds sweep it from the course steered by an amount which, under favorable conditions, is known. The aviator can thus determine his position by dead reckoning and occasionally check it by observation on the sun or a star just as is done in marine navigation.

Here the analogy ends, and in the remaining considerations the disadvantage lies almost wholly with the aircraft. In spite of certain advantages which are unique to flight, the higher rates of speed of the craft and of the supporting medium, the impossibility of charting the winds as the ocean currents are charted, the dangers of landing in fog or darkness, and the added difficulties due to the fact that we have three dimensions involved instead of two, all tend to make the navigation of aircraft more difficult and hazardous than navigation on the sea. This entails the development of new methods and instruments for aerial use as well as the modification of those already in existence.

The methods of aerial navigation which stand out prominently are three in number: "Dead reckoning," "astronomical observation," and "directional wireless telegraphy," if we omit (1) the common practice of flying by map, consisting simply of steering one's course by means of objects on the ground which can be identified on the map, and (2) the possible use of a gyroscopic device for indicating latitude and longitude. Several attempts have been made to construct such an instrument, but it is extremely doubtful whether in the present state of development of gyroscopic apparatus sufficient accuracy can be attained in this way.

No one method or combination of methods has been found preeminently suitable for exclusive use under all conditions. What might be well suited for a transatlantic flight would not be adapted to use on a cross-country trip of a few hundred miles. Lighter-than-air and heavier-than-air craft each have their own characteristics which involve different variations of the fundamental methods of navigation.

DEAD RECKONING.

Of the three fundamental methods of navigation, that by dead reckoning is unquestionably the most universally used. It is always required in some form, however simple, even when reliance is placed on one or both of the other methods. Although dead reckoning may be partly superseded by the radio direction finder, it can never be dispensed with entirely.

Navigation by dead reckoning consists in determining the position and course of any craft from its known or estimated direction and speed over the surface of the earth. In aerial navigation the direction in which the craft is heading and its speed relative to the air can be found with comparative ease, but unfortunately for simplicity, the winds are usually present to complicate the problem. In nearly every case this means that both the speed and the direction of motion with respect to the ground differ from that shown by the air-speed indicator and the compass. If the direction and the speed of the wind are known, the problem still remains simple, involving merely the solution of a velocity triangle; but the wind is rarely known accurately at the start of a flight and only too often no information is available. In this case the navigator is forced to rely on observations which he can make while in the air to give him a knowledge of the drift caused by the wind.

In time a more intimate knowledge of the atmosphere and a well-organized meteorological service will undoubtedly serve to give aviators more complete and accurate data to aid them in their navigation, but even then it will still be necessary to check the course actually followed by means of drift and ground speed observations.

From what has been said, it can be seen that the measurement of four different quantities—heading, air speed, ground speed, and drift—may be involved in navigation by dead reckoning. For the purposes of this paper the instruments used in measuring these quantities and in reducing them to useful form may be classified as (*a*) Direction Indicators; (*b*) Speed and Drift Indicators; and (*c*) Computers and Plotting Accessories.

USE OF DIRECTION INSTRUMENTS.

COMPASSES.

The determination of direction is the most fundamental process in navigation. A knowledge of his speed may not be absolutely essential to the navigator, but without the sense of direction he may be completely lost. Aside from the limited possibilities of directing one's course by map, by rough measurement of angles from the sun or stars, or by the aid of the radio direction finder, the compass is an absolute necessity to navigation, whether aerial or marine.

Owing to its supreme importance, this instrument had already been brought to a high state of development for marine use before the days of modern aviation. Its first applications to aerial use, however, showed that an entirely new development was needed before it would function properly in aircraft. It was not simply a question of reducing the size and weight of the instrument but of altering the design radically, in order to make the compass function properly when subjected to the severe conditions found in aviation.

It had already been learned that the dry compass ordinarily used on shipboard was unsuitable for small boats where the rolling and pitching in bad weather and the engine vibrations were very pronounced. Under these conditions the compass card was found to lose its stability and to oscillate in both horizontal and vertical planes, the amplitude often increasing until continuous rotation about the vertical axis resulted. Furthermore, the vibration damaged the pivot point, and an increase in pivot friction resulted which made the error of the instrument still greater. These considerations led to the adoption of the liquid compass for small boats. The marked similarity between the conditions encountered here and those met with in the air explain the almost universal adoption of this type of compass for aerial work.

The liquid in the bowl serves a double purpose: It damps the motion of the card, reducing the oscillations and thus improving the stability; and it removes a portion of the weight on the pivot point and so tends to preserve the latter, owing to the reduction of the intensity of the shocks given it. The decrease in the apparent weight of the card is also beneficial in that it reduces the pivot friction.

Inseparable from the advantages due to the liquid are several disadvantages. The liquid serves as a damping medium, but it also forms a viscous connection between the bowl and the card, and when the former turns the latter is dragged around, too. The same property of the liquid makes the compass sluggish, with the result that it responds but slowly to changes of direction.

In adapting the liquid compass to aerial use a number of modifications were made, the most important of which was probably the adoption of a short period. The period of the marine compass is in the neighborhood of 50 seconds, and experience in the air brought protests from the pilots that a compass having so great a period took too long to come to rest after a turn. Consequently compasses having a period of from 10 to 20 seconds were adopted. Other changes included the use of stronger magnetic moments in an attempt to overcome the sluggishness produced by the liquid and the attaching of the pivot to the card instead of the bowl in order to reduce the turning effect due to vibration.

During the early days of its use the peculiar action of the short-period compass caused the instrument to fall into discredit. It was found that on banked turns from northerly courses the compass swung wildly and often indicated a turn in the wrong direction. A study of this "northerly-turning error," as it was called, led to the temporary adoption of a long-period compass in an effort to reduce the effect.

Errors of compasses.—The compass, as used in aerial navigation, is subject to errors due to (*a*) calibration, (*b*) pivot friction, (*c*) permanent magnetism of the craft, (*d*) induced magnetism, (*e*) vibration, (*f*) tilting of the card, (*g*) linear acceleration of the craft, and (*h*) turning of the craft.

(*a*) and (*b*) The sum of the calibration and pivot friction errors should not be greater than about 2° in a good compass, such as is needed for navigation. The friction error can be reduced to a very small amount by keeping down the weight of the card. These errors are discussed in Part III of Report No. 128 of this series.

(*c*) The deviation due to permanent magnetism is caused by the fact that the steel used in the construction of the airplane acquires a certain amount of permanent magnetism due to vibration and shocks. The field thus set up lies in a general way along the longitudinal axis of the ship. The deviation caused by this field changes sign when the aircraft changes its azimuth by 180°; hence it is called the "semicircular deviation." The permanent magnetism thus produced is not constant over an extended period owing to the vibration and shocks incidental to the use of the ship.

(*d*) The earth's field induces magnetism in the soft iron of the craft, producing a field which varies in direction as the azimuth of the airplane is altered. This field causes a deviation which changes in sign for each change in azimuth of 90°, so the name "quadrantal deviation" is applied to it.

The deviations due to the causes just discussed can be largely eliminated by "swinging the airplane" and compensating the compass.[2] The semicircular deviation, the more important of the two, is corrected by heading the craft successively north, east, south, and west and reducing the deviations found in this way to the smallest possible amounts by the use of small compensating magnets. The quadrantal deviations are corrected by heading the airplane successively northeast, southeast, southwest, and northwest and reducing the deviations found in these directions to a minimum by means of two soft iron spheres properly placed. After this process has been completed the airplane is "swung" again and the remaining deviations are found for the points of the compass. A table of these deviations can then be prepared for use when flying. Owing to the change in the permanent magnetism of the ship, the semicircular deviation should be compensated at frequent intervals.

The errors which have been considered above can be determined fairly well and corrected under ordinary conditions. On short flights or on flights over territory which is known and visible they are not very important, as the course can be corrected at frequent intervals. On

[1] "Aerial Navigation," by J. E. Dumbleton.

long flights and particularly on transoceanic flights the accuracy of the compass is of supreme importance, however. A glance at the map of the Atlantic Ocean will emphasize at once the accurate navigation which would be required to enable a craft flying, say, to the Azores to attain its objective. An error of 2° in following the true course might cause it to miss its destination with the chances greatly in favor of its continuing on into another vast expanse of ocean.

The errors which remain to be discussed can not be accurately determined and compensated as can the others, although they can often be reduced in amount. The error caused by the electrical circuits in the engine is a good example of this. This disturbance may be serious if the compass is near the engine, but it may be overcome by removing the compass from the field thus set up.

(*e*) Rotary vibration in a horizontal plane has a very serious effect upon compasses in which the pivot is fastened to the bowl and the cup is fastened to the card. Under the influence of such vibrations the pivot will carry the card around in the direction of the vibration. In tests carried on at the Royal Aircraft Factory with a British Admiralty standard compass,[3] deviations due to this cause were found to range from 7° at 1,400 R. P. M. to 42° at 1,600 R. P. M. Reversing the positions of the pivot and cup reduced this deviation to 1° at 1,100 R. P. M. and 4° at 1,600 R. P. M.

The suspension of the bowl was also carefully investigated at the same time. It was concluded after extensive tests that rubber stops, unless they are too loose to maintain the proper direction of the bowl with respect to the case, do not damp out excessive vibrations. Spring mountings were also found unsatisfactory owing to resonance at certain speeds. The method finally adopted was to support the bowl on a polished plate with a felt pad between the two. Flat leaf springs mounted on the case supported the bowl and maintained its direction but allowed it to slide on the felt.

When a compass with the standard arrangement of pivot and cup and with this mounting was tested in an airplane on the ground with the engine running, the deviation due to vibration was reduced to a maximum of 2° at 1,600 R. P. M. The results obtained improve rapidly as the distance of the compass from the engine increases. This is due both to a reduction of vibration and to a lessening of the effect of the engine circuits.

(*f*) If the axis about which the card rotates is inclined to the vertical while the craft is flying a straight course, a large error in the compass reading may be introduced. This is due to the vertical component of the earth's magnetic field, which then tends to rotate the card about its axis.

(*g*) Linear accelerations of the aircraft while flying a straight course other than magnetic north or south produce an error in the compass reading. Owing to the mass placed on the south side of the card to balance the force due to the vertical component of the earth's magnetic field, the center of gravity of the needle does not lie in the vertical line through the point of support. A linear acceleration produces a moment acting about the point of support to deflect the needle.

(*h*) Perhaps the most difficult error to contend with and the one that has had the most serious consequences is the "northerly turning error," which occurs when the aircraft veers from a northerly course. The short-period compass under such conditions deviates widely from the correct reading, and, what is worse, often indicates a turn in the opposite direction to that which is actually being made. When this occurs in a cloud or in a fog, the results are likely to be disastrous, for the pilot loses all sense of the horizontal under these circumstances and must depend on his instruments to keep the ship in equilibrium.

It is possible to derive the equation of motion for the compass card for specified turns from a given course.[4] In its general form this equation can not be integrated, but, by keeping the values of the variables sufficiently small, approximations can be made which allow the integration to be performed. Thus it is possible to investigate the initial motion of the compass

[3] "Report on the Errors of Compasses on Aeroplanes," by Keith Lucas, British Advisory Committee for Aeronautics, vol. 7, 1915–16, p. 554.

[4] Report of the British Advisory Committee for Aeronautics, "The Northerly Turning Error of the Magnetic Compass," F. A. Lindeman, 1916–17, pp. 678–698.

in a turn from the north, for example, and a large turn can be investigated by a step-by-step process, keeping the variables small for each step.

It is usually assumed that the liquid does not rotate with the bowl during a turn. This of course, is not true, even with a spherical bowl such as is used in the R. A. F. Mark II compass. The condition for rotation of the liquid with the bowl can be put into the equation of motion and its effect thus determined. The true state of affairs must lie between these two extremes, and this has been corroborated by actual tests.

The cause of the "northerly turning error" is discussed in Part III of Report No. 128 of this series.

The development of the turn indicator has made it possible to neglect the compass entirely during the turn. The latter can now be utilized in its proper rôle, that of giving the direction of a straight course, while the former can be used exclusively for turns.

A detailed description of the different compasses in aerial use will be found in Part IV of Report No. 128 of this series.

TURN INDICATORS.

While aerial navigation in a strict sense may consist simply of the geometrical problems involved in flying from one point to another, in a broader sense it may be considered to include other problems which are essential to carrying out the flight from a practical standpoint, such as the determination of the radius of action or the ability to fly through fog. Commercial aviation will never be a success, however accurately courses and distances can be determined, so long as the craft is likely to fall out of a cloud in a tail spin, owing to the inability of the pilot to keep his equilibrium.

Before the turn indicator was developed it was an all too common experience for the pilot upon entering a cloud to notice the compass needle beginning to swing. This meant that the airplane was turning and he would immediately try to correct the course. With all sense of the horizontal lost, all he could do, however, was to adapt his bank to the turn, since the usual type of banking indicator gives only the apparent vertical. So he would get onto a sharper and sharper turn, particularly if turning from the north with a short-period compass. The airspeed would now creep up and, in correcting this, he would stall the machine, which would then fall out of control until a view of the ground enabled him to regain his balance and level off or until he crashed. A conspicuous example of such an occurrence is to be found in Sir Arthur Whitten Brown's "Flying the Atlantic in Sixteen Hours," page 59.

With a turn indicator in front of him, the pilot can recognize a turn as soon as it commences and can correct it. The banking indicator will then give the true vertical and the ship can be kept on an even keel and flown in a straight line.

Turn indicators are described in Part V of Report No. 128 of this series, so brief reference only will be made to them here.

The static head turn indicator.—This turn indicator consists of two static pressure nozzles mounted one at each end of the wings of the airplane and connected to opposite sides of the diaphragm of a very sensitive pressure gage. When the airplane banks for a turn, the two static nozzles are at different elevations, and consequently are at points of different static pressure. Without discussing the other forces acting upon the pressure indicator, since the subject is thoroughly treated elsewhere in this report, it will be said that the resultant force upon the diaphragm may be considered due to the difference in barometric pressure between the levels of the static nozzles or due to the centrifugal force of the air in the tubes connecting the indicator to the static nozzles. The differential pressure acting upon the indicator is thus a measure of the rate at which the aircraft is turning. The Darwin static head turn indicator is the only instrument of this type in common use.

The gyroscopic turn indicator.—The operation of this type of turn indicator is based on the action of a gyroscope. If the axis of spin is fore and aft, the precession then takes place about an axis parallel to the lateral axis of the ship. Springs limit the precession to a small amount and the deflection is magnified and read on a dial.

A number of gyroscopic turn indicators have been developed. In most cases the gyro wheel is in the form of an air turbine which is driven either by the blast of the air stream or by the suction developed by a Venturi tube mounted on a wing. In some cases an electrical drive is used.

Errors of turn indicators.—The static pressure turn indicator works well under favorable conditions if properly mounted. It has, however, several serious defects. (*a*) The instrument is said to read lateral accelerations almost independently of the bank. Thus it is liable to be in error whenever side-slipping occurs. (*b*) Obviously the differential pressure involved is extremely small. This makes a very delicate indicator necessary and increases the liability to error due to local pressure disturbances. (*c*) It has an excessive lag. (*d*) Under bad weather conditions the nozzles may freeze up solid and the instrument will cease to function.

The best gyro turn indicators with an adjustment for sensitivity are unquestionably more satisfactory than the static pressure type of instrument. They are practically instantaneous in action, can be adjusted to the desired sensitivity, start functioning again immediately after the plane has been stunted, and can be made strong and rugged, owing to the large forces involved.

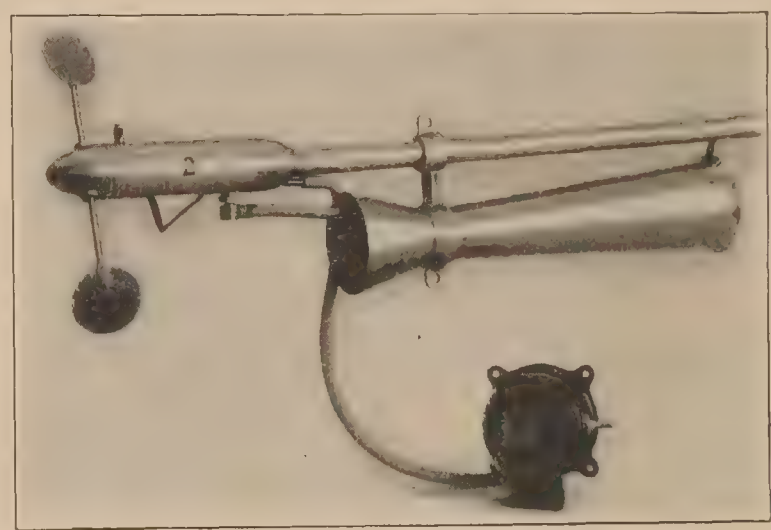

FIG. 1.—Pioneer air-distance recorder.

USE OF SPEED INSTRUMENTS.

Under this heading are included air-speed indicators, ground-speed indicators, and drift indicators.

AIR-SPEED INDICATORS.

Little will be said of air-speed indicators in this paper. The instruments are well known, forming a standard part of the equipment of all aircraft, and have been treated thoroughly in other parts of this report. The reader is referred to Parts I and II of Report No. 127 of this series, also National Advisory Committee for Aeronautics Report No. 110, "The Altitude Effect on Air-speed Indicators," by M. D. Hersey, F. L. Hunt, and H. N. Eaton.

PIONEER AIR-DISTANCE RECORDER.

This instrument (fig. 1) is designed to record the air distance traveled. A small propeller is mounted on one of the forward wing struts. This propeller makes a definite number of revolutions for each mile of air which passes it, and at the end of each mile operates a valve which admits the suction created by a Venturi tube to the recorder, thus operating pneumatically the mechanism which adds 1 mile to the reading. Since the propeller has very little resistance to its motion, the apparatus is very nearly independent of the density of the air. The makers claim that the maximum error of the instrument due to the change in air density is less than 1 per cent at 20,000 feet.

GROUND-SPEED INDICATORS.

The two principal types of ground-speed indicators which have been developed for use on aircraft are based upon optical and dynamical principles. The optical type is used to follow and measure the relative motion of objects upon the ground with respect to the aircraft and so is dependent upon the visibility of the ground from the craft. The dynamical type integrates the accelerations of the aircraft, and consequently is independent of visibility. A discussion of the principles upon which the various types of ground-speed indicators are based is to be found in Part III of Report 127 of this series.

While the measurement of ground speed is a distinct operation, practically it is often convenient to combine the optical ground-speed indicator, the most commonly used type, with a drift indicator owing to the fact that in this type of ground-speed indicator the angle of drift is found, whether or not it is read. On this account the description of drift indicators which follows will include descriptions of several simple ground-speed indicators of the optical type.

THEORY OF DRIFT MEASUREMENT.

Just as in marine navigation it is necessary to take into account the speed and direction of ocean currents in order to follow a given course, so in aerial navigation the motion of the supporting medium relative to the earth must be considered. And in the latter case the difficulties involved in taking full account of the motion of the medium are immensely greater. Not only are the velocities of the winds many times those of the ocean currents, but they are changeable, varying from day to day, and even from hour to hour. Unlike ships, which are limited to motion upon the surface of the sea, aircraft often fly at elevations differing by several miles, and it is a well-known fact that the winds usually vary both in amount and direction with altitude. It is probable that meteorological information will in time reach such a state that the conditions which the aviator will be likely to meet in a given flight can be predicted quite accurately in advance, but it will never be possible to chart the winds as the ocean currents are charted.

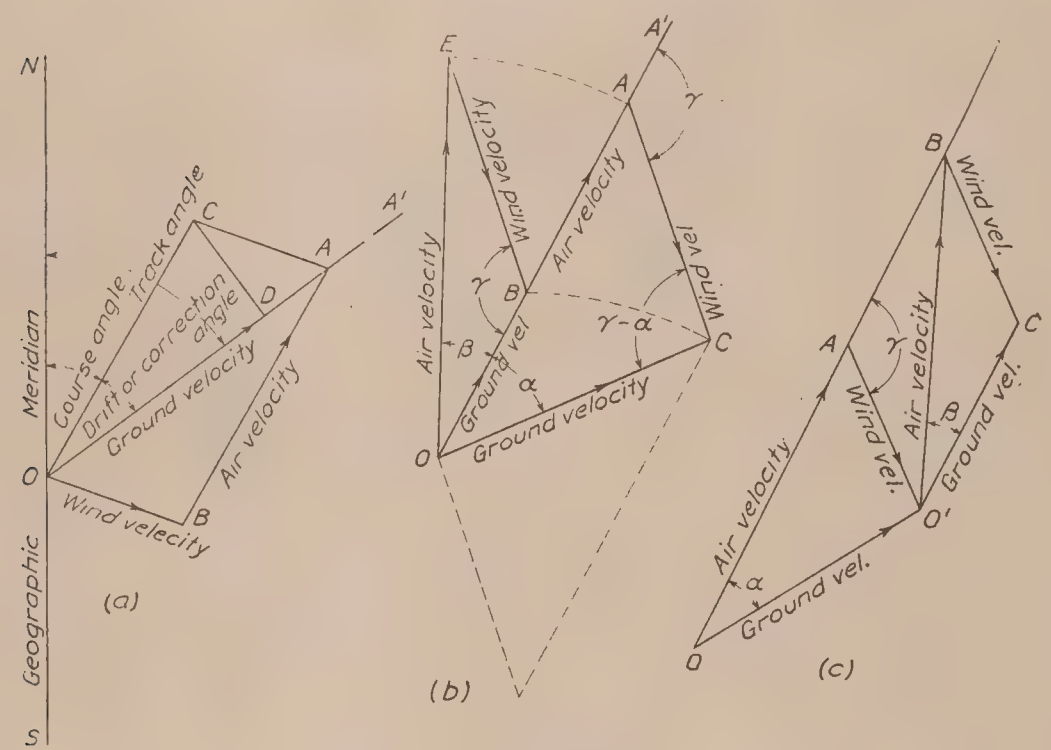

Fig. 2.—Drift diagrams.

The value of meteorological data for commercial aviation can hardly be overestimated. By choosing his altitude properly, the aviator can often obtain a favorable wind in both going to and returning from his objective. The lack of such information may often add greatly to the time of his flight and, if visibility is poor, he may be swept miles from his course. A historic instance of such an occurrence is that of a fleet of Zeppelins which, after a raid on England on October 19, 1917, ascended into an air current very different from that existing at the lower levels. The crews of the dirigibles, not knowing of this change in the wind, made a wrong allowance for its effect, with the result that the ships were swept over France, where a number of them were destroyed.[5]

The fact that the wind usually alters the compass course which should be steered to reach a definite point on the surface of the earth makes it necessary to determine the angle by which the heading of the plane must be altered in order to follow a particular course over the ground. The fundamental problem involved in all cases is the determination of enough quantities to solve the velocity triangle whose sides represent in amount and direction the air speed, wind speed, and ground speed.

In figure 2(*a*) the triangle (OAC) represents the velocities involved. Suppose the aircraft starts from (O) to reach a point (A′). The angle (NOA′) which the course to be followed makes with the geographic meridian can be determined from the map. This will be called the "track angle" and the course (OA) which the aircraft actually follows over the ground will be called the "track," or the "course-made-good," in accordance with customary usage. The wind is

[5] Manual of Meteorology, Part IV, page 73. "The Relation of Wind to the Distribution of Barometric Pressure," by Sir Napier Shaw.

represented in amount and direction by the vector (OB). Let us assume that (OB) represents the distance which the wind moves in unit time. Now, if we take (B) as center and with a radius equal to the distance which the craft moves through the air in unit time swing an arc cutting the straight line from (O) to (A′), we obtain point (A) which represents the point which the aircraft can reach in unit time if so headed as actually to fly over the ground from (O) toward (A′) in a straight line. Now, drawing (OC) parallel to (BA) and (CA) parallel to (OB), we have the triangle of velocities which is to be solved. Knowing (OC), (CA), and angle (OAC), it is a simple matter to solve for the distance covered with respect to the ground in unit time and for the drift or correction angle (AOC). (See next paragraph.) The angle between the course to be steered (OC) and the geographic meridian (SN) is now known; consequently the compass course can be corrected to counteract the drift due to wind. The sides of the velocity triangle thus drawn are proportional to the air speed, wind speed, and ground speed.

It will be advantageous to carry the discussion of drift a little further at this point and to give arbitrary definitions, for the sake of clearness, to the names "drift angle" and "correction angle," both of which will be involved in the following description of drift indicators.

In figure 2(*b*) let (OA′) be the course the pilot wishes to make good and let (O) be his starting point. Also let (OA) represent his air speed and (AC) the wind speed in both direction and amount. The triangle (OAC) can now be drawn, since both (OA) and (AC) are completely known. The ground speed (OC) and the angle (α), which is the angular amount by which the aircraft is deviated from its desired course and which we shall call the "drift angle," are now known. This triangle is often solved in various ways by the navigator while in the air in order to determine the wind or his ground speed. But the pilot desires actually to follow the course (OA′) and to do this he must steer the compass course (OE). For this purpose it is not sufficient for him simply to apply the drift angle (α) to the direction of the course to be made good, but a new triangle must be solved. In other words, he must determine angle (β), which is the true correction to be applied to his compass course in order to overcome the component of motion due to the wind. (β) will be called the "correction angle" and can be found in terms of the angles (α) and (γ), where (γ) is the angle between the desired track and the direction of the wind.

Applying the law of sines to each triangle and remembering that the ratio of the air speed to the wind speed is the same in each, we have

$$\sin \beta = \frac{\sin \alpha \sin \gamma}{\sin (\gamma - \alpha)}.$$

The "drift angle" will thus be defined as the angle by which the craft is deviated from its compass course by the wind, while the name "correction angle" will be applied to a particular drift angle, namely, the drift angle which exists when the craft is following the desired track.

DRIFT INDICATORS.

Drift indicators are a class of instruments whose use is dependent on a view of the ground or the ocean. The usual type is based on the fact that it is possible to tell when points on the ground, floating objects on the surface of water, or patches of foam move parallel to a line which can be rotated in a horizontal plane in the aircraft.

Webster drift sight.—Probably the simplest drift sight in use is that devised by D. L. Webster. Fixed wires radiate at 10 degree intervals from a point so placed that the pilot can sight along them at the ground. By watching the points on the ground as they move by these wires, the pilot can estimate the drift angle to about 2°. A movable sighting wire which can be operated from the cockpit is sometimes used to replace the fixed wires.

Sperry synchronized drift set.—This device (fig. 3) is a combination compass and drift indicator so connected that the observer can move the lubber line on the pilot's compass and thus correct the compass course for drift without having direct communication with the pilot.

The drift indicator consists of a prismatic telescope having a series of parallel lines in the focal plane of the eyepiece. These lines can be turned by a tiller wheel until they are parallel to the apparent motion of objects on the ground. A pointer indicates on a fixed scale the angle which these lines make with the axis of the ship; i. e., the drift angle.

A wire cord is wrapped around the tiller wheel so that as the observer sets the drift lines in the telescope he moves this cord, which is attached to another tiller wheel on the compass. This second tiller wheel turns the movable lubber line, and so the pilot, who merely keeps the ship headed along the indicated compass course, automatically corrects for the drift.

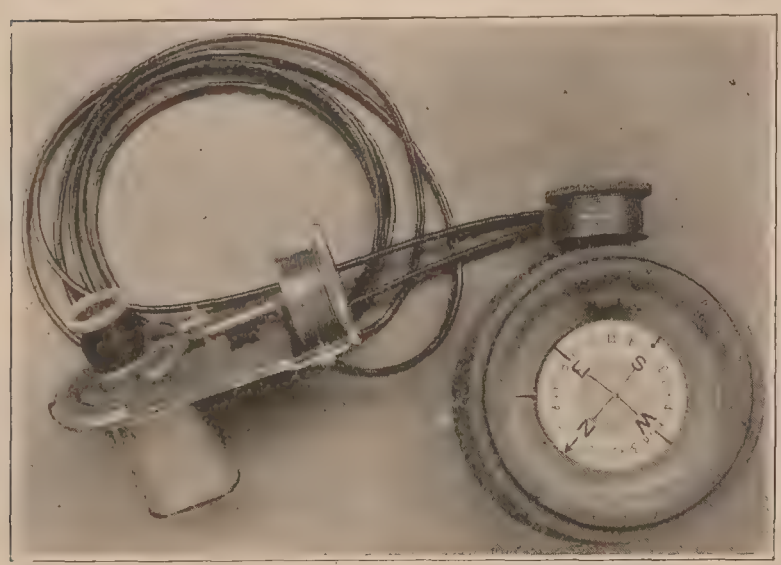

FIG. 3.—Sperry synchronized drift set.

The correction thus made is not exact, since the drift angle, as distinguished from the correction angle, is laid off. Practically the two angles are nearly equal and successive corrections for drift counteract the slight error involved.

Angus compass course indicator.—This instrument (fig. 4), designed by Capt. Angus, R. A. F., is intended to solve the velocity triangle when the air speed, the compass course, and the amount and direction of the wind are known.

A square of transparent celluloid (S) rests on the map, its center at the point of departure. An arrow at the midpoint of one of the sides marks the geographic north. The square is oriented by means of a parallel rule (R), which can be set to coincide with a convenient meridian. At the center of the square of celluloid is a circular celluloid protractor (P), which can be revolved. The points of the compass are engraved on this card. Three graduated arms (T), (A), and (W), which can be suitably adjusted relative to each other, serve to solve the velocity triangle mechanically.

The first of these, a long radial arm (T), called the track scale, is pivoted at the center of the protractor. One edge of this arm, if prolonged, would pass through the pivot (O) and is graduated in miles per hour with the zero point at (O).

Pivoted also at (O) is another arm (A). This has a longitudinal slot, so that it can be slipped by the pivot (O) and clamped there by means of a thumb screw. One end of the arm carries a pin which can be set into the holes in the third arm (W), which will be described shortly. The arm (A) is used to represent the air speed and is graduated in miles per hour with the zero at the pin.

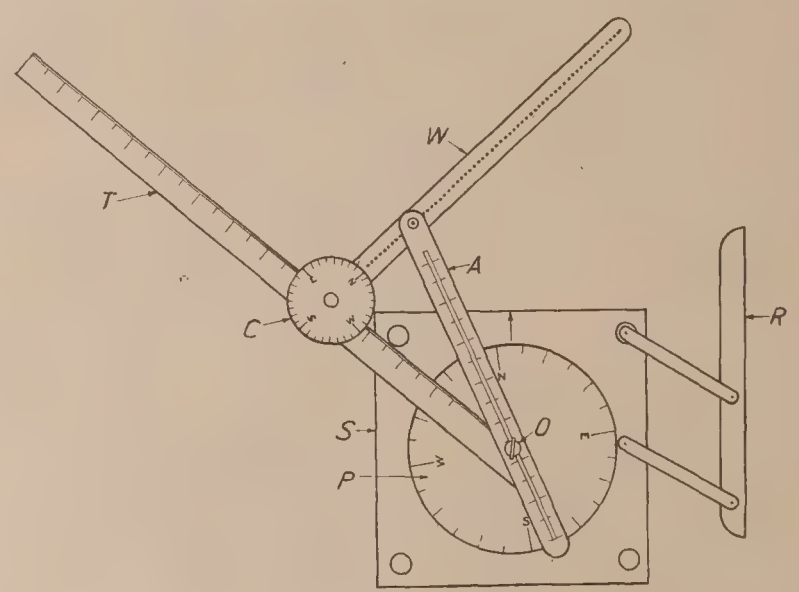

FIG. 4.—Angus compass course indicator.

The third arm (W) is called the wind scale. It slides on the track scale, while its direction with respect to the latter can be set by means of a small compass disk (C) attached rigidly to it. The wind scale is graduated in miles per hour and has small holes drilled in it at each graduation to receive the pin on the air-speed scale.

Although this apparatus will solve several different problems involving the triangle of velocities, its principal use is to determine the compass course to be steered in order to follow a fixed route and the ground speed which will be attained over this route when the air speed, the desired track, and the amount and direction of the wind are known.

The instrument is placed on the map with the center (O) over the point of departure and is then oriented by means of the parallel rule. The protractor is turned until the declination is laid off (i. e., a magnetic needle suspended at (O) would read zero). The track scale is laid along the course to be followed and the wind scale is set to the proper direction by means of the compass card (C), the arm being turned until the graduation on the card corresponding to the known wind direction is parallel to any convenient meridian line. The pin at the end of the air-speed scale is now placed in the hole on the wind scale corresponding to the wind speed. The air-speed scale is slid past (O) until the graduation at (O) represents the air speed. As this is done, the wind scale slides on the track scale and when the air speed is properly set, the ground speed can be read on the track scale. The course to be steered to overcome the drift due to the wind is given by the reading of the air-speed scale on the protractor.

The angle between the track and the air-speed scales corresponds to the correction angle of figure 2(*a*).

Drift or aero-bearing plate.—This instrument may consist merely of a rotating graduated circle with diametral wires for measuring the drift or it may combine with this a ground-speed attachment. (See fig. 5.) Only the latter type will be described as it includes the former.

The frame of the instrument is attached rigidly to the fuselage with the line through the index (I_1) and the center of the fixed circle parallel to the longitudinal axis of the plane. An inner circle (C), graduated clockwise from 0° to 360° and with the cardinal points of the compass engraved on it, slides in the fixed frame (F) and can be set in any desired position. An outer movable circle (M) carries two diametral parallel lines engraved on strips of celluloid (L) together with an index (I) reading on the inner graduated circle. In measuring the drift angle, the inner circle is set with its zero at the index (I_1) of the fixed circle; i. e., in the direction in which the plane is heading. The outer circle is now rotated until points on the ground move parallel to the diametral lines (L). The index (I) on the outer circle now reads the drift angle.

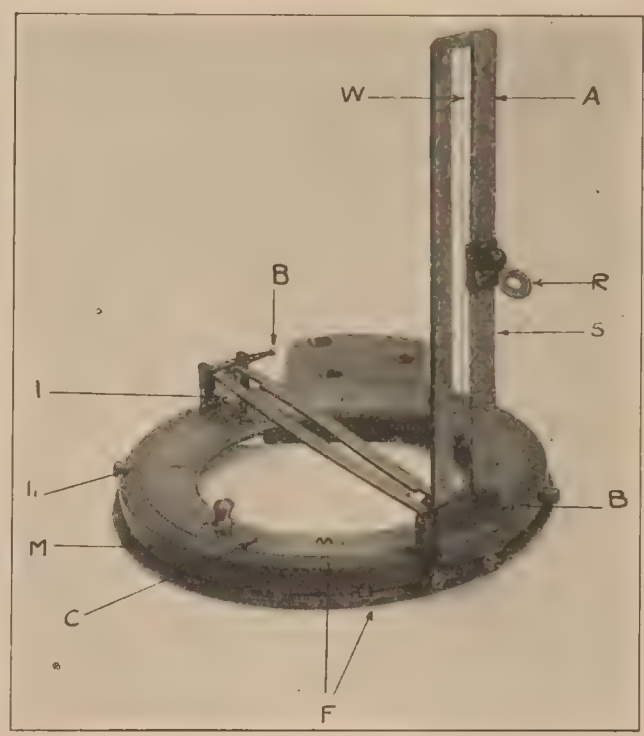

FIG. 5.—Drift bearing plate.

The standards at the ends of the celluloid strips carry two ball sights (B) in a horizontal plane. One of the standards also carries a vertical folding arm (A) with a central vertical wire (W), and a sliding ring sight (R) whose center is in the same vertical plane as the two ball sights and is directly over one of them. This ring sight carries an index which reads on an altitude scale (S) graduated from 0 to 6,000 meters on the vertical arm. By setting the index of the ring sight to read the altitude of the aircraft, setting the outer circle so that the plane of the sights is in the direction of motion over the ground, and timing the passage of an object between the two ball sights, the ground speed can be computed from the known length of base line on the ground. (See Report No. 127 of this series.) In the case of the particular instrument just described, this base line is 5 miles or 8 kilometers.

Wind-gauge bearing plate.[6]—This is an ingenious instrument devised by Maj. H. E. Wimperis, R. A. F., to measure the drift angle by means of tail bearings and to determine the ground speed and the direction and amount of the wind by using the principle of the "wind star."

Suppose the navigator has a horizontal graduated bearing plate with a transparent glass or celluloid center. (See fig. 6.) A straightedge (AC) is pivoted at (A), (AB) being the center line of the device fixed parallel to the longitudinal axis of the airplane. The straightedge (AC)

[6] "Air Navigation," by H. E. Wimperis. Constable & Co., Limited, London.

can be adjusted until it is parallel to the direction of motion over the ground. A line is now drawn along this straightedge on the transparent center of the plate which should be oriented with the aid of the compass. Now, suppose that (AO) represents the air speed, which is known. Then in the velocity triangle (*c*), which is properly oriented to illustrate (*a*), the line just drawn includes the side representing the ground speed (AP). The position of the point (P) on this line (AC) is not known, however, as the ground speed has not yet been determined.

Now, suppose the airplane is deviated from its course by, say, 30° or 40°. Reorient the bearing plate. Now set the straightedge parallel to the drift lines again and draw a new line (A′C′) (fig. 6b). (A′C′) will intersect (AC) at (P), thus determining the second side of the velocity triangle representing the ground speed. The closing side (OP) represents the wind. (P) is called the "wind point" and the diagram is called a "wind star."

In designing an instrument to use the method just described, it is necessary to make some provision for varying the distance (AO) for different air speeds. Wimperis provided for this by making the pivot (A) movable along the line (AB), its motion being controlled by a thumbscrew and the proper length being determined by an air-speed scale. The drift angle can be read by an index sliding over a scale at the end of this arm. The center of the bearing plate is of glass and has concentric circles representing wind speeds differing by 10-mile intervals. The pivoted straightedge or drift bar carries speed and time scales and has a beveled edge for drawing lines on the glass plate with a glass-marking pencil. A diametral wind arrow carrying a sliding red wind point can be rotated in the fixed circle of the bearing plate. The bearing plate is graduated and has an index which enables the observer to orient it as above described.

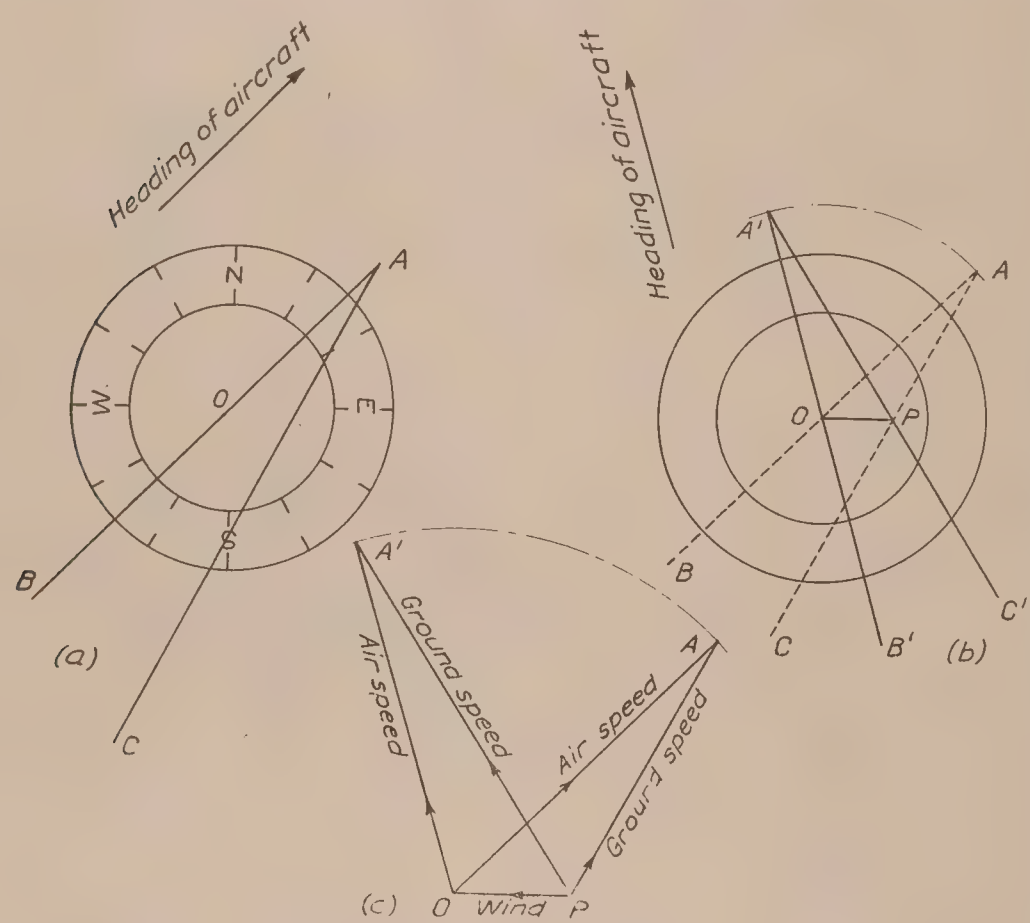

FIG. 6.—Wind star diagram.

A ground-speed attachment is also combined with this bearing plate. It consists of a vertical arm as described for the aero bearing plate and two horizontal wires, each carrying two head sights. One of these wires gives a base line on the ground of one-half mile, while the other is arranged to give the number of minutes taken to fly 30 miles.

Course-setting sight.—Another instrument invented by Wimperis is the course-setting sight designed for use in the nose of the aircraft to set off the course to be followed. Here are combined a vector triangle linkage and a bearing plate, the latter commonly, although not necessarily, containing a compass needle. Briefly, the action of this instrument is as follows: The air speed is set off on the air-speed bar. The machine is then flown directly up the wind, this being done by adjusting the course until the sight shows no drift. The bearing plate is oriented by means of the compass needle. The aircraft is then turned off this course by about 90°. The bearing plate is reoriented by the needle and the wind scale adjusted until points on the ground appear to travel along the drift wires. The ground speed and wind speed can now be read.

The geometry of the solution is as follows (see fig. 2*a*): By flying up the wind we determine the direction (CA) of the wind vector. For any course we know the direction and amount of the air-speed vector (OC). Now, if we observe the drift, we know the direction (OA) of of the ground-speed vector, and the intersection of a line through (C) in the direction of the wind vector and a line through (O) in the direction of the ground-speed vector gives the point (A), and the wind and ground speed are then completely determined.

Christmas drift indicator.—This instrument (fig. 7), invented by Pilot Valdemar de Christmas, of the French Air Service, fits over the map board used in the French airplanes and has an adjustment at right angles to the motion of the map. With the aid of this instrument, the pilot can take the air without knowing the wind and by a single observation can obtain the correction angle, thus enabling him to steer so as to follow the desired track.

The apparatus is composed essentially of two circular limbs, an inner one (A) rotating inside of the other (B), which is fixed. The outer one is graduated from 0° to 360° clockwise in the usual manner. The interior limb carries a transparent sheet of mica laid off in kilometer squares to the scale of the map. A wedge-shaped pointer (P), sliding on the circumference of the outer circle, reaches to the center and is also graduated in kilometers from the center outward.

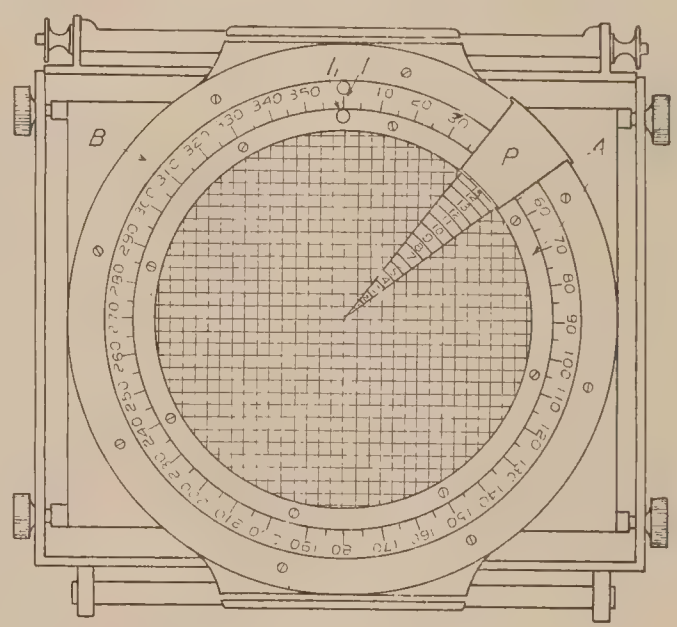

Fig. 7.—Christmas drift indicator.

Before leaving the airdrome the navigator marks on the map the route to be followed and the geographic meridian is drawn through the starting point. The drift indicator is mounted on the map case and is adjusted, together with the map, until its center is over the starting point. The zero graduation (I) of the outer limb is set to the meridian on the map and the index of the interior limb (I_1) is now set on the desired track, thus indicating on the outer limb the track angle. Making the necessary correction for declination, the pilot lays off the track angle on his compass by means of a movable index and flies so that the compass needle is directed toward the index.

At the end of a suitable period—three minutes, for example—he notes the point directly beneath him on the ground and marks it on the map. This new point (O′) is now taken as a center for the drift indicator and the slide is turned until the graduation corresponding to the air distance covered during the three-minute interval cuts the route line on the map in point (B). (See fig. 2*c*.) Angle (α) is here the drift angle and angle (β) the correction angle. Now, if the ship is headed at an angle (β) to the route to be made good and to the windward of the latter, it will move over the ground, following a route parallel to the route to be made good at a ground speed represented by (O′C). Consequently the duration of the trip can be computed.

Since the center of the drift indicator is at the point (O′) to which the ship has drifted at the end of the three-minute interval, by turning the slide until it cuts the point (A), where the ship would have been at the end of three minutes had there been no wind, the pilot can read from the slide the distance which the wind has traveled in three minutes and then from the graduated circle the angle ($180° + \gamma$). Or by first moving the drift indicator until its center is at A, he can read the angle (γ) and the wind speed directly. The ground speed represented by (O′C) can be determined by measuring the equal distance (AB) with the aid of the graduated inner limb.

Crocco drift and ground-speed indicator.—This instrument (see fig. 8), designed by Col. Crocco, of the Italian Air Service, has been used in a modified form by the United States Naval Air Service and is being adopted in this form for dirigibles by the United States Army Air Service.

It performs two distinct operations: It measures the ground speed and the angle of drift, and it solves the velocity triangle. The instrument is set in the tail with the long side of the frame parallel to the longitudinal axis of the ship. The arm (A) at the extreme right of the photograph can be turned by means of the large knurled screw (S) until objects on the ground appear to move along the longitudinal wire (W), when the angle between the objects and the axis of the plane—i. e., the drift angle—can be read on the graduated drift sector (C) at the base of the upright (U) by means of the index (I) shown on the right. If it is desired to pick up a particular object on the ground, the arm (A) can be moved without altering its angle with respect to the axis of the airplane by the parallel motion (P) connecting the arm to the base. An adjustable eyepiece (E) on the upright serves to keep the eye in the proper position and to hold it steady.

The ground speed can be determined by measuring with a stop watch the time it takes an object astern on the ground to travel from the fixed wire (F) to the movable wire (M) when the eye is at the eyepiece (E). The movable wire (M) is set to correspond to the altitude by means of the scale on the ground-speed arm (A). Two scales are engraved, one on either side of the arm, the first giving altitudes from 600 to 3,000 meters and the other from 3,050 to 6,000 meters. To these scales correspond base lines on the ground of 600 and 1,200 meters, respectively, so that when the movable wire is properly set for altitude, the distance actually traveled, when a point on the ground moves from one wire to the other, is known.

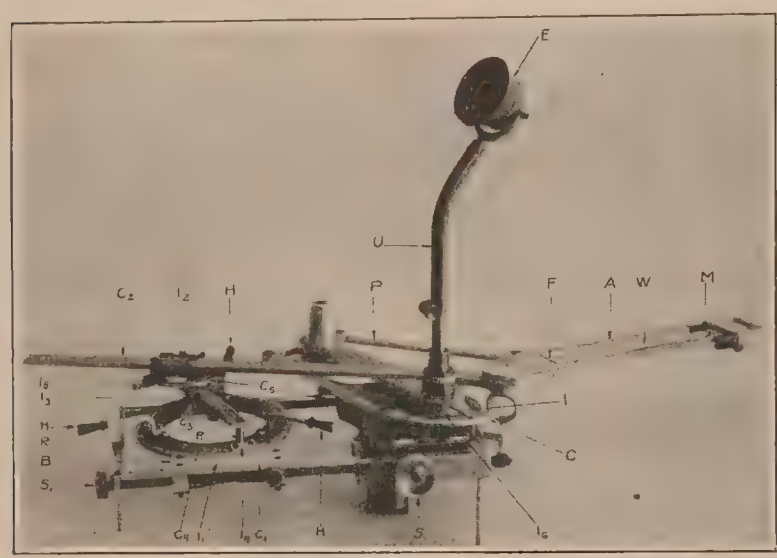

Fig. 8.—Crocco drift indicator.

It has already been explained how the ground speed and the drift angle are determined. The air speed, also, is known from the reading of the air-speed indicator corrected, if necessary, for air density. Once these three quantities are known, the velocity triangle can be solved for the direction and amount of the wind by means of the instrument.

The air speed is first set on the scale (C_1) by moving the sliding base (B) parallel to the longitudinal axis of the aircraft by means of the thumbscrew (S_1) until the index (I_1) reads the air speed.

The ground speed is now laid off on the ground-speed or track scale (C_2) by sliding the index (I_2) to the proper reading. The track arm (C_2) carries three scales, the two outer ones corresponding to the two altitude scales on the ground-speed arm (A) and graduated in seconds, so that the time taken for an object to pass between the wires (F) and (M) can be laid off directly without the necessity of computing the ground speed. The central scale on (C_2) is in meters per second, and gives the ground speed corresponding to the measured number of seconds.

The graduated arm (C_3) represents the wind in direction and amount. This arm is mounted on two posts (P_1) (only one of which is visible in the figure) attached to the ring (R), which can be rotated relative to the graduated circle (C_4), a part of the base (B). (C_3) can slide past the track arm (C_2) and can change its direction relative to the latter. Holding the index (I_2) against the ground speed on the track arm, the navigator turns the wind arm (C_3) by means of the handles (H) and the rotating ring (R) until the index (I_6), which is rigidly attached to the arm (C_2) and which reads zero when the track arm is parallel to the longitudinal axis of the ship, coincides with the index (I). As (I) is set for the drift, this procedure amounts to setting the arm (C_2) for the drift.

The procedure outlined above has brought the wind arm into such a position that the amount and direction of the wind can be read directly. The index (I_3) indicates on scale (C_3) the speed of the wind. The direction of the wind with respect to the longitudinal axis of the

plane can be read on scale (C_4) by means of index (I_4). The semicircular double scale (C_5) and the index (I_5) give the angle between the track and the wind.

The process above described solves completely the velocity triangle. Now, if the navigator wishes to determine the correction angle which will enable him to follow a chosen track over the ground, he can reverse this process. He first sets off his air speed on scale (C_1). He then sets on scale (C_3) the wind speed just found, and turns the scale by means of the handles (H) until index (I_5) reads the known angle between his desired track and the wind. The correction angle can then be read on the drift sector (C) by means of index (I_6).

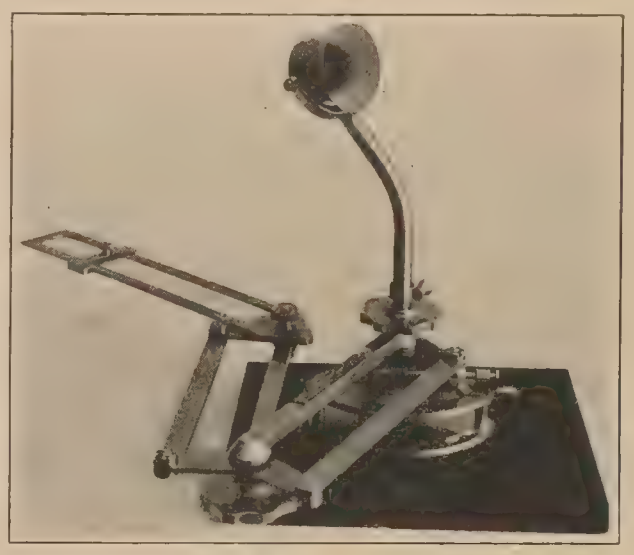

FIG. 9.—Pioneer drift indicator.

The Crocco drift indicator is essentially an instrument for dirigibles, where the matter of size and weight are not so important as in heavier-than-air craft. However, by the substitution of a computer for the portion of the instrument which solves the velocity triangle we can make the indicator much lighter and more compact. Drift indicators modeled after the Crocco, but altered as just suggested, were used successfully in the flight of the United States Navy *NC* boats across the Atlantic.

Pioneer drift indicator.—A still later modification of the Crocco drift indicator is to be found in the Pioneer drift and ground-speed indicator (fig. 9). This instrument measures the ground speed in a manner similar to that described above. The arm has two altitude scales, having corresponding base lines on the ground of one-tenth and four-tenths nautical miles.

Le Prieur navigraph.—This instrument (fig. 10) is a synchronized drift set. It is comprised of two parts: A derivograph, with which the navigator plots a drift line, and so determines the drift angle and the wind vector, and a repeating disk placed in front of the pilot connected to the derivograph by means of a flexible cord, so that its indications are synchronous with the latter, thus enabling the navigator to indicate to the pilot the proper course to follow.

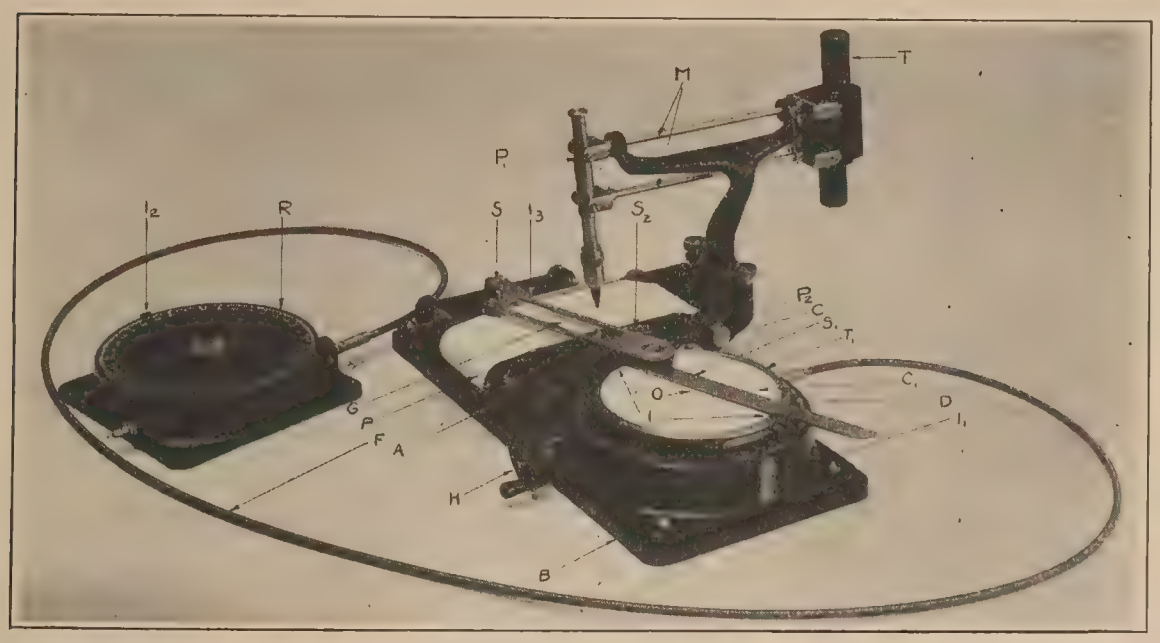

FIG. 10.—Le Prieur navigraph.

A feature of the navigraph which is not found in the other drift-measuring instruments described in this paper is the means provided for determining the drift more accurately than can be done by sighting along a wire and estimating the direction of motion over the ground. Owing to the yawing, rolling, and pitching of the aircraft, points on the ground appear to follow a sinuous line whose general direction is often hard to estimate.

With the navigraph an object upon the ground is followed with a telescope (T) (fig. 10) and a series of points is plotted on the sheet of paper (P) with the pencil (P_1), which is connected to the telescope (T) by means of the parallel motion (M). In this way a drift line is obtained on (S) for the object on the ground whose relative motion is being followed. The advantage of this drift line lies in the fact that it enables the navigator to obtain the mean

drift by averaging the irregularities due to the unavoidable pitching and yawing of the aircraft. In practice several drift lines may be obtained from as many objects on the ground and the mean of these lines taken. The sheet of paper (P) is mounted on rollers and so can be set to any desired position.

The operation of the derivograph is based upon the "wind star" principle as is the Wimperis wind-gage bearing plate. By measuring the drift angle for two different courses the navigator can determine the wind vector and the correction angle which should be applied to the bearing of the desired track to give the proper compass course.

To make the following discussion of the principle and operation of the navigraph more concrete, the following problem will be assumed:

The navigator wishes to follow a track whose direction is 40° (N. 40° E.). His air speed is 100 M. P. H. How can he determine the compass course to be followed?

The derivograph is fixed with the long side of its base (B) parallel to the longitudinal axis of the aircraft. On the base are mounted (1) a movable carriage (C) supporting the roll of paper and the telescope, pencil, and parallel motion already described, and (2) a graduated disk (D), which can be rotated about a vertical axis by means of the handle (H). This disk is graduated in degrees clockwise from 0° to 360° and the cardinal points of the compass are engraved upon it: north corresponding to 0°, east to 90°, south to 180°, and west to 270°. A circular table (T_1) surmounts the disk (D) and can be moved relative to (D) against friction. (T_1) carries two indices (I) 180° apart, and a fixed index (I_1) is mounted on the longitudinal center line of the base (B) so as to read on the graduated scale on (D).

A circular chart (C), on which are drawn a series of concentric circles and parallel lines, is fastened to (T_1) with its diametral line coinciding with the indices (I). (T_1) is then turned by friction relative to the disk (D) until one index (I) reads the bearing of the desired track; in this case, 40°. The table (T_1) is kept in this position relative to (D) throughout the flight. It will be observed that, if the aircraft is headed along the course indicated on (D) by the index (I_1), (D) is properly oriented and the indices (I) and the parallel lines on (C_1) lie in the direction of the desired track. This relation is maintained during the flight with the aid of the repeater disk (R) as follows:

The graduated disk (D) is connected to the repeater disk (R), which is graduated exactly like (D), by means of a flexible cord (F). (R) is so set that the index (I_2) reads the bearing indicated by (I), corrected for magnetic declination. The pilot watches the repeater (R) and steers his ship so that his compass reads the bearing shown by the repeater.

At the start of the flight the disk (D) is turned until the bearing of the desired track (40°) is opposite the index (I_1). The pilot, following the instructions given by the repeater, heads the ship parallel to the desired track. The navigator plots a drift line on the paper (P) for an object on the ground. He then sets the movable carriage (C) so that the index (I_3) (not visible in fig. 10) reads the air speed on the scale (S). When this is done the center (O) of the chart (C_1) is opposite the graduation giving the air speed on scale (S_1) on the arm (A), which is pivoted at (P_2). The arm (A) can be turned about (P_2) so that the line ruled parallel to the fiducial edge of (A) on the glass (G) coincides with the drift line just plotted. The drift angle can then be read on the scale (S_2).

When the arm is set for zero drift angle and so passes through the center (O) of the chart (C_1), it represents the air-speed vector of the velocity triangle. The pivot (P_2) corresponds to the air-speed—ground-speed vertex of the velocity triangle. When the arm is set to the proper drift angle, it then represents the ground-speed vector in direction, but the length of this vector is not known unless either the ground-speed or the wind vector is known.

The principle of the navigraph is shown in figure 11. The inner circle (BDGH) represents the chart (C_1). The outer circle (NESW) represents the graduated compass disk (D). The outer arc (AFC) represents the locus of positions of the pivot (P_2). In the actual instrument the compass disk and chart rotate while the pivot remains fixed, but in figure 11, for simplicity, the disk and chart are considered as fixed and the pivot is considered to move in the arc (AFC). This is justifiable since the relative motion is the same.

When the navigator has set the arm (A) according to the drift line on (P) he draws along the fiducial edge of the arm (A) the line (GB), which is known to include the point (E). The intersection of the wind and ground-speed vectors is not yet known. To determine the position of (E) on the line (GB), the navigator first arbitrarily selects a new course (in this case 320° or N. 40° W.) and rotates the compass disk by means of the handle (H) until the index (I_1) reads 320°. This corresponds in figure 11 to rotating the pivot (P_2) from (A) to (C). The navigator now determines a new drift line on (P), sets the arm (A) to this drift line, and draws the line (HD) (fig. 11). The intersection of (GB) and (HD) determines the wind point (E) and the line from (O) to (E) represents the wind vector.

Now that the wind vector is known, the navigator can correct his compass course so as actually to follow the desired track. If he rotates the compass disk (D), holding the arm (A) in contact with the wind point (E) until the arm is parallel to the lines on the chart, the reading of the index (I_1) will be the course which should be followed. This is shown in figure 11. The rotation of the compass disk is represented by the shifting of point (C) to (F), where (FE) is parallel to the system of lines on the chart. These lines point always in the direction of the desired track, and therefore, when the arm (A) is adjusted as explained, it is parallel to the desired track. But since (A) also passes through the point (E), it includes the ground-speed vector for the desired track and so should coincide with the direction of motion of points over the ground. This can be checked by observing the motion of suitable points. Usually a small discrepancy will be observed owing to instrumental errors, observational errors, and changes in the wind. In this case a second trial will usually suffice to give the proper course with sufficient accuracy.

When the arm (A) passes through the wind point (E), the graduation on the arm opposite this point gives the ground speed.

FIG. 11.—Principle of navigraph.

COMPUTERS AND PLOTTING DEVICES.

Campbell-Harrison course and distance calculator.—This computer (fig. 12, A and B) is a simplified modification of the naval Battenberg course and distance indicator. It consists of a circular disk rotating inside a ring which is graduated from 0° to 360° clockwise. The disk is laid off in squares of suitable size, reading from 0 to 140 by intervals of 10 units from the center of the circumference. Two rotating arms graduated from 0 to 140 to the same scale as the disk and carrying slides complete the device.

In the form just described, the computer can be used for solving the velocity triangle and can be used in an approximate way as a slide rule for computing times and speeds.

Appleyard course and distance calculator.—This computer is similar to the one just described except that it includes also a computing dial or circular slide rule on the circumference. (See fig. 12C.)

The computers shown in figures 12 A and B have central disks about 7 inches in diameter, while that of the Appleyard computer is considerably smaller. The tendency in this country is to increase the size of the disk and the number of graduations in order to make a more accurate instrument.

The use of these computers is illustrated by the following examples:

(1) (fig. 12A). The course steered is 104° and the air speed is 110 miles per hour. The wind is blowing from 205° at the rate of 40 miles per hour. What is the direction of the track and what is the ground speed?

It may be said, in general, that the arms should be used for the known quantities and the disk for the unknown. This rule is applicable to nearly all cases. Proceeding in this way, we set one arm to 205° and move slide to 40. Set the other arm to 104° and set slide to 110. Rotate the disk until the arrow is parallel to the line through the indices of the two slides, the sense of direction being obtained from the air-speed arm. The direction of the track is then read opposite the arrow as 85° and the ground speed is given by the distance between the two slides as $20+105=125$ miles per hour. The two arms and the line through the indices of the slides form the velocity triangle, which is solved with the aid of the graduations on the computer.

(2) Given the course steered as 120°, air speed 90 miles per hour, observed track 95°, ground speed 100 miles per hour. What is the amount and direction of the wind? (See fig. 12C.) Lay off on one arm the ground speed 100 miles per hour and turn the arm until it reads 95°, the observed track. Lay off on the other arm, 90 miles per hour, the air speed, and turn this arm until it reads 120°, the compass course. Turn disk until the lines are parallel to the line of the indices. The arrow then reads 31°. The sense of direction is given by the relative directions of the course steered and the observed track. In this case the wind is blowing

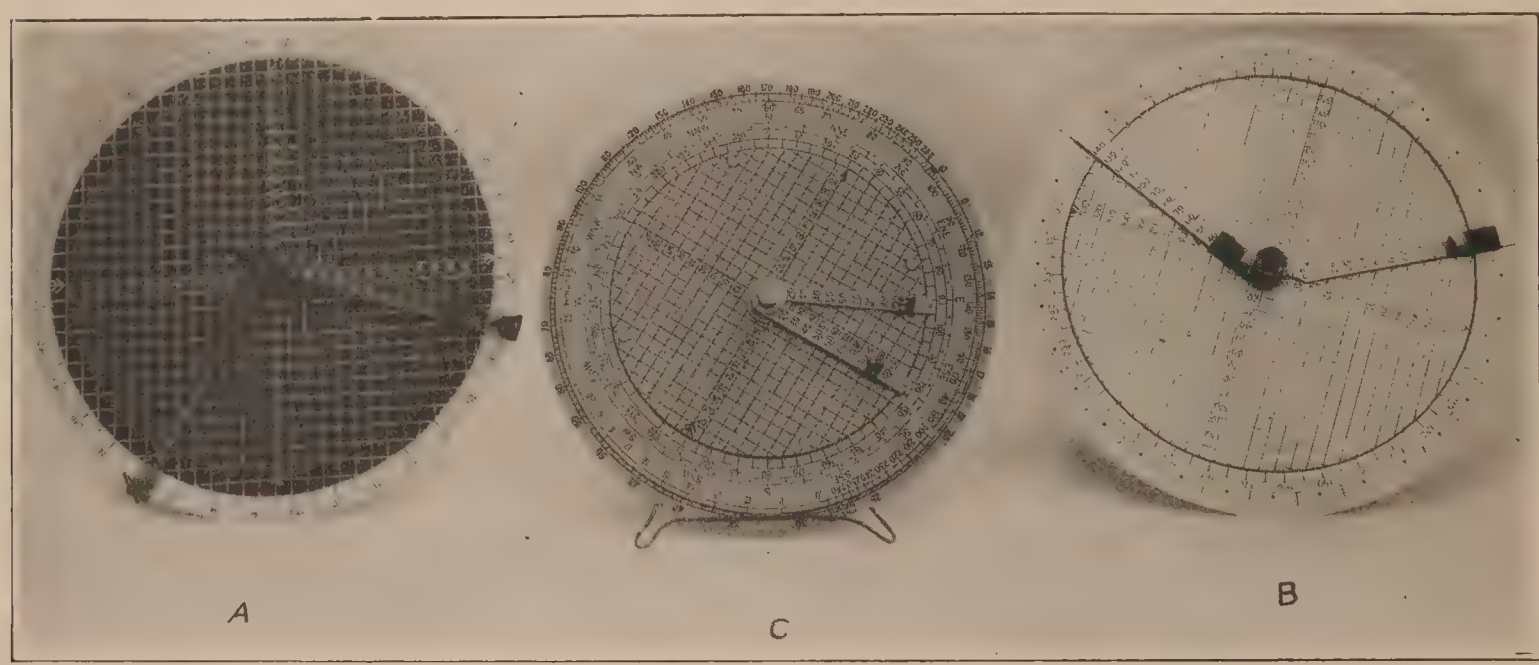

Fig. 12.—Course and distance calculators.

toward 31° or from 211°. The speed of the wind is found from the distance between the two indices, in this case about 42 miles per hour.

(3) Given the same data as in (1), how long will it take to fly 480 miles? The proportion used is

$$\frac{\text{time required in minutes}}{60}=\frac{\text{distance covered in miles}}{\text{ground speed in miles per hour}}$$

or in this case

$$\frac{\text{time}}{60}=\frac{480}{125}$$

(Fig. 12B.) Set slide on one arm to 125 and turn arm until the index lies on the line through 60. The other arm is not used. Read line on disk corresponding to 480 on arm. In this case we obtain 230 minutes or 3 hours 50 minutes.

The Bigsworth protractor, parallels, and chart board.—This is one of the most convenient available outfits (fig. 13) for plotting and determining courses, finding position, etc. Two sizes are provided, 14 and 17 inches square, depending on the room available. A number of charts can be carried under the celluloid upper surface of the board and held in place by clips with the one desired for use placed uppermost.

A parallel motion carrying a celluloid protractor and straightedge can be clamped to the side of the board in any desired position. The parallels are fitted with a hinge, so that the protractor can be lifted off the board if so desired.

The protractor can be set with its arrow pointing magnetic north by means of the compass rose or by a convenient meridian and the known variation for the region.

For measuring distances, two interchangeable protractors are provided, one graduated in inches and tenths for the standard charts which are drawn to the scales 10 nautical miles to the inch for General Charts and 3 nautical miles to the inch for Coastal Charts (these specifica-

FIG. 13.—Bigsworth chart board.

tions are for the British Air Service), and the other graduated in miles and halves on a scale of $\frac{1}{200000}$.

A special pencil is used for drawing on the celluloid surface, the marks being easily erased by rubbing with the finger.

Map cases.—A number of map cases have been devised, most of which are quite similar in form. They consist of small rectangular cases with the map, which is cut in a long narrow strip, including the route to be covered, mounted on two rollers which can be revolved by hand, thus bringing the desired portion of the map into view. A celluloid cover protects the visible surface of the map. A light is sometimes provided beneath the portion of the map which is in use to illuminate the latter for work at night.

Protractors.—Various protractors have been devised for aerial use. These consist usually of a circular or square piece of celluloid with angles laid off on the circumference or the sides and often with squares ruled on them to the scale of map used. A string or movable arm is used to determine courses.

ASTRONOMICAL OBSERVATION.

GENERAL PRINCIPLES.

Astronomical observations are made for the purpose of checking or correcting the position as found by dead reckoning. The observations involve the measurement of either the altitude or azimuth, or both, of the sun or some other heavenly body. In the majority of cases the altitude alone is measured, since this leads to the customary simple "Sumner line" method, which has been used for about 80 years in marine navigation.

It was natural that in the first attempts at aerial navigation by astronomical observation the methods and instruments which long experience had shown to be the best for marine navigation should be adopted. The aviator, however, is concerned with somewhat different requirements and conditions from those affecting the mariner. He has greater need of an artificial horizon for use with his sextant. Owing to the much higher speed of his craft, he must be able to reduce his observations quickly. Fortunately, a slight sacrifice of accuracy is permissible in accomplishing this, since there are no dangerous rocks and shoals to endanger the aircraft, and an error in position which might increase the duration of the voyage for a mariner by an hour would mean but a few minutes additional for the aviator. Only in exceptional cases is it necessary for the aviator to know his position with great accuracy, aside from landing in a fog.

The above considerations have already brought about for aerial use the modification of the marine sextant, new methods of reducing observations, and the use of special maps to facilitate the graphical work involved in plotting position. The nature of the observations made still remains practically the same as in marine navigation, but the urgent need of simplifying the work has led to the attempt to obtain latitude and longitude more directly than is possible with the orthodox methods.

Sumner line method.—In using the "Sumner line" method, the observer measures the altitude of some celestial body, say the sun. He then knows that he is on a small circle whose radius in nautical miles is equal to the zenith distance of the sun in minutes of arc and whose center is the subsolar point. The position of the subsolar point can be computed from the Greenwich mean solar time, the equation of time, and the declination of the sun. The first of these quantities is read from a chronometer and the second and third are obtained from tables in the Nautical Almanac. If observing a star, the observer makes use of the right ascension of the star as determined from the Nautical Almanac.

In practice he usually knows his location within 2°, so only a small portion of the curve is needed. This small portion is assumed to be a straight line tangent to the circle and is called a "Sumner line."

St. Hilaire method.—A modification of the above procedure is found in the Marcq St. Hilaire method. The observer lays down on the chart his dead reckoning (D. R.) position. If actually at this point, he would at a definite instant see the sun at a particular altitude and azimuth, both of which can be computed. The observer performs these computations and measures the true altitude of the sun. His computed altitude will usually differ from his observed altitude by a few minutes of arc, depending on the accuracy of the D. R. position assumed. A line is drawn through the D. R. position in the computed direction of the sun from the observer. The Sumner line is now drawn at right angles to this line and at a distance from the D. R. position of as many nautical miles as there are minutes of arc difference between the observed and computed altitudes. If the observed altitude is greater than the computed, the Sumner line is nearer the sun than the D. R. position; if less, the reverse is true. The most probable position is at the intersection of the Sumner line and the D. R. azimuth line, since the D. R. position was the most probable location of the ship before the altitude of the sun was measured, but all that is positively known is that the craft is at some point on the Sumner line. If the observer has both the sun and moon available, or if two suitable stars are visible, he can measure the altitudes of the two bodies, thus obtaining two Sumner lines whose intersection will fix his position. Or if he can measure the azimuth of the sun, as well as its altitude, his position is determined upon the Sumner line.

Direct measurement of sidereal time.—A radically different and theoretically much simpler method of determining position has been suggested for aerial use. In this method the observer measures the altitude of Polaris and the angle through which the vernal equinox has turned since upper transit at the observer's present meridian. The corrected altitude of Polaris gives directly the latitude of the observer. The rotation of the vernal equinox is determined from β Cassiopeiæ, which has approximately zero right ascension. Measuring this angle in the equatorial plane, the observer obtains local sidereal time. Then if he has a chronometer giving Greenwich sidereal time, he can find his longitude by taking the difference of these two times.

SEXTANTS.

DIFFICULTIES OF USE IN AIRCRAFT.

The sextant, as might be expected, is at present the universal instrument for astronomical observation from aircraft as it has been for seagoing craft. The conditions under which it is used in aeronautics are often much more adverse than in marine navigation. A very appreciable difficulty is found in using it in an airplane. The force of the wind makes it difficult to hold the instrument and affects the vision, the sun or the particular piece of horizon beneath the sun may be obscured by the wings or by a strut, the observer's fingers may be numbed with the cold; and when we add to these a cramped position, it can be seen that the navigator's task is not an easy one.

NATURAL HORIZONS.

The horizon used as a basis for measuring the altitude of the celestial body may at times present serious problems both as to its visibility and as to the magnitude of the dip correction. The most favorable conditions are found when the sea horizon is visible or one formed by flat land is available. Observations can then be made with nearly the precision obtained on shipboard.

A small error is introduced, however, in the reduction of the observations owing to the uncertainty as to the correction for the dip of the horizon. Theory tells us that the dip in minutes of arc is almost exactly equal to the square root of the height in feet and this relation has been confirmed up to an altitude of about 12,000 feet.[7] The error involved in the dip correction is thus dependent upon the accuracy with which the height of the aircraft above the surface of the sea or the land can be determined. This height is obtained from the altimeter carried on board the craft. Very few altimeters which have been properly tested and accepted, unless damaged by rough handling, will show an error greater than from 200 to 300 feet at an altitude of 12,000 feet and this maximum error diminishes with decrease in altitude to about 100 to 120 feet at 1,000 feet altitude. Under these circumstances the greatest error in determining the dip correction would be about 1.5 minutes at altitudes between 1,000 and 12,000 feet. This accuracy will not be attained, however, unless the altimeter is set at the start of the flight to read the altitude corresponding to the existing atmospheric pressure at the starting point and unless the reading is occasionally checked during long flights by dropping down close to the level of the sea in order to take account of local changes in atmospheric pressure. It seems reasonable to assume that in the future we may expect to see the same degree of intelligence exercised in aerial navigation as is found in marine navigation, and in this case the correction for dip should never be in error by more than two or three minutes, an amount which, when added to the error of observation, is small compared with the errors which are involved when artificial horizons are used.

In the clearest weather the sea horizon may be visible up to an altitude of about 10,000 feet, and a horizon formed by flat land, owing to its greater contrast, has been seen up to 12,000 feet.[8] Ordinarily no such range of visibility is found on account of haze in the atmosphere. Even when the horizon is seen distinctly at sea level, it is usually found that the line between sea and sky becomes less and less clear with increasing altitude, until, at a height of from 1,000

[7] "Report on the Navigation of Aircraft by Sextant Observations," by H. N. Russell, Proceedings Astronomical Society of the Pacific, June, 1919 vol. 31, No. 181, pp. 129–149.

[8] H. N. Russell, loc. cit

to 2,000 feet, depending on atmospheric conditions, it is indistinguishable. However, by climbing still higher, the aviator often finds that a horizon reappears distinctly at from 4,000 to 6,000 feet. The new horizon is formed by the upper surface of the haze or mist which lies over the water and which is not opaque to the observer at sea level, since the horizon seen from this height is only about 5 miles away. With increasing altitude this distance becomes rapidly greater, until at 1,000 feet the observer is sighting through over 30 miles of haze through which rays of light from the horizon may be unable to penetrate. By continuing to climb he may reach a point where the upper surface of the haze furnishes another natural horizon, but one whose altitude is not usually known.

In some cases it is possible to determine the altitude of the upper surface of the haze when climbing through it. In such a case it is possible to determine the dip correction with fair accuracy if it is known that the horizon is level. Russell in his work at Langley Field (loc. cit.) found that during the months of August and September observations made on haze horizons were almost without exception reliable, whereas in November similar observations were valueless. He concluded from his work that the layers of haze were nearly level in summer, while in the late autumn irregularities existed. In several cases his observations indicated that he had sighted on a ridge of haze instead of a true horizon.

Under such conditions the use of an artificial horizon is an absolute necessity. If the haze furnishes a level horizon we can dispense with a knowledge of its height by using a sextant like the Baker, or possibly by using a dip-measuring instrument such as has been occasionally used at sea. But if the navigator does not feel confident as to the uniformity of the horizon, he had better discard it and avail himself of an artificial horizon instead.

Clouds occasionally furnish good natural horizons, but, as in the case of haze horizons, they must be used with caution.

ARTIFICIAL HORIZONS.

In probably a majority of cases it will be found necessary to use an artificial horizon in making astronomical observations from aircraft. In this event a vertical reference line is necessary.[9] Since we must have recourse to gravity in order to establish and maintain a vertical line in an airplane, whatever device we use for the purpose, such as a level or plumb bob, will also be influenced by the accelerations to which the airplane is subjected. Of these accelerations the most important are the linear accelerations in the line of flight and the angular accelerations accompanying a change in course. Even in straightaway horizontal flight in smooth air, oscillations in pitch accompanied by changes in speed are going on continuously. Such oscillations may reach an amplitude of 1° or more with a period of approximately 20 seconds, the latter depending upon the free period of oscillation of the airplane used.

A fore-and-aft level will not indicate accurately the oscillations of the airplane in pitch, owing to the linear accelerations in the flight path which accompany the angular motion. Lucas[10] has shown experimentally with the aid of a solar reflector that a longitudinal level indicates a deflection of only about one-third of the true deflection of the airplane, and, furthermore, that its movement is one-quarter of a period out of phase with the movement of the airplane.

In a turn, a cross-level bubble tends to set itself normal to the resultant of the centrifugal force and the gravitational component, and if the bank is well executed the bubble remains at the center of the glass, even though the airplane may be inclined 50° or more. Consequently the indication at any instant of a level or short pendulum can be accepted as indicating a vertical line in an airplane only if the airplane is known to be free from accelerations.

The application of gyroscopic principles to artificial horizons has also been attempted. A gyroscope mounted in gimbals with its spin axis vertical can be given a very long period of oscillation by suitably adjusting the position of the center of gravity. A period of 100 seconds or more can readily be secured in this way, corresponding to a pendulum 5 miles long. Since

[9] The assistance rendered by Dr. L. J. Briggs in writing the portion of this report dealing with the establishment of a vertical reference line in aircraft is acknowledged.

[10] "The Oscillations of an Aeroplane in Flight," K. Lucas (British Advisory Committee for Aeronautics). Report 1915–16; 267–277.

this period is large compared with the oscillation periods of the airplane, the chances of progressively building up a large oscillation of the gyroscope from the oscillations of the airship are minimized. The spin axis of the gyroscope thus tends to hold a vertical position in straightaway flight, even though the airplane is constantly oscillating. The system containing the gyroscope is then said to be stabilized.

Careful piloting is an absolute necessity if an artificial horizon is used. If this condition is met, the errors of observation will be found to average from 10 to 20 minutes of arc, while with careless piloting the errors become so great as to make the results useless.

The development of a satisfactory artificial horizon has proceeded along two general lines: First, the attempt to build an auxiliary horizon to be used with a marine type of sextant much as the mercury horizon has been used, and second, to attach the artificial horizon to the sextant.

Pendulum artificial horizon.—A horizon of this type was designed and used by H. N. Russell (loc. cit.) in connection with his investigation of the navigation of aircraft by sextant observations. It consists of a pendulum about 10 inches long mounted in gimbals and having a speculum mirror set horizontally on top of it. The bob is arranged to swing in a pot containing a viscous liquid for damping vibrations. The whole is protected from the wind by a case.

Satisfactory observations upon both the sun and the moon were secured with the aid of this instrument, while provisional tests at night indicated that it could be used for star observations.

Engine vibration caused little trouble when the instrument was mounted upon a suitable shock-absorbing substance. Probably the most serious objection to an artificial horizon of this type, whether gravitational or gyroscopic in its nature, is that it is heavy and cumbersome and has to be moved to different positions for different bodies.

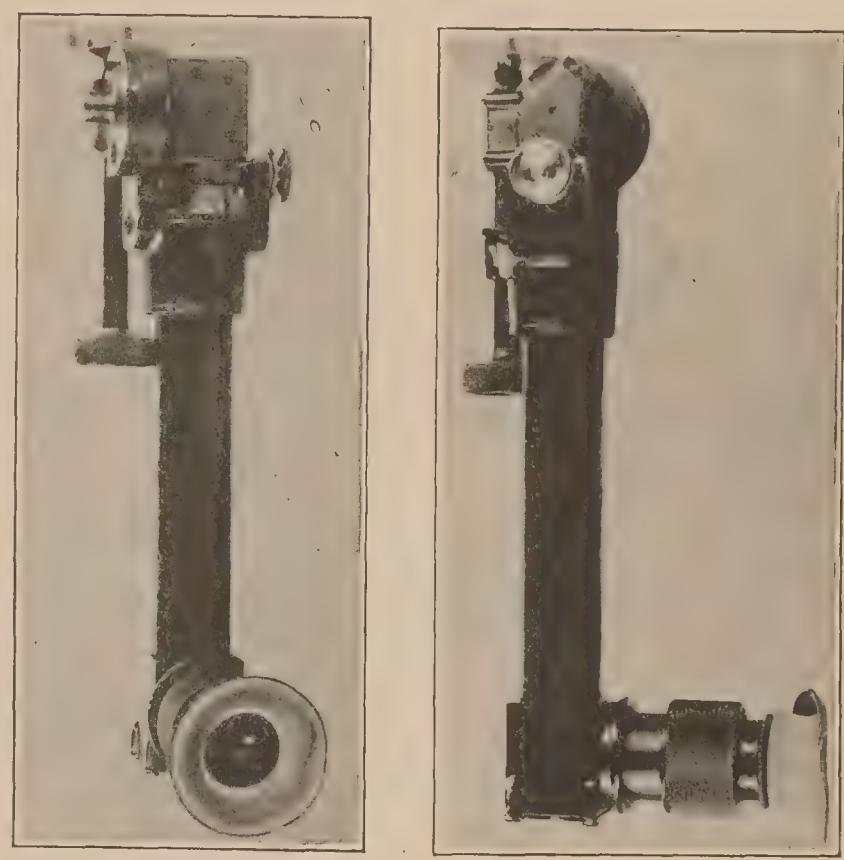

FIG. 14.—Baker sextant.

SEXTANTS USING NATURAL HORIZONS.

In the following description of sextants it will be assumed that the reader is familiar with the principle and the operation of the ordinary sextant. Only the sextants which are particularly adapted to aerial work will be described.

Marine sextants.—These are of the ordinary type used in marine navigation and will not be described here. For aerial use a telescope of low power and large field should be used. The graduations on the arc should be heavy and the vernier should be easily read without the aid of a reading glass.

The Baker sextant.—This instrument (figs. 14 and 15) possesses the unique feature that it eliminates both the correction for the dip of the horizon (when the horizon is level) and for the semidiameter of the body. This not only adds to the rapidity of obtaining the altitude, but it removes the uncertainty due to the lack of knowledge as to the actual height of the aircraft above the horizon used.

The elimination of these two corrections is accomplished by reflecting into the field with the image of the sun the image of the horizon directly under the sun and that of the horizon opposite it. The observer sees the two horizons, one of which is inverted, with an open space between, the width of this space depending on the sum of the dips of the two horizons. Now, if he bisects

the space between the two horizons with the image of the sun, he eliminates the dip and semi-diameter corrections.

It has already been pointed out that this sextant can be used only when the horizon is level. This fact makes it useful in only a few cases when the ordinary type of sextant can not be used.

Fig. 15 illustrates the principle of the Baker sextant. The fixed horizon prisms (H) are set at the top of a vertical tube (V) high enough to reach above the observer's head as he looks into the eyepiece (E). This is done so that both back and front horizons may be visible simultaneously. These prisms reflect rays of light from the horizons down through the vertical tube (V) to the prism (P), where they are reflected horizontally to the eye at (E). An index prism (I), rotating about a horizontal axis perpendicular to the plane of the paper, is controlled by a knurled thumbscrew carrying a micrometer head, which operates the altitude scale as well. This index prism catches rays of light from the celestial body, reflects them through a small circular hole drilled vertically through the joint between the prisms (H) down parallel to the rays from the horizons until they are reflected by the prism (P) to the eye. The sextant itself is shown in figure 14.

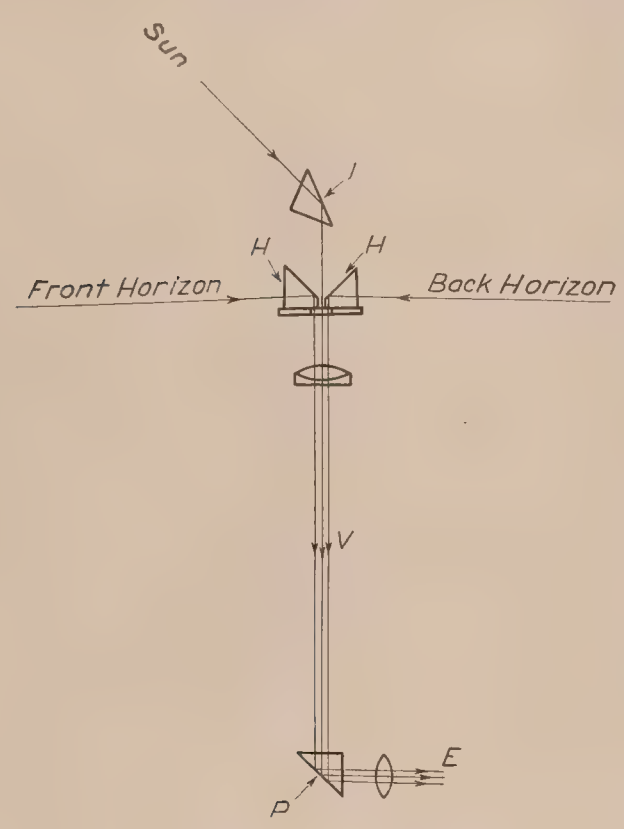

Fig. 15.—Principle of Baker sextant.

BUBBLE SEXTANTS.

The artificial horizon sextants most commonly used are of the bubble type (fig. 16). A bubble tube of the ordinary type or of the circular type is used and the image of the celestial body is brought into coincidence with that of the bubble when the bubble is at the center of the tube.

Byrd bubble sextant.—A sextant of this type (fig. 16A) used considerably in naval aviation has been developed by the United States Navy. This sextant is very similar in appearance to the ordinary marine sextant and can, in fact, be used with a natural horizon by depressing the mirror (M), which reflects the bubble through the horizon glass (H) to the telescope (T).

The bubble is contained in an ordinary spirit-filled tube set into the metal tube (B), which is mounted rigidly to the frame. For day observations, the light entering through the bottom of the tube is sufficient to make the bubble visible to the observer. At night the lamp (L) is used to illuminate the bubble. In order to bring the image of the bubble to a focus at the same point as the image of the celestial body, a half lens is used to focus the rays of the former while the rays from the body pass through the open half.

A convenient means is provided for making the first rough setting of the arm (A). Underneath the arc which carries the scale is a rack into which fits a worm gear operated by the micrometer head (D). By moving the lever (L_1) down, the worm gear is taken out of mesh with the rack and the arm can be set to its approximate position. The lever is then released, the worm gear again meshes with the rack, and the fine adjustment is made with the micrometer head. The head reads directly to the nearest 30 seconds. A lamp (L_2) is used to illuminate the scale at night.

The switch (S) is used for the lamp (L_2), which illuminates the scale, while the lamp (L), illuminating the bubble, is operated by the push button (P). These two switches are located on the handle of the sextant, where they can be operated by the fingers of the hand holding the instrument. The handle contains the battery used for the lamps.

With this instrument, the images of the bubble and the celestial body move in the same direction when the sextant is inclined longitudinally.

Willson bubble sextant.—A similar form of bubble sextant (fig. 16B) has been designed by R. W. Willson, of Harvard University. The bubble level in this case is of the spherical type, so that the leveling of the sextant laterally is facilitated. In the latest model of this instrument the focal lengths of the eyepiece, the objective, and of a lens just over the bubble and the radius of curvature of the spherical surface of the bubble tube are so arranged that the images of the bubble and of the celestial body move the same amount in the same direction for a given inclination of the axis of the telescope. The two images may not move at the same rates; actually the bubble appears to move faster than the object in the instruments which the writer has so far examined. The motion of the bubble can be slowed down by the use of a more viscous liquid, but such a liquid would be seriously affected by low temperatures.

In the Willson sextant the bubble is contained in the lower portion of the vertical tube (B) attached to the telescope. An electric lamp (L) is mounted at the top of this tube to illuminate the bubble at night. A cap with a small central hole is placed just under the lamp to limit the illumination when sighting on a star. If this cap is removed, the illumination from the lamp is suitable for sun observations, or if both the lamp and the cap are removed, the light from the sky is sufficient.

The rays of light pass through the bubble tube, and then are reflected to the eye from a semisilvered surface set at 45° to the axis of the telescope. The rays from the object pass through this surface on their way to the eye.

The arm of this sextant is controlled by a rack, worm gear, and micrometer head, reading to 30 seconds of arc, just as described for the Byrd sextant. A lamp (L′) is provided for illuminating the scale at night.

The two lamps are controlled by two push buttons (P) conveniently situated on the handle (H) of the sextant. A battery is set into the handle to furnish power for the lamps.

The sunglasses (S) should be placed in front of the horizon glass (G) in order to shut out the view through the unsilvered portion of this mirror. This instrument can be used as an ordinary sextant by turning the sunglasses behind the horizon glass down out of the way and using a natural horizon instead.

Schwartzchild bubble sextant.—This sextant (fig. 16C), of German origin, was loaned by the United States Coast and Geodetic Survey for examination in connection with this report. The bubble is of the ordinary tube type and is located just above the lamp (L) in the case (C). The rays of light from the bubble are reflected through the hole in the horizon glass (G) to the eye. For day observations, the lower portion of the case (C) can be folded back as shown in the figure, and the white celluloid reflector (W) used to illuminate the bubble by means of ordinary daylight. For night observations, the bottom of the case is folded down so as to inclose the lamp. A shield in front of the lamp prevents the direct rays of the latter from reaching the bubble. Only the rays reflected from the sides illuminate the bubble so that the intensity is reduced.

The battery for the lamp is contained in the handle (H). The current is turned on by means of a plug at the end of the handle hidden behind the case (C). This plug is arranged with a variable resistance, so that the current flowing depends on the amount which the plug is pulled out.

The images of the sun and bubble move in the same direction when the inclination of the sextant is changed and the image of the bubble moves at approximately the same rate as that of the sun.

The arm is set by means of the rack (R) and a pinion operated by the knurled head (K). A vernier (V), reading to two minutes, is mounted on the arm.

R. A. E. bubble sextant.—This instrument (fig. 17) was designed by the British Royal Aircraft Establishment. For sun observations the eye is usually placed at (E_1). The rays from the sun are reflected from the sunglass (S), while the rays from the bubble pass through both the mirror (M) of plain glass and the sunglass. For star observations, the sunglass is rotated back out of the way and the eye is placed at (E_2). The rays from the star then pass directly through the mirror to the eye, while the rays from the bubble are partially reflected by the mirror.

The bubble level (B) is of the spherical type. The lamp (L) provides the illumination for the bubble and the rays are reflected by the prism (P) up through the lens system to the mirror.

The lamp (L_1) illuminates the scale which is on the large knurled head (H). The batteries for the lamps are in the handle of the sextant and are operated by push-buttons located conveniently on the handle.

It is claimed[11] that the focal lengths of the lenses and the curvature of the upper surface of the level are such that the bubble appears to move with the sun or star when the sextant is tipped, thus facilitating observations.

A later model of this instrument includes a number of improvements, including a vapor bubble and a handle mounted on the side of the sextant. The vapor bubble is formed by filling completely the bubble tube and adjusting chamber with the liquid used, and then increasing the volume of the chamber. Since the liquid is unable to expand to fill the increased volume, a bubble is formed which is filled with vapor from the liquid.

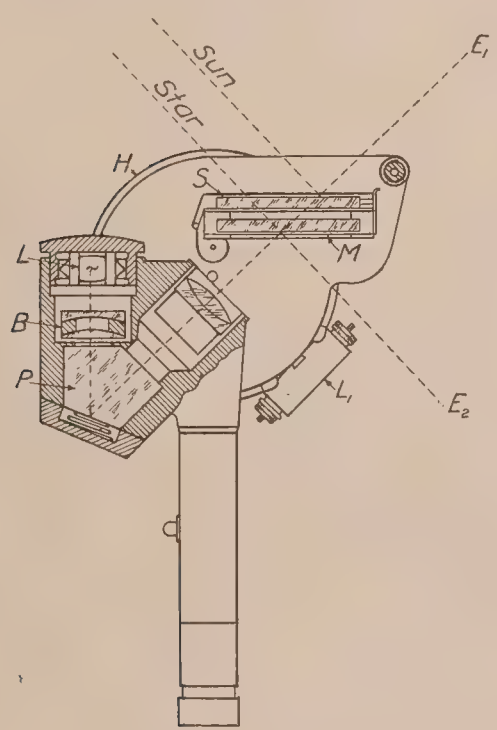

Fig. 17.—R. A. E. bubble sextant.

PENDULUM SEXTANT.

As yet the pendulum sextant has not been utilized in aerial navigation to the same extent as has the bubble type. Several such instruments have been constructed, however, and used successfully on the ground. In at least one case the extreme delicacy of the mechanism is a handicap as regards aerial work.

As a matter of historical interest attention is called to the fact that a sextant utilizing a pendulum artificial horizon was carried on the dirigible *America* when Wellman and Vaniman attempted to cross the Atlantic in 1910.[12]

Pulfrich sextant.—This instrument has been developed by Dr. Carl Pulfrich, of Jena. The telescope is mounted on the arm which moves over the graduated sector, and the whole is so balanced on a pivot that the arc always hangs in exactly the same position regardless of the position of the arm. The telescope is pointed directly at the celestial object. A reflecting prism covers a portion of the field, so that two images of the body are seen—one direct and one reflected. As the instrument oscillates, the two images move in opposite directions and, when they are momentarily in coincidence, the pressure of a finger on a spring stops the oscillation and the reading can be taken. It is claimed that this sextant is both accurate and light in weight.

GYROSCOPIC SEXTANTS.

Two French gyro-sextants are available for use in aerial navigation.

Fleuriais sextant.—This instrument (figs. 18 and 19) was developed some years ago for use in the French Navy by Admiral Fleuriais.[13] It consists of a sextant of the usual form with a gyroscopic horizon attached directly behind the horizon glass.

The horizon is mounted on the upper surface of a wind-driven gyroscope (G), about 2 inches in diameter, which is inclosed in a case. (See fig. 19.) Air is sucked out of the case by means of a pump. Atmospheric pressure then forces air into the case through a small hole so arranged that the air stream impinges upon vanes arranged around the circumference of the gyro, which is thus driven at a high rate of speed. When the gyro has attained the proper speed, valves are closed so as to seal the case hermetically. This is done in order to reduce air friction, so that the gyro may not slow down too much while the altitude is being measured.

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[11] "Design of Instruments for the Navigation of Aircraft," by G. M. B. Dobson, Geographical Journal Vol. LVI, No. 5, p. 370, November, 1920; Royal Geographical Society of London.

[12] "The Aerial Age," loc. cit.

[13] "The Search for Instrumental Means to Enable Navigators to Observe the Altitude of a Celestial Body when the Horizon is not Visible," by G. M. Littlehales. Proceedings U. S. Naval Institute, vol. 44, No. 8, August, 1918. Also "Gyrostatics and Rotational Motion," by Andrew Gray, p. 129.

The horizon consists of a series of equally spaced horizontal lines engraved on a glass plate (H), the central or horizon line more prominent than the others. Rays of light from these lines are made parallel by means of a plano-convex lens (L) so that the lines can be brought to a focus in the plane with the image of the celestial body. As the horizon system rotates with the gyroscope at a high rate of speed, the persistence of vision makes it appear to the observer that he is viewing a fixed horizon.

The image of the celestial body is superimposed on the image of the system of horizontal lines as shown in figure 19. As the gyro precesses, the image of the body appears to oscillate up and down between the lines on the glass. The mean position can be estimated and the reading of the arc corrected accordingly.

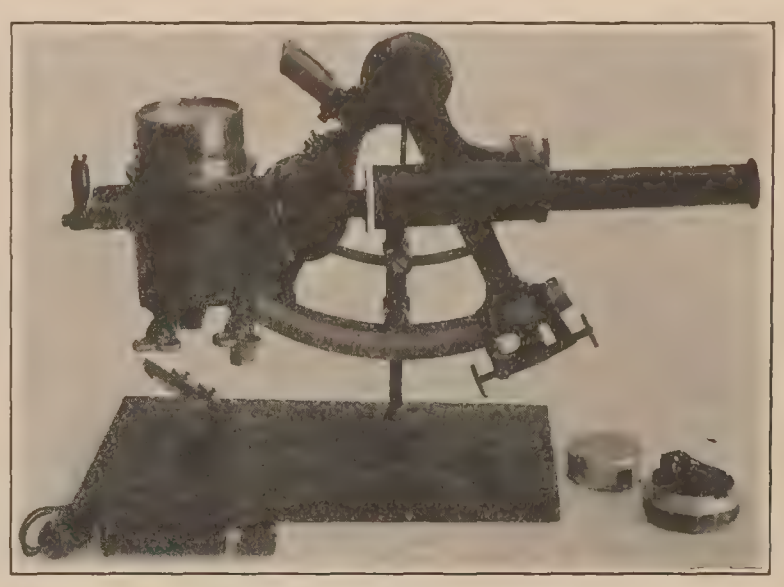

FIG. 18.—Fleuriais gyroscopic sextant.

Derrien sextant.—This is a French sextant (fig. 20) utilizing a gyroscope to furnish an artificial horizon. A telescopic line of sight (T) is mounted rigidly to the frame. The movable arm (A) carries a horizon glass (H) and a gyroscope (G). The latter has a mirror (M) mounted horizontally on its upper surface. The gyroscope serves to keep the mirror (M) horizontal when taking observations.

The operation of this sextant is very simple. The telescope is pointed directly at the celestial body, and the rays of light pass through the unsilvered part of the horizon glass. Rays of light from the object are also reflected as shown in the figure from the gyro mirror, which is horizontal. However, these rays will not be reflected by the horizon glass parallel to the direct rays unless the horizon glass is horizontal, or, in other words, parallel to the gyro mirror. So the arm (A) is moved until the direct and reflected images coincide, when it is known that the horizon

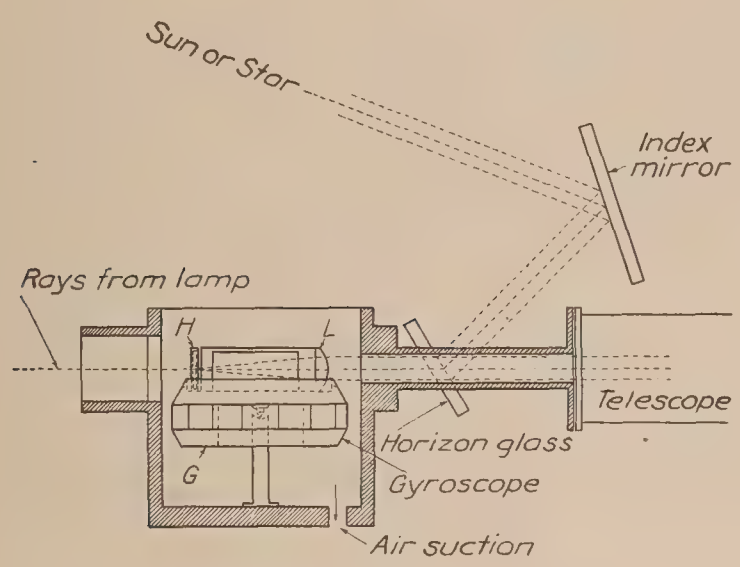

FIG. 19.—Diagram of Fleuriais sextant.

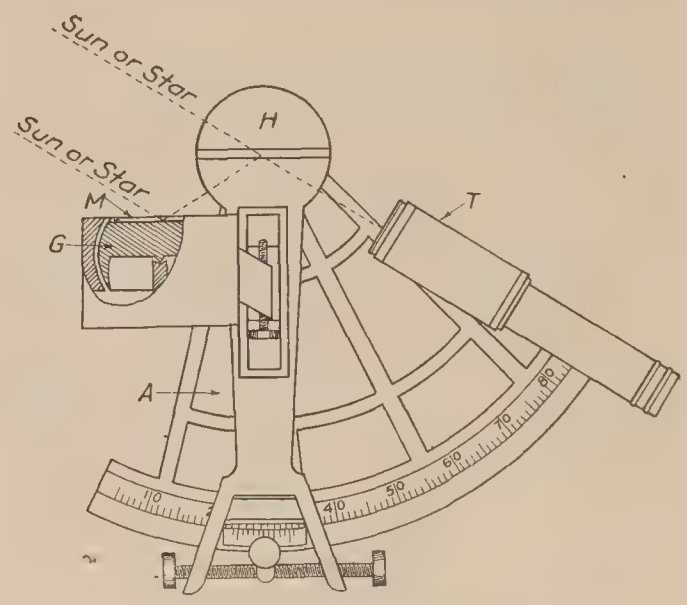

FIG. 20.—Derrien gyroscopic sextant.

glass is parallel to the gyro mirror. The altitude of the body can then be read. The graduations on the scale are equal to the actual angular distances on the arc instead of being compressed to one-half the true length, as in most sextants.

ACCURACY OBTAINABLE.

The question next arises as to what accuracy can be obtained in determining one's position by sextant observations from the air and what accuracy is essential. In the first place, it should be realized that the aerial navigator need not know his location as exactly as the mariner must.

The aircraft is in general not subject to dangers from shoals and hidden rocks, and its superior speed makes an error of from 10 to 20 miles at the end of its journey of little moment, since that distance represents a flight of only a few minutes. For finding the exact location of the landing field other methods are available, and so we can consider that an error of from 10 to 20 nautical miles, corresponding to the same number of minutes of arc, can be permitted.

The usual errors (not including the known corrections for semidiameter and parallax) which may affect the measurement of altitudes with the sextant are (*a*) dip, (*b*) refraction, (*c*) index error, (*d*) bubble error, and (*e*) acceleration errors. It should be observed that the artificial horizon sextant has no dip error, while, on the other hand, the marine sextant can have no bubble error.

(*a*) *Dip.*—The nature of the dip error has already been discussed and it has been pointed out that the dip in minutes of arc may be taken equal to the square root of the height in feet of the aircraft above the horizon used. Terrestrial refraction is included in the law, as above stated, as has been determined by tests made at altitudes up to 12,000 feet.

(*b*) *Refraction.*—Astronomical refraction makes the observed celestial body appear nearer the zenith than it actually is, except in the limiting case, when the body is at the zenith. The magnitude of this error increases as the altitude of the body decreases. Assuming that there will be no occasion to measure altitudes less than 20°, the maximum value of this error is found to be slightly greater than 2.5 minutes. This is for an observation made at sea level, but at 12,000 feet the refraction error is less than 2 minutes. At an altitude of 45° there exist corresponding errors of approximately 1 minute at the ground and 30 seconds at 12,000 feet. For most aerial work this error can then be neglected.

(*c*) *Index error.*—This error occurs if the horizon glass is not exactly parallel to the index mirror when the vernier reads zero. The magnitude of the error can be determined by sighting on a distant object and bringing the direct and reflected images into coincidence. The vernier should then read zero, and if it does not, the reading as thus determined should be subtracted from any altitude measured subsequently.

(*d*) *Bubble error.*—This error is due to the line of sight not being horizontal when the bubble is in the center of the tube. Its magnitude can be determined while the observer is in the air by measuring the dip of the horizon from a known height and comparing the value thus obtained with the true value. The index correction must be applied to the measured dip before comparison with the true dip. Both the bubble and the index errors should be determined for each flight.

(*e*) *Acceleration error.*—Errors due to acceleration are of primary importance when an artificial horizon is used. Only by the best piloting can satisfactory results be obtained. The oscillations of the craft affect the pendulum or the bubble, and if the latter should happen to have about the same period as that of the oscillations of the plane, resonance occurs, and it is hopeless to try to get a reading. It has been reported from the United States naval air station at Pensacola, Fla., that students undergoing NC training have taken sights, using the bubble, with an average error of about 8.5 minutes, and a maximum error which is rarely greater than 15 minutes. If these results represent average conditions, they are excellent. Russell (loc. cit.) reports errors in single readings of from 12 to 14 minutes, using a Willson bubble sextant under what are probably typical flying conditions.

Other less important errors due to the bubble are caused by vibration of the aircraft, unsteadiness of the observer's muscles, and surface tension which may cause the bubble to stick and then move suddenly. Vibration makes it difficult to determine the tangency of the celestial body or its coincidence with the bubble and shakes the liquid in the tube. Unsteadiness of the muscles may be at least partly remedied by a support of some kind attached to the body of the observer.

It is obviously impossible to predict exactly what errors may be reasonably expected under various conditions of flight using different types of instruments. Nevertheless, it is believed that an estimation of the errors involved based on available data may be useful.

	Error of a single observation.		Dip error.
	Good piloting.	Ordinary piloting.	
Naval sextant—Sea horizon	2 to 4 minutes		2 to 4 minutes.
Naval sextant—Level cloud horizon	do		Do.
Naval sextant—Average cloud horizon	15 minutes		Do.
Naval sextant with auxiliary artificial horizon	15 to 20 minutes	40 minutes+	
Bubble sextant	10 to 15 minutes	do	
Gyroscopic sextant	No data		

The above table emphasizes the fact that the ordinary sextant used with a horizon which is known to be level is superior to an instrument utilizing an artificial horizon. It is doubtful whether increased accuracy can be attained by taking a series of observations on a cloud horizon, as the error is usually largely due to the clouds not being exactly level. When an artificial horizon is used, however, no single reading should be relied on unless it is impossible to take a series of observations, since in this case the errors are largely due to accelerations of the craft.

CHRONOMETERS.

While chronometers are among the most essential instruments used in aerial navigation, they are so common a type of instrument that no description will be given here. It is advisable to carry at least two chronometers on board the craft, one keeping sidereal time, the other solar time, both for the meridian of Greenwich. The chronometer keeping solar time may be set either to apparent or to mean time. In the former case a slight error will be involved owing to the continual variation in the equation of time, but in the case of flights which last for less than 24 hours this error will be small.

Extreme accuracy in the chronometers is not essential where time signals can be picked up by wireless. If these are not available, it will be advisable to carry a standard chronometer with which the other two can be compared.

REDUCTION OF OBSERVATIONS.

PRECOMPUTATION.

Computation in the air is always difficult, except possibly in the case of a large dirigible. In an open airplane, the force of the wind, the necessity for wearing gloves in most cases, and the speed with which the results must be obtained make quick methods for determining the geographical position of the aircraft from the measured altitude an absolute essential.

Under such circumstances it is often advantageous to compute the altitude and azimuth of the sun, or of any other celestial body which is likely to be used, for certain assumed positions of the aircraft which can be predicted in advance. These positions correspond to the dead reckoning position assumed in the St. Hilaire method, and while they may be in error by from 50 to 100 miles, this fact will have little effect upon the accuracy of the result provided a suitable map is used for plotting the position of the craft. The altitudes and azimuths of the body as it would be seen from each assumed position are computed and tabulated in convenient form for different times during an interval which is so chosen as to include the time when the aircraft will pass near the point.

The work involved in plotting the position can be partly anticipated by drawing from the assumed positions radiating lines in the direction of the sun's azimuth as computed for various times.

The work which must be performed in the air is then very simple. The altitude of the body is measured, the difference in minutes of arc between the measured and the computed altitude is laid off in nautical miles along the proper azimuth line as determined from the table, and a line drawn at right angles to the azimuth line through the point so formed is the desired line of position.

COMPUTATION IN THE AIR.

Precomputation will not always serve one's purpose on account of weather conditions which prevent observations at the proper time on the body whose altitude has been computed. Consequently much effort has been made to provide suitable means, graphical or mechanical, to make any desired computation in the air. A description of these methods will be included in the discussion of the reduction of observations which follows.

The fundamental equation to be solved in reducing an observation is

(*a*) $\sin h = \sin L \sin D + \cos L \cos D \cos t$

where L is the latitude of the observer,
h is the altitude of the celestial body,
D is the declination of the celestial body,
t is the hour angle of the celestial body.

For convenience in computation this can be transformed to the cosine-haversine form, in which all the quantities are positive and the chance of error is thereby reduced.

(*b*) $\text{hav } Z = \cos L \cos D \text{ hav } t + \text{hav } (L \pm D)$

where Z is the zenith distance of the celestial body
and the haversine is defined as the sine squared of one-half the angle.

The reduction of astronomical observations in aerial navigation may be accomplished in four distinct ways:

1. Logarithmic computation.
2. Use of tables.
3. Graphical methods.
4. Use of mechanical calculators.

LOGARITHMIC COMPUTATION.

Equation (*a*) is probably the most useful for logarithmic computation, as the ordinary trigonometric tables can be used. Aerial work, however, does not warrant this high degree of accuracy, which can be attained only by a comparatively laborious process.

USE OF TABLES.

For the more accurate work required in the air, special tables can be used, such as the "Altitude, Azimuth, and Line of Position Tables," published by the United States Hydrographic Office. These tables contain all that is necessary to determine the altitude and the azimuth, the former accurate to one minute of arc.

The reduction may be made by the use of the double entry "Altitude and Azimuth Tables" of de Aquino. These tables, which may also be found in the above-mentioned publication of the Hydrographic Office, avoid the necessity for logarithmic computation, but involve more chances of error.[14]

The altitude and azimuth of the celestial body may be obtained from Hydrographic Office Publication No. 201, "Simultaneous Altitudes and Azimuths of Celestial Bodies." These tables give simultaneous values of altitude and azimuth for celestial bodies having declinations within the range $\pm 24°$ and for latitudes up to 60° north and south. When using these tables the observer often finds it possible to assume his D. R. position on an even degree of latitude and in such a longitude as will make the hour angle of the observed body a multiple of 10 minutes from his meridian, and so can reduce the labor of interpolation to a minimum.

The Hydrographic Office is preparing tables for use in aerial navigation comprising simultaneous values of the hour angle and azimuth of the celestial body for values of the latitude of the observer and the declination of the observed body. These tables allow of a quick determination of the Sumner line with very little interpolation.

[14] "Simplified Navigation," by Charles Lane Poor.

GRAPHICAL METHODS.

Altitude, azimuth, and hour-angle diagram.—Littlehales has made use of the haversine formula for solving the astronomical triangle graphically.[15] This form of the equation is

$$(c)\ \text{hav}\ Z = \text{hav}\ (L \pm D) + [\text{hav}\ (180° - (L+D)) - \text{hav}\ (L-D)]\ \text{hav}\ t$$

The quantities involved in this equation have already been defined.

If we now let—

$$y = \text{hav}\ Z$$
$$x = \text{hav}\ t$$
$$a = \text{hav}\ (L \pm d)$$
$$m = \frac{\text{hav}\ (180° - (L+d)) - \text{hav}\ (L-d)}{\text{hav}\ 180°}$$

equation (c) can be written in the form

$$y = mx + a$$

which is the equation of a straight line in terms of its slope and its intercept on the Y axis.

A diagram is now prepared as shown in figure 21 with the haversines of angles from 0° to 180° laid off along the X and Y axes, representing hav t and hav Z, respectively. Now, if we lay off on OY hav $(L-d)$ and on $O'\ Y'$ hav $[180° - (L+d)]$ and connect these points by a straight line, we can immediately solve for t when Z is known, or vice versa.

To do this, we project up from the proper value of t on the X axis to the line just drawn and from this intersection project across to the Y axis where the value of Z may be read.

A relation analogous to equation (c) connecting the azimuth and polar distance $(90° - D)$ exists and may be solved by the same diagram as that for the zenith distance and hour angle.

We have —

$$\text{hav}\ (90° \pm D) = \text{hav}\ (L-h) + \{\text{hav}\ [180° - (L+h)] - \text{hav}\ (L-h)\}\ \text{hav}\ A.$$

Where A is the azimuth of the celestial body and the other quantities have already been defined.

By laying off $(L-h)$ and $[180° - (L+h)]$ in place of $(L-D)$ and $[180° - (L+D)]$ and proceeding as before we can determine the azimuth if the polar distance is given or vice versa.

ILLUSTRATIVE PROBLEM.

Latitude by D. R. = 50° N.
Declination of star = 30° N.
Computed hour angle of star = 105° = 7 hours.
Measured altitude (corrected) 13° 56′.

(a) Altitude:

$$L+D = 80°,\ L-D = 20°.$$

Laying off $(L+D)$ on the $O'Y'$ scale, $L-D$ on the OY scale, connecting these points with a straight line, projecting up to this line from $t = 7$ hours on OX, then from this intersection projecting horizontally to OY we find

$$Z = 76° \text{ or } h = 90° - 76° = 14°.$$

(b) Azimuth:

Using the value for the altitude as just found, we obtain $L+h = 64°$, $L-h = 35°$, also $90° - D = 60°$.

Laying off $L+h$ on the $O'Y'$ scale, $L-h$ on the OY scale, connecting these points with a straight line, projecting horizontally from $(90° - D) = 60°$ to this line, then projecting down from this intersection we obtain $A = 59°\ 45'$.

We now have the information necessary to draw the Sumner line on the map.

Accuracy.—This diagram, if so constructed 15 inches square, will give an accuracy of from 5 to 10 minutes of arc.

[15] "Altitude, Azimuth, and Hour-Angle Diagram," by G. W. Littlehales. Proceedings U. S. Naval Institute, November, 1917.

Rotation of astronomical triangle.—Suppose, for the moment, that we have available a sphere with hour circles and the equator drawn upon it (fig. 22). The hour circles will be graduated from 0° at the pole to 90° at the equator and the equator will be graduated from 0° at one intersection of the meridian, which will be taken as the meridian of the observer, in both directions to 180° at the other intersection of the same meridian with the equator.

Now, in figure 22, let (P) be the pole; (Z) the zenith of the observer; (S) the celestial body; (PD), (PS), and (PC) three hour circles, (PD) being also the meridian of the observer, and (DCB) the equator. (PZS) is the astronomical triangle which is to be solved for (ZS) the zenith distance and angle (PZS) the azimuth of the body.

Let us now rotate the astronomical triangle about the equatorial axis (AB), which is perpendicular to the plane of the observer's meridian, by sliding it over the surface of the sphere, the side (ZP) remaining in the observer's meridian, until (Z) coincides with the pole (P) of the sphere. (S) rotates into the point (S′), and since the angle (PZS) is unchanged by the rotation, (P′PS′) is equal to (PZS). But we can read the angle (P′PS′) from the grad-

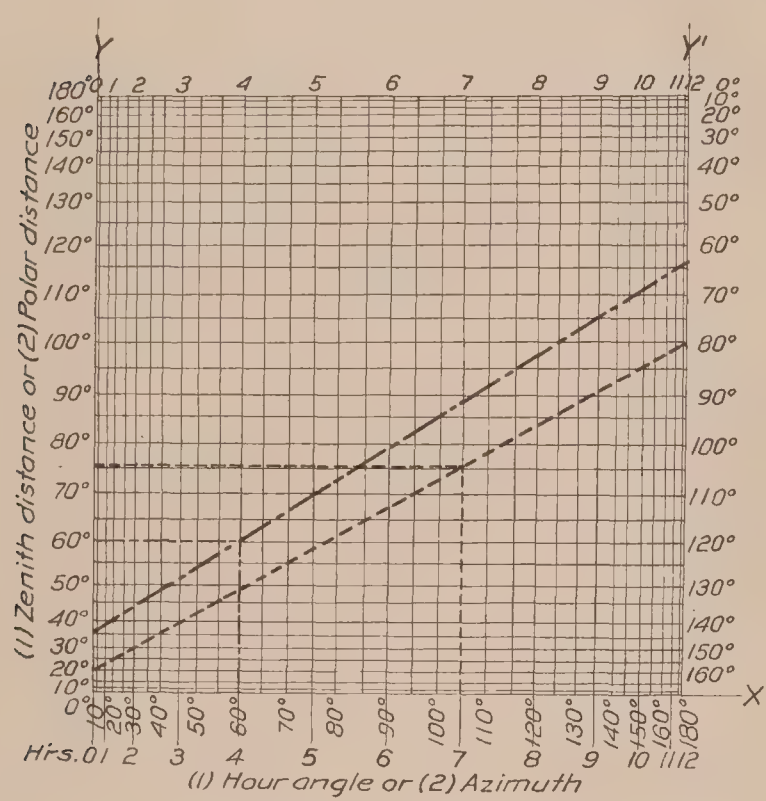

FIG. 21.—Altitude, azimuth, and hour-angle diagram.

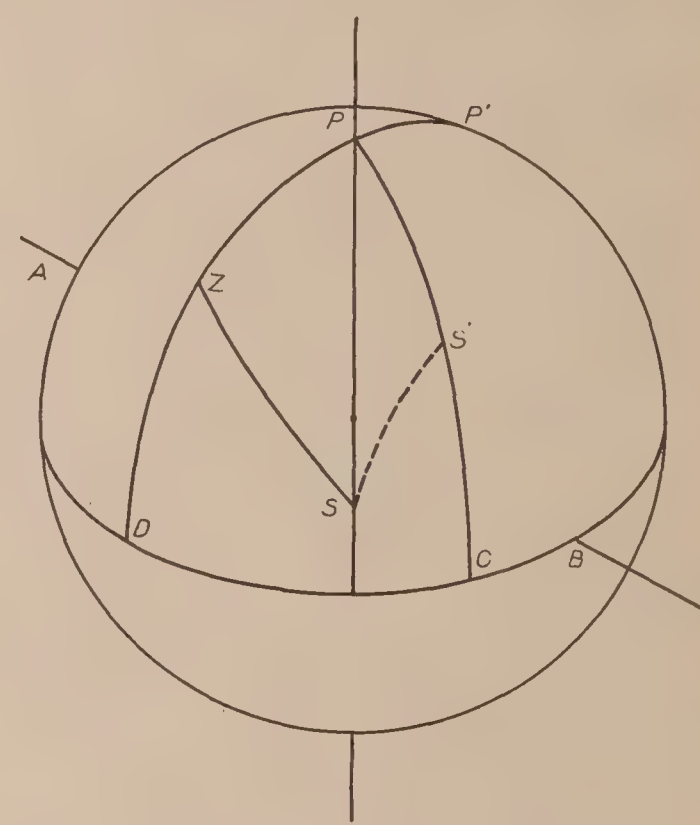

FIG. 22.—Rotation of spherical triangle.

uated equator, for the sides of the angle lie along hour circles. We can also read the arc (PS′) from the graduated hour circle (PC). The desired quantities have thus been found.

To obtain the necessary accuracy, it would be necessary to use a sphere or a portion of a sphere from 10 to 15 feet in diameter. This is plainly impracticable. Consequently several methods have been devised for performing the rotation of the triangle on a projection of the sphere instead of on the sphere itself.

Littlehales method.—Littlehales has used stereographic meridian projection for this purpose.[16] Figure 23 shows the stereographic projection of the hemisphere determined by the observer's meridian (DZPP′) and including the astronomical triangle upon the plane of this meridian. (PZ) is the colatitude of the observer, (PS) the codeclination of the celestial body, and (ZPS) the hour angle of the body. We are to determine (ZS) the zenith distance and angle (PZS) the azimuth of the body.

To do this we rotate the astronomical triangle upon the projection just as we did upon the sphere. This is made possible by superimposing upon the system of meridians and parallels of latitude a network of equally spaced concentric circles drawn about (O), the center of the

[16] "Altitude, Azimuth, and Geographical Position," by G. W. Littlehales. Published by Lippincott.

projection, and of radial lines through (O). It should be observed that (O) is the projection of the equatorial axis (AB) of the sphere (fig. 22). The system of circles is numbered consecutively from the center (O) to the bounding meridian of the projection, and the radial lines are marked by numbers which indicate their angular distance in minutes of arc from the meridian (ON).

In practice it is merely necessary to rotate point (S). Note the numbers of the radial and the circumference passing through (S). Add to the number of the radial the colatitude (ZP) in minutes of arc to find the radial upon which (S′), the rotated position of (S), will lie. Find the intersection (S′) of the new radial with the same concentric circle that passes through (S). The rotation is now completed and the azimuth (P′PS′) and the zenith distance (PS′) can be read from the graduated arcs.

To attain the necessary accuracy, the projection has been made for a sphere 12 feet in diameter. The projection is subdivided into 368 overlapping sections, which are numbered in order and arranged in a compact form. A key diagram is provided showing the whole projection with its subdivision into sections. In this way the navigator can easily observe in what sections the points (S) and (S′) lie, and can turn to the corresponding plates in order to read off the desired values.

In figure 23 the meridians and parallels of latitude are shown as full lines. The superimposed network of concentric circles and radials used for performing the rotation is shown in light dash lines in the upper half of the projection. The astronomical triangle is shown in its original position in heavy full lines; it is shown in its rotated position in heavy dash lines. The subdivision of the projection into overlapping sections is not indicated in the figure as the addition of another system of lines would be confusing.

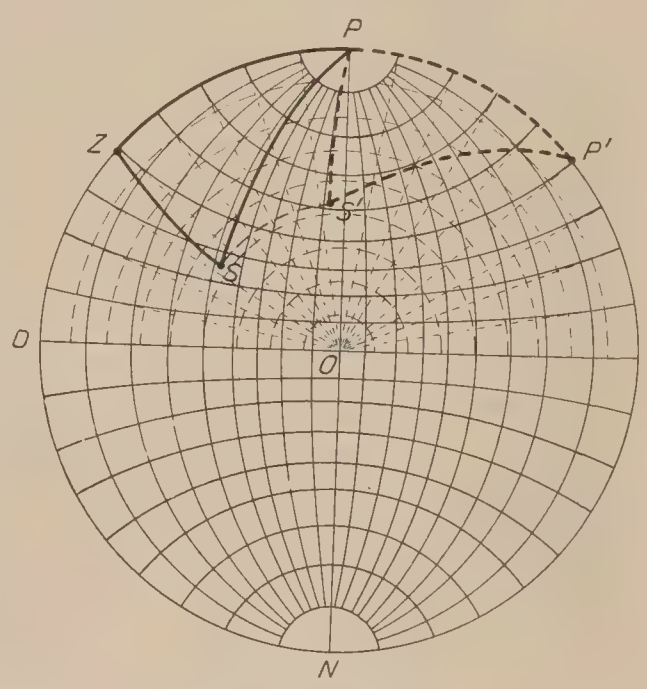

FIG. 23.—Littlehales diagram.

Veater diagram.—Commander Veater, of the Royal British Navy, has used the Mercator projection for this same purpose.[17] The observer's meridian is taken as the equator of the projection. Thus rotation about the equatorial axis AB of the sphere (fig. 22) becomes translation parallel to the observer's meridian.

Here, as in the Littlehales method, the projection is subdivided into a larger number of overlapping plates and a key diagram to these plates is provided.

MECHANICAL DEVICES.

Line-of-position computer.—This slide rule (fig. 24), designed by Charles Lane Poor, for solving the cosine-haversine formula, is in the form of a circular metal plate engraved with a group of scales necessary to solve for the altitude and azimuth. This plate is surmounted by a movable celluloid disk carrying a radial reference line and by an arm which can be rotated to any point on the circumference of the disk and clamped there. The instrument gives an easy and rapid reduction, one which is amply accurate for aerial work. A description of this computer and its use and a comparison of the results obtained by the logarithmic, Aquino, and mechanical methods is to be found in "Simplified Navigation" (loc. cit.).

Bygrave slide rule.—A description of the construction and use of this computer can be found in "Air Navigation" (loc. cit.). The slide rule is designed to solve the two right-angled spherical triangles into which the astronomical triangle can be divided. It is in the form of two concentric cylinders, about 6 inches long and 3 inches in diameter, each carrying a spiral scale. It is claimed that an accuracy of one minute of arc is attainable in nearly all cases.

[17] "Air Navigation," loc. cit.

Nomogram slide rule.[18]—A slide rule for solving the formula on which the altitude, azimuth, and hour-angle diagram is based has been devised by Wimperis and Horsley. The name of the slide rule is explained by the fact that the British use this same diagram under the name of the "Spherical Triangle Nomogram."

Mechanical navigator.—Instruments which are mechanical representations of the celestial sphere have been built to solve either a single astronomical triangle or two triangles simul-

FIG. 24.—Line-of-position computer.

taneously. Graduated circles which can be turned and rotated with respect to each other represent the equator, the meridians of the observer and the celestial body, and the great circle passing through the positions of the observer and the celestial body. Verniers are provided for reading the angles.

In one instrument of this type which the writer examined, the verniers read to 30 seconds of arc. It was clear that if the required accuracy were reduced to, say, three minutes of arc,

[18] "Air navigation," loc cit.

which is sufficient for aerial purposes, the instrument could be sufficiently reduced both in size and weight for use on dirigibles.

METHODS OF PLOTTING POSITION.

The usual method of plotting position has already been described. The dead reckoning position is plotted on the map, a line is drawn through this point in the computed direction of the sun, the difference between the observed altitude and the computed altitude in minutes of arc is laid off along the azimuth line in nautical miles. If the observed altitude is greater than the computed, the true position is nearer the geographical position of the sun than the assumed dead reckoning position. A line drawn through the point thus found perpendicular to the azimuth line is the desired line of position. If we have an observation on only one body, the intersection of the azimuth line and the line of position is the most probable position of the craft.

Littlehales method.—Littlehales has originated a simplified method of plotting position.[19] He selects a suitable position within the limits of the chart and computes the simultaneous altitudes and azimuths of selected celestial bodies as they would appear if viewed from the chosen position at definite intervals of time. These quantities are tabulated for use in the air.

The observer, knowing himself to be within the limits of his map, measures the altitude of one of the selected celestial bodies. Comparing the measured altitude with the altitude computed for the body at the given instant if viewed from the chosen position on the map, he may find a difference as great as several degrees, depending on the area of the earth's surface included in his map. He measures off along a line drawn through the chosen point in the direction of the celestial body a distance which equals in nautical miles the altitude difference expressed in minutes of arc. Then he knows himself to be on the line of position drawn through the point so found perpendicular to the azimuth line.

Figure 25 is a representation of the earth. The area included in the rectangular figure (MM′NN′) is that included in the observer's map. (O) is the chosen point of this area for which the simultaneous altitudes and azimuths are calculated. A compass diagram is drawn with (O) as center to facilitate laying off the azimuth line for the body the altitude of which is to be measured.

Suppose that at a given instant the altitude of a star whose geographical position is shown at (S_1)is measured. Assume that this measurement gives a zenith distance (S_1A_1) as against a computed zenith distance (S_1O). The observer then takes the difference of the measured and computed zenith distances (or altitudes) and lays off the corresponding distance (OA_1) along the computed azimuth line of the star, measuring toward the star if the measured altitude is greater than the computed, and away from the star if the reverse is true. He knows that he is on the arc of the small circle having (S_1) for center and passing through (A_1). Notice that (A_1), however, is not the most probable position of the craft since (O) was not necessarily the most probable position before the altitude was measured.

If we can measure the altitude of another star whose geographical position is, for example, (S_2), we can establish another small circle, this one having (S_2) for center and passing through (A_2) where (OA_2) is the altitude difference for the second body. The two small circles intersect at (P), which is then the location of the observer.

For ease in plotting position by this method, it is essential that we use a projection by means of which great circle arcs plot practically as straight lines within the required limits. The Mercator projection is not satisfactory for a case where the altitude difference exceeds a few minutes of arc. Littlehales has selected the ordinary or American polyconic projection and the Lambert zenithal projection for this particular use as possessing the desired characteristics. With these projections it is possible to replace the arcs of the circles through A_1 and A_2

[19] "The Chart as a Means of Finding Geographical Position by Observations of Celestial Bodies in Aerial and Marine Navigation," by G. W. Littlehales. Proceedings of the U. S. Naval Institute, March, 1918.

by straight lines without causing too great an error in the position of the intersection. The Lambert zenithal projection was used in the aerial navigation chart of the North Atlantic Ocean which was prepared by Littlehales for use on the transoceanic flight of the United States Navy *NC* boats in 1919.

Favé method.—Favé has used stereographic projection in a way somewhat similar to that described above. He selects the central point of the map as the fixed point for which the simultaneous altitudes and azimuths of suitable stars are computed. He then constructs on a sheet of paper a series of arcs of circles of equal altitudes for the chosen stars. The circles of equal altitudes plot as circles on the stereographic projection. A straight line is drawn through the centers of the arcs.

The map is constructed on transparent paper. It is laid over the paper on which the series of arcs is drawn and is oriented; i. e., it is turned until the straight line through the centers of the arcs becomes the azimuth line for the observed star from the central point of the map. It is then moved parallel to the azimuth line until the altitude circle passing through the central point corresponds to the altitude given by the table. The observer can then trace on the map the required portion of the altitude circle corresponding to the measured altitude to obtain the line on which he is located. Figure 25 will aid in making this method clear.

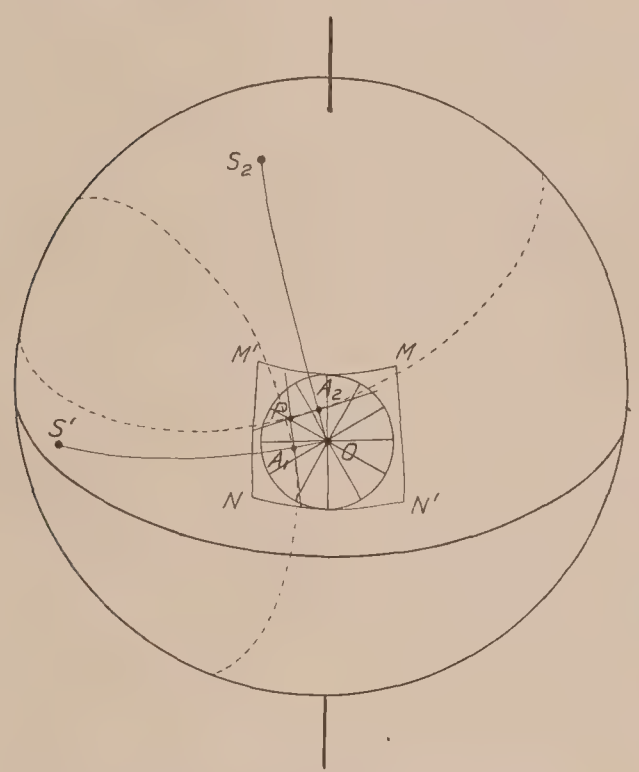

Fig. 25.—Littlehales method of plotting position.

Baker navigating machine.—An instrument for obtaining the curve of position without the necessity of computing an altitude and azimuth has been developed in England.

This machine carries in a case the map of the region over which the flight is to take place with a special chart superimposed on the map and capable of being moved over the latter by means of rollers. This chart carries on its surface lines of equal altitudes for the sun. The equal altitude lines are the Mercator projections of portions of the equal altitude circles which may be imagined to exist on the earth's surface, having for center the subsolar point (assuming the earth to be a sphere). A moment's reflection will make it clear that these circles move westward over the earth as the latter rotates and north or south as the declination of the sun changes. Only the east to west motion is reproduced on the map by sliding the chart over it. Changes in declination cause changes in the spacing of the curves and so it is necessary to make different charts for different declinations of the sun. This has been done from 24° N. to 24° S. by 4 degree intervals. Consequently the declination for which the chart is drawn need never be more than 2° from the true declination. To take account of the small difference which may then exist, a first-order correction is made to the observed altitude by multiplying the difference in the declination by the cosine of the hour angle. This computation is made by means of a slide rule attached to the navigating machine.

The only operations necessary to obtain the Sumner line on the map by this method when the altitude of the celestial body has been determined are to slide a special transparent chart over the map until the proper time as indicated on the time scale of this chart is opposite the meridian, to correct the indicated altitude for the difference in declination (of the chart and the celestial body) by means of a slide rule attached to the machine, and to mark the Sumner line on the map by means of carbon paper.

A chart carrying equal altitude curves for six first magnitude stars has also been prepared for the machine. For stars the change in declination is negligible, so that one chart is sufficient and no correction need be applied.

Brill method.—Brill, in Germany, has used azimuthal (or zenithal) projection in a very similar manner. Brill constructs on a strip of tracing cloth curves of position corresponding to altitudes of selected bodies. This is mounted on rollers so that it can be slid under the map, which is drawn on transparent paper. By orienting the map according to the azimuth taken from the table, and sliding the strip of cloth to a position which depends upon the altitude given by the table, the observer can trace on his map the curve of position corresponding to the measured altitude.

DIRECTIONAL WIRELESS TELEGRAPHY.

The radio direction finder is a recent development which is useful to both mariners and aviators. It will be of particular value in commercial aviation for position determination and for landing purposes.

Previous to the war with Germany a small amount of work had been done toward determining the position of aircraft by means of the directive properties of a radio receiving coil, but it was only during the war that this method came to be used in a practical way.

FUNDAMENTAL PRINCIPLE.

If a rectangular coil is mounted in a vertical plane so that it can be revolved about a vertical axis and its ends connected to suitable receiving instruments, the device can be utilized to determine the direction from which the received waves are coming. The directive properties of the coil can perhaps be most easily understood by considering the effect on the coil of the magnetic force which accompanies the radio waves. This force is horizontal and at right angles to the direction of travel of the waves. Now, if the plane of the coil is parallel to the direction in which the waves are traveling, the coil will be threaded momentarily by magnetic lines of force which vary with the phase of the wave from a maximum in one direction to a maximum in the opposite direction. This change in the magnetic lines of force sets up in the coil an induced current which is a maximum for the assumed position of the coil but which decreases to zero as the latter is rotated until its plane is parallel to the direction of magnetic force, or, in other words, is at right angles to the direction of travel of the waves. This is the fundamental principle involved in the radio direction finder.

METHODS OF USE.

There are two general methods of navigation by means of the radio direction finder. In the first the craft transmits and fixed radio direction finder stations on the ground determine its direction, compute the position, and transmit it by radio back to the ship. In the second method the ship carries its own direction finder, with which it determines the direction of two or more fixed beacon stations, which transmit upon request, or, as will probably be the case in the future, send out signals at regular intervals. The first method is desirable for aerial use, since the apparatus which must be carried is lighter, but it has several disadvantages. As the demands upon the ground stations grow heavy, considerable delay and confusion may be experienced in keeping the necessary number of aircraft informed as to their location. For military use, too, this method is impracticable, since the call for position would immediately reveal the position of the craft to the enemy. The Germans used this method in connection with the Zeppelin raids on England, and consequently the British were able to trace the progress of the big dirigibles as they crossed the North Sea.[2] Present indications are that the future will see the direction finder carried on board the aircraft with beacon stations sending signals at regular time intervals.

A number of modifications of the apparatus for use in the air or on the ocean are possible. It is obvious that either the maximum or the minimum intensity of signal can be used to determine the direction. Theoretically the minimum method is more desirable owing to the fact that the point of extinction of the signal is more clearly defined than the point of maximum intensity. Practically, however, the noise in the receiving set caused by the ignition currents of the engines masks the point of extinction, so that instead of a point a zone of extinction is found. The operator finds the limits of this zone and bisects the angle.

[2] Aeronautical Journal, August, 1919, p. 465.

To overcome this objection to the minimum method, the British have adopted the double-coil aerial. Two coils at right angles are used. The approximate direction is found by means of one coil using the maximum method. Then the second coil is put in the circuit by throwing a switch and, if this coil is not exactly in the position for minimum signal, its effect will be to add or to subtract from the signal received by the maximum coil. If the second coil is in the position for minimum signal, throwing the switch produces no change in the intensity of the signal received on the maximum coil from the direction of which the position of the transmitting station is then noted. It will be observed that this method preserves the sharpness of the minimum method, at the same time yielding a loud signal.

The single-coil system is favored by the United States Naval Air Service. By improved methods of eliminating the disturbances due to the magneto, it has been found possible to use the minimum method with a single coil, this having the advantage of being much more rapid than the double-coil system.

Another modification is possible in the choice of either the fixed or the rotating coil aerial. When the first is used, a large fixed coil is mounted on the wing and the aircraft must itself be turned until it is headed toward the beacon station. This arrangement is particularly useful where it is desired to fly directly toward a particular station. It should be remembered that this procedure will not give the shortest path to the station, as the drift due to the wind will sweep the ship to one side and thus cause it to approach the station following a "curve of pursuit."

A fixed double-coil aerial may be used on the wing in combination with two smaller coils mounted at right angles to one another in the fuselage and within which a small "search coil" is rotated. Rotating the search coil is equivalent to turning the large aerial, but the much smaller size and weight of the former, as well as its sheltered position, make it far easier to manipulate. This arrangement increases the difficulties of tuning.

When the rotating aerial, either single or double, is used, it is mounted in the fuselage.

SOURCES OF ERROR—ACCURACY ATTAINABLE.

It may be said that outside of instrumental errors which can be compensated there are two general sources of error encountered: (*a*) Distortion due to neighboring objects, such as the engine and other metallic parts of the craft, and (*b*) a change in the direction of the advancing wave front owing to atmospheric conditions. Strays and static disturbances may cause a great deal of trouble, but they can hardly be classed as errors. Distortion due to neighboring objects is practically all due to the craft itself when flying. It can be determined by swinging the ship, just as is done in compensating the compass. The atmospheric effect is very marked at night in the case of long wave lengths. Deviations of the order of 90° may occur in the case of very long continuous waves,[21] but this effect is fortunately quite small for the shorter wave lengths which are ordinarily used for radio communication with aircraft.

A difficulty is found in the electrical disturbance caused by the magneto. It has been found necessary to cut out the short wave lengths given off by the latter in order to decrease the noise which these waves cause in the telephones. This may be accomplished by screening the whole magneto system or by reducing the radiating surfaces.

The magneto was a serious source of trouble in the transatlantic flight of the *NC* boats. Owing to the haste with which the project was organized, there was not sufficient time to develop the wireless direction-finding apparatus to the proper point, and, in the case of the *NC-3* at least, it was found impossible to use the system for distances of more than 10 miles because of disturbances caused by the engines.

The British claim to have obtained during a number of test flights an average error of $1\frac{3}{4}°$ in bearing using beacon stations at distances ranging from 20 to 500 miles. This corresponds to an average error in position of 7 miles. The sensitivity of the system was increased to $\frac{3}{4}°$ by

[21] Bureau of Standards Scientific Paper No. 353, "Variation in Direction of Propagation of Long Electro-Magnetic Waves," by Lieut. Commander A. Hoyt Taylor, U. S. N. R. F.

making the area turns on the minimum coil about eight times as great as those on the maximum coil.

For a fuller description of the principles and application of the radio direction finder the reader is referred to the footnotes below.[22]

METHODS OF PLOTTING POSITION.

The charts used for marine navigation are usually based on the Mercator projection. The path followed by the wireless wave reaching the aircraft from a fixed station is the arc of a great circle and, on the Mercator projection, great circles plot as curved lines. So to plot the position of the aircraft the navigator should draw through each fixed station a curve, which is the arc of a great circle and which will have the proper direction at the point finally determined as the location of the ship. There is no direct geometrical solution of this problem known.

Three methods may be used to determine the position: (*a*) A conversion angle may be computed and applied to the observed bearing to get the rhumb line (the straight line joining two points on the Mercator projection), (*b*) the Weir azimuth diagram may be used to plot the position on the Mercator projection, or (*c*) a different projection, such that great circles plot as straight lines, may be used.

(*a*) *Conversion angle.*[23]—A conversion angle may be computed and applied to the bearing of the great circle passing through the fixed station and the craft to give the approximate bearing of the rhumb line through the same two points. This angle, shown in figure 26, is equal approximately to half the difference of the longitudes of the station and the ship multiplied by the sine of their mean latitude. So, by using the dead reckoning position of the craft, the conversion angle can be computed and the rhumb line drawn. A second rhumb line through another beacon station and intersecting the first rhumb line gives the position. This method can be used up to distances of about one thousand miles.

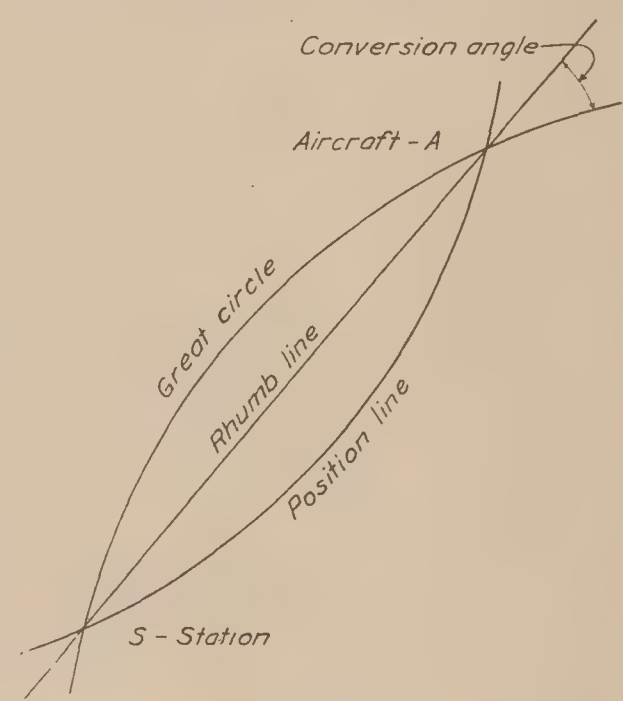

FIG. 26.—Conversion angle.

(*b*) *Weir azimuth diagram.*—In figure 26 are shown the great circle, the rhumb line, and the position line cutting the locations of the beacon station (S) and the aircraft (A) as plotted on a Mercator chart. The position line is the locus of points such that all great circles passing through (S) and cutting this line at consecutive points make the same angle with the meridians through these points. Hence we know that the aircraft is located somewhere on the position line (SA), but we do not know the point on this line. Now, if we obtain another position line from a second station, the intersection of the two will locate the aircraft.

The Weir diagram can be used to facilitate the plotting of the position lines on the Mercator projection owing to the fact that these lines are straight on this diagram. The sending station is marked on the zero hour angle line of the Weir diagram at the proper latitude. A straight line is then drawn from this point with a bearing equal to the observed bearing. The points in which this position line cuts the hour circles and the latitude circles are transferred to the Mercator projection. The line drawn through these points then determines the position line on the latter. Proceeding in the same way, the navigator can draw a second position line on the Mercator chart, and thus determine the position from the intersection of the two.

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[22] Bureau of Standards Scientific Paper No. 354, "Principles of Radio Transmission and Reception with Antenna and Coil Aerials," by J. H. Dellinger.

Bureau of Standards publication (forthcoming), "The Radio Direction Finder and Its Application to Navigation," by F. A. Kolster and F. W. Dunmore.

"Direction and Position Finding," by H. J. Round, Journal Institution of Electrical Engineers (British), No. 58, pp. 224-247 and 247-257, March, 1920.

"Wireless Navigation for Aircraft," by J. Robinson, Electrician, No. 83, p. 420, October, 1919.

[23] Air Navigation (loc. cit.).

(c) *Gnomonic projection.*—The determination of position by the radio compass may be simplified by use of the gnomonic projection. A gnomonic chart is obtained by projecting points on the earth's surface onto a tangent plane by means of lines drawn from the center of the earth through the points. Thus it can be seen that great circles plot as straight lines on this projection.

Then, if the bearing of the ship is measured from the fixed station, it is simply necessary to plot a straight line through the station with a bearing equal to the observed bearing and it is known that the ship is located on this line. A similar line from another station will give an intersection which locates the ship definitely.

If, however, the bearing of the shore station is measured at the ship the problem is more complicated, since the shore-to-vessel bearing is not the same as the vessel-to-shore bearing on account of the fact that great circles do not cut successive meridians at the same angle. This difficulty has been overcome, it is claimed, in a method devised by E. B. Collins, of the United States Hydrographic Office.[24] He makes use of the gnomonic projection, together with a specially arranged compass diagram for each fixed station, so that the observed bearing can be laid off from the fixed station.

Still another method has been developed by Littlehales,[25] in which the navigator determines a position line by means of an observed vessel-to-shore bearing and two or more assumed latitudes.

Littlehales also points out in his discussion of the method just mentioned that Hydrographic Office Publications Nos. 71 and 120 (Time Azimuth tables) can be utilized to furnish a simple and accurate method of plotting a position line for the ship when the shore-to-vessel bearing from a given fixed station has been determined and transmitted to the ship by radio. The approximate latitude of the vessel will be known, so that certain limiting latitudes can be determined. Then, by the use of the above-mentioned tables, taking for latitude the latitude of the fixed station, for declination the latitude of the vessel, and for azimuth the shore-to-vessel bearing, the hour angle (or longitude of the vessel measured from the fixed station) corresponding to the assumed declination can be read. Thus one point of the position line can be plotted. By determining the hour angles corresponding to the limiting latitudes (or declinations) of the vessel and to as many intermediate latitudes as may be necessary for purposes of plotting, the navigator can obtain a sufficient number of points to enable him to plot the position line.

The use of the gnomonic projection with special compass diagrams appears to be the best for use in aerial navigation. The work of plotting the position line is extremely simple and the map can be reduced to a size which is suitable for use either in airplanes or dirigibles.

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[24] "Position Plotting by Radio Bearings," E. B. Collins, U. S. Naval Institute Proceedings, vol. 47, No. 2, February, 1921.

[25] "The Prospective Utilization of Vessel-to-Shore Radio-Compass Bearings in Aerial Navigation." G. W. Littlehales Journal American ciety of Naval Engineers, vol. 32 No. 1, February 1920 pp. 38–44.

REPORT No. 132

AERONAUTIC INSTRUMENTS
SECTION VIII

RECENT DEVELOPMENTS AND OUTSTANDING PROBLEMS

BY F. L. HUNT
Bureau of Standards

REPORT No. 132.

AERONAUTIC INSTRUMENTS.

RECENT DEVELOPMENTS AND OUTSTANDING PROBLEMS.

By Franklin L. Hunt.

INTRODUCTION.

This report is Section VIII of a series of reports on aeronautic instruments (Technical Reports Nos. 125 to 132, inclusive) prepared by the Aeronautic Instruments Section of the Bureau of Standards under research authorizations formulated and recommended by the Subcommittee on Aerodynamics and approved by the National Advisory Committee for Aeronautics.

The preceding reports in this series have discussed in detail the various types of aeronautic instruments which have reached a state of practical development such that they have already found extensive use. It is the purpose of this paper to discuss briefly some of the more recent developments in the field of aeronautic instrument design and to suggest some of the outstanding problems awaiting solution. The different types of instruments will be considered as far as possible in the order in which they are discussed in the preceding papers. Practically all the new designs mentioned which are not otherwise accredited have been developed by the Bureau of Standards at the request of and with funds provided by the War and Navy Departments and the National Advisory Committee for Aeronautics.

ALTITUDE INSTRUMENTS.

PRECISION ALTIMETER.

The altitude of aircraft is determined in practice almost exclusively by the use of aneroid barometers, which have been described in detail in Report No. 126. The essential working element in these instruments is an exhausted metal capsule made ordinarily by soldering together two thin corrugated metal disks. The capsule is kept from collapse by the use of internal or external springs. Owing to the imperfect elasticity of the springs and the metal of which the aneroid capsule is made (usually German silver or bronze), the diaphragm, after the distention caused by rising to an altitude, fails to recover its initial position on descent, and the pointer of the altimeter fails to return quite to zero. This is a serious imperfection, particularly when it is desired to use the altimeter in bombing operations, in landing, or in other cases where the altitude must be accurately known. Experiments have led to the development of altimeters in which this difficulty is almost completely overcome. The error due to inelasticity or hysteresis, as it is ordinarily called, has been found to be due almost entirely to the imperfect elasticity of the diaphragm itself. By making the spring so stiff compared with the diaphragm that the elastic effect of the latter is negligible, altimeters have been constructed in which the elastic effect is confined almost entirely to the spring itself. With proper design the hysteresis of the distending spring can be made negligible. In this way altimeters have been made which show a hysteresis error not exceeding 10 feet in altitude, while in ordinary instruments this error may amount to several hundred feet. One of these altimeters is described in Report No. 126 of this series. In a more recent model the mainspring is mounted on knife-edges instead of being suspended, as in the earlier instrument, and phosphor-bronze strips and pulleys are used instead of a sector and pinion in the transfer mechanism.

JENKINS NIGHT ALTITUDE INDICATOR.

A radically different altitude indicator, a device of an optical type, known as the Jenkins night altitude indicator, has recently been developed in England. This consists essentially of two telescopic projectors, which throw spots of light on the ground below the aircraft. One of the projectors is maintained vertical, the other is rotatable about a horizontal axis so that for any altitude the spots from both projectors can be brought into coincidence. The angle subtended by the projectors is then a measure of the altitude. In the actual instrument the rotatable telescope projects on the ground the figures representing the altitude in feet at the time of observation. The range is limited to a few hundred feet, as it is intended for use in landing at night.

PRECISION BAROGRAPH.

When a record of the altitude attained by aircraft is desired, barographs or recording altimeters are ordinarily used. In the most usual types the actuating mechanism consists of a battery of aneroid diaphragms, such as are used in altimeters, which operates a lever arm with recording pen. The hysteresis effects above mentioned in the case of altimeters are equally troublesome in barographs. It has been recently shown, however, that the hysteresis error can be greatly diminished by using instead of the usual battery of ordinary diaphragms a so-called "sylphon" diaphragm element, which is spun in the form of a cylindrical bellows from a single piece of bronze tubing. The "sylphon" diaphragm is maintained distended by an internal coiled spring. In this case, as in that of the improved type of altimeter above described, the elastic effect is concentrated almost entirely in the spring.

THERMOMETRIC ALTIMETER.

Aside from the instrumental errors of altimeters and barographs due to mechanical imperfections, altitude determinations are also subject to temperature corrections resulting from the variations of the temperature of the atmosphere. The scales of ordinary barometers and barographs are graduated either on the assumption that the temperature remains at some arbitrary constant value, ordinarily 10° C., or varies with the altitude in some empirical manner. In either case the readings are subject to corrections, due to the inevitable deviations of the temperature experienced in flight from the standard temperature which was assumed in laying out the pressure altitude scale for the instruments. To overcome this difficulty, an altimeter with temperature compensation has been made, which greatly reduces the errors due to temperature change. This instrument consists of an altimeter of the type described above with aneroid capsule and stiff external spring. In addition, it is provided with a thermometric element, which consists of a bimetallic bar attached to the indicating system, so as to vary the deflection of the pointer according to the surrounding temperature.

"SYLPHON" RATE-OF-CLIMB INDICATOR.

Rate-of-climb indicators generally depend for their operation upon the differential pressures set up between the external atmosphere and a confined volume of air within the instrument. The forces available for operating the indicating mechanism are very slight, consequently if metal diaphragms are used to measure this pressure they must be extremely thin. Recent experiments indicate the possibility of substituting for the ordinary corrugated metal diaphragm a very thin diaphragm of the "sylphon" type. In this way the necessary motion can be obtained while at the same time the volume of the confined air, and consequently the size of the instrument, can be considerably reduced.

RECORDING RATE-OF-CLIMB INDICATOR.

Experiments are also now being conducted on recording rate-of-climb indicators. These instruments give a permanent record of the rate of ascent or descent throughout the period of flight. Here again the problem is made difficult because of the feeble forces available for operating the recording mechanism. On this account an optical record is used, the light from a small incandescent lamp being reflected by a suitable lens and mirror system onto the surface of a light-sensitive paper on the surface of a revolving drum.

COMBINED STATOSCOPE AND RATE-OF-CLIMB INDICATOR.

In certain instances it is desirable to have on aircraft not only a sensitive statoscope to indicate qualitatively very slight changes in altitude but also a less sensitive instrument to indicate quantitatively rates of ascent and descent. To fill this need a combined statoscope and rate-of-climb indicator has been made which can be used either as a sensitive statoscope or as a rate-of-climb indicator. The device consists of a model III Bureau of Standards rate-of-climb indicator described in Report No. 126 provided with a special cut-off valve between the capillary tube and the chamber and a breaking bubble of the ordinary statoscope type. By rotating a thumbscrew the capillary opening can be closed and the bubble tube connected, in which case the instrument becomes a statoscope and the pointer continues to move away from the zero in one direction or the other, according to whether the instrument is ascending or descending, until the bubble breaks. Under the dial and concentric with it is a rotatable sector which is also connected to the thumbscrew. The words "statoscope" and "rate-of-climb indicator" are printed on the sector, and one or the other legend is exposed through a slit in the dial according to the setting of the thumbscrew, so that the aviator always knows which element of the combined instrument is functioning.

AIRCRAFT SPEED INSTRUMENTS.

LOW SPEED AIR-SPEED INDICATOR.

In the navigating of lighter-than-air craft under certain circumstances it is desirable to measure accurately air speeds of less than 10 miles per hour. Air-speed indicators of the ordinary type such as are used on heavier-than-air craft are not suitable for this purpose. The two methods of approach which have been tried recently to satisfy this need include (1) using a Pitot tube in connection with an indicator much more sensitive than is ordinarily made and (2) using sensitive rotating anemometers. It has been found that Pitot indicators with doped fabric diaphragms and thin flat springs for the elastic element can be made sufficiently sensitive for the purpose, but they get out of adjustment easily. The problem of developing an air-speed indicator of the anemometer type was therefore attacked. This is complicated by the difficulty of transferring the motion of the anemometer to the instrument board without putting a sufficient load on the anemometer to cause it to slip appreciably. Synchronizing commutators have been tried in the French Favre-Bulle indicator, which is of this type. Another method which has been applied in a recent model is that of automatically charging a condenser and discharging it by means of a suitably constructed commutator through a sensitive two-pivot galvanometer at rapid intervals proportional to the rate of revolution of the anemometer. The effective current and consequently the deflection of the galvanometer is then proportional to the speed of the anemometer. This method requires a constant voltage which is maintained by using a potentiometer in connection with the generator or batteries available as standard equipment on aircraft.

GROUND-SPEED INDICATOR.

One of the problems which is now demanding a great deal of attention is that of developing an accurate ground-speed indicator. The various methods which have been proposed are discussed in Report No. 127. While it is theoretically possible to make instruments of the absolute integrating type, the practical difficulties encountered are almost insuperable. If the ground is visible, however, optical devices can be used. A ground-speed indicator of this type in which the apparent motion of the ground is neutralized by the rotation of a hexagonal prism about an axis perpendicular to the direction of motion of the aircraft relative to the ground has been constructed recently. Knowing the altitude and the rate of rotation of the prism required to cause the objects on the ground to appear to remain still, the ground speed can be determined. In the instrument in question the ground speed is read directly from the dial of a tachometer connected to the rotating prism. The readings of the instrument may also be automatically corrected for the effect of altitude.

DIRECTION INSTRUMENTS.

COMPASSES.

The compass still remains one of the least satisfactory aeronautic instruments. Aside from the inherent difficulties due to the smallness of the directional force of the earth's magnetic field, its location on the instrument board of the airplane in the proximity of large moving masses of magnetic material seriously limits its dependability. Airplane compasses of comparatively long period and high damping have been used. These are less subject to disturbance due to momentary accelerations and the northerly turning error than short-period quick-acting compasses, but in general the latter type are preferred, especially when used in connection with turn indicators of which several satisfactory types have recently been developed. One of the difficulties encountered is that due to the unsteadiness and swirling of the liquid set up by the maneuvering of the airplane. Efforts have been made to overcome this difficulty by mounting the compass element proper in the center of a spherical bowl. Another method is to make the needle system very light by eliminating the ordinary card and mounting the needles on a light spider made of small straight wires projecting radially in the damping liquid from the point of support. The effect of the light radial arms is to damp the motion so heavily that the system is aperiodic and when displaced returns to its equilibrium position without oscillation. An instrument of this type known as an aperiodic compass has recently been developed in England and is described in Report No. 128.

An effort to overcome the difficulties caused by the disturbing effect of masses of iron in the vicinity of the instrument board has been made in a distant reading compass recently developed in Germany. It is described in the paper just mentioned. In its present form this compass, with its auxiliary course setting and rotating devices, is very heavy compared with compasses of the ordinary type. Flight tests have shown it to be insensitive. It is also complicated and has not been used sufficiently in practice to determine its reliability under continued use.

In another type of distant reading compass the current developed in a coil rotating in the earth's magnetic field is used to indicate direction. It is known as the earth inductor compass. An instrument of this type for use on aircraft has recently been developed for the United States Army Air Service. It differs from previous instruments in that the indications are by a null method instead of the usual galvanometer deflection method. A course-setting device of novel type is also provided in which by turning a movable dial at the instrument board the galvanometer connections are so arranged that the indicating pointer reads zero when the aircraft assumes the desired direction. A method is also provided for eliminating the effect due to rolling and pitching on the effective component of the earth's magnetic field. A more detailed description of this instrument is found in Report No. 128.

TURN INDICATORS.

As previously stated, the development of satisfactory turn indicators has made the weaknesses of the compass a matter of less serious concern to the aviator since the turn indicator enables him to maintain the direction of his course when the compass is subjected to disturbing accelerations which make its readings temporarily unreliable. The most successful types of instruments of this class include as the essential directing element a gyroscopic rotor. The two distinct types which have reached a state of practical development include (1) instruments in which the gyroscopic element is driven by the impact of a wind stream (Sperry type; Pioneer type) and (2) instruments in which the rotor is an induction motor (Drexler type). In the wind-driven type the differential pressure is usually maintained by a Venturi tube. This is a simple and effective way of applying the necessary power, but not an economical one from the point of view of power consumption, since experiments show that a large amount of energy is absorbed in driving the Venturi through the air. Electrical types usually depend upon a small auxiliary generator driven by a wind propeller for their power. It has also been proposed to operate the wind-driven instrument by an electrically operated pump connected to

the generator of the airplane. Still another suggestion is to develop the necessary differential air pressure by the use of a hollow propeller with a tube extending longitudinally inside each blade (Pioneer type). These tubes are connected through a hollow axle to the turn indicator. The centrifugal force set up by the rotation of the propeller forces the air out radially through the propeller tubes and a differential pressure is thereby established. The difficulty here is to secure a sufficient pressure difference without rotating the propeller too fast for safety. The wind-driven type of turn indicator has the advantage that it can be made of nonmagnetic metal, so that it can be located on the instrument board without disturbing the compass. It is also simpler and lighter in construction than the electrically driven type. On the other hand, the electric type can be made to give more precise indications and is not subject to disturbing precessions which are present in the wind-driven type.

POWER PLANT INSTRUMENTS.

TACHOMETERS.

Of the various types of airplane tachometers which have been proposed the only two which have thus far been in extensive use are the centrifugal and chronometric. Laboratory experiments have shown that the chronometric type in general gives more accurate indications than the centrifugal instruments, but it is of complicated construction, inclined to get out of order easily, and is difficult to repair. On the other hand, the centrifugal type has the distinct advantage of greater simplicity of construction and greater durability. It, therefore, has appeared advisable to undertake experiments on centrifugal tachometers in the hope of overcoming the defects in existing instruments of this type. Investigations have led to the development of a centrifugal tachometer of the double inclined weight type, which is balanced to eliminate the disturbing effect of accelerations of the airplane. The differences between the readings with increasing and decreasing scale deflection have also been reduced considerably below that of ordinary instruments of this type.

DISTANT CONTROL METHODS.

The increase in the size of airplanes and the installation of tachometers in large airships necessitates the development of methods of distant control for tachometers other than the use of flexible shafts. This problem still awaits a satisfactory practical solution. Various methods have been proposed. One is to attach a small three-phase generator to the engine shaft and connect it to a synchronous motor, with an ordinary tachometer attached, located on the instrument board. A modification of this method is to attach to the engine shaft a synchronizing commutator so constructed as to convert direct current into three-phase alternating current, the frequency of which is proportional to the speed of revolution of the engine. By supplying this current to a three-phase synchronous motor with attached tachometer the indications of the instrument can be read at any desired distance from the engine itself. The French Favre-Bulle tachometer is of this type. Still another possibility is the method suggested previously in this paper (see aircraft speed instruments), in which the frequency of charging and discharging of a condenser through a galvanometer is determined by the rate of revolution of the engine. The deflection of the galvanometer would then be proportional to the rate of revolution of the engine. It has also been proposed to operate a tachometer on the instrument board by air pulses transmitted through tubing connecting the pulsator on the engine shaft to the tachometer, the rapidity of the pulsations being proportional to the rate of revolution of the engine (Hamilton type).

ELECTRIC THERMOMETER.

A similar problem of distant transmission for large aircraft has necessitated the development of distant reading thermometers other than those of the liquid-filled or vapor-pressure type. The most successful of the distant reading instruments are electrical resistance thermometers. A Hartman and Braun instrument of this type has been extensively used in Germany. The thermometric element consists of silk covered platinum wire wound on micanite. This unit is inclosed in a metal tube which is soldered into the engine radiator. The indicator is a small moving coil galvanometer.

OXYGEN INSTRUMENTS.

DEVELOPMENT OF LIQUID TYPE.

The necessity of supplying aviators with artificial oxygen at the high altitudes now attained in flight has resulted in the development of apparatus for the use of oxygen, both as compressed gas and in liquid form. A detailed description of these instruments has been given in Report No. 130. During the recent war compressed oxygen was used almost entirely by the Allies and liquid oxygen by the Germans. The weight and bulk of the supply tanks required by instruments of the compressed oxygen type has led recently to the further development of liquid oxygen apparatus which becomes particularly advantageous for use during long flights or where several persons must be supplied. The disadvantages of instruments of the liquid type is that the oxygen must be used at comparatively short distances from the source of supply and within a comparatively short time after liquefaction. In most of these instruments the evaporation losses are reduced as much as possible by the use of vacuum jacketed containers, and hand operated or automatically controlled cut-off valves are used to regulate the supply to the aviators.

NAVIGATING INSTRUMENTS.

SEXTANTS.

The practicability of long distance flying and night flying have recently emphasized the necessity of developing types of instruments for navigating aircraft analogous to those used in determining the position of ships at sea. One of the most important instruments of this class is the sextant. A number of aircraft sextants have recently been developed. In general they resemble an ordinary marine sextant except that an artificial horizon is used. The artificial horizon is ordinarily of the bubble level type. It has also been proposed to use for this purpose a small pendulum or gyroscopic rotor. A serious objection to sextants of the ordinary type from the point of view of aircraft navigation is their weight and size. Efforts have recently been made to design a sextant especially suited to the conditions experienced in air navigation. This has resulted in the development of an instrument in which a small sector with micrometer head or a rotating drum has been substituted for the large sextant scale of the ordinary type of instrument. Means for adjusting the size of the bubble, which varies greatly because of the wide variations in temperature experienced in flight, are also provided. This is effected by making one side of the bubble chamber a flexible diaphragm controlled by a thumbscrew. By turning the thumbscrew the effective volume of the bubble chamber can be altered at will. Artificial illumination for the bubble and scales is also provided for reading at night.

ASTRONOMICAL POSITION FINDER.

The Engineering Division of the Air Service has recently suggested the development of an instrument for determining the position of aircraft from the position of any two known celestial objects, such as two stars, and the Greenwich siderial time. This in effect is an optical instrument for determining the altitude of the pole and hour angle of one of the two celestial objects chosen. Such an instrument under construction consists essentially of a telescope and two prisms or mirrors to reflect the light of the two objects chosen into the telescope. The mirrors are each independently rotatable about the axis of the telescope and about an axis at right angles to the telescope. A bubble horizon is used.

HORIZONTAL ANGLE INDICATORS.

The use of aircraft in connection with range finding has emphasized the importance of developing horizontal angle indicators suitable for use on aircraft. A number of instruments of this class have been designed and are now under construction. They consist essentially of graduated quadrants or sextants, and mirrors or prism systems so oriented as to make it possible to bring the images of the two objects under observation first into the same horizontal plane and then into coincidence.

SPECIAL PROBLEMS.

DIAPHRAGM INVESTIGATION.

Reference has been made in previous papers of this report to the difficulty encountered in all aeronautic instruments depending for their action upon metallic diaphragms which results from the imperfect elasticity of the metal of which the diaphragms are made. A study of the conditions determining the magnitude of this effect has been for some time in progress It is found that the effect varies markedly not only with the composition of the metal but also with the mechanical processes and temperature treatment to which it has been subjected. For example, spun diaphragms in general exhibit better elastic properties than pressed or stamped samples, probably because spinning hardens the metal uniformly while the stamping tends to crush and tear the surface layers. It has also been found that subjecting the diaphragms several thousand times to deflections such as they would experience in use decreases appreciably the elastic lag or hysteresis effect. A similar result is also obtained by heating the diaphragms for a time, approximately a half hour, at temperatures depending on the composition of the metal but varying from 200° to 400° C.

In connection with this investigation the behavior of slack diaphragms of rubber and doped fabric is also being undertaken. Since it is possible with instruments of this type to concentrate the elastic resistance almost entirely in a spring it should be possible to make the hysteresis effect small in accordance with the principle applied in the precision altimeter already described. However, this is not found to be the case with instruments of the slack-diaphragm type now in use which show a hysteresis effect as large as that of instruments with metallic diaphragms. The porosity of diaphragms of this type is an important factor and is being determined experimentally. The stiffening of such diaphragms at low temperatures has also to be considered and is under investigation.

BOURDON TUBE INVESTIGATION.

It has been proposed that Bourdon tubes be substituted for metallic diaphragms in aeronautic instruments. Experiments on the elastic properties of such tubes have also been undertaken. Since these tubes are made of metal similar to that used in diaphragms and therefore imperfectly elastic, it is reasonable to assume that they would display hysteresis effects. Experiments indicate such to be the case and from the results thus far at hand it is not evident that the effect is appreciably less than in the case of metallic diaphragms.

BIMETALLIC BARS.

Still another problem presented in the development of aeronautic instruments relates to the behavior of bimetallic bars which are used in temperature compensation and also in certain cases as temperature-measuring elements. Here again the imperfect elasticity of the metals of which the strips are made has to be taken into consideration. This is not ordinarily a serious factor where the bimetallic bars are used for temperature compensation since a comparatively small corrective factor is involved, but it becomes important in temperature-measuring instruments based upon the bimetallic principle and warrants careful investigation.

ALTITUDE EFFECT ON AIR-SPEED INDICATORS.

The behavior of air-speed measuring instruments, particularly of the Pitot or Venturi type, under the conditions of reduced pressure and low temperature experienced in high altitude flight is difficult if not impossible to investigate in the airplane itself. The possibility of simulating in the laboratory the conditions of actual use have therefore recently been considered. For some time past an investigation has been in progress in a reduced pressure wind tunnel which consists essentially of a chamber containing a small wind channel so arranged that the air can be partially exhausted from the chamber. The results thus far obtained show that for Venturi nozzles there are appreciable deviations from the ρV^2 law for high altitudes or low speeds.[1]

[1] See The Altitude Effect on Air Speed Indicators; Report No. 110 of N. A. C. A. 1920, by M. D. Hersey, F. L. Hunt, and H. N. Eaton.

The behavior of instruments of the anemometer and wind propeller type should be investigated at reduced pressures. While it is ordinarily assumed that the indications of such instruments are independent of the density this is known not to be strictly true.

INK SUBSTITUTES FOR RECORDING INSTRUMENTS.

In connection with the development of recording aeronautic instruments it has been found that ink records which are ordinarily made by such instruments are not entirely satisfactory for use in aircraft because the ink coagulates at the excessively low temperatures encountered at high altitudes. Other possibilities have therefore been surveyed.

Records can be made on smoked surfaces with very light contact by the recording pen but the records are easily effaced and can be made permanent only by coating them with paraffin, spraying with shellac, or by using some similar adhesive agent. A mixture of alcohol, white shellac, and glycerin has been found satisfactory in our experiments. Paraffin-coated carbon copy paper may also be used with a fine jewel point recorder. These records, except at extreme temperatures, are more permanent than those obtained on smoked paper but require greater pressure by the recording pen with consequent decrease in the sensitiveness of response of the instrument to slight changes of reading. Paper coated with zinc oxide, such as is used for engine indicator diagrams, was also considered. This is used with a rather blunt brass recording point. The pressure required is excessive in view of the sensitiveness of response required in the recording mechanism.

Perforated records can be made by using an auxiliary clockwork mechanism to bring the recording point forcibly in contact with the record and thus perforate it several times per minute. The pointer ordinarily would not touch the record paper. This method is positive and sure but discontinuous, and it involves the added complication of an auxiliary clock or similar driving mechanism.

Records may also be obtained by perforation with an electric spark by connecting the metal drum on which the paper is fastened to one terminal of an induction coil, connecting the other terminal to the recording point and attaching an auxiliary contact breaker to the driving clock mechanism to make and break the induction coil circuit several times a minute. A device of this kind was made and attached to a barograph. It has the advantage that the sensitiveness of response of the pointer is not affected by contact with the record. On the other hand it requires the accessory of an induction coil, and the spark sometimes jumps sidewise a short though appreciable distance from the point.

Record paper previously impregnated with starch and potassium iodide can be used and the potassium iodide decomposed by passing an electric current at low voltage through the record paper from the pointer, which in this case is in contact with the record, to the drum. A purple record is obtained. The color, however, is inclined to spread, which makes the record abnormally broad.

Where dark-room facilities are available, records on photographic films can be obtained by allowing a beam of light to fall upon the record after it has been deflected by a suitable optical system attached to the indicating mechanism of the instrument. This may be done either by reflecting light from a light source, which may be a small incandescent lamp, from a mirror onto the film or by use of a suitable lens system. The disadvantage of this method is the necessity of developing the record and its extreme sensitiveness to extraneous light.

Experiments have also been carried out with a view to the possibility of using a small quantity of radioactive material on the end of the pointer of recording instruments. A metallic screen, which was perforated by a pin hole, was mounted on the pointer between the radioactive material and the recording film, to avoid diffuseness in the record. Satisfactory records on ordinary photographic films were obtained in this way. This method has the advantage of simplicity and reliability.

The possibility of photographing ordinary dial indicating instrument at frequents intervals with a magazine or motion-picture camera also presents itself. This offers the advantage of obtaining permanent simultaneous records of a large number of instruments which would be useful both in performance tests of airplanes and on investigating instruments themselves.

DUMMY OBSERVER

The last-mentioned method has been applied in the development of an automatic photographic apparatus known as a dummy observer, to be installed in an airplane for airplane performance tests. Records of the speed of the engine, air speed, altitude, temperature, pressure gage readings, and time were desired. Most of the instruments ordinarily employed for the measurement of these factors involve such small forces as to make the use of relays or amplifiers to operate recording mechanisms necessary if graphical records of the ordinary type are to be obtained. The advantage of the photographic method is that a simultaneous record of all the instruments can be made at intervals, which may be varied at will according to the conditions of flight down to approximately 20 exposures per second. The exact time of successive exposures can be recorded on the negatives by including among the instruments photographed a clock with hour, minute, and second hands.

The instantaneous behavior of the airplane with respect to each of the factors under investigation can thus be had as a permanent record at intervals of a fraction of a second throughout the flight and the history of the flight afterwards studied by magnifying the negatives and tabulating and plotting at leisure the readings of the various instruments. The advantages of this method are the absolute coordination in time of all the records; the possibility of controlling the time interval between the records and of detecting sudden changes and studying such changes in greater detail by making rapid records while these changes are taking place.

The apparatus as it was finally worked out consisted of a Mark I gun camera mounted at one end of a box approximately 14 inches square and 18 inches long. At the other end of the box were mounted the instruments to be photographed. The instruments were illuminated by a series of electric lamps inside the box, and the camera mechanism was modified to operate automatically at rates varying from 2 to 10 exposures per minute by the use of a centrifugal governor. Unperforated motion-picture film was used. To prevent excessive heating the lamps were automatically illuminated at the moment when the exposure was made. The instruments used were of the ordinary dial type, but it was found necessary to use white dials with black graduations for greater contrast. To avoid troublesome reflections the cover glasses were removed on all the instruments, except the air-speed indicator, where the glass dial is necessary to maintain the air pressure in the case. Care was taken to arrange the instruments to occupy as small an area as possible. An improved model of the dummy observer is being developed by the Engineering Division of the Air Service.

CONCLUSION.

One of the most important of the outstanding problems in the field of aeronautic instruments at present relates to the development of navigating instruments for use in long-distance flights and for flying at night and landing in fog. These include not only development of optical instruments used in connection with astronomical position finding but also improvements in altimeter, compass, and turn indicator construction. There is also reason to believe that commercial flying may soon be undertaken at very high altitudes to take advantage of favorable winds of high velocity. In this connection the importance of oxygen instruments will unquestionably be emphasized. The extensive use of large aircraft, particularly of lighter-than-air craft, requires the development of satisfactory distant reading instruments of various types. The proposed methods applicable to distant reading compasses, air-speed indicators, tachometers, and thermometers, previously discussed in this paper, are still in a preliminary state and have not been used extensively in practice.

The development of greater precision in all of the aeronautic instruments should be attained. While errors of from 1 to 5 per cent which are now common in instruments as actually installed in the airplane are not fatal under ordinary circumstances, such errors are much greater than could be desired. In certain instruments, particularly those used in navigation and in landing, errors of this magnitude constitute a serious if not fatal defect. Moreover, under certain special conditions, for instance, in aircraft performance tests, it is essential that the instruments have the greatest accuracy obtainable.

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